A Tower WGC from IR consistency

Stefano Andriolo

based on SA, D. Junghans, T. Noumi, G. Shiu 1802.04287

Kavli IPMU
Tokyo University

20th September 2019

香港科技大學
THE HONG KONG UNIVERSITY OF
SCIENCE AND TECHNOLOGY

Outline

The Weak Gravity Conjecture (WGC)

- Landscape and Swampland
- The WGC (and CHC)

Infrared Consistency

- Causality, Analyticity
- Relation to WGC

WGC and dimensional reduction

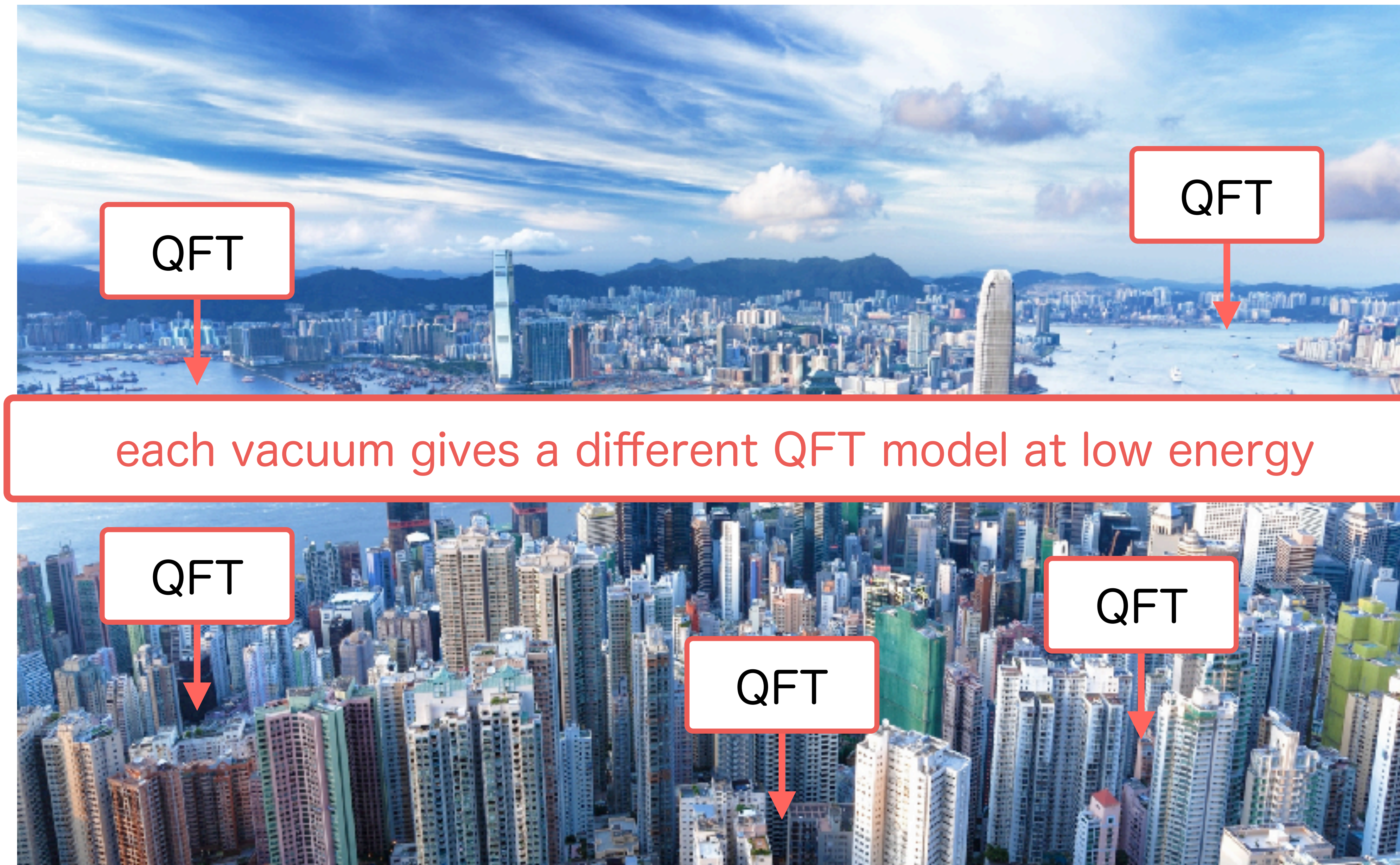
- The Lattice WGC
- Tower WGC

Summary & Outlook

Landscape and Swampland

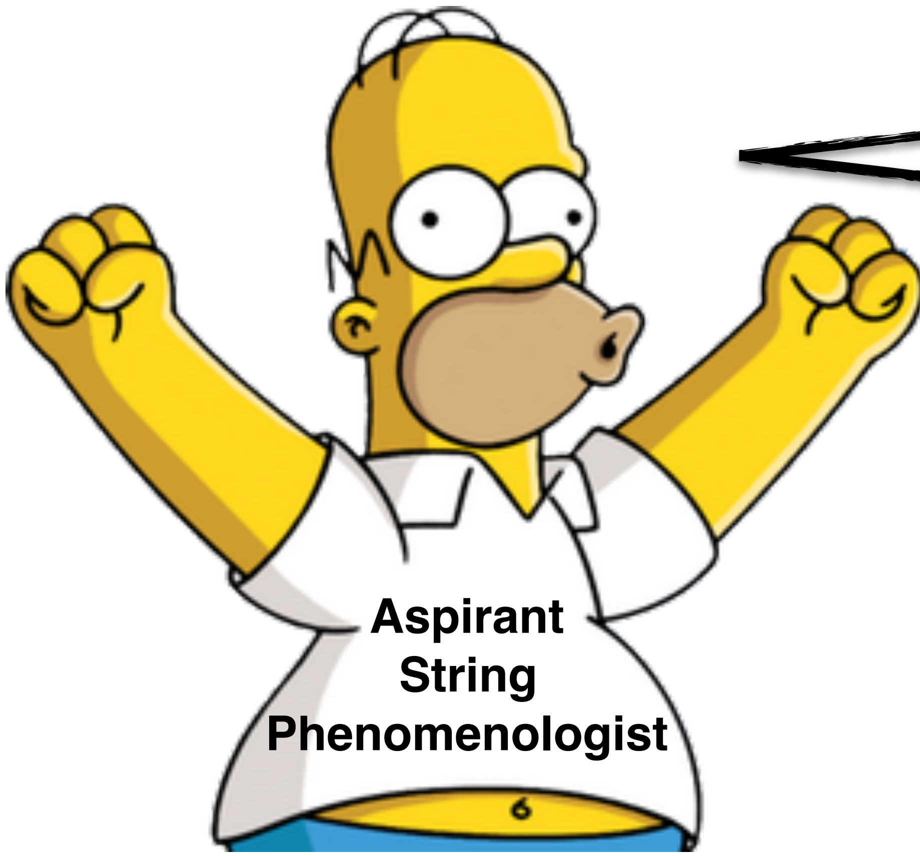
Observation: String Theory allows for enormous amount of different vacua (“Landscape”)

- how to compactify, type/amount fluxes, how to put D-branes/localised objs…



each vacuum gives a different QFT model at low energy

Landscape and Swampland



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pretending it is a LE manifestation
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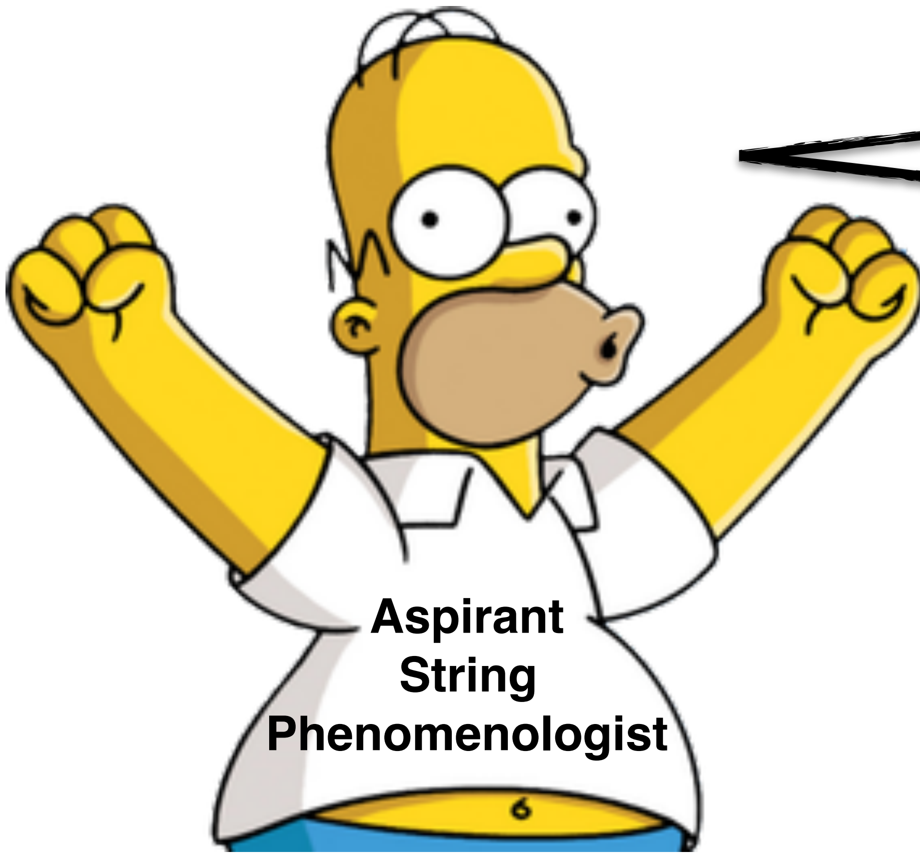

