# Stochastic gravitational wave background from preheating

based on arXiv:1905.00371 in collaboration with
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# What are gravitational waves?

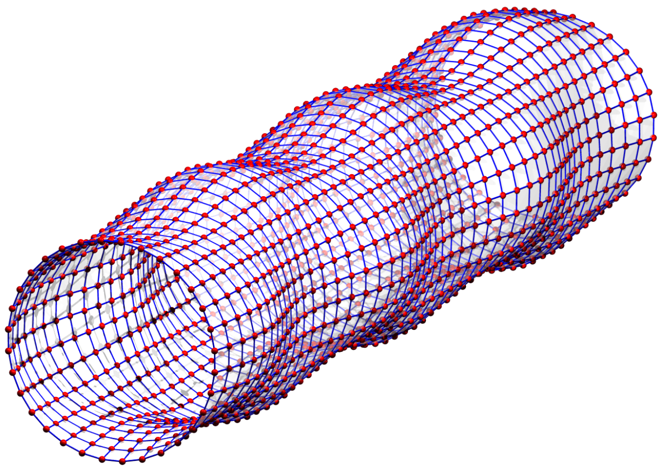
- General Theory of Relativity: Gravity is a manifestation of matter curving space-time geometry

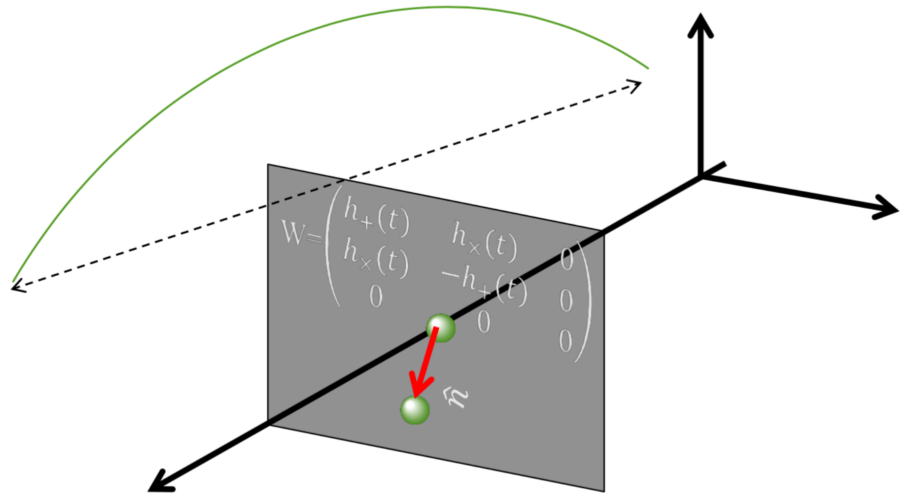
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Curvature Matter

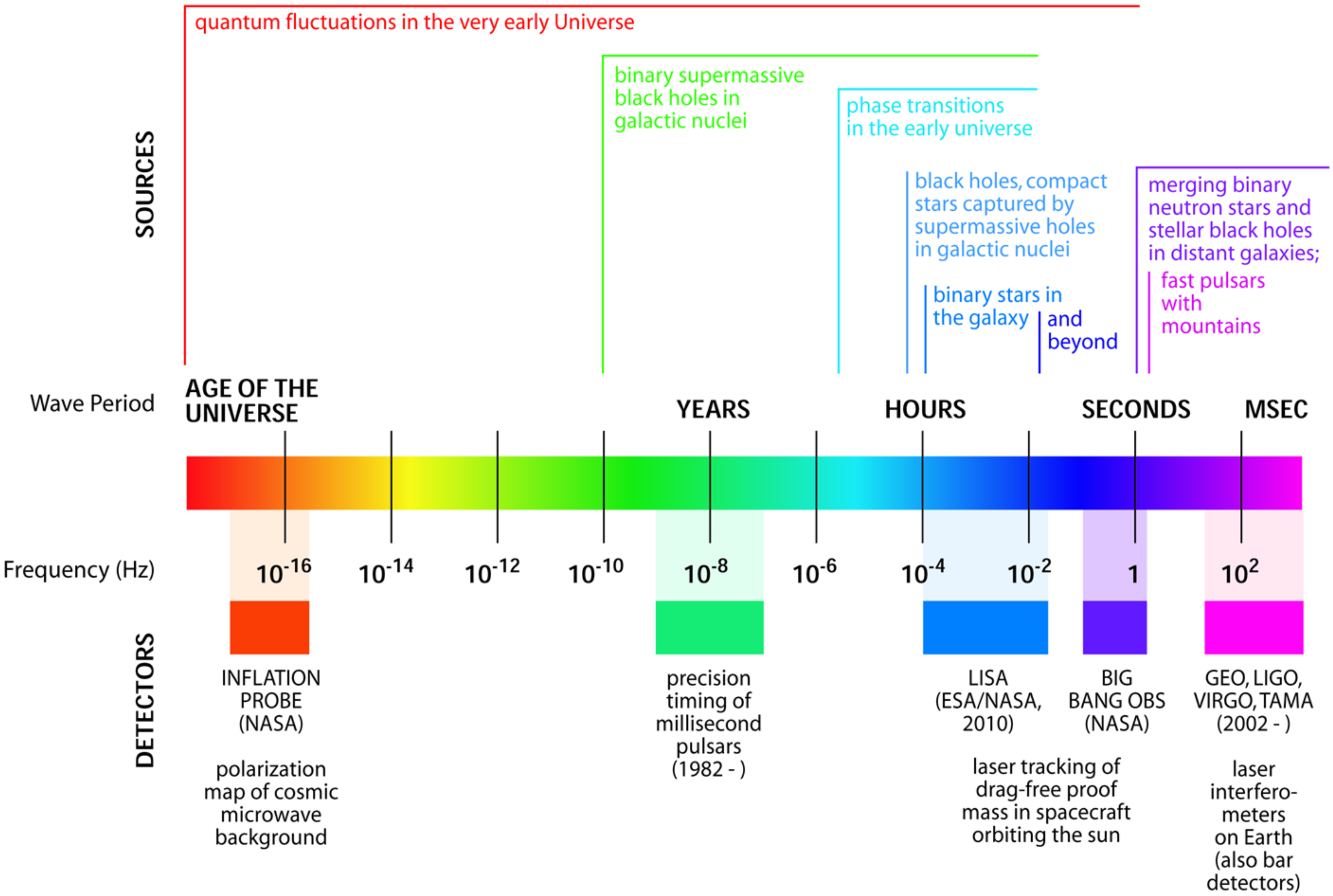
- Changes in matter distribution appear to distant observers as a time dependent change in their local space-time geometry
  Local expansion and contraction of space

