

Potential applications of optical lattice clocks in geodesy -an introduction-

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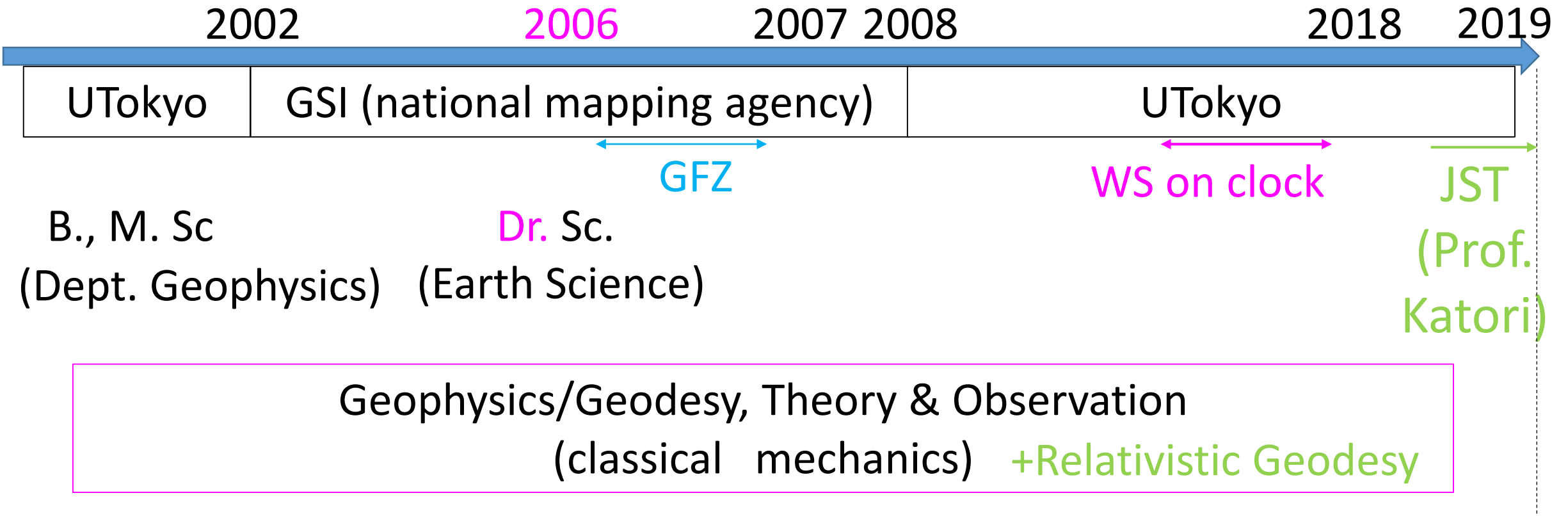
Key words:

geodesy, reference coordinate system,
gravity observation, relativity, clock

Japan Science and
Technology Agency



Self-introduction



The purpose of today's talk

Fundamental Theories of Physics

{ General relativity
Quantum mechanics

Violation of EEP

Dark matter/energy/fluid?
How is the universe changing?

Observations

Cosmology
Space

Physicists
e.g., Altschul+(2015)

Geodesy
Near the Earth

Physicists
Geodesists

Basic knowledge

- How do new optical clocks impact the confirmation of those theories?

- Altschul et al., 2015. Quantum Tests of the Einstein Equivalence Principle with the STE-QUEST Space Mission, Adv.Space Res. 55, 501-524 (physics)
- Mehlstäubler et al., 2018. Atomic clocks for geodesy, Rep. Prog. Phys. 81, 064401 (physics & geodesy)
- Müller et al., 2018. High Performance Clocks and Gravity Field Determination, Space Sci Rev, 214:5 (geodesy)

Outline

- Basic concept of geodesy (Part I)
 - Geophysical signals and measurement methods
 - Reference coordinate systems for geodesy
 - Height and geoid
- Chronometric geodesy using optical clocks (Part II)
 - Progress in Europe
 - Progress in Japan
- A few topics on clock and relativity (short)
 - Issue on the realization of the SI second
 - Relativistic effects in geodetic measurements

Basic concept of Geodesy

- Helmert (1843-1917) developed mathematical theories of geodesy.
“Geodesy is the science of measuring and representing the Earth’s surface.”
- Scientific and practical motivations

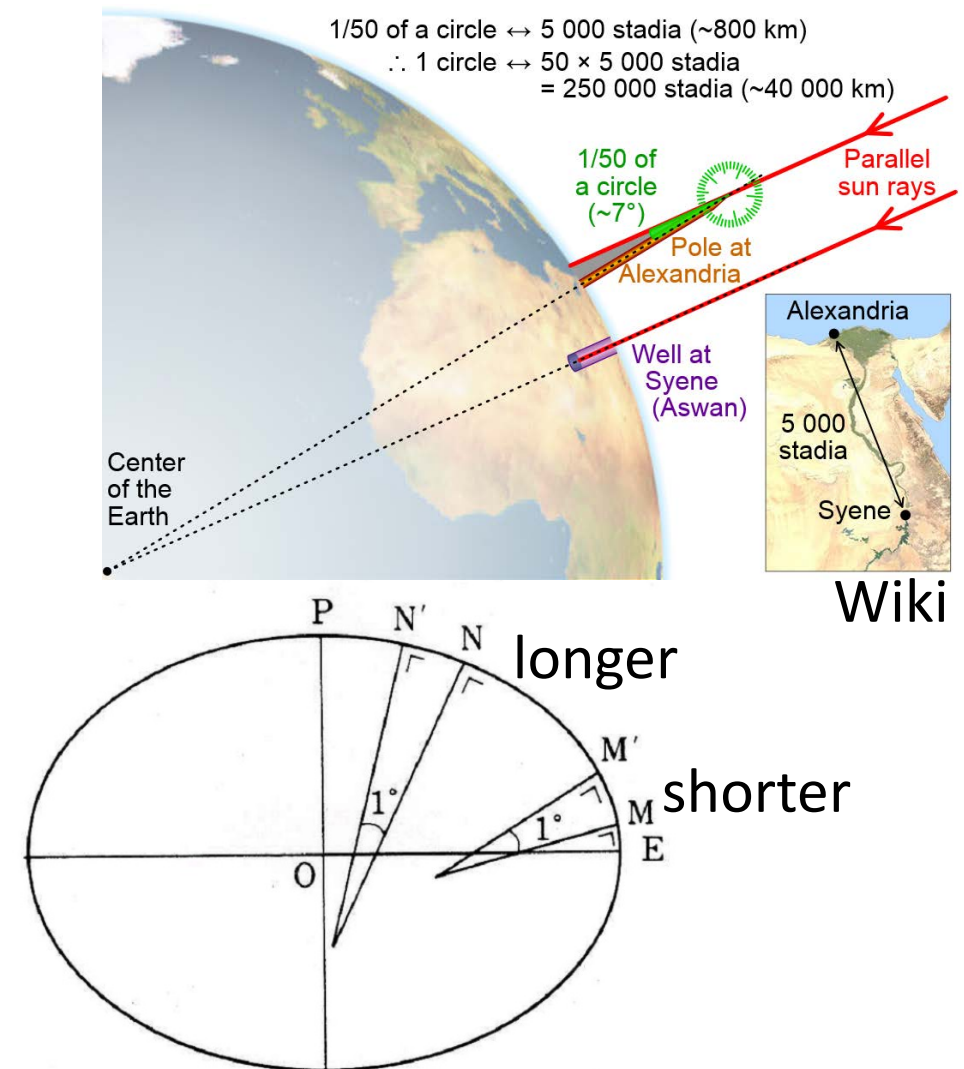


German Research Center for Geoscience (GFZ Potsdam), Dept. 1

Geodesy as Science

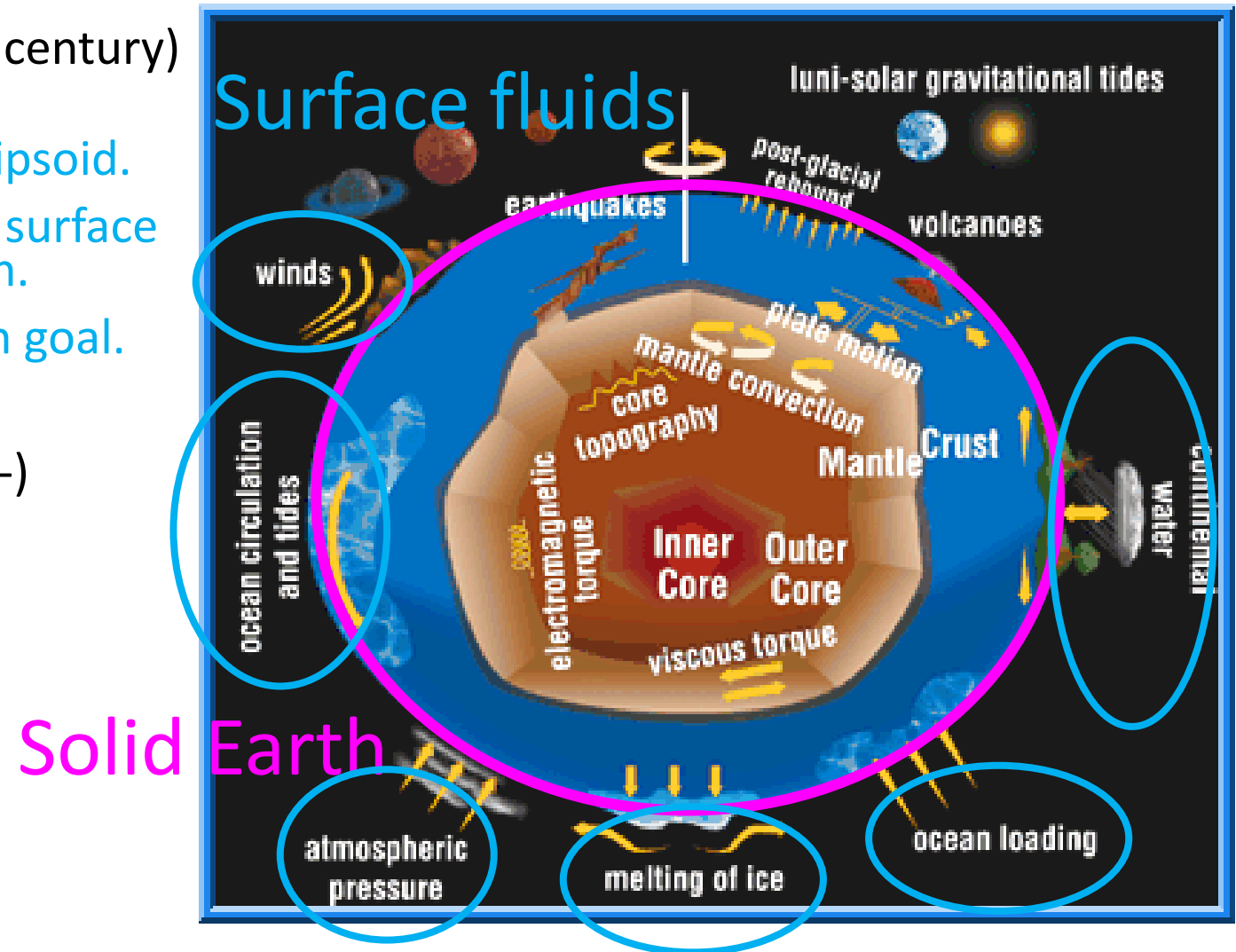
History of “the Earth’s shape” Soffel (1989)

- Phase A (Spherical Earth model: 200 BC – mid 17th century)
 - Eratosthenes: $\Delta r/r=16\%$ (BC)
 - Snellius: 3.4%, Picard: 0.1%, telescope and triangulation
- Phase B (Ellipsoidal Earth model: mid 17th – mid 19th century)
 - Richter: pendulum oscillation to find gravity increase toward the poles
 - Newton: rotational ellipsoid, which was confirmed by survey



Geodesy as Science

- Phase C (Geoid model, mid-19th –mid 20th century)
 - Laplace, Gauss & Bessel found that the equipotential surface deviates from the ellipsoid.
 - Concept of geoid as level (equipotential) surface approximating the mean sea level was born.
 - Determination of geoid became the main goal.
- Phase D (Dynamic model, mid 20th century-)
 - Space (SLR, VLBI, GPS...) and terrestrial (superconducting gravimeter...) geodetic techniques
 - Static → Dynamic
 - Earth → Earth system
 - Newtonian → Relativistic corrections



Gravity changes due to geophysical phenomena

Static

(mass distribution)

Dynamic

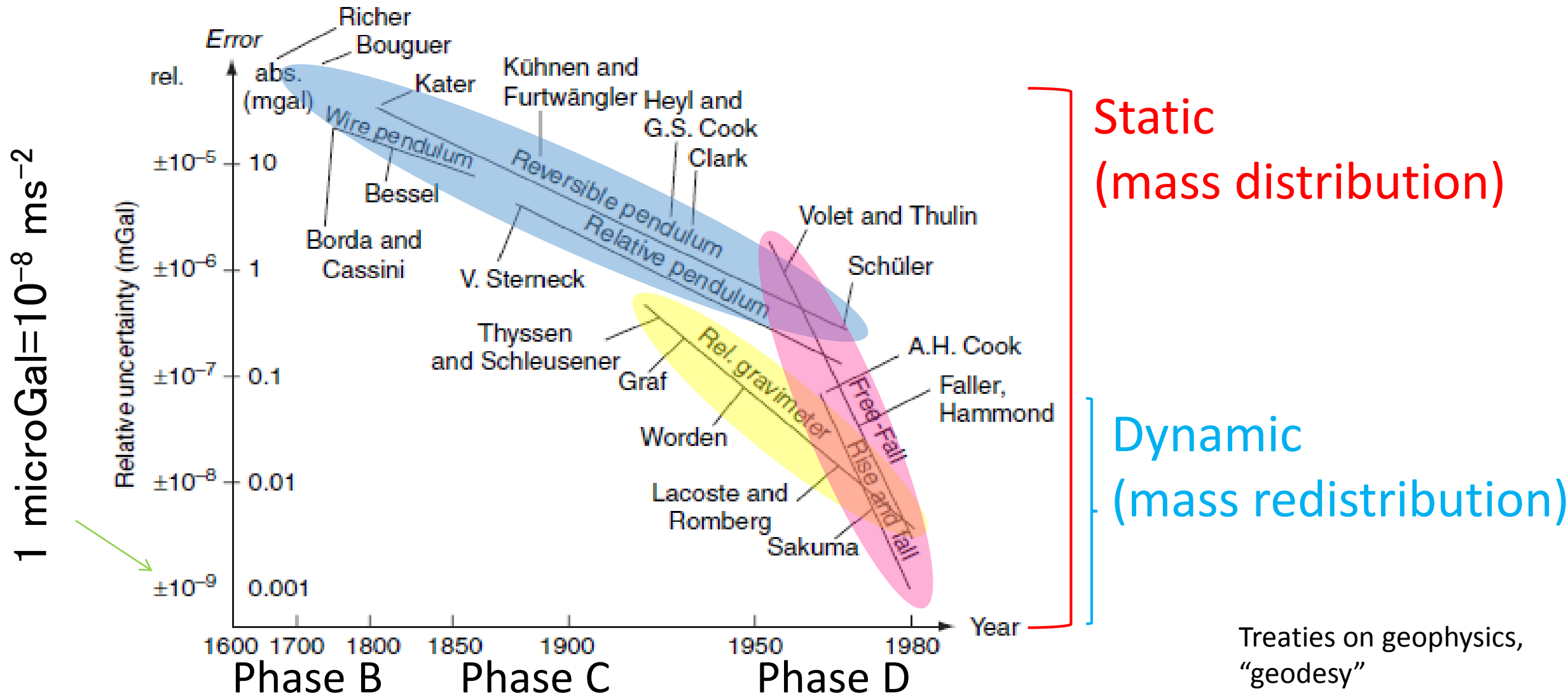
(mass redistribution)

$g = 9.791234567... \text{ m/s}^2$

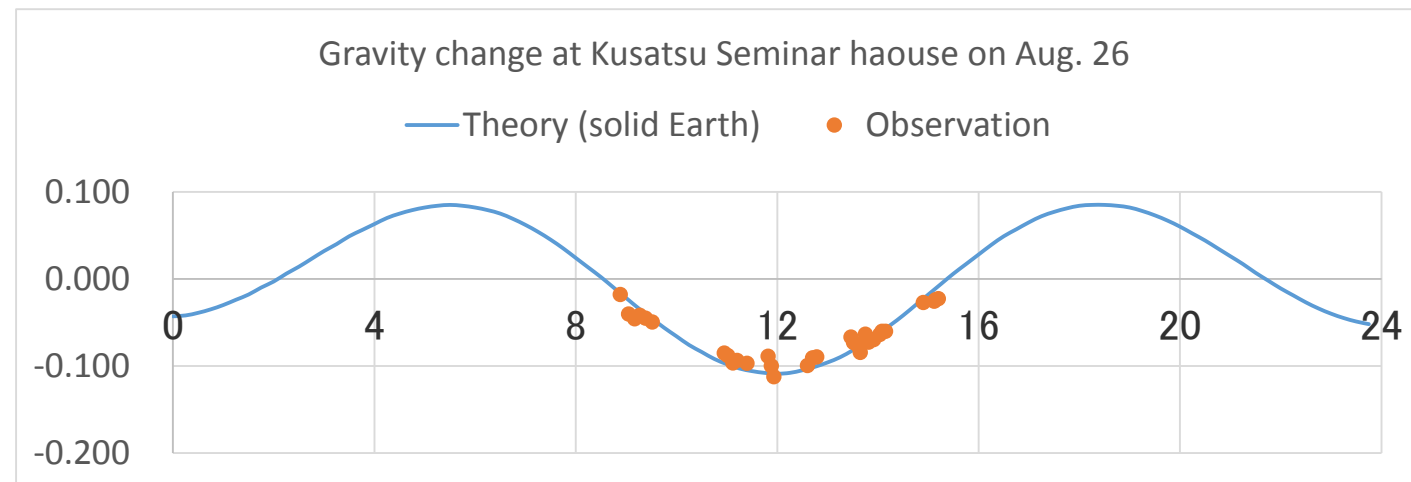
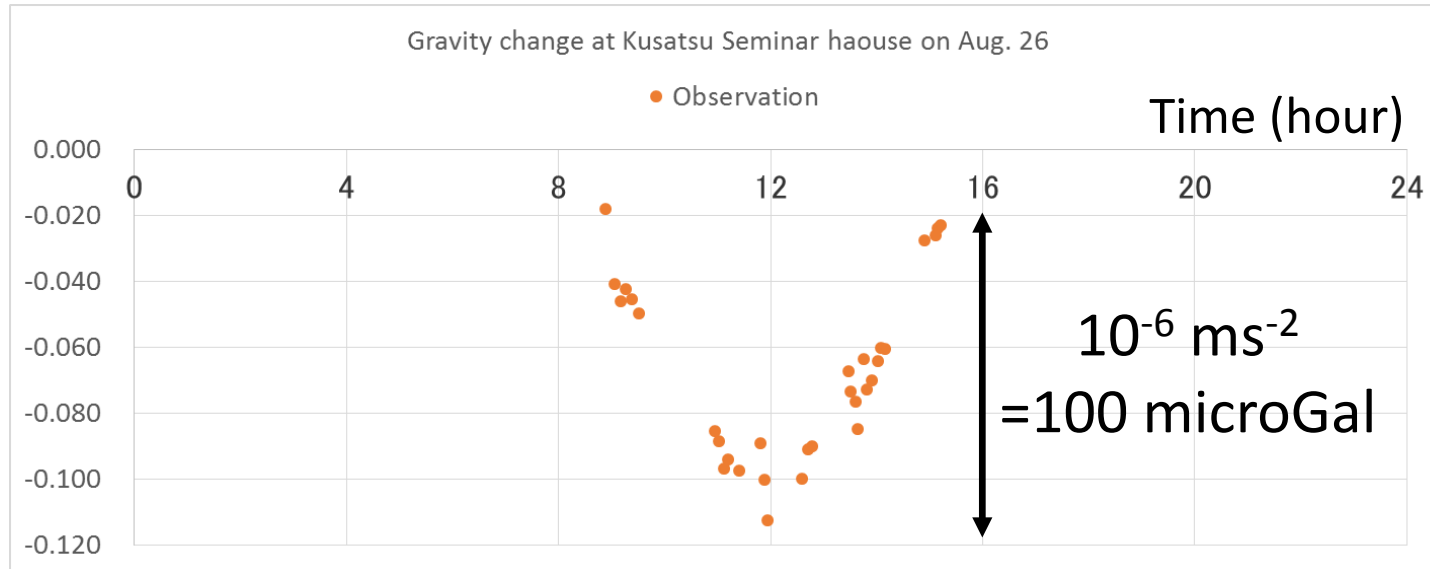
Flattening, rotation
Mountain, ocean trench (1km height)
Internal mass distribution
Solid Earth and ocean tides (1 m height)
Hydrology
Ocean topography, polar motion
Relativity, 1 mm height

