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New approach to the cosmological constant problem: q–theory

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Introduction

The first part of this talk (track 1) is for an audience of non-experts:

- skip technical details;
- focus on basic ideas.

The second part of this talk (track 2) gives some technical details.

But this second part is also kept as simple as possible and should be accessible to interested non-experts.

Introduction

Consider, then, the biggest problem of modern physics:

the **Cosmological Constant Problem** (CCP).

The ‘biggest’ problem for, at least, two reasons:

1. magnitude of the problem: $|\Lambda^{\text{theo}}|/|\Lambda^{\text{exp}}| \geq 10^{42}$
2. size of the problem: the Universe.

The CCP (see, e.g., [1] for a review) can be phrased as follows:

why does the zero-point energy of the vacuum not produce naturally a large cosmological constant Λ in the Einstein field equations?

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[1] S. Weinberg, RMP 61, 1 (1989).

Introduction

Indeed, it is known that QCD involves a vacuum energy density (e.g., gluon condensate or bag constant) of order

$$|\epsilon_{\text{QCD}}| \sim (100 \text{ MeV})^4 \sim 10^{32} \text{ eV}^4 \,.$$

Moreover, this energy density can be expected to change as the temperature T of the Universe drops,

$$\epsilon_{\text{QCD}} = \epsilon_{\text{QCD}}(T) \,.$$

How can it, then, be that the Universe ends up with a vacuum energy density certainly less than

$$|\epsilon_{\text{present}}| < 10^{-28} \text{ g cm}^{-3} \sim 10^{-10} \text{ eV}^4 \,?$$

Here, there are 42 orders of magnitude to explain!

Introduction

Even more CCPs after the discovery of the "accelerating Universe" :

CCP1 – why $|\Lambda| \ll (E_{\text{QCD}})^4 \ll (E_{\text{electroweak}})^4 \ll (E_{\text{UV}})^4$?

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}}\big|_{\text{present}} \sim 10^{-11}\ \text{eV}^4$?

Hundreds of interesting papers published on CCP2, but, most likely, CCP1 needs to be solved first before CCP2 can be addressed.

Here, a particular approach to CCP1 will be reviewed, which goes under the name of q–theory [2,3].

At the end, some brief (and tantalizing) remarks about CCP2.

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[2] F.R. Klinkhamer & G.E. Volovik, PRD 77, 085015 (2008), arXiv:0711.3170.
[3] F.R. Klinkhamer & G.E. Volovik, JETPL 91, 259 (2010), arXiv:0907.4887.

CCP1 – track1

Crucial insight [2]: *there is vacuum energy and vacuum energy.*

More specifically and introducing an appropriate notation:

the vacuum energy density ϵ appearing in the action
need not be the same as
the vacuum energy density ρ_V in the Einstein field equations.

How can this happen . . .

CCP1 – track1

One physical picture is to consider the full quantum vacuum as a type of **self-sustained medium** (just like a droplet of water in free fall).

That medium would be characterized by some **conserved charge**.

Then, consider macroscopic equations of this conserved microscopic variable (later called q), whose precise nature need not be known.

This quantity q is similar to the mass density in liquids, which describes a microscopic quantity – the number density of atoms – but obeys the macroscopic equations of hydrodynamics, because of particle-number conservation.

However, is the quantum vacuum just like a normal fluid?

CCP1 – track1

No, as the vacuum is known to be Lorentz invariant
(cf. experimental limits at the 10^{-15} level in the photon sector [4,5,6]).

The Lorentz invariance of the vacuum rules out the standard type of charge density which arises from the <u>time</u> component j_0 of a conserved vector current j_μ.

Needed is a new type of **relativistic conserved** charge, called the vacuum variable q.

In other words, look for a relativistic generalization (q) of the number density (n) which characterizes the known material fluids.

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[4] A. Kostelecký and M. Mewes, PRD 66, 056005 (2002), arXiv:hep-ph/0205211.

[5] F.R. Klinkhamer and M. Risse, PRD 77, 117901 (2008), arXiv:0709.2502

[6] F.R. Klinkhamer and M. Schreck, PRD 78, 085026 (2008), arXiv:0809.3217.

CCP1 – track1

With such a variable q, take the microscopic vacuum energy density

$$\epsilon = \epsilon(q)\,, \tag{1}$$

which may include a constant term due to the zero-point energies of the fields of the Standard Model (SM), $\epsilon(q) = \Lambda_{\text{bare}} + \epsilon_{\text{var}}(q)$.

