Recent developments in $\mathrm{AdS}_6/\mathrm{CFT}_5$

Christoph Uhlemann
UCLA

Kavli IPMU
April 2019

Recent developments in AdS_6/CFT_5

Outline

- AdS_6/CFT_5 dualities in Type IIB
- Matching "stringy" operators
- Sphere partition functions
- Counting black hole microstates

AdS_6/CFT_5 dualities in Type IIB

Why 5d SCFTs?

Intrinsic interest:

- mathematical consistency allows SCFTs for $d \leq 6$ [Nahm '78], but conventional Lagrangian constructions fail for $d > 4$
- string theory evidence that SCFTs in $d = 5, 6$ do exist

More pragmatically: unified perspective on lower-dim. QFT

- new QFTs in $d \leq 4$ from compactification of higher-dimensional ones (4d class $\mathcal{S}$, 3d class $\mathcal{F}$)
- new dualities and natural explanations for known dualities (S-duality, AGT, Argyres-Seiberg duality)

5d SCFTs from asymptotically safe gauge theories

Gauge theories in $d > 4$ perturbatively non-renormalizable ($\sim$ 4d GR). Naively make sense only as effective low-energy theories.

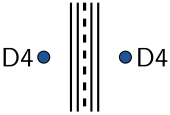
But they may make sense non-perturbatively, and flow to strongly-coupled UV fixed point $\sim$ asymptotic safety.

5d SCFTs from asymptotically safe gauge theories

Gauge theories in $d > 4$ perturbatively non-renormalizable ($\sim$ 4d GR). Naively make sense only as effective low-energy theories.

But they may make sense non-perturbatively, and flow to strongly-coupled UV fixed point $\sim$ asymptotic safety.

– [Seiberg '96]: SU(2) $\mathcal{N}=1$ gauge theory with $N_f \leq 7$ may flow to strongly-coupled UV fixed point $\rightarrow$ asymptotically safe



O8 + N_f D8

(i) convex prepotential on Coulomb Branch

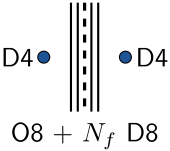
(ii) construction in Type I' string theory

5d SCFTs from asymptotically safe gauge theories

Gauge theories in $d > 4$ perturbatively non-renormalizable ($\sim$ 4d GR). Naively make sense only as effective low-energy theories.

But they may make sense non-perturbatively, and flow to strongly-coupled UV fixed point $\sim$ asymptotic safety.

- [Seiberg '96]: SU(2) $\mathcal{N}=1$ gauge theory with $N_f \leq 7$ may flow to strongly-coupled UV fixed point $\rightarrow$ asymptotically safe



(i) convex prepotential on Coulomb Branch

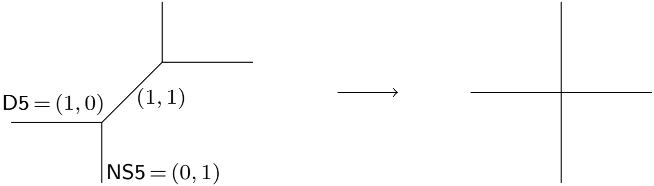
(ii) construction in Type I' string theory

Gauge theories with (i) classified in [Intriligator,Morrison,Seiberg].
Even more theories realized by (p, q) 5-brane webs in Type IIB...

5-brane webs in Type IIB

[Aharony,Hanany,Kol '97]

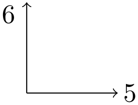
5-brane web: planar arrangement of (p,q) 5-branes at angles fixed by (p,q), junctions w/ conserved charges



free massive hypermultiplet

free massless hypermultiplet

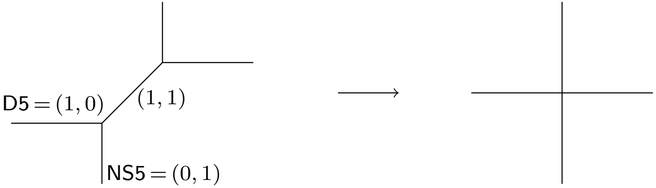
	0	1	2	3	4	5	6	7	8	9
D5	×	×	×	×	×	×				
NS5	×	×	×	×	×		×			



5-brane webs in Type IIB

[Aharony,Hanany,Kol '97]

5-brane web: planar arrangement of (p,q) 5-branes at angles fixed by (p,q), junctions w/ conserved charges



free massive hypermultiplet

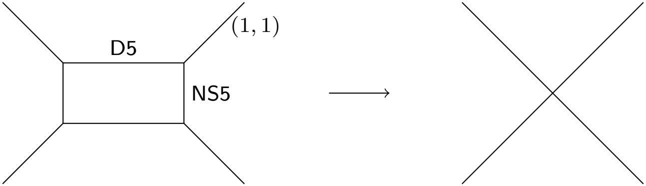
free massless hypermultiplet

Length scales in brane web $\leftrightarrow$ mass parameters in field theory.
UV fixed point: all lengths $\to 0$, intersection at a point.

5-brane webs in Type IIB

[Aharony,Hanany,Kol '97]

5-brane web: planar arrangement of (p, q) 5-branes at angles fixed by (p, q), junctions w/ conserved charges



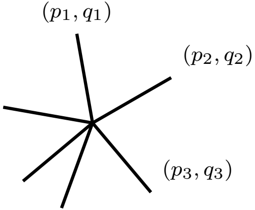
SU(2) Coulomb branch
finite gauge coupling

UV fixed point SCFT

Length scales in brane web $\leftrightarrow$ mass parameters in field theory.
UV fixed point: all lengths $\to 0$, intersection at a point.

5d SCFTs from 5-brane junctions

General picture: any planar 5-brane junction realizes a 5d SCFT on the intersection point



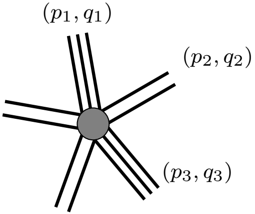
$$p_i, q_i \in \mathbb{Z}$$

$$\sum p_i = \sum q_i = 0$$

Characterized entirely by external 5-brane charges. No standard Lagrangian. May or may not have gauge theory deformations.