1 microGal
 $= 10^{-8} \text{ m/s}^2 \doteq$
1 ppb x g (9.8 ms^{-2})

Uncertainties of gravity measurements



Examples of terrestrial gravity measurements



Spring-type relatively
cheap, portable gravimeter

Measurement precision
 $\sim 10 \text{ microGal}$

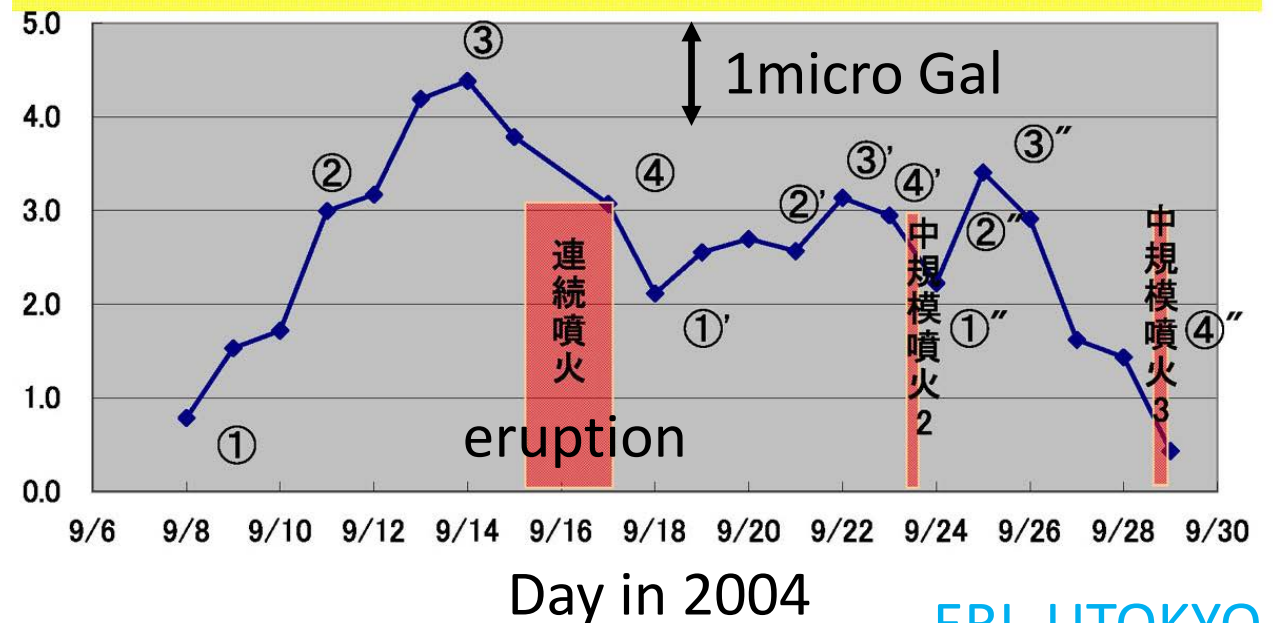
Only relative gravity change

Absolute gravimeter

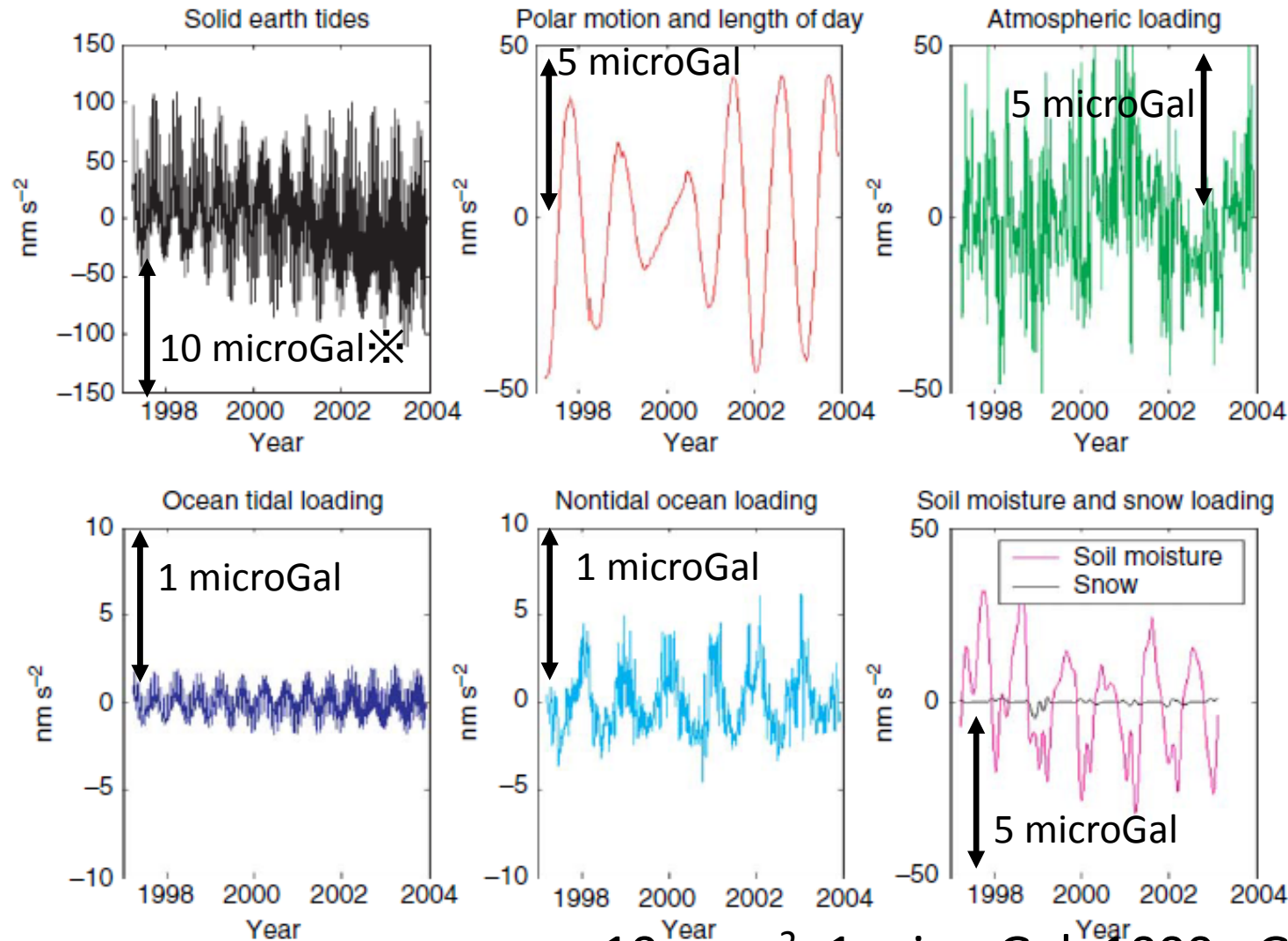
- Free-fall, corner cube reflector, laser interferometry, Rubidium clock
 - 1 drop/10 seconds
 - Measurement precision ~ 1 microGal for 24 h
 - Absolute value (e.g. 9.80...) can be determined.



Gravity change observed at Asama Volcano



Examples of terrestrial gravity measurements



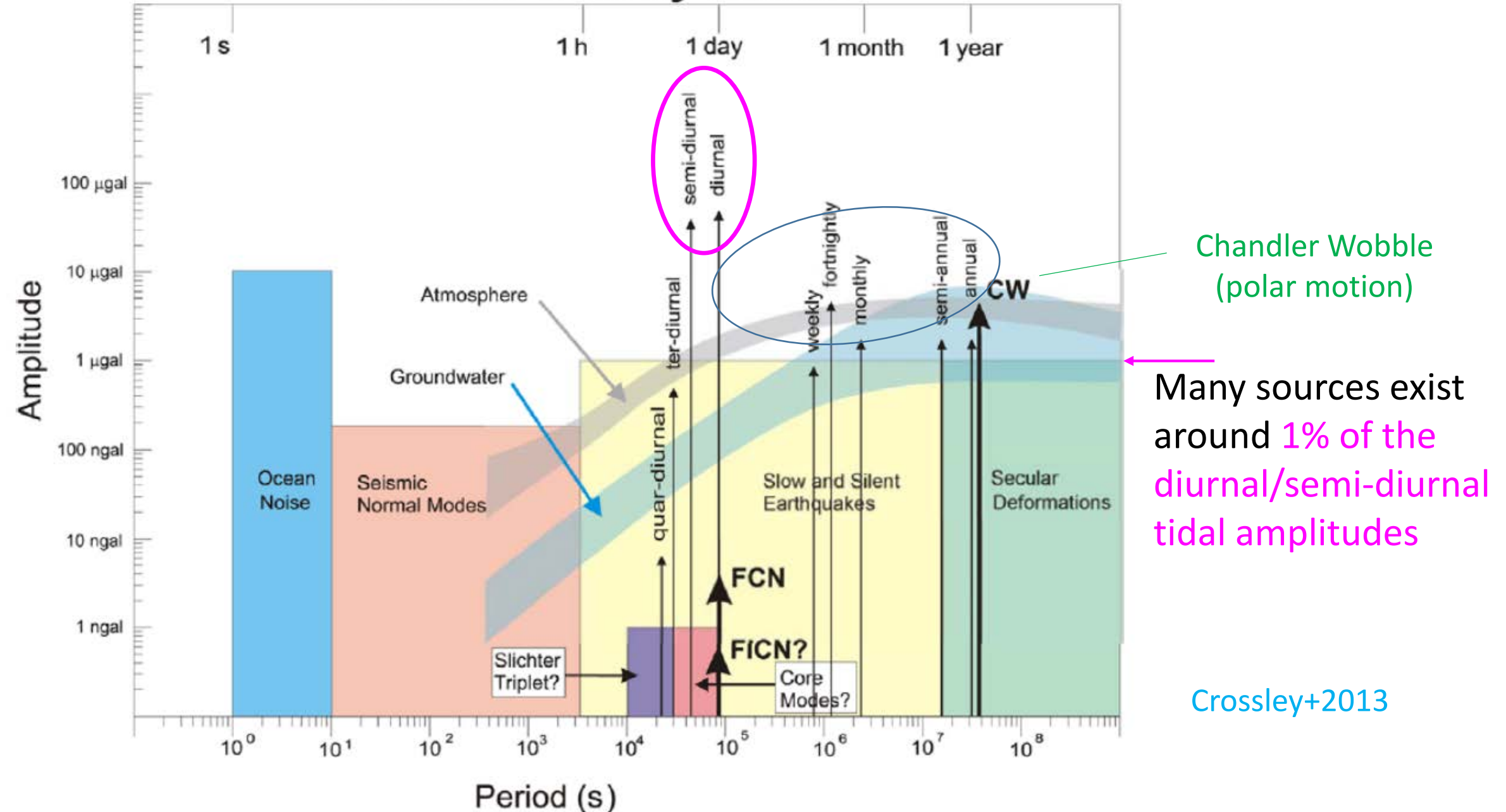
Superconducting gravimeter at Strasbourg (J.-P. Boy.)

Measurement precision $\sim 10 \text{ nGal}$
Only relative gravity change

※ Daily tides being averaged out in this plot

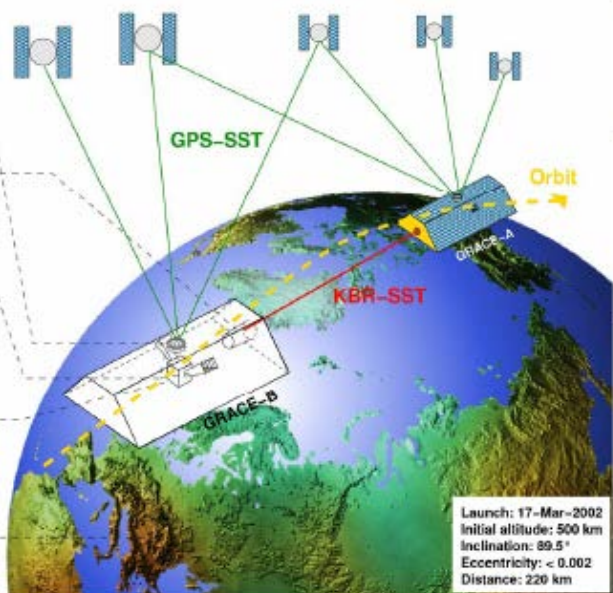
Treaties on geophysics, "geodesy" $10 \text{ nms}^{-2} = 1 \text{ microGal} = 1000 \text{ nGal}$

Surface Gravity Effect

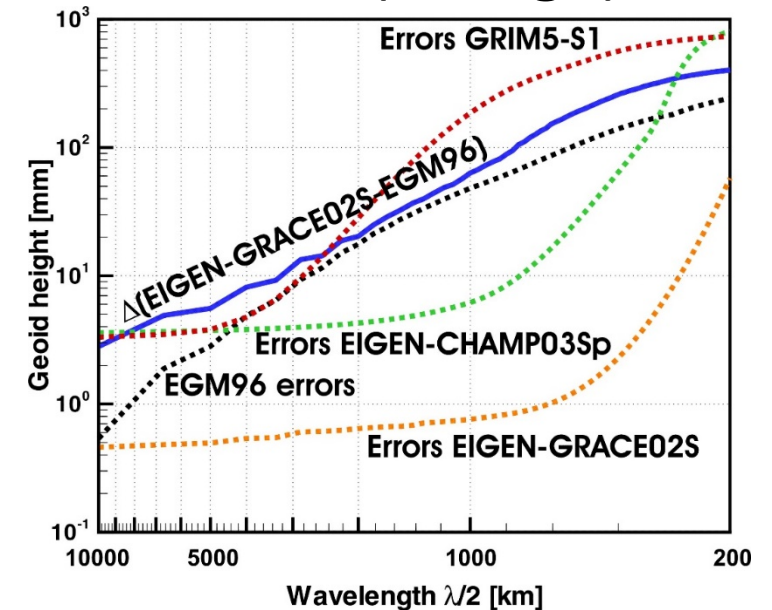
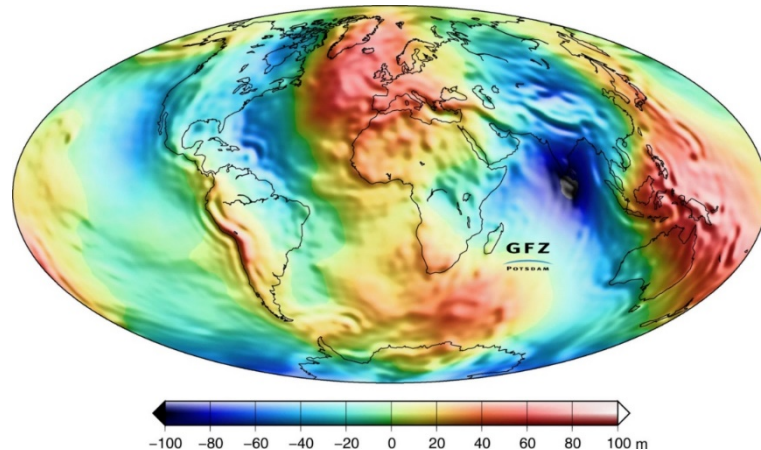


Examples of space-borne gravity measurements

- GRACE mission, 2002-present, altitude ~ 500 km, GPS and microwave (Shared+2012, J. Geodesy, 86, 1083-1095)
- Global gravity change at temporal resolution of 1 month
- Precision ~ 1 microGal or 1 mm at spatial scales of ~ 300 km (orange)