From ① thermodynamics and ② Lorentz invariance, it then follows that

$$P_V \stackrel{①}{=} -\left(\epsilon - q\,\frac{d\,\epsilon}{d\,q}\right) \stackrel{②}{=} -\rho_V \neq -\epsilon\,, \tag{2}$$

with the first equality corresponding to an integrated form of the Gibbs–Duhem equation (with chemical potential $\mu \equiv d\epsilon/dq$).

Recall GD-eq: $N\,d\mu = V\,dP - S\,dT \Rightarrow dP = (N/V)\,d\mu$ for $dT = 0$.

CCP1 – track1

Both terms entering ρ_V from (2) can be of order $(E_{\text{UV}})^4$, but they can cancel exactly for an appropriate value q_0 of the vacuum variable q.

Hence. for a generic function $\epsilon(q)$,

$$\exists\, q_0 \;:\quad \Lambda \equiv \rho_V = \left[\epsilon(q) - q\, \frac{d\,\epsilon(q)}{d\,q} \right]_{q=q_0} = 0\,, \tag{3}$$

with constant vacuum variable q_0 [a similar constant variable is known to play a role for the Larkin–Pikin effect (1969) in solid-state physics].

Great, CCP1 solved in principle....

However, is a relativistic vacuum variable q possible at all?

Yes, there exist several theories which harbor such a q. $\longrightarrow$ track2

CCP1 – track1

Two important remarks [3]:

1. The adjustment-type solution (3) of the CCP1 circumvents Weinberg's no–go theorem [1].

   Crux: q is a non-fundamental scalar field. $\rightarrow$ track2

2. Next question is how the Universe got the right value q_0?
   [For the track–2 example $(q_0)^2/E_{\text{UV}}^4 = 17.84531678067157\ldots$]

   Possible answer via a generalization of q–theory,
   for which the correct value q_0 arises dynamically. $\rightarrow$ track2

CCP1 – track1

To summarize, q–theory approach to CCP1 provides a solution.

For the moment, this is only a possible solution, because it is not known for sure that the “beyond-the-Standard-Model” physics does have a q–type variable.

Still, better to have one possible solution than none.

CCP2 – track1

Now briefly, the remaining problems:

CCP2a – why $\Lambda \neq 0$?

CCP2b – why $\Lambda \sim \rho_{\text{matter}}\big|_{\text{now}} \sim 10^{-29}$ g cm^{-3} $\sim 10^{-11}$ eV^4 ?

CCP2b also goes under the name of ‘cosmic coincidence puzzle’ (ccp).

CCP2 – track1

Interesting suggestion by Arkani-Hamed, Hall, Kolda, and Murayama [7]:

In order to solve CCP2b, just two fundamental energy scales may suffice,

the electroweak scale $E_{\text{ew}} \sim 1$ TeV and
the gravitational scale $E_{\text{Planck}} \equiv (8\pi G_N)^{-1/2} \sim 10^{15}$ TeV.

But this only holds provided the effective cosmological constant (remnant vacuum energy density) has the following parametric form:

$$\Lambda \equiv \rho_{\text{V, remnant}} \sim \Big((E_{\text{ew}})^2 \,/ E_{\text{Planck}} \Big)^4 \sim (10^{-3}\ \text{eV})^4 \sim 10^{-12}\ \text{eV}^4\,. \qquad (4)$$

[7] N. Arkani-Hamed et al., PRL 85, 4434 (2000), arXiv:astro-ph/0005111.

CCP2 – track1

Remarkably, the suggested expression $\Lambda \sim (E_{\text{ew}})^8 / (E_{\text{Planck}})^4$ may result from quantum-dissipative effects in q–theory operating at a cosmic time set by the mass scale $M \sim E_{\text{ew}}$ of massive particles with electroweak interactions [8].

In a Keplerian approach, it is possible to postulate cosmological ODEs which give the correct order of magnitude for $\rho_{\text{V, remnant}}$ [9].

Combined with the observed value of Λ, a first estimate of the required value of the energy scale E_{ew} ranges from 3 to 15 TeV, depending on the number of massive particles and assuming a dissipative coupling constant of order unity.

If correct, this estimate implies the existence of new TeV–scale physics beyond the Standard Model... $\rightarrow$ track2

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[8] F.R. Klinkhamer and G.E. Volovik, PRD 80, 083001 (2009), arXiv:0905.1919.