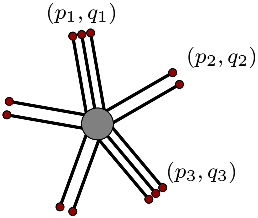
5-brane junctions with 7-branes [DeWolfe,Hanany,Iqbal,Katz '99]

More general theories realized by 5-junctions with 7-branes:



5-brane junctions with 7-branes [DeWolfe,Hanany,Iqbal,Katz '99]

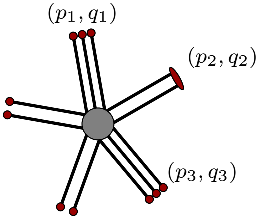
More general theories realized by 5-junctions with 7-branes:



– 5-branes can terminate on 7-branes without changing the theory

5-brane junctions with 7-branes [DeWolfe,Hanany,Iqbal,Katz '99]

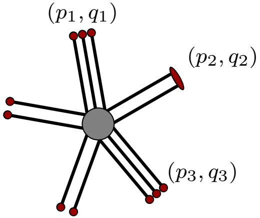
More general theories realized by 5-junctions with 7-branes:



- 5-branes can terminate on 7-branes without changing the theory
- multiple 5-branes may end on same 7-brane $\rightarrow$ s-rule constraints [Benini,Benvenuti,Tachikawa]
- related to unconstrained junction by Higgs-branch flow

5-brane junctions with 7-branes [DeWolfe,Hanany,Iqbal,Katz '99]

More general theories realized by 5-junctions with 7-branes:



- 5-branes can terminate on 7-branes without changing the theory
- multiple 5-branes may end on same 7-brane $\rightarrow$ s-rule constraints [Benini,Benvenuti,Tachikawa]
- related to unconstrained junction by Higgs-branch flow

Additional data for 5-brane junctions w/ 7-branes: partition of like-charged 5-branes into subgroups ending on same 7-brane

Holographic duals for 5d SCFTs

Brane webs realize Coulomb and Higgs branches, RG flows, ... geometrically. AdS/CFT to access superconformal fixed points?

Needs AdS_6 solutions in Type IIB supergravity:

- Unique superconformal algebra $F(4)$, 8_Q supercharges. No maximally supersymmetric solutions (unlike $d \neq 5$).
- Fully localized intersections, not a standard near-horizon limit.

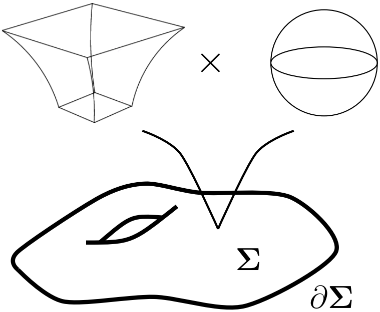
BPS equations studied by [Apruzzi, Fazzi, Passias, Rosa, Tomasiello '14; Kim, Kim, Suh '15; Kim, Kim '16].

Symmetries and ansatz [D'Hoker,Gutperle,Karch,CFU arXiv:1606.01254]

$$\text{AdS}_6 + 16 \text{ susies} \;\rightarrow\; \text{F}(4) \;\supset\; \text{bosonic } \underbrace{\text{SO}(2,5)}_{\text{AdS}_6} \oplus \underbrace{\text{SO}(3)}_{\text{S}^2}$$

Symmetries and ansatz [D'Hoker,Gutperle,Karch,CFU arXiv:1606.01254]

AdS_6 + 16 susies $\rightarrow$ F(4) $\supset$ bosonic $\underbrace{\mathrm{SO}(2,5)}_{\mathrm{AdS}_6} \oplus \underbrace{\mathrm{SO}(3)}_{\mathrm{S}^2}$



General ansatz: AdS_6 and S^2 warped over Riemann surface Σ

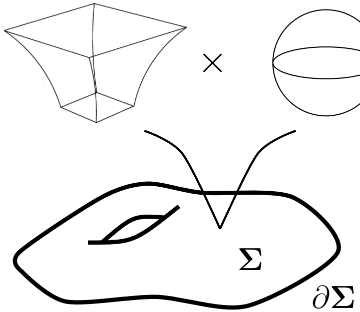
$$\mathcal{M} = (\mathrm{AdS}_6 \times \mathrm{S}^2) \times_{\mathrm{w}} \Sigma$$

Symmetries and ansatz

[D'Hoker,Gutperle,Karch,CFU arXiv:1606.01254]

AdS_6 + 16 susies $\rightarrow$ F(4) $\supset$ bosonic SO(2,5) $\oplus$ SO(3)

$\nearrow$ AdS_6 $\qquad$ $\nwarrow$ S^2



General ansatz: AdS_6 and S^2 warped over Riemann surface Σ

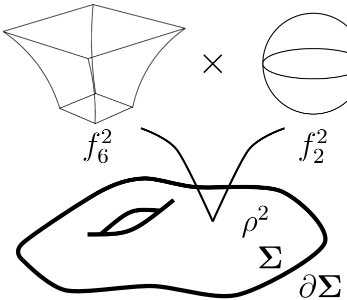
$$\mathcal{M} = (\mathrm{AdS}_6 \times \mathrm{S}^2) \times_{\mathrm{w}} \Sigma$$

$$ds^2 = f_6(w,\bar{w})^2 ds^2_{\mathrm{AdS}_6} + f_2(w,\bar{w})^2 ds^2_{\mathrm{S}^2} + 4\rho(w,\bar{w})^2 |dw|^2$$

$$C_{(4)} = 0 \qquad B_2 + iC^{\mathrm{RR}}_{(2)} = \mathcal{C}(w,\bar{w}) \mathrm{vol}_{\mathrm{S}^2} \qquad \tau = \tau(w,\bar{w})$$

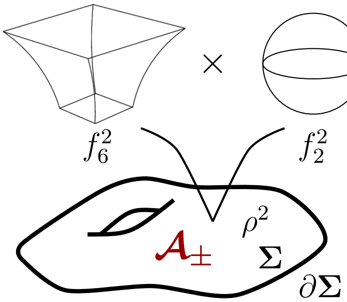
General local solution [D'Hoker,Gutperle,Karch,CFU arXiv:1606.01254]

BPS eq. $\rightarrow$ coupled non-linear PDEs on Σ. General local solution parametrized by two locally holomorphic functions on Σ:



General local solution [D'Hoker,Gutperle,Karch,CFU arXiv:1606.01254]

BPS eq. $\rightarrow$ coupled non-linear PDEs on Σ. General local solution parametrized by two locally holomorphic functions on Σ:



Arbitrary locally holomorphic

$$\mathcal{A}_\pm : \Sigma \rightarrow \mathbb{C}$$

yield metric functions, axio-dilaton, two-form field and Killing spinors

SU$(1,1)$ transforming $\partial\mathcal{A}_\pm$ induces SL(2,$\mathbb{R}$) on supergravity fields

General local solution [D'Hoker,Gutperle,Karch,CFU arXiv:1606.01254]

General local solution to BPS eq. parametrized by two locally holomorphic functions on Σ:

$$f_6^2 = \sqrt{6\mathcal{G}T} \qquad f_2^2 = \frac{1}{9}\sqrt{\frac{6\mathcal{G}}{T^3}} \qquad \rho^2 = \kappa^2\sqrt{\frac{T}{6\mathcal{G}}}$$

$$B = \frac{1+i\tau}{1-i\tau} = \frac{\partial_w \mathcal{A}_+\,\partial_{\bar w}\mathcal{G} - R\,\partial_{\bar w}\bar{\mathcal{A}}_-\partial_w\mathcal{G}}{R\,\partial_{\bar w}\bar{\mathcal{A}}_+\partial_w\mathcal{G} - \partial_w\mathcal{A}_-\partial_{\bar w}\mathcal{G}}$$

$$\mathcal{C} = \frac{2i}{3}\left(\frac{\partial_{\bar w}\mathcal{G}\partial_w\mathcal{A}_+ + \partial_w\mathcal{G}\partial_{\bar w}\bar{\mathcal{A}}_-}{3\kappa^2 T^2} - \bar{\mathcal{A}}_- - \mathcal{A}_+\right)$$

with composite quantities

$$\kappa^2 = -|\partial_w\mathcal{A}_+|^2 + |\partial_w\mathcal{A}_-|^2 \qquad \partial_w\mathcal{B} = \mathcal{A}_+\partial_w\mathcal{A}_- - \mathcal{A}_-\partial_w\mathcal{A}_+$$

$$\mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}} \qquad T^2 = \left[\frac{1+R}{1-R}\right]^2 = 1 + \frac{2\kappa^2\,\mathcal{G}}{3|\partial_w\mathcal{G}|^2}$$