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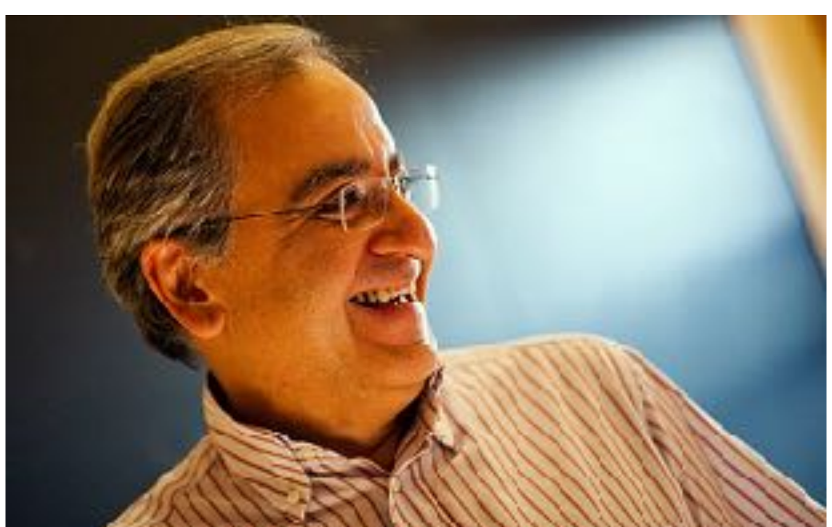
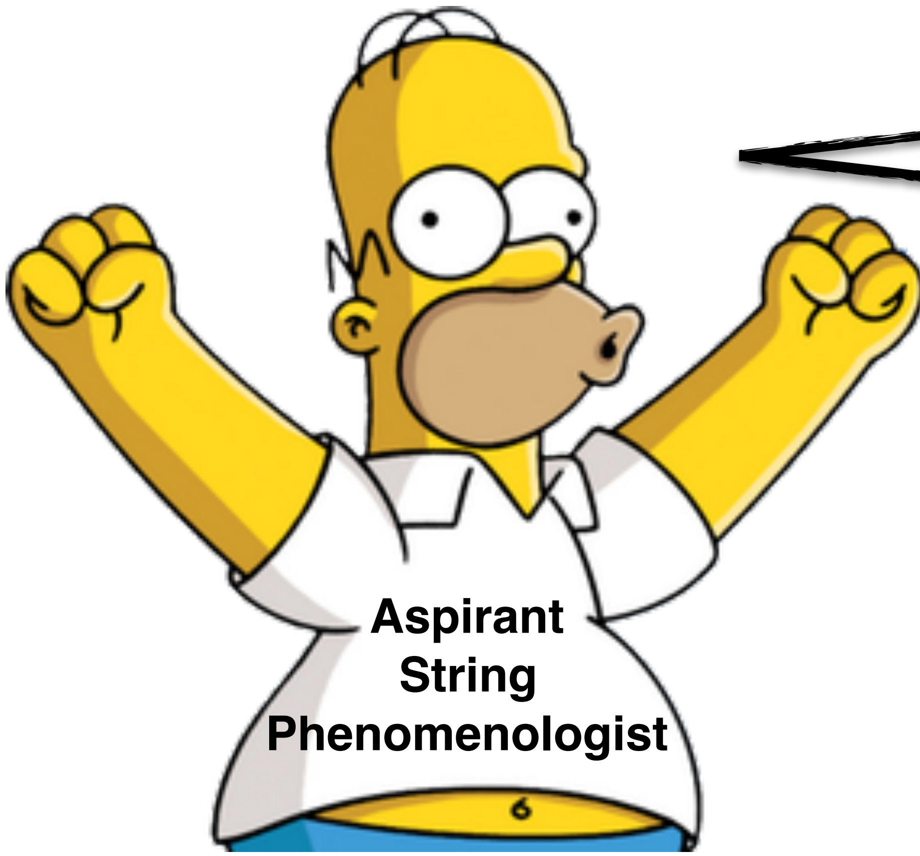


[Vafa 0509212]

Landscape and Swampland



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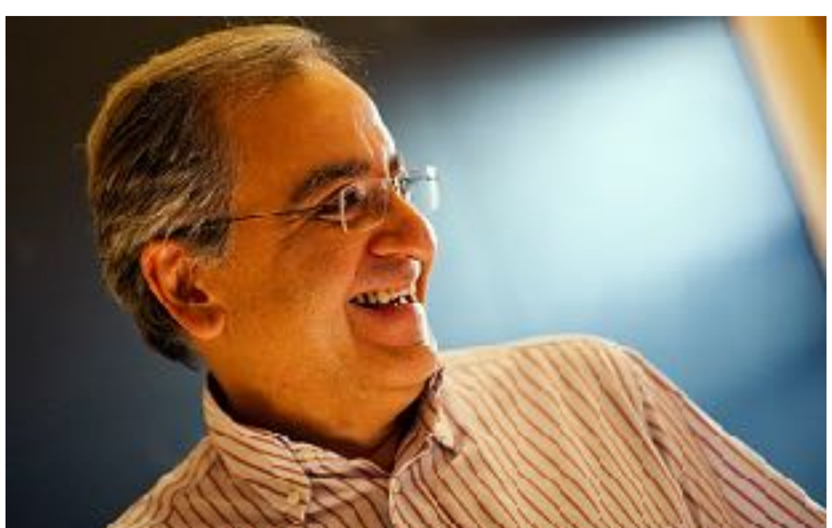


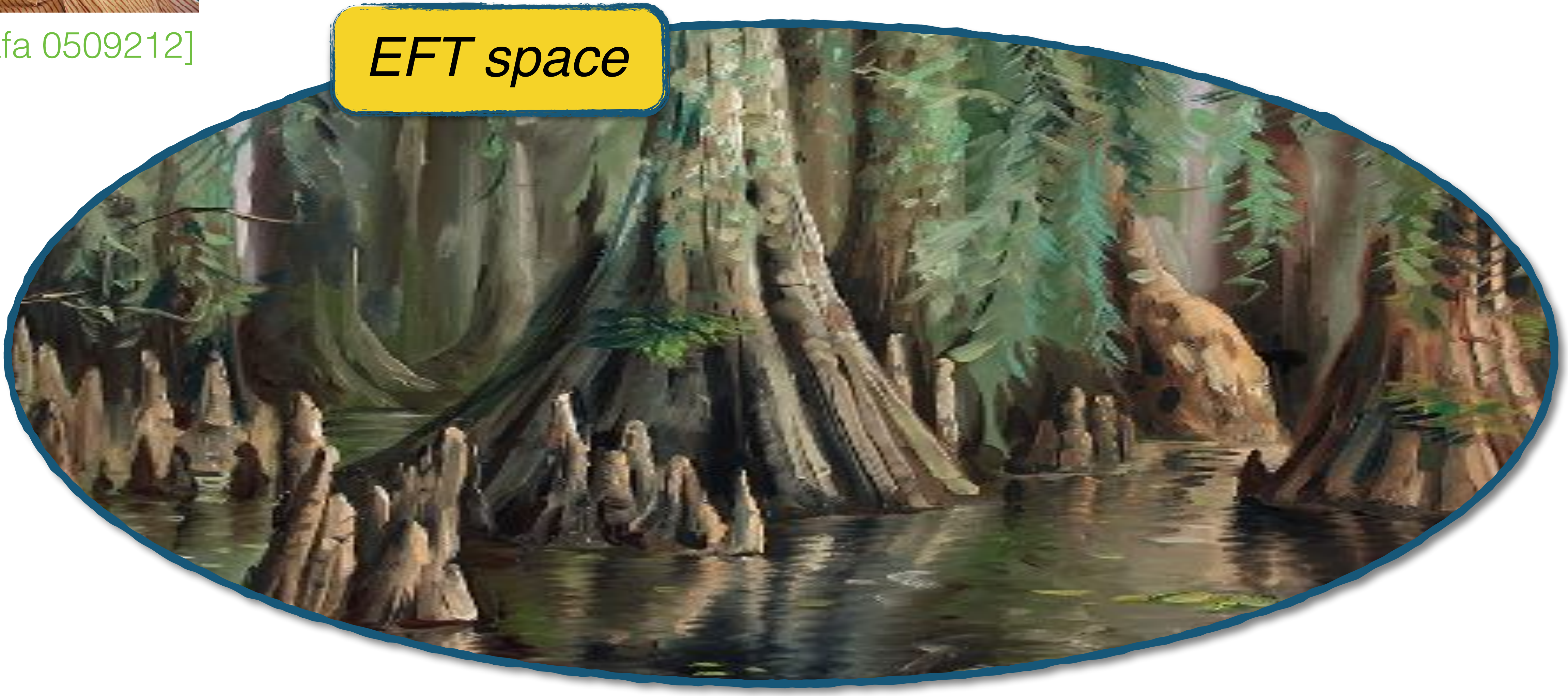
[Vafa 0509212]

If an EFT comes from QG (ST), then it must inherit and manifest some particular QG (stringy) features, which are absent in a generic EFT!

