Linearizing a Deep Learning Model for Interpretable Classification of Two-Prong Jets

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Machine Learning Seminar, IPMU
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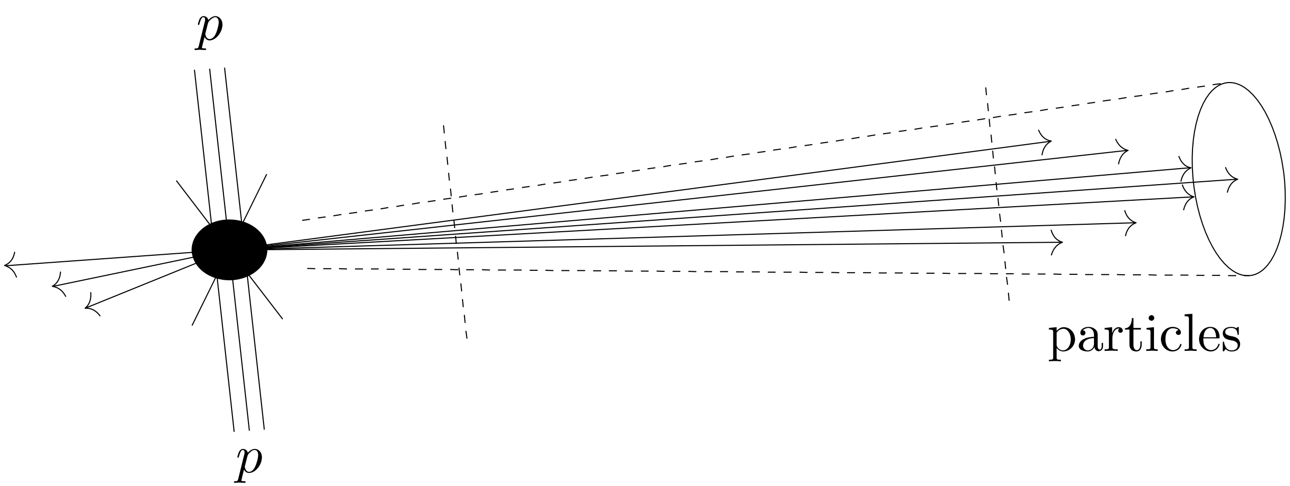
based on **S. H. Lim**, M. M. Nojiri, arXiv:1807.03312, JHEP10(2018)181.
A. Chakraborty, **S. H. Lim**, M. M. Nojiri, arXiv:1904.02092.

Preface

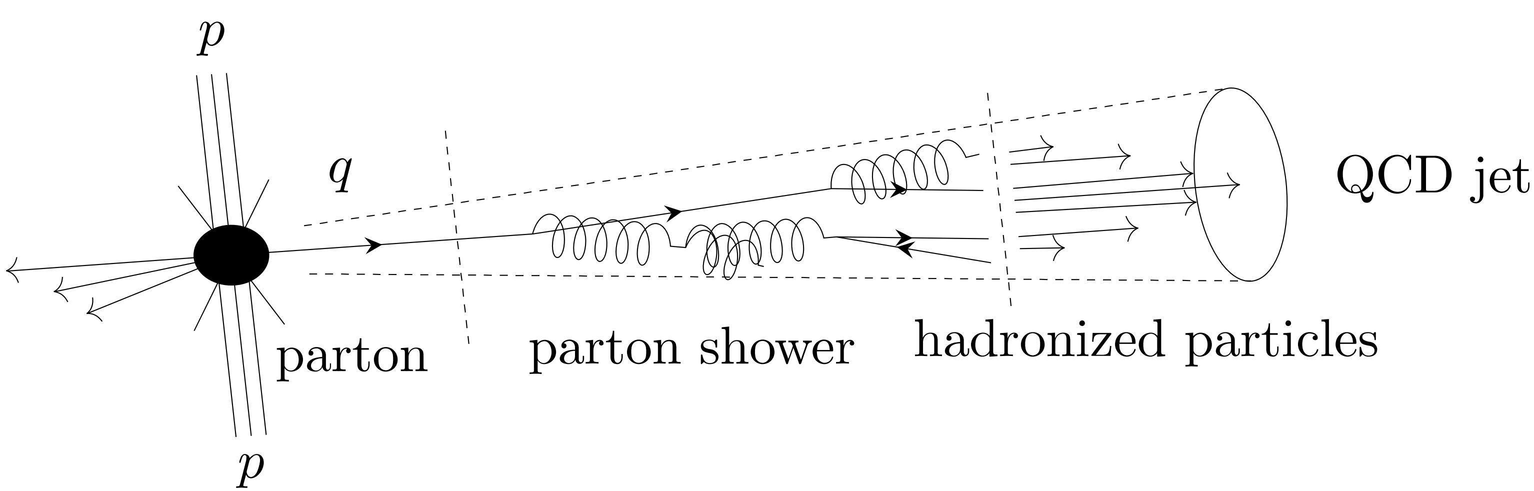
- This is not a full theoretical introduction, so this slide may contain lots of sloppy logical jumps.
- Feel free to interrupt me anytime if you don’t understand or have any questions.
- This is a talk focused on machine learning aspect of our work. For physics introduction, please refer Nojiri-san’s talk in last Johns-Hopkins workshop.

Jet?

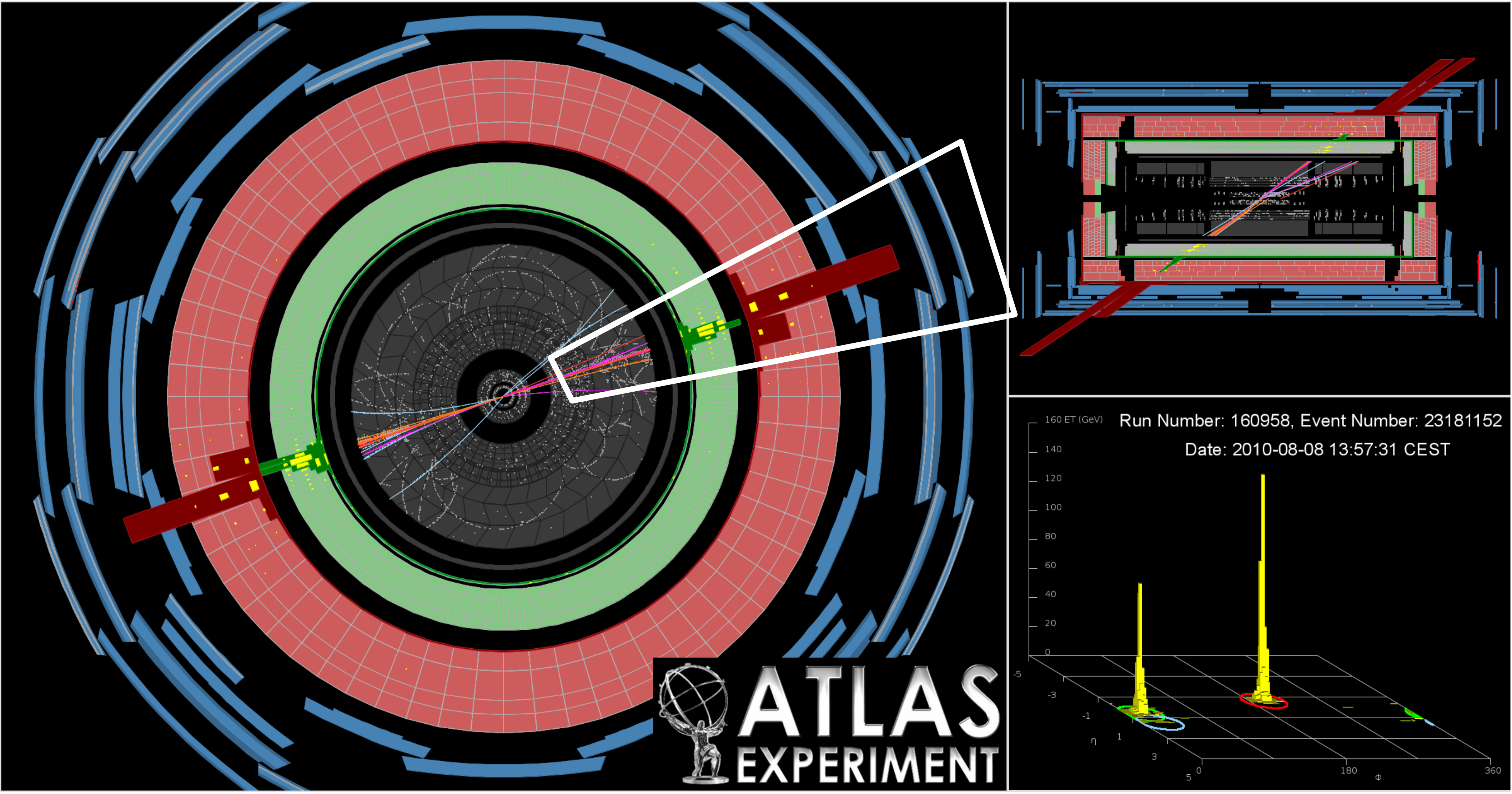
- At a hadron-hadron collider, such as LHC, a collimated particle cluster called a jet often appears.



- Such cluster can be interpreted as particles produced after hadronization of colored parton due to color confinement.



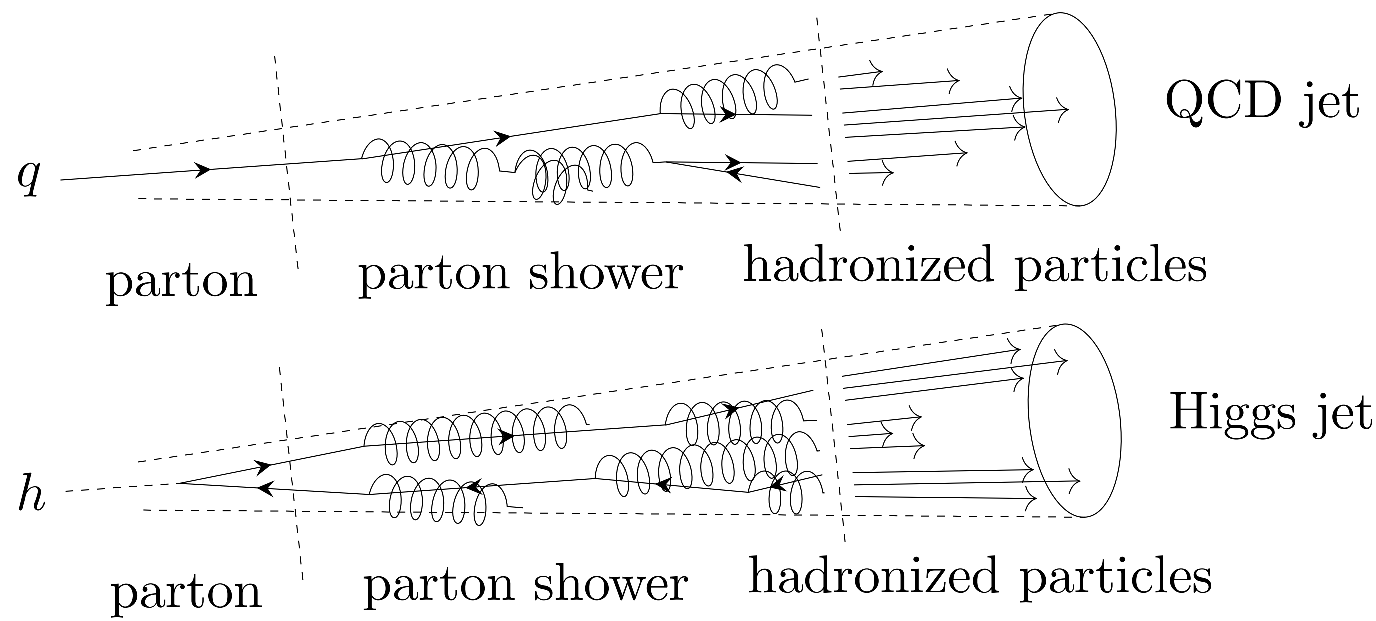
Jet?



http://cds.cern.ch/record/1697057/files

Boosted Jets: Jets have substructure!