General local solution [D'Hoker,Gutperle,Karch,CFU arXiv:1606.01254]

General local solution to BPS eq. parametrized by two locally holomorphic functions on Σ:

$$f_6^2 = \sqrt{6\mathcal{G}T} \qquad f_2^2 = \frac{1}{9}\sqrt{\frac{6\mathcal{G}}{T^3}} \qquad \rho^2 = \kappa^2\sqrt{\frac{T}{6\mathcal{G}}}$$

$$B = \frac{1+i\tau}{1-i\tau} = \frac{\partial_w\mathcal{A}_+\,\partial_{\bar w}\mathcal{G} - R\,\partial_{\bar w}\bar{\mathcal{A}}_-\partial_w\mathcal{G}}{R\,\partial_{\bar w}\bar{\mathcal{A}}_+\partial_w\mathcal{G} - \partial_w\mathcal{A}_-\partial_{\bar w}\mathcal{G}}$$

$$\mathcal{C} = \frac{2i}{3}\left(\frac{\partial_{\bar w}\mathcal{G}\partial_w\mathcal{A}_+ + \partial_w\mathcal{G}\partial_{\bar w}\bar{\mathcal{A}}_-}{3\kappa^2T^2} - \bar{\mathcal{A}}_- - \mathcal{A}_+\right)$$

with composite quantities

$$\kappa^2 = -|\partial_w\mathcal{A}_+|^2 + |\partial_w\mathcal{A}_-|^2 \qquad \partial_w\mathcal{B} = \mathcal{A}_+\partial_w\mathcal{A}_- - \mathcal{A}_-\partial_w\mathcal{A}_+$$

$$\mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}} \qquad T^2 = \left[\frac{1+R}{1-R}\right]^2 = 1 + \frac{2\kappa^2\,\mathcal{G}}{3|\partial_w\mathcal{G}|^2}$$

General local solution [D'Hoker,Gutperle,Karch,CFU arXiv:1606.01254]

General local form of Type IIB solution with geometry $AdS_6 \times S^2$ warped over Riemann surface and 16 supersymmetries.

Generic choices of $\mathcal{A}_\pm$ do not lead to physically regular solutions.

Near-horizon limit of (p, q) 5-brane junctions? Implement global regularity constraints. . .

Regularity conditions

[D'Hoker,Gutperle,CFU arXiv:1703.08186]

Real geometry with consistent spacetime signature, $\mathrm{Im}(\tau) > 0$:

$$\kappa^2\big|_{\mathrm{int}(\Sigma)} > 0 \qquad\qquad \mathcal{G}\big|_{\mathrm{int}(\Sigma)} > 0$$

$\rightarrow \Sigma$ must have a boundary ($\partial_w \partial_{\bar{w}} \mathcal{G} = -\kappa^2$ by construction)

Regularity conditions

[D'Hoker,Gutperle,CFU arXiv:1703.08186]

Real geometry with consistent spacetime signature, $\mathrm{Im}(\tau) > 0$:

$$\kappa^2\big|_{\mathsf{int}(\Sigma)} > 0 \qquad\qquad \mathcal{G}\big|_{\mathsf{int}(\Sigma)} > 0$$

$\rightarrow \Sigma$ must have a boundary ($\partial_w \partial_{\bar{w}} \mathcal{G} = -\kappa^2$ by construction)

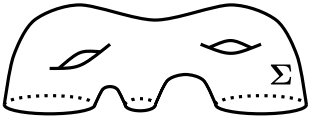
For 10d geometry w/o boundary, collapse S^2 on $\partial\Sigma$ (AdS_6 finite):

$$\kappa^2\big|_{\partial\Sigma} = 0 \qquad\qquad \mathcal{G}\big|_{\partial\Sigma} = 0$$

Global regularity conditions, to be realized by holomorphic $\mathcal{A}_\pm$.

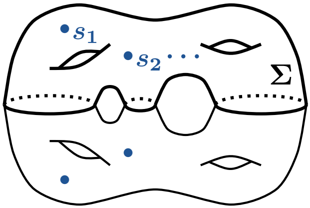
Solving the regularity conditions [D'Hoker,Gutperle,CFU arXiv:1703.08186]

Strategy to solve regularity conditions for Σ of arbitrary topology:



Solving the regularity conditions [D'Hoker,Gutperle,CFU arXiv:1703.08186]

Strategy to solve regularity conditions for Σ of arbitrary topology:

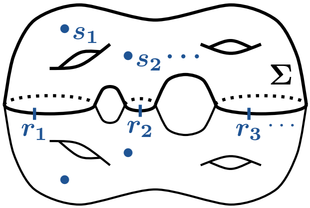


1) regular κ^2: electrostatics potential $\Phi \equiv -\ln|\partial_w \mathcal{A}_+/\partial_w \mathcal{A}_-|$

positive **charges s_n** in Σ + mirror charges $\sim$ zeros of $\partial\mathcal{A}_\pm$

Solving the regularity conditions [D'Hoker,Gutperle,CFU arXiv:1703.08186]

Strategy to solve regularity conditions for Σ of arbitrary topology:



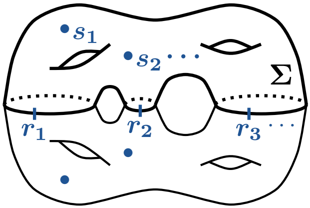
1) regular κ^2: electrostatics potential $\Phi \equiv -\ln|\partial_w \mathcal{A}_+ / \partial_w \mathcal{A}_-|$

positive **charges** s_n in Σ + mirror charges $\sim$ zeros of $\partial\mathcal{A}_\pm$

regular differentials need **poles** r_ℓ on $\partial\Sigma$

Solving the regularity conditions [D'Hoker,Gutperle,CFU arXiv:1703.08186]

Strategy to solve regularity conditions for Σ of arbitrary topology:

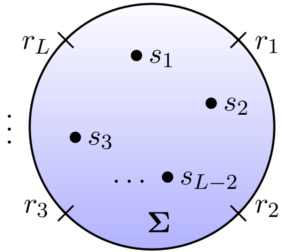


1) regular κ^2: electrostatics potential $\Phi \equiv -\ln|\partial_w \mathcal{A}_+ / \partial_w \mathcal{A}_-|$
positive **charges** s_n in Σ + mirror charges $\sim$ zeros of $\partial\mathcal{A}_\pm$
regular differentials need **poles** r_ℓ on $\partial\Sigma$

