Calculating Multi-Leg One Loop amplitudes with Unitarity Cut Method (Part I)

Zoltan Kunszt

IPMU, March 25, 2010

Donnerstag, 25. März 2010

Goal: automated precision predictions for the LHC

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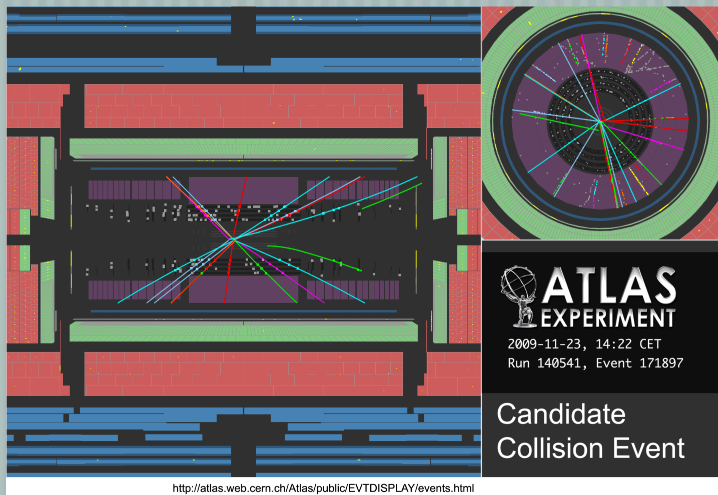

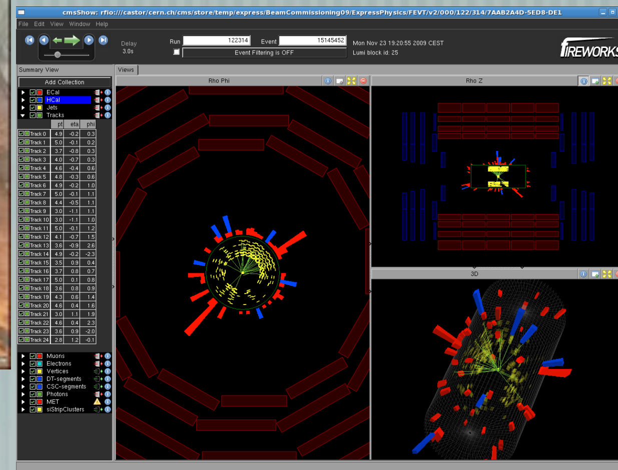

CMS

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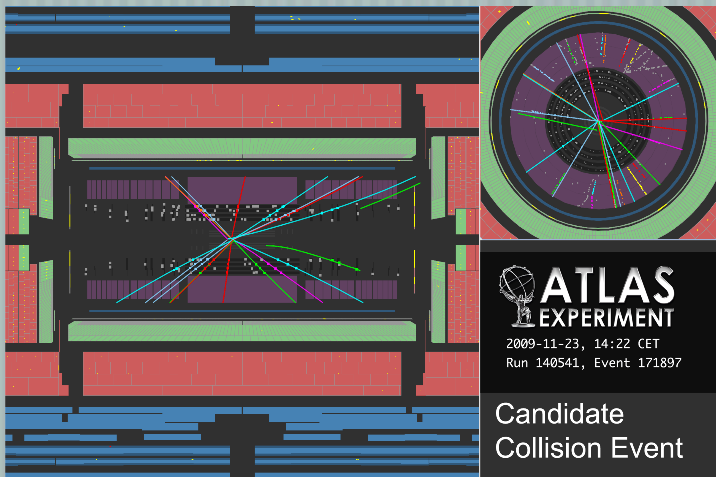
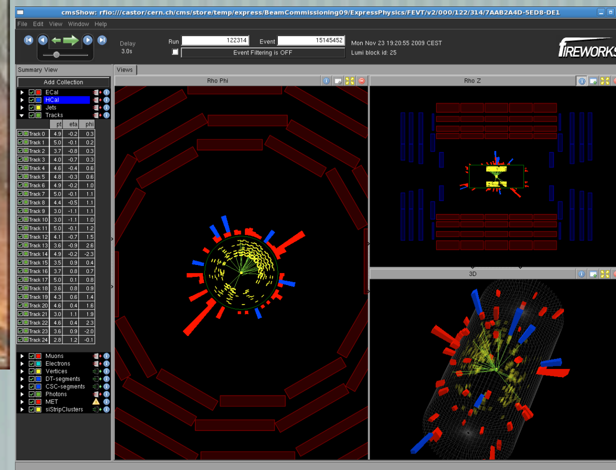


http://atlas.web.cern.ch/Atlas/public/EVTDISPLAY/events.html

Large Hadron Collider



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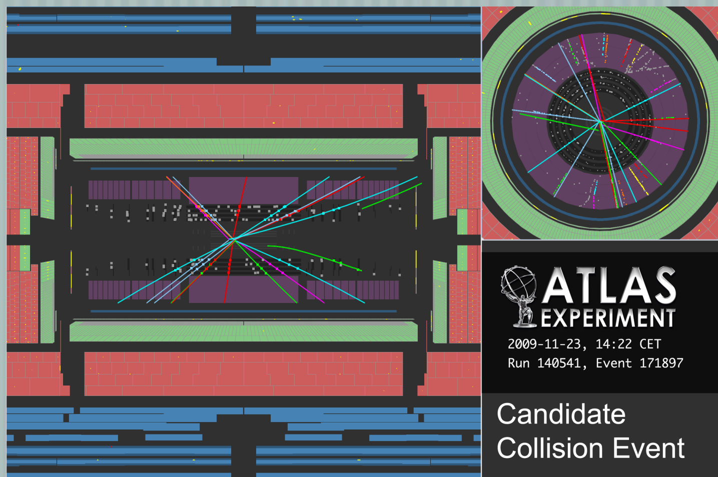

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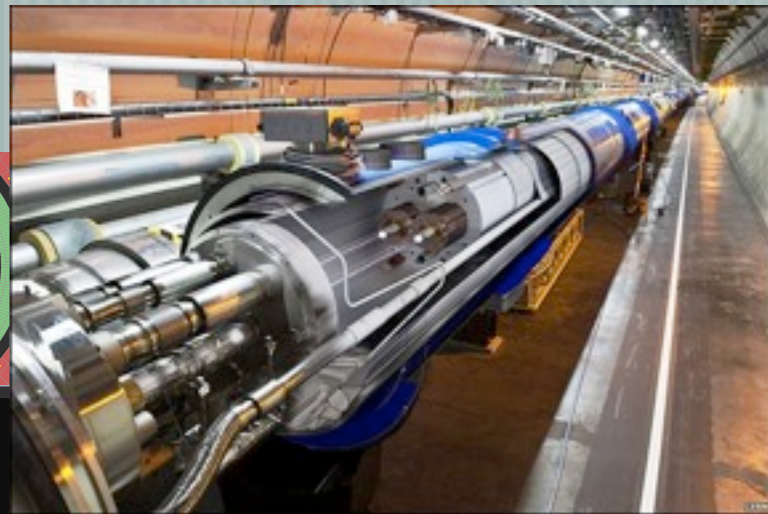

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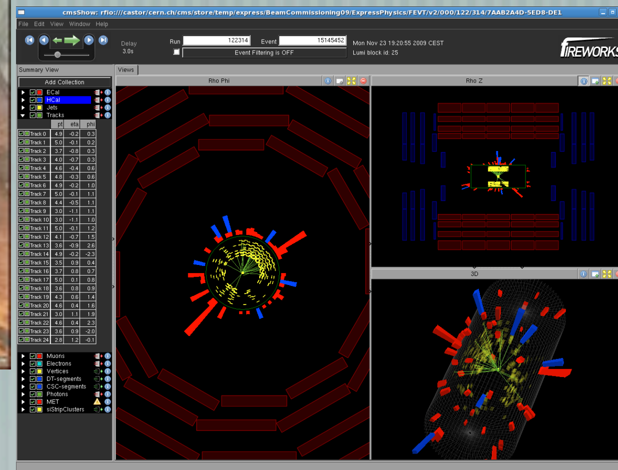
$$E_{\rm cm} = 7 \text{ TeV} \ldots 14 \text{ TeV}$$



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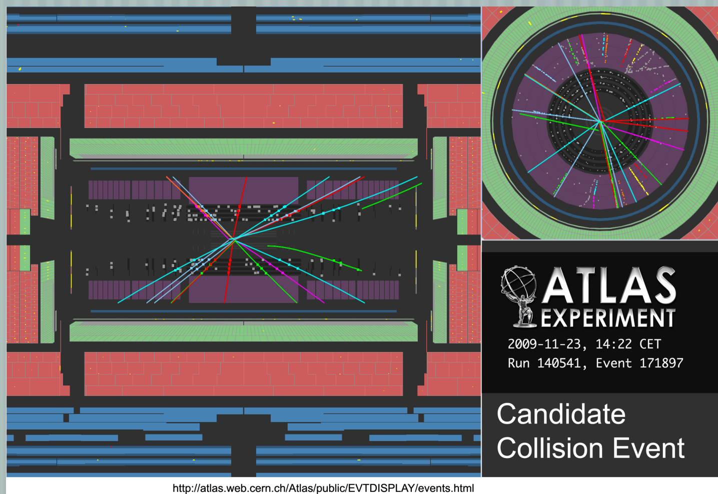

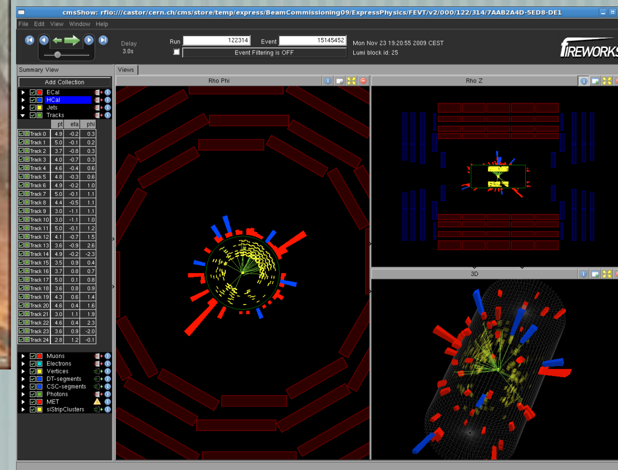

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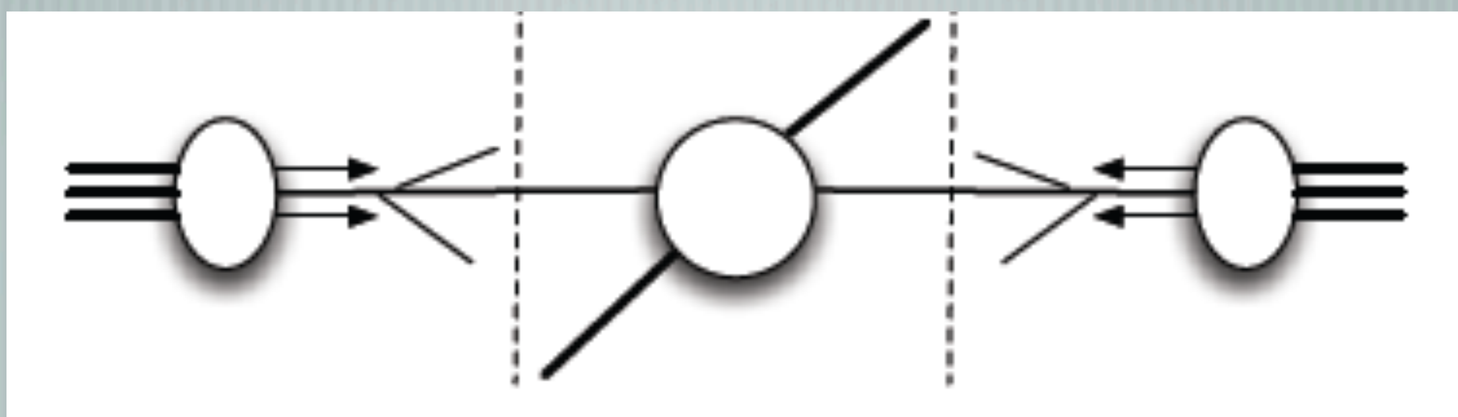

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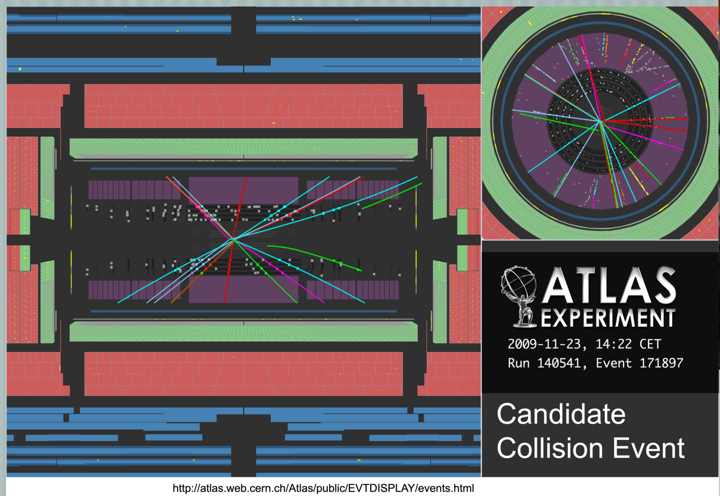

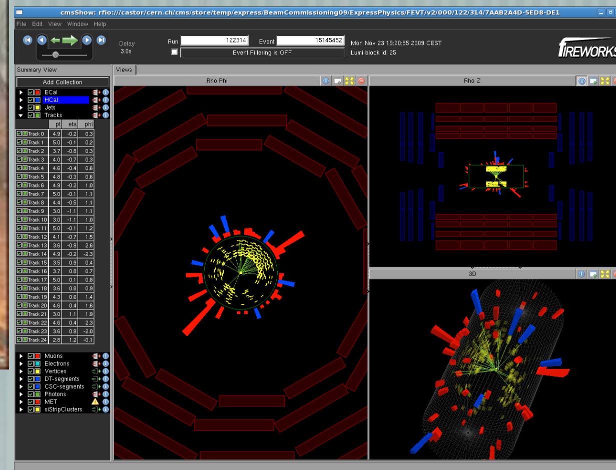

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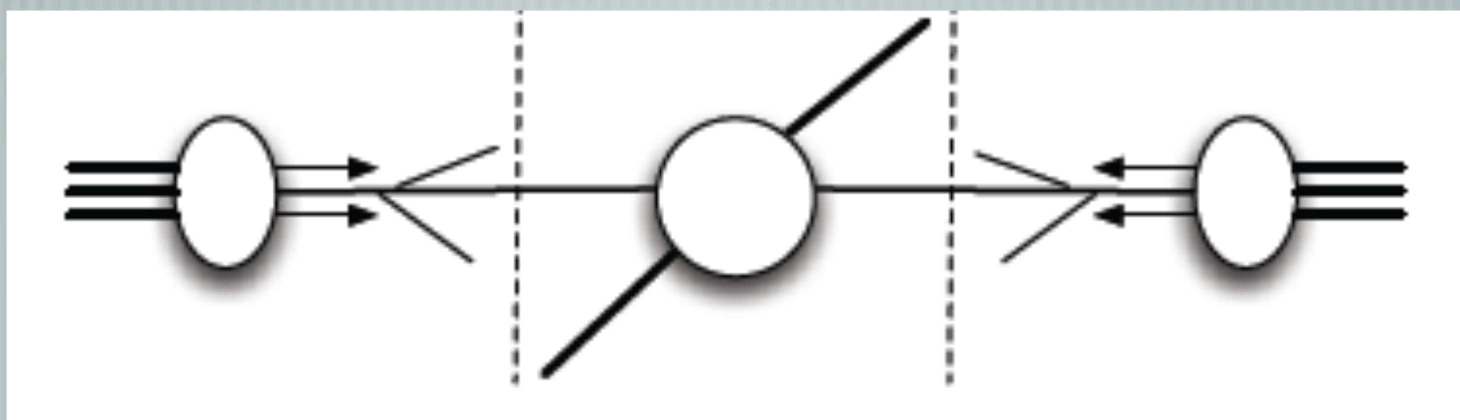