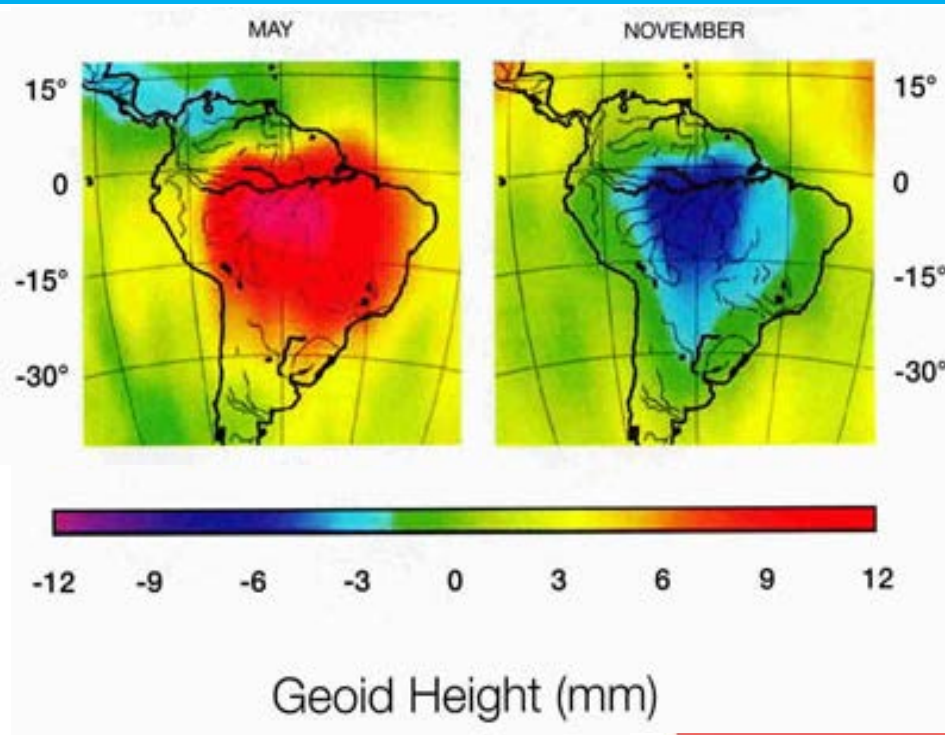
<1 mm at 10,000 km



[http://op.gfz-potsdam.de/grace/
results/grav/g002_eigen-grace02s.html](http://op.gfz-potsdam.de/grace/results/grav/g002_eigen-grace02s.html)

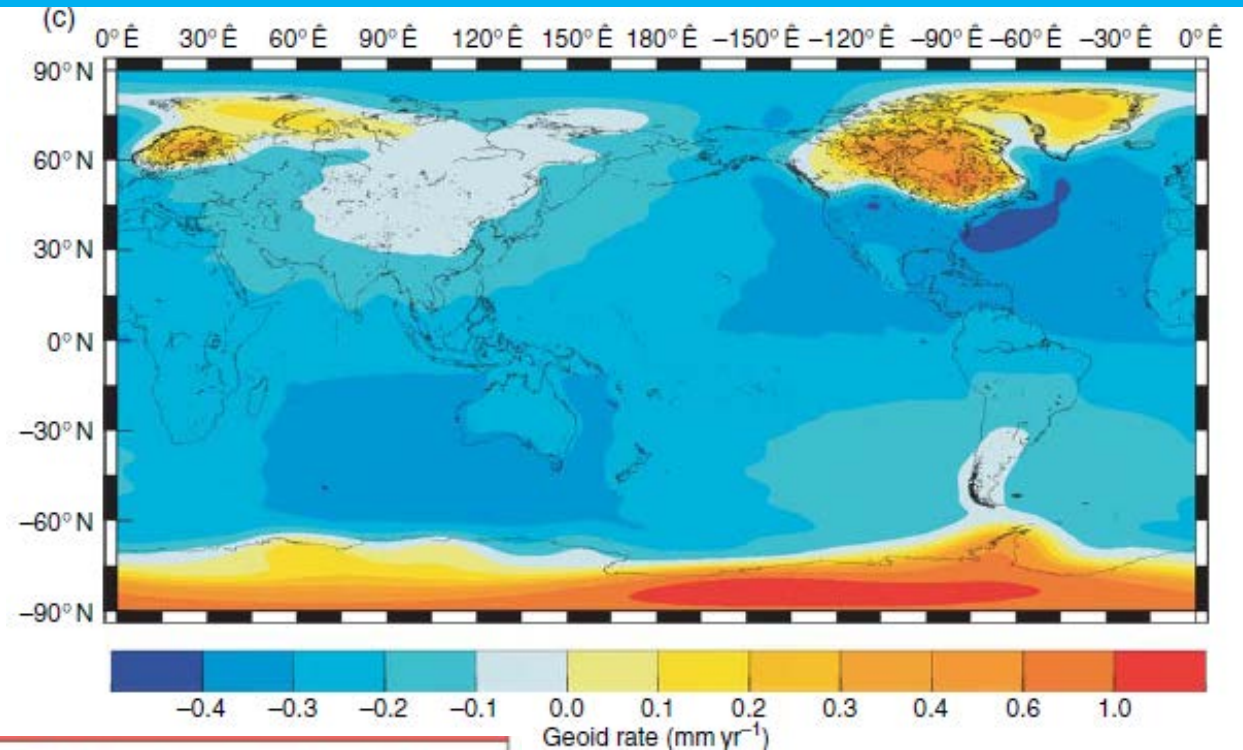
Geoid ($\hat{=}$ surface potential/ 9.8 ms^{-2}) undulation ± 100 [m]

Examples of temporal gravity variations observed by GRACE



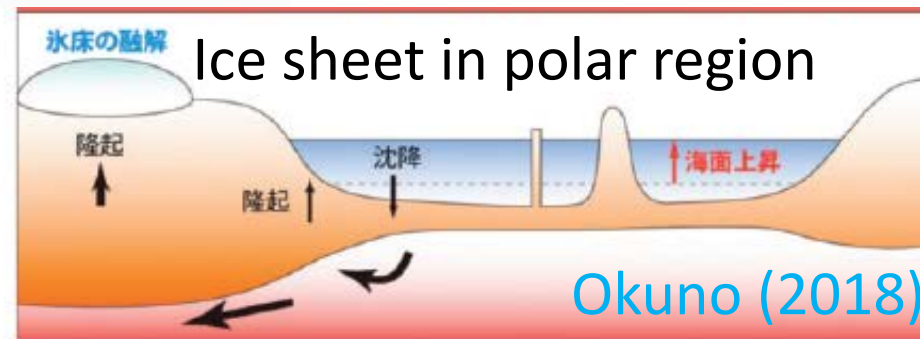
Continental water
(annual variation)

Vergos & Tziavos (2008)



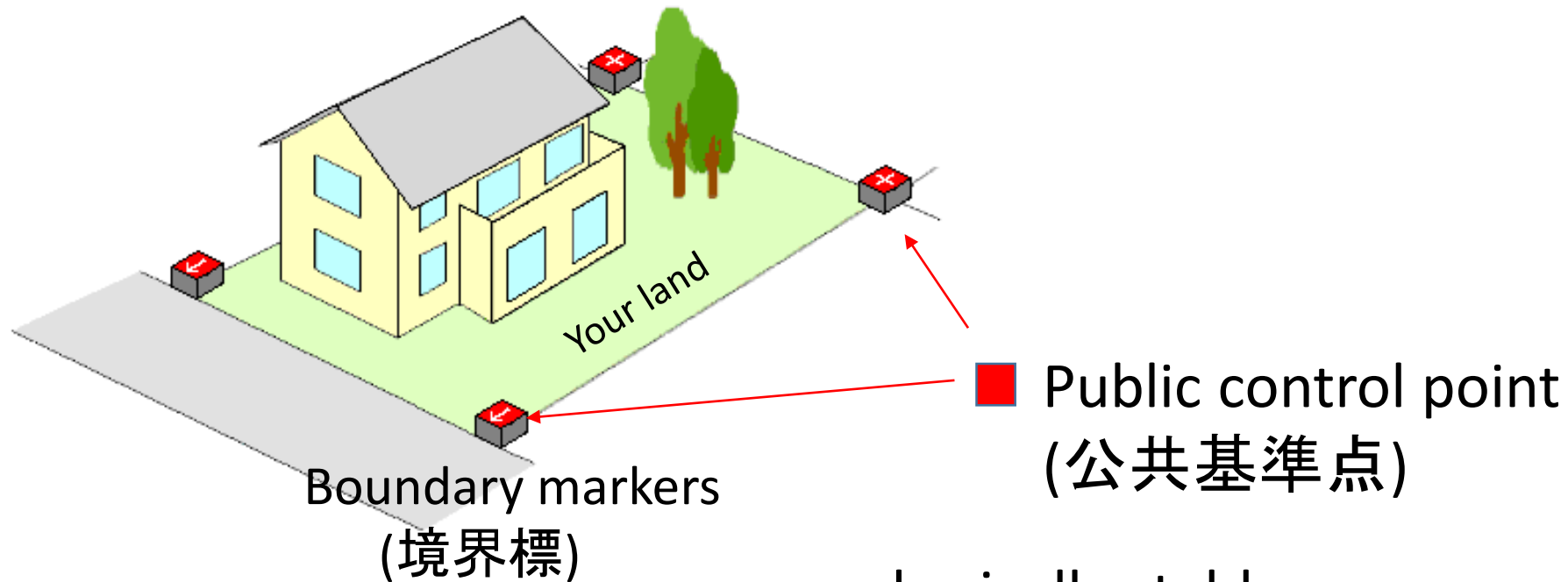
Glacial Isostatic Adjustment
(almost secular variation)

Treaties on geophysics, "geodesy"



Practical aspect of Geodesy

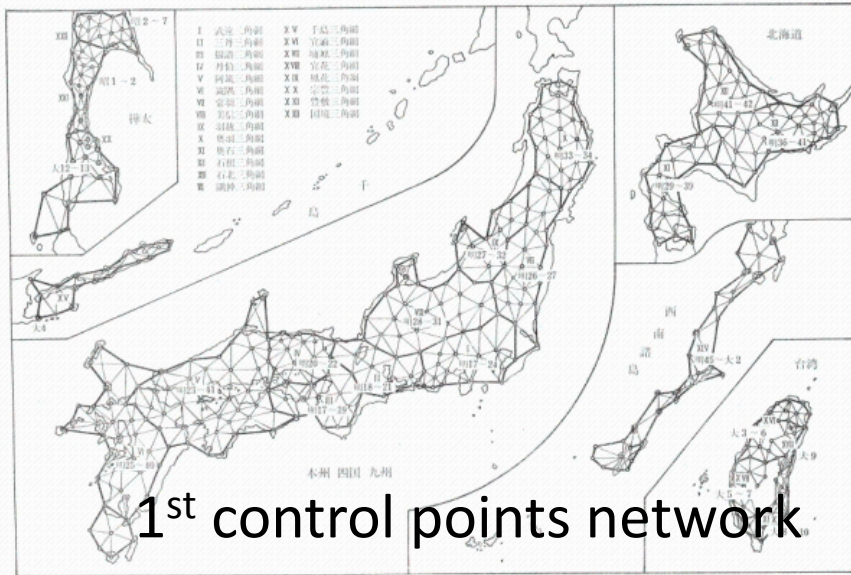
- Geodesy provides “the” reference frame to ensure the reproducibility of measured positions with certain accuracy.



<https://ameblo.jp/kitakyushu-touki/entry-12368104946.html>

physically stable, accuracy known
(A set of control points = a reference frame)

Hierarchy of control points



GSI



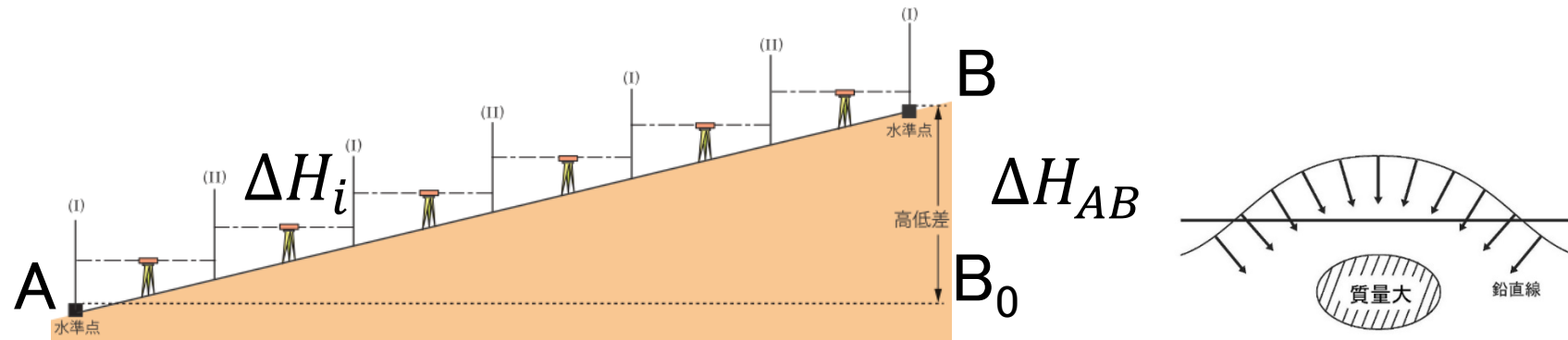
<https://blog.goo.ne.jp/yoshikatsuisobemirimir1018/d/20190609>

- Above public control points, national control points (1st to 4th grades, the higher the grade becomes, the more precise and sparse), the position is determined by the GSI
- The origin of longitude and latitude is located in Azabu in Tokyo (right figure)
- Relative consistency to the origin in positions within a country is ensured (within survey errors prescribed by the government)

Definition of “height” in Geodesy

- We define height, based on the gravitational potential.

Hofmann-Wellenhof & Moritz (1967)



$\sum_i \Delta H_i$ is not unique (different curvature of level surfaces).



$$\Delta W_{AB} \cong - \sum_i g_i \Delta H_i$$

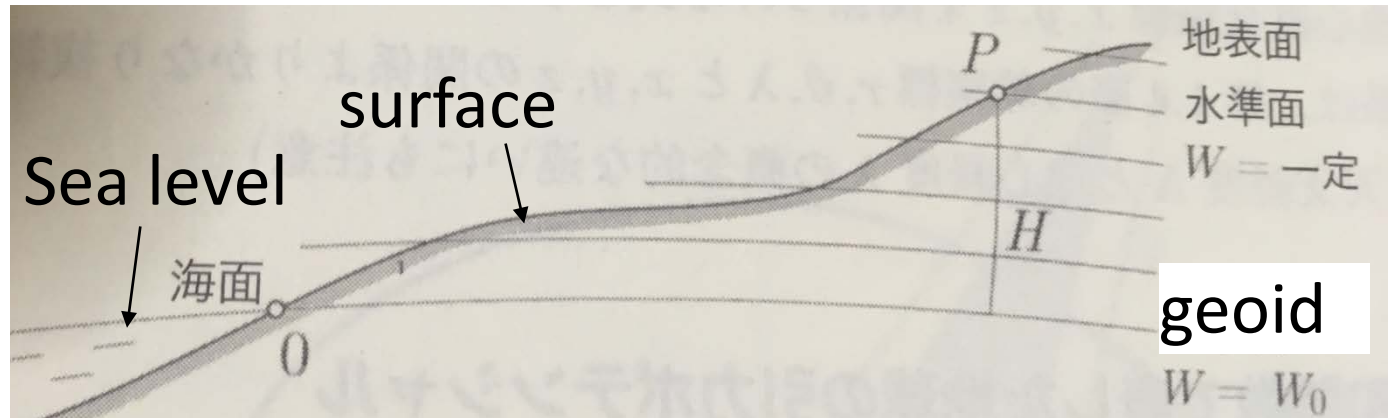
$$\Delta H_{AB} \equiv \frac{-\Delta W_{AB}}{\bar{g}} \left[\frac{\text{m}^2}{\text{s}^2} \right] \rightarrow [\text{m}] \quad (\text{orthometric height})$$

$\bar{g}$ average gravity along B-B₀

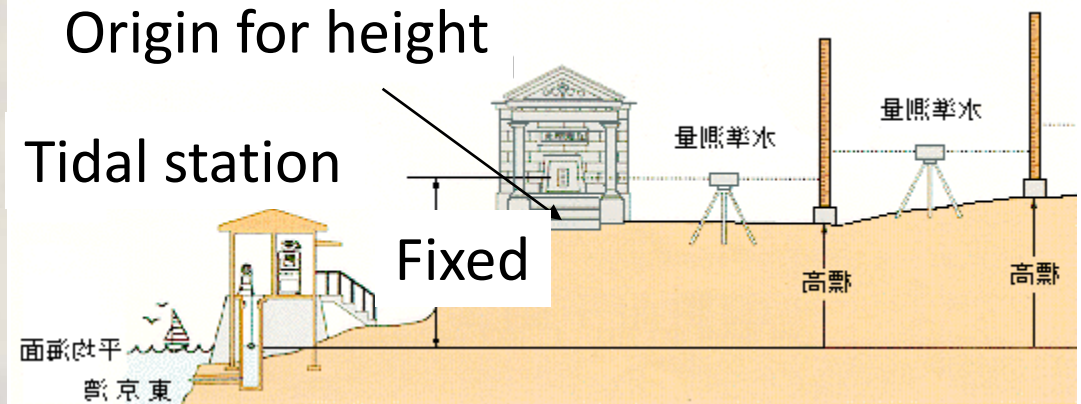


How to realize the origin for height?

- “zero height” → Equipotential surface approximating a mean sea level, extended under the ground



Hofmann-Wellenhof & Moritz (1967)

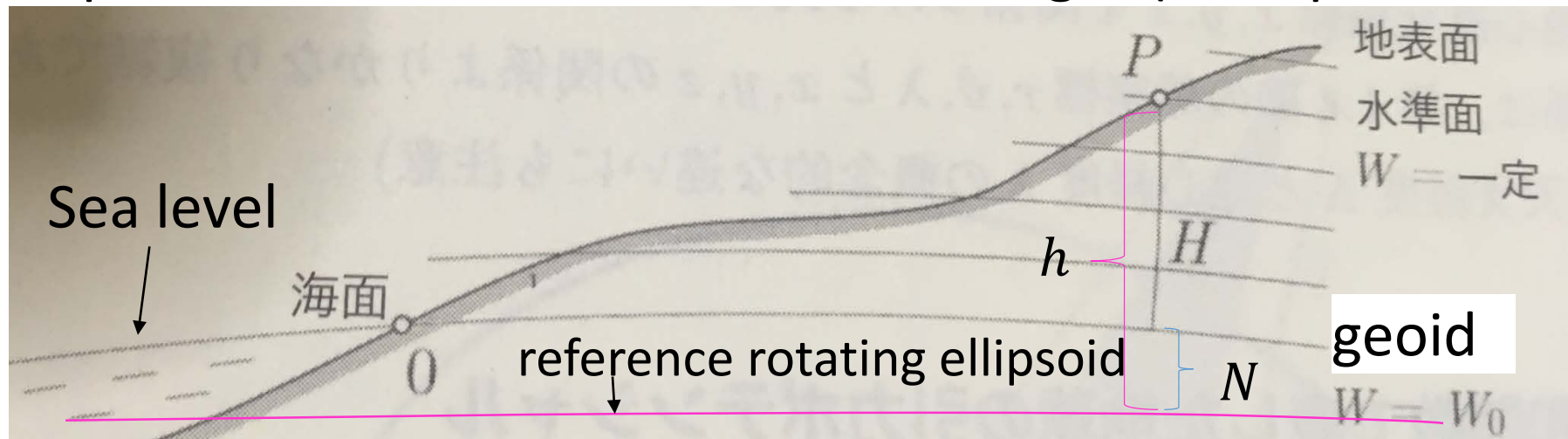


GSI

- Actual sea surface can vary in time, which is inconvenient as a reference.
- We locate an average sea level at the tide gauge during some time period and measure an offset between the tide gauge and the origin. This offset is kept constant (e.g. 24.4140 m).
- $W(x, y, z) = W_0$ prescribing the geoid (as a height reference) differs among countries.