- As LHC stacking up multi TeV center-of-mass energy events, boosted heavy particles is produced and forms a single collimated cluster of particles similar to the QCD jets. $(\, m_{\rm EW}/\sqrt{\hat{s}} \approx \mathcal{O}[0.1] \,)$

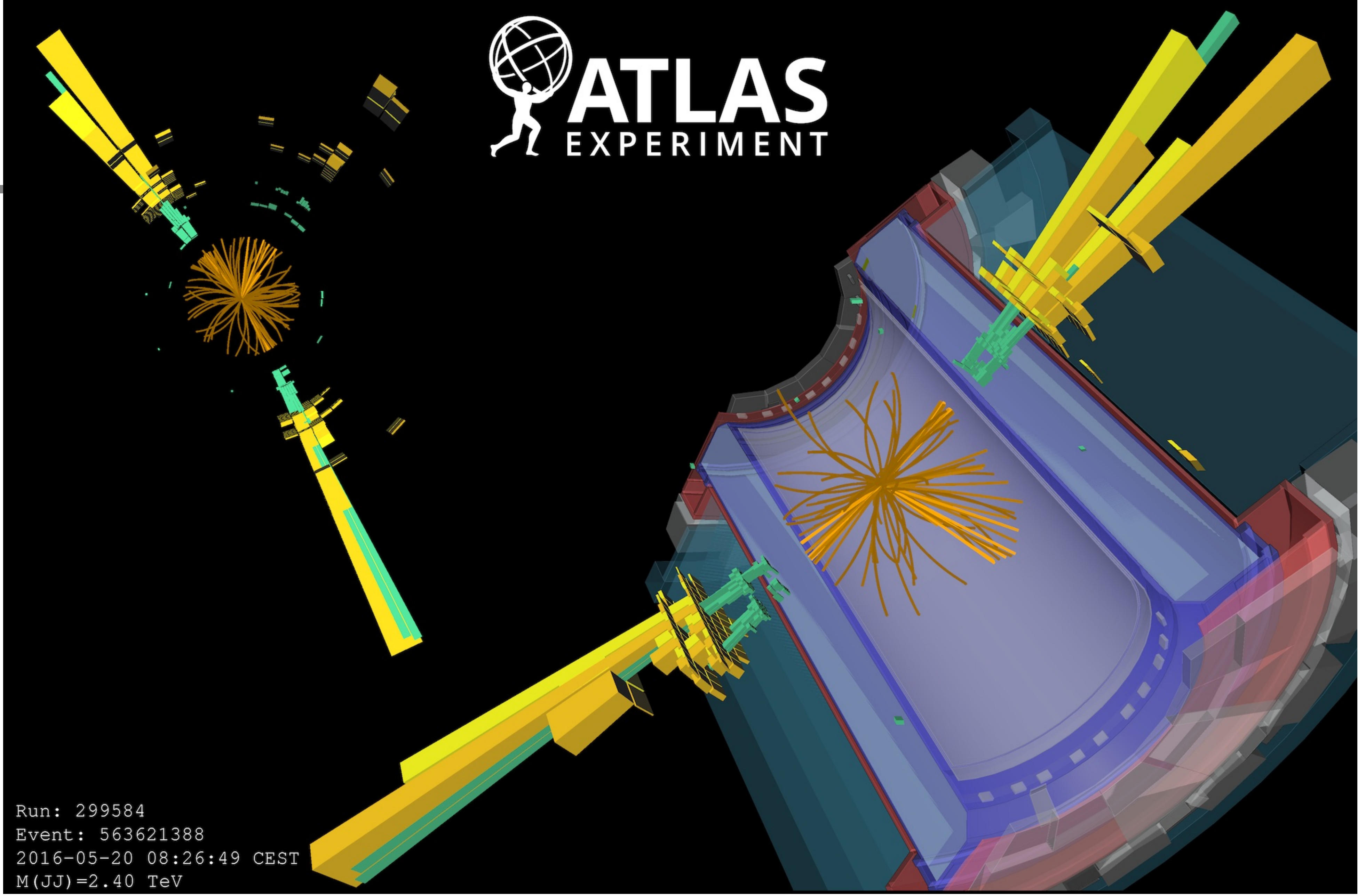


- Angular scale of the relativistic beaming of Higgs jet decay:

$$R_{b\bar{b}} \geq \frac{2m_h}{p_{T,h}} \qquad \begin{aligned} p_{T,h} &= 300\,\mathrm{GeV} \rightarrow 0.83 \\ p_{T,h} &= 500\,\mathrm{GeV} \rightarrow 0.5 \end{aligned}$$

- Typical jet radius used in various analyses

$$R_{\rm jet} = \begin{cases} 0.4 & \text{narrow jet analysis} \\ \sim 1.0 & \text{fat jet analysis} \end{cases}$$



https://atlas.cern/updates/physics-briefing/hunting-new-physics-boosted-bosons

- We have to differentiate these non-QCD jets from QCD jets to maximize sensitivity of channels involving boosted particles.

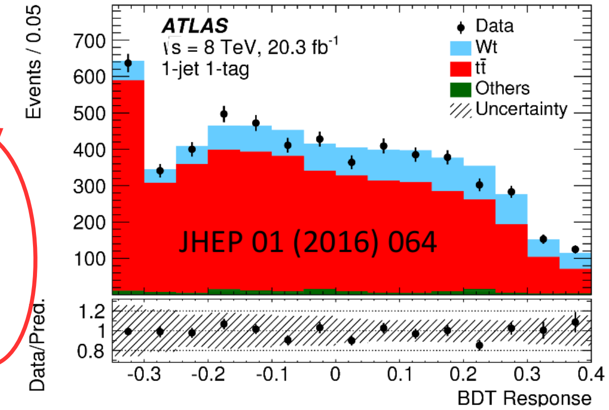
Slide from M. Kagan (SLAC), https://indico.cern.ch/event/619370/

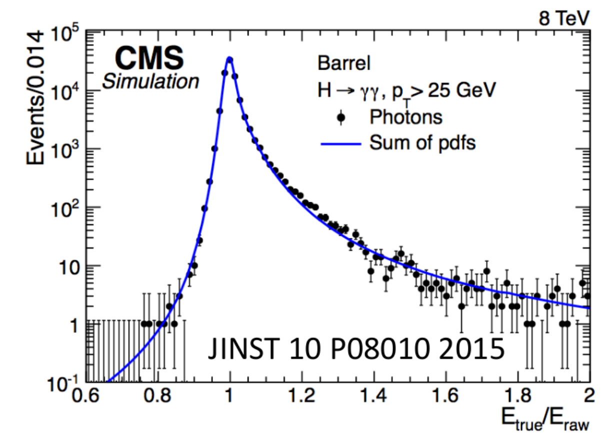
Machine Learning Applied Widely in HEP

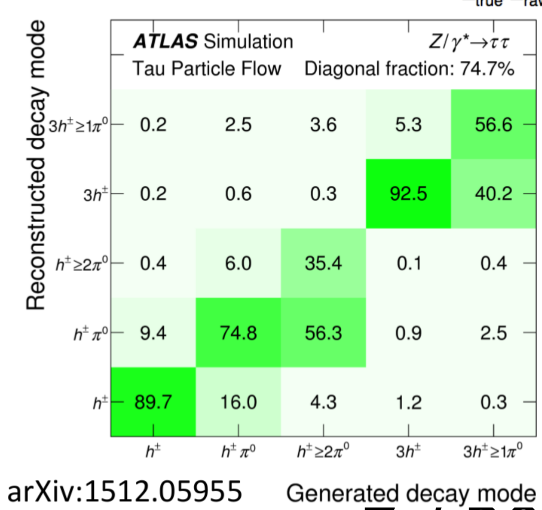
- **In analysis**:
  - Classifying signal from background, especially in complex final states
  - Reconstructing heavy particles and improving the energy / mass resolution
  - …

Today's topic!

- **In reconstruction**:
  - Improving detector level inputs to reconstruction
  - Particle identification tasks
  - Energy / direction calibration
  - …
- **In the trigger**:
  - Quickly identifying complex final states
  - …
- **In computing**:
  - Estimating dataset popularity, and determining how number and location of dataset replicas
  - …







arXiv:1512.05955

Classification Problem

Can you distinguish <u>cats</u> and <u>dogs</u> from **photos**?

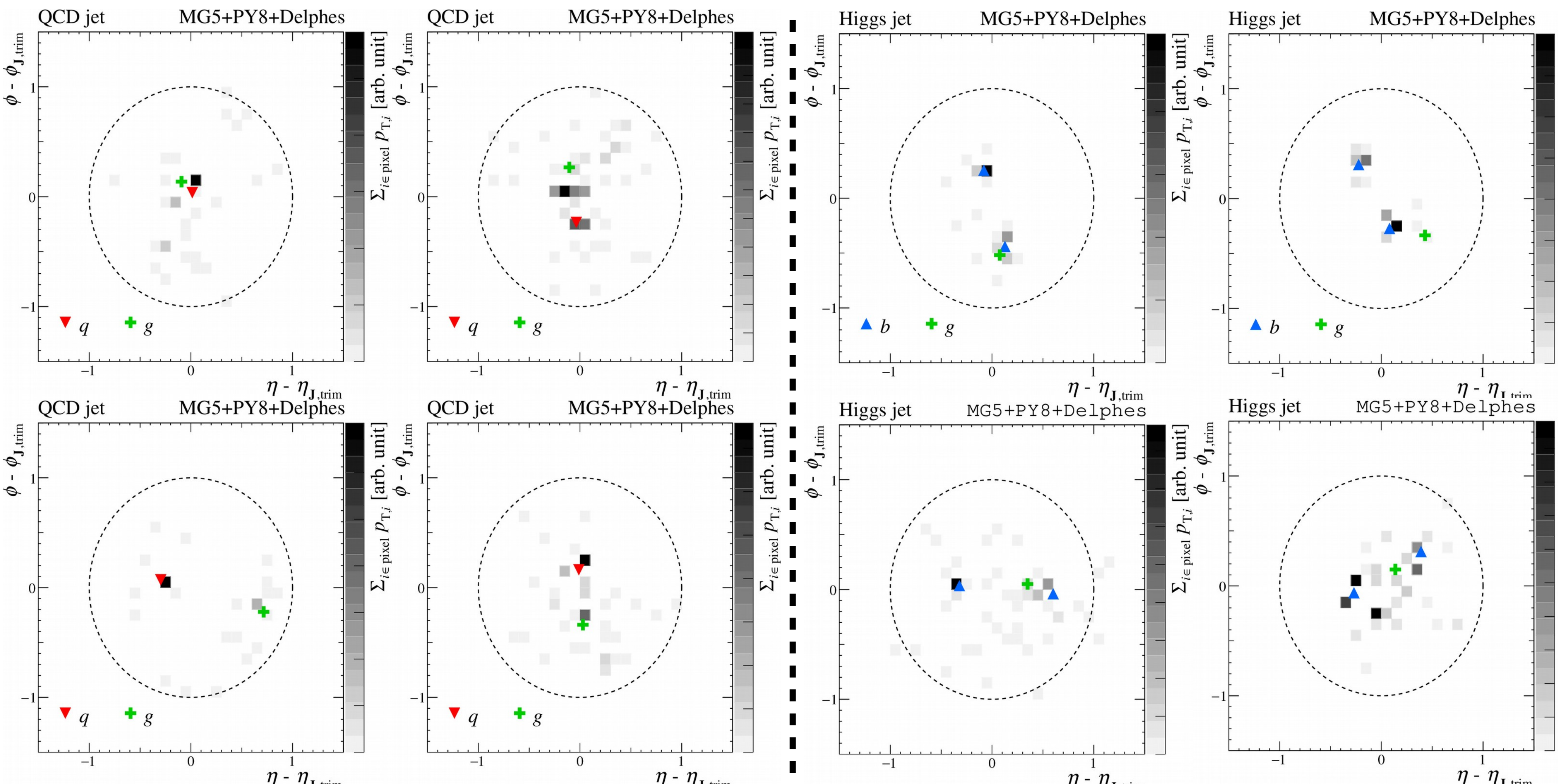
<u>Cats (?)</u>

<u>Dogs (?)</u>

Check Fabian's slides for the review of classification with deep learning!

Classification Problem

Can you distinguish QCD jets and Higgs jets from **reconstructed particles**?