Proton beams = non-monochromatic beams of quarks and gluons

The main goals of the LHC

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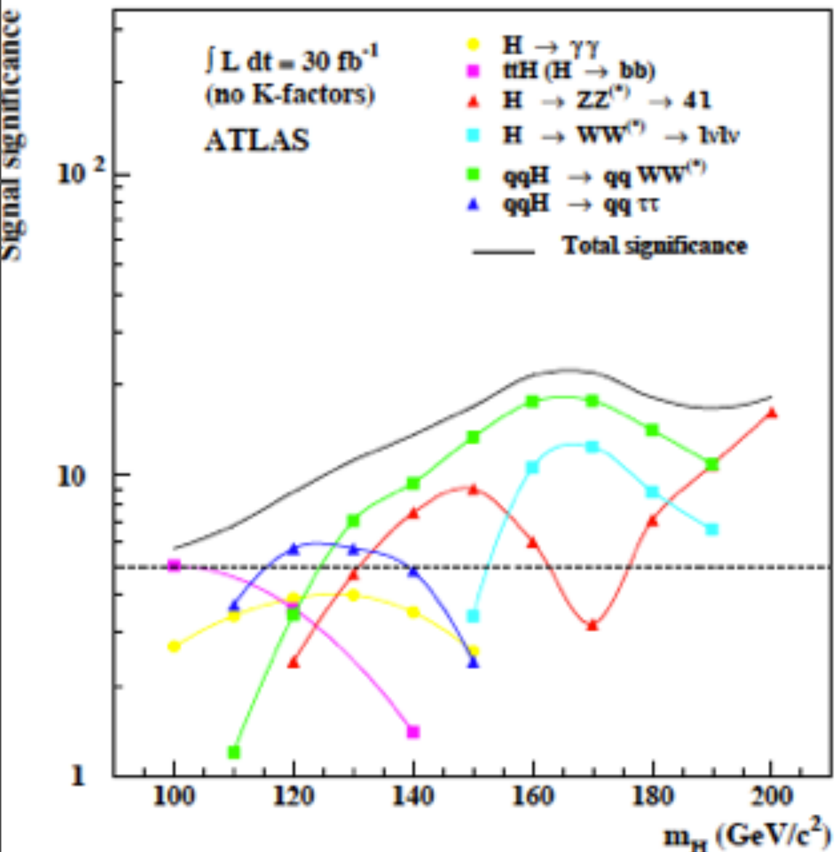

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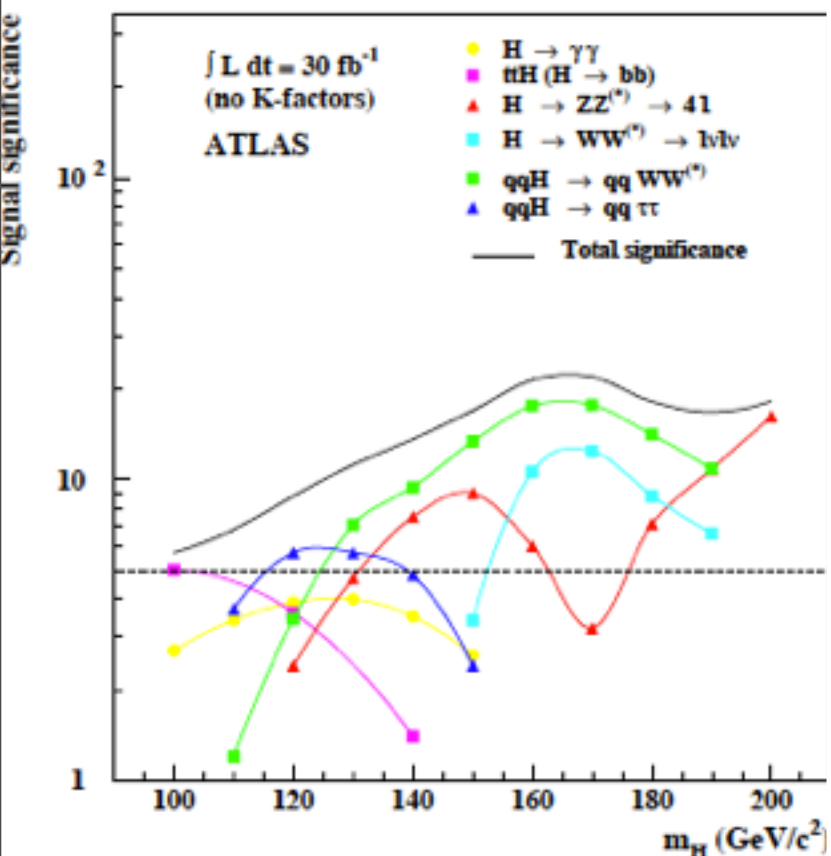

Scenario A: sort splices out in one go

Mike Lamont

- Two years at 3.5 TeV
- 2010: should peak at 10^{32} and yield up to 0.5 fb^{-1}
- 2011: ~1 fb^{-1} at 3.5 TeV
- 2012: splice consolidation (and cryo collimator prep.)
- 2013: 6.5 TeV - 25% nominal intensity
- 2014: 7 TeV – 50% nominal intensity

(2013–2014: Aggressive)

Year	Months	energy	beta	ib	nb	Peak Lumi	Lumi per month	Int Lumi Year	Int Lumi Cul
2010	8	3.5	2.5	7 e10	720	1.2 e32	-	0.2	0.2
2011	8	3.5	2.5	7 e10	720	1.2 e32	0.1	0.8	1.0
2012									
2013	6	6.5	1	1.1 e11	720	1.4 e33	1.1	7	8
2014	7	7	1	1.1 e11	1404	3.0 e33	2.3	16	24

29/01/10 LHC luminosity estimates 15

Other scenarios with stepwise splice consolidation were clearly not favored

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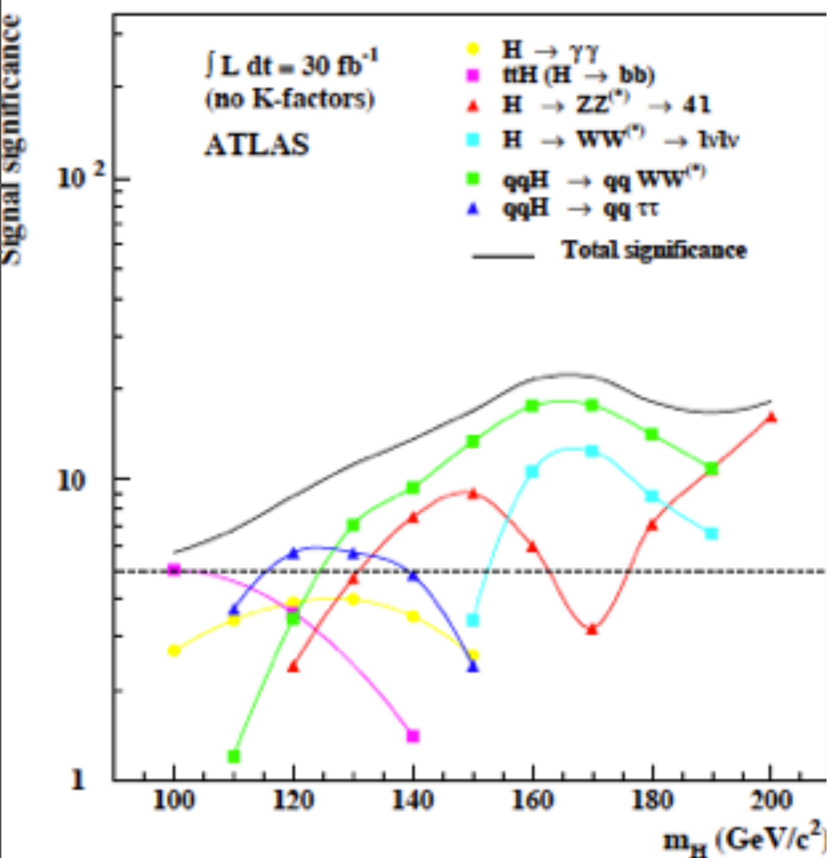

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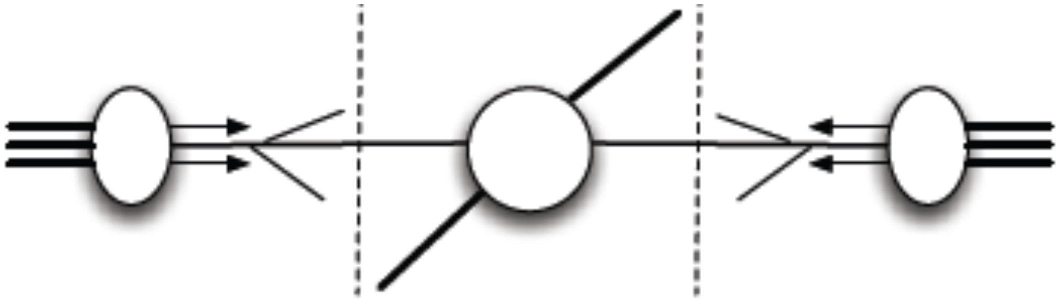
Quantitative description in QCD with perturbation theory



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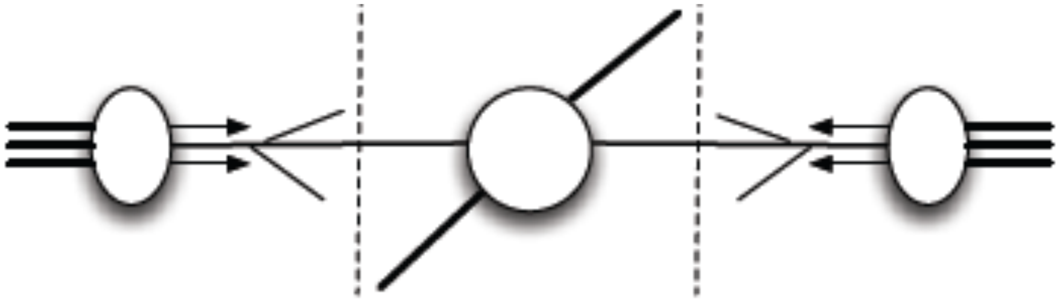

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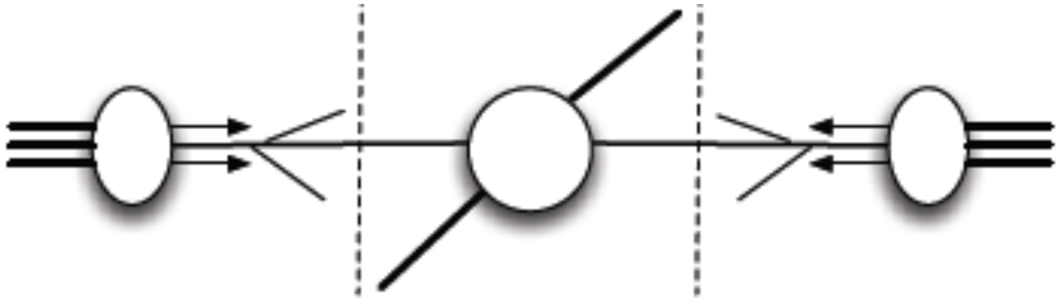

FACTORIZATION

$$\frac{d\sigma_{\mathrm{pp}\to\mathrm{hadrons}}}{dX} = \sum_{a,b} \int dx_1 dx_2 f_a(x_1,\mu_F) f_b(x_2,\mu_F) \times \frac{d\hat{\sigma}_{\mathrm{ab}\to\mathrm{partons}}(\alpha_s(\mu_R),\mu_R,\mu_F)}{dX} + \mathcal{O}\left(\frac{\Lambda^n_{\mathrm{QCD}}}{Q^n}\right)$$

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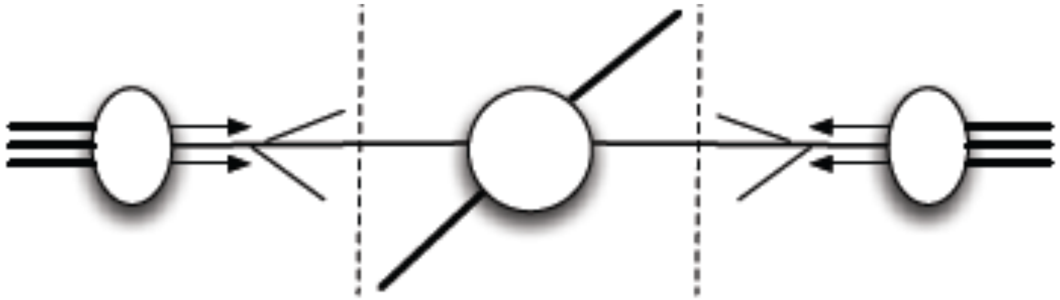

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scale dependence calculated
using perturbative AP-equation

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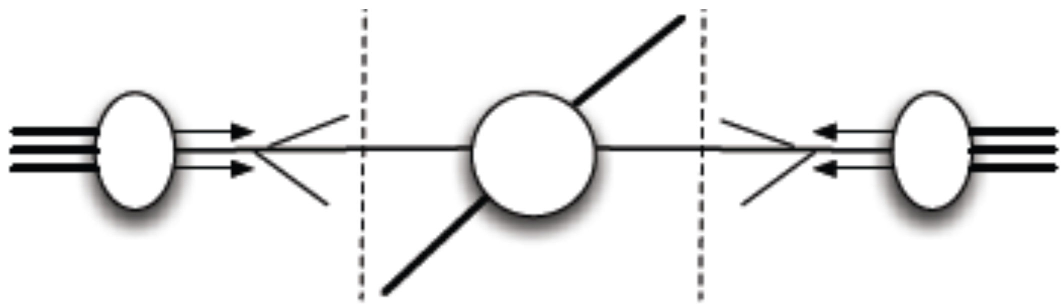

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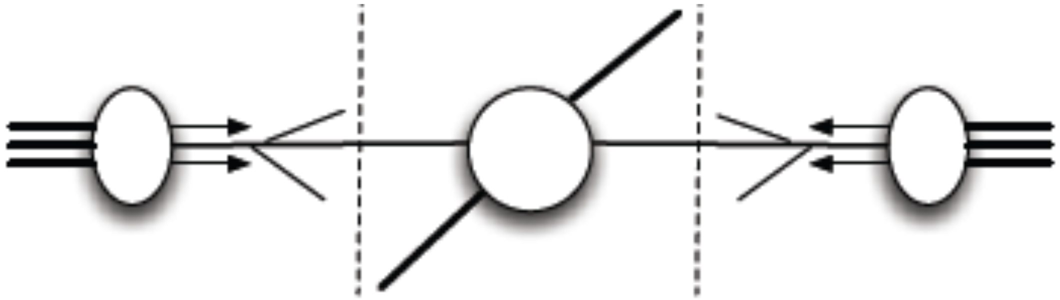