Landscape and Swampland

[Vafa 0509212]

In other words, I believe the picture should be:



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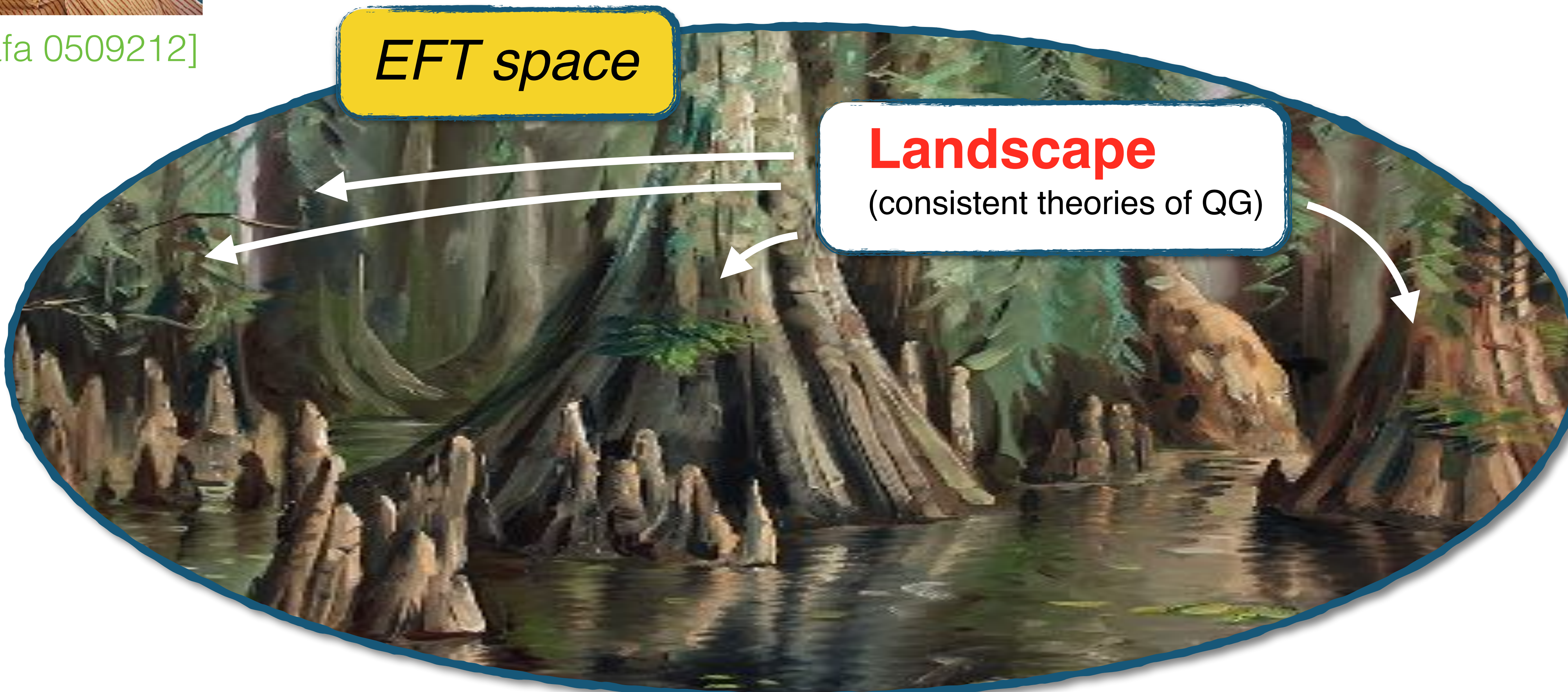

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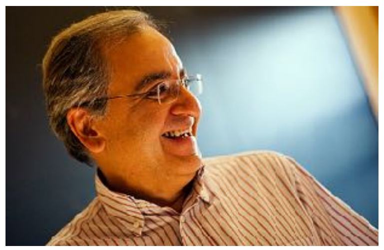

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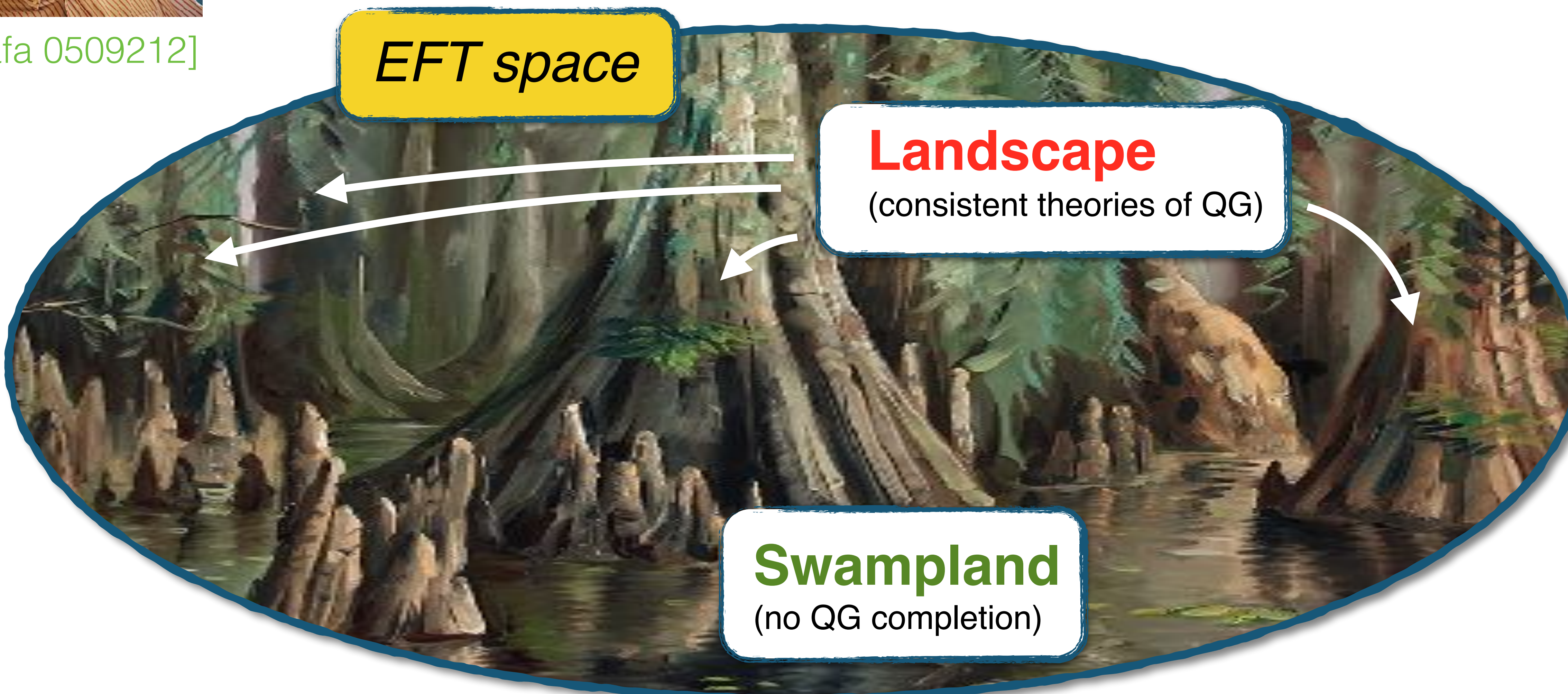

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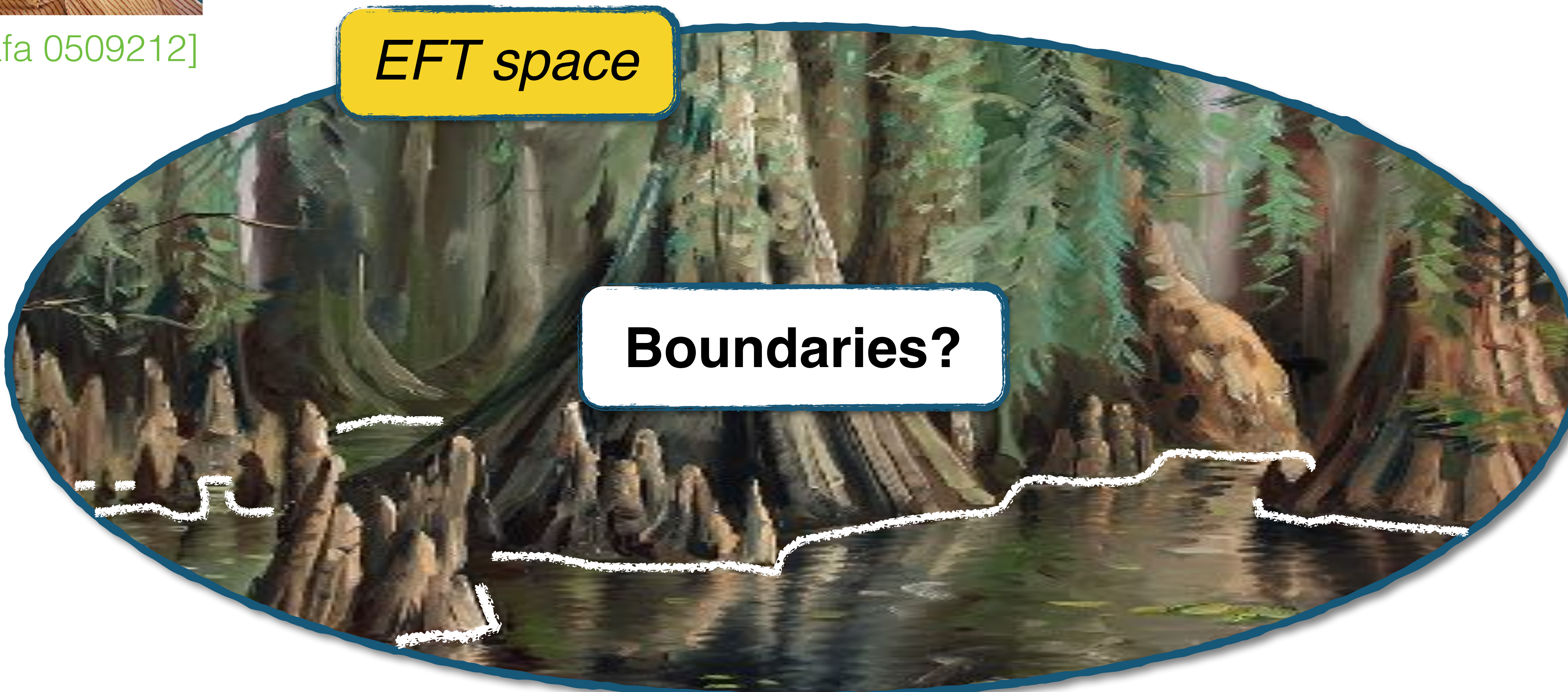

Criteria to distinguish
Swampland/Landscape?