How to determine the geoid?

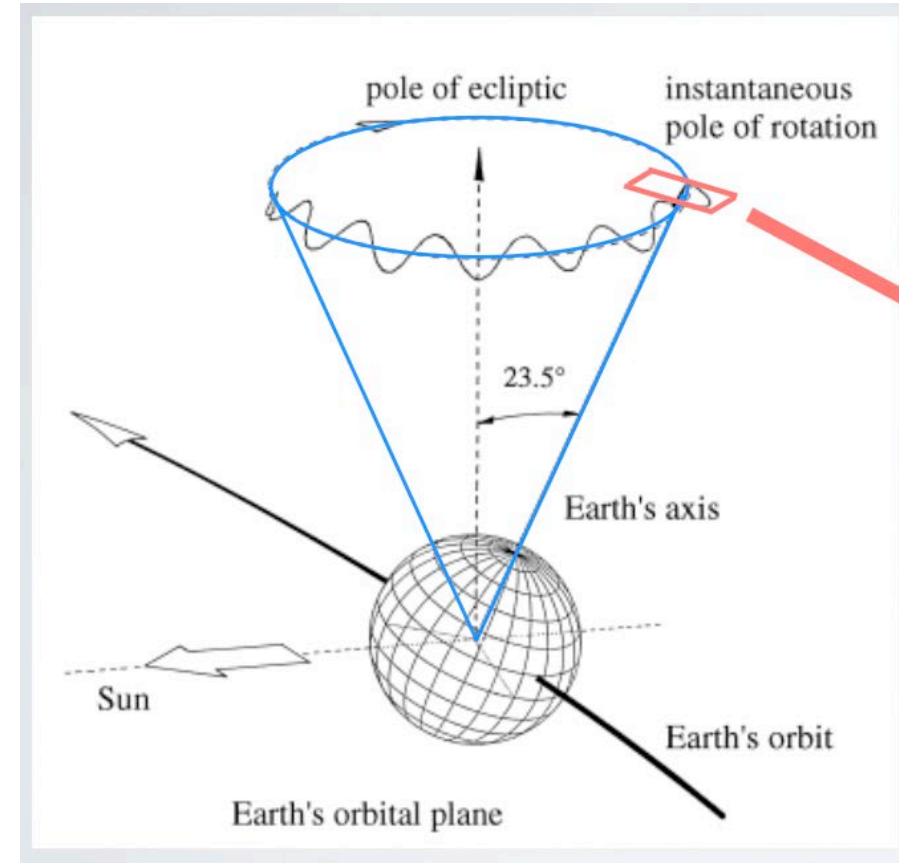
- The geoid is the difference of the potential from the equipotential surface of the reference ellipsoid (which locally best approximates the geoid)
- Expressed in terms of dimension of height (c.f. equilibrium tide)



- Method 1: Boundary Value Problem from 3D position and gravity at sites
→ Surface can be mathematically determined by a least-square method
- Method 2: Direct observation combining GNSS and leveling: $h - H = N$
→ the geoid height at a site is obtained

From local to global reference frame

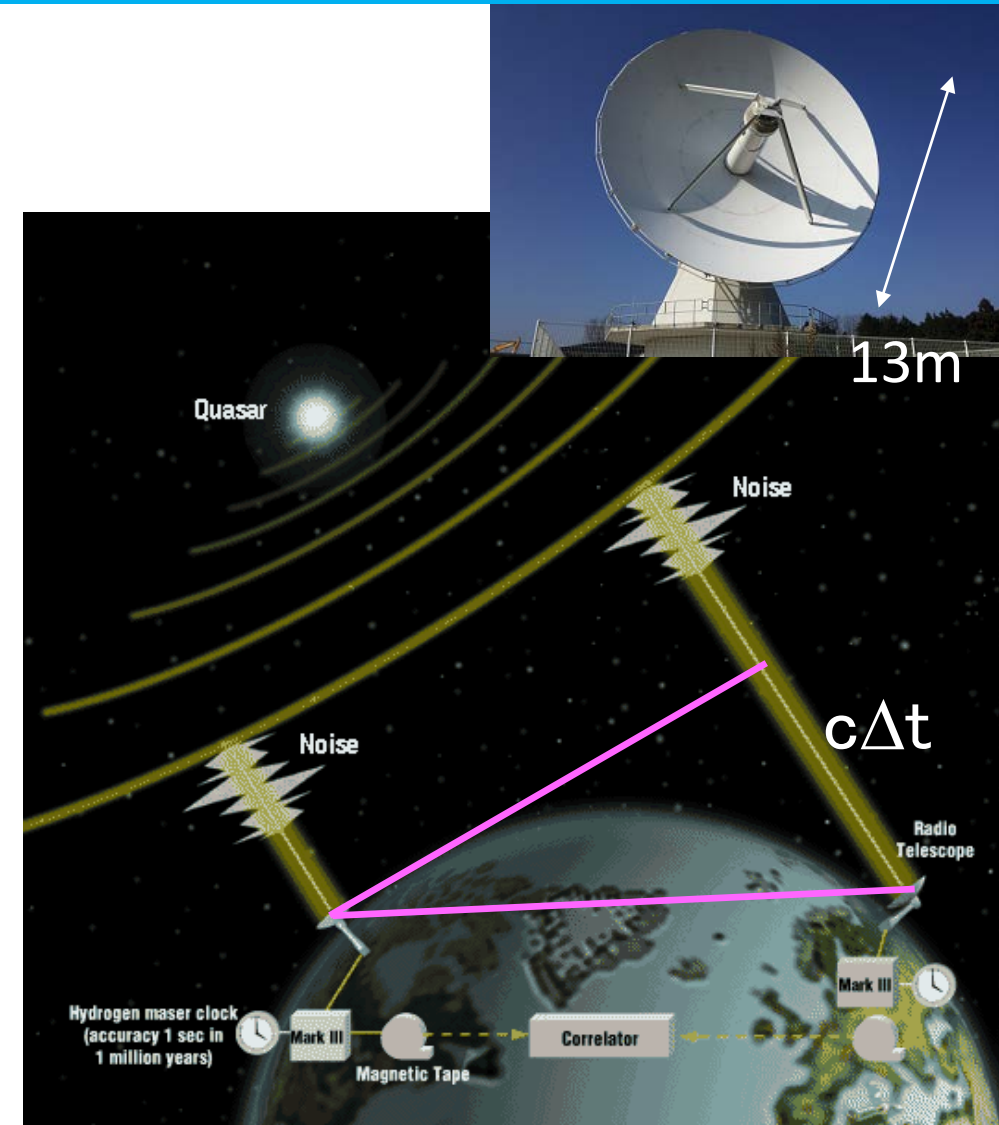
- Needed to uniquely describe the positions of cars, ships, airplanes, artificial satellites etc.
- Put artificial satellites as control points?
- A geocentric rotating frame is a natural choice.
- Precession and nutation, length of day (LOD)
- Satellite orbits follow Kepler's law, suitable to use an "inertial frame" rather than Earth-fixed frame
- Otherwise, precession of satellites due to the equatorial bulge cannot be separated from that of the Earth.
- "Absolute" orientation of the Earth has to be known accurately and conveniently to describe satellite motions.



<https://sites.ualberta.ca/~dumberry/>

Substantial inertial system and its orientation

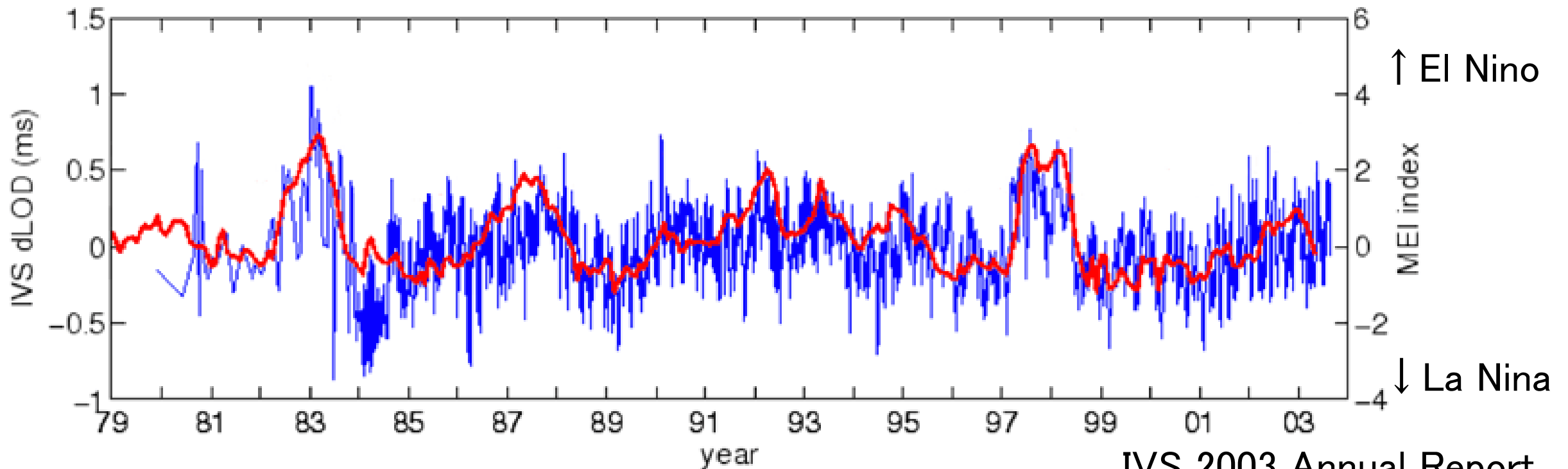
- About 200 **quasars** for geodetic applications
- VLBI since 1960s, >30 stations in the world
- Hydrogen maser: 10^{-11} second
- Precisions:
 - Distance: 10^{-9} (1 mm for 1,000 km) or better
 - Rotation speed: 0.01 msec
 - Orientation: 1 marcsec
- At the equator, 1700 km/h (470 m/s) 1 msec=47 cm
- 0.01 msec corresponds to 5 mm in position at the eq.



An example of Earth rotation data

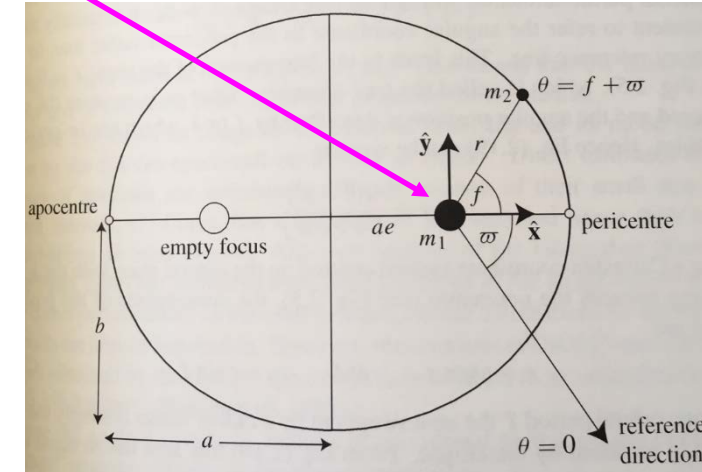
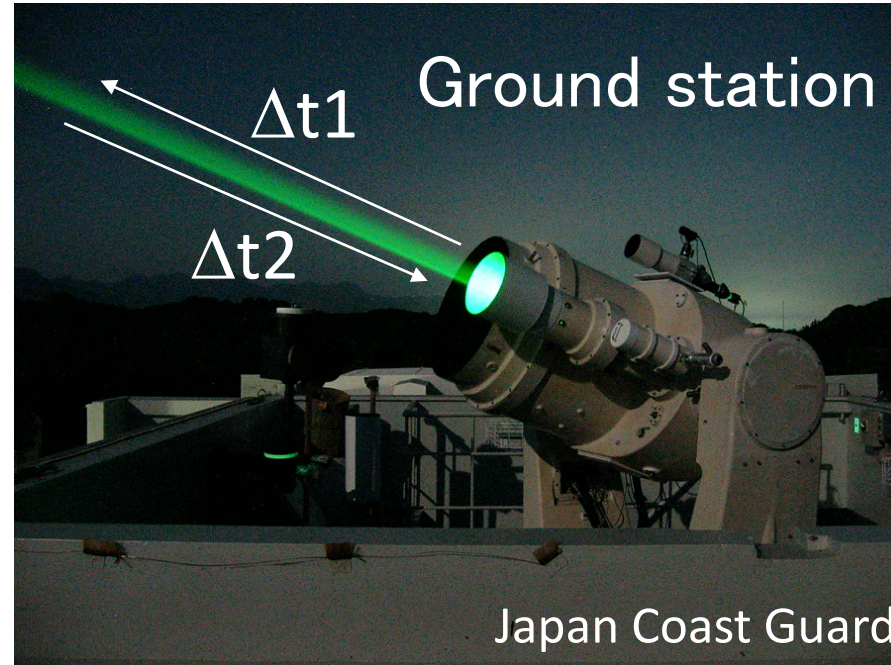
- Blue: Length of Day (LOD), band-pass filtered
- Red: Multivariate ENSO Index: larger values correspond to El Nino
- Reflecting the conservation of angular momentum

(When eastward wind speed increases, rotation speed decreases)



The origin of the global reference system

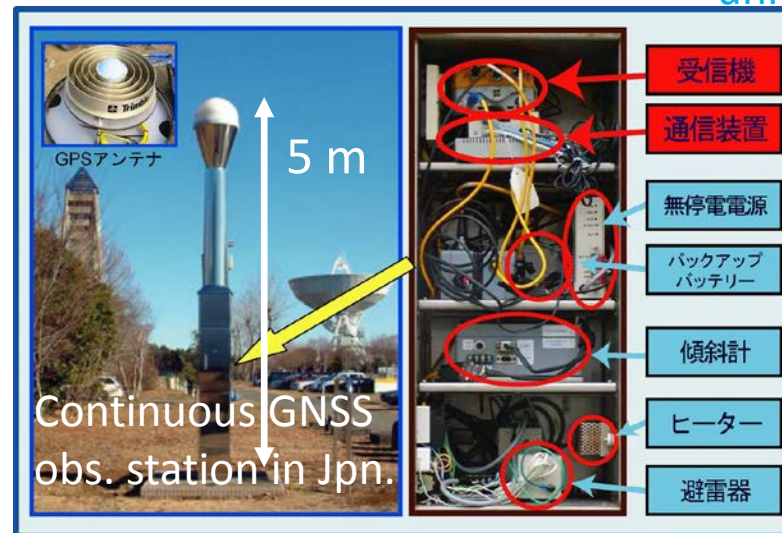
- Satellite Laser Ranging (SLR), since 1970s, >40 stations over the world
- Altitude $\sim 6,000$ km, $\lambda \sim 500$ nm, corner reflectors, measure $\Delta t = \Delta t_1 + \Delta t_2$
- Distance: 10^{-9} (1 mm for 1,000 km) or better, many stations $\rightarrow$ 3D position
- 1 of the 2 focuses of the satellite orbit is the center of mass (=geocenter)



Global Navigation Satellite Systems (GNSSs)

- Geometrical positioning of 3D coordinates in the inertial reference system
- GNSSs (GPS since 1980s, Galileo, etc.), total >80 satellites
- **Altitude: $\sim 20,000$ km**, Rubidium or Caesium clock on board, microwaves
- It densifies the global reference frame on the Earth, determined by SLR and VLBI etc., thanks to the simpler receiver system.

Voluntary federation of over 200 self-funding agencies, universities, and research institutions!