[9] F.R. Klinkhamer, arXiv:1001.1939v6.

Conclusions

CCP1: self-adjustment of a particular type of vacuum variable q can give $\rho_V(q_0) = 0$ in the equilibrium state $q = q_0$;

CCP2: finite remnant value of $\rho_V(t)$ may result from quantum-dissipative effects operating at a cosmic time t_{ew} set by the scale $E_{\text{ew}} \sim$ TeV of massive particles with electroweak interactions;

Hint: required E_{ew} value ranges from 3 to 15 TeV, which, if correct, implies new TeV–scale physics beyond the SM.

after the break $\rightarrow$ track2

CCP1 – track1 – repeat

Two important remarks [3]:

1. The adjustment-type solution (3) of the CCP1 circumvents Weinberg's no–go theorem [1].

   Crux: q is a non-fundamental scalar field.

2. ...

CCP1 – track2

Explicit realization of vacuum variable q via a 3–form gauge field A [10,11].

Effective action of GR+SM,

$$S^{\text{eff}}[g,\psi] = -\int_{\mathbb{R}^4} d^4x\,\sqrt{-\det g}\,\Big(K_{\text{N}}\,R[g] + \Lambda_{SM} + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big)\,, \quad (5)$$

with $K_{\text{N}} \equiv 1/(16\pi G_{\text{N}})$ and $\hbar = c = 1$, is replaced by

$$\widetilde{S}^{\text{eff}}[A,g,\psi] \;=\; -\int_{\mathbb{R}^4} d^4x\,\sqrt{-\det g}\,\Big(K_{\text{N}}\,R[g] + \widetilde{\epsilon}(q) + \mathcal{L}^{\text{eff}}_{\text{SM}}[\psi,g]\Big)\,, \quad (6a)$$

$$F_{\alpha\beta\gamma\delta} \;=\; \nabla_{[\alpha}A_{\beta\gamma\delta]} \equiv q\,\epsilon_{\alpha\beta\gamma\delta}\,\sqrt{-\det g}\,, \quad (6b)$$

$$q^2 \;\equiv\; -\frac{1}{24}\,F_{\alpha\beta\gamma\delta}\,F^{\alpha\beta\gamma\delta}\,. \quad (6c)$$

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[10] M.J. Duff and P. van Nieuwenhuizen, PLB 94, 179 (1980).

[11] A. Aurilia, H. Nicolai, and P.K. Townsend, NPB 176, 509 (1980).

CCP1 – track2

Then, variational principle (without any appeal to thermodynamics) gives the Einstein equations with a vacuum energy density term

$$\rho_V = \widetilde{\epsilon} - q \, \frac{d\,\widetilde{\epsilon}}{d\,q} \,, \tag{7}$$

which is precisely of the form argued before, cf. (2).

Concrete example:

$$\widetilde{\epsilon}(q) = \Lambda_{\text{bare}} + \tfrac{1}{2}\,(E_{\text{UV}})^4 \, \sin\left[\, q^2/(E_{\text{UV}})^4 \,\right] , \tag{8a}$$

$$\Lambda_{\text{bare}} \equiv \Lambda_{\text{SM}} + \Lambda_{\text{UV}} \,, \tag{8b}$$

with $|\Lambda_{\text{SM}}| \sim (E_{\text{ew}})^4$ and $|\Lambda_{\text{UV}}| \sim (E_{\text{UV}})^4$.