2) $\mathcal{G}|_{\partial\Sigma} = 0$ reduces to one local constraint per pole + one non-local for each extra boundary; $\mathcal{G} > 0$ in int(Σ) automatic

Regular solutions on the disc [D'Hoker,Gutperle,CFU arXiv:1703.08186]

Σ =disc/upper half plane: $L \geq 3$ poles, $L-2$ "charges"



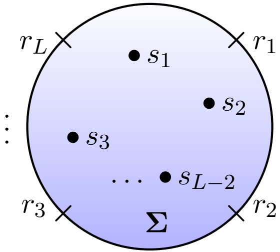
$$\mathcal{A}_\pm = \mathcal{A}^0_\pm + \sum_{\ell=1}^{L} Z^\ell_\pm \ln(w - r_\ell)$$

$$Z^\ell_+ = \sigma \prod_{n=1}^{L-2} (r_\ell - s_n) \prod_{k\neq\ell}^{L} \frac{1}{r_\ell - r_k}$$

$$\mathcal{A}_-(w) = -\overline{\mathcal{A}_+(\bar{w})} \qquad \sum_{\ell=1}^{L} Z^\ell_+ = 0$$

Regular solutions on the disc [D'Hoker,Gutperle,CFU arXiv:1703.08186]

$\Sigma =$disc/upper half plane: $L \geq 3$ poles, $L-2$ "charges"



$$\mathcal{A}_\pm = \mathcal{A}^0_\pm + \sum_{\ell=1}^{L} Z^\ell_\pm \ln(w - r_\ell)$$

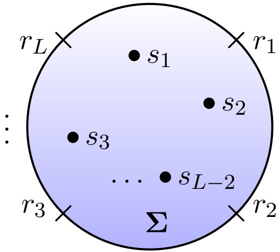
$$Z^\ell_+ = \sigma \prod_{n=1}^{L-2} (r_\ell - s_n) \prod_{k \neq \ell}^{L} \frac{1}{r_\ell - r_k}$$

$$\mathcal{A}_-(w) = -\overline{\mathcal{A}_+(\bar{w})} \qquad \sum_{\ell=1}^{L} Z^\ell_+ = 0$$

Solving remaining regularity conditions leaves $2L-2$ free real parameter $\sim$ choice of Z^ℓ_+ with $\sum Z^\ell_+ = 0$.

Regular solutions on the disc [D'Hoker,Gutperle,CFU arXiv:1703.08186]

Σ =disc/upper half plane: $L \geq 3$ poles, $L-2$ "charges"



$$\mathcal{A}_\pm = \mathcal{A}^0_\pm + \sum_{\ell=1}^{L} Z^\ell_\pm \ln(w - r_\ell)$$

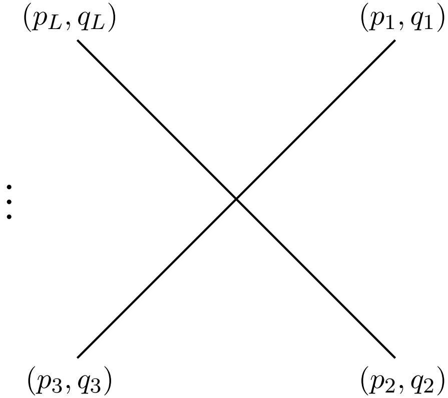
$$Z^\ell_+ = \sigma \prod_{n=1}^{L-2} (r_\ell - s_n) \prod_{k \neq \ell}^{L} \frac{1}{r_\ell - r_k}$$

$$\mathcal{A}_-(w) = -\overline{\mathcal{A}_+(\bar{w})} \qquad \sum_{\ell=1}^{L} Z^\ell_+ = 0$$

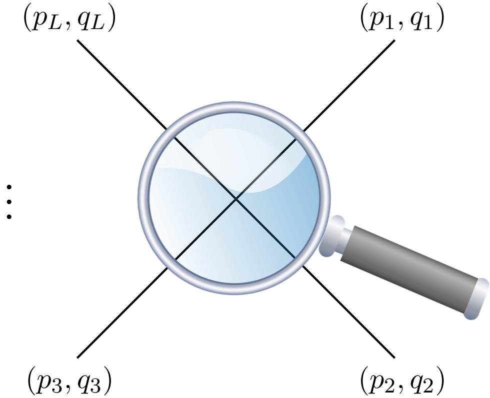
Solving remaining regularity conditions leaves $2L-2$ free real parameter $\sim$ choice of Z^ℓ_+ with $\sum Z^\ell_+ = 0$.

S^2 collapses to each side of each pole $\rightarrow$ 3-cycles w/ 3-form flux.
Entire solution near poles matches (p,q) 5-brane w/ $q + ip \sim Z^m_+$.

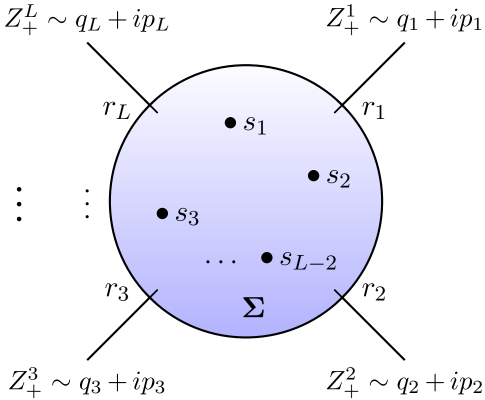
5-brane web picture



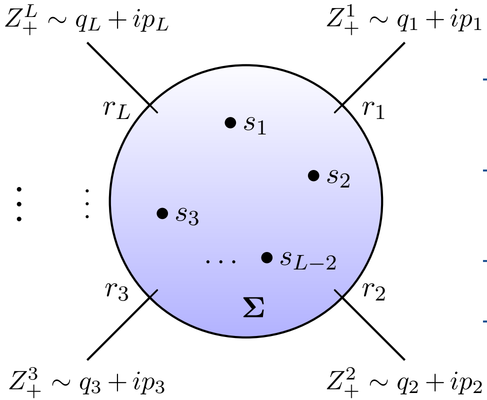
5-brane web picture



5-brane web picture

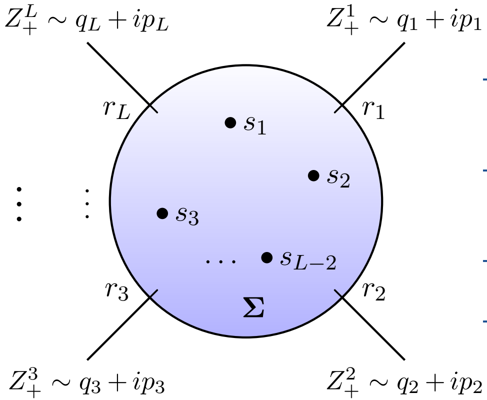


5-brane web picture



- external 5-branes explicitly (p, q) charge conserved
- parametrized by choice of residues mod charge cons.
- $\mathrm{AdS}_6 + 16$ susies $= F(4)$
- need $L \geq 3$, p and q charge

