$T\bar{T}$, $J\bar{T}$, $T\bar{J}$ Partition Sums from String Theory

with David Kutasov

1907.07221 + ...

- What is $T\bar{T}$?

Start with QFT in 1+1

$$T_{\mu\nu}(x^\lambda)$$

$$S \to S + \underbrace{\mu}_{L^{+2}} \int d^2x \left(T_{zz} T_{\bar{z}\bar{z}} - \left(T_{z\bar{z}}\right)^2 \right)$$

Irrelevant

Zamolodchikov: System actually make sense (in a way I will now Review.)

ON $R \times S^1$, ONE can determine the Energy spectrum

$$E_n(\mu) \quad \text{if}$$

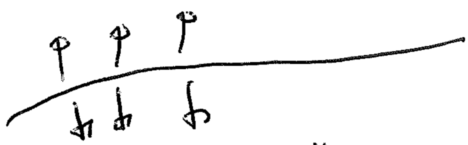
$$E_n(\mu = 0) \quad \text{is known}$$

1) IR : undeformed spectrum

2) UV : Hagedorn spectrum

$$\left(E \gtrsim 1/\mu^{1/2}\right)$$

INTERESTING because



- Extends the "space of field theories"
- appears to connect with string theory

- More Review

$$\sum_n \frac{d}{d\mu} \langle n | e^{-\beta E_n(x)} | n \rangle$$

$$= \langle n | T_{00} T_{11} - T_{01}^2 | n \rangle$$

$$\hookrightarrow \quad \frac{dE_n}{d\mu} + E_n \frac{\partial E_n}{\partial R} + \frac{p^2}{R} = 0$$

"Inviscid Burger's Equation"

EXACT solution

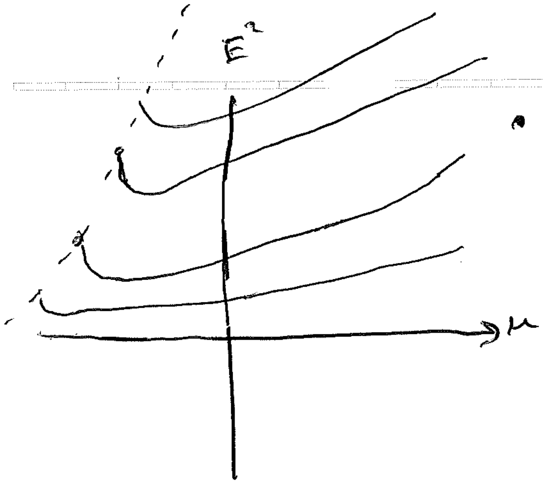
$$E_n'(\mu) = \frac{R}{\mu} \left(\sqrt{1 + \frac{\mu E_n}{R} + \frac{\mu^2}{R^2} p_n^2} - 1 \right)$$

- $\mu \to 0$: $E_n(\mu) \to E_n(0)$

- if $\mu > 0$ & if E_n is condy then Hagedorn in UV

- if $\mu \leq 0$ spectrum imaginary for

$$\mu \sim -\frac{R}{E_{min}}$$



- Verlinde et. al. interpreted $\mu \leq 0$ case as Lorentz invariant UV cut-off (?)

- Tateo et. al. showed that $\mu \geq 0$ in bosons is interpretable as the spectrum of Nambu-Goto string

$$RE(\mu) = \frac{R^2}{\mu}\left(\sqrt{1 + \frac{\mu(\Delta+\bar{\Delta})}{R^2} + \frac{\mu^2}{R^4}(\Delta-\bar{\Delta})^2} - 1\right)$$

$$\Delta \sim L_0 - \tfrac{c}{24} \qquad \bar{\Delta} \sim \bar{L}_0 - \tfrac{c}{24}$$

with $\mu \sim \alpha'$ $\quad c = n-2$

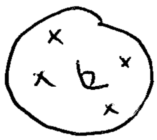
(classical action)

Does not require $c = 24$

Weren't we supposed to only quantize Nambu-Goto in critical dimension?