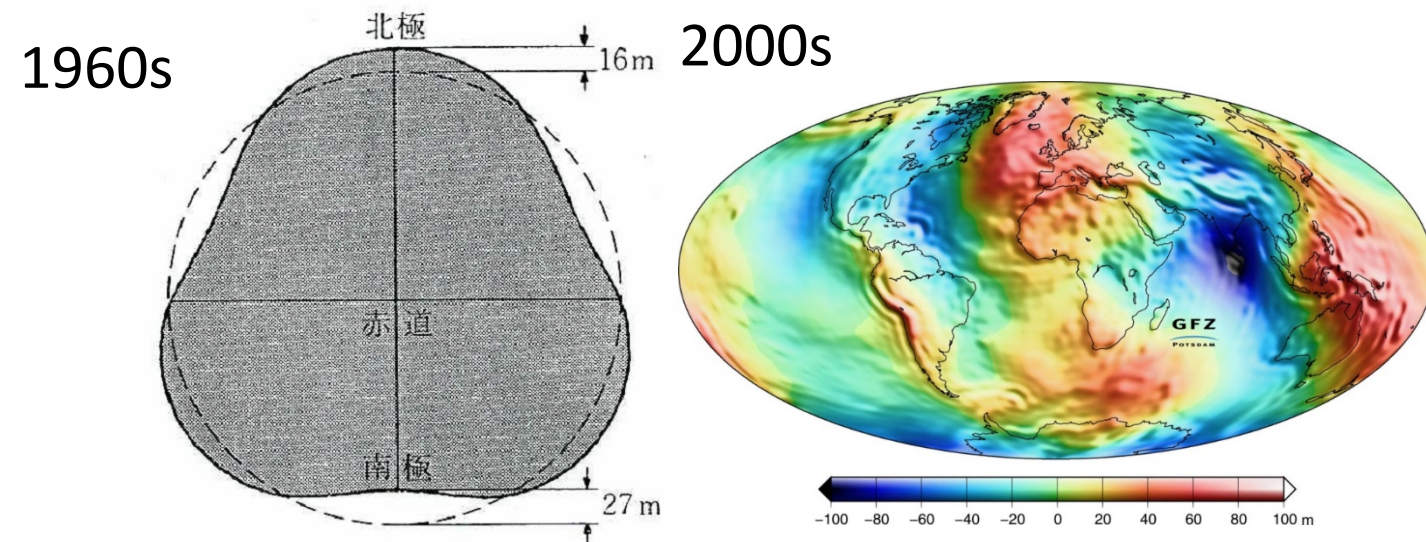


Global height reference system

- To uniquely define height across the countries, a common reference ellipsoid and a global geoid must be established.
- SLR/Low Earth Orbit satellites (e.g. GRACE) for longer/shorter wavelengths
- $W(x, y, z) = W_0 = U_0$
gravimetric obs. geometrical obs.

a, b, GM, ω

ref. ellipsoid is prescribed by these 4 parameters



Origin & orientation defined

- X, Y, Z at the surface measured
- the ellipsoid best fits the surface
- U_0 is calculated from the ellipsoid
- The equipotential surface by satellites satisfying $W_0 = U_0$ is the geoid

Global Geodetic Observation System (GGOS)

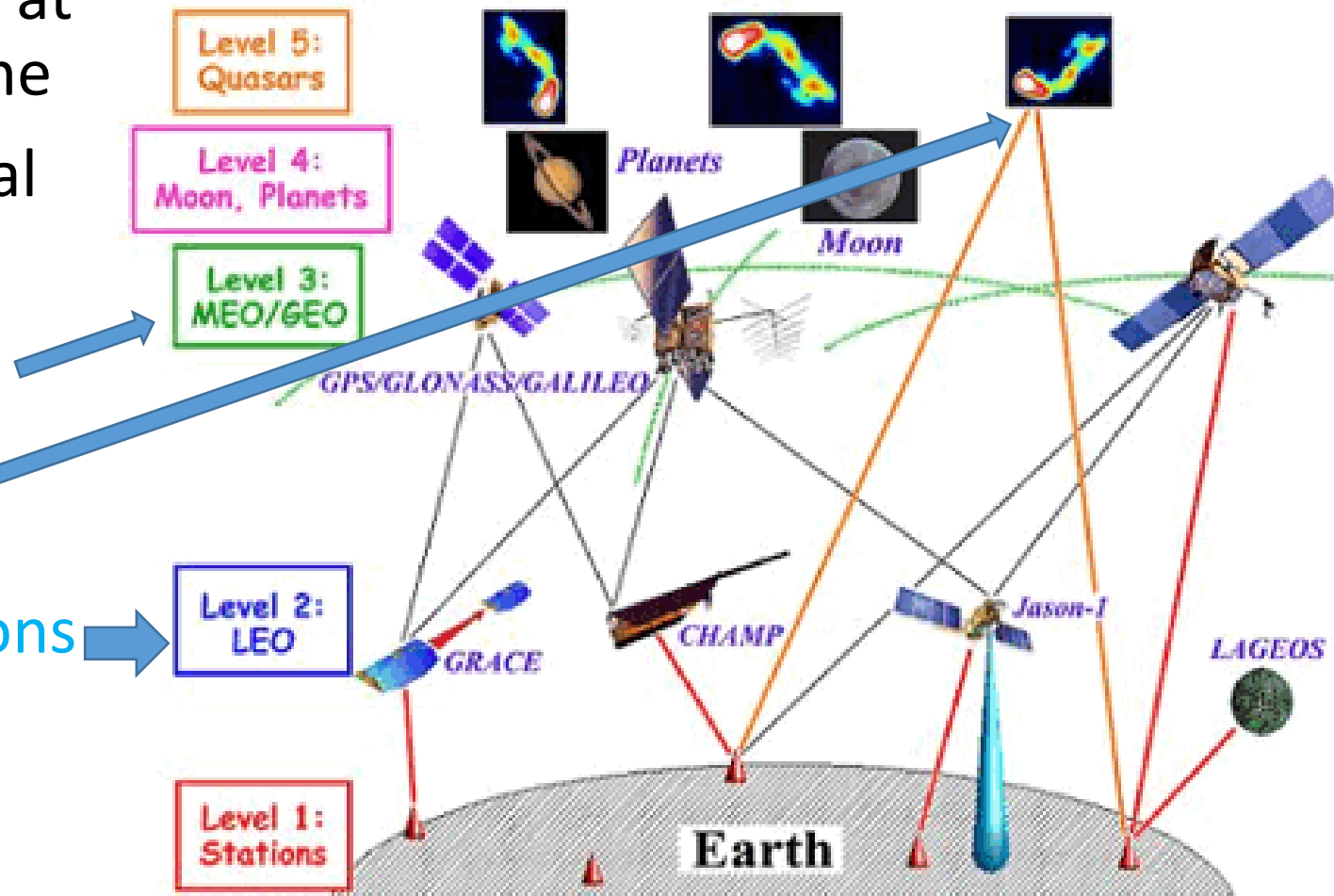
- International Association of Geodesy (IAG)'s goal: positioning at 1 mm accuracy, anywhere anytime
- 3 pillars for maintaining the global reference frame

Geometry: monitoring station position, tracking satellite position

Rotation: LOD, orientation

Gravity: geoid, mass redistributions

- Data are published, used to interdisciplinary researches



Outline

- Basic concept of geodesy (Part I)
 - Geophysical signals and measurement methods
 - Reference coordinate systems for geodesy
 - Height and geoid
- Chronometric geodesy using optical clocks (Part II)
 - Progress in Europe Use of atomic clocks for unifying height reference systems
 - Progress in Japan Potential applications in crustal deformation monitoring
- A few topics on clock and relativity (short)
 - Issue on the realization of the SI second
 - Relativistic effects in geodetic measurements

How to determine the geoid

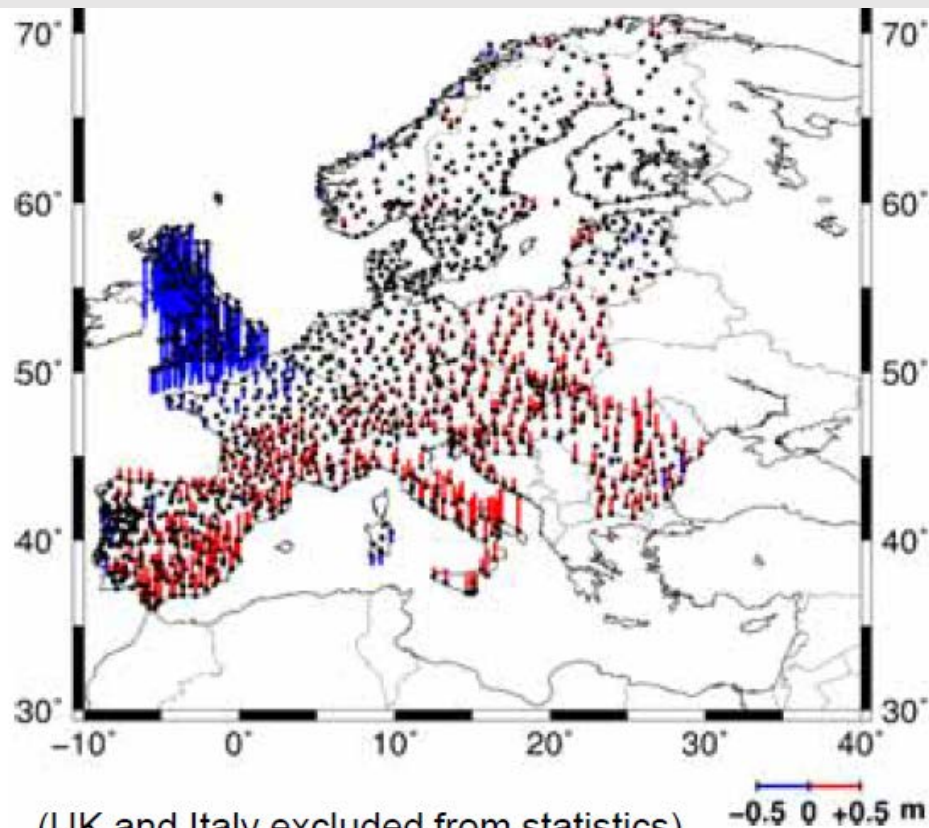
- Usually 2 or 3 approaches are combined:

Approaches	Obs.	Uncertainty etc.
a. Leveling + gravity	g, H	Better for shorter wavelengths
b. GNSS + leveling	$h - H$	More accurate but only at a point
c. Satellite (GRACE)	L	Better for longer wavelengths

- Measurement of H is most expensive and difficult in mountainous areas.
- Inconsistency >10 cm can be seen in the existing height reference systems between countries, motivating the unification of height reference systems

International inconsistency in height reference systems

Difference between approaches a. and b.



(UK and Italy excluded from statistics)

Rms: ± 0.072 m; Min: -0.259 m; Max: +0.396 m

(Denker+2018, web document)

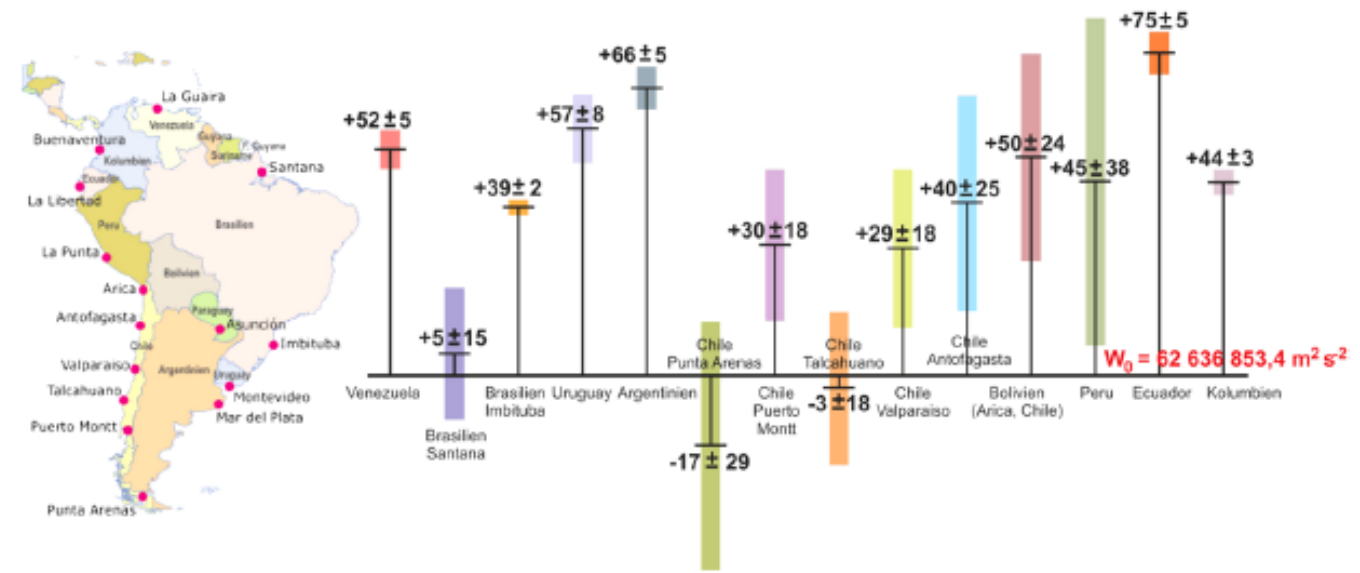


Fig. 3 Difference of regional vertical reference levels to a global one (related to W_0) in South America derived from geodetic measurements (gravity, GNSS, levelling, satellite altimetry), unit: cm (figure is taken from Sánchez 2015)

Chronometric geodesy

- The forth method is to use atomic clocks based on the gravitational red shift in general relativity, which can replace leveling in approaches a. & b.
- Post Newtonian approximation valid (weak gravitational field and slow velocity)
- TCG (Geocentric coordinate time), theoretically calculable time with the metric tensor, is used as coordinate time
- For terrestrial two fiber-linked clocks at rest,

$$\frac{d\tau}{dt} = 1 + W/c^2$$

τ , t proper time, TCG

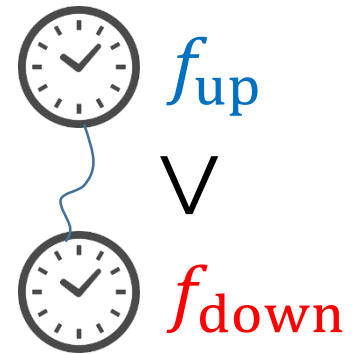
W gravitational +
centrifugal potential

$$\Delta W = -g\Delta H \quad g \cong 9.8 \text{ ms}^{-2}$$

$$\Delta W/c^2 \approx -1.1 \times 10^{-18} \Delta H [\text{cm}]$$

“chronometric geodesy”

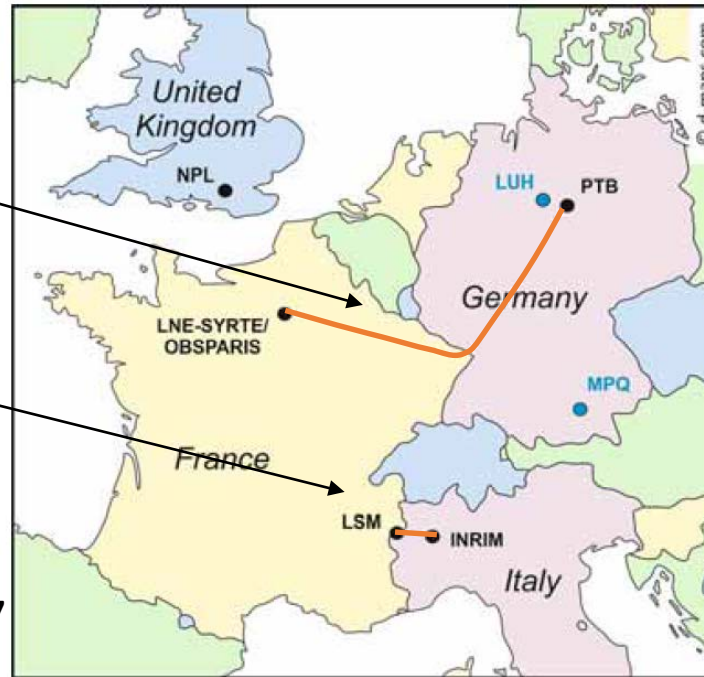
(e.g., Delva+ 2019)



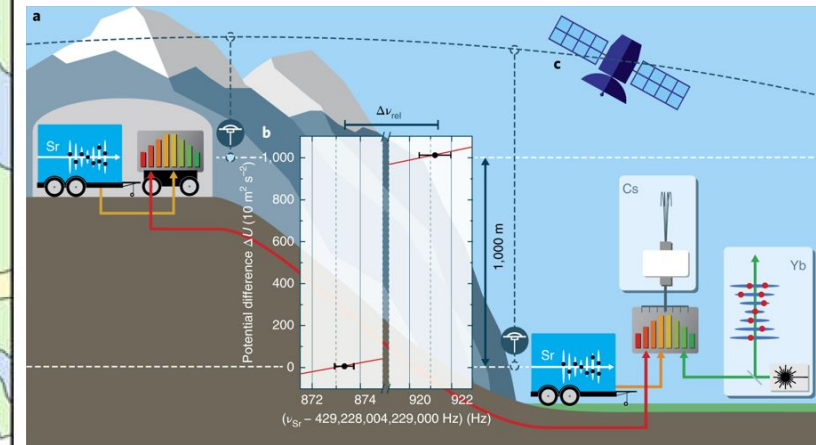
Chronometric geodesy in Europe

- Clocks can help unify the height reference systems.
- International project (ITOC, International Timescales with Optical Clocks, etc.)
- “Geodetic methods to determine the relativistic redshift” (Denker+ 2018)

- Optical clock comparisons
 - ◆ uncertainty of 5×10^{-17} (Lisdat+2016)
 - ◆ Agreement at 2×10^{-17} between chronometric and geodetic results (Grotti+2018)
 - ◆ Transportable clock, 7×10^{-17} (Koller+2017)