QCD jets

Higgs jets

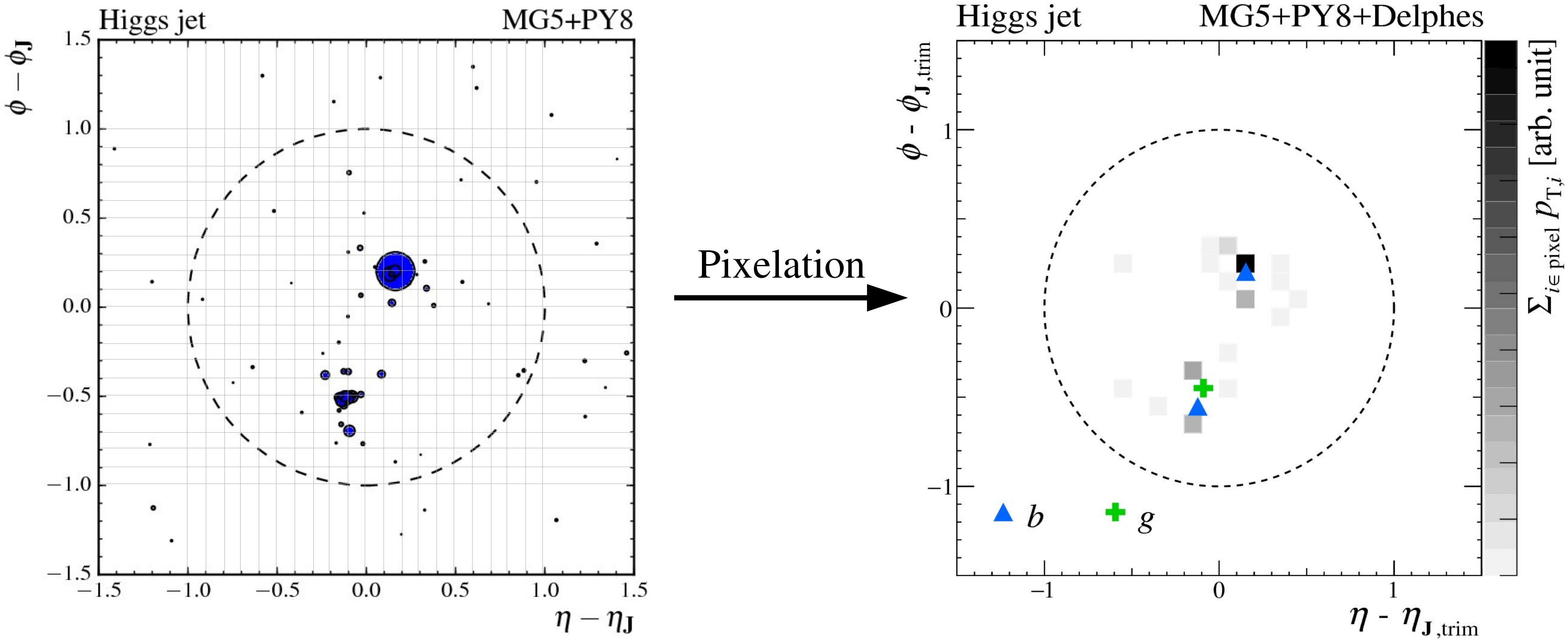
Check Fabian's slides for the review of classification with deep learning!

Jet Image

⋆ jet image $\neq$ calorimeter image

1407.5675

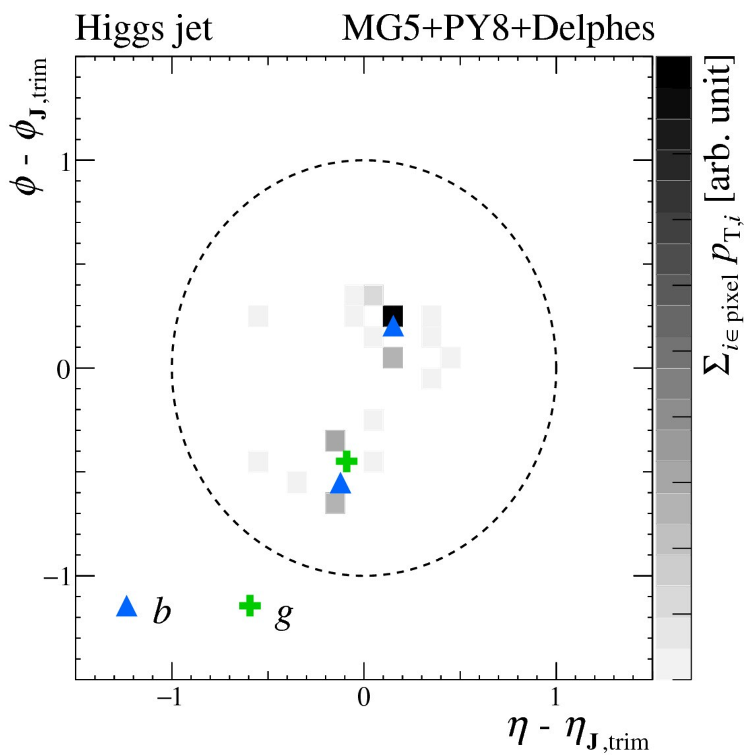
An artificial neural network can handle many data, but we have to process the raw data suitable for the given NN architecture.
For example, CNN requires an image on a rectangular grid:

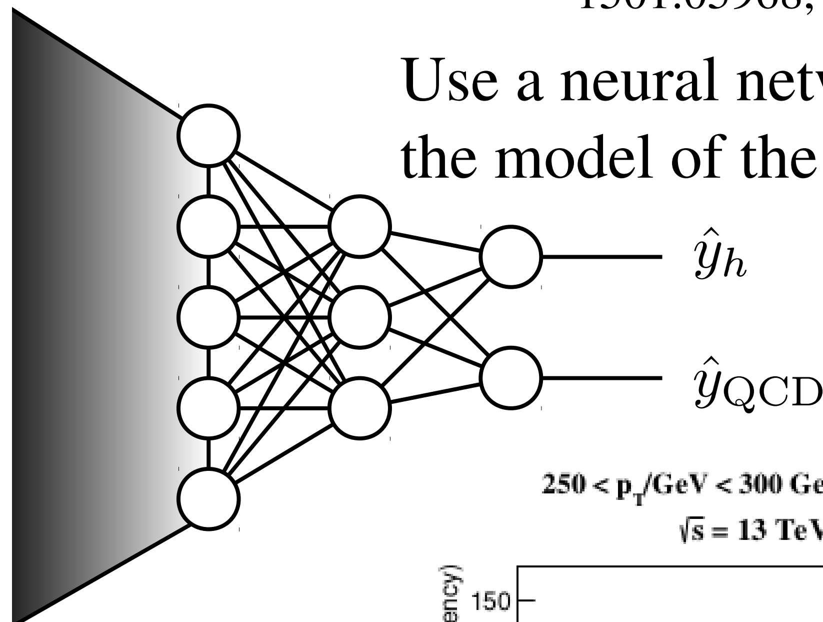


$$\text{Energy deposit on pixel} = \sum_{i\in\text{pixel}} p_{T,i}$$

Jet imaging is motivated for
using image analysis techniques in jet substructure analysis.

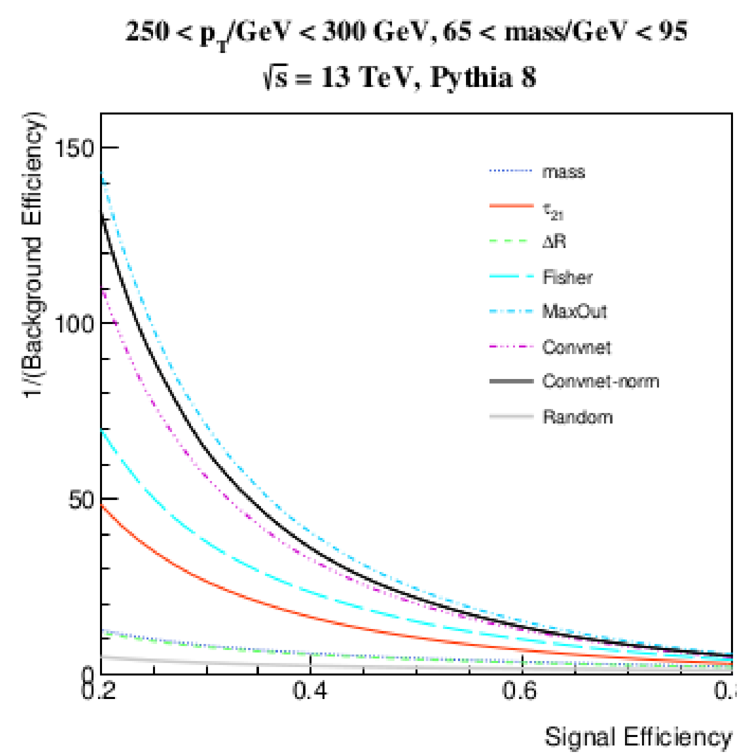
Classification with Jet Images





1501.05968, 1511.05190

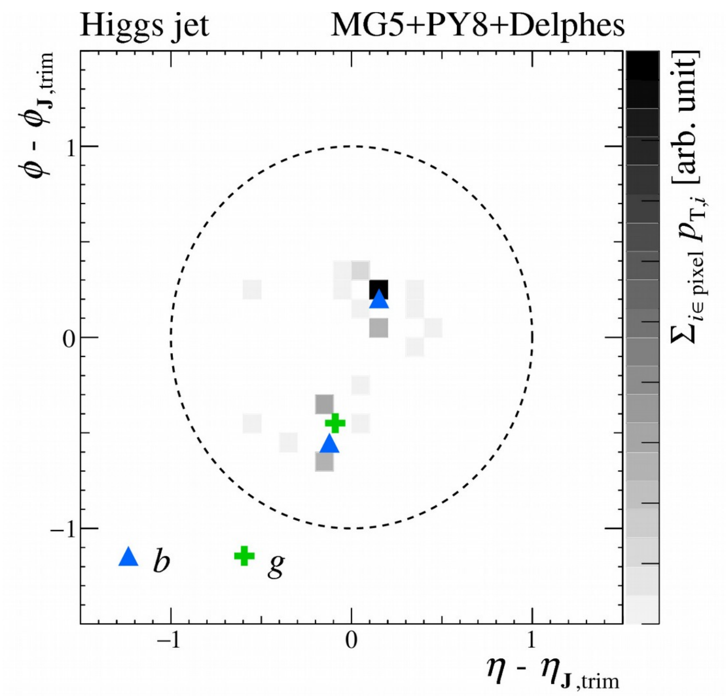
Use a neural network as the model of the classifier

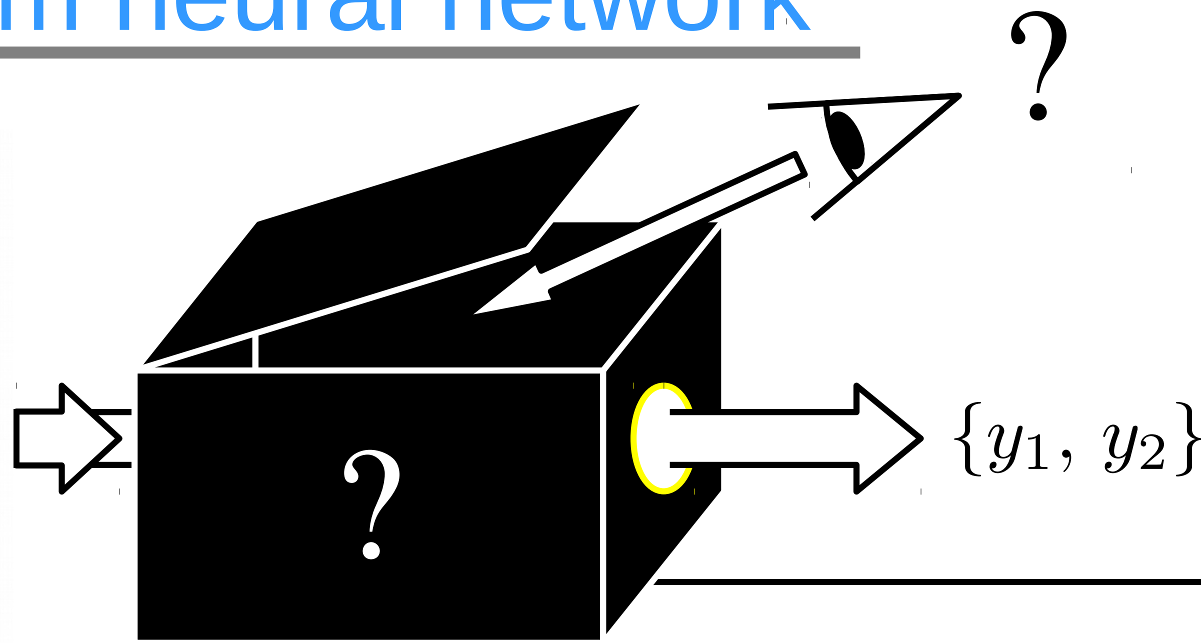


Deep learning is more powerful than a few physics motivated classifiers!

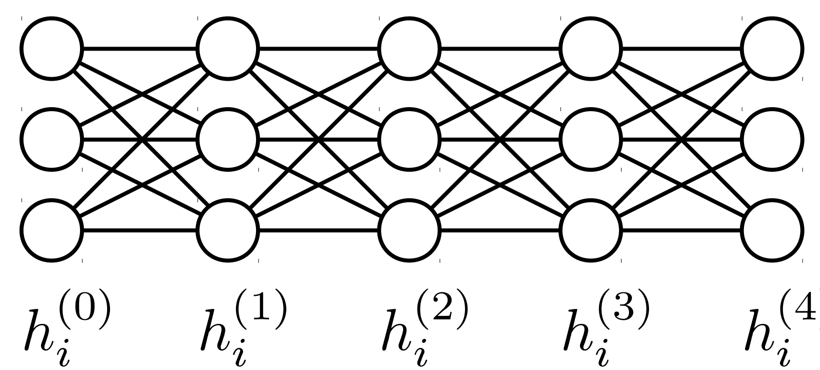
Disclaimer: physics motivated classifiers use particular features only.