Swampland criteria (conjectures)

[Reviews:
Brennan, Carta, Vafa 1711.00864
Palti 1903.06239]

An effective field theory coupled to gravity:

- **No free parameters**

ST: only scale=string scale,
no free parameters

e.g. string coupling

$$g_s = e^{\phi}$$

$\Rightarrow$

IR: EFT parameters=field vevs

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Rencent findings concern

- **Distance conjecture** (new Physics from the boundaries of moduli space) [Ooguri, Palti, Shiu, Vafa 1810.05506]
- **(A)dS Conjecture(s)** [Obied, Ooguri, Spodyneiko, Vafa 1806.08362] [Lust, Palti, Vafa 1906.05225]
- **Brane-y constraints** [Kim, Shiu, Vafa 1905.08261]

Swampland criteria (conjectures)

An effective field theory coupled to gravity:

▶ **No (exact) global symmetries** [Matt Reece ICTP lectures 2019]

- true in ST: all global symmetries turn out to be gauged [Banks, Dixon '88]
- true in theories with asymptotically AdS (holography)

[Harlow, Ooguri 1810.05338]

Q: More generically, is this true with *any* QG completion ?

A: **BH Physics**

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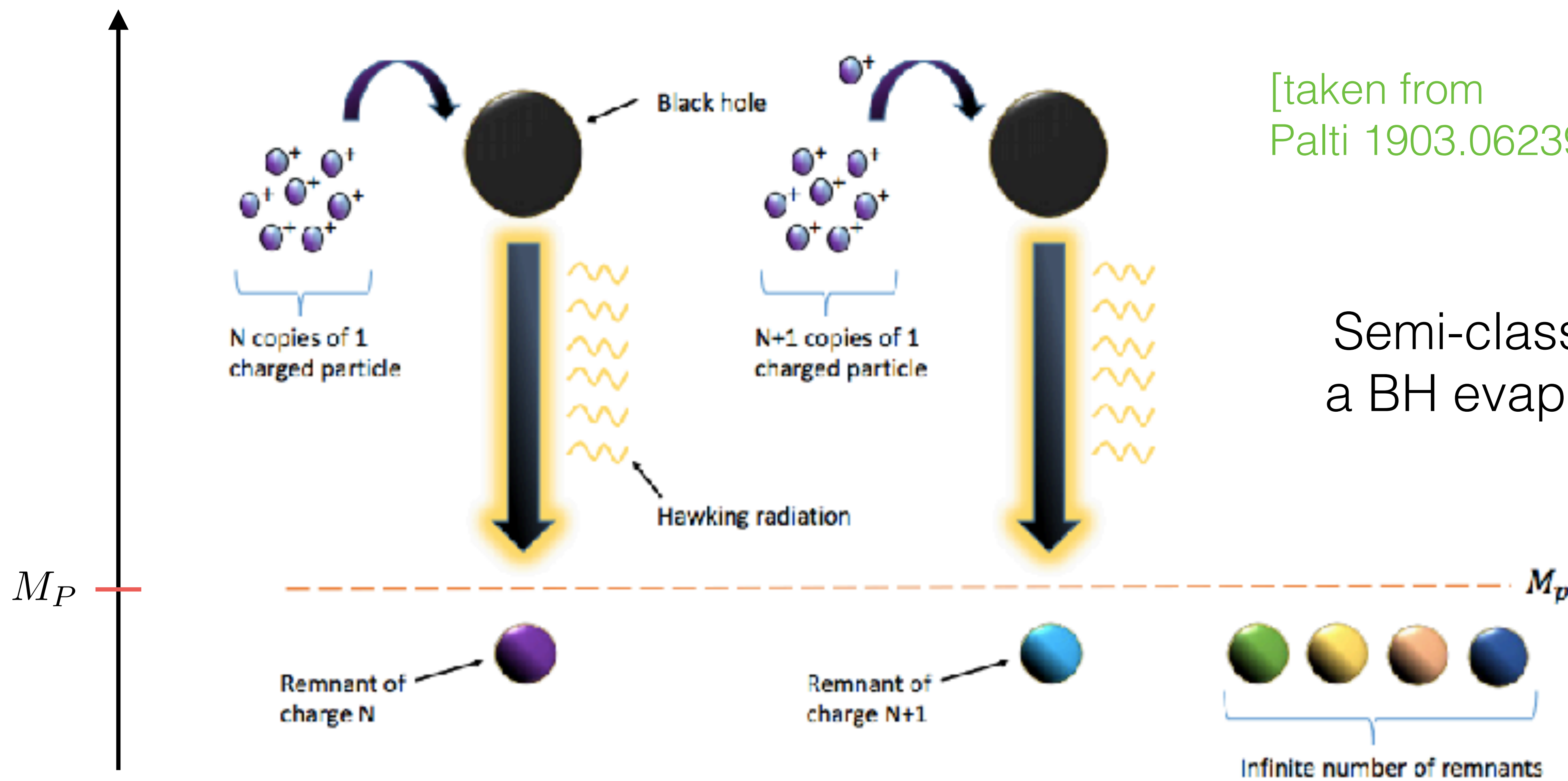
Rough arguments follow. Please bear with me!

No (exact) global symmetries

Suppose you have global symmetry (e.g.U(1)), and a particle charged under it (q)

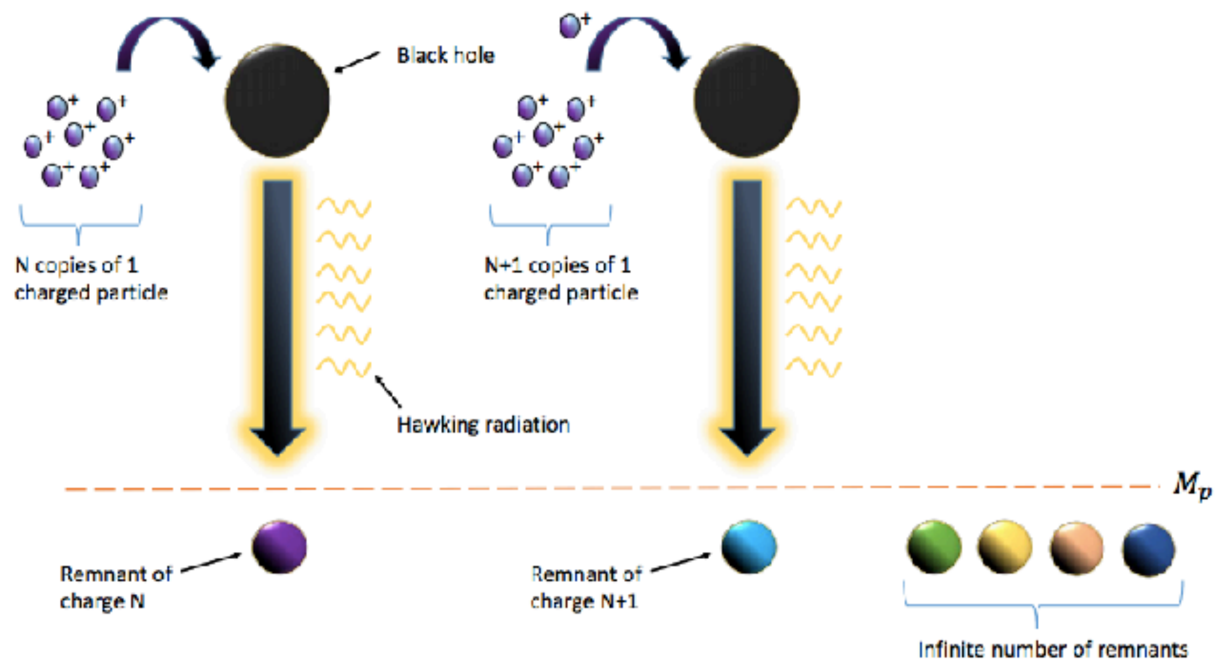


[taken from Palti 1903.06239]

Semi-classically a BH evaporates

Problems with **infinite** number of (semi-classically stable) charged remnants?

Problem at **low energies**…



M_P

at lower energies we have to integrate out all these states: they contribute to renormalising Newton constant

$\Lambda \ll M_P$

deep-IR: EFT with $G_N = 0$

The problem is avoided if the symmetry is **gauged**:

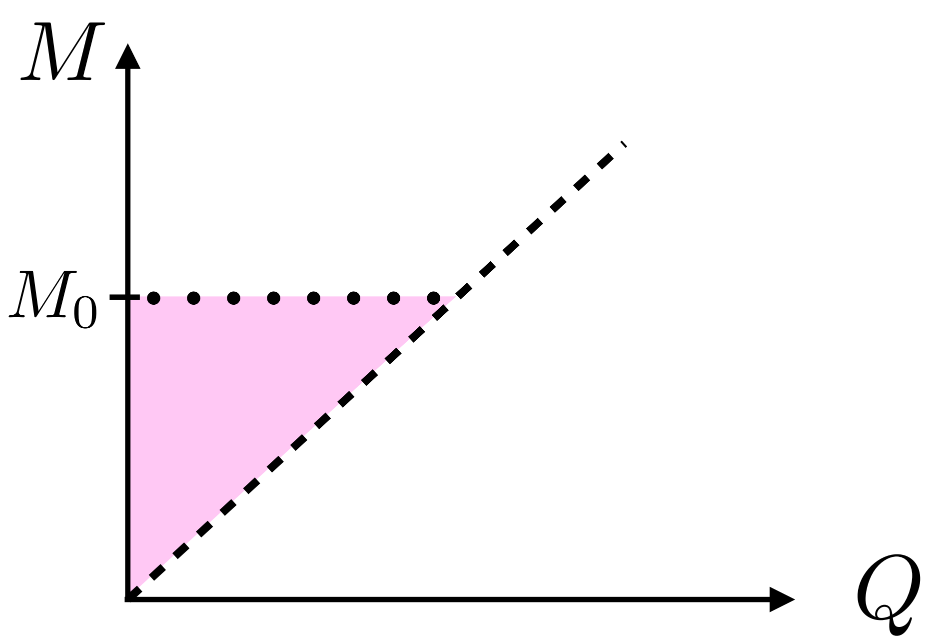
charged BH solutions

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2 d\Omega^2\,, \qquad f(r) = 1 - \frac{2M}{r} + \frac{2g^2Q^2}{r^2}$$

Extremality bound

$$M \geq \sqrt{2} g Q M_P$$

(Violation=naked singularities)



Below any given mass,
there is a **finite** number of EBH's

$$N_{EBH} \sim \frac{M_0}{g M_p}$$

Maximum charge with M_0

Problem solved!