5-brane web picture



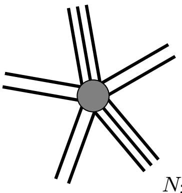
- external 5-branes explicitly (p, q) charge conserved
- parametrized by choice of residues mod charge cons.
- $\mathrm{AdS}_6 + 16$ susies $= F(4)$
- need $L \geq 3$, p and q charge

Supergravity solutions for arbitrary fully localized 5-brane junctions:
5-branes $\leftrightarrow$ poles, (p, q) charges $\leftrightarrow$ residues Z_+^ℓ ✓

$\mathrm{AdS}_6/\mathrm{CFT}_5$ in Type IIB

$\mathrm{AdS}_6/\mathrm{CFT}_5$: Type IIB string theory on warped AdS_6 solution $\cong$ 5d SCFT realized on associated (p, q) 5-brane junction.

"Large-N": junctions of large groups of like-charged 5-branes



$p_i, q_i \in \mathbb{Z}$, relatively prime

$N_i \gg 1 \ \forall i$

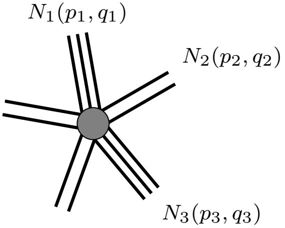
$Z_+^\ell \sim N_\ell(q_\ell + ip_\ell)$

$\rightarrow$ classical supergravity w/ continuous $Z_+^\ell \sim N_\ell(q_\ell + ip_\ell)$

AdS$_6$/CFT$_5$ in Type IIB

AdS$_6$/CFT$_5$: Type IIB string theory on warped AdS$_6$ solution $\cong$ 5d SCFT realized on associated (p, q) 5-brane junction.

"Large-N": junctions of large groups of like-charged 5-branes



$p_i, q_i \in \mathbb{Z}$, relatively prime

$N_i \gg 1 \; \forall i$

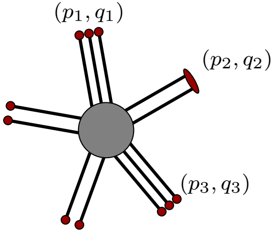
$Z_+^\ell \sim N_\ell(q_\ell + ip_\ell)$

$\rightarrow$ classical supergravity w/ continuous $Z_+^\ell \sim N_\ell(q_\ell + ip_\ell)$

Unconstrained junctions: no 7-branes (or one 5-brane per 7-brane)

Extension to constrained junctions [D'Hoker,Gutperle,CFU 1706.00433]

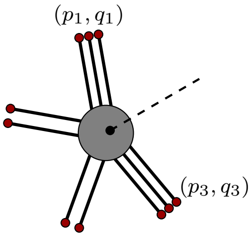
Supergravity solutions for "constrained" junctions with 7-branes:



- 5-branes can terminate on 7-branes without changing the theory
- multiple 5-branes ending on same 7-brane: s-rule constraints

Extension to constrained junctions [D'Hoker,Gutperle,CFU 1706.00433]

Supergravity solutions for "constrained" junctions with 7-branes:

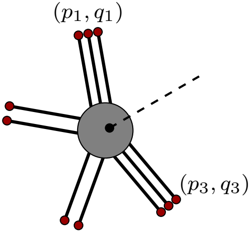


- 5-branes can terminate on 7-branes without changing the theory
- multiple 5-branes ending on same 7-brane: s-rule constraints

Mass-deform to open up brane web, Hanany-Witten transitions to move 7-branes w/ multiple 5-branes into the web, take UV limit.

Extension to constrained junctions [D'Hoker,Gutperle,CFU 1706.00433]

Supergravity solutions for "constrained" junctions with 7-branes:



- 5-branes can terminate on 7-branes without changing the theory
- multiple 5-branes ending on same 7-brane: s-rule constraints

Mass-deform to open up brane web, Hanany-Witten transitions to move 7-branes w/ multiple 5-branes into the web, take UV limit.

$\rightarrow \Sigma =$ disc with punctures & $SL(2,\mathbb{R})$ monodromy $=$ 7-branes. Worked out for commuting monodromies, so far.

Holographic duals for 5d SCFTs

General warped $AdS_6 \times S^2 \times \Sigma$ solution with 16 supersymmetries in Type IIB. ✓

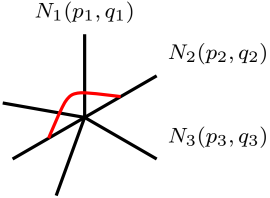
Physically regular solutions for $\Sigma =$disc with single-valued $\mathcal{A}_\pm$. Identified with unconstrained (p,q) 5-brane junctions. ✓

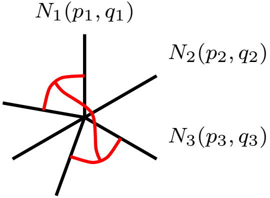
Regular solutions for punctured disc with commuting $SL(2,\mathbb{R})$ monodromies. Describe constrained 5-brane junctions. ✓

– Matching stringy operators –

Matching stringy operators [Bergman,Rodriguez-Gomez,CFU 1806.07898]

5-brane picture: gauge invariant operators from strings and string junctions connecting external 5-branes

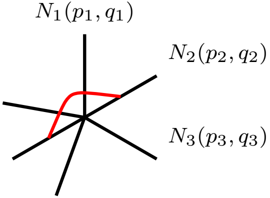


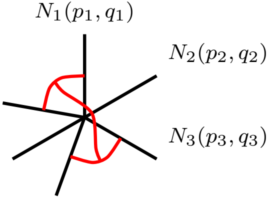


Supergravity: probe string (junctions) $\leftrightarrow \Delta = \mathcal{O}(N)$ operators

Matching stringy operators [Bergman,Rodriguez-Gomez,CFU 1806.07898]

5-brane picture: gauge invariant operators from strings and string junctions connecting external 5-branes

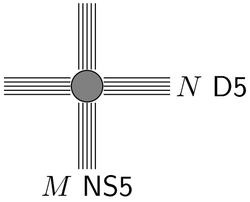




Supergravity: probe string (junctions) $\leftrightarrow \Delta = \mathcal{O}(N)$ operators

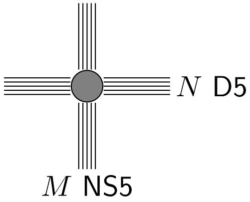
Strategy: identify stringy BPS operators in gauge theory deformations, extrapolate charges and scaling dim to SCFT