Difficulties on understanding the results from neural network





Neural network is often considered as a **black box** because studying its internal information barely gives you an insight about the decision...



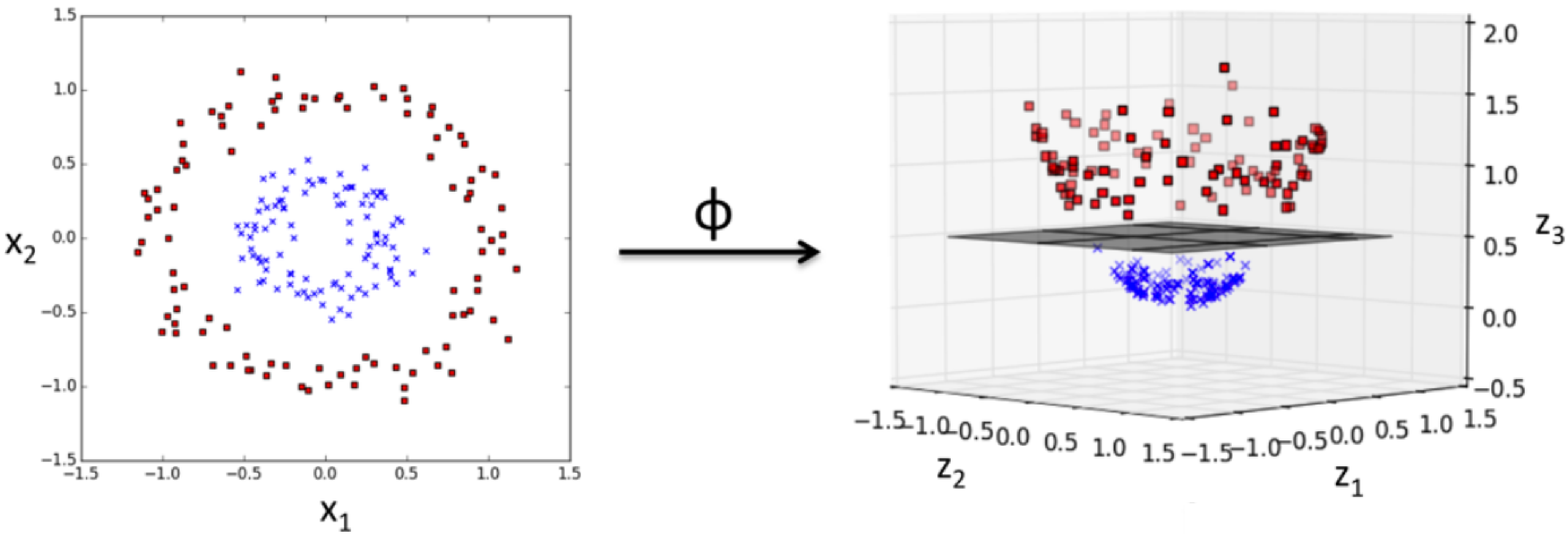
$$h_i^{(n)} = \varphi(w_{ij}^{(n)} h_j^{(n-1)} + b_i^{(n)})$$

We need some special treatment, and we already studied one **trick**...

Slide from Ting-Wen's talk

The Kernel Trick

$$\vec{h} = \Phi(x_1, x_2) = (z_1, z_2, z_3) = (x_1, x_2, x_1^2 + x_2^2)$$



Non-linear classification

<u>Linear</u> classification

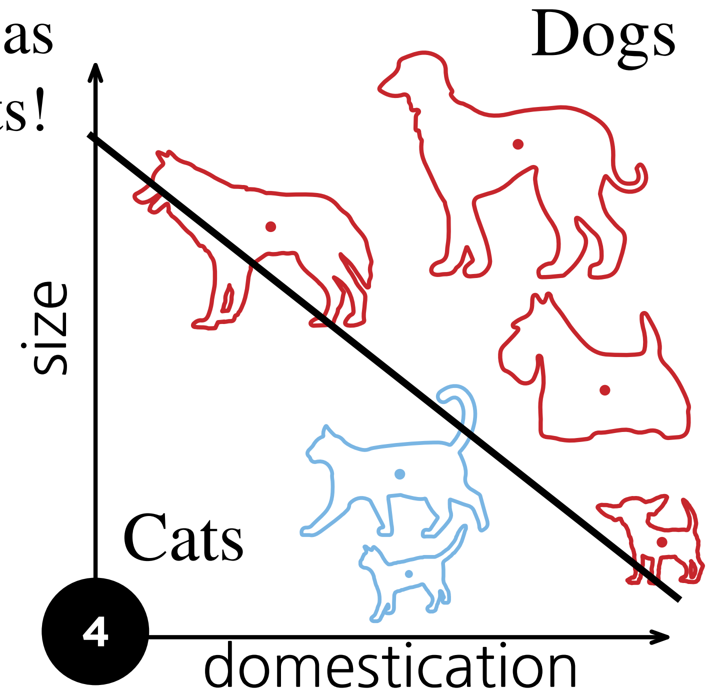
$$\vec{w} = (0, 0, 1)$$

https://github.com/rasbt/python-machine-learning-book/blob/master/code/ch03/ch03.ipynb

Why linear classification is good?

In a linear classifier, the weights can be interpreted as a measure of importance of the corresponding inputs!

$$h = \sum_i w_i \, x_i$$



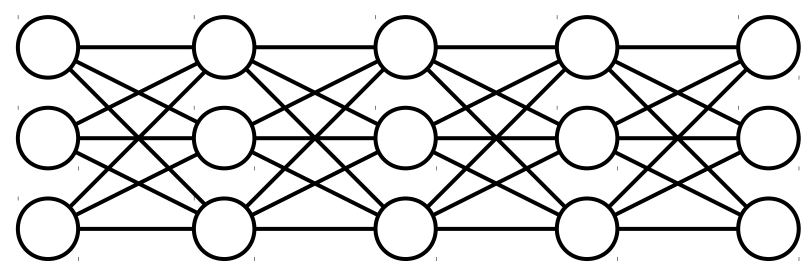
The normal vector of the hyperplane is the weights.

$$\vec{w} = (1, 1)$$

Kernel trick transforms the problem into linearly separable problem!

ϕ_k is a problem-specific feature map.

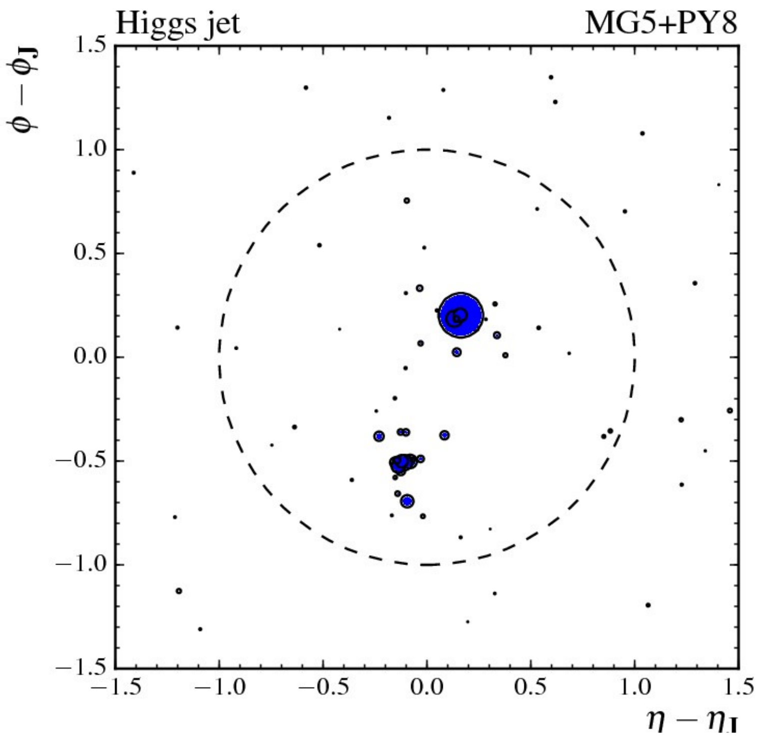
$$\tilde{f}(\{x_i\}) = \sum_k w_k \phi_k(\{x_i\}) \approx$$



To find out a good feature map for the kernel trick, we need a formal description of our model first.

Energy Flow

The energy flow is a representation of set of particles,
which is a distribution of energy on rapidity-azimuth plane.



$$P_T(\vec{R}) = \sum_{i \in \mathbf{J}} p_{T,i}\, \delta(\vec{R} - \vec{R}_i)$$

$$\vec{R} = (y, \phi)$$

The CNN trained on jet images can be considered as a discrete realization of a classifier functional $\Phi[P_T]$.

Output of convolutional layer:

$$(w * P_T)(\vec{R}) = \int d\vec{R}' w(\vec{R}') P_T(\vec{R} - \vec{R}')$$

$$(\mathrm{CONV} \circ P_T)(\vec{R}) = \varphi(w * P_T(\vec{R}) + b)$$

Functional Taylor Expansion

One easy method for the linearization is the Taylor expansion.

$$\Phi[P_T] = w^{(0)} + \int d\vec{R}\; P_T(\vec{R}) w^{(1)}(\vec{R}) + \frac{1}{2!} \int d\vec{R}_1 d\vec{R}_2\; P_T(\vec{R}_1) P_T(\vec{R}_2) w^{(2)}(\vec{R}_1, \vec{R}_2) + \cdots$$

Since $\Phi[P_T]$ is a scalar and doesn't have any additional parameters, the first nontrivial term is

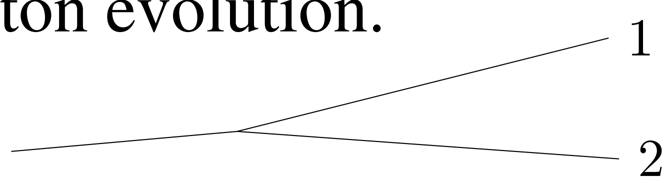
$$\Phi[P_T] = \int dR\; S_2(R) w^{(2)}(R) + \cdots$$

$$S_2(R) = \int d\vec{R}_1\, d\vec{R}_2\; P_T(\vec{R}_1) P_T(\vec{R}_2) \delta(R - R_{12})$$

This new variable looks okay,
but is it linearly separating Higgs jets from QCD jets?

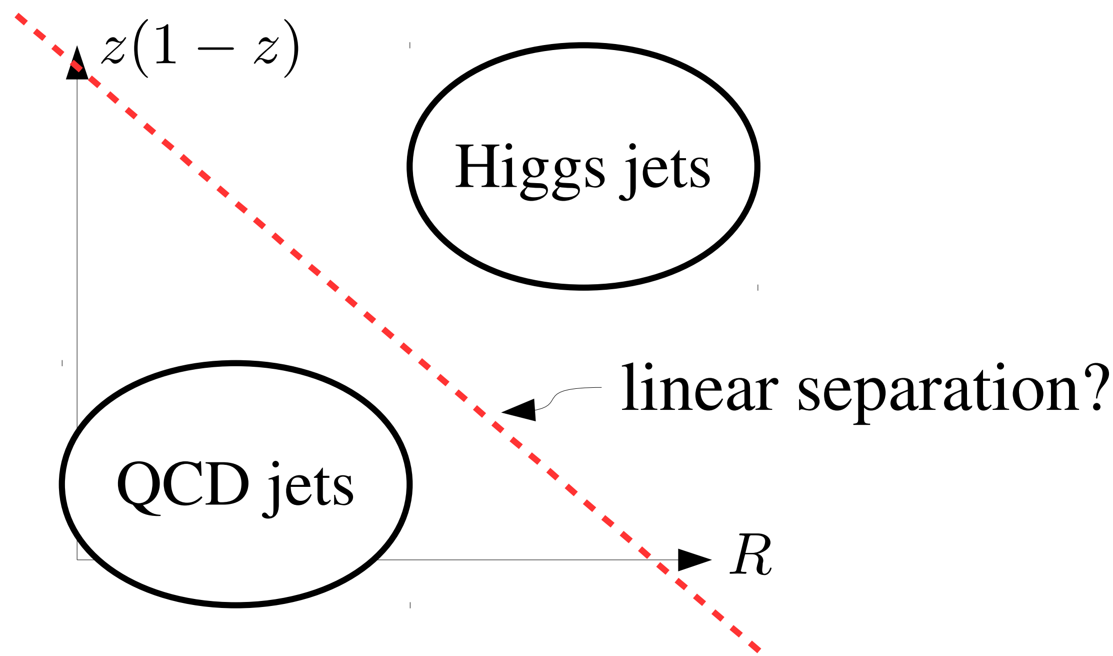
Kinematics inside Jet

The parameter set (p_T, z, R) determines the kinematics of parton evolution.