- The changes propagate away from the source as a wave travelling at the speed of light

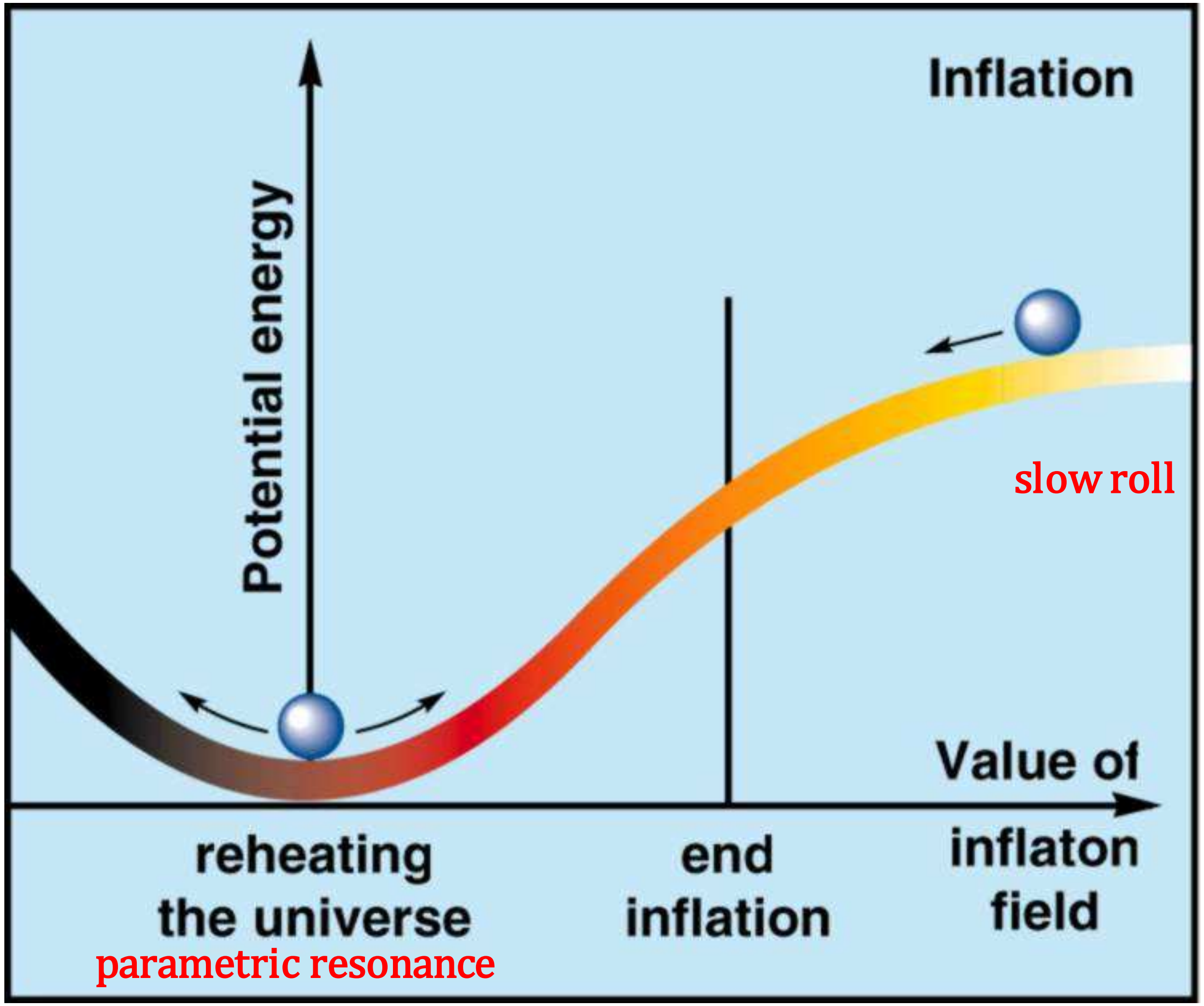




# THE GRAVITATIONAL WAVE SPECTRUM



# Dynamics during and after inflation




slow roll $\frac{\dot{\phi}^2}{2} << V(\phi)$

J. Garcia-Bellido, Phil. Trans. R. Soc. Lond. A 357 (1999) 3237.
J. Garcia-Bellido. astro-ph/0502139

# Parametric resonance: example 1. two field inflation

Let us assume that inflation is driven by a single field, but is coupled to a subdominant field,