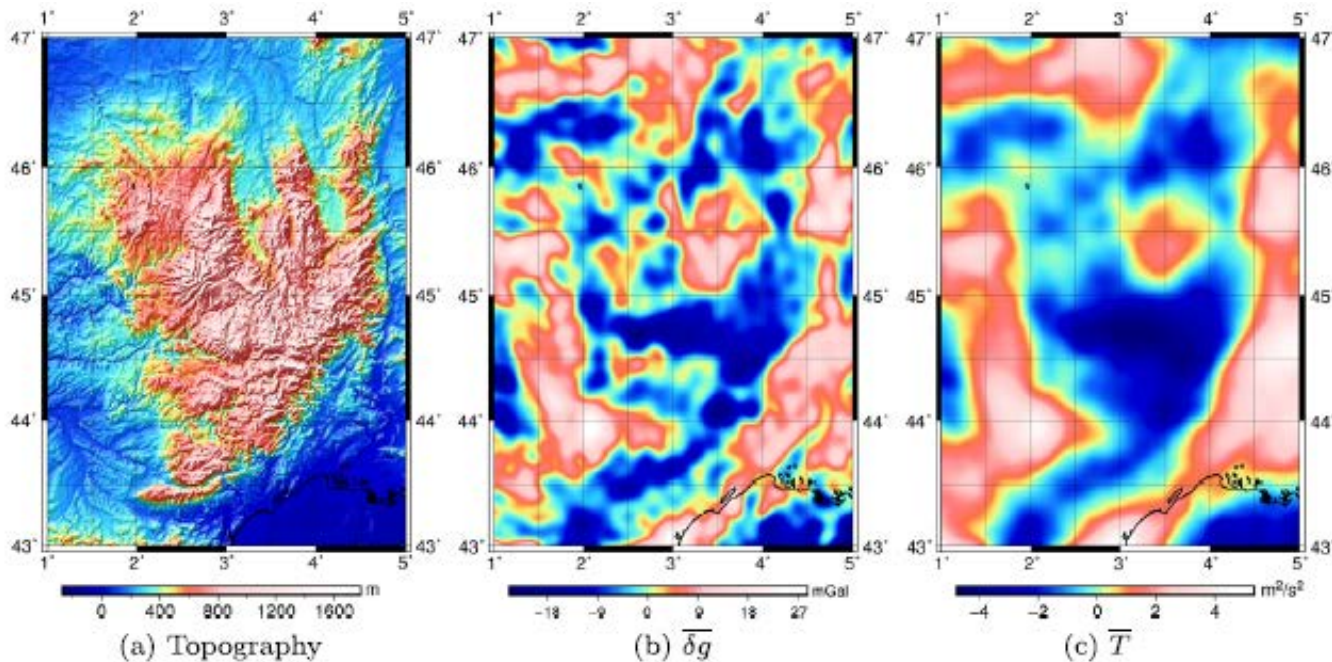


ITOC & geo-Q sites



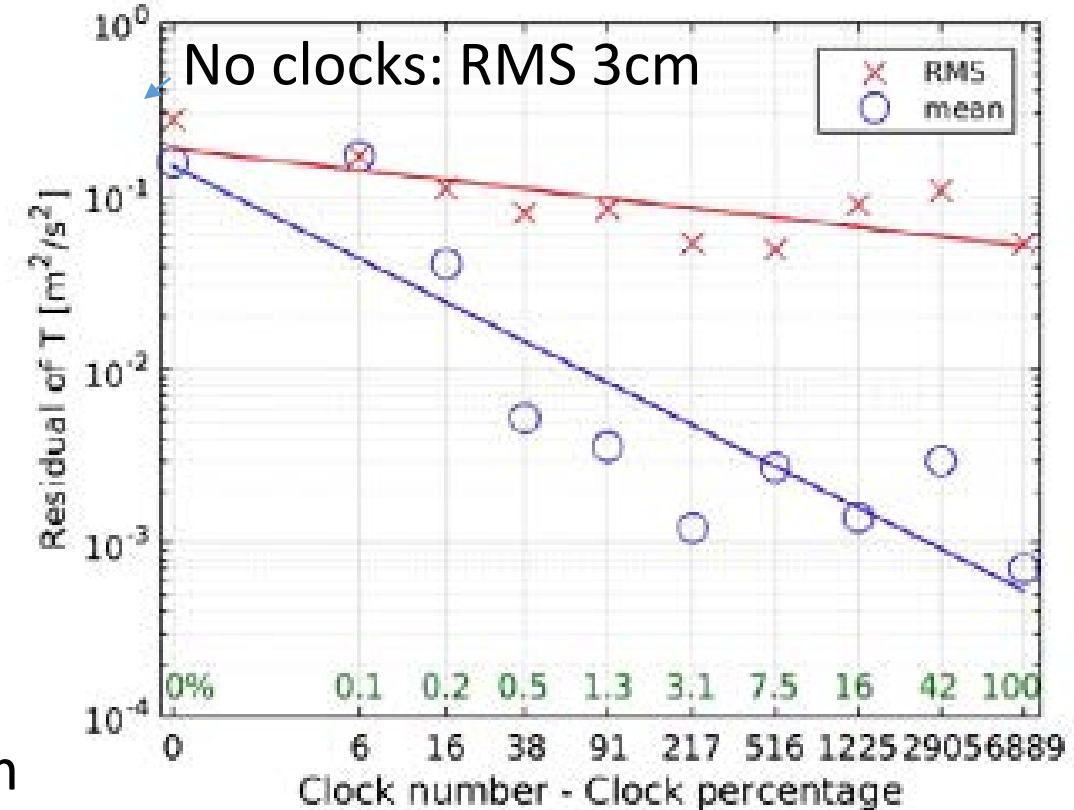
Grotti+2018

Determination of the geoid with clocks



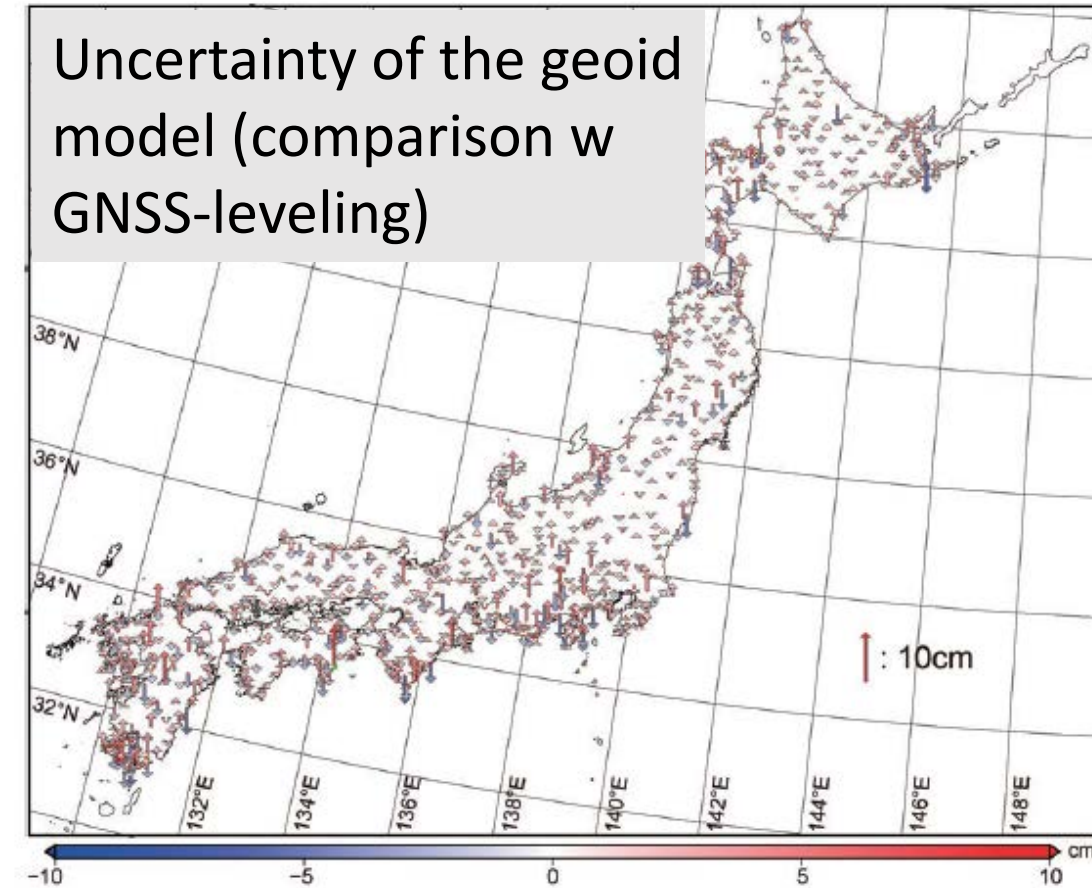
- Clocks can be also used to infer the geoid
- A simulation study shows that adding clocks with 1 cm (10^{-18}) accuracy in the BVP greatly improves the result with only 10% coverage of the gravity observation sites.
- Clocks can constrain larger scale mass anomalies than gravity measurements

(Müller+2018)



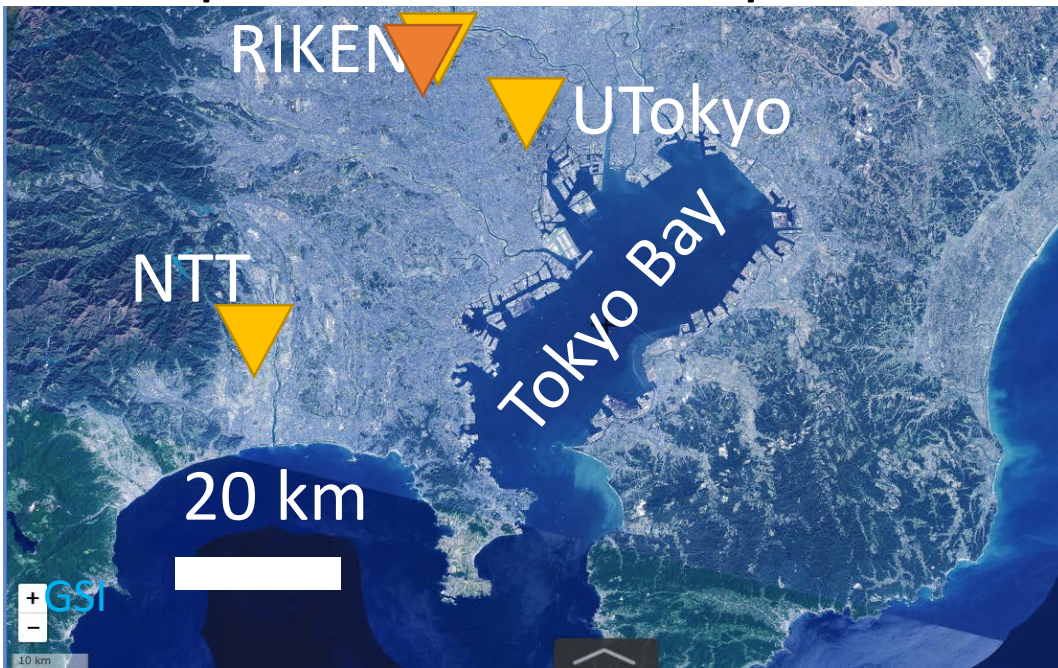
Height system and geoid model in Japan

- Orthometric height is adopted.
- No international inconsistency
- Geoid model by the Geospatial Information Authority of Japan (GSI) ([Miyahara+ 2014](#)), SD=1.8 cm
- 1300 cGNSS stations, first-order leveling over 18,000 km
- Terrestrial data for shorter wavelengths (a.), GNSS/leveling approach for longer (b.)



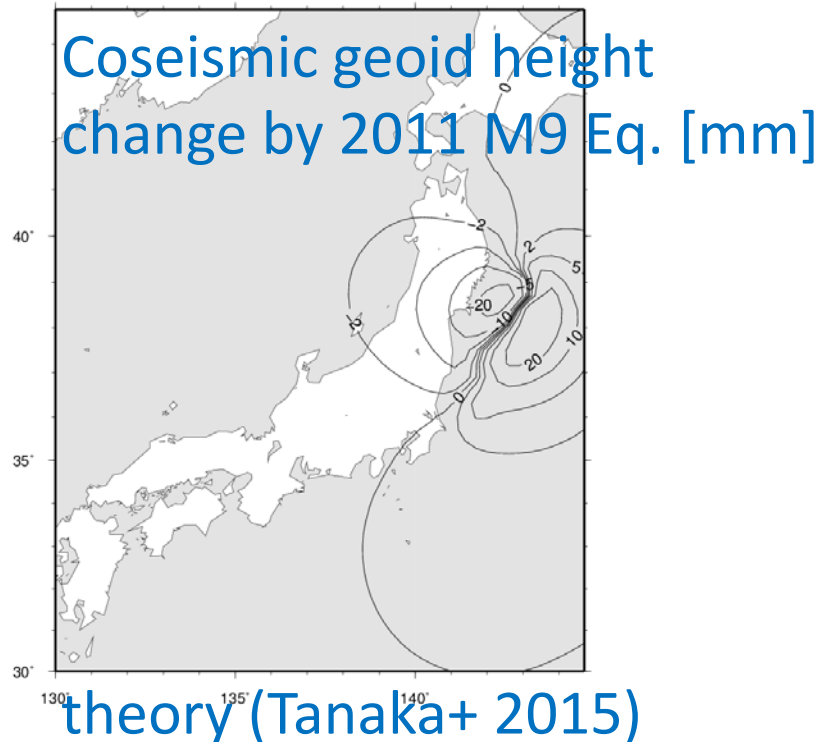
Progress in chronometric geodesy in Japan

- Prof. Katori's group has developed OLCs and compared with geodetic results (GSI).
 - (a) Riken-Univ. Tokyo: 5×10^{-18} (Takano+ 2016)
 - (b) The red shift due to 450 m height difference at Tokyo Sky Tree (under analysis)
- A larger-scale project started in Nov. 2018. Another OLC at NTT.
- First transportable OLC in Japan between NTT and RIKEN under planning.



Detecting vertical deformation

- “The space-time information platform with a cloud of optical lattice clocks”
project leader: Hidetoshi Katori (RIKEN, UTokyo)
- Relativistic Geodesy group aims at detecting tectonic vertical deformation.
- Observed potential changes $\cong$ height changes at clocks (e.g., Bondarescu+ 2015).



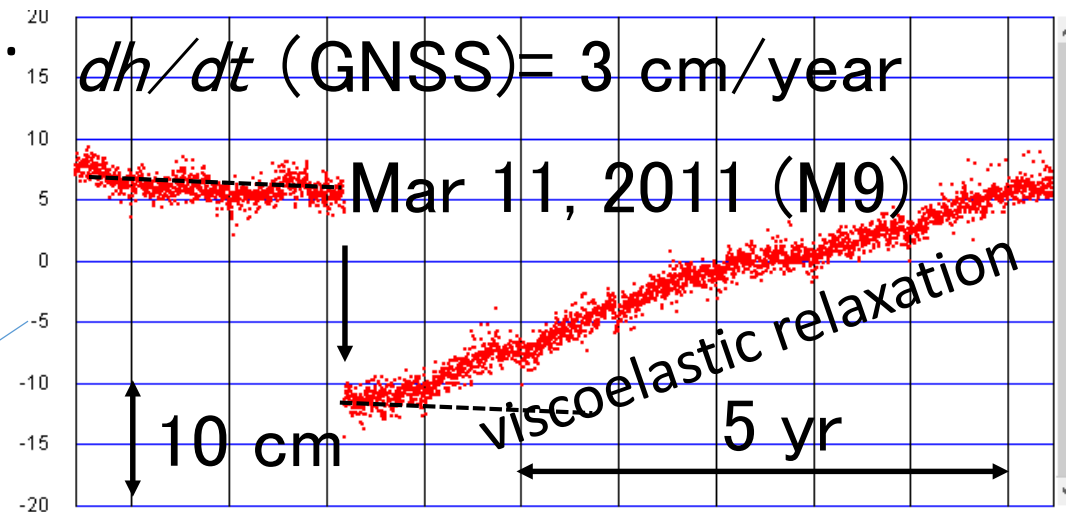
Max. vertical deformation > 10 m
Max. geoid height change = 20 mm

$\Delta h = \Delta H + \Delta N$ Δ : difference
GNSS between two sites

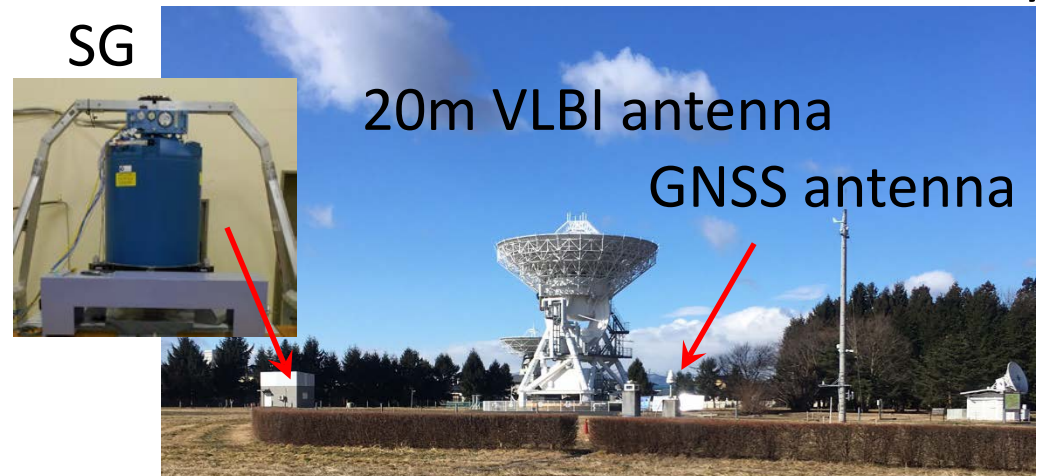
$$\frac{d}{dt} \Delta h \cong \frac{d}{dt} \Delta H = -\frac{d}{dt} \Delta W / g$$

Main target of the RG group

- Postseismic deformation by the Tohoku Eq.
- 400 km fiber link between Mizusawa and Tokyo (under development)
- 1×10^{-18} in 3h, better than GNSS
- 5×10^{-19} in 3h (near future)