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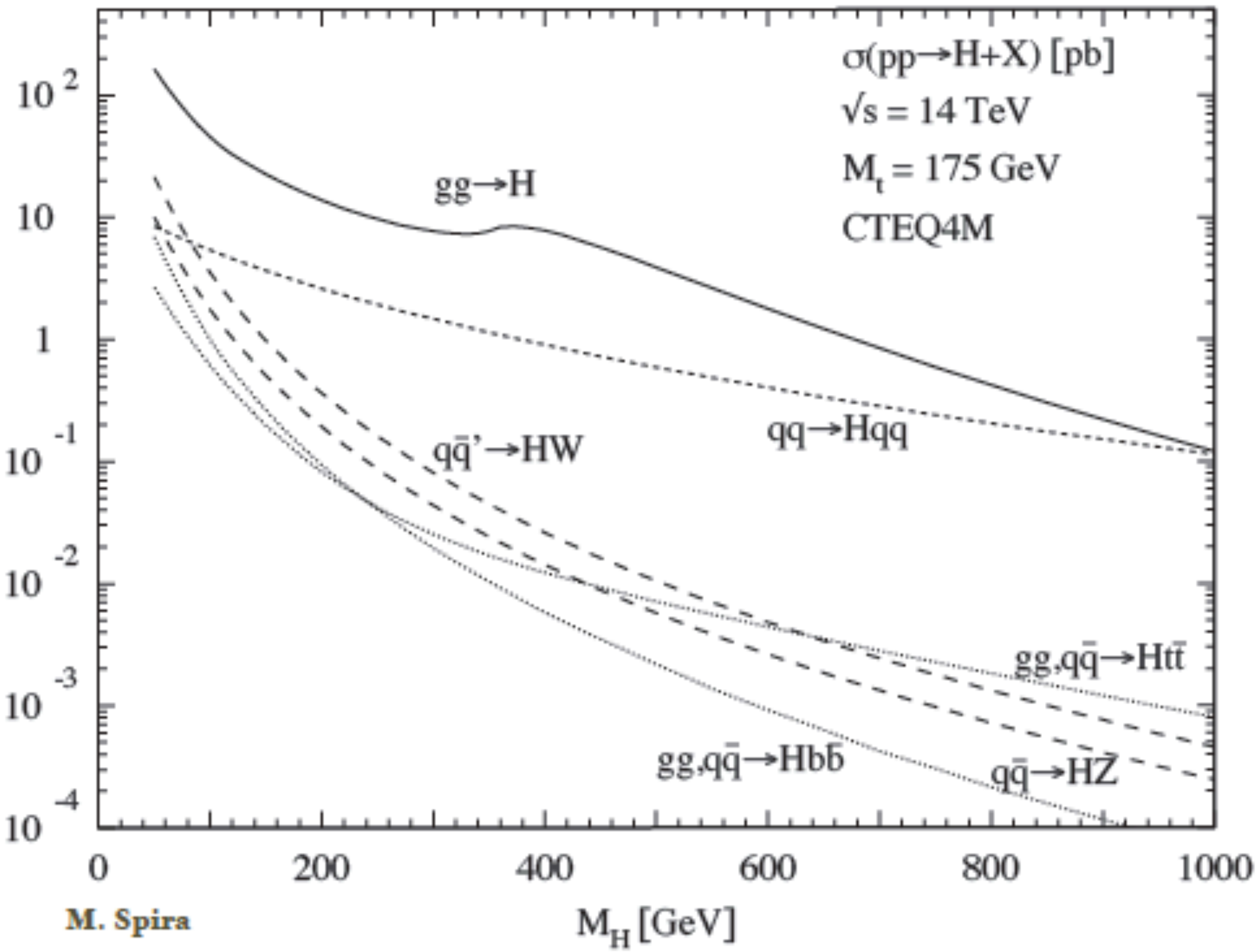
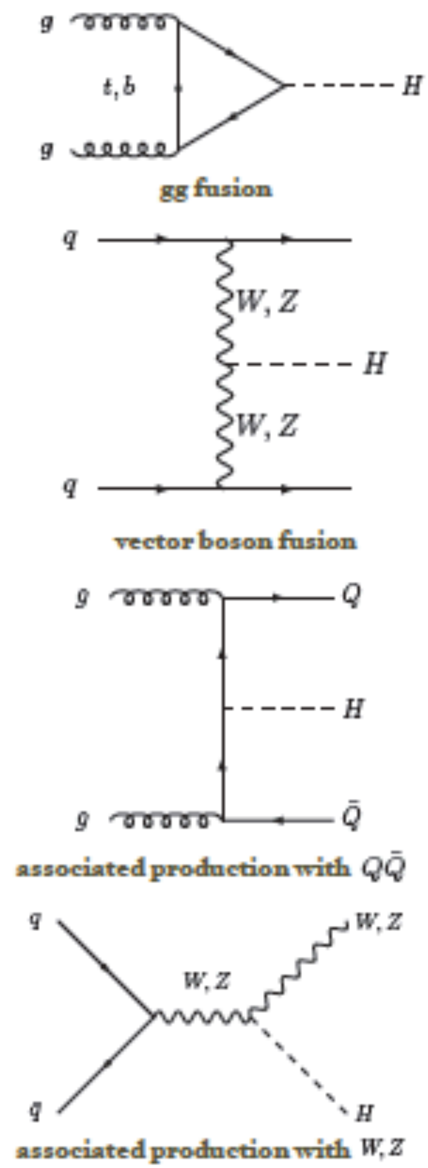
SCALE DEPENDENCE

Higgs production at the LHC





Large gluon luminosity → gg fusion is the dominant production channel over the whole range of M_H

LO,NLO,NNLO accuracy

parton number densities
extracted from the data +evolution
evolution kernel at NNLO: Moch Vermaseren Vogt

hard parton scattering expanded
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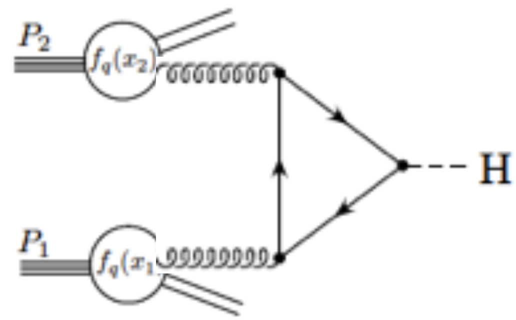

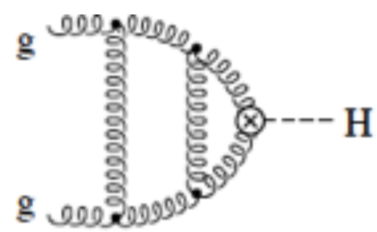

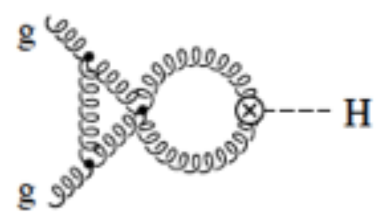

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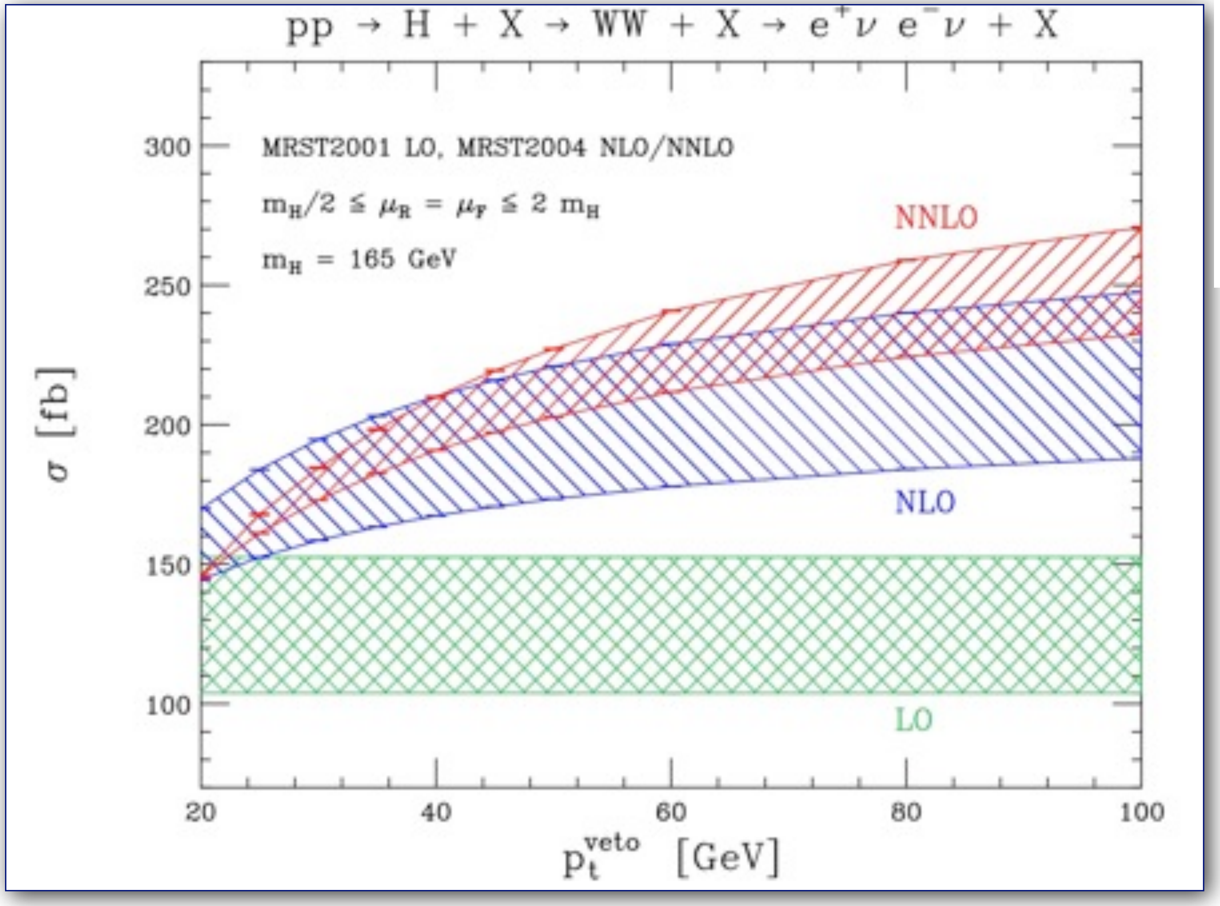

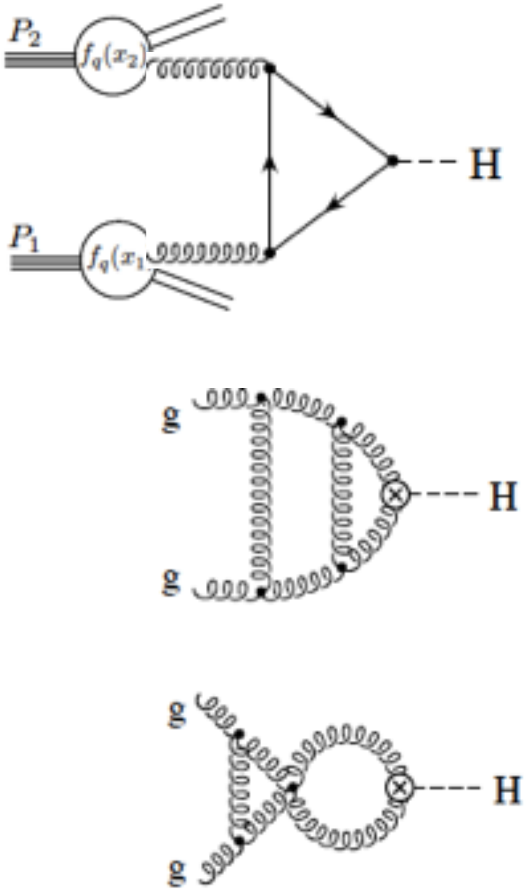

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Minimal Supersymmetric Standard Model

SUSY searches: gluino pair production

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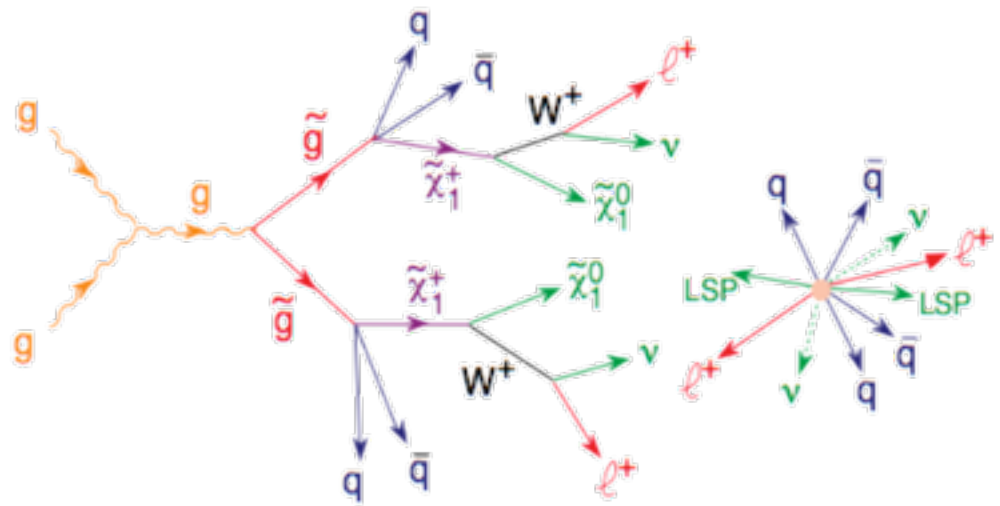

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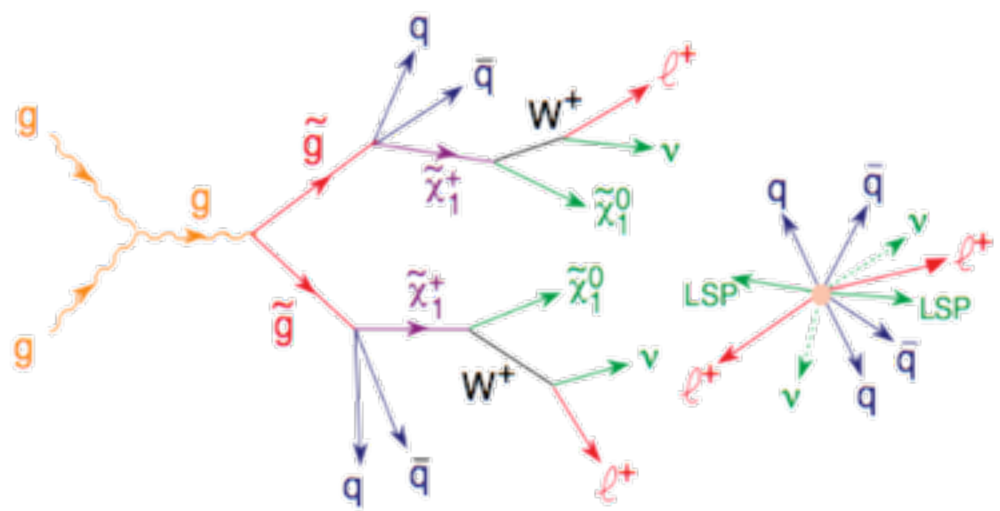

Observable signals:

New physics search, complex final states

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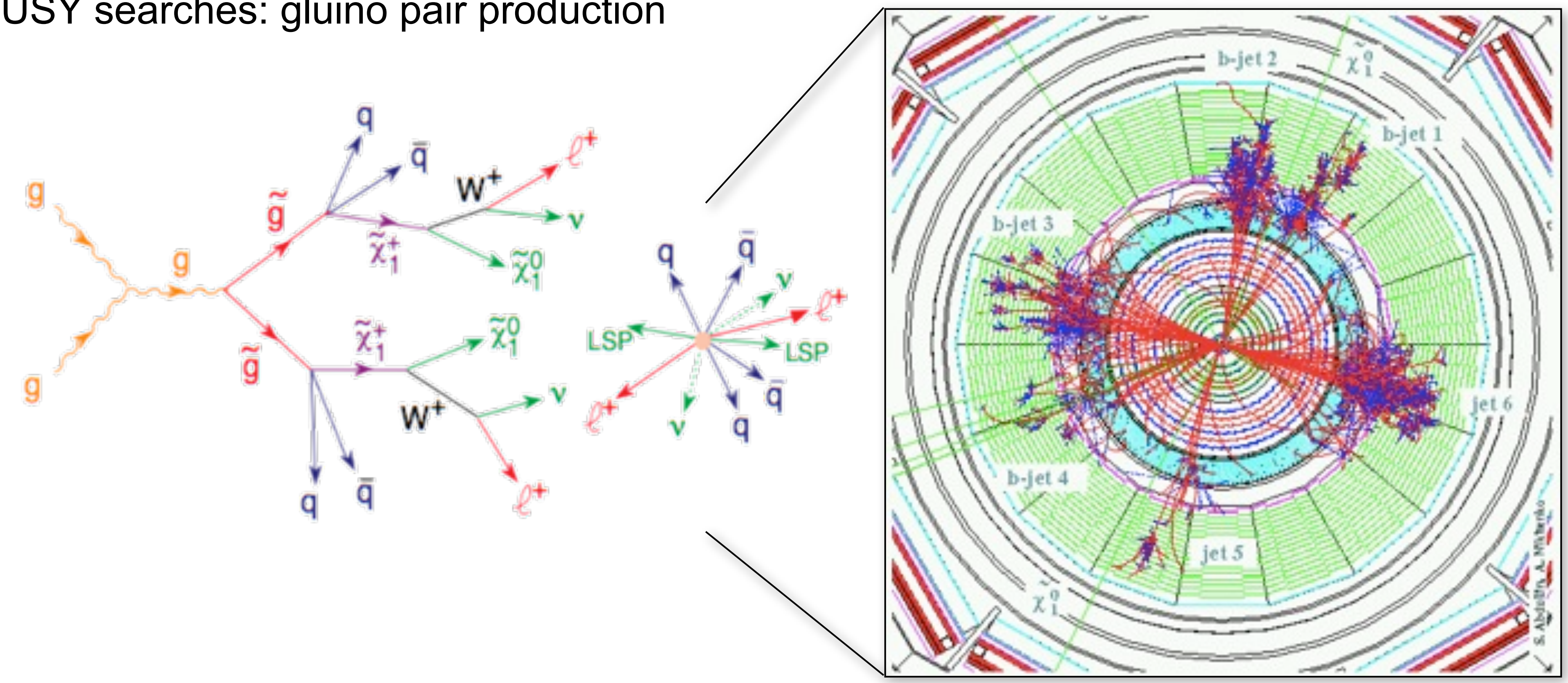
Typical event signature in CMS at LHC:



Observable signals: n leptons + n jets + missing E_T

Large number of SM processes: background to new physics

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An experimenter's wishlist

■ Hadron collider cross-sections one would like to know at NLO

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

Campbell

Next-to-Leading Order QCD Tools: Status and Prospects – p.5/29

Automated leading order calculations

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Engine: generates fortran/C++ codes for tree level matrix element squared, phase space integrations, graphics

Output: as requested (plots, tables, user friendly)