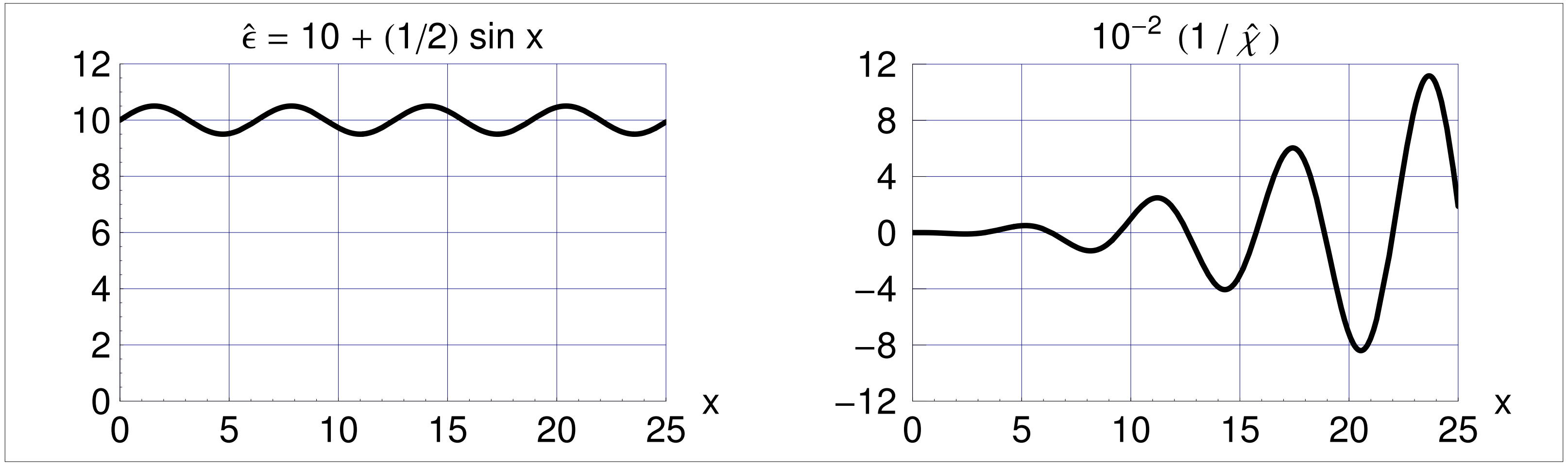
CCP1 – track2

Defining $\widehat{q} \equiv q/(E_{\text{UV}})^2$ and $\lambda \equiv \Lambda_{\text{bare}}/(E_{\text{UV}})^4$, our example has dimensionless vacuum energy density and compressibility

$$\widehat{\epsilon} \equiv \widetilde{\epsilon}(q)/(E_{\text{UV}})^4 = \lambda + \tfrac{1}{2} \sin \widehat{q}^{\,2} , \quad (9a)$$

$$\widehat{\chi} = 1/\left[\widehat{q}^{\,2} \cos \widehat{q}^{\,2} - 2\,\widehat{q}^{\,4} \sin \widehat{q}^{\,2}\right] . \quad (9b)$$

With $\lambda = 10$ and $x \equiv \widehat{q}^{\,2}$, have:

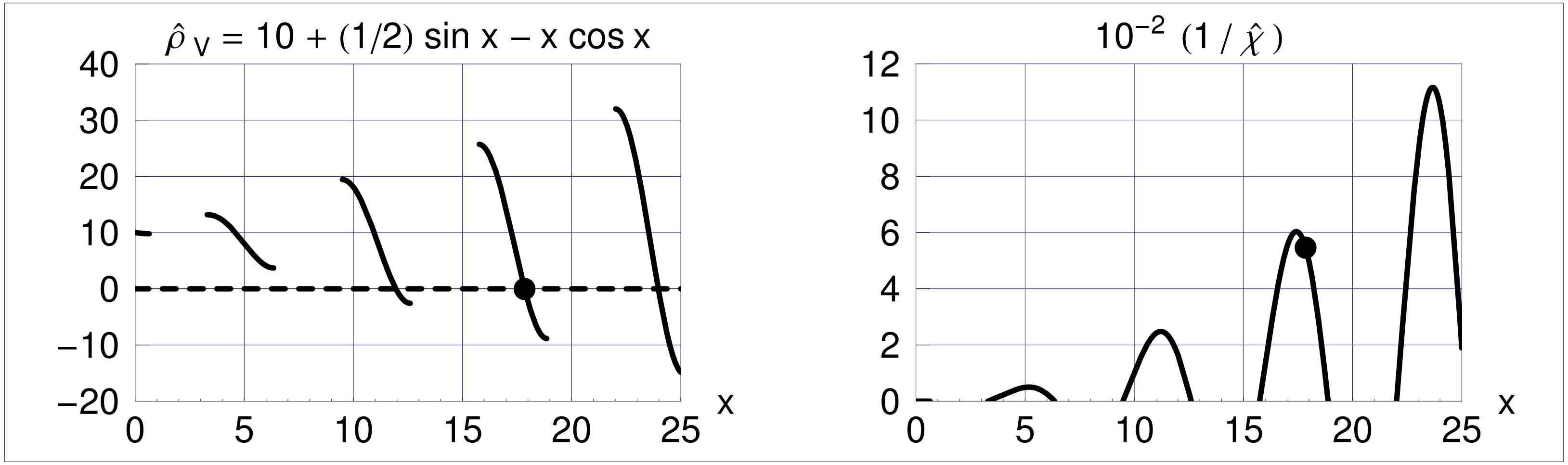


CCP1 – track2

Equilibrium and stability conditions (with $\lambda = 10$ for the example):