- Insisting on working outside critical dimension usually necessitates Liouville
- Or the strings no longer Nambu-Goto

But these are issues having to do with string interactions.



But spectrum only involves cylinder / torus.

JT gravity + matter is an abstraction which appears to capture the physics in such a restricted setup

$$S = \int d^2x \left[\phi R + \text{matter} \right]$$

- Physical Observables

1) Scattering Amplitudes

$2 \to 2$

$$S = \mathbb{1}\, e^{i\delta(s)}$$

straight forward, almost trivial

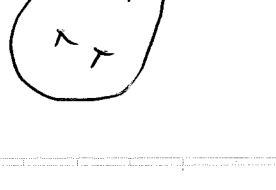
"CCD phase"

2) Correlation functions

$$\langle T_{\mu\nu}(x) \quad T_{\rho\sigma}(y) \rangle$$

computed as expansion in μ

but one does not expect these observables to exist at the non-perturbative level.



Aharony et. al.

$$c \to \infty$$
$$\mu \to 0 \qquad t = c\mu \quad \text{fixed}$$

Leads to Local correlation function

even though spectrum is still Hagedorn

(strong interaction effect)

- Partition function
  - $T\bar{T}$ deformed CFT on $S_1 \times S_1 \leftarrow \tau$

- Can be inferred from the known spectrum

- Cardy showed diffusion interpretation

$$\sim \frac{d}{d\mu} \sim \frac{d}{dL}\frac{d}{dL'}$$

- Dubovsky et. al.
  JT gravity

SAME ANSWER

Aharony 1808.02492

work perturbatively in $\lambda = \frac{\mu}{R^2}$

- $Z[\tau, \bar{\tau}]$ is modular invariant for $\lambda = 0$

- at linear order in λ

$Z[\tau, \bar{\tau}, \lambda]$ is modular invariant

if $\tau \to \frac{a\tau + b}{c\tau + d}$

$$\lambda \to \frac{\lambda}{|c\tau + d|^2}$$

- if one Requires

1) $Z[\tau, \bar{\tau}, 0]$ as initial condition
2) modular invariant
3) $E_n(\lambda)$ depends only on λ & E_n

$$\partial_\lambda Z(\tau, \bar{\tau}, \lambda) = \left[\tau_2 \frac{\partial}{\partial \tau} \frac{\partial}{\partial \bar{\tau}} + \frac{1}{2} \left(i(\partial_\tau - \partial_{\bar{\tau}}) - \frac{1}{\tau_2} \right) \lambda \partial_\lambda \right] Z(\tau, \bar{\tau}, \lambda)$$

is determined uniquely

2 more issues

- $T\bar{T}$ generalizes to $J\bar{T}$

$$\lambda T\bar{T} + \epsilon_+ J\bar{T} + \epsilon_- T\bar{J} + \overbrace{\eta J\bar{J}}^{\text{Trivial}}$$

Spectrum known

modular properties / geometric interpretation

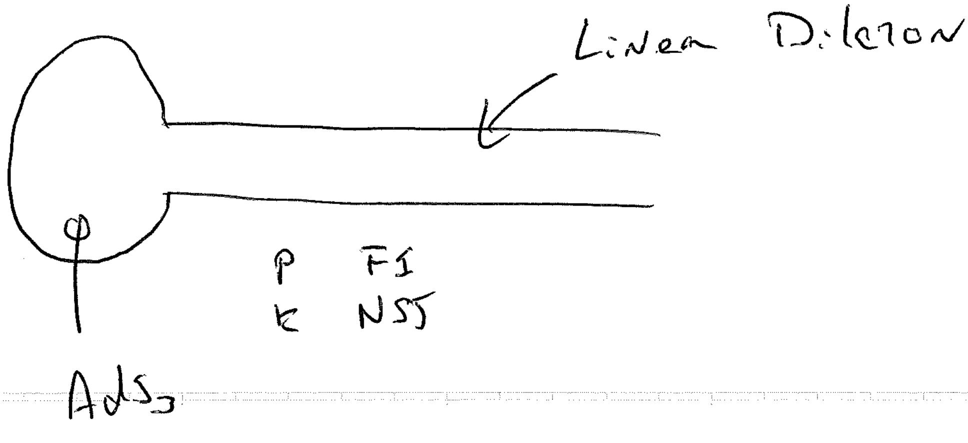
less known

- For symmetric product CFT's

$$\left(\sum T_i\right)\left(\sum \bar{T}_j\right) \Leftarrow \text{double trace}$$