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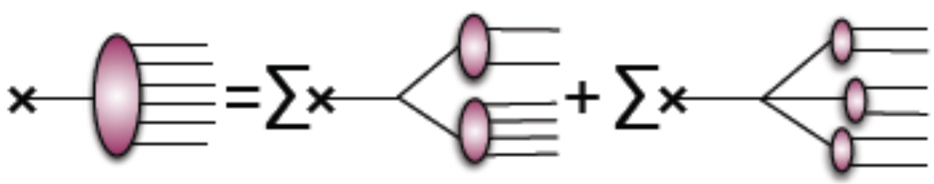

Berends-Giele

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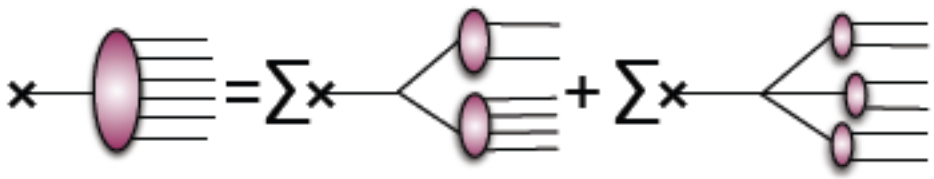

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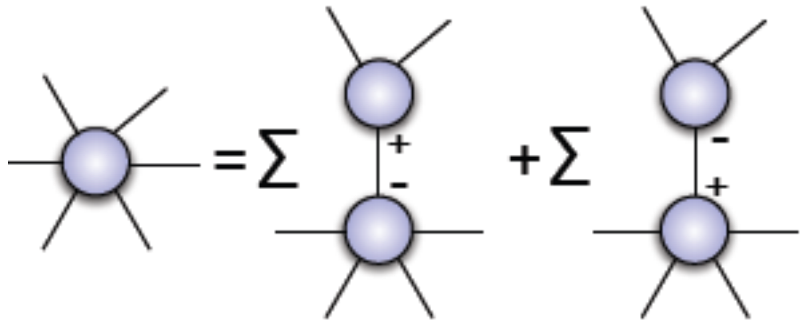

Britto Cachazo Feng

When and why LO and NLO

LO:

- quick testing of new ideas, fully automated,
- large scale dependence, e.g. x-section of W+4jet production is $\approx \alpha_s^4(\mu^2)$

NLO:

- normalization is fixed only at NLO
- more detailed description, better modeling of jets
- if the data

At LHC NLO accuracy is mandatory

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$$d\sigma_n^{(1)} \approx |M_n^{(0)}|^2 d\Phi_{n-2} + 2\mathrm{Re}(M_n^{(0)\dagger} M_n^{(1)}) d\Phi_{n-2} + |M_{n+1}^{(0)}|^2 d\Phi_{n-1}$$

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tree amplitude squared

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automated LO generators

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Virtual contributions

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automated LO generators

semi-automated generators

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Virtual contributions

subtracted tree amplitude squared

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automated LO generators

semi-automated generators

automated dipole, FKS generators

At LHC NLO accuracy is mandatory

$$d\sigma_n^{(1)} \approx |M_n^{(0)}|^2 d\Phi_{n-2} + 2\mathrm{Re}(M_n^{(0)\dagger} M_n^{(1)}) d\Phi_{n-2} + |M_{n+1}^{(0)}|^2 d\Phi_{n-1}$$

tree amplitude squared	Virtual contributions	subtracted tree amplitude squared
$\lvert M_n^{(0)}\rvert^2 d\Phi_{n-2}$	$2\mathrm{Re}(M_n^{(0)\dagger} M_n^{(1)}) d\Phi_{n-2}$	$\lvert M_{n+1}^{(0)}\rvert^2 d\Phi_{n-1}$
automated LO generators	semi-automated generators	automated dipole, FKS generators

↕

Generalized unitarity method is most suitable for automation

At LHC NLO accuracy is mandatory

$$d\sigma_n^{(1)} \approx |M_n^{(0)}|^2 d\Phi_{n-2} + 2\mathrm{Re}(M_n^{(0)\dagger} M_n^{(1)}) d\Phi_{n-2} + |M_{n+1}^{(0)}|^2 d\Phi_{n-1}$$

tree amplitude squared — automated LO generators

Virtual contributions — semi-automated generators

subtracted tree amplitude squared — automated dipole, FKS generators

↕

Generalized unitarity method is most suitable for automation

We counts cuts and not Feynman diagrams. The number of the cuts with n grows polynomially. Automatic calculation up to n=8,...,10 is possible

D-Dimensional unitarity method with OPP reduction

D-Dimensional unitarity method with OPP reduction

color and flavor ordered primitive amplitude

D-Dimensional unitarity method with OPP reduction

color and flavor ordered primitive amplitude

$$\mathcal{A}_N(p_1, p_2, \ldots, p_N; l) = \frac{\mathcal{N}(p_1, p_2, \ldots, p_N; l)}{d_1 d_2 \cdots d_N} = \sum_{i_1 \le i_2 \cdots \le i_5 \le N} \frac{e_{i_1 i_2 i_3 i_4 i_5}}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} +$$

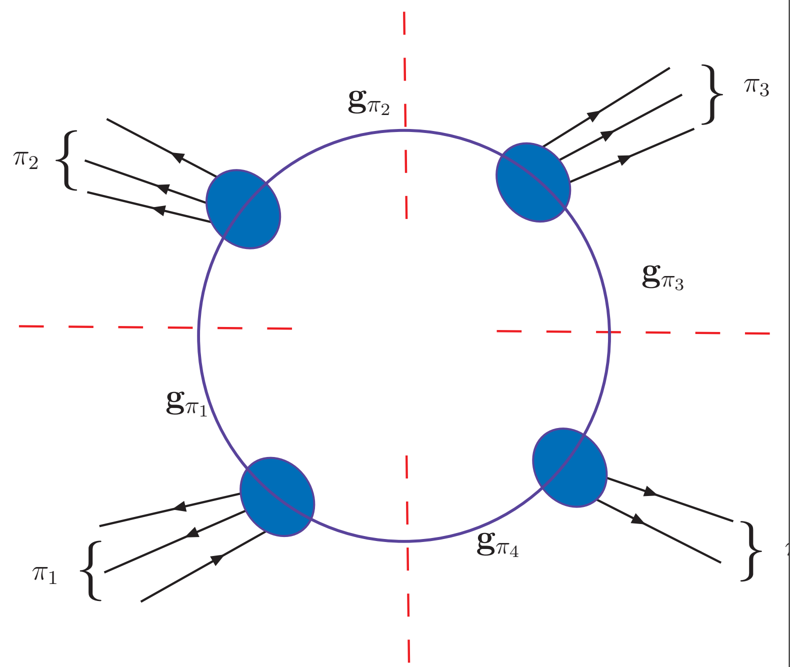
$$\sum_{1 \le i_1 < i_2 < i_3 < i_4 \le N} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1 \le i_1 < i_2 < i_3 \le N} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1 \le i_1 < i_2 \le N} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1 \le i_1 \le N} \frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

D-Dimensional unitarity method with OPP reduction

color and flavor ordered primitive amplitude

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l) = \frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1 d_2 \cdots d_N} = \sum_{i_1 \le i_2 \cdots \le i_5 \le N} \frac{e_{i_1 i_2 i_3 i_4 i_5}}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} +$$

$$\sum_{1\le i_1<i_2<i_3<i_4\le N} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1\le i_1<i_2<i_3\le N} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1\le i_1<i_2\le N} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1\le i_1\le N} \frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$



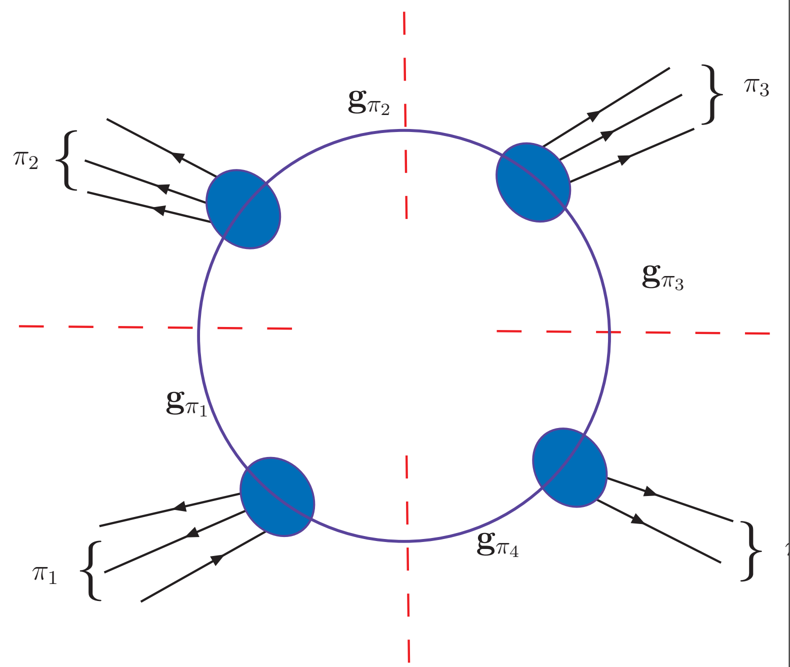
D-Dimensional unitarity method with OPP reduction

color and flavor ordered primitive amplitude

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l) = \frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1 d_2 \cdots d_N} = \sum_{i_1\le i_2 \cdots \le i_5 \le N} \frac{e_{i_1 i_2 i_3 i_4 i_5}}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} +$$

$$\sum_{1\le i_1<i_2<i_3<i_4\le N} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1\le i_1<i_2<i_3\le N} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1\le i_1<i_2\le N} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1\le i_1\le N} \frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

$$\mathbf{d_{ijkl}(l) = d^{(0)}_{ijk} + d^{(1)}_{ijkl}(n_1 l)}$$



D-Dimensional unitarity method with OPP reduction

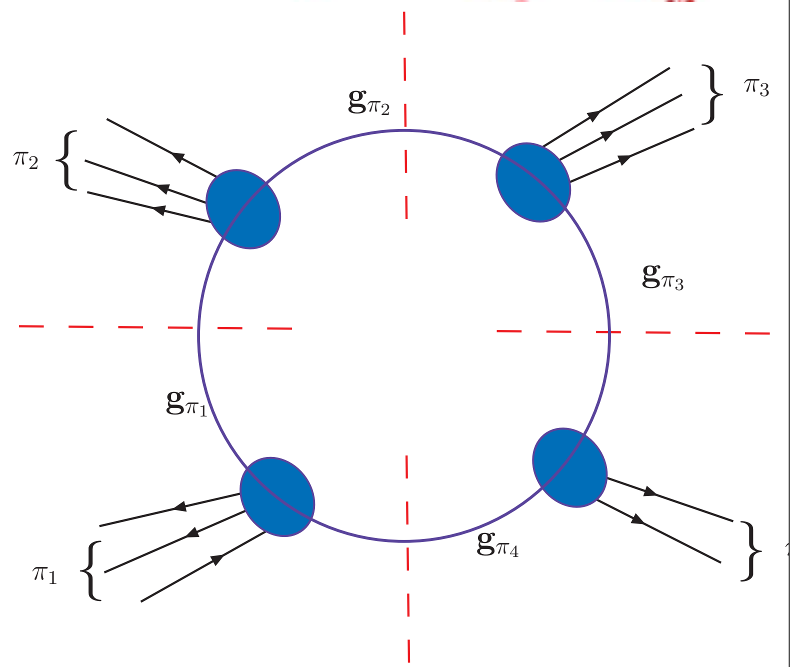
color and flavor ordered primitive amplitude

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l) = \frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1 d_2 \cdots d_N} = \sum_{i_1\le i_2\cdots\le i_5\le N} \frac{e_{i_1 i_2 i_3 i_4 i_5}}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} +$$

$$\sum_{1\le i_1<i_2<i_3<i_4\le N} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1\le i_1<i_2<i_3\le N} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1\le i_1<i_2\le N} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1\le i_1\le N} \frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

$$\mathbf{d_{ijkl}(l) = d^{(0)}_{ijk} + d^{(1)}_{ijkl}(n_1 l)}$$

$$I_{i_1\cdots i_M} = \int [d\,l]\, \frac{1}{d_{i_1}\cdots d_{i_M}}$$



D-Dimensional unitarity method with OPP reduction

color and flavor ordered primitive amplitude

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l)=\frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1d_2\cdots d_N}=\sum_{i_1\le i_2\cdots\le i_5\le N}\frac{e_{i_1i_2i_3i_4i_5}}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}d_{i_5}}+$$

$$\sum_{1\le i_1<i_2<i_3<i_4\le N}\frac{\bar{d}_{i_1i_2i_3i_4}(l)}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}}+\sum_{1\le i_1<i_2<i_3\le N}\frac{\bar{c}_{i_1i_2i_3}(l)}{d_{i_1}d_{i_2}d_{i_3}}+\sum_{1\le i_1<i_2\le N}\frac{\bar{b}_{i_1i_2}(l)}{d_{i_1}d_{i_2}}+\sum_{1\le i_1\le N}\frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

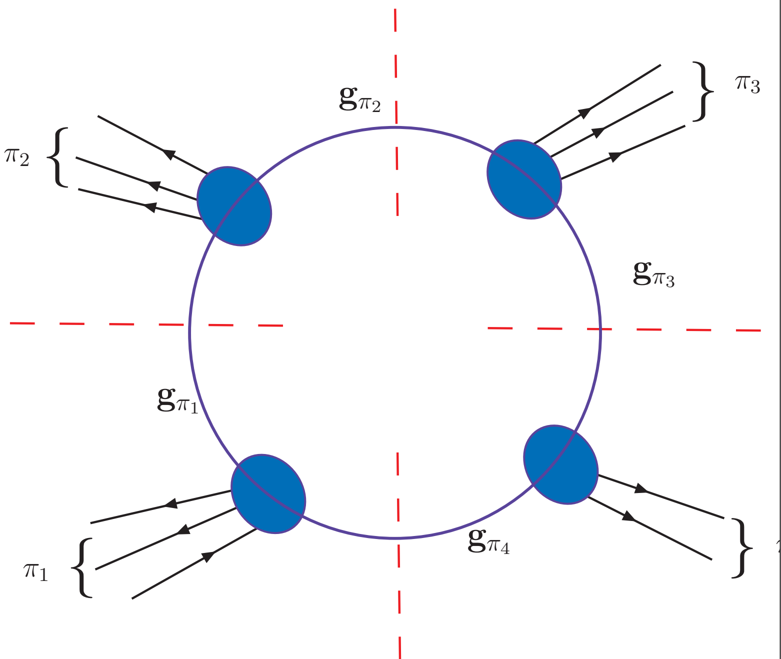
$$\mathbf{d_{ijkl}(l)=d^{(0)}_{ijk}+d^{(1)}_{ijkl}(n_1 l)}$$

$$I_{i_1\cdots i_M}=\int[d\,l]\,\frac{1}{d_{i_1}\cdots d_{i_M}}$$

$$\text{Res}_{ij\cdots k}\left[F(l)\right]\equiv\left[d_i(l)d_j(l)\cdots d_k(l)F(l)\right]\Big|_{l=l_{ij\cdots k}}.$$

$$\bar{d}_{ijkl}(l)=\text{Res}_{ijkl}\left(\mathcal{A}_N(l)\right)=\sum A_i\times A_j\times A_k\times A_l$$

$$\bar{c}_{ijk}(l)=\text{Res}_{ijk}\left(\mathcal{A}_N(l)-\sum_{l\neq i,j,k}\frac{\bar{d}_{ijkl}(l)}{d_id_jd_kd_l}\right)$$