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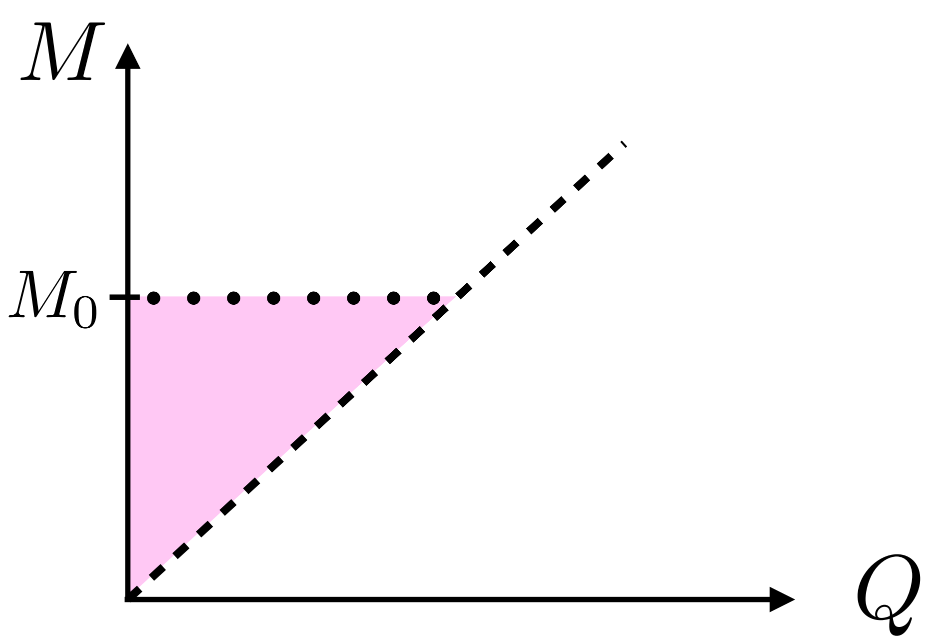

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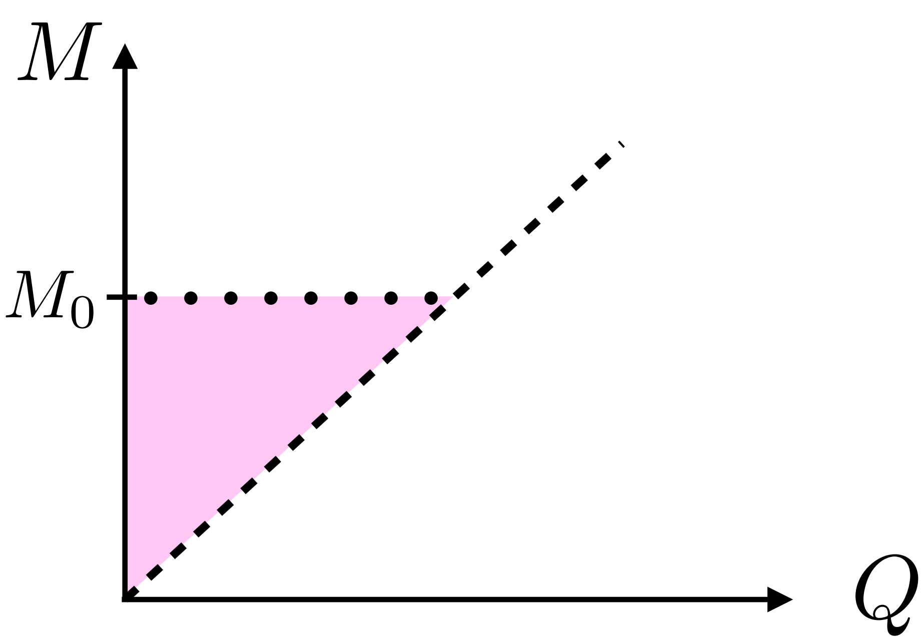

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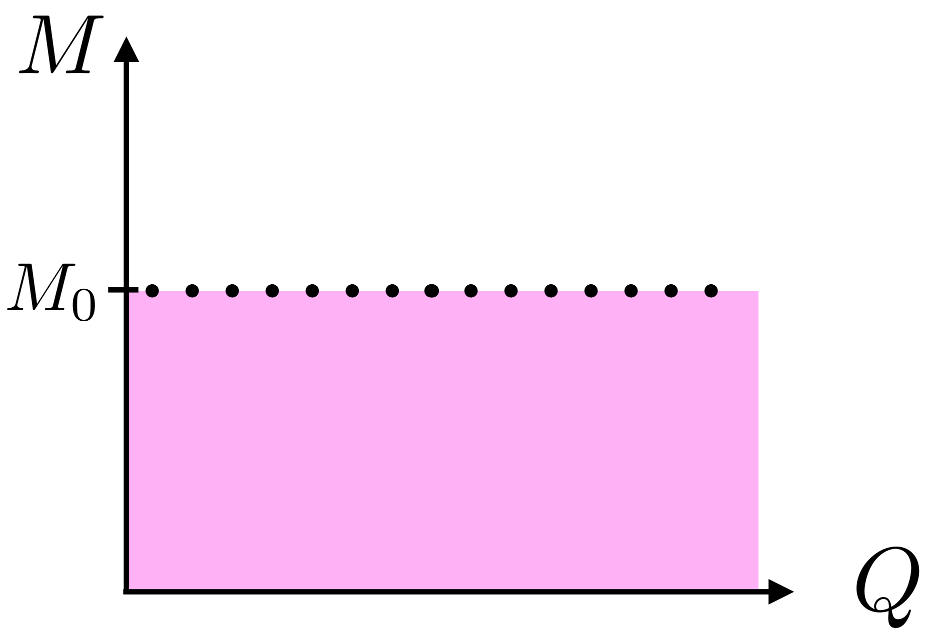
What about a gauge theory in the limit $g \to 0$

For $g \to 0$ the problem is **recovered**!

Extremality bound

$$M \geq \sqrt{2} g Q M_P \rightarrow 0$$



Below any given mass,
there is a **infinite** number of BH's

$$N_{EBH} \sim \frac{M_0}{g M_p} \to \infty$$

Basically, switching off the coupling g corresponds to approaching the **GLOBAL** symmetry with its issues…

If we insist in having an EFT with finite G_N and no naked singularities,
Then it must be:

- Impossible to take $g \to 0$? but this seems valid in ST…

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OR

- Something off with our understanding of BHs

  (indeed above reasonings use the semi-classical approximation)

not correct to use semi-classical description of BH's up to M_P :
Neglected quantum gravity effects may render BH's **unstable** and decay!

In such a case, they do not contribute to catastrophic renormalisation of G_N

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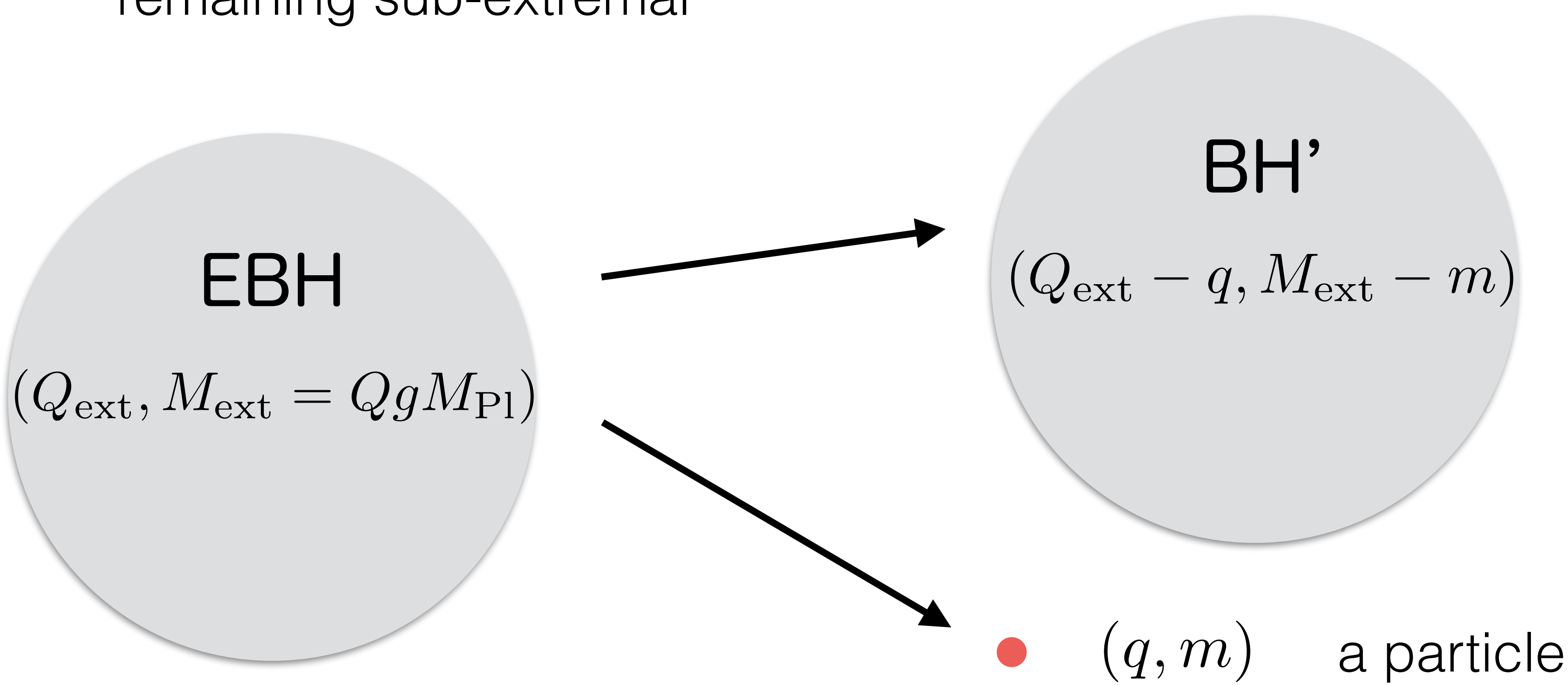