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}g^2\phi^2\chi^2$$

$$\langle\chi\rangle = 0$$

In this case the linearized equations of motion for the fluctuations are

$$\delta\ddot{\phi}_{\mathbf{k}} + (k^2 + m_\phi^2)\delta\phi_{\mathbf{k}} = 0$$ a simple harmonic oscillator $\delta\phi_{\mathbf{k}} \sim e^{i\sqrt{k^2+m_\phi^2}t}$

$$\delta\ddot{\chi}_{\mathbf{k}} + (k^2 + m_\chi^2 + g^2\varphi^2)\delta\chi_{\mathbf{k}} = 0$$

$$\omega_k^2(t) = k^2 + m_\chi^2 + g^2\varphi^2(t)$$ a time-dependent frequency

$$\ddot{\varphi} + V_{,\phi} \simeq 0 \qquad \varphi(t) = \Phi\sin(m_\phi t)$$

# Parametric resonance: example 1: two field inflation

Equations of motion for the fluctuation

$$\delta\ddot{\chi}_{\mathbf{k}} + (k^2 + m_\chi^2 + g^2\varphi^2)\delta\chi_{\mathbf{k}} = 0$$

periodic frequency: Hill's equation

Typically written in the form

$$\frac{d^2 y_{\mathbf{k}}}{dz^2} + [A_k + qF(z)]y_{\mathbf{k}}(z) = 0$$

$$z = m_\phi t \qquad q = g^2\Phi^2/(2m_\phi^2)$$

$$A_k = (k^2 + m_\chi^2 + \tfrac{1}{2}g^2\Phi^2)/m_\phi^2$$

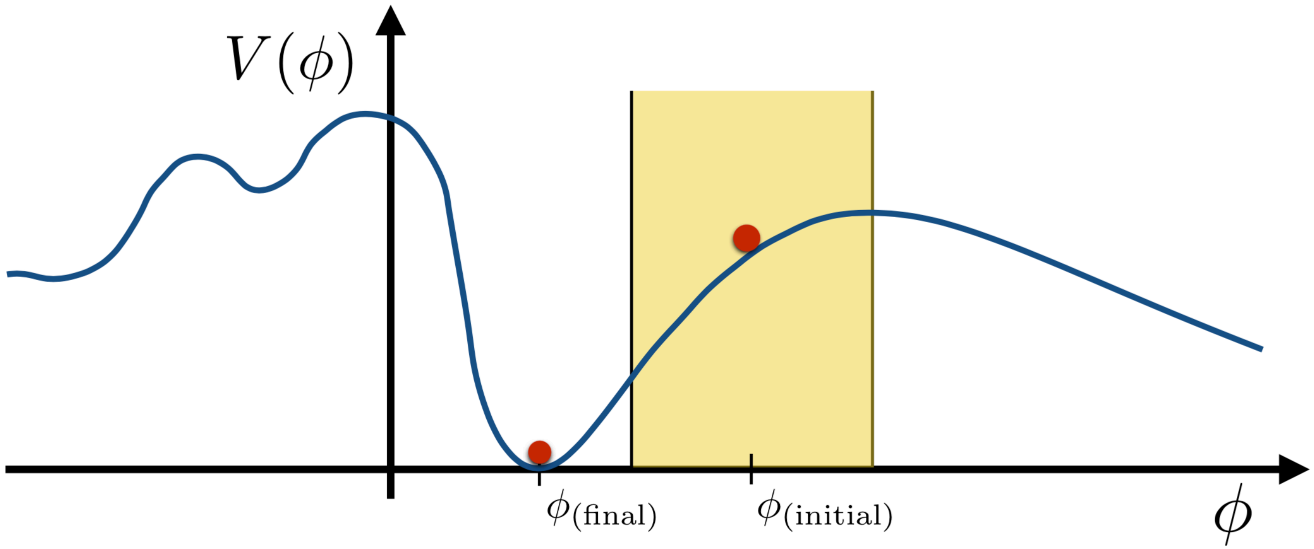
$$y_k(z) = e^{\tilde{\mu}_k z} g_1(z) + e^{-\tilde{\mu}_k z} g_2(z)$$

Wavenumbers k with $\Re[\tilde{\mu}_k] > 0$ $\delta\chi_{\mathbf{k}}$ grow exponentially, parametric resonance

$\tilde{\mu}_k$ pure imaginary, modes are stable and no parametric resonance

# Example 2. single field inflation

$$\delta\ddot{\phi}_{\mathbf{k}} + \left[k^2 + V_{,\phi\phi}(\varphi)\right]\delta\phi_{\mathbf{k}} = 0$$




Tachyonic preheating
Tachyonic oscillations

$$k^2/a^2 + \partial^2 V/\partial\phi^2 < 0$$

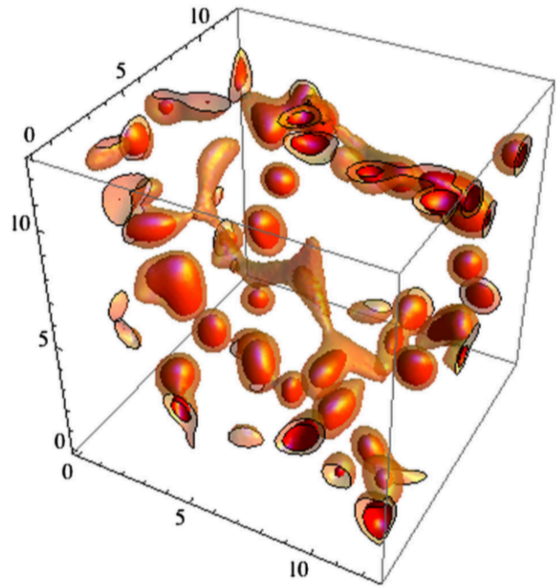
Parametric resonance

periodic frequency: Hill's equation

# Perturbation amplification and oscillon formation

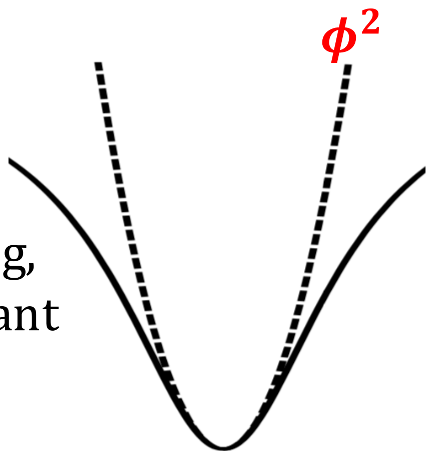
Oscillons:

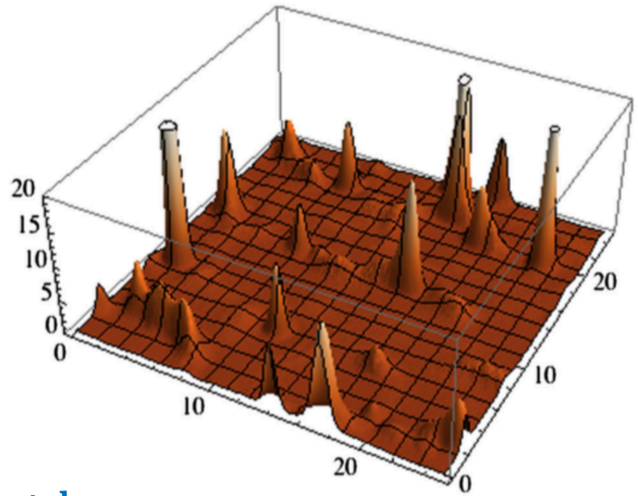
- spatially localised oscillating scalar field configurations with large amplitude
- extremely long-lived
- radiate energy
- often tend to be spherical



Formation conditions:

- growth of perturbations is strong, non-liner interactions is important
- potential opens up away from it minimum



Possible consequences:

- affect expansion history
- production of gravitational waves

Figure credit: M. A. Amin, R. Easther, H. Finkel, R. Flauger and M. P. Hertzberg, Phys. Rev. Lett. 108, 241302 (2012)

# Gravitational wave background from preheating

General characteristics:

- Peak frequency: $f \propto V^{1/4}$
- Amplitude: independent of energy scale of inflation
- Shape: detailed function of physics

# Gravitational wave background from preheating

Example: $$V(\phi, \chi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2$$

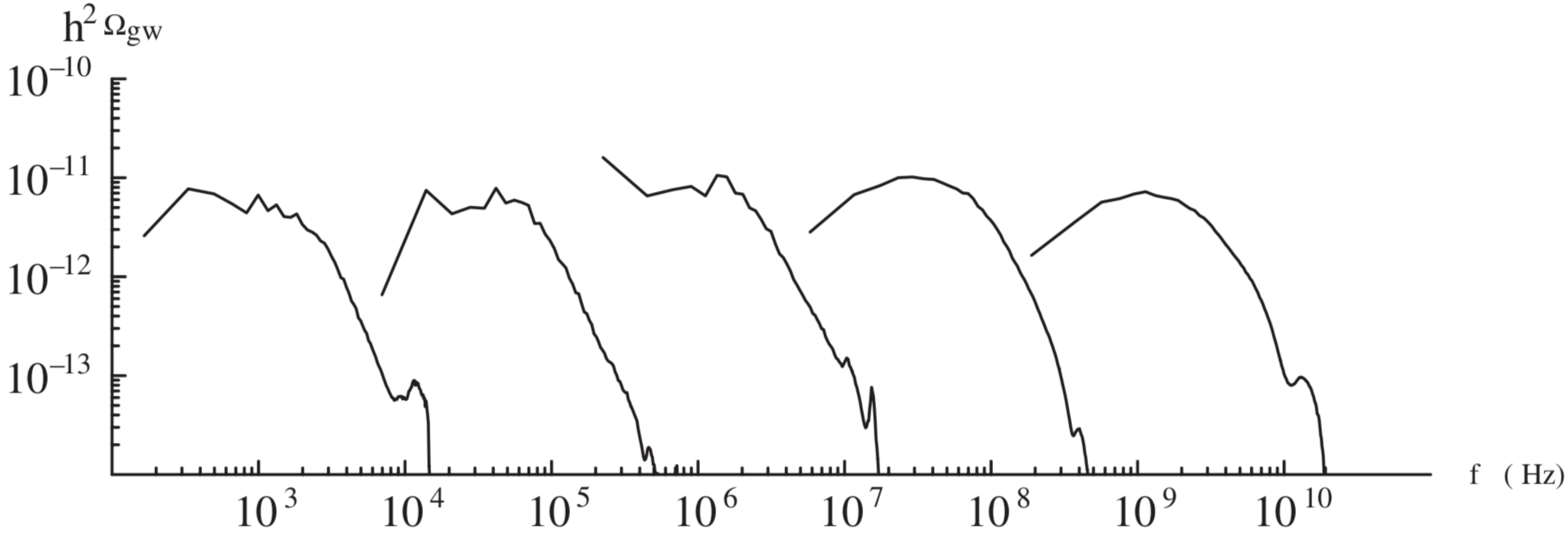



FIG. 1. We plot the spectrum of gravitational radiation produced during resonance with $\mu = 10^{-18}$ (left) through to $10^{-6}$ (right) in units where $m_{\rm Pl} \approx 10^{19}$ GeV $= 1$, where each spectrum has a value of $\mu 10^3$ times larger than the one immediately to the left. The corresponding initial energy densities run from $(4.5 \times 10^9\ \mathrm{GeV})^4$ to $(4.5 \times 10^{15}\ \mathrm{GeV})^4$ for our choice of $\phi_0$. The plots are made on $128^3$ grids, and the "feature" at high frequency is a numerical artifact.