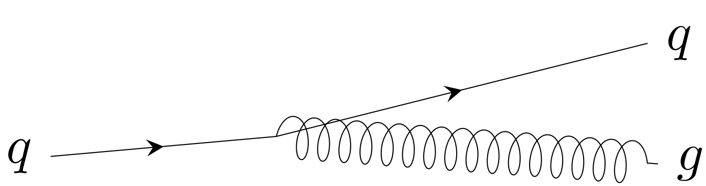


$$p_{T,1} = p_{T,0} z \qquad p_{T,2} = p_{T,0}(1-z)$$

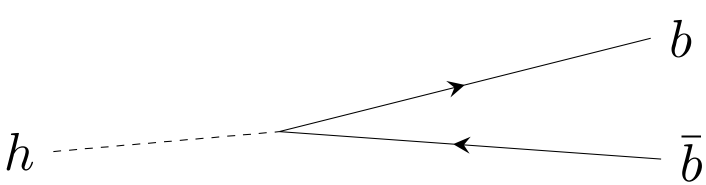
$$R^2 = (y_1 - y_2)^2 + (\phi_1 - \phi_2)^2$$



- parton shower



  - Collinear: small R
  - Soft: z ~ 0 or 1
- particle decay



$$S_2(R) = \int d\vec{R}_1\, d\vec{R}_2\, P_T(\vec{R}_1) P_T(\vec{R}_2) \delta(R - R_{12})$$

$$\ni z(1-z)\delta(R - R_{12})$$

- Transverse decay:
  R ~ 2m/pt and z ~ 0.5

Two-Point Correlation Spectrum

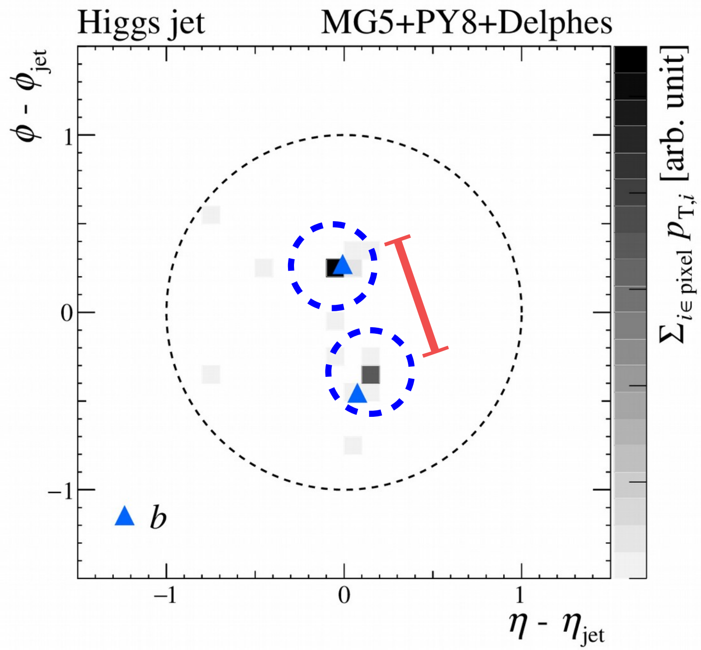
We introduce an IRC safe (binned) spectral function:

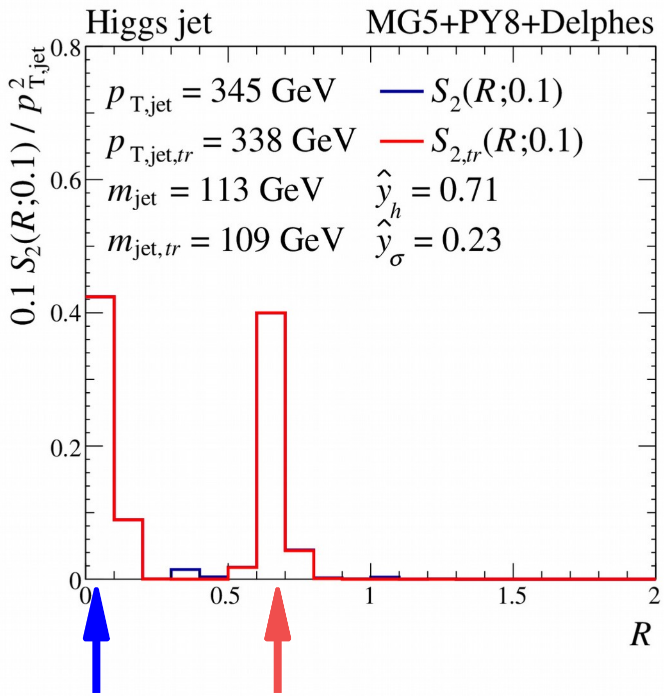
$$S_2(R;\Delta R) = \frac{1}{\Delta R} \sum_{\substack{i,j \in \text{jet} \\ R_{ij} \in [R, R+\Delta R]}} p_{T,i} p_{T,j}$$

Example: spectrum of a two-prong jet with two constituents

$$S_2(R) = (p_{T,b}^2 + p_{T,\bar{b}}^2) \cdot \delta(R) + 2 p_{T,b} p_{T,\bar{b}} \cdot \delta(R - R_{b\bar{b}})$$

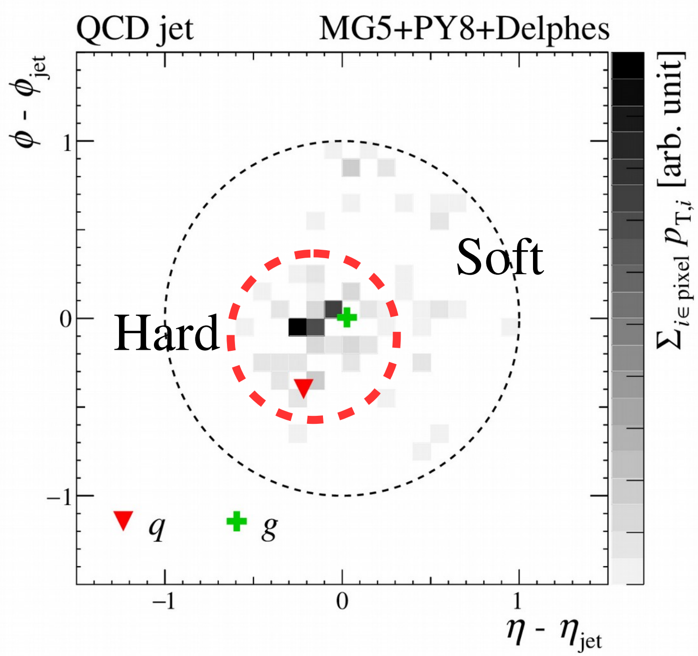
Autocorrelation Cross-Correlation

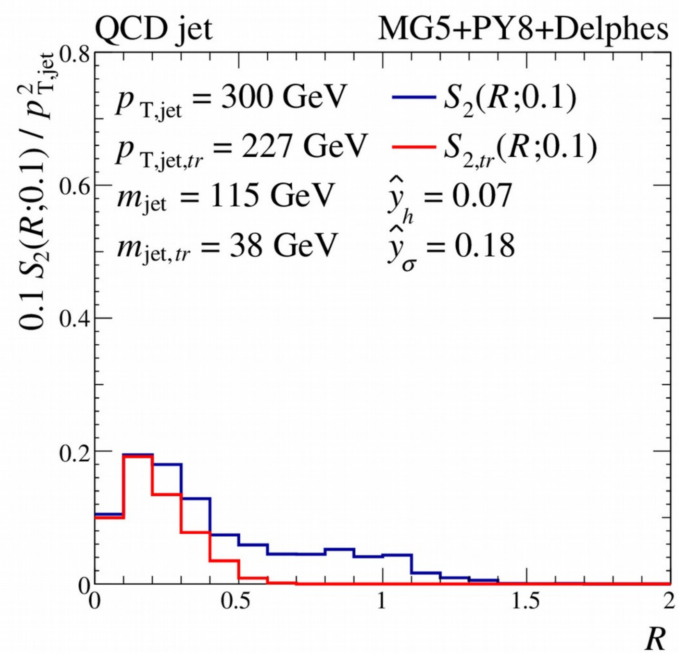




Two-Point Correlation Spectrum

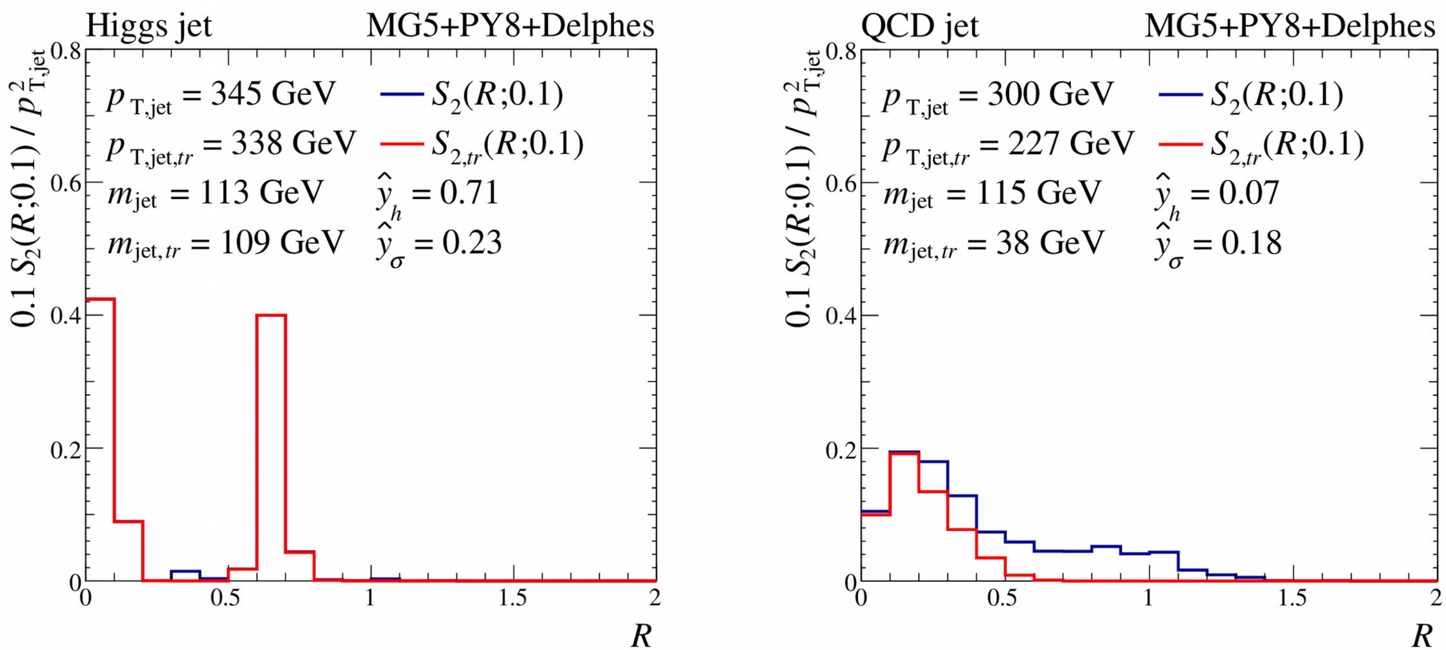
- QCD jets are mostly one-prong jets with surrounding soft particles.
- Its spectrum has a smoothly falling behavior.





Logistic Regression with S2

QCD jets and Higgs jets occupy different phase space of S2.