D-Dimensional unitarity method with OPP reduction

color and flavor ordered primitive amplitude

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l) = \frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1 d_2\cdots d_N} = \sum_{i_1\le i_2\cdots\le i_5\le N} \frac{e_{i_1 i_2 i_3 i_4 i_5}}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}d_{i_5}} +$$

$$\sum_{1\le i_1<i_2<i_3<i_4\le N} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}} + \sum_{1\le i_1<i_2<i_3\le N} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1}d_{i_2}d_{i_3}} + \sum_{1\le i_1<i_2\le N} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1}d_{i_2}} + \sum_{1\le i_1\le N} \frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

$$\mathbf{d_{ijkl}(l) = d^{(0)}_{ijk} + d^{(1)}_{ijkl}(n_1 l)}$$

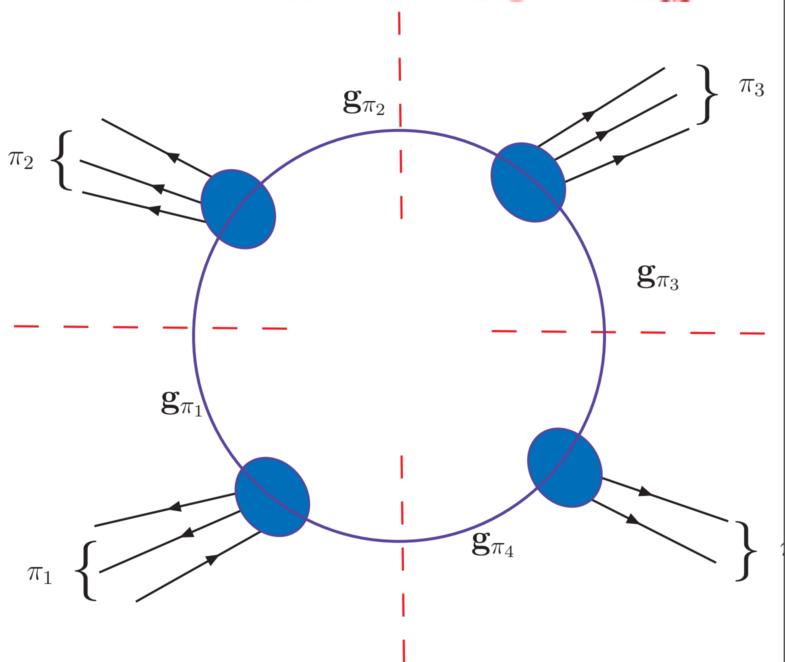
$$I_{i_1\cdots i_M} = \int [d\,l]\, \frac{1}{d_{i_1}\cdots d_{i_M}}$$

$$\mathrm{Res}_{ij\cdots k}\left[F(l)\right] \equiv \left[d_i(l)d_j(l)\cdots d_k(l)F(l)\right]\Big|_{l=l_{ij\cdots k}}.$$

$$\bar{d}_{ijkl}(l) = \mathrm{Res}_{ijkl}\left(\mathcal{A}_N(l)\right) = \sum A_i \times A_j \times A_k \times A_l$$

$$\bar{c}_{ijk}(l) = \mathrm{Res}_{ijk}\left(\mathcal{A}_N(l) - \sum_{l\ne i,j,k} \frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$

$$\mathcal{N}^{(D_s)}(l) = \mathcal{N}_0(l) + (D_s-4)\mathcal{N}_1(l)$$



D-Dimensional unitarity method with OPP reduction

color and flavor ordered primitive amplitude

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l)=\frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1 d_2\cdots d_N}=\sum_{i_1\le i_2\cdots\le i_5\le N}\frac{e_{i_1i_2i_3i_4i_5}}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}d_{i_5}}+$$

$$\sum_{1\le i_1<i_2<i_3<i_4\le N}\frac{\bar{d}_{i_1i_2i_3i_4}(l)}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}}+\sum_{1\le i_1<i_2<i_3\le N}\frac{\bar{c}_{i_1i_2i_3}(l)}{d_{i_1}d_{i_2}d_{i_3}}+\sum_{1\le i_1<i_2\le N}\frac{\bar{b}_{i_1i_2}(l)}{d_{i_1}d_{i_2}}+\sum_{1\le i_1\le N}\frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

$$\mathbf{d_{ijkl}(l)=d^{(0)}_{ijk}+d^{(1)}_{ijkl}(n_1 l)}$$

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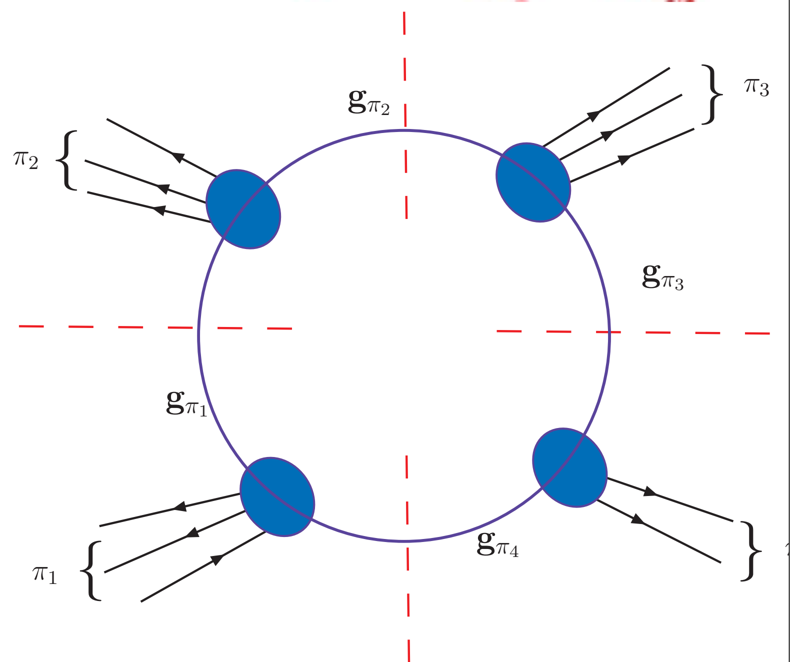


$$\mathcal{N}^{(D_s)}(l)=\mathcal{N}_0(l)+(D_s-4)\mathcal{N}_1(l)$$

$$\mathcal{A}^{\mathrm{FDH}}=2\mathcal{A}_{(D,D_s=5)}-\mathcal{A}_{(D,D_s=6)}$$

D-Dimensional unitarity method with OPP reduction

color and flavor ordered primitive amplitude

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l)=\frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1d_2\cdots d_N}=\sum_{i_1\le i_2\cdots\le i_5\le N}\frac{e_{i_1i_2i_3i_4i_5}}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}d_{i_5}}+$$

$$\sum_{1\le i_1<i_2<i_3<i_4\le N}\frac{\bar{d}_{i_1i_2i_3i_4}(l)}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}}+\sum_{1\le i_1<i_2<i_3\le N}\frac{\bar{c}_{i_1i_2i_3}(l)}{d_{i_1}d_{i_2}d_{i_3}}+\sum_{1\le i_1<i_2\le N}\frac{\bar{b}_{i_1i_2}(l)}{d_{i_1}d_{i_2}}+\sum_{1\le i_1\le N}\frac{\bar{a}_{i_1}(l)}{d_{i_1}}$$

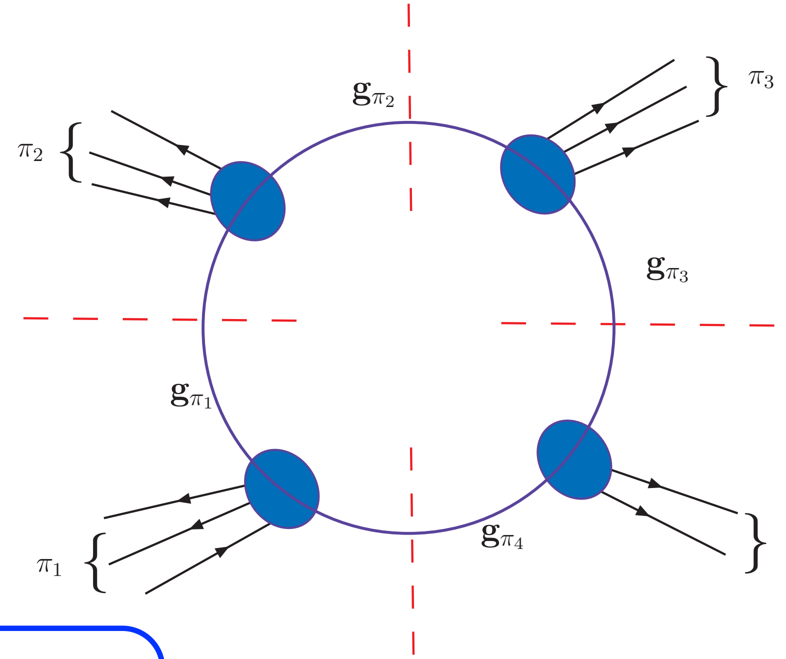
$$\mathbf{d_{ijkl}(l)=d^{(0)}_{ijk}+d^{(1)}_{ijkl}(n_1 l)}$$

$$I_{i_1\cdots i_M}=\int[d\,l]\,\frac{1}{d_{i_1}\cdots d_{i_M}}$$

$$\mathrm{Res}_{ij\cdots k}\left[F(l)\right]\equiv\left[d_i(l)d_j(l)\cdots d_k(l)F(l)\right]\Big|_{l=l_{ij\cdots k}}.$$

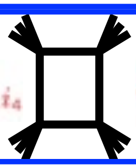
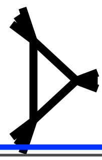
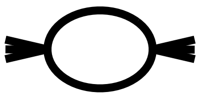
$$\bar{d}_{ijkl}(l)=\mathrm{Res}_{ijkl}\left(\mathcal{A}_N(l)\right)=\sum A_i\times A_j\times A_k\times A_l$$

$$\bar{c}_{ijk}(l)=\mathrm{Res}_{ijk}\left(\mathcal{A}_N(l)-\sum_{l\ne i,j,k}\frac{\bar{d}_{ijkl}(l)}{d_id_jd_kd_l}\right)$$



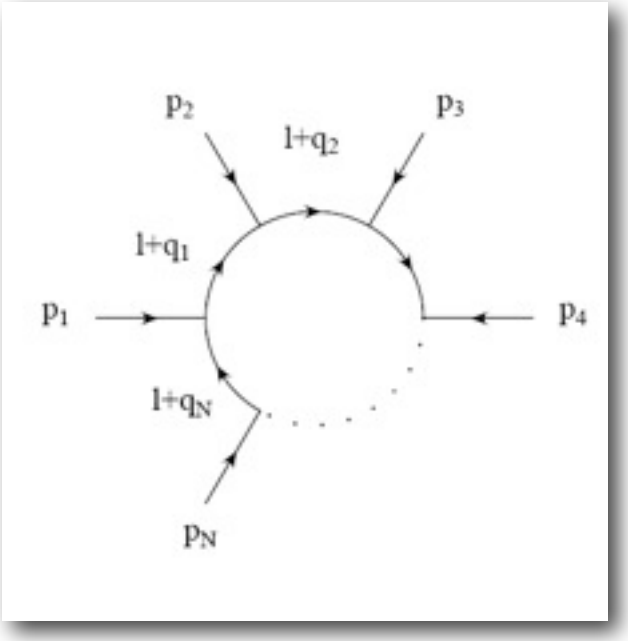
$$\mathcal{N}^{(D_s)}(l)=\mathcal{N}_0(l)+(D_s-4)\mathcal{N}_1(l)$$

$$\mathcal{A}^{\mathrm{FDH}}=2\mathcal{A}_{(D,D_s=5)}-\mathcal{A}_{(D,D_s=6)}$$

$\mathcal{A}_N(\{p_i\})=\sum d_{i_1i_2i_3i_4}$  $+\sum c_{i_1i_2i_3}$  $+\sum b_{i_1i_2}$  $+\mathcal{R}$

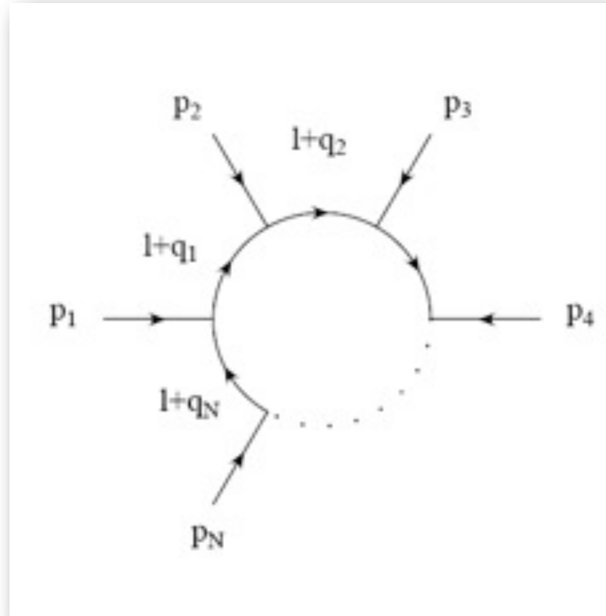
Scaling of the computer time with the number of the external legs

Scaling of the computer time with the number of the external legs



Scaling of the computer time with the number of the external legs

CPU time: n^4 (tree recursive), n^9 (loop ordered colorless, generalized unitarity,
7^n (color included), (n!!)^2 (Feynman diagram based)



Scaling of the computer time with the number of the external legs

CPU time: n^4 (tree recursive), n^9 (loop ordered colorless, generalized unitarity,
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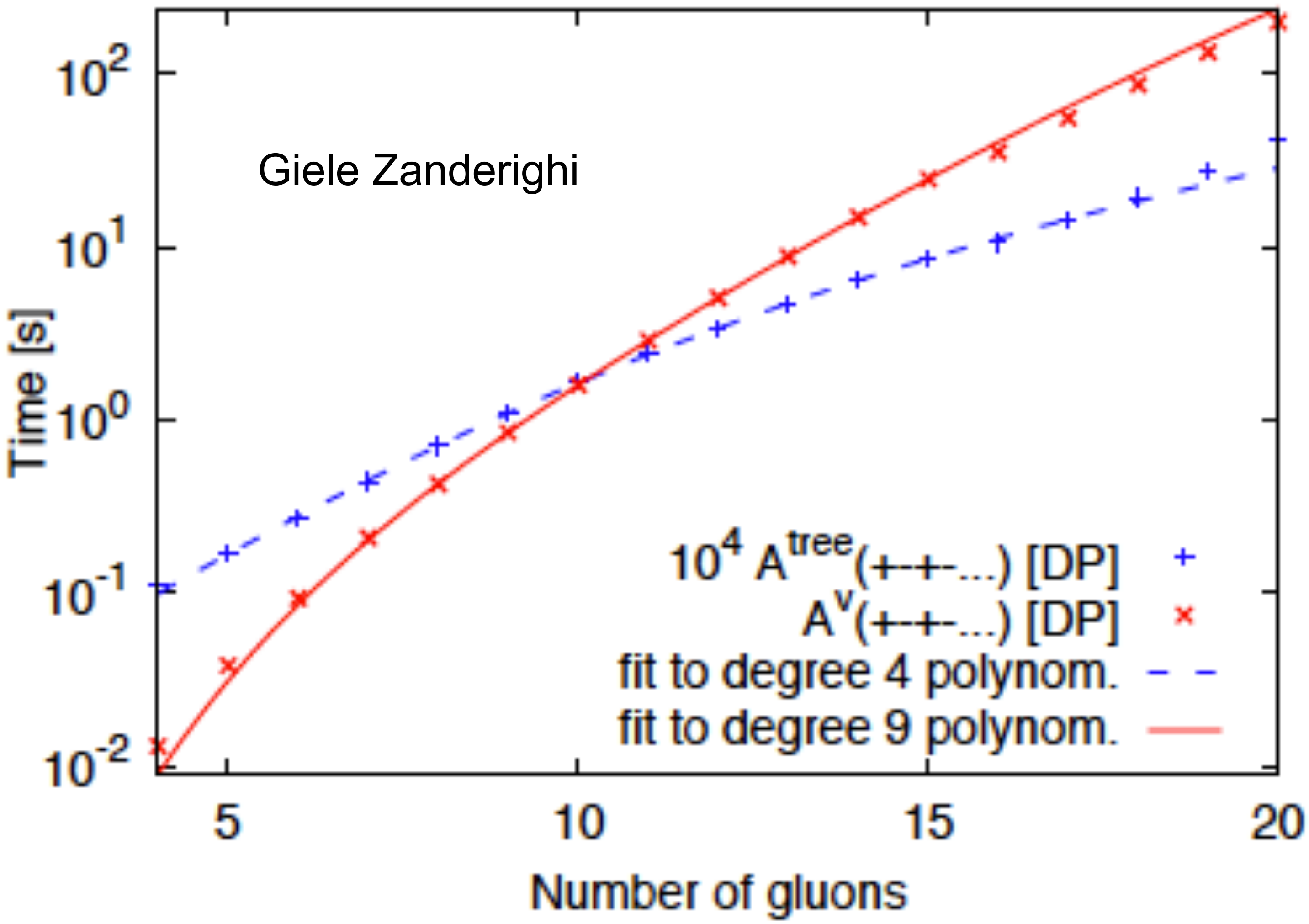
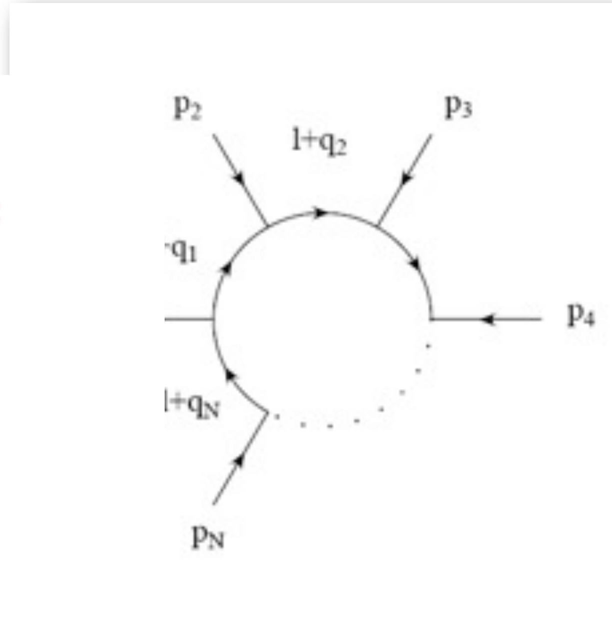
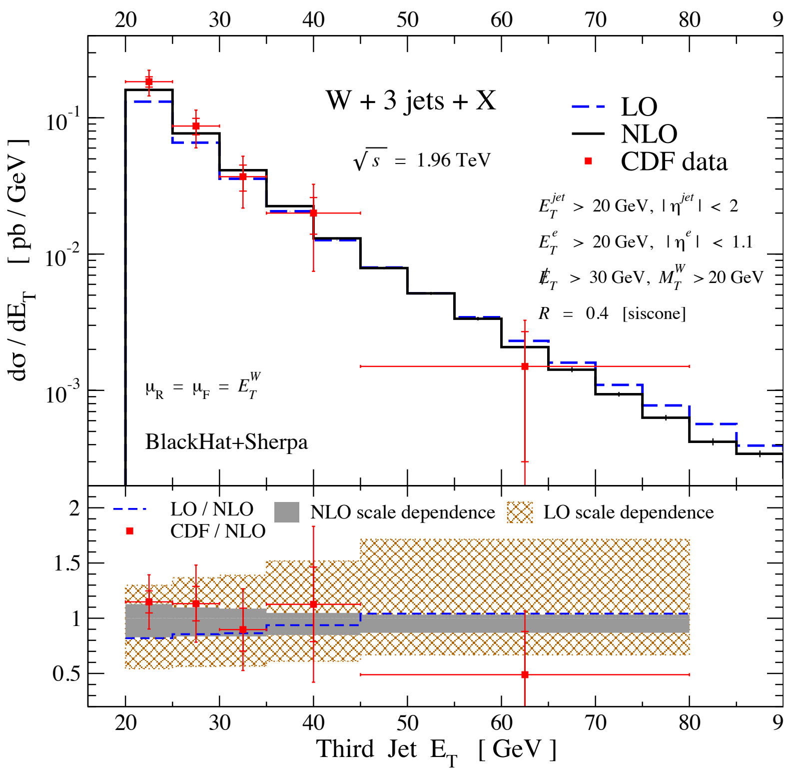
Color ordered amplitudes with n-gluons