(Sub-extremal BH “decay” into EBH via radiation, but EBH’s are stable, need another decay process)

$$g(Q_{\rm ext} - q) \leq (M_{\rm ext} - m)/M_{\rm Pl} \leftrightarrows gq \geq m/M_{\rm Pl}$$

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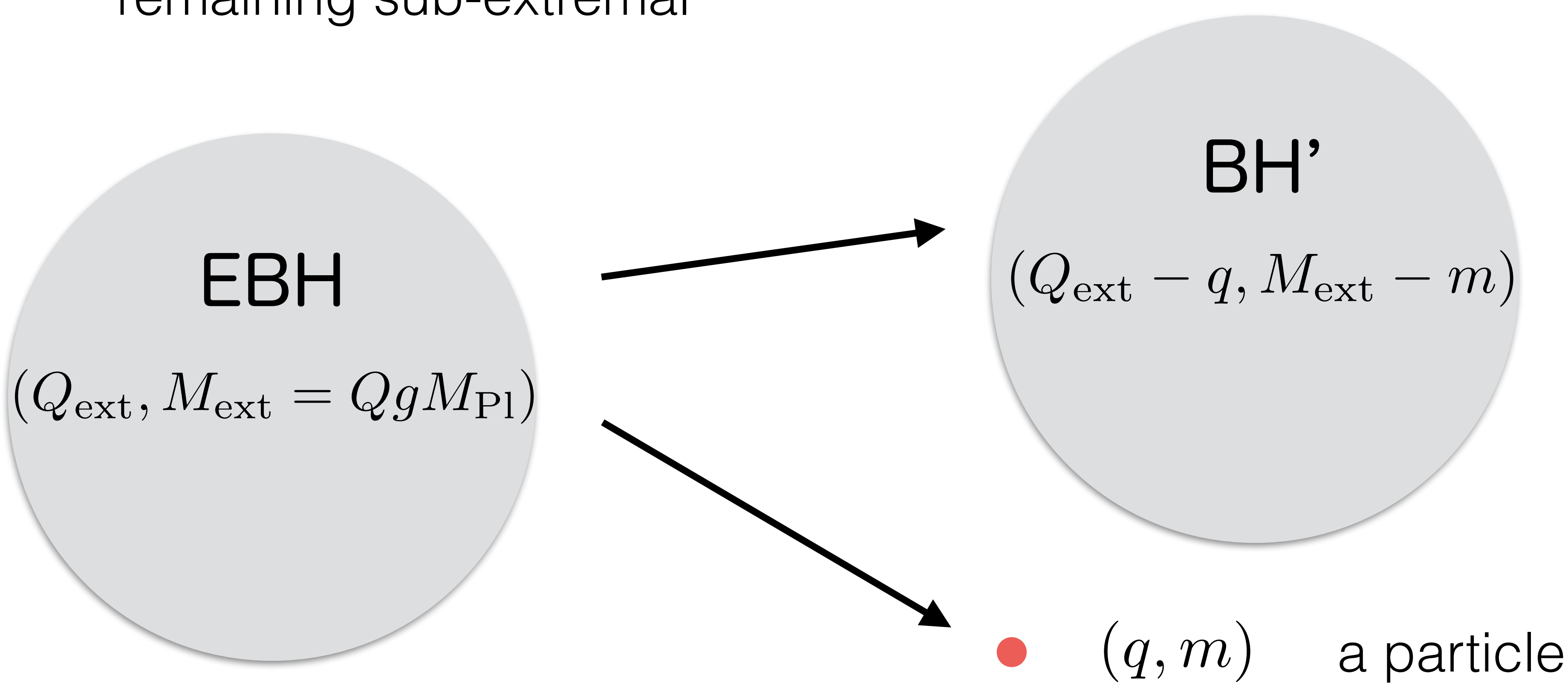

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z_{EBH}

→ require existence of a particle satisfying $\boxed{z \equiv \dfrac{gqM_P}{m} \geq 1}$

WGC

The Weak Gravity Conjecture (WGC)

In any consistent EFT of gauge U(1) coupled to gravity
There must exist a particle (m,q) with charge-mass ratio

$$z \equiv \frac{gqM_P}{m} \geq 1$$

In particular, it has to be true also for magnetic monopoles

$$m_{mag} \leq g_{mag} M_p \sim \frac{M_p}{g_{el}}$$

but we know

$$m_{mag} \sim \frac{\Lambda}{g_{el}^2}$$

UV CUT-OFF

The cutoff scale Λ of the effective theory is bounded
from above approximately by the gauge coupling

$$\Lambda \lesssim g_{el} M_P$$

A posteriori, motivated by many
examples in ST

NEW CUT-OFF below M_p
when weakly coupled $g_{el} \ll 1$

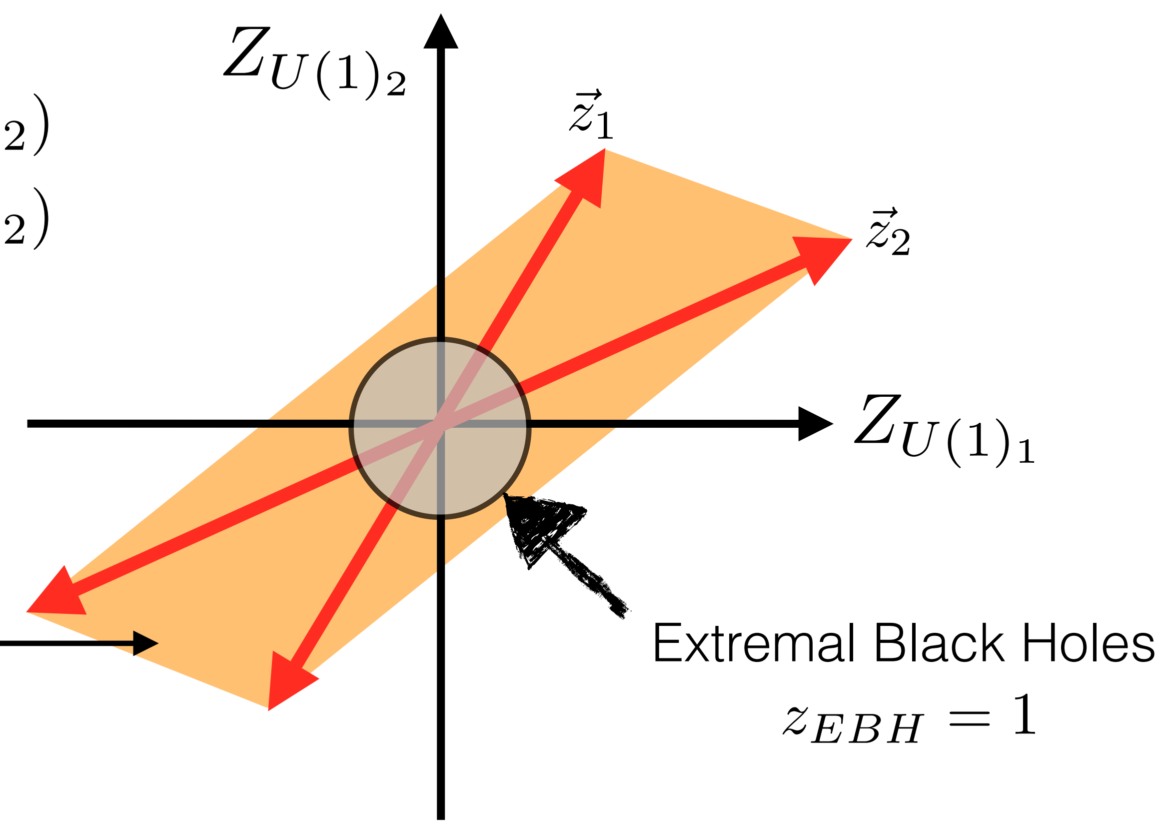
WGC: The convex hull condition (CHC)

In case we have **multiple U(1)'s**, obj can decay if their charge-to-mass ratio lies inside the convex hull (in the z-space) [Cheung, Remmen 1402.2287]

(sub-)Extremal Black holes can decay if they lie inside the Convex Hull!

Ex.: $U(1)^2$, 2 particles

$$\vec{z_1} = (z_{11}, z_{12})$$
$$\vec{z_2} = (z_{21}, z_{22})$$



Convex Hull is the region of instability: any Z inside, decays

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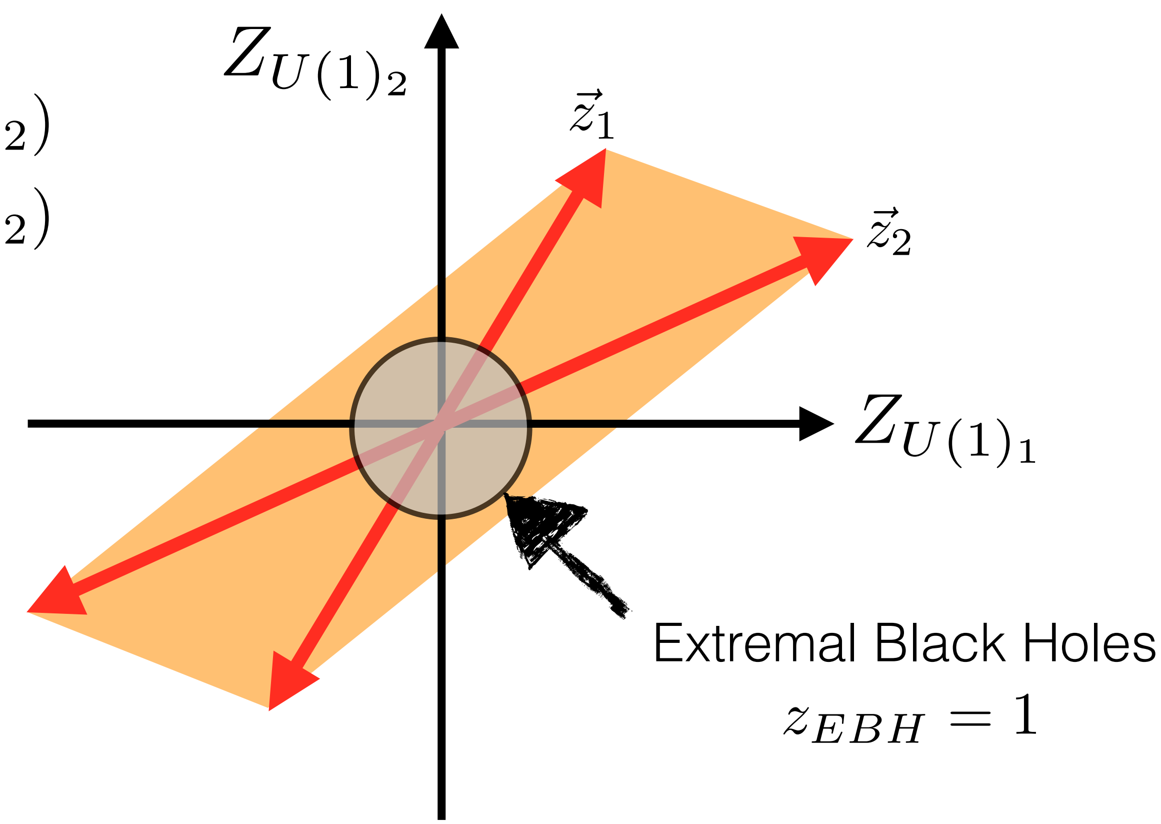