R. Easther, J. T. Giblin, Jr. and E. A. Lim, Phys. Rev. Lett. 99, 221301 (2007)

# Lattice simulation: Equations & Methods

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + \frac{\partial V}{\partial \phi} = 0$$

$$H^2 = \frac{1}{3M_{\rm pl}^2}\left(V + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}\left|\nabla\phi\right|^2\right)$$

| | | |
|---|---|---|
| Time evolution | Staggered Leapfrog Method | RK4 Method |
| Spatial gradient & Laplace | Finite difference Method | pseudo-spectral method |
| | LATTICEEASY | PSpectRe |
| | Gary Felder, Igor Tkachev | Richard Easther, Hal Finkel, Nathaniel Roth |

# Lattice simulation: Methods

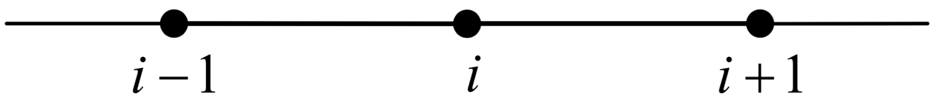
| | LATTICEEASY | PSpectRe |
|---|---|---|
| Time evolution | Staggered Leapfrog Method (Second order integration) | RK4 Method (Fourth order Runge-Kutta integration) |

$$
\begin{aligned}
f(t) &= f(t-dt) + dt\,\dot{f}(t-dt/2) \\
\dot{f}(t+dt/2) &= \dot{f}(t-dt/2) + dt\,\ddot{f}[f(t)] \\
f(t+dt) &= f(t) + dt\,\dot{f}(t+dt/2) \\
&\dots
\end{aligned}
$$

$$
\begin{aligned}
k_1 &= hf(x_n, y_n) \\
k_2 &= hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_1) \\
k_3 &= hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_2) \\
k_4 &= hf(x_n + h, y_n + k_3) \\
y_{n+1} &= y_n + \tfrac{1}{6}k_1 + \tfrac{1}{3}k_2 + \tfrac{1}{3}k_3 + \tfrac{1}{6}k_4 + O(h^5)
\end{aligned}
$$

# Lattice simulation: Methods

| | LATTICEEASY | PSpectRe |
|---|---|---|
| Spatial gradient & Laplace | Finite difference Method | pseudo-spectral method |




$$\nabla^2 \longrightarrow -\vec{k}\cdot\vec{k}$$

$$\frac{d^2 y}{dx^2} \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{(\Delta x)^2}$$

free of differencing noise

# Lattice simulation: Gravitational wave

Gravitational waves are described by transverse and traceless (TT) part of the metric perturbation in the synchronous gauge

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

Gravitational waves are sourced by TT-part of the anisotropic stress of the scalar field

$$\Pi_{ij} = [\partial_i \phi \partial_j \phi]^{\mathrm{TT}}$$

The TT-part of the anisotropic stress can be extracted by multiplied by projection tensor

$$P_{il}(\hat{\mathbf{k}}) P_{jm}(\hat{\mathbf{k}}) - \frac{1}{2} P_{ij}(\hat{\mathbf{k}}) P_{lm}(\hat{\mathbf{k}})$$

where

$$P_{ij}(\hat{\mathbf{k}}) \equiv \delta_{ij} - \hat{k}_i \hat{k}_j$$

$$\hat{k}_i \equiv k_i / |\mathbf{k}|$$

# Lattice simulation: Gravitational wave

The evolution of the GW is given by

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm pl}^2 a^2}\Pi_{ij}$$

The initial conditions for the metric perturbations are

$$h_{ij}(0) = \dot{h}_{ij}(0) = 0$$

The energy density of the GW is

$$\rho_{\rm GW}(t) = \frac{M_{\rm Pl}^2}{4}\left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle_{\rm V}$$

The spectrum of GW per logarithmic momentum interval is

$$\Omega_{\rm GW}(k) = \frac{1}{\rho_{\rm c}}\, k\, \frac{d\,\rho_{\rm GW}}{dk}$$

# Lattice simulation: Gravitational wave

Take into account the expansion history of the Universe between the emission of the GWs and today, the observable spectrum today is

$$\Omega_{\mathrm{gw},0}h_0^2 = \Omega_{\mathrm{gw}}(a_{\mathrm{e}}) \times \left(\frac{g_{\mathrm{th}}}{g_0}\right)^{-1/3}\Omega_{\mathrm{r},0}h_0^2$$

The observed frequency corresponding to a wave vector k is

$$f \approx 2.7 \times 10^{10}\,\frac{k}{a_{\mathrm{e}}\sqrt{m_{\mathrm{pl}}H_{\mathrm{e}}}}\mathrm{Hz}$$

# Lattice simulation: Gravitational wave

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + \frac{\partial V}{\partial \phi} = 0$$

$$H^2 = \frac{1}{3M_{\rm pl}^2}\left(V + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}\left|\nabla\phi\right|^2\right)$$