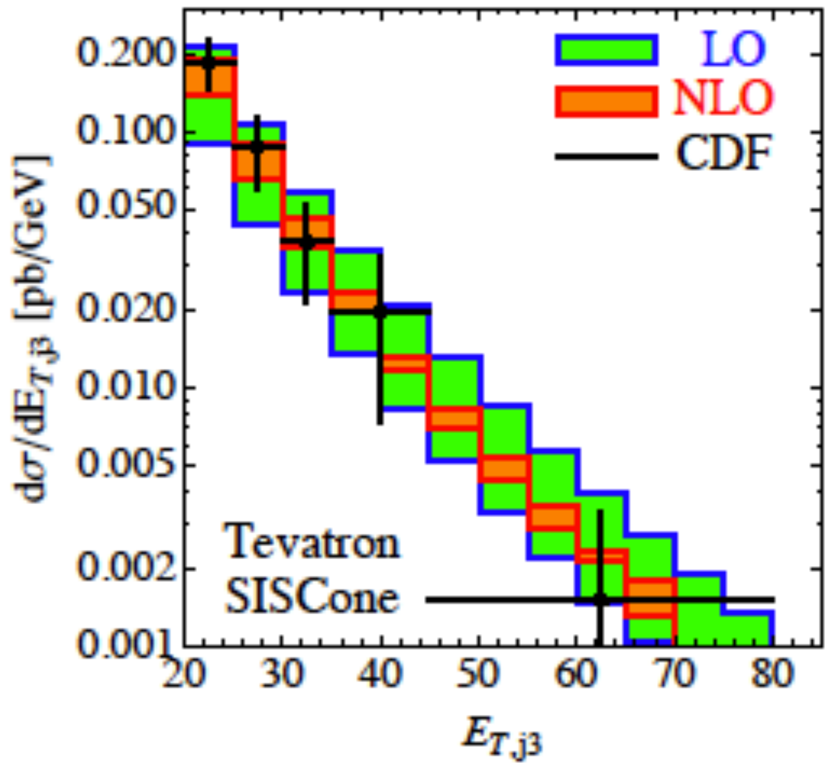




Result for physical cross-sections of 2->4 processes

* NLO W+3jes cross section, leading color
  - Ellis, Melnikov, Zanderighi(2009)   **hep-ph/0901.4101**
  - BlackHat+Sherpa(Gleisberg) **arXiv:0902.1835**, I
* NLO W+3jets cross section, full result
  - BlackHat+Sherpa **arXiv:0907.1984**
* NLO ttbar cross section with decay and full spin correlation
  - Melnikov, Schultze , **arXiv:0907.3090**
* .... Z+3jets, 4jets, W/Z+4jets, ttbar+2jets (with top decay)





2->4 processes, three approaches

1.Traditional methods:

Automatic generation of Feynman one-loop diagrams (FeynArts and QGRAF)
Passarino-Veltman reduction of Tensor integrals (FormCalc).
Algebraic manipulations Mathematica, Maple or Form.
Many special results for 2->2, 2->3 processes (see for example MCFM).

2->4 processes are very challenging: too many diagrams, to many terms given by the reduction of tensor integrals.
Denner, Dittmairer, Pozzorini: p+p->t+tbar+ b+bar. Special effort for effective reduction of the spin structure, color structure and the tensor integrals. Partial result by GOLEM (Binoth et.al.)

2. Helac-CutTools-OPP: some Feynman diagrams are still generated
color flow representation

van Hameren, Papadopoulos ,Pittau, Bequila, Czakon, Worek

3. Generalized unitarity method : loop amplitudes from tree amplitudes
Rocket: Ellis, Giele, ZK, Melnikov, Zanderighi
Black Hat: C.F.Berger, Z. Bern, L.J. Dixon,F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D.A. Kosower, D. Maitre;
winter, lazopoulos

NLO calculations for 2->5 processes in progress

Calculating Multi-Leg One Loop amplitudes with Unitarity Cut Method (Part II)

Zoltan Kunszt

IPMU, March 25, 2010

The Unitarity Method: efficient P(E)-algorithm at NLO

Gauge theory one-loop amplitudes from tree amplitudes many analytic results, with applications to collider physics Bern,Dixon, Kosower, 1994-2007,...

Generalized cuts, box coefficients Britto, Cachazo, Feng 2004
Recursion relations for the rational parts Beger et.al. 2005

Parametric integration method of Ossola, Papadopoulos, Pittau (2006)
Numerical algorithmic implementation of generalized unitarity Ellis, Giele, ZK (2007)
Generalized D-dimensional unitarity Giele, ZK, Melnikov (2008),
Numerical algorithmic implementation by Black Hat Collaboration (2008)

Explicit NLO generators for calculating NLO cross-sections of 6-leg process for LHC and Tevatron Ellis,Melnikov,Zanderighi; BlackHatCollaboration;
van Hameren et.al., Bevilaqua (2009)

Complementary analytic methods Badger (2008), Mastrolia (2009)
Independent numerical implementations (Giele, Winter (2008), Lazoploulos (2008), etc)

with R.K. Ellis, W. Giele, K. Melnikov, J. Winter, G. Zanderighi

Incomplete list of references

JHEP 0803:003,2008 (Ellis, Giele)– cut constructible (completely determined by their unitarity cuts)
JHEP 0804:049,2008 (Giele, Melnikov)– rational part from 5/6 dimensional cuts
Nucl. Phys. B822:270-282,2009 (Ellis, Giele, Melnikov)– extensions to massive external fermions
JHEP 0901:012,2009 (Ellis, Giele, Melnikov, Zandrighi)– W+3jets one loop amplitudes
arXiv:0911.1962, Nucl. Phys.B (to appear) ˚ (Giele, Winter)– color-dressed

Z. Bern, L. J. Dixon, D.C. Dunbar and D.A. Kosower, Nucl. Phys. B 435,59,(1995)
E. Witten, Comm. Math. Phys. 252,189 (2004)
R. Britto, F. Cachazo and B. Feng, Nucl. Phys. B 725, 275 (2005)
G. Ossola, C.G. Papadopoulos and R. Pittau, Nucl. Phys. B763 (2007)
} cut constructible amplitudes

D. Forde, Phys.Rev.D75, 125019 (2007)
C. Berger, Z. Bern, L.J. Dixon, D. Forde, D.A. Kosower, Phys. Rev.D 74 (2006) — loop amplitudes via recursion
R.K.Ellis, K. Melnikov, G. Zanderighi Phys.Rev.D.80.094002 (2009) — W+3jet physical cross sections
C.F.Berger, Z. Bern, L.J. Dixon,F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D.A. Kosower, D. Maitre (BLACK HAT)
Phys.Rev.Lett. 102.222001 (2009); Phys.Rev. D80:074036,(2009)
Bevilacqua, Czakon, Papadopoulos, M. Worek, arXiv:1002.4009 — t+tbar +2jets

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WItten: Perturbative gauge theories in twistor space, Comm. Math. Phys. 252,189, 2004; on-shell amplitudes with complex momentum components
Passarino&Veltman, Reduction to scalar integrals, Nucl. Phys. B 160 (1979)
van Neerven&Vermaseren Large Loop Integrals, Phys. Lett. 137B (1984)
van Neerven, Nucl. Phys. B268, 453 (1986)
Z. Bern&A.G. Morgan, Massive loop amplitudes from unitarity, Nucl. Phys. B467 (1996)

OPP method for the cut constructible part of the amplitude

OPP method for the cut constructible part of the amplitude

parametric integral over the loop momentum

OPP method for the cut constructible part of the amplitude

parametric integral over the loop momentum

Lemma 1.

OPP method for the cut constructible part of the amplitude

parametric integral over the loop momentum

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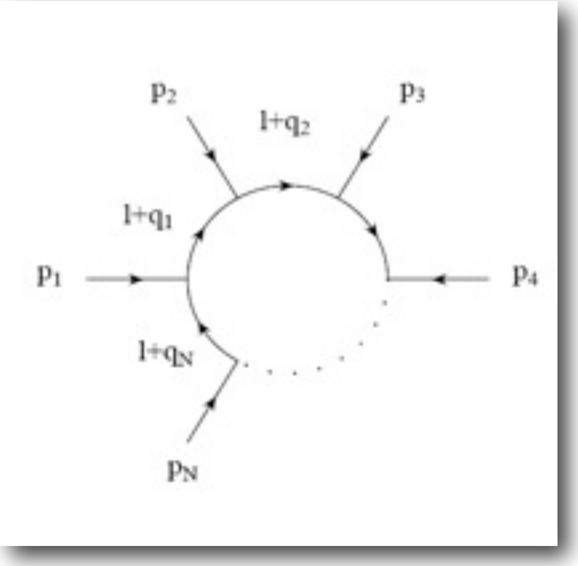
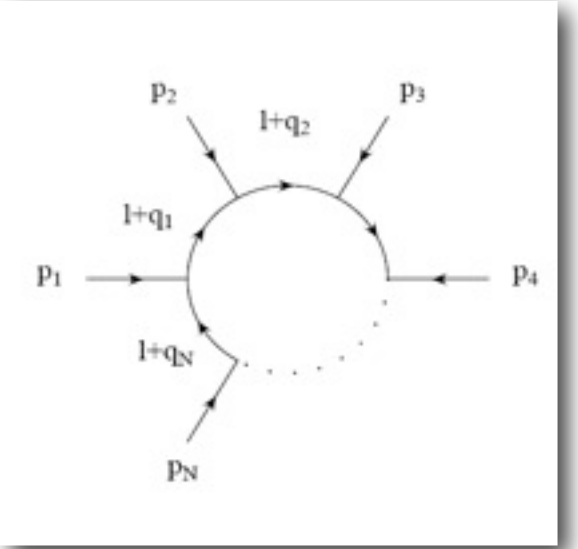


OPP method for the cut constructible part of the amplitude

parametric integral over the loop momentum

Lemma 1.

Assuming four-dimensional space time,
the integrand of any one-loop Feynman amplitude with arbitrary number of external legs can be written in the standard form of linear combination of quadro-,triple-,double-,single-pole terms



OPP method for the cut constructible part of the amplitude

parametric integral over the loop momentum

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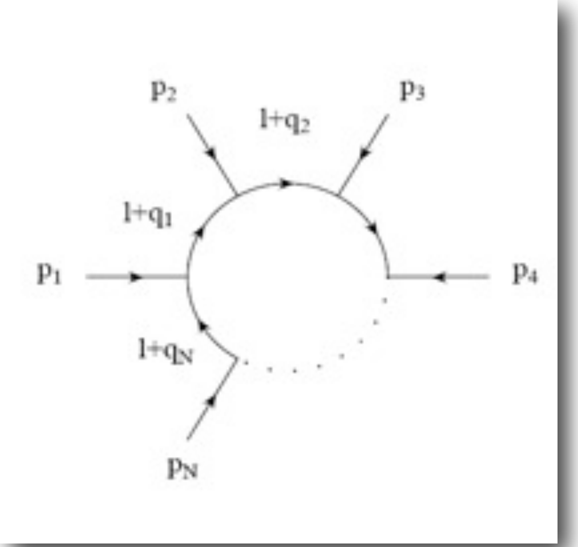

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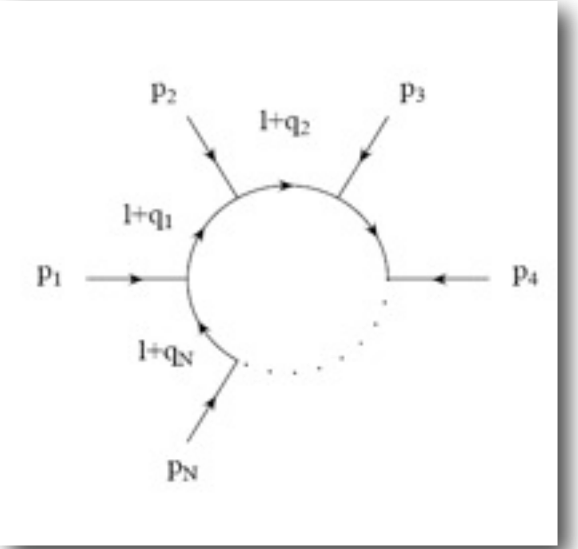

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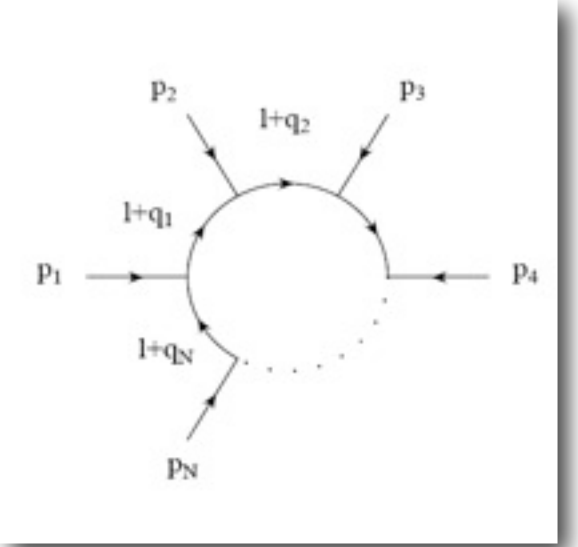

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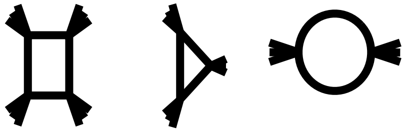

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Reduction of scalar triangle integrand to bubble integrand in D=2

van Neerven Vermaseren

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devide second equation by D_0, D_1, D_2

Donnerstag, 25. März 2010

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$$b_0 = \frac{1}{2} \left(\frac{1}{(l_c^+ + q_2)^2 - m_2^2} + \frac{1}{(l_c^- + q_2)^2 - m_2^2} \right) \quad b_1 = \frac{1}{2\alpha^+} \left(\frac{1}{(l_c^+ + q_2)^2 - m_2^2)} - \frac{1}{(l_c^- + q_2)^2 - m_2^2} \right)$$