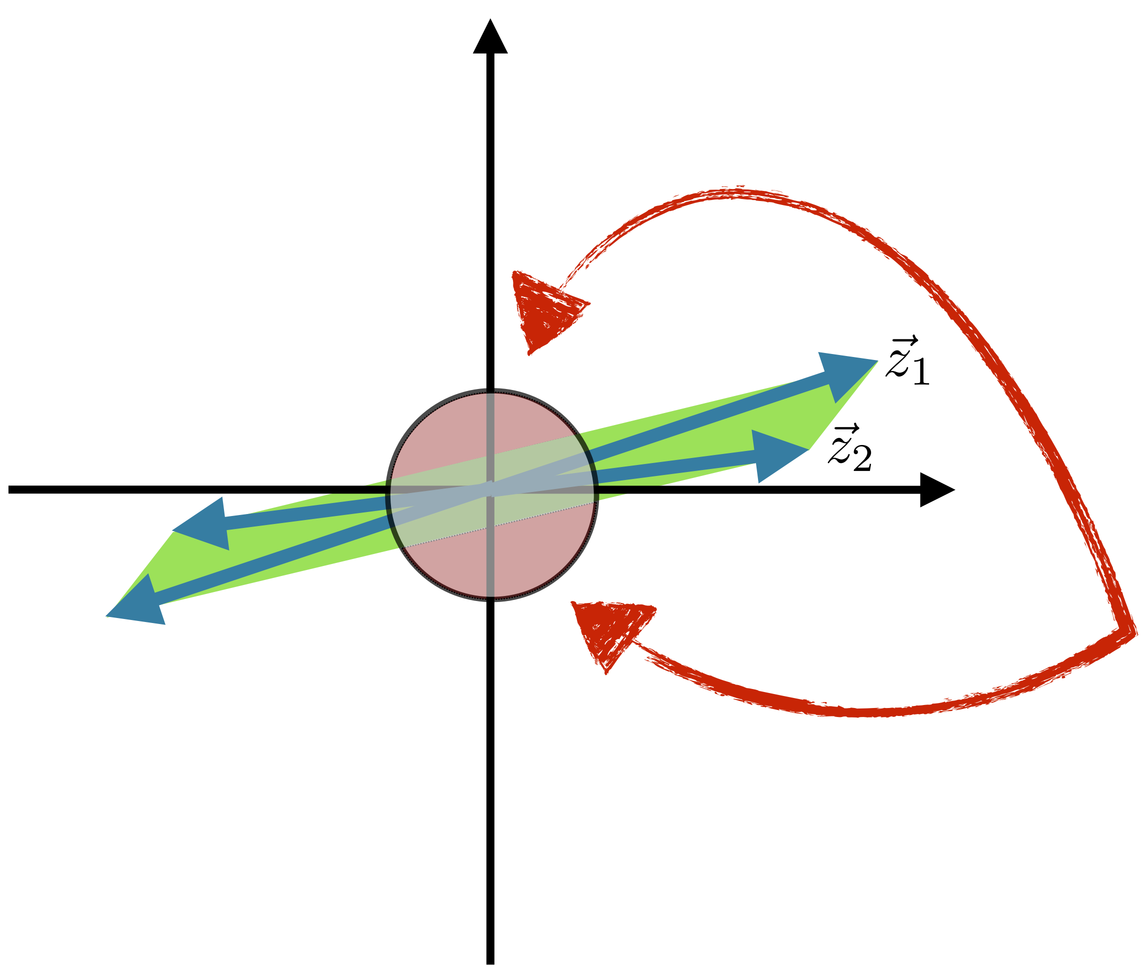

There are no states which can discharge Black Holes in the red regions!

recent directions:

1. how to evade WGC and realize axion inflation models

[De la Fuente et al '14, Bachlechner et al '15, Choi-Kim '15, Conlon-Krippendorf '16,…]

2. constraints on particle physics models (ex. neutrino masses)

[Ooguri-Vafa '16, Ibanez, MartinLozano-Valenzuela '17, Hamada-Shiu '17 …]

3. better understanding & towards a proof of WGC

- lessons from string theory examples

[Brown et al '15, Heidenreich et al '15, Hebecker-Soler '17, Montero et al '17,…]

- use of AdS/CFT (holography)

[Nakayama-Nomura '15, Harlow '15, Benjamin et al '16, Montero et al '16, Montero '18,…]

- **relation to positivity bounds**

[Cheung-Remmen '14, Andriolo-Junghans-Noumi-Shiu '18, Hamada-Noumi-Shiu '18,…]

Positivity bounds (IR)

An EFT with HO operators = 4-derivative corrections

Eg. $$\mathcal{L}_{1-loop} = \frac{M_P}{2} R - \frac{F^2}{4e^2} + CF^4 \; + \ldots$$

is “IR consistent” i.e. respects: causality

analyticity of S-matrix

if $$\boxed{C > 0}$$

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi 0602178]

Causality - eg 3D

First, dualisation: $\mathcal{L}_1 = \frac{M_3}{2}R - \frac{(\partial\phi)^2}{2} + 4C(\partial\phi)^4$ $\quad F \sim *d\phi$

We want to study the the speed of propagation of fluctuations $\phi = \bar{\phi} + \varphi$
and require it is sub-luminal in any locally flat frame η_{ab}

i.e., we require $\quad v \equiv \frac{k_0}{|\vec{k}|} < 1$

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1) Compute the corrected EOMs, locally go to Fourier space and obtain (background-dependent) dispersion relation

$$\left(\eta_{ab} + 16C\ \overline{\partial_a\phi\partial_b\phi}\right)\ k^a k^b = 0$$

True for any bg choice

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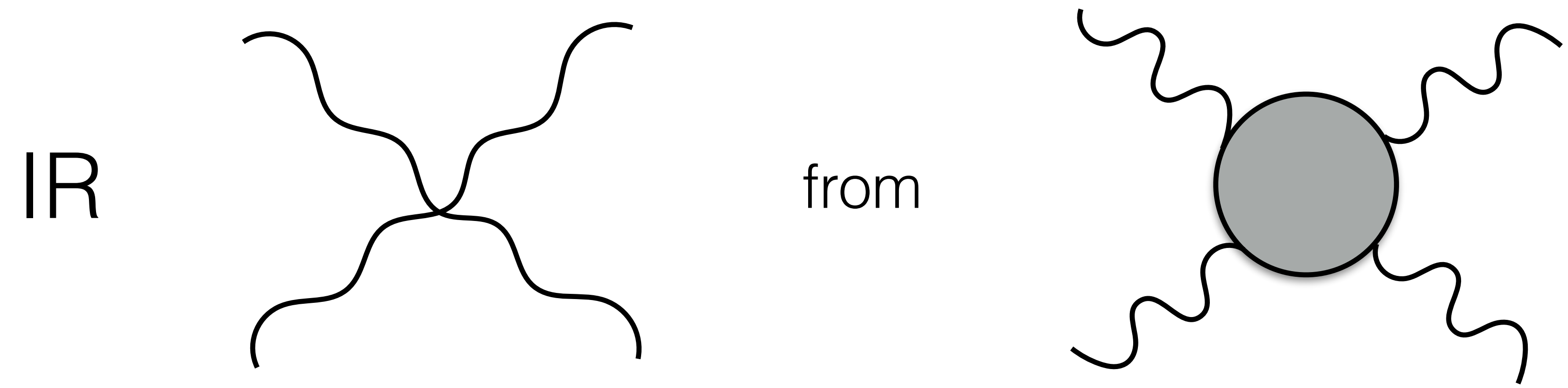
2) Simplest choice of bg: constant EM field $\overline{\partial_a\phi} = w_a = (w_0, \vec{w}) = const$

$$v = \frac{k_0}{|\vec{k}|} = 1 - 8C(w_0 - \vec{w}\cdot\hat{k})^2 \quad \rightarrow \quad \boxed{C > 0}$$

Analyticity - eg 3D

Consider 4-pts photon scatterings

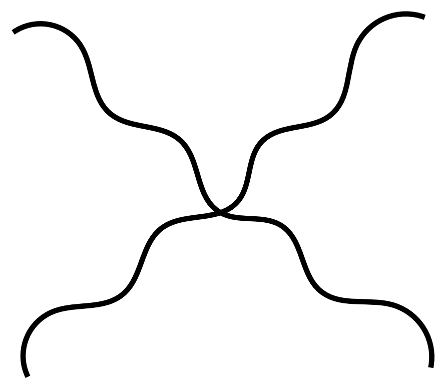
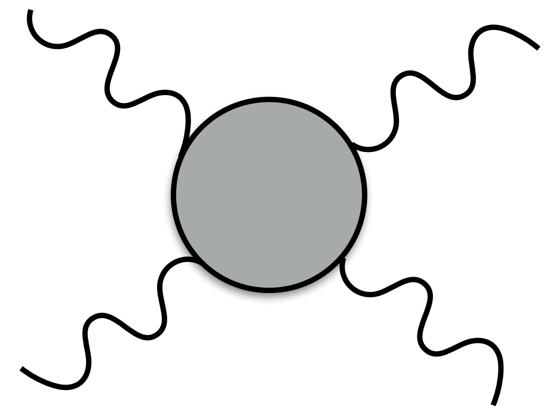