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm pl}^2 a^2}\Pi_{ij}$$

LATTICEEASY or PSpectRe

+

GW evolution extension

=

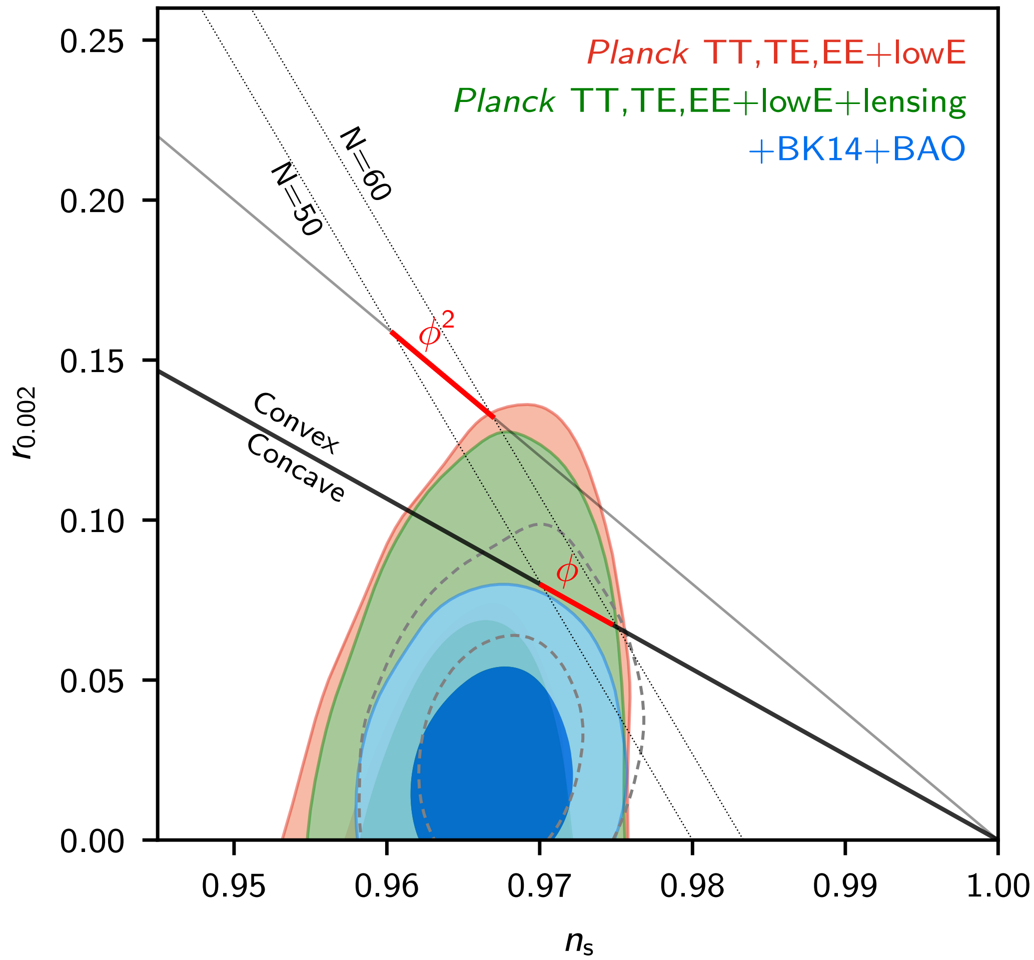
LATTICEEASY-GW

PSpectRe-GW

# Single field slow-roll infaltion model

$$S = \int d^4x\sqrt{-g}\left[\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$$V(\phi) \propto \phi^p$$




Planck 2018: arXiv:1807.06209

PHYSICAL REVIEW LETTERS **120,** 031301 (2018)

# Gravitational Waves from Oscillons with Cuspy Potentials

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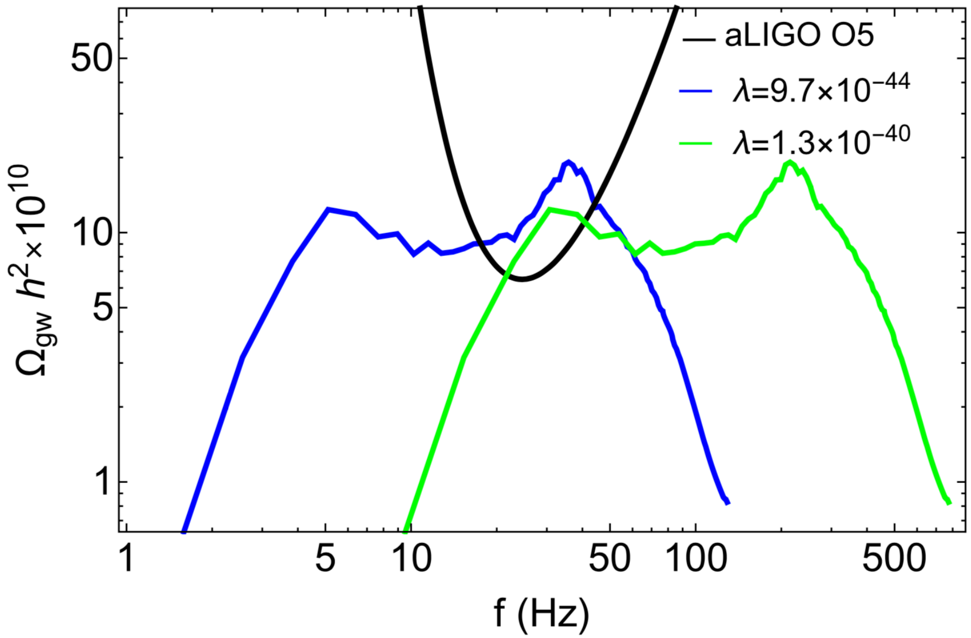
We study the production of gravitational waves during oscillations of the inflaton around the minimum of a cuspy potential after inflation. We find that a cusp in the potential can trigger copious oscillon formation, which sources a characteristic energy spectrum of gravitational waves with double peaks. The discovery of such a double-peak spectrum could test the underlying inflationary physics.