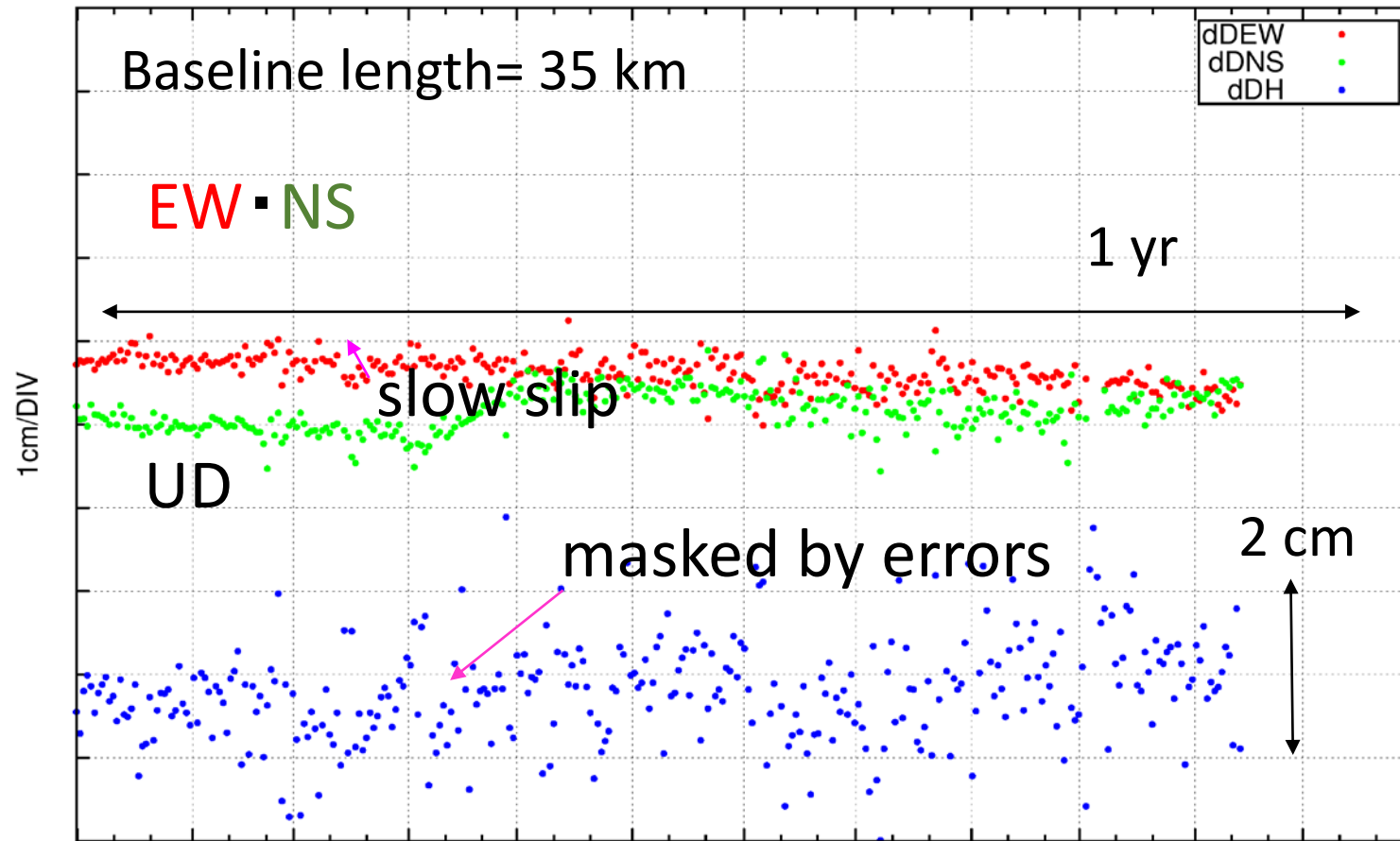


National Astronomical Observatory



Future applications of 5×10^{-19} clocks

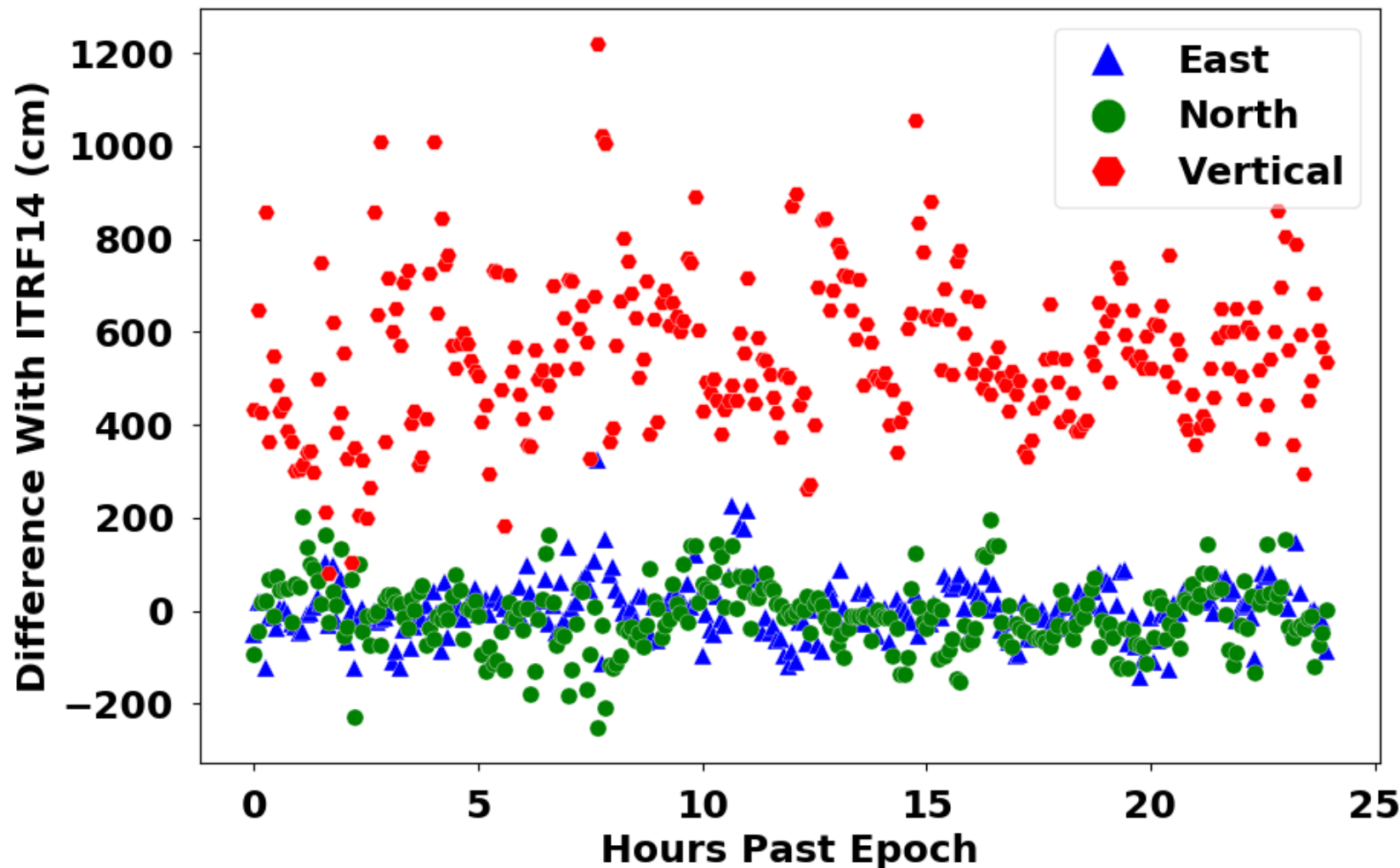
- Example of GNSS daily coordinates



- Atmospheric delays and other unmolded effects (e.g., surface loading, orbit, reference frame) lead to deviations of 1 cm for 24 hour average.

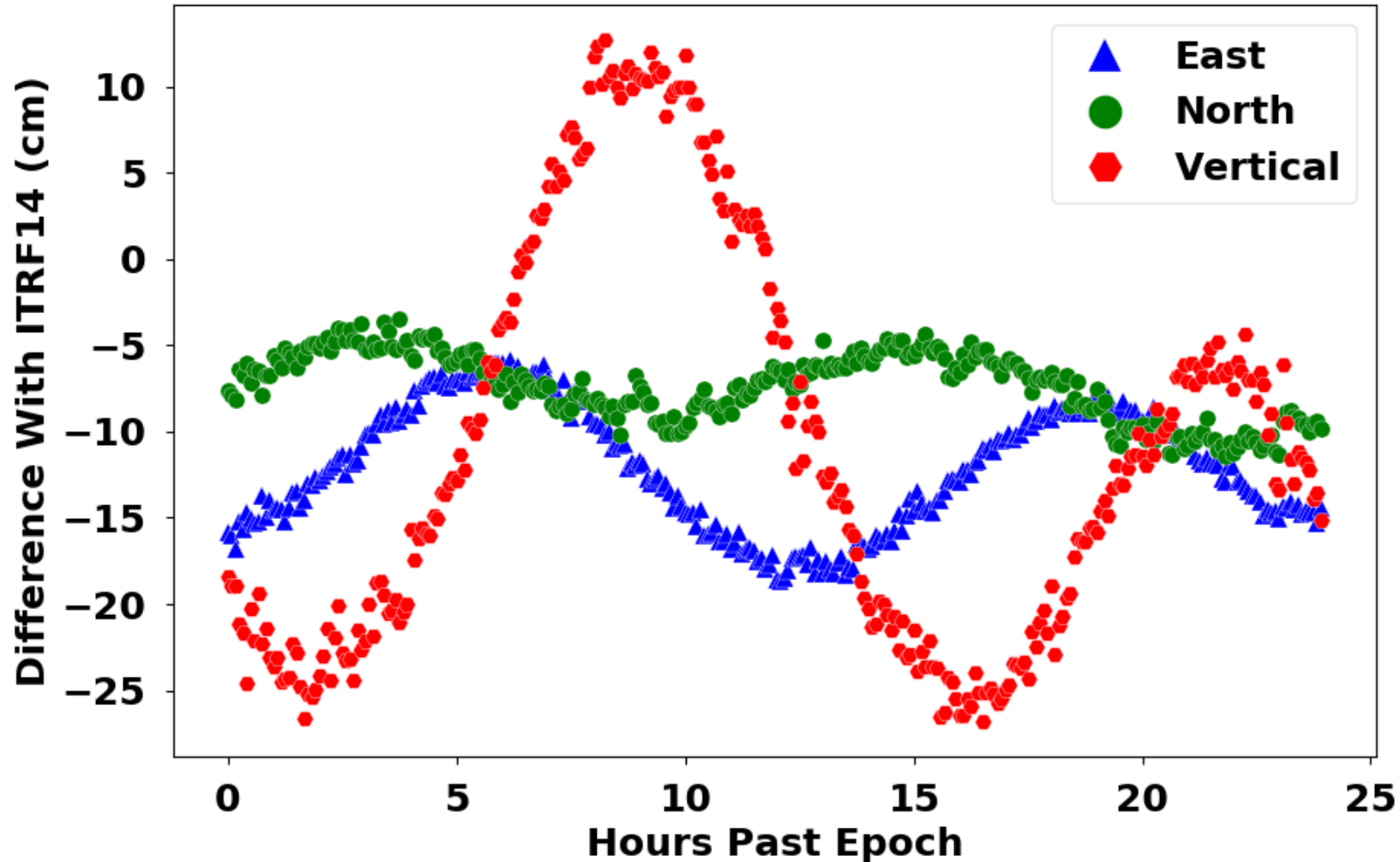
Atmospheric delay in the GNSS measurement

- 5 min. average (before correction for the atmospheric delay), PPP



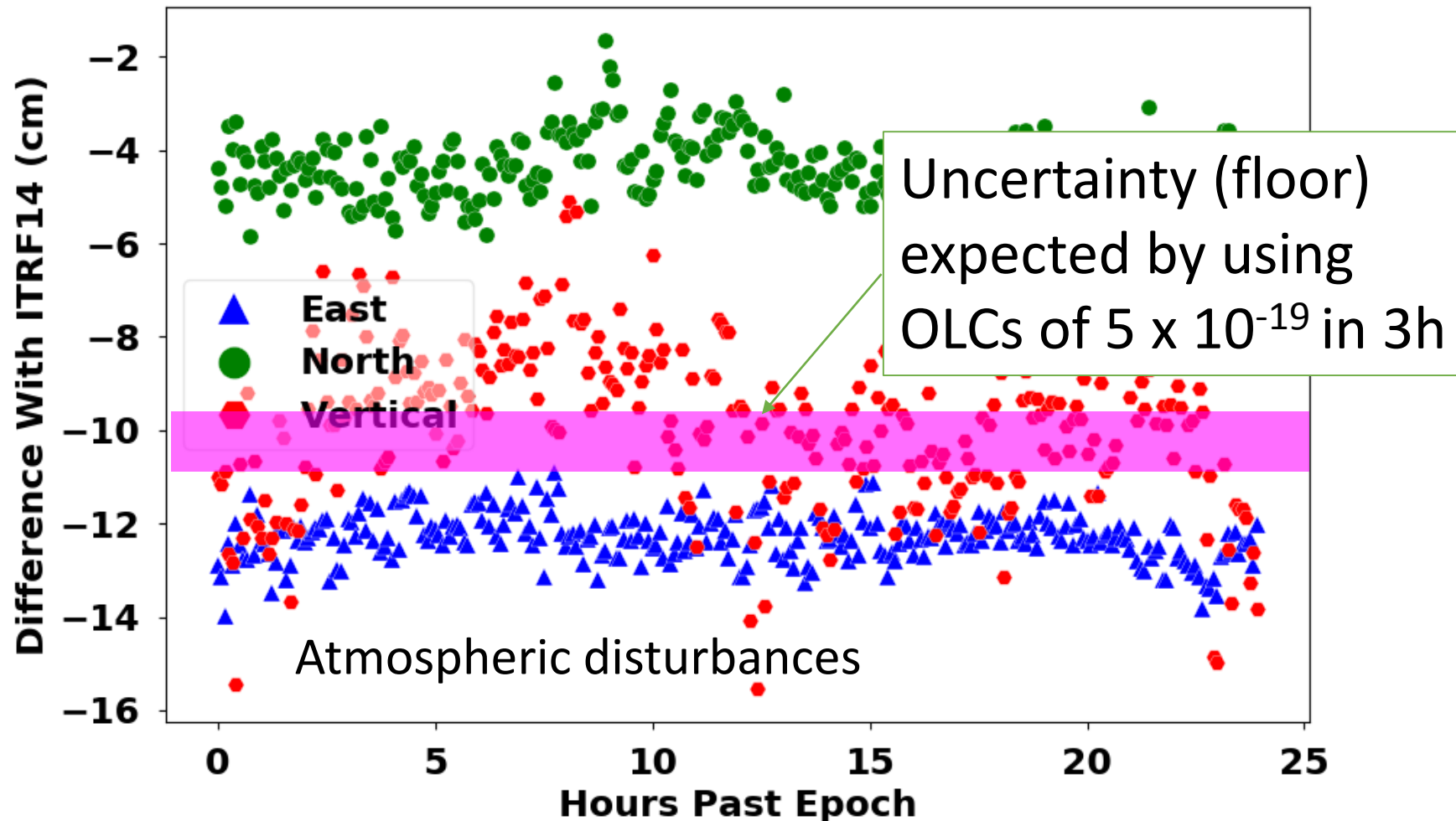
Atmospheric delay in the GNSS measurement

- 5 min. average (after correction for the atmospheric delay)



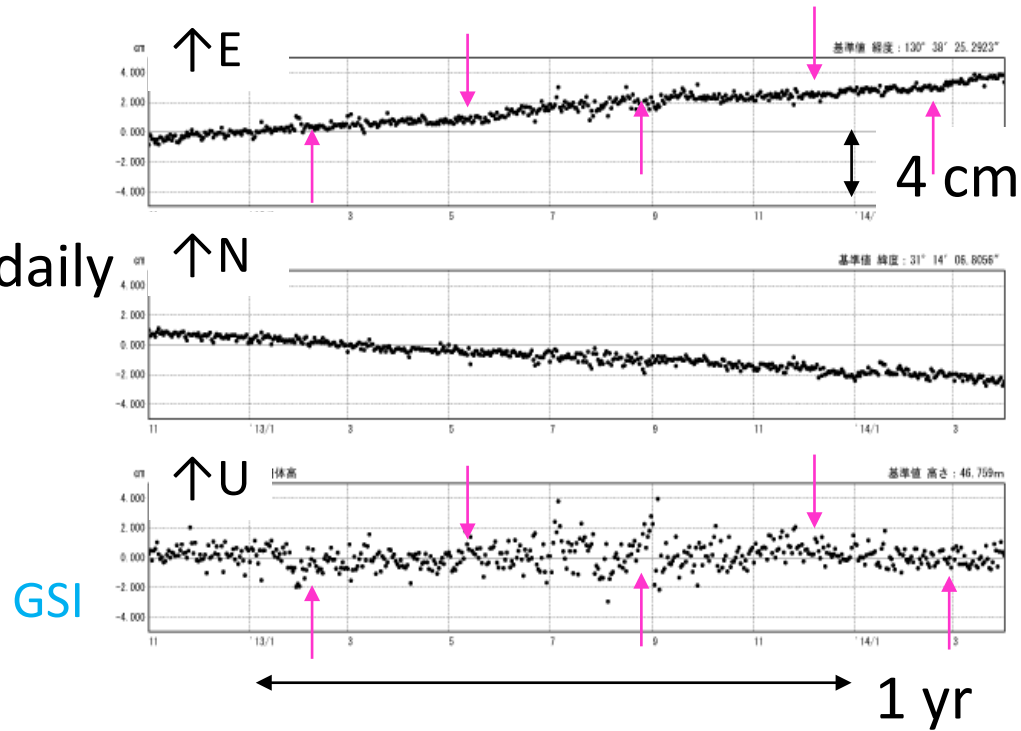
Improvement of real-time monitoring

- Example of GNSS coordinates, 5 min average, tides being removed, PPP

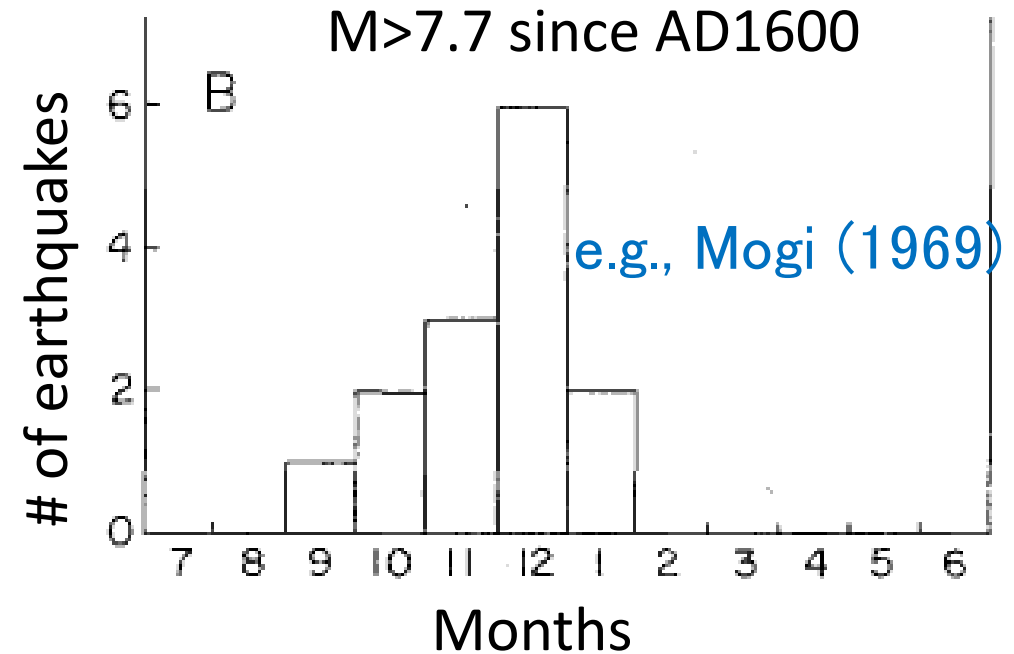


Reveal crustal behavior in seasonal bands

GNSS daily
coord.



- Even horizontal components suffer from seasonal noises (or signals?)



- Great earthquakes in SW Japan tend to occur in Autumn and Winter

Is there a seasonal strain accumulation on the plate subduction interface?
Ground-based OLCs may provide a clue.

Summary of Part II

- Gravitational potential differences can be measured by ground-based, fiber-linked optical lattice clocks at uncertainties of 10^{-17} to 10^{-18} (10 cm to 1 cm).
- A large part of temporal gravitational potential variations due to tectonic phenomena is caused by height changes at the measurement sites.
- A new project started in Japan, using OLCs with uncertainties better than GNSS (1×10^{-18} in 3h), and the RG group aims at detection of vertical crustal deformation (e.g., postseismic deformation by the 2011 Tohoku event).
- OLCs may enable us to monitor more rapid/seasonal-scale inter-plate coupling through an improved quantification of vertical motions.