$$\widehat{\rho}_V(x) \equiv \widehat{\epsilon} - 2\,x\;d\widehat{\epsilon}/d\,x = \lambda + \tfrac{1}{2}\,\sin x - x\,\cos x = 0\,, \qquad \text{(10a)}$$

$$\widehat{\chi}^{\,-1}(x) = x\cos x - 2\,x^2\sin x \quad > 0\,, \qquad \text{(10b)}$$



$\Rightarrow$ One possible Minkowski vacuum with $(\widehat{q}_0)^2 \approx 17.8$ and $\widehat{\chi}_0 \approx 1/547$.

CCP1 – track1 – repeat

Two important remarks [3]:

1. . . .

2. Next question is how the Universe got the right value q_0?
   [For the track–2 example $(q_0)^2/E_{\text{UV}}^4 = 17.84531678067157\ldots$]

   Possible answer via a generalization of q–theory, for which the correct value q_0 arises dynamically.

CCP1 – track2

Realization of vacuum variable q via an aether-type velocity field u_β [12,13], setting $E_{\text{UV}} = E_{\text{Planck}}$. For a flat FRW metric with cosmic time t, there is an asymptotic solution for $u_\beta = (u_0, u_b)$ and Hubble parameter $H(t)$:

$$u_0(t) \to q_0\, t\,, \quad u_b(t) = 0\,, \quad H(t) \to 1/t\,. \tag{11}$$

Define $v \equiv u_0/E_{\text{Planck}}$, $\tau \equiv t\, E_{\text{Planck}}$, $h \equiv H/E_{\text{Planck}}$, and $\lambda \equiv \Lambda/(E_{\text{Planck}})^4$. Then, the field equations are [12]:

$$\ddot{v} + 3\, h\, \dot{v} - 3\, h^2\, v = 0\,, \tag{12a}$$

$$2\,\lambda - (\dot{v})^2 - 3\,(h\, v)^2 = 6\, h^2\,, \tag{12b}$$

with the overdot standing for differentiation with respect to τ.

Starting from a de-Sitter universe with $\lambda > 0$, there is a unique value of $\widehat{q}_0 \equiv q_0/(E_{\text{Planck}})^2$ to end up with a static Minkowski spacetime, $\widehat{q}_0 = \sqrt{\lambda/2}$.

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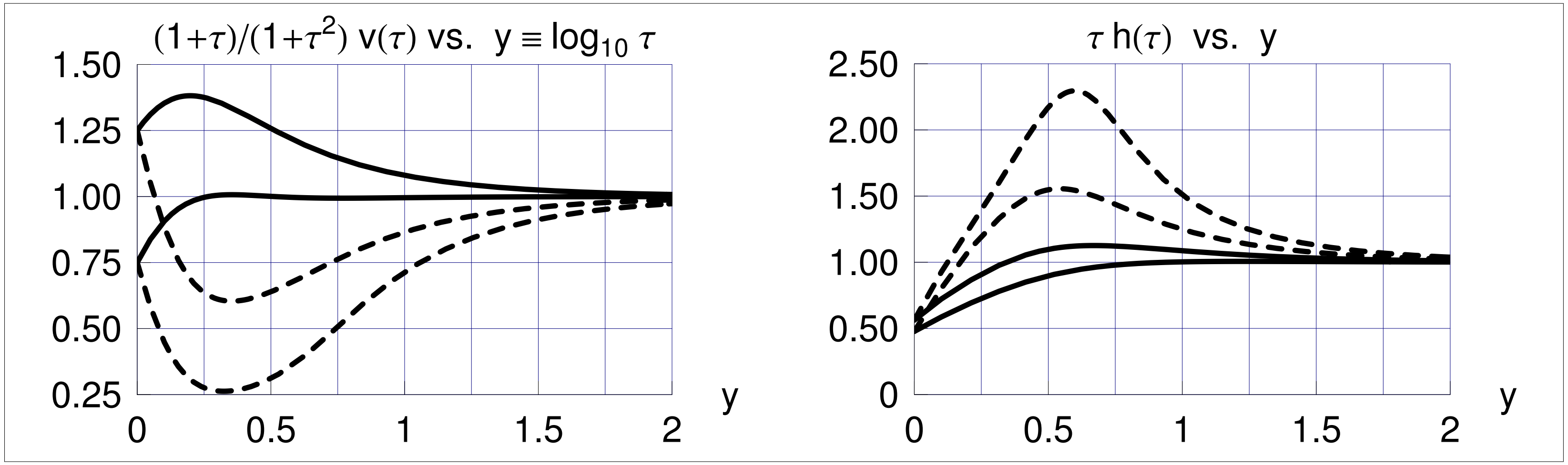
[12] A.D. Dolgov, PRD 55, 5881 (1997), arXiv:astro-ph/9608175.