$$V(\phi) = \lambda M_{\rm pl}^{4-p}|\phi|^p$$

$$p = 1, 2/3, 2/5$$

Code modified from LATTICEEASY



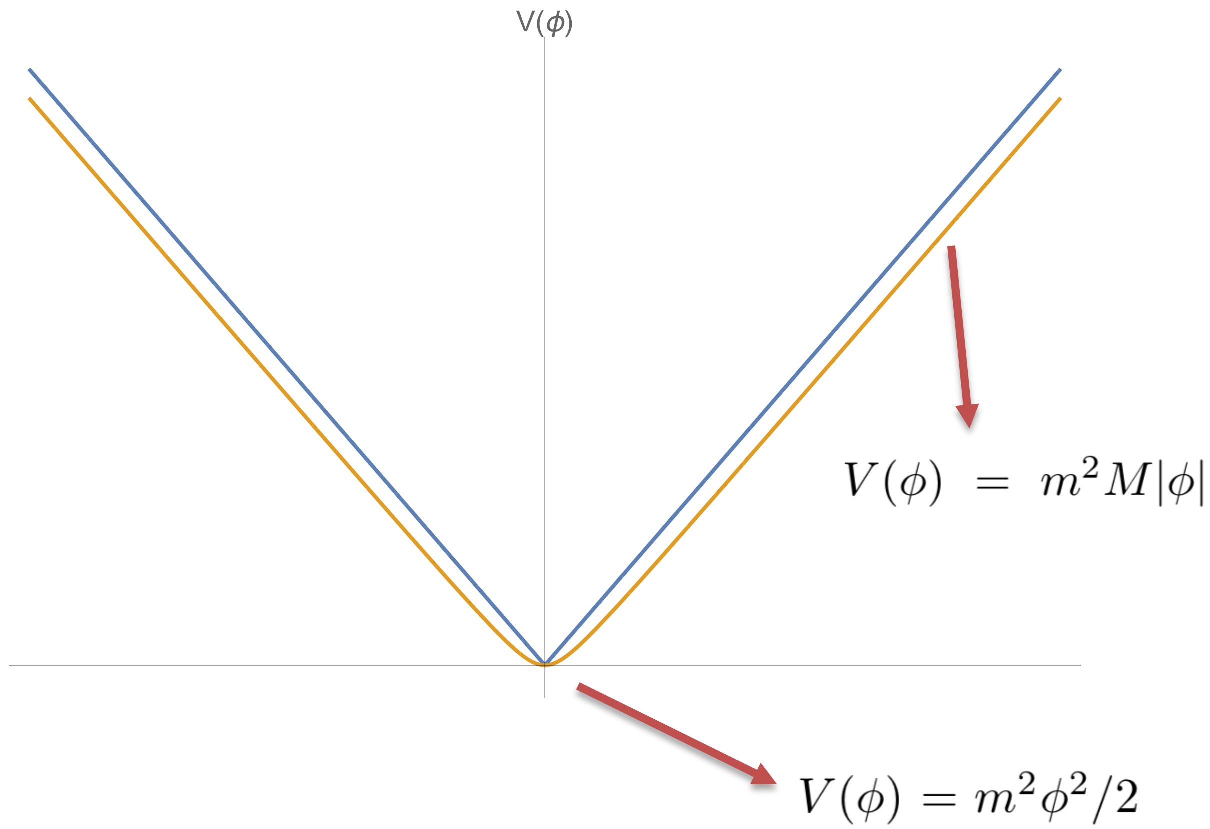
# Axion monodromy inflation with asymptotic linear potential

$$\mathcal{L}(\phi) = -\frac{(\partial\phi)^2}{2} - m^2 M^2 \left[ \left(1 + \frac{\phi^2}{M^2}\right)^{1/2} - 1 \right]$$

L. McAllister, E. Silverstein and A. Westphal, Phys. Rev. D 82, 046003 (2010)

# Cuspy potential

$$V(\phi) = \lambda_1 M_{\rm pl}^3 |\phi|$$



# Axion monodromy inflation with asymptotic $\phi^{2/3}$ potential

$$\mathcal{L}(\psi) = -\sqrt{1+\left(\frac{\psi}{\psi_c}\right)^2}\frac{(\partial\psi)^2}{2} - \lambda M_{\rm pl}^4\left(\sqrt{1+\left(\frac{\psi}{\psi_c}\right)^2}-1\right)$$

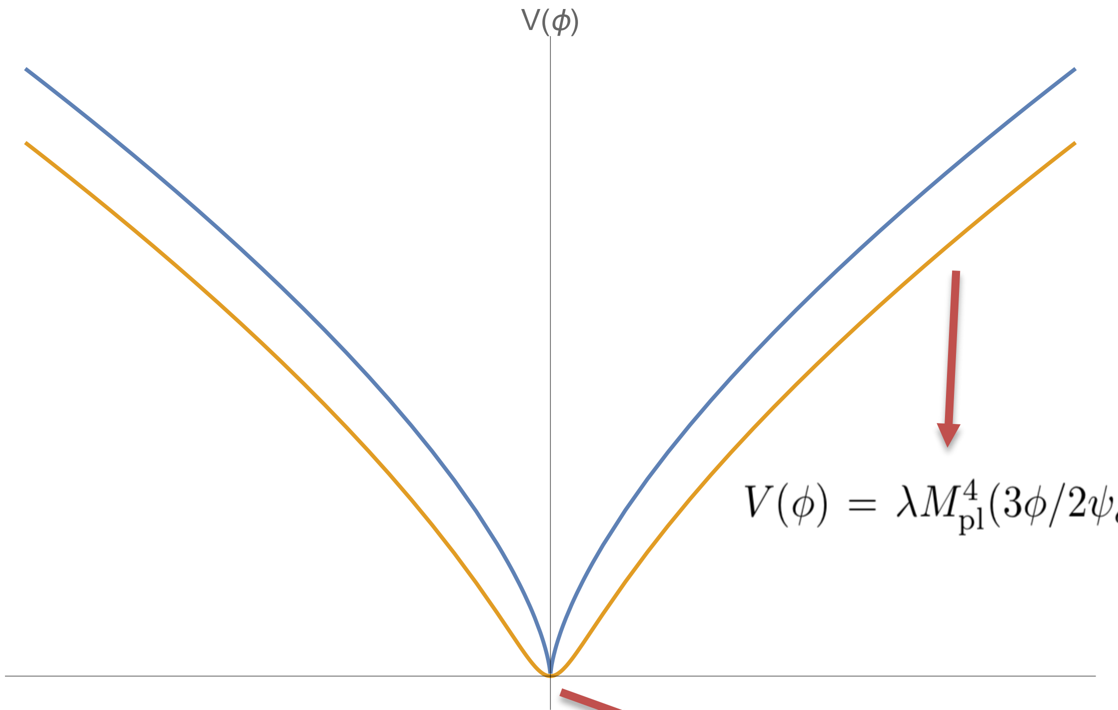
E. Silverstein and A. Westphal, Phys. Rev. D 78, 106003 (2008)

$$d\phi = \left[1+\left(\frac{\psi}{\psi_c}\right)^2\right]^{1/4} d\psi.$$

$$\mathcal{L}(\phi) = -\frac{(\partial\phi)^2}{2} - V(\phi)$$

# Cuspy potential

$$V(\phi) = \lambda_2 M_{\rm pl}^{10/3}|\phi|^{2/3}$$




$$V(\phi) = \lambda M_{\rm pl}^4 (3\phi/2\psi_c)^{2/3}$$

$$V(\phi) = \lambda M_{\rm pl}^4 \phi^2/2\psi_c^2$$

# Lattice simulation

- PSpectRe-GW
- pseudo-spectral algorithms
- fourth-order-in-time Runge-Kutta integration
- $256^3$ lattice
- initial conditions