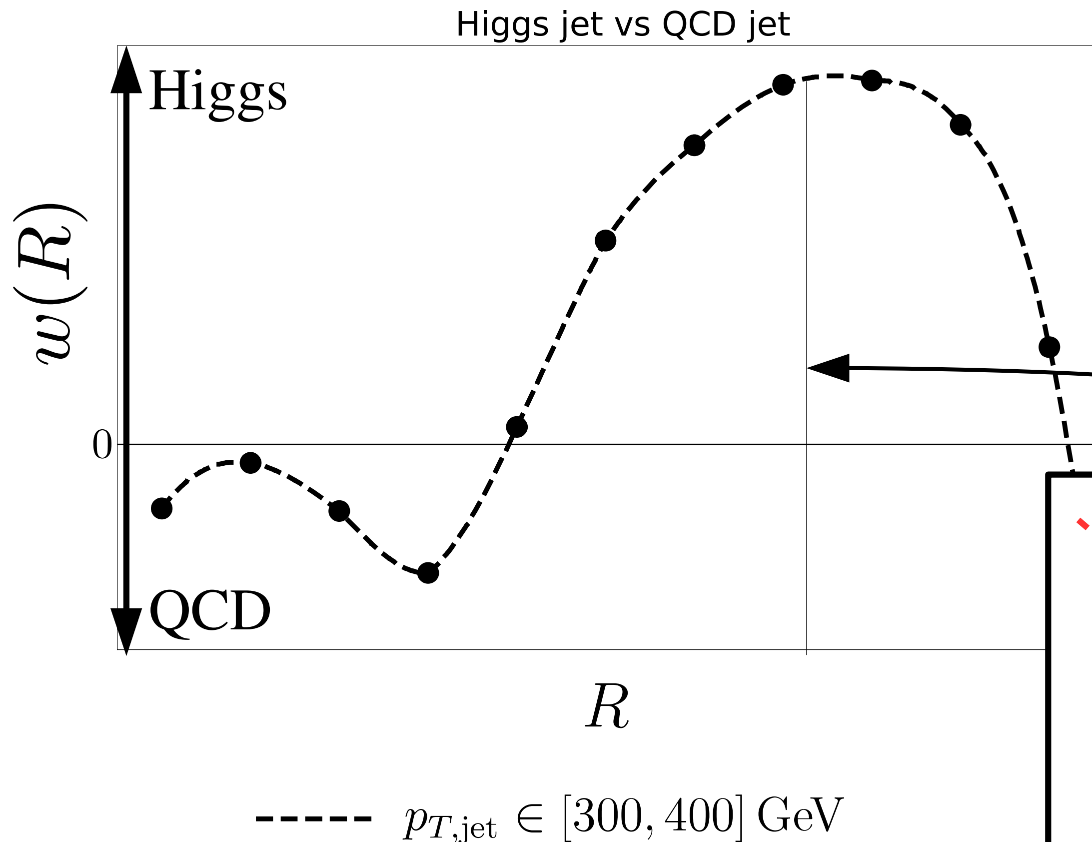


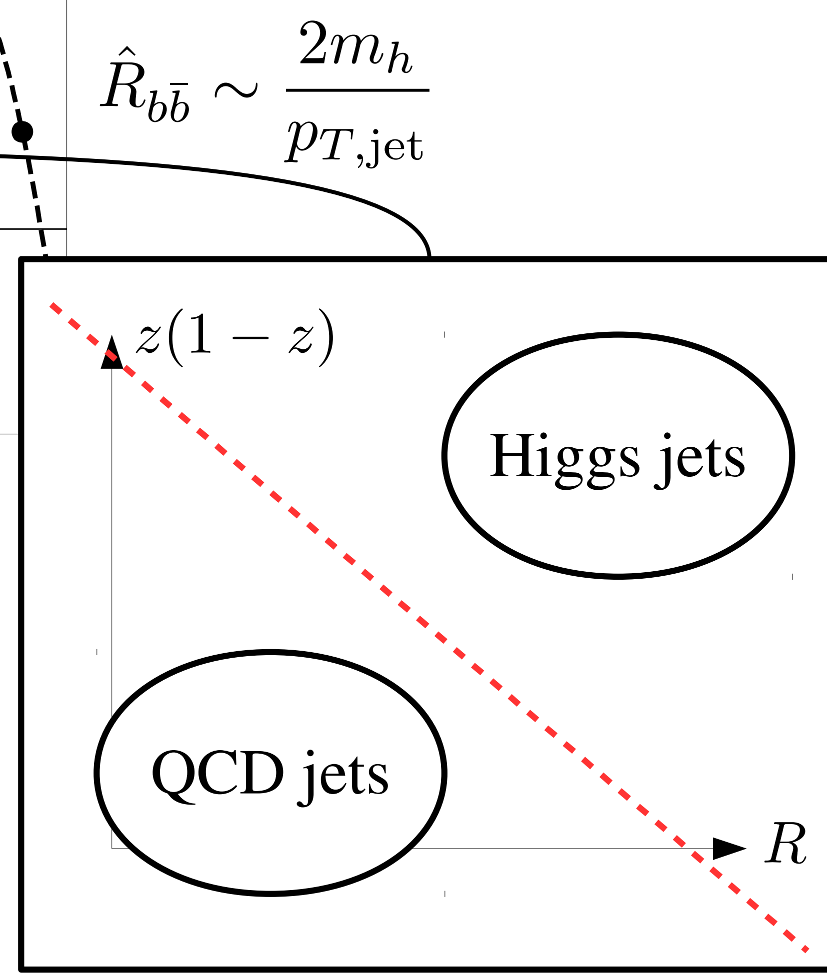
It looks like S2 separates two kinds of jets linearly.
We may try the logistic regression with S2.

$$h = \sum_k S_2^k w^k, \quad y_h = \phi_{\mathrm{sigmoid}}(h)$$

Results: logistic

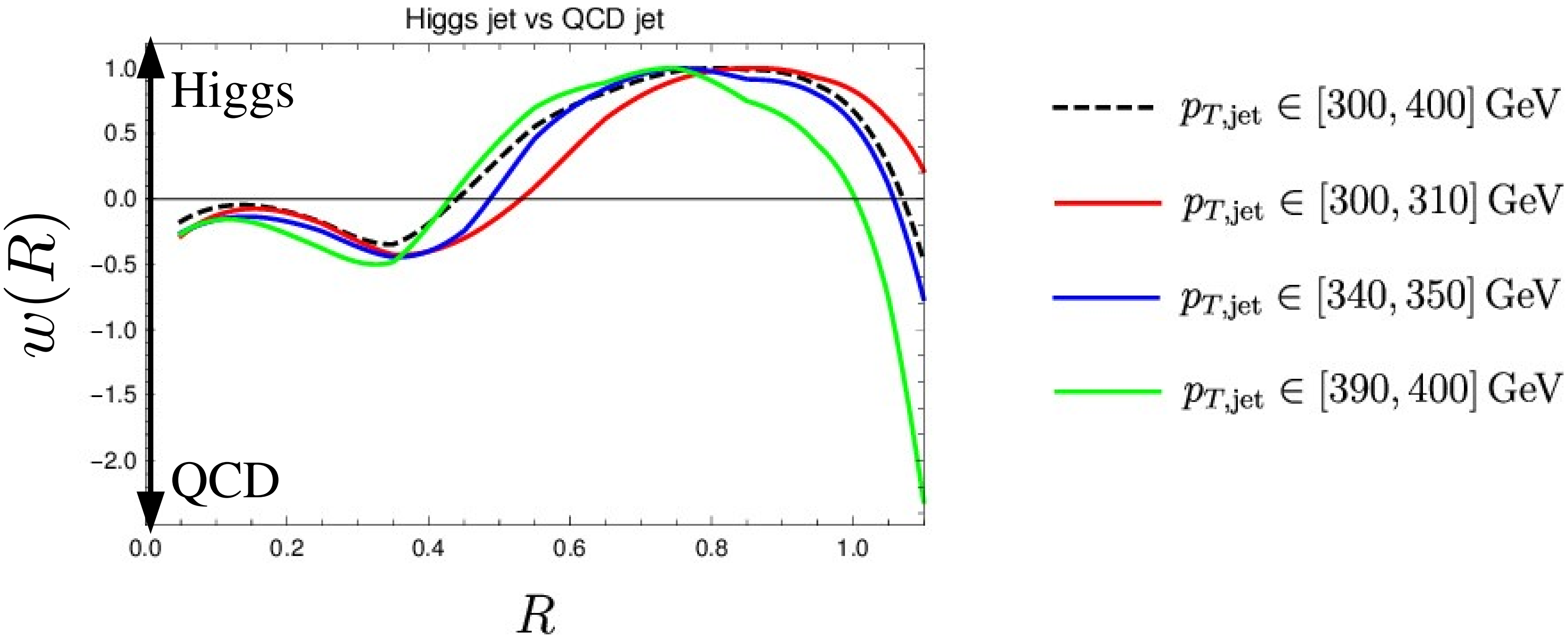
$$h = \sum_k S_2^k w^k, \qquad y_h = \phi_{\text{sigmoid}}(h)$$





The logistic classifier found out the expected hyperplane. However…

Results: logistic



The neural network learned the average of features for the classification over whole phase space region...
This small failure is happened because the peak of Rbb depends PT of the jet...

$$\hat{R}_{b\bar{b}} \sim \frac{2m_h}{p_{T,\mathrm{jet}}}$$

~~Abandon S2?~~

Improve S2?

Two-level setup

The logistic classifier works well, but not enough.

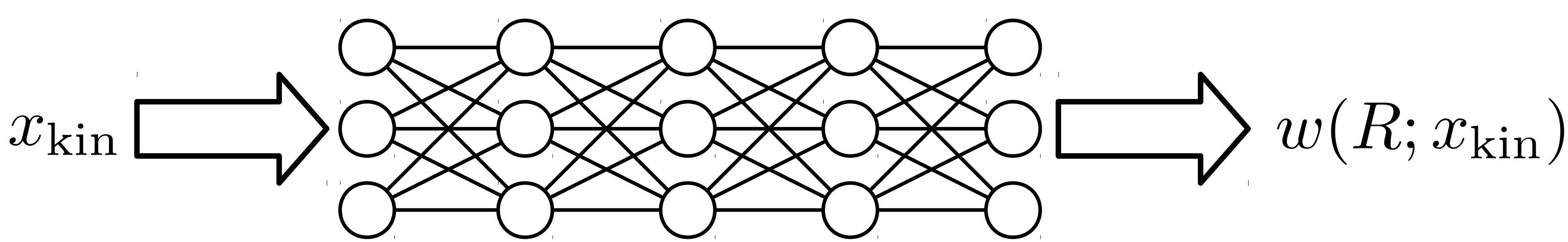
$$\Phi[P_T] = \int dR\, S_2(R) w(R)$$

The weights should learn the change of phase space!

$$\Phi[P_T; x_{\text{kin}}] = \int dR\, S_2(R) w(R; x_{\text{kin}})$$

$$x_{\text{kin}} = \{p_{T,\text{jet}}, m_{\text{jet}}\}$$

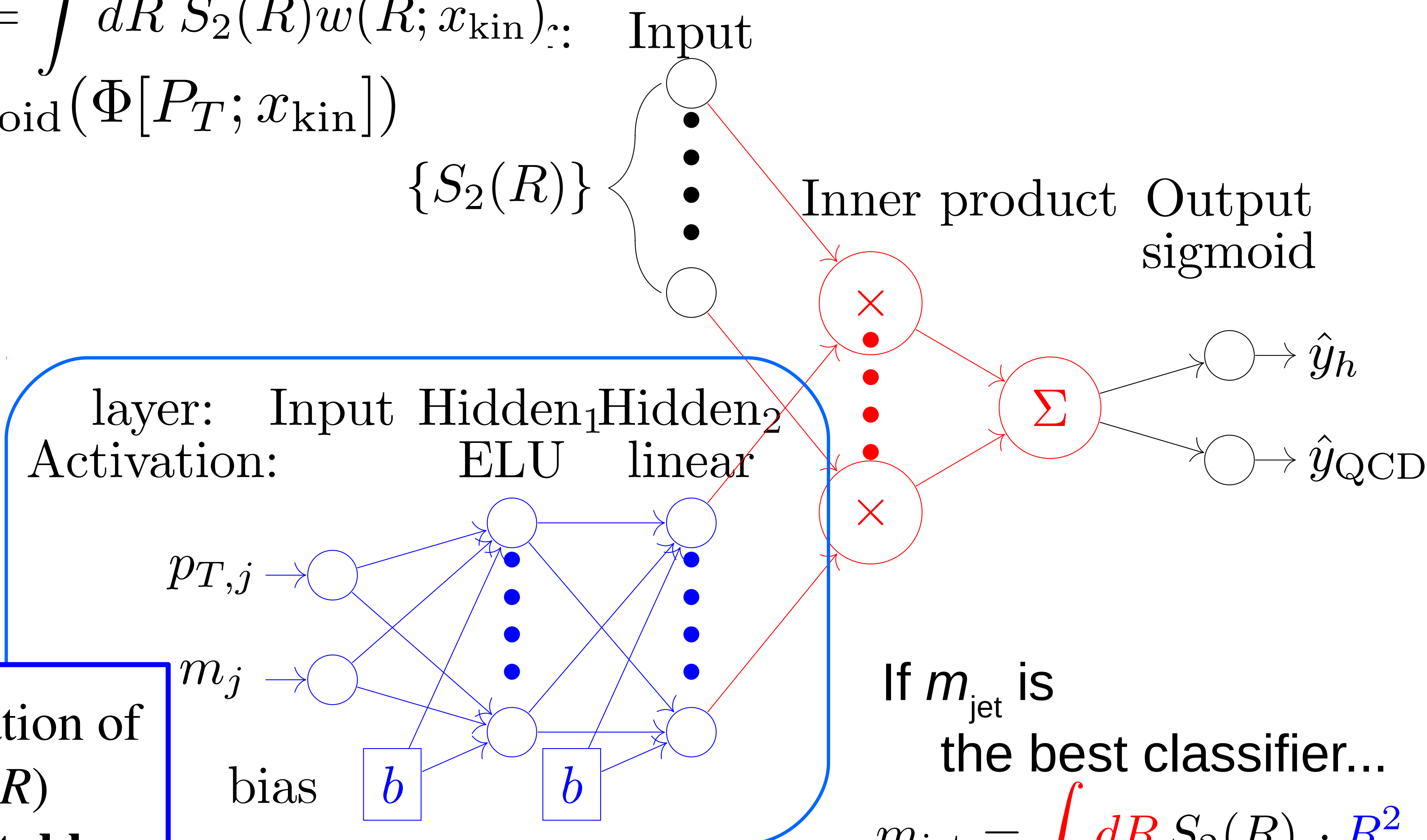
Use neural network to approximate the weight function.