[13] T. Jacobson, PoS QG-PH, 020 (2007), arXiv:0801.1547.

CCP1 – track2

Numerical solutions of ODEs (12ab) for $\lambda = 2$ and boundary conditions $v(1) = 1 \pm 0.25$ and $\dot{v}(1) = \pm 1.25$, with dashed curves for negative $\dot{v}(1)$:



$\Rightarrow$ required Minkowski value $\widehat{q}_0 = \sqrt{\lambda/2} = 1$ arises dynamically [see left panel].

$\Rightarrow$ Minkowski spacetime is an attractor.

CCP2 – track1 – repeat

Remarkably, the suggested expression $\Lambda \sim (E_{\text{ew}})^8 / (E_{\text{Planck}})^4$ may result from quantum-dissipative effects in q–theory operating at a cosmic time set by the mass scale $M \sim E_{\text{ew}}$ of massive particles with electroweak interactions [8].

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[8] F.R. Klinkhamer and G.E. Volovik, PRD 80, 083001 (2009), arXiv:0905.1919.

[9] F.R. Klinkhamer, arXiv:1001.1939v6.

CCP2 – track2

Mechanism of electroweak "kick" in standard q–theory and basic structure of modified ODEs explained in another talk (Waseda U). Here, focus on the physics implications.

Theoretical value of the effective cosmological constant given by

$$\Lambda^{\text{theory}} \equiv \lim_{t\to\infty} \rho_V^{\text{theory}}(t) = r_V^{\text{num}} \left(E_{\text{ew}}\right)^8 / \left(E_{\text{Planck}}\right)^4 , \tag{13}$$

with r_V^{num} a number obtained by numerically solving the relevant ODEs. Equating this to the experimental value $\Lambda^{\text{exp}} \approx (2\ \text{meV})^4$ gives

$$E_{\text{ew}} = \left(\frac{\Lambda^{\text{exp}}}{r_V^{\text{num}}}\right)^{1/8} \left(E_{\text{Planck}}\right)^{1/2} \approx 3.8\ \text{TeV} \left(\frac{0.013}{r_V^{\text{num}}}\right)^{1/8} . \tag{14}$$

Analytic bound: $r_V^{\text{num}} \lesssim 1 \Rightarrow E_{\text{ew}} \gtrsim 2$ TeV.
Numerical results for r_V^{num} give the E_{ew} estimates in the Table.

CCP2 – track2

Preliminary estimates [7] of the energy scale E_{ew} from a simple model. Both massive type–1 and massless type–2 particles are assumed to have been in thermal equilibrium before the "kick" and the effective number of type–2 particles is taken as $N_{\text{eff},2} = 10^2$.
Left: artificial kick with type–1 particles of equal mass $M = E_{\text{ew}}$ and, for fixed coupling constant $\zeta = 2$, E_{ew} shown as a function of the number of degrees of freedom $N_{\text{eff},1}$.
Right: dynamic kick with case–A type–1 mass spectrum $(N_{1a}, M_{1a}; N_{1b}, M_{1b}) = (40, 2 \times E_{\text{ew}}; 60, 1/3 \times E_{\text{ew}})$ and $E_{\text{ew}} = \langle M_{1i} \rangle$ shown as a function of ζ.

$N_{\text{eff},1}$	E_{ew} [TeV]
1	8.5
10^1	4.9
10^2	$\boxed{3.2}$
10^3	2.8
10^4	2.7

ζ	E_{ew} [TeV]
0.2	14.8
2	$\boxed{3.8}$
20	5.6

Conclusions – repeat

CCP1: self-adjustment of a particular type of vacuum variable q can give $\rho_V(q_0) = 0$ in the equilibrium state $q = q_0$;

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