Matching stringy operators: $+_{N,M}$ theory

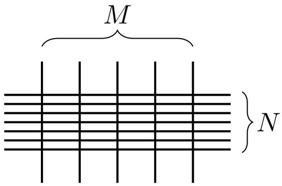


$\mathsf{SU}(N)^2 \times \mathsf{SU}(M)^2 \times \mathsf{U}(1)$
global symmetry

Matching stringy operators: $+_{N,M}$ theory



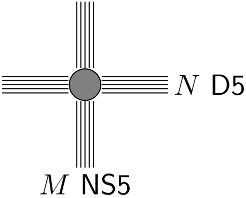
SU$(N)^2\times$SU$(M)^2\times$U(1)
global symmetry



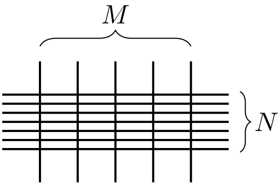
gauge theory deformation:

$$[N] \overset{x_1}{-} \mathrm{SU}(N) \overset{x_2}{-} \cdots \overset{x_{M-1}}{-} \mathrm{SU}(N) \overset{x_M}{-} [N]$$

Matching stringy operators: $+_{N,M}$ theory



$\mathsf{SU}(N)^2 \times \mathsf{SU}(M)^2 \times \mathsf{U}(1)$
global symmetry



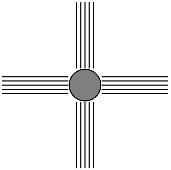
gauge theory deformation:

$$[N] \overset{x_1}{-} \mathrm{SU}(N) \overset{x_2}{-} \cdots \overset{x_{M-1}}{-} \mathrm{SU}(N) \overset{x_M}{-} [N]$$

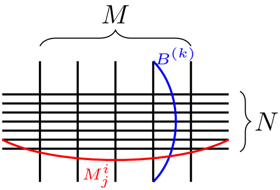
$$M^i_j = (x^{(1)} \cdots x^{(M)})^i_j \qquad (\mathbf{N}, \bar{\mathbf{N}}, \mathbf{1}, \mathbf{1}) \qquad \Delta = \frac{3}{2} M \qquad Q = \frac{1}{2} M$$

$$B^{(k)} = \det(x^{(k)}) \qquad \subset (\mathbf{1}, \mathbf{1}, \mathbf{M}, \bar{\mathbf{M}}) \qquad \Delta = \frac{3}{2} N \qquad Q = \frac{1}{2} N$$

Matching stringy operators: $+_{N,M}$ theory



SU$(N)^2\times$SU$(M)^2\times$U(1)
global symmetry



gauge theory deformation:

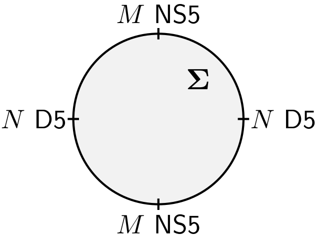
$$[N] \overset{x_1}{-} \mathrm{SU}(N) \overset{x_2}{-} \cdots \overset{x_{M-1}}{-} \mathrm{SU}(N) \overset{x_M}{-} [N]$$

$$M^i_j = (x^{(1)}\cdots x^{(M)})^i_j \qquad (\mathbf{N},\bar{\mathbf{N}},\mathbf{1},\mathbf{1}) \qquad \Delta = \frac{3}{2}M \qquad Q = \frac{1}{2}M$$

$$B^{(k)} = \det(x^{(k)}) \qquad \subset (\mathbf{1},\mathbf{1},\mathbf{M},\bar{\mathbf{M}}) \qquad \Delta = \frac{3}{2}N \qquad Q = \frac{1}{2}N$$

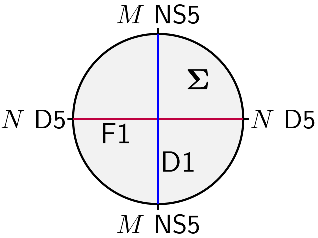
$M^i_j \sim$ F1 between D5, $B^{(k)} \subset$ D1 between NS5

Matching stringy operators: $+_{N,M}$ theory



Supergravity: 4-pole solution

Matching stringy operators: $+_{N,M}$ theory



Supergravity: 4-pole solution

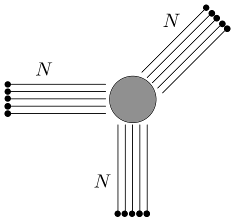
F1 between D5 poles, D1 between NS5 poles:

$$\Delta_{\mathrm{F1}} = \frac{3}{2}M \qquad\qquad \Delta_{\mathrm{D1}} = \frac{3}{2}N$$

Solve EOM and are BPS, scaling dim. match field theory ✓✓

Matching stringy operators: T_N theory

T_N: N D5, N NS5, N $(1,1)$ 5-branes [Benini,Benvenuti,Tachikawa '09]

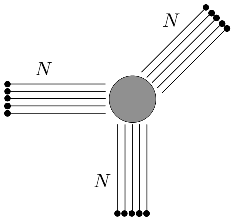


- $\mathrm{SU}(N)^3$ global symmetry
  E_6 theory for $N=3$

- reduce on S^1 to 4d $T_N \sim$ 6d $\mathcal{N}{=}(2,0)$ on 3-punctured sphere [Gaiotto '09]

Matching stringy operators: T_N theory

T_N: N D5, N NS5, N $(1,1)$ 5-branes [Benini,Benvenuti,Tachikawa '09]

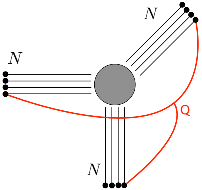


- $\mathrm{SU}(N)^3$ global symmetry
  E_6 theory for $N = 3$
- reduce on S^1 to 4d $T_N \sim$ 6d $\mathcal{N}=(2,0)$ on 3-punctured sphere [Gaiotto '09]

Gauge theory deformation of 5d T_N:

$$N - SU(N-1) - \ldots - SU(2) - 2$$

Matching stringy operators: T_N theory

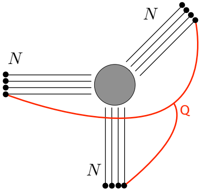


external 5-branes can be connected with D1-F1-$(1,1)$ triple string junction

$(\mathbf{N},\mathbf{N},\mathbf{N})$ of $SU(N)^3$; contains meson in

$$N - SU(N-1) - \ldots - SU(2) - 2$$

Matching stringy operators: T_N theory



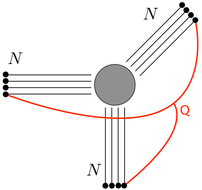
external 5-branes can be connected with D1-F1-$(1,1)$ triple string junction

$(\mathbf{N},\mathbf{N},\mathbf{N})$ of $SU(N)^3$; contains meson in

$$N - SU(N-1) - \ldots - SU(2) - 2$$

$$\Longrightarrow \ \Delta = \frac{3}{2}(N-1) \qquad Q = \frac{1}{2}(N-1)$$

Matching stringy operators: T_N theory



external 5-branes can be connected with D1-F1-$(1,1)$ triple string junction

$(\mathbf{N},\mathbf{N},\mathbf{N})$ of $SU(N)^3$; contains meson in

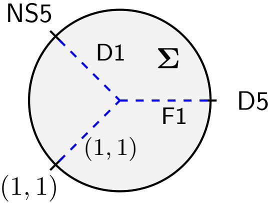
$$N - SU(N-1) - \ldots - SU(2) - 2$$

$$\Longrightarrow \ \Delta = \frac{3}{2}(N-1) \qquad Q = \frac{1}{2}(N-1)$$

Triple junction in supergravity:

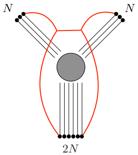
$$\Delta = \frac{3}{2}N \qquad Q = \frac{1}{2}N$$

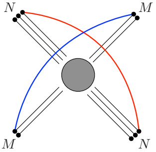
Agrees with T_N operator at large N. ✓✓

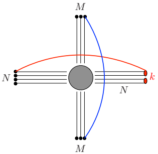


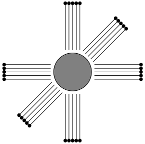
Matching stringy operators

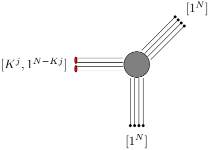
Similar quantitative matches of field theory and supergravity for

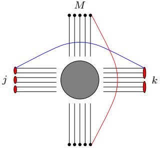










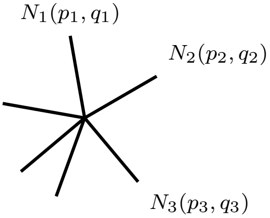


Predictions for various operators not easily seen in gauge theory, and for more exotic junctions, e.g. $\Delta = \frac{9}{2}N$ in large-N E_0 theory.

– Sphere partition functions –

S^5 partition functions [Gutperle,Marasinou,Trivella,CFU 1705.01561]

Holographically: F_{S^5} from on-shell action (with zero 5-form) or via $S_{\mathrm{EE,disc}}$ from 8d minimal surface.



– results agree, poles unproblematic for both computations

– generically non-trivial dependence of F_{S^5} on all (p, q) charges

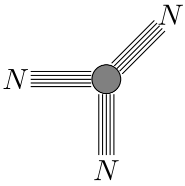
Homogeneous rescaling of all (p, q) charges:

$$N_i \to nN_i \;\; \forall i \qquad \Longrightarrow \qquad \mathcal{F}(S^5) \to n^4 \mathcal{F}(S^5)$$

Steeper than $n^{5/2}$ for $USp(N)$ theory from D4/D8/O8 [Jafferis,Pufu]

$+_{N,M}$ and T_N theories

Supergravity results for T_N and $+_{N,M}$ theories:



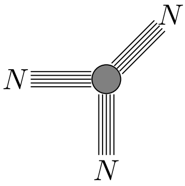
5d T_N theory w/ gauge theory deformation

$$N - SU(N-1) \times \cdots \times SU(2) - 2$$

$$\mathcal{F}_{\text{sugra}}(S^5) = -\frac{27}{8\pi^2}\,\zeta(3)N^4$$

$+_{N,M}$ and T_N theories

Supergravity results for T_N and $+_{N,M}$ theories:



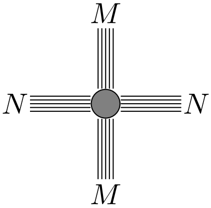
5d T_N theory w/ gauge theory deformation

$$N - SU(N-1) \times \cdots \times SU(2) - 2$$

$$\mathcal{F}_{\text{sugra}}(S^5) = -\frac{27}{8\pi^2}\,\zeta(3)N^4$$

D5/NS5 intersection: $N - SU(N)^{M-1} - N$

$$\mathcal{F}_{\text{sugra}}(S^5) = -\frac{189}{16\pi^2}\,\zeta(3)N^2M^2$$



$+_{N,M}$ and T_N in field theory [Fluder, CFU 1806.08374]

Supersymmetric localization in large-N gauge theory: instantons exponentially suppressed, saddle point approximation exact.

Extrapolate to SCFT assuming higher-dim operators Q-exact.

$+_{N,M}$ and T_N in field theory [Fluder, CFU 1806.08374]

Supersymmetric localization in large-N gauge theory: instantons exponentially suppressed, saddle point approximation exact.

Extrapolate to SCFT assuming higher-dim operators Q-exact.

Remaining challenge: long quivers

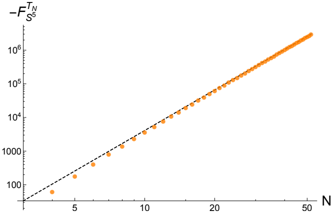
$$\mathcal{Z}_0 = \int \prod_{i,j} \mathrm{d}\lambda_i^{(j)} \exp\left(-\mathcal{F}\right)$$

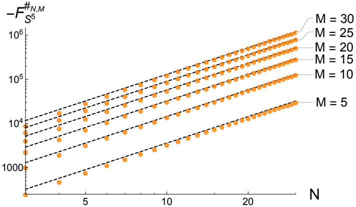
Gauge node becomes effectively continuous parameter at large N.

$+_{N,M}$ and T_N in field theory [Fluder, CFU 1806.08374]

Numerical evaluation [Herzog,Klebanov,Pufu,Tesileanu]: Replace saddle point eq. by set of particles w/ coordinates $\lambda_i^{(j)}$ in potential $\mathcal{F}$

$\Rightarrow \mathcal{F}_{S^5}$ numerically, $N \leq 50$ for T_N, $N, M \leq 30$ for $+_{N,M}$





Confirms N^4 and N^2M^2 scaling predicted from supergravity, coefficients of leading terms agree to $1‰$ ✓✓

– Counting AdS_6 black hole microstates –

Counting black hole microstates [Fluder,Hosseini,CFU 1902.05074]

$\mathrm{AdS}_6 \times S^2 \times \Sigma$ solution in Type IIB $\longleftrightarrow$ 5d SCFT encoded in $(\Sigma, \mathcal{A}_\pm)$ in conformal vacuum on $\mathbb{R}^{1,4}$, S^5, ..., finite T,...

Consistent KK reduction to 6d F(4) sugra based on these general AdS_6 solutions: [Hong,Liu,Mayerson '18;Malek,Samtleben,Vall Camell '18]

$\rightarrow$ any (bosonic) solution to 6d F(4) supergravity combined with any choice of $(\Sigma, \mathcal{A}_\pm)$ uplifts to 10d solution of Type IIB

Counting black hole microstates [Fluder,Hosseini,CFU 1902.05074]

Magnetically charged AdS_6 black holes in 6d F(4) supergravity with $\mathrm{AdS}_2 \times \Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$, near-horizon limit [Suh '18].

Analytic near-horizon solution including B_2 (missing in [Naka '02])

$$ds^2 = ds^2_{AdS_2} + ds^2_{\Sigma_{\mathfrak{g}_1}} + ds^2_{\Sigma_{\mathfrak{g}_2}} \qquad \phi = \text{const}$$

$$F^3 \sim \text{vol}_{\Sigma_{\mathfrak{g}_1}} + \text{vol}_{\Sigma_{\mathfrak{g}_2}} \qquad B_2 \sim \text{vol}_{AdS_2}$$

$\Sigma_{\mathfrak{g}_i}$ constant curvature Riemann surfaces of genus g_i, $\mathfrak{g}_1, \mathfrak{g}_2 > 1$.