Outline

- Basic concept of geodesy (Part I)
 - Geophysical signals
 - Reference coordinate systems for geodesy
 - Height and geoid
- Chronometric geodesy using optical clocks (Part II)
 - Progress in Europe Use of atomic clocks for unifying height reference systems
 - Progress in Japan Potential applications in crustal deformation monitoring
- A few topics on clock and relativity (short)
 - Issue on the realization of the SI second
 - Relativistic effects in geodetic measurements (VLBI and satellites)

Issue on the realization of the SI second

- TAI (International Atomic Time)=UTC (Coordinated Universal Time)-[37 sec] is a coordinate time scale defined in a geocentric reference frame with **the SI second as realized on the rotating geoid** as the scale unit.
- Present clocks in use (e.g. 10^{-13} in NICT) require the red shift correction at only ~ 10 m (10^{-15}) accuracy (c.f. Caesium primary frequency standards can achieve 10^{-16} level)
- To use 10^{-18} clocks requires the red shift correction at <1 cm accuracy
 - (A) uncertainties for height and geoid $\ll 10$ m, but > 1 cm
 - (B) temporal variations in W_0 (solid Earth tides >10 cm, secular $\sim +3$ mm/yr)
- The current definition in the IAG assumes constant W_0 : WG established for solving this
 - The geoid is the equipotential surface closest to the mean sea level
 - The value of the gravity potential on the geoid is W_0

Space-borne high-precision clocks

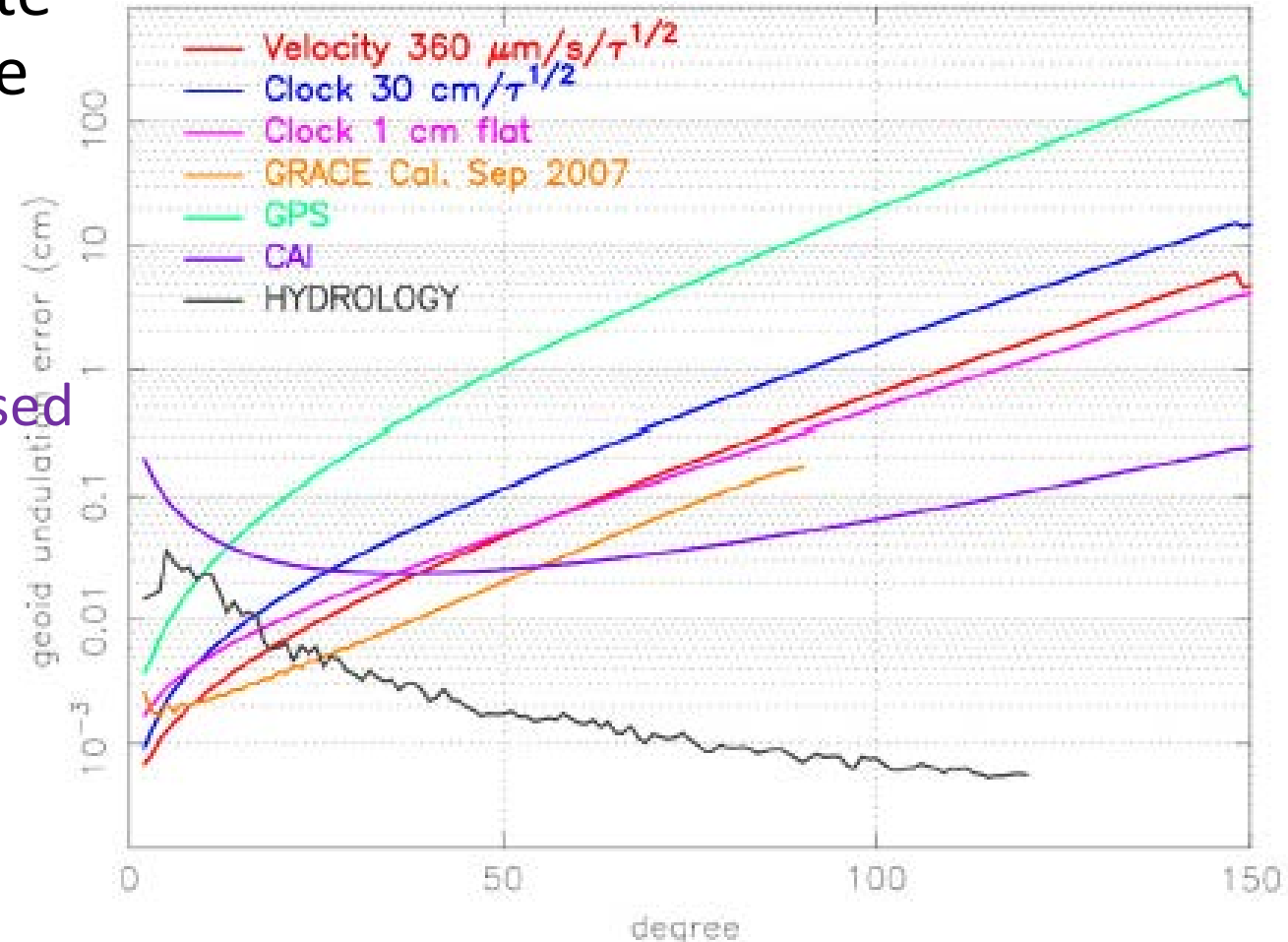
- Clock comparison between LEO satellite and ground station for determining the Earth's gravity field

- $$\frac{\Delta V_E}{g} = \frac{1}{g} \frac{\mu}{r} \left\{ 1 + \sum_{l=2}^{\infty} \sum_{m=0}^l \left(\frac{a_e}{r} \right)^l (\Delta \bar{C}_{lm} \cos m\lambda + \Delta \bar{S}_{lm} \sin m\lambda) \bar{P}_{lm}(\sin \phi) \right\}$$

Blue: 10^{-18} clock Purple: 3D gradiometer based on cold atom interferometry

Orange: GRACE Black: hydrological signal

- The satellite motion (or velocity) must be known extremely accurately, unlike the case for clocks at rest

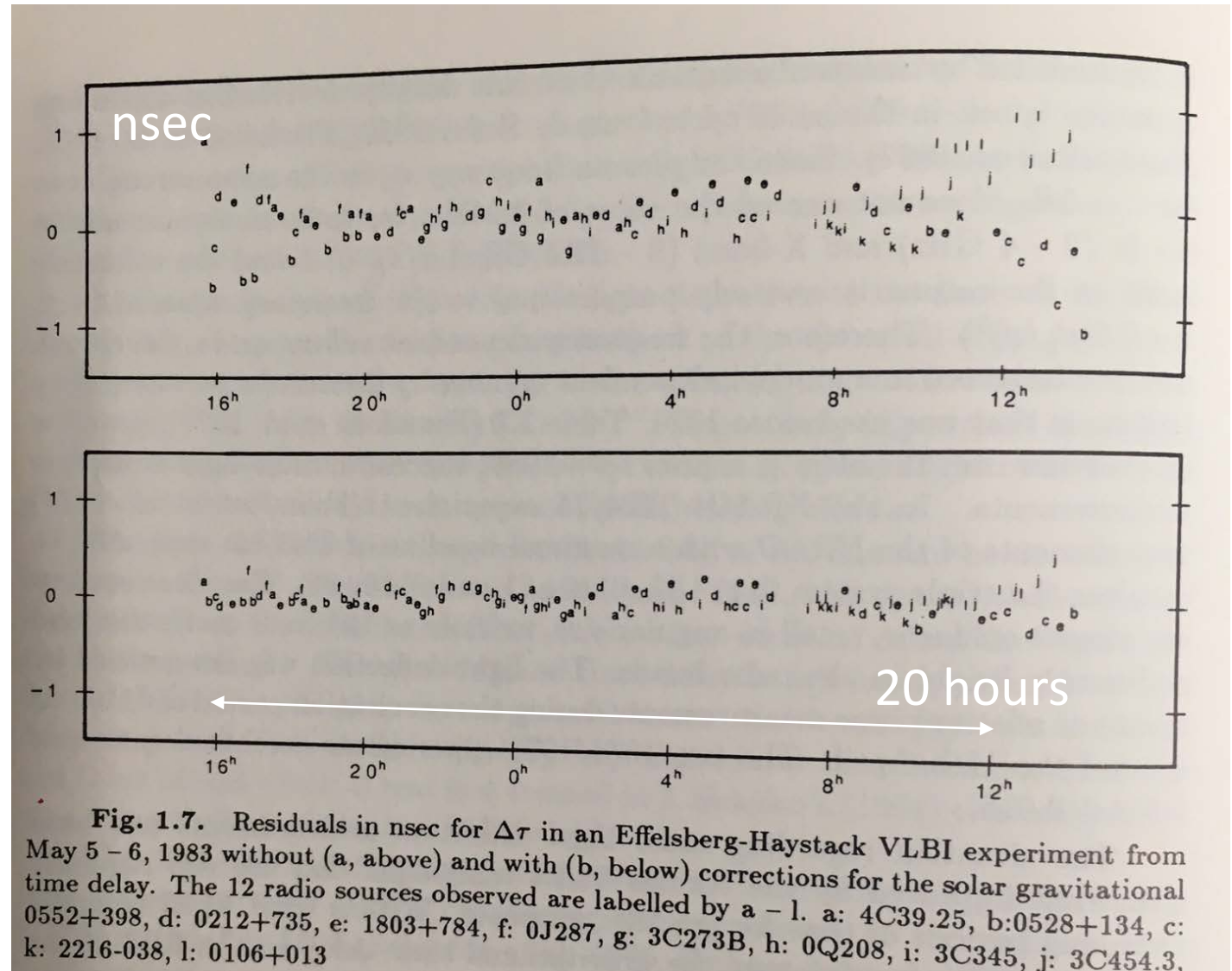


Examples of relativistic effects in geodesy

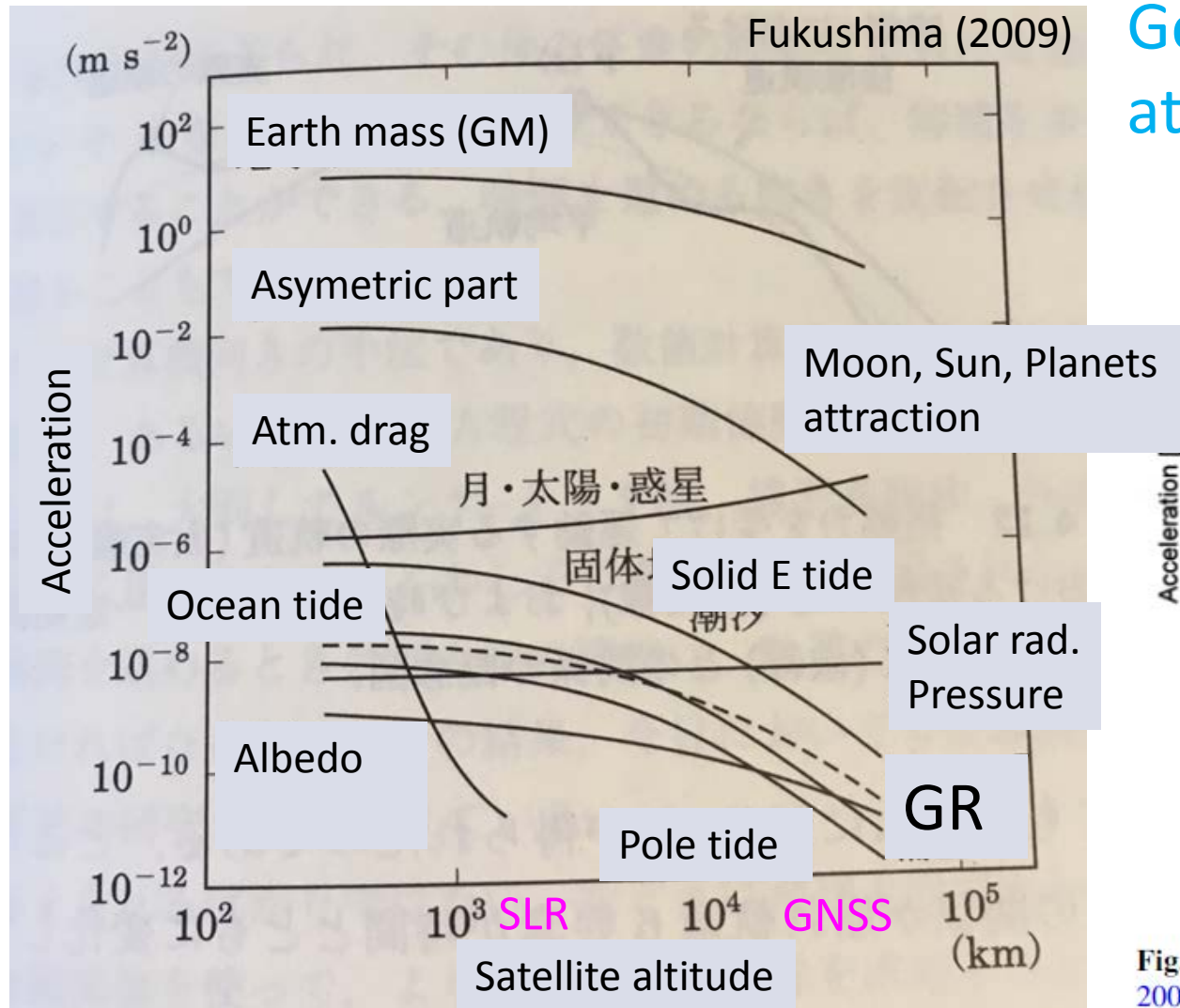
Solar gravitational delay in VLBI

- Time difference $\Delta\tau$ between Effelsberg and Haystack for 12 quasars (a-l)
- Plotting residuals in $\Delta\tau$
- Upper: before correction
- Lower: after correction

Soffel (1989)



Examples of relativistic effects in geodesy



Geophysical and GR effects on acceleration at various satellite altitudes

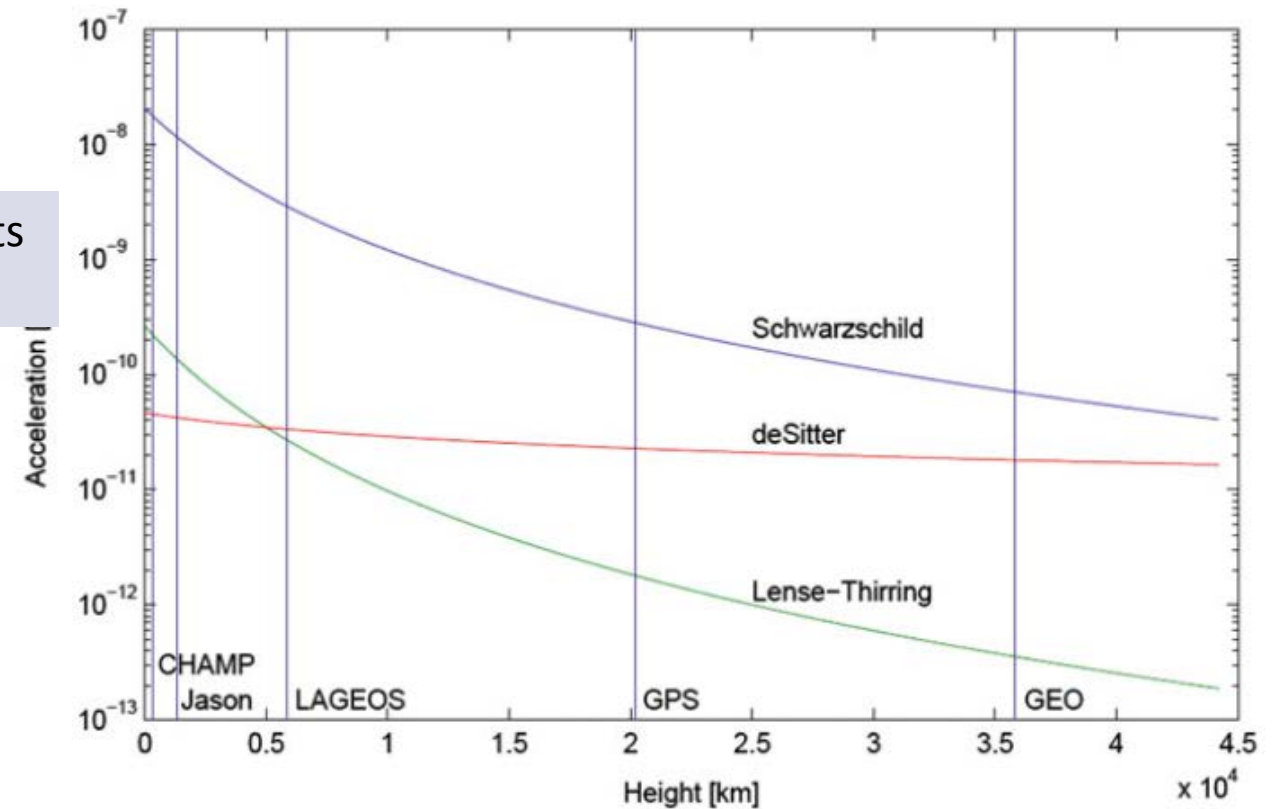


Fig. 2.1 Radial acceleration as a function of satellite height for circular orbits (Hugentobler 2008)

- The orbital motion of SLR can be represented as

$$\ddot{\vec{r}} = -\frac{GM_{\oplus}}{r^3}\vec{r} + \vec{f}$$

Moon, Sun, Planets Solar and Earth radiations
 $\vec{f} = \vec{f}_{\text{NS}} + \vec{f}_{\text{TC}} + \vec{f}_{3B} + \vec{f}_g + \vec{f}_{\text{Drag}} + \vec{f}_{\text{SRP}} + \vec{f}_{\text{ERP}} + \vec{f}_{\text{other}} + \vec{f}_{\text{Emp}}$
Static and temporal Once/cycle empirical cor.
Gravity field

$$\Delta\ddot{\vec{r}} = \frac{GM_{\oplus}}{c^2 r^3} \left\{ \left[2(\beta + \gamma) \frac{GM_{\oplus}}{r} - \gamma (\ddot{\vec{r}} \cdot \ddot{\vec{r}}) \right] \ddot{\vec{r}} + 2(1 + \gamma) (\ddot{\vec{r}} \cdot \ddot{\vec{r}}) \ddot{\vec{r}} \right\} \quad \text{Schwarzschild}$$

$$+ (1 + \gamma) \frac{GM_{\oplus}}{c^2 r^3} \left[\frac{3}{r^2} (\ddot{\vec{r}} \times \ddot{\vec{r}}) (\ddot{\vec{r}} \cdot \vec{J}) + (\ddot{\vec{r}} \times \vec{J}) \right] \quad \text{Lense-Thirring precession}$$

$$+ \left\{ (1 + 2\gamma) \left[\ddot{\vec{R}} \times \left(\frac{-GM_s \ddot{\vec{R}}}{c^2 R^3} \right) \right] \times \ddot{\vec{r}} \right\} \quad \text{de Sitter precession}$$

PPN (Eddington 1923)

- All perturbations except for the GR effects are modeled for a LSQ estimation of β and γ from the observed motion: $\gamma-1, \beta-1 \sim 10^{-3}$
 - These parameters can be better constrained by Lunar Laser Ranging (LLR)
- Combrinck (2013) General Relativity and Space Geodesy

Summary

- Geodesy provides a practically available geocentric reference frame.
- Geophysical signals concentrate around 1% of the tidal signals in amplitude. These have been observed and various models have been proposed.
- Ground-based fiber-linked optical lattice clocks are useful for unifying the height reference systems and geoid models as well as crustal deformation monitoring.
- Other uses (e.g. space-borne clocks) are under consideration (IAG WG)