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + \frac{\partial V}{\partial \phi} = 0$$

$$H^2 = \frac{1}{3M_{\rm pl}^2}\left(V + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}\left|\nabla\phi\right|^2\right)$$

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm pl}^2 a^2}\Pi_{ij}$$

$$\Pi_{ij} = \left[\partial_i\phi\partial_j\phi\right]^{\rm TT}$$

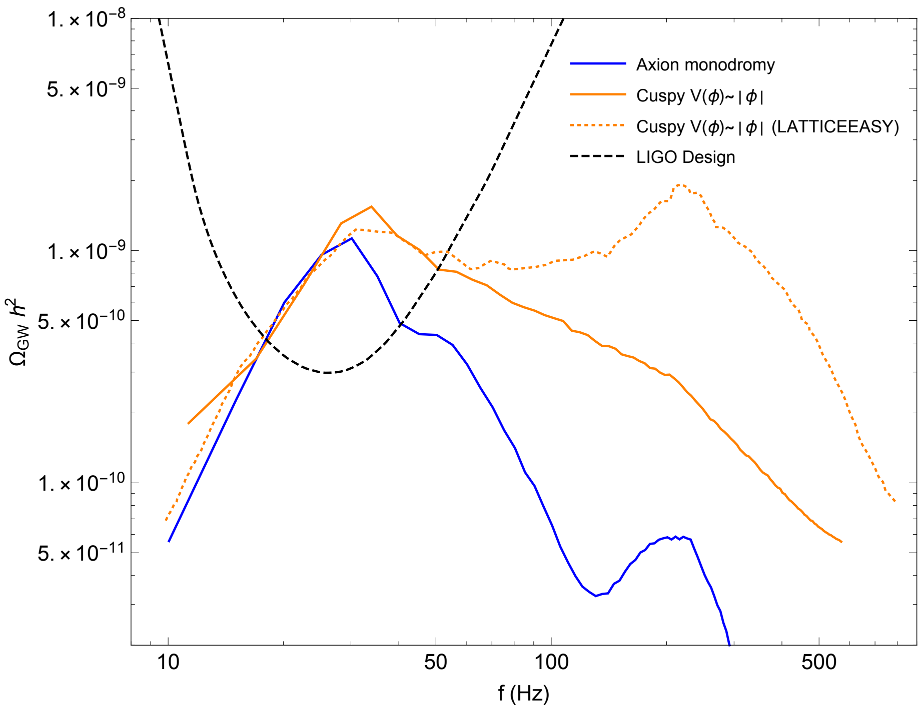
$$\rho_{\rm GW}(t) = \frac{M_{\rm Pl}^2}{4}\left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle_{\rm V}$$

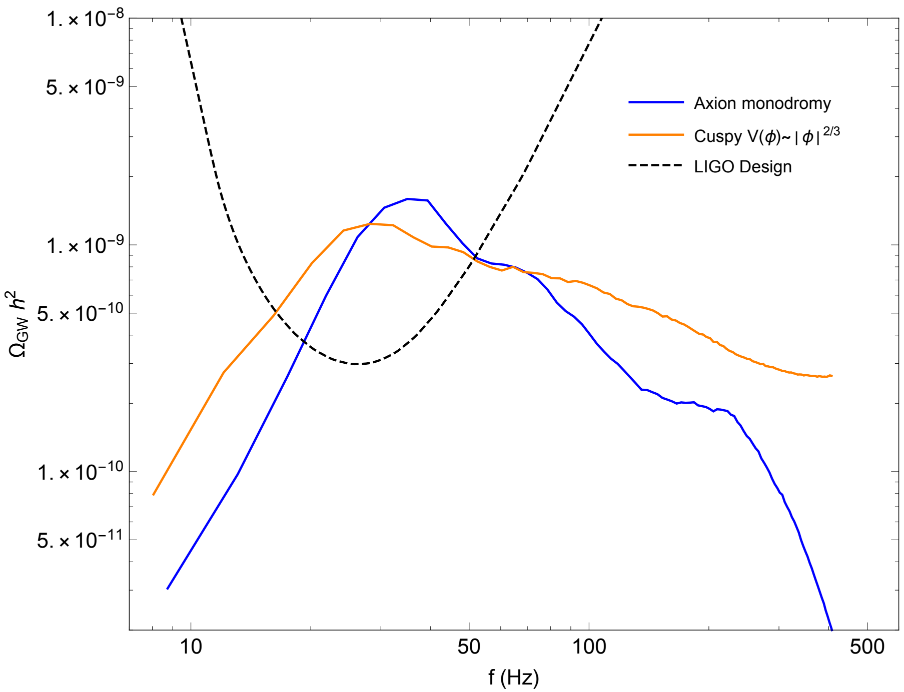
$$\Omega_{\rm GW}(k) = \frac{1}{\rho_{\rm c}}\, k\,\frac{d\,\rho_{\rm GW}}{dk}$$

# GW from axion monodromy inflation with asymptotic linear and $\phi^{2/3}$ potential

$$\mathcal{L}(\phi) = -\frac{(\partial\phi)^2}{2} - m^2 M^2 \left[ \left(1 + \frac{\phi^2}{M^2}\right)^{1/2} - 1 \right]$$

$$\mathcal{L}(\psi) = -\sqrt{1 + \left(\frac{\psi}{\psi_c}\right)^2} \frac{(\partial\psi)^2}{2} - \lambda M_{\rm pl}^4 \left( \sqrt{1 + \left(\frac{\psi}{\psi_c}\right)^2} - 1 \right)$$



# Summary

- Parametric resonance as a mechanism for GW production
- Two lattice simulation methods:
  based on LATTICEEASY and PSpectRe
- GW from axion monodromy inflation during Preheating

# Thank you