Uplift to AdS_6 black hole solutions in IIB with near-horizon limit $(\mathrm{AdS}_2 \times \Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}) \times S^2$ warped over IIB Riemann surface Σ.

Counting black hole microstates [Fluder,Hosseini,CFU 1902.05074]

Family of Type IIB AdS_6 black hole solutions for each (regular) choice $(\Sigma, \mathcal{A}_\pm)$, labeled by $(\mathfrak{g}_1, \mathfrak{g}_2)$. Bekenstein-Hawking entropy:

$$S_{\text{BH}} = -\frac{8}{9}(1-\mathfrak{g}_1)(1-\mathfrak{g}_2)\mathcal{F}_{S^5}$$

$$\mathcal{F}_{S^5} = -\frac{4}{9}\pi^3 \int_\Sigma d^2w |\partial_w \mathcal{G}|^2$$

$$\partial_w \mathcal{G} = (\bar{\mathcal{A}}_+ - \mathcal{A}_-)\partial_w \mathcal{A}_+ + (\mathcal{A}_+ - \bar{\mathcal{A}}_-)\partial_w \mathcal{A}_-$$

Near-horizon solution describes 5d SCFT characterized by $(\Sigma, \mathcal{A}_\pm)$ compactified on $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2} \times S^1$ with topological twist.

Counting black hole microstates [Fluder,Hosseini,CFU 1902.05074]

Partition function $\mathcal{Z}_{S^1\times\Sigma_{\mathfrak{g}_1}\times\Sigma_{\mathfrak{g}_2}}$ from localization [Hosseini,Yaakov, Zaffaroni '18, Crichigno,Jain,Willett '18] $\sim$ 5d top. twisted index.

Large-N prescription of [Hosseini,Yaakov, Zaffaroni '18]:

$$Z_{\Sigma_{\mathfrak{g}_1}\times\Sigma_{\mathfrak{g}_2}\times S^1} = \sum_{k=0}^{\infty}\sum_{\mathfrak{m},\mathfrak{n}}\oint_{\mathcal{C}} q^k Z_{\text{int}}^{(k\text{-instantons})}(\mathfrak{m},\mathfrak{n},a;\dots)$$

(i) exchange sum over flux on $\Sigma_{\mathfrak{g}_1}$ with integration and resum

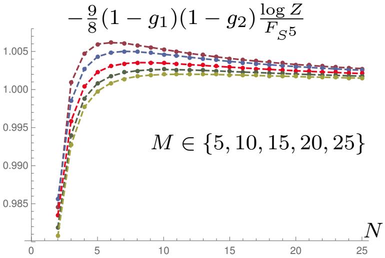
(ii) (pole, flux on $\Sigma_{\mathfrak{g}_2}$) dominating remaining integral+sum from

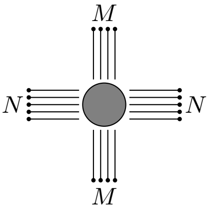
$$\exp\left(i\frac{\partial\widetilde{\mathcal{W}}(a,\mathfrak{n};\Delta,\mathfrak{t})}{\partial a_\ell}\right)=1 \qquad \exp\left(\frac{2\pi i}{\hbar}\frac{\partial\mathcal{F}(a)}{\partial a_\ell}\right)=1$$

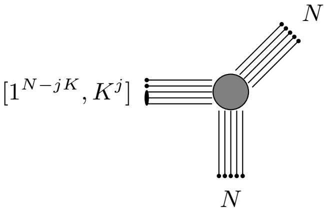
Agrees with holographic prediction for 5d $USp(N)$ theory [Suh '18]

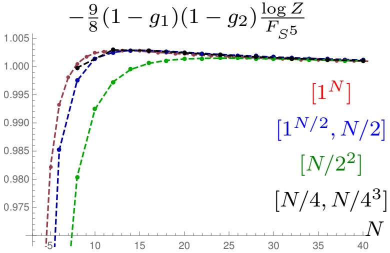
Counting black hole microstates

[Fluder,Hosseini,CFU 1902.05074]

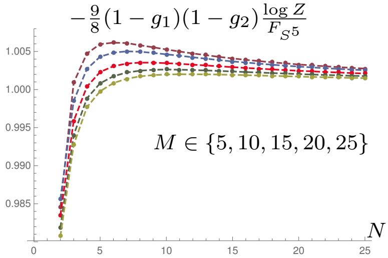


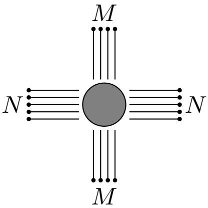


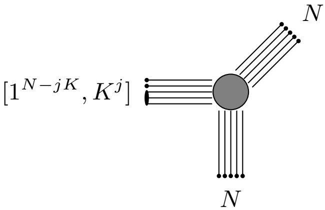


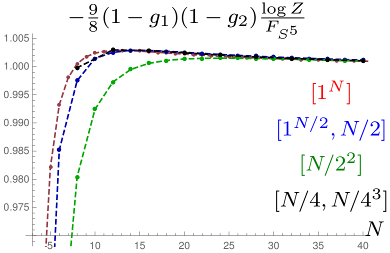


Counting black hole microstates [Fluder,Hosseini,CFU 1902.05074]









Supports index computation, KK reductions, AdS_6/CFT_5 dualities.

– Summary & Outlook–

Summary

Supergravity solutions for fully localized 5-brane junctions in Type IIB. Holographic duals for the corresponding 5d SCFTs.

Quantitative tests of proposed $\mathrm{AdS}_6/\mathrm{CFT}_5$ dualities: spectrum of stringy operators, S^5 partition functions, top. twisted indices.

Supports existence of 5d SCFTs and SCFT interpretation of 5-brane junctions.

N^4 scaling of # d.o.f. from sphere partition functions, results consistent with conjectured 5d F-theorem

Outlook

More quantitative studies of 5d SCFTs: spectrum, correlators, non-local operators, finite T, ...

Lessons for $d \leq 4$: boundaries and defects, compactification

Further solutions: mutually non-local 7-branes?

Similar story for closely related $\mathrm{AdS}_2 \times \mathrm{S}^6 \times \Sigma$ solutions?
[Corbino,D'Hoker,CFU 1712.04463], [Corbino,D'Hoker,Kaidi,CFU 1812.10206]

Outlook

More quantitative studies of 5d SCFTs: spectrum, correlators, non-local operators, finite T, ...

Lessons for $d \leq 4$: boundaries and defects, compactification

Further solutions: mutually non-local 7-branes?

Similar story for closely related $\mathrm{AdS}_2 \times \mathrm{S}^6 \times \Sigma$ solutions?
[Corbino,D'Hoker,CFU 1712.04463], [Corbino,D'Hoker,Kaidi,CFU 1812.10206]

Thank you!