Concrete realization of the neural network architecture

$$\Phi[P_T; x_{\rm kin}] = \int dR\, S_2(R) w(R; x_{\rm kin}),$$

$$y_h = \phi_{\rm sigmoid}(\Phi[P_T; x_{\rm kin}])$$

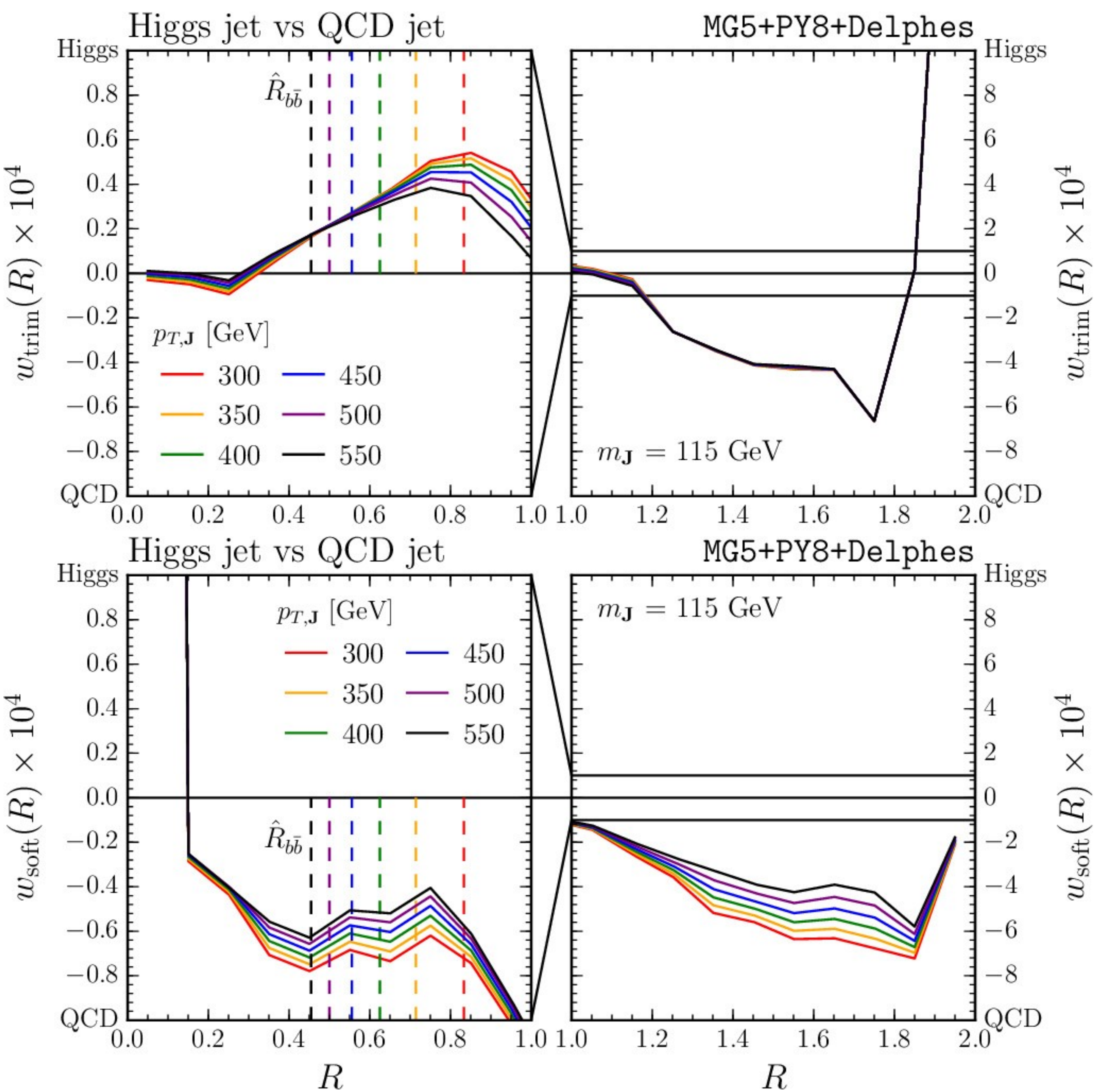


NN representation of kernel $w(R)$
→ **interpretable**

If $m_{\rm jet}$ is the best classifier...

$$m_{\rm jet} = \int dR\, S_2(R) \cdot R^2$$

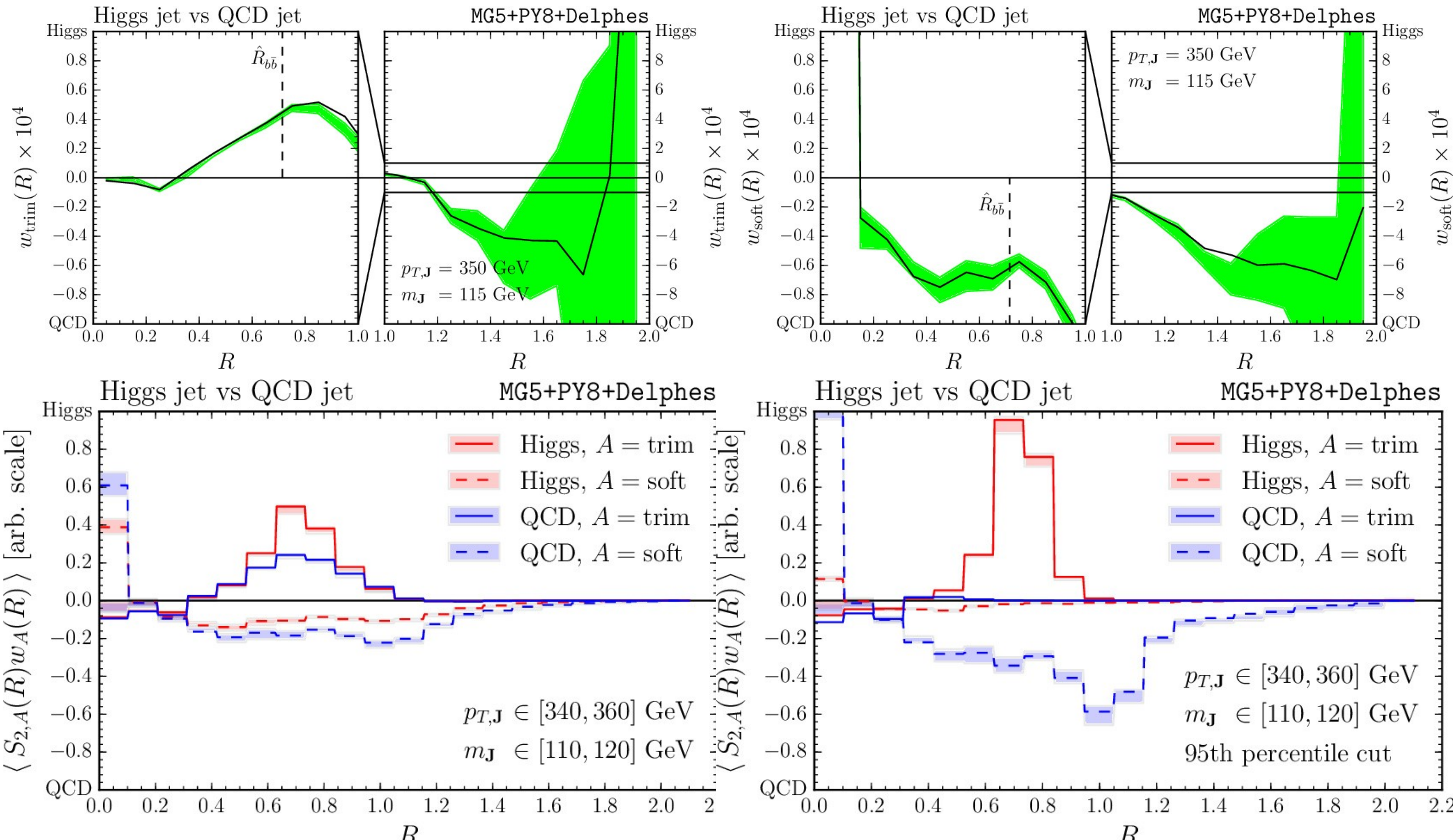
Results: Two-level Setup



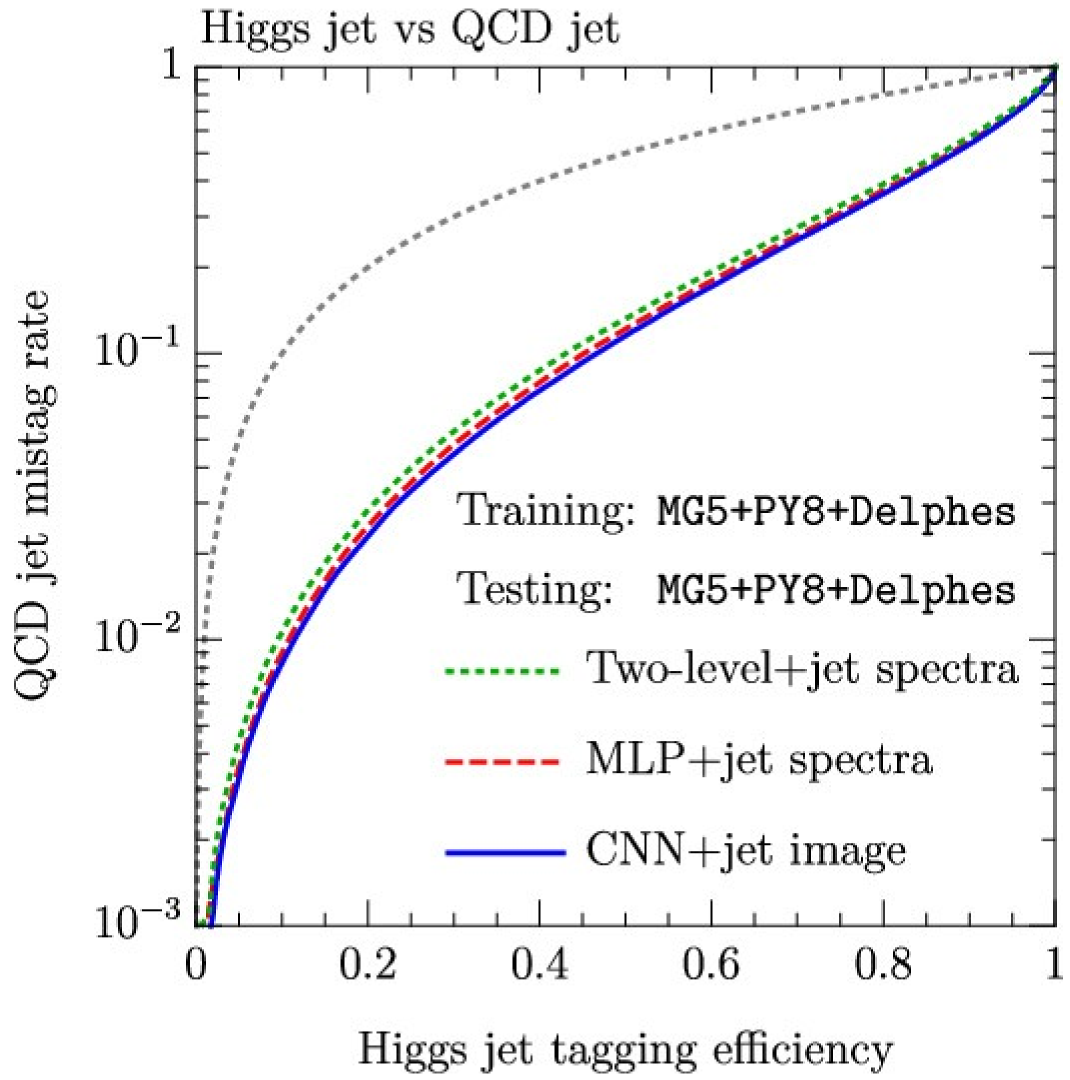
Note that high w(R) does not mean the high importance of the corresponding S2(R). For that purpose, we have to check...