$$\sum_i \left(T_i \bar{T}_i\right) \Leftarrow \text{single trace}$$

$$AdS_3 \times N \subset T_3 \times N$$

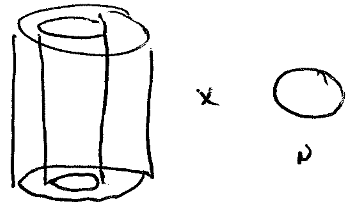


Finally, my story

start with AdS_3 (p, k) $\times N$

$p \gg 1$ deformed to $T_3 \times N$

consider long string



world volume theory

$\mathbb{R}_\phi \times N$

ϕ linear dilaton

critical. spectrum Dense

$\sim \mathbb{R} \times N$

TORUS Amplitude

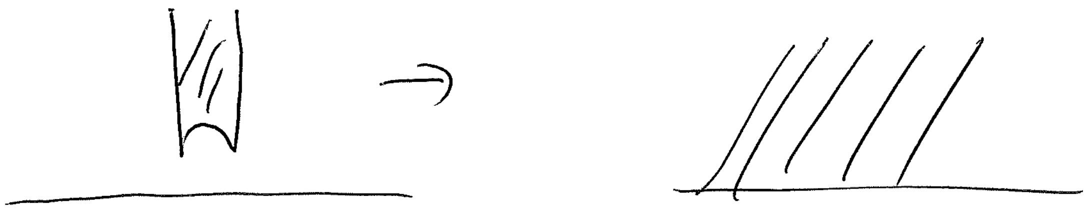
$$Z = \sum_{\substack{m_1,w_1\\ m_2 w_2}} \int_F \frac{d\tau\, d\bar\tau}{\tau_2^2}\, e^{-S_{m,w}(\xi,\tau)}\, Z_R(\tau,\bar\tau)\, Z_N(\tau,\bar\tau)$$

φ

Restrict to $N = \det\begin{pmatrix} m_1 & w_1\\ m_2 & w_2\end{pmatrix} = 1$

Restrict to $\begin{pmatrix} m_1 & w_1 \\ m_2 & w_2\end{pmatrix} = \begin{pmatrix}1 & \\ & 1\end{pmatrix}$

& integrate



$$Z = \int d\tau\, d\bar\tau\, \frac{1}{\tau_2^2}\, \frac{\xi_2}{\lambda}\, e^{-\frac{1}{\lambda\tau_2}(\xi-\tau)(\bar\xi-\bar\tau)} \times \underbrace{Z_R(\tau,\bar\tau)\, Z_N(\tau,\bar\tau)}_{\text{Replace with general } Z_{CFT}(\tau,\bar\tau)}$$

Same answer as Cardy, Dubovski

τ integral can be done in closed form

$$\frac{dt}{t^{3/2}} \, e^{\left(\frac{1}{t} a + b t\right)} \sim e^{\sqrt{ab}}$$

$$Z = \sum_n e^{-\epsilon_n E_n(\alpha) + i \epsilon_1 P_n}$$

to read off the spectrum

$T\bar{T}$ def: smearing of Z_{CFT}

Hagedorn $\Rightarrow$ maximum T

Conclusion

- mathematical structure familiar to string theorists
- modular behavior $\Longleftrightarrow$ T duality
- extend to $T\bar{T} + J\bar{T} + T\bar{J}$ by working in $T_2 \times S \times \hat{N}$
- $N \neq 1 \Rightarrow$ single trace $T\bar{T}$ deformation of symmetric product
- guide for identifying other observables