Parameterization of the loop momentum, general case

The loop momenta is decomposed in terms of VN basis vectors

we define: **a set of dual momenta** v_i $\quad v_i p_j = \delta_{ij}$

and : **a set of orthogonal unit vectors** $n_i, \quad n_i p_j = 0$

$$l^\mu = V_R^\mu + \sum_{i=1}^{D_P} \frac{1}{2}(d_i - d_{i-1})\, v_i^\mu + \sum_{i=1}^{D_T} \alpha_i\, n_i^\mu \,, \qquad V_R^\mu = -\frac{1}{2}\sum_{i=1}^{D_P}\left((q_i^2 - m_i^2) - (q_{i-1}^2 - m_{i-1}^2)\right) v_i^\mu$$

$$v_i^\mu(k_1,\ldots,k_{D_P}) \equiv \frac{\delta^{k_1\ldots k_{i-1}\mu k_{i+1}\ldots k_{D_P}}_{k_1\ldots k_{i-1}k_i k_{i+1}\ldots k_{D_P}}}{\Delta(k_1,\ldots,k_{D_P})}\,, \qquad \delta^{\mu_1\mu_2\cdots\mu_R}_{\nu_1\nu_2\cdots\nu_R} = \begin{vmatrix} \delta^{\mu_1}_{\nu_1} & \delta^{\mu_1}_{\nu_2} & \ldots & \delta^{\mu_1}_{\nu_R} \\ \delta^{\mu_2}_{\nu_1} & \delta^{\mu_2}_{\nu_2} & \ldots & \delta^{\mu_2}_{\nu_R} \\ \vdots & \vdots & & \vdots \\ \delta^{\mu_R}_{\nu_1} & \delta^{\mu_R}_{\nu_2} & \ldots & \delta^{\mu_R}_{\nu_R} \end{vmatrix}\,,$$

Gram-determinant

Van Neerven-Vermaseren: reduction at the integrand level

loop momenta on the cut $d_j = 0$

1. Quadrupole cut $d_i=d_j=d_k=d_l=0$ (two solutions)

$$l^\mu = V_4^\mu + \alpha_1 n_1^\mu$$

$$l_\pm^\mu = V_4^\mu \pm i\sqrt{V_4^2 - m_l^2} \times n_1^\mu$$

Complex valued loop momenta

2. Triple cut, infinite number of solutions (on a circle circle)

$$l^\mu = V_3^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu$$

$$l_{\alpha_1\alpha_2}^\mu = V_3^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu;\ \alpha_1^2 + \alpha_2^2 = -(V_3^2 - m_k^2)$$

3. Double cut, infinite number of solutions (on a “sphere”)

$$l^\mu = V_2^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu + \alpha_3 n_3^\mu$$

$$l_{\alpha_1\alpha_2\alpha_3}^\mu = V_2^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu + \alpha_3 n_3^\mu;\ \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = -(V_2^2 - m_j^2)\ .$$

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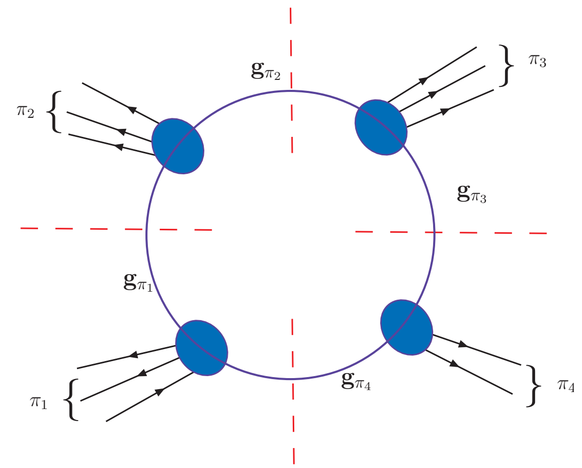

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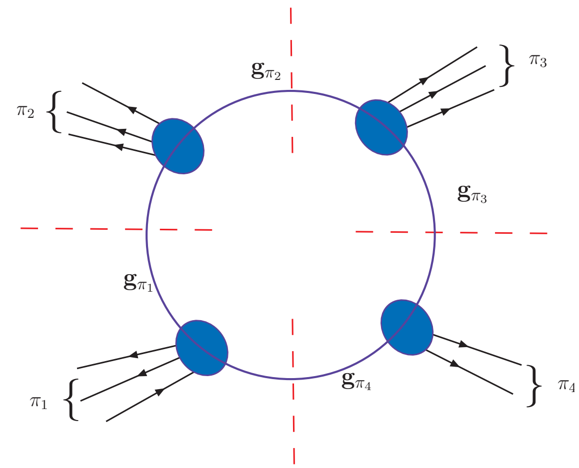
residue of 4,3,2,1 simultaneous poles
factorize to product of tree amplitudes



The parameters are fixed by linear algebraic equations

choose a cut type

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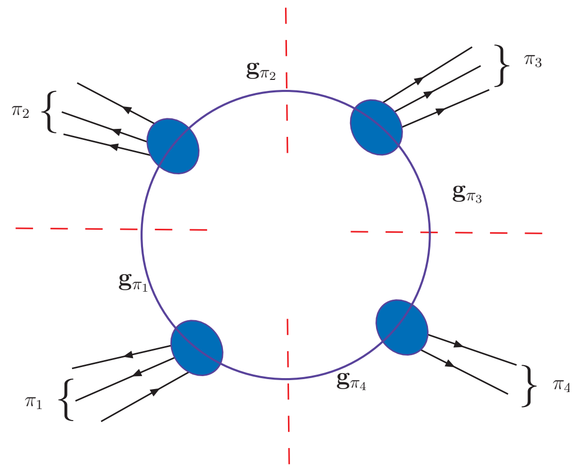
Right side: product of tree amplitudes
at special values of loop momenta
Left side side: linear combination of coefficients



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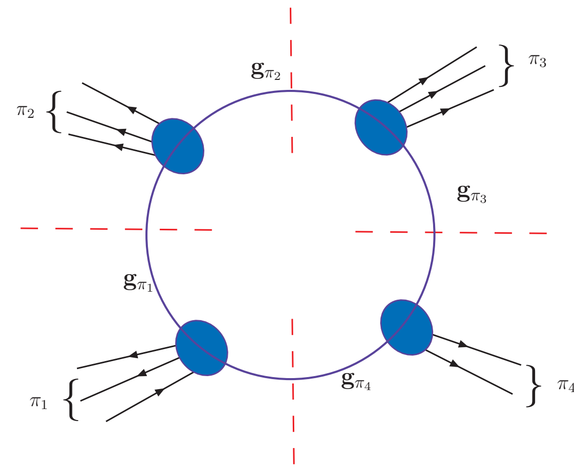

$\bar{d}_{ijkl}(l) = \text{Res}_{ijkl}\left(\mathcal{A}_N(l)\right)$ $d_i=d_j=d_k=d_l=0$ two solutions, 4-cut

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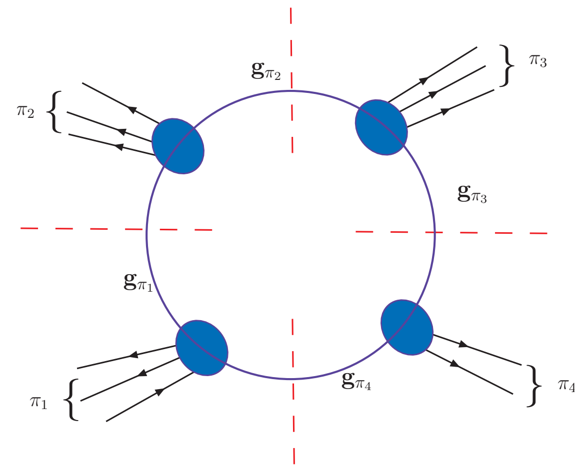

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$d_i=d_j=d_k=0$ infinite # of solutions

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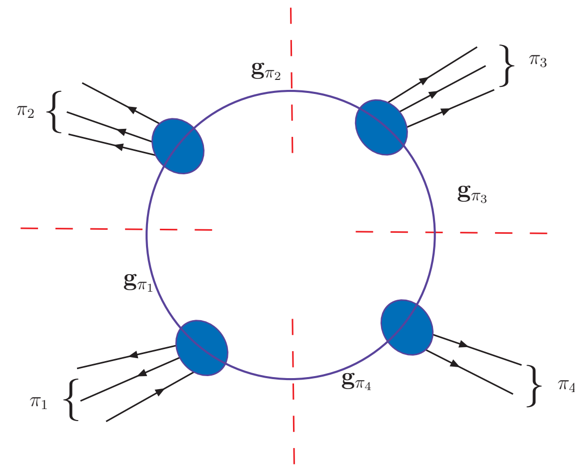

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$d_i=d_j=d_k=0$ infinite # of solutions

$$\bar{b}_{ij}(l) = \mathrm{Res}_{ij}\left(\mathcal{A}_N(l) - \sum_{k\neq i,j}\frac{\bar{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{2!}\sum_{k,l\neq i,j}\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$

$d_i=d_j=0$

infinite # of solutions

The box residue

The box residue

$$\text{Res}_{2346}\left(\mathcal{A}_6(l^\pm)\right) = \mathcal{M}_4^{(0)}(l_6^\pm; p_1, p_2; -l_2^\pm) \times \mathcal{M}_3^{(0)}(l_2^\pm; p_3; -l_3^\pm)\mathcal{M}_3^{(0)}(l_3^\pm; p_4; -l_4^\pm)$$

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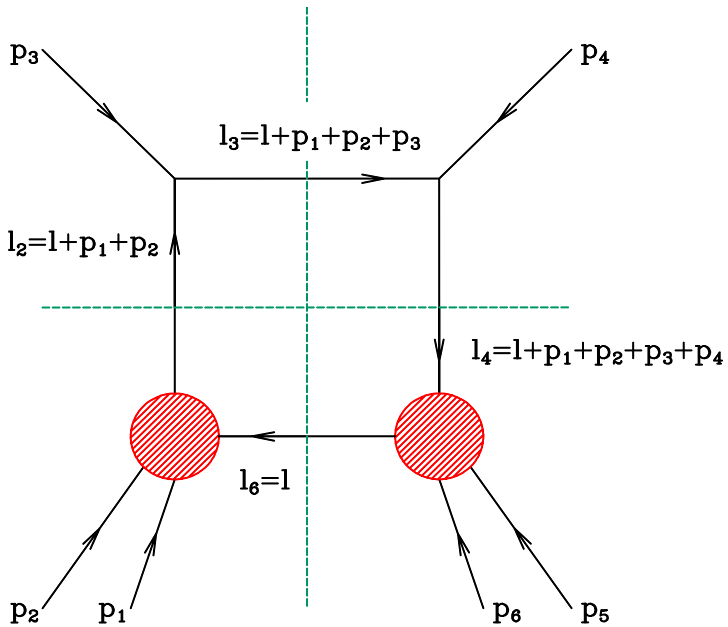

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$$d_{ijkl} = \frac{\text{Res}_{ijkl}\big(\mathcal{A}_N(l^+)\big) + \text{Res}_{ijkl}\big(\mathcal{A}_N(l^-)\big)}{2}$$

$$\tilde{d}_{ijkl} = \frac{\text{Res}_{ijkl}\big(\mathcal{A}_N(l^+)\big) - \text{Res}_{ijkl}\big(\mathcal{A}_N(l^-)\big)}{2i\sqrt{V_4^2 - m_l^2}}$$

Unitarity in D-dimension: uniform treatment of the cut constructible and rational parts (GKM)

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Two sources of D-dependence

i) spin-polarization states live in D_s .

ii) loop momentum component live in D. (D_s>D)

$$\mathcal{A}_{(D,D_s)}(\{p_i\},\{J_i\}) = \int \frac{d^D l}{i(\pi)^{D/2}} \frac{\mathcal{N}^{(D_s)}(\{p_i\},\{J_i\};l)}{d_1 d_2 \cdots d_N}.$$

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We can calculate the D_s dependence before carrying out the integral over the loop momentum

$$\sum_{i=1}^{D_s-2} e_\mu^{(i)}(l) e_\nu^{(i)}(l) = -g_{\mu\nu}^{(D_s)} + \frac{l_\mu b_\nu + b_\mu l_\nu}{l \cdot b},$$

$$l^2 = \bar{l}^2 - \tilde{l}^2 = l_1^2 - l_2^2 - l_3^2 - l_4^2 - \sum_{i=5}^{D} l_i^2$$

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$$\mathcal{N}^{(D_s)}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

full D_s dependence

- **Choose two integer values $D_s = D_1$ and $D_s = D_2$ to reconstruct the full D_s dependence.**
- **Suitable for numerical implementation**
- **$D_s=4-2\varepsilon$ 't Hooft Veltman scheme, $D_s=4$ FDHS** (Bern, Koswer)
- **for closed fermion loops** $\mathcal{N}^{D_s}(l) = 2^{(D_s-4)/2}\mathcal{N}_0(l)$

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The loop momentum effectively has only 4+1 component

$$\mathcal{N}(l) = \mathcal{N}(\tilde{l}, \mu), \qquad l^2 = \tilde{l}^2 - \mu^2$$

maximum 5 unitarity constraints: pentagon cuts

Loop integrals are in $D < D_s$ dimensions $D = 4 - 2\epsilon$

New structures and new integrals

$$\bar{e}^{(D_s)}_{ijkmn}(l) = e^{(D_s,(0))}_{ijkmn}$$

no new scalar integrals

$$\bar{d}^{\mathrm{FDH}}_{ijkn}(l) = d^{(0)}_{ijkn} + d^{(1)}_{ijkn} s_1 + (d^{(2)}_{ijkn} + d^{(3)}_{ijkn} s_1) s_e^2 + d^{(4)}_{ijkn} s_e^4,$$

two new scalar integrals

$$\bar{c}^{\mathrm{FDH}}_{ijk}(l) = \ldots + c^{(7)}_{ijk} s_1 s_e^2 + c^{(8)}_{ijk} s_2 s_e^2 + c^{(9)}_{ijk} s_e^2,$$

one new scalar integrals

$$\bar{b}^{\mathrm{FDH}}_{ij}(l) = \ldots + b^{(9)}_{ij} s_e^2$$

one new scalar integrals

$$s_e^2 = -\sum_{i=5}^{D} (l \cdot n_i)^2 = -\sum_{i=5}^{D} (\tilde{l} \cdot n_i)^2$$

The full ϵ -dependence is obtained via the integrals

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = -\frac{D-4}{2} I^{D+2}_{i_1 i_2 i_3 i_4},$$

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^4}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = \frac{(D-2)(D-4)}{4} I^{D+4}_{i_1 i_2 i_3 i_4},$$

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3}} = -\frac{(D-4)}{2} I^{D+2}_{i_1 i_2 i_3},$$

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2}} = -\frac{(D-4)}{2} I^{D+2}_{i_1 i_2}.$$

$$\lim_{D\to 4} \frac{(D-4)}{2} I^{(D+2)}_{i_1 i_2 i_3 i_4} = 0,$$

$$\lim_{D\to 4} \frac{(D-4)(D-2)}{4} I^{(D+4)}_{i_1 i_2 i_3 i_4} = -\frac{1}{3},$$

$$\lim_{D\to 4} \frac{(D-4)}{2} I^{(D+2)}_{i_1 i_2 i_3} = \frac{1}{2},$$

$$\lim_{D\to 4} \frac{(D-4)}{2} I^{(D+2)}_{i_1 i_2} = -\frac{m_{i_1}^2 + m_{i_2}^2}{2} + \frac{1}{6}\left(q_{i_1} - q_{i_2}\right)^2 .$$

One-loop amplitudes up to terms of order ε

One loop amplitudes as sum of cut-constructible and rational parts:

$$\mathcal{A}_N = \mathcal{A}_N^{CC} + R_N .$$

The cut constructible part is as before (EGK):

$$\mathcal{A}_N^{CC} = \sum_{[i_1|i_4]} \tilde{d}^{(0)}_{i_1 i_2 i_3 i_4} I^{(4-2\epsilon)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3} I^{(4-2\epsilon)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2} I^{(4-2\epsilon)}_{i_1 i_2} + \sum_{i_1=1}^{N} a^{(0)}_{i_1} I^{(4-2\epsilon)}_{i_1} ,$$

The rational part is new (GKM):

$$R_N = -\sum_{[i_1|i_4]} \frac{d^{(4)}_{i_1 i_2 i_3 i_4}}{6} + \sum_{[i_1|i_3]} \frac{c^{(7)}_{i_1 i_2 i_3}}{2} - \sum_{[i_1|i_2]} \left(\frac{(q_{i_1} - q_{i_2})^2}{6} - \frac{m_{i_1}^2 + m_{i_2}^2}{2} \right) b^{(9)}_{i_1 i_2} ,$$

D-dimensional unitary algorithm for massive fermions (EGKM)

Application to ggtt and gggtt

- We have to choose even values for D_s

$$\mathcal{A}^{\mathrm{FDH}} = 2\mathcal{A}_{(D,D_s=6)} - \mathcal{A}_{(D,D_s=8)}$$