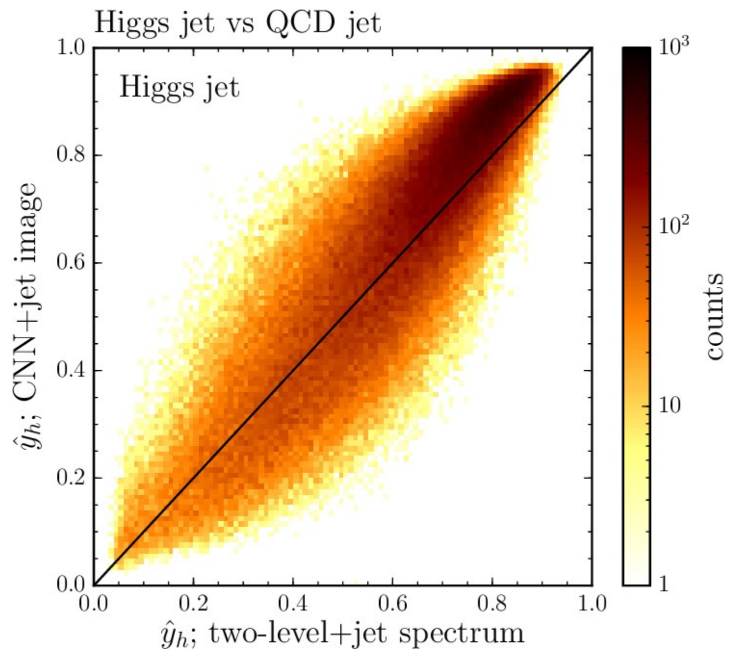
$$S_2(R)w(R)$$

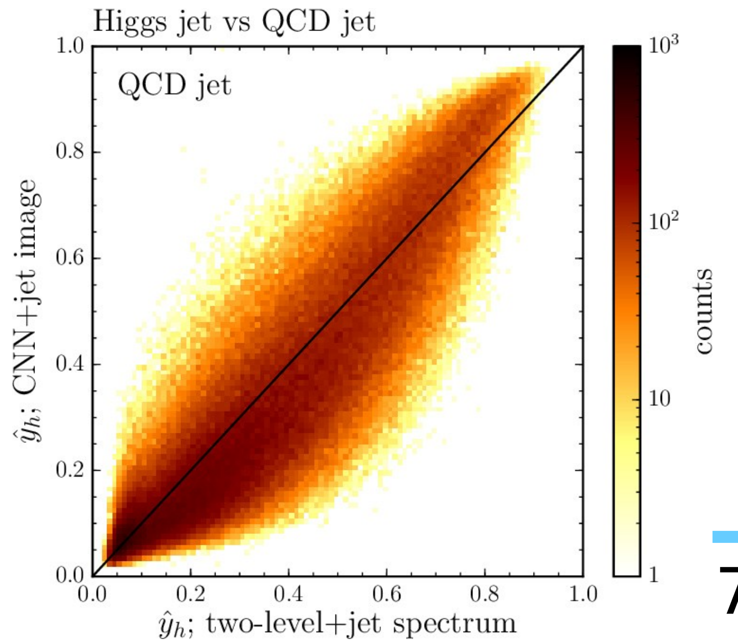
Results: Two-level Setup



Comparison with CNN



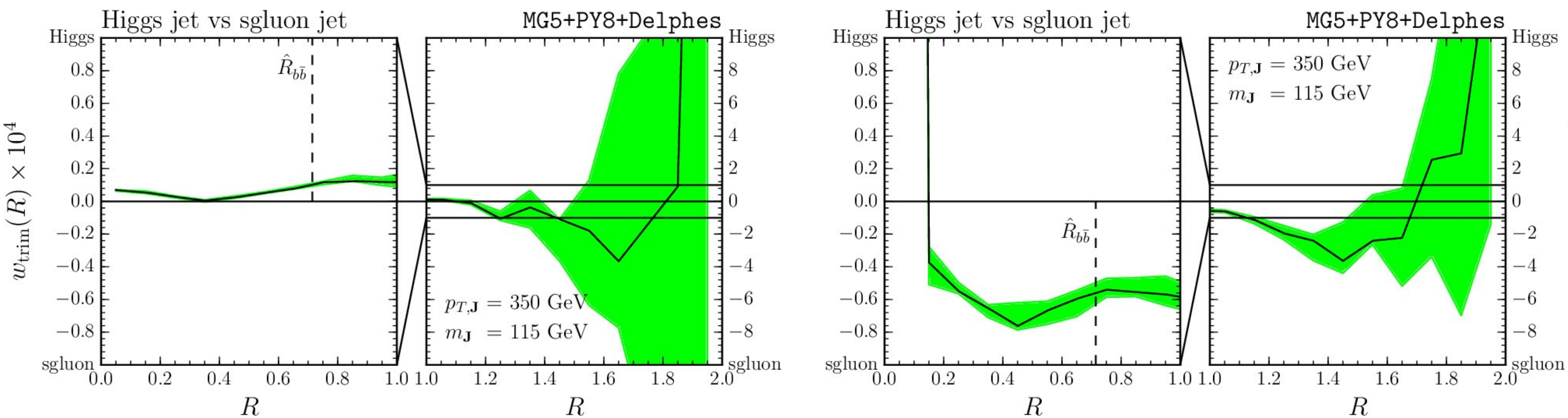




Other physics results

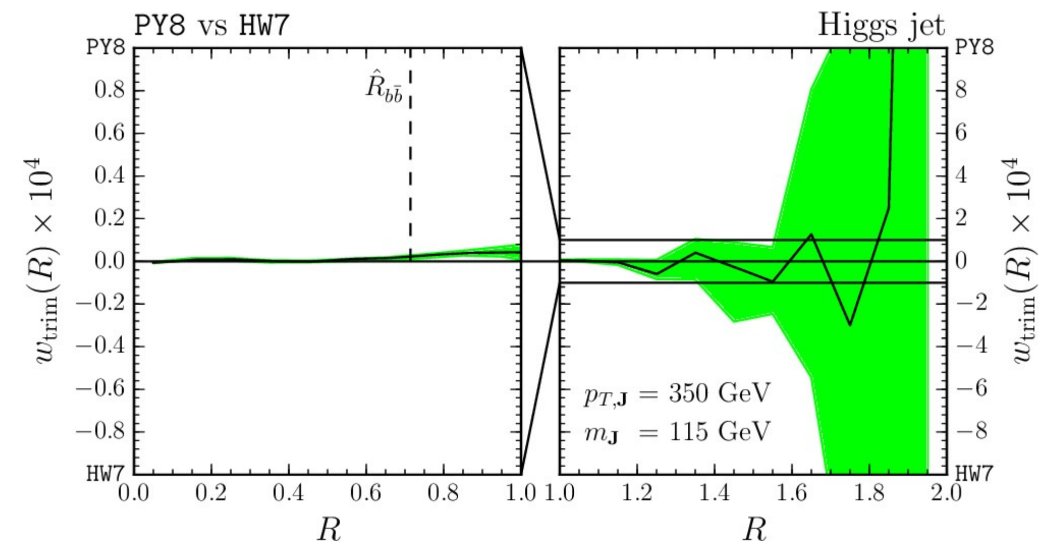
- During this talk, I have focused on explaining the results based on parton description, but the two-point correlation encodes other physical difference between Higgs jet and QCD jet.
- This architecture can be also used for distinguishing other objects:

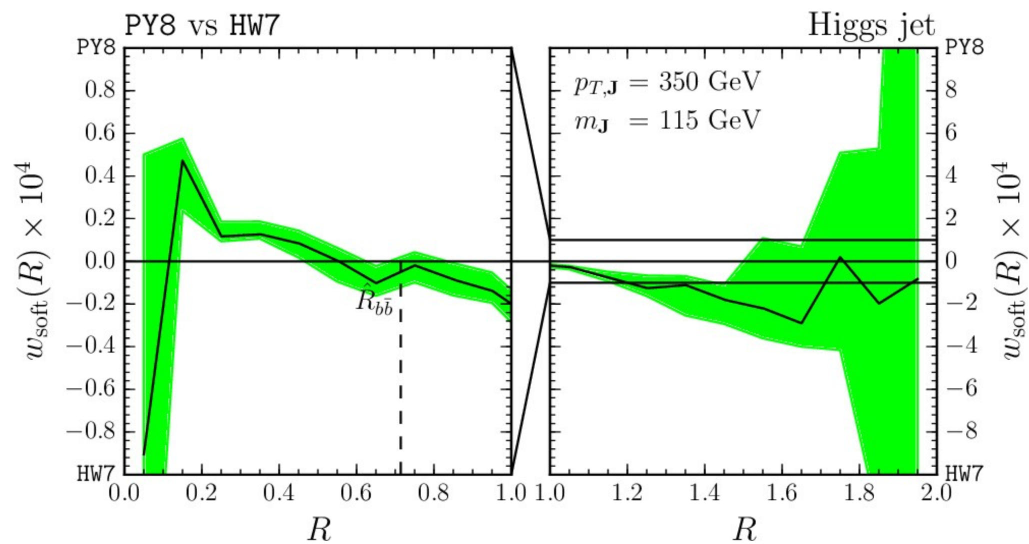
Identification of color of originating parton: **1** vs **8**



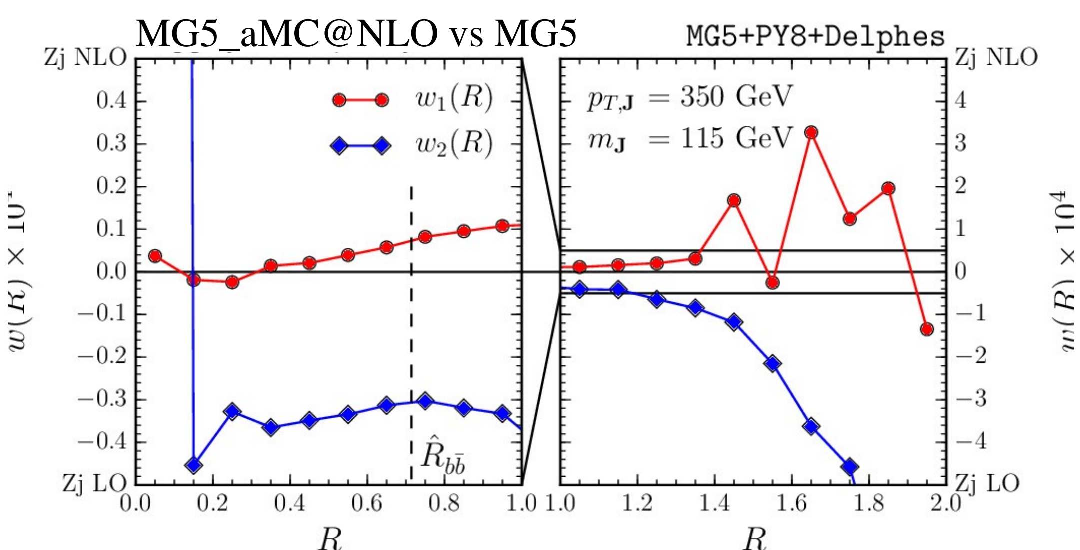
Other physics results

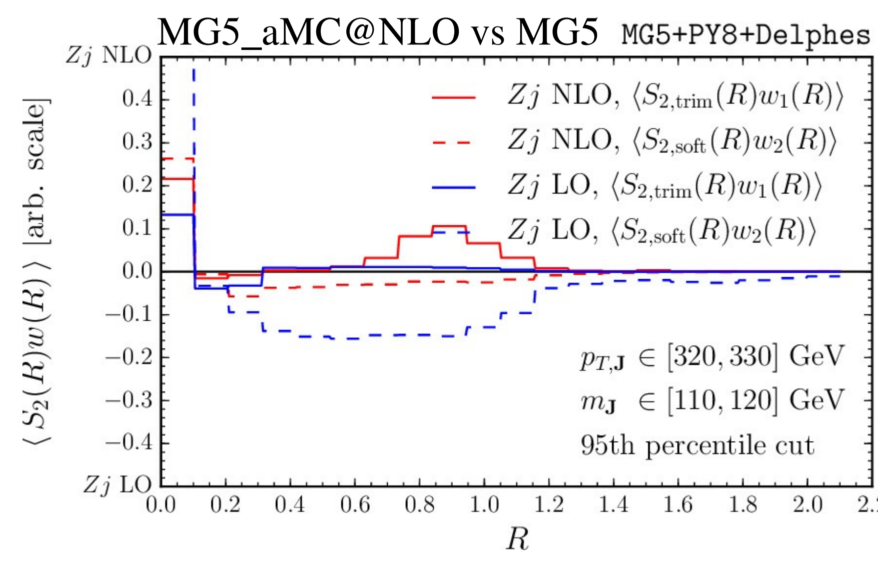
Comparison between parton shower monte carlo simulators:





Comparison between parton simulators:





Summary

- Deep Learning is a hot topic in jet physics.
- We developed a machine learning framework using **two-point correlation spectrum** for analyzing jet substructures.
- The spectrum can be used for the kernel trick and resulting linear classifier gives us an **interpretable normal vector**.

- We are currently doing analysis on more complicated objects, such as top jets.
- Since we have interpretable architecture, we are also interested in different type of machine learning (semi-supervised or unsupervised) for solving problems in jet physics and physics problems in other areas.

Please stay tuned!