- Pentagon, box, triangle ,bubble and tadpole cuts
- The treatment of bubble and tadpole cuts is more subtle:
  i) light-like bubbles, tadpoles
  ii) (1,n-1) partitioning of the n-legs has to be included
  unitarity has difficulty with self-energy insertions on external legs
- Particles of different flavors: more sophisticated bookkeeping
- Color and "flavor ordered" primitive amplitudes
- More master integrals (use QCDLoop, Ellis, Zanderighi)

Dirac spinors in 6 dimensions

gamma-matrices in $D_s = 4$ $\{\gamma^0, \gamma^1, \gamma^2, \gamma^3, \gamma^5\}$

gamma-matrices in $D_s = 6$ $\Gamma^0 = \begin{pmatrix} \gamma^0 & 0 \\ 0 & \gamma^0 \end{pmatrix}, \quad \Gamma^{i=1,2,3} = \begin{pmatrix} \gamma^i & 0 \\ 0 & \gamma^i \end{pmatrix}, \quad \Gamma^4 = \begin{pmatrix} 0 & \gamma^5 \\ -\gamma^5 & 0 \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & i\gamma^5 \\ i\gamma^5 & 0 \end{pmatrix}$

Dirac spinors in D_s dimensions

$$u^{(s)}(l,m) = \frac{(l_\mu \Gamma^\mu + m)}{\sqrt{l_0 + m}} \eta^{(s)}_{D_s}, \quad s = 1, \ldots, 2^{D_s/2-1} . \quad \text{in } D_s = 4: \quad \eta_4^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \eta_4^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},$$

in $D_s = 6$, they are constructed recursively

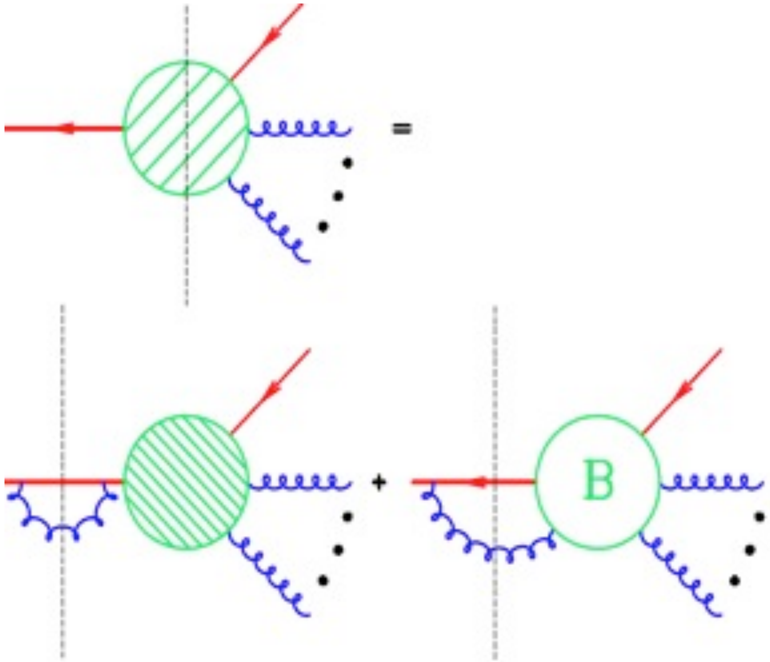
$$\eta_6^{(1)} = \begin{pmatrix} \eta_4^{(1)} \\ \mathbf{0} \end{pmatrix}, \quad \eta_6^{(2)} = \begin{pmatrix} \eta_4^{(2)} \\ \mathbf{0} \end{pmatrix}, \quad \eta_6^{(3)} = \begin{pmatrix} \mathbf{0} \\ \eta_4^{(1)} \end{pmatrix}, \quad \eta_6^{(4)} = \begin{pmatrix} \mathbf{0} \\ \eta_4^{(2)} \end{pmatrix}.$$

conjugate spinors:

$$\bar{u}^{(s)}(l,m) = \bar{\eta}^{(s)}_{D_s} \frac{(l_\mu \Gamma^\mu + m)}{\sqrt{l_0 + m}}$$

l_μ is not conjugated

Self-energy on external massive fermion leg



For massless line: vanishing contributions

Tree amplitude on the right hand side is not well defined

$$\mathrm{Res}\left[\mathcal{A}^{[1]}(t, g_1, \ldots, g_n, \bar{t})\right] \sim \sum_{\text{states}} \mathcal{A}^{[0]}(t, g^*, \bar{t}^*) \times \mathcal{A}^{[0]}(t^*, g^*, g_1, \ldots, g_n, \bar{t}) \ ,$$

$$\mathcal{A}^{[0]}(t^*, g^*, g_1, \ldots, g_n, \bar{t}) = \frac{R(t^*, g^*, g_1, \ldots, g_n, \bar{t})}{(p_{t*} + p_{g^*})^2 - m_t^2} + B(t^*, g^*, g_1, \ldots, g_n, \bar{t}).$$

Self-energy contribution, gauge invariance and generic conflict with unitarity

Feynman diagram calculation:

i) one particle reducible self-energy corrections on external legs are discarded

ii) Their effects are included by wave-function renormalization constants (Z_2)

Follow the same path:

i) discard the term in the tree amplitude generating one particle reducible diagrams

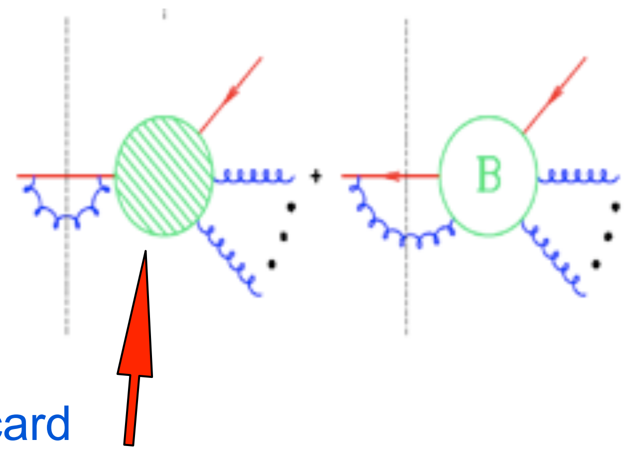
BG recursion relations can accommodate it by truncating the recursive steps

ii) It is taken into account by adding later wave function renormalization

The remaining part of the amplitude (B) is not gauge invariant

iii) **The gauges used to calculate Z_2 and B must be the same**

It mildly violates "unitarity ": sum over non-physical states



discard

C++ code for dressed amplitudes

- Implemented algorithm based on previous implementation of ordered amplitudes
  Giele and Winter, arXiv:0902.0094

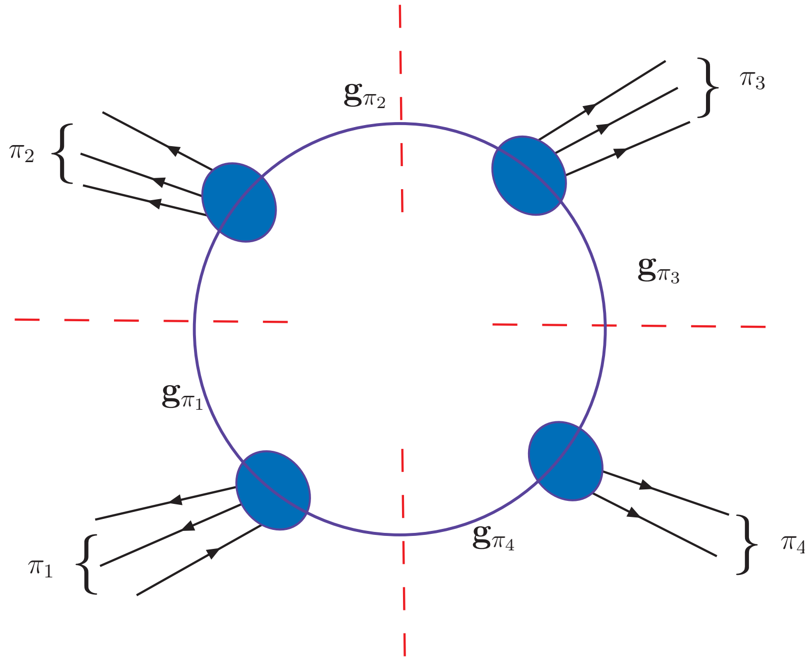
- New features:

• Sum over ordered cuts changes to sum over partitions including non-cyclic, non-reflective permutations of the external partons

$$\sum_{[i_1|i_k]} \rightarrow \sum_{RP_{\pi_1 \dots \pi_k}(1,2,..,N)}$$

• Residues are given by color dressed tree amplitudes and summed over internal color and spin

• Symmetry factor, e.g. 1/2 for bubble

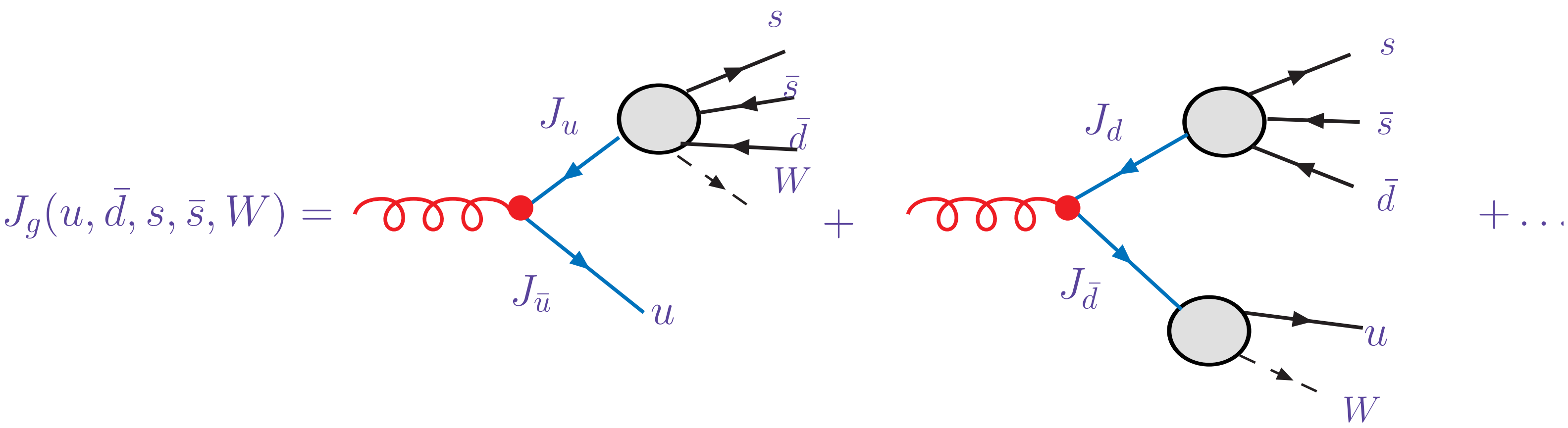


- **Sum over ordered cuts changes to sum over partitions including non-cyclic, non-reflective premutations of the external partons**

$$\mathcal{A}^{(1)}\left(\mathbf{f}_1,\ldots,\mathbf{f}_n \mid \ell\right) = \sum_{k=1}^{C_{\max}} \sum_{RP_{\pi_1\cdots\pi_k}(1,2,\ldots,n)}^{\max\left(1,\frac{1}{2}(k-1)!\right)\mathcal{S}_2(n,k)} \sum_{g_{\Pi_1},\ldots,g_{\Pi_k}} \frac{\mathcal{P}_k\left(\vec{C}_{g_{\Pi_1}\cdots g_{\Pi_k}} \mid \ell\right)}{d_{g_{\Pi_1}}(\ell)\, d_{g_{\Pi_2}}(\ell)\cdots d_{g_{\Pi_k}}(\ell)}\,,$$

- sum over the propagator flavors $g_{\Pi_1},\ldots,g_{\Pi_k}$ is required as these are not uniquely defined for unordered amplitudes

BG recursion relations:



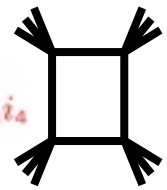
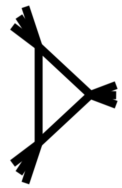
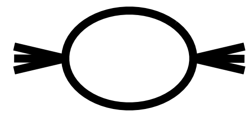
$$
\begin{aligned}
P_{\mathbf{g}}\left[D\left[J(\mathbf{f}_{\pi_1}),J(\mathbf{f}_{\pi_2})\right]\right] \quad &= \quad \frac{1}{K^2_{\Pi_2}} D^{i;IJ;j}_{s_1\mu s_2} \times J^i_{s_1}(\mathbf{f}_{\pi_1}) \times J^j_{s_2}(\mathbf{f}_{\pi_2}) \\
&+ \quad \frac{1}{K^2_{\Pi_2}} D^{IJ;i_1j_1;i_2j_2}_{\mu\mu_1\mu_2}(-K_{\pi_1\cup\pi_2},K_{\pi_1},K_{\pi_2}) \times J^{\mu_1}_{i_1j_1}(\mathbf{f}_{\pi_1}) \times J^{\mu_2}_{i_2j_2}(\mathbf{f}_{\pi_2}) \ .
\end{aligned}
$$

Automation of the calculation of the subtracted radiative x-section

subtracted tree amplitude squared

$$d\sigma_n^{(1)} \approx |M_n^{(0)}|^2 d\Phi_{n-2} + 2\mathrm{Re}(M_n^{(0)\dagger} M_n^{(1)}) d\Phi_{n-2} + |M_{n+1}^{(0)}|^2 d\Phi_{n-1}$$

automated dipole, FKS generators

$M_n^{(1)}(\{p_i\}) = \sum d_{i_1 i_2 i_3 i_4}$  $+ \sum c_{i_1 i_2 i_3}$  $+ \sum b_{i_1 i_2}$  $+\mathcal{R}$

$$M_n^{(1)} = c_\Gamma \left(\frac{N}{\epsilon^2} + \left(\sum_{i=1}^{n} \ln \frac{-s_{i,i+1}}{\mu^2} - \frac{11}{3} \right) \right) M_n^{(0)}$$

$$\sigma^{\mathrm{NLO}} = \int d\Phi_{n-2}^{(4)} \sigma^{\mathrm{B}} + \int d\Phi_{n-2}^{(4)} \sigma^{\mathrm{V}} + \int d\Phi_{n-1}^{(4-2\epsilon)} \sigma^{R}$$

UV renormalized virtual corrections are infrared divergent
it is cancelled by the real contribution for IR safe quantities

Automated FKS subtraction in MadEvent

If the emitted gluon index i is soft and collinear with parton j the matrix element blows up as

$$|M_{n+1}^{(1)}|^2 \rightarrow \frac{1}{\xi_i^2}\frac{1}{1-y_{ij}} \qquad \xi_i = \frac{E_i}{\sqrt{\hat{s}}} \qquad y_{ij} = \cos\Theta_{ij}$$

Partition the phase space into sectors such that each sector has at most one soft and one collinear singularity

$$d\sigma^{\mathrm{R}} = \sum_{ij} S_{ij}|M_{n+1}|^2 d\Phi_{n-1} \qquad \sum_{ij} S_{ij} = 1$$

Use plus distributions to regulate the singularities

$$d\tilde{\sigma}^{\mathrm{R}} = \sum_{ij}\left(\frac{1}{\xi_i}_+\right)\left(\frac{1}{1-y_{ij}}\right)_+ \xi_i(1-y_{ij})S_{ij}|M_{n+1}|^2 d\Phi_{n-1}$$

MadFKS + dipole subtractions

MadFKS + dipole subtractions

- Given the (n+1) process, it generates the real subtracted x- section and the Born x-section

- For an NLO computation we need to input only the finite parts of virtual corrections for the requested process

Input from Black Hat and Rocket

- Phase-space integration for n and (n+1) body process is carried out at the same time

MadFKS + dipole subtractions

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Input from Black Hat and Rocket

- Phase-space integration for n and (n+1) body process is carried out at the same time

Automated Dipole subtraction

Gleisberg, Krauss (Black Hat)

Independent implementations of Dipole Subtraction

(Ellis et.al.,Czakon et.al.)

Concluding remarks

- We have an efficient method to calculate the full one-loop amplitude from tree amplitudes for SM and BSM theories

- Cut-constructible part and rational part is treated uniformly

- A significant number of one-loop virtual amplitudes are implemented in F90 and C++ codes.
  First results for W+3jet cross-sections are encouraging

- The Rocket/Black Hat/CutTool-Helac
  can be developed to fully automated NLO
  generators for SM and BSM up to 7 (?) leg processes

- Generalized unitarity provides the most robust algorithm for full automation. NLO codes likely will be integrated to existin LO codes (ALPGEN, MADGRAPH,SHERPA etc.)