In the forward limit $\mathcal{M}(s,t) = 8C(s^2+t^2+u^2) \underset{t\to 0}{\rightarrow} \mathcal{M}(s) = 16Cs^2$

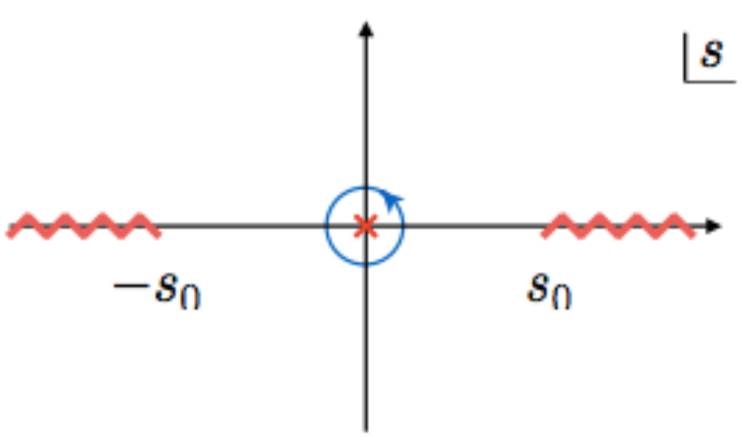
We can extract C from

Analyticity - 3D/4D

Consider 4-pts photon scatterings

IR  from  $\mathcal{M}(s) = 16Cs^2$

$$16C = \oint_\gamma \frac{ds}{2\pi i} \frac{\mathcal{M}(s)}{s^3}$$



Analyticity - 3D/4D

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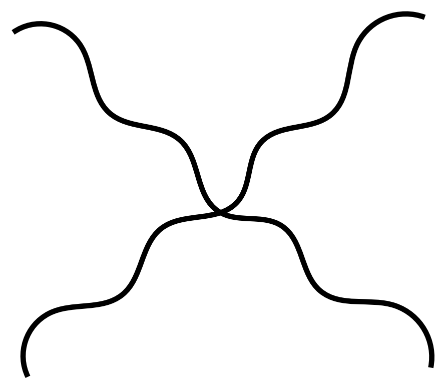
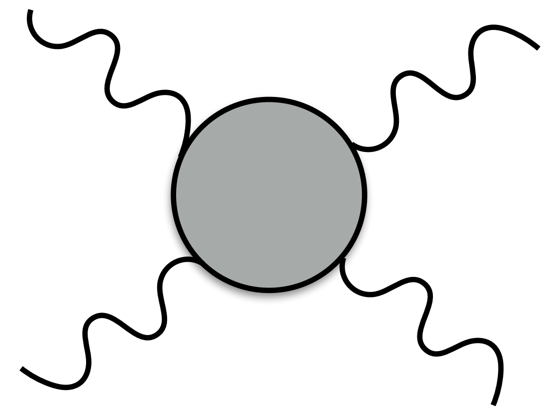
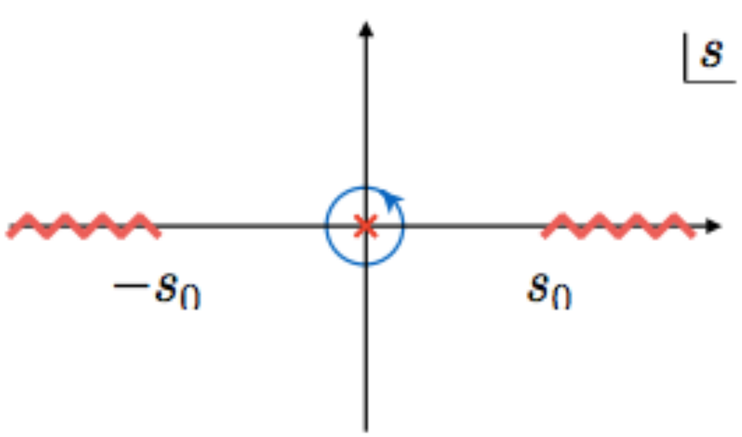
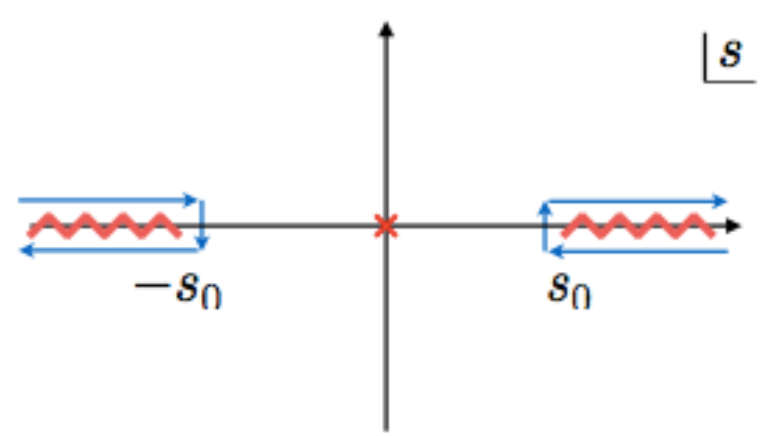
IR  from 

$$16C = \oint_{\gamma} \frac{ds}{2\pi i}\frac{\mathcal{M}(s)}{s^3} = \left(\int_{-\infty}^{-s_0} + \int_{s_0}^{\infty}\right)\frac{ds}{2\pi i}\frac{\mathrm{Disc}[\mathcal{M}(s)]}{s^3}$$

contour def
+ **analyticity**
+ Froissart bound



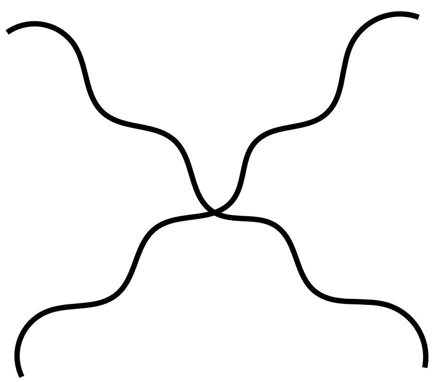


The S-matrix is **analytical** along the real axis $|s| < s_0$, up to the lowest energy where on-shell intermediate states are created (= red discontinuities)

Analyticity - 3D/4D

Consider 4-pts photon scatterings

IR



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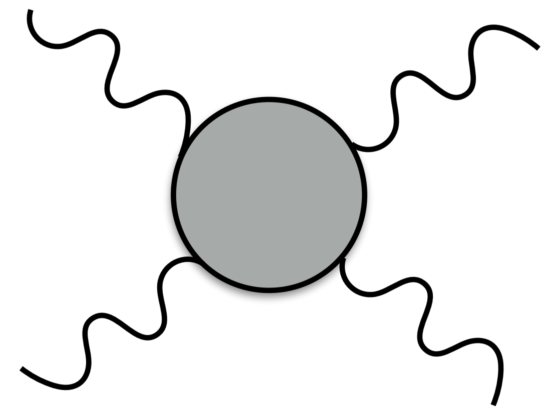

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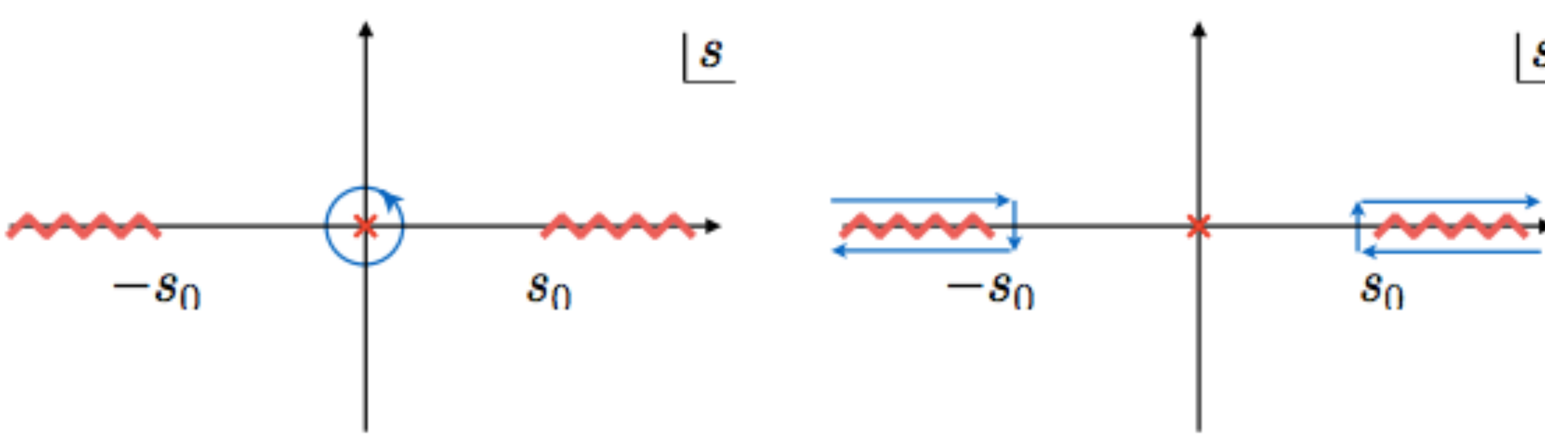
contour def
+ **analyticity**
+ Froissart bound

analyticity
+ Schwarz reflection
+ crossing symm

Opt Th
$\mathrm{Im}\mathcal{M}(s) = s\sigma(s)$



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$$\boxed{C > 0}$$

Positivity bounds $\longleftrightarrow$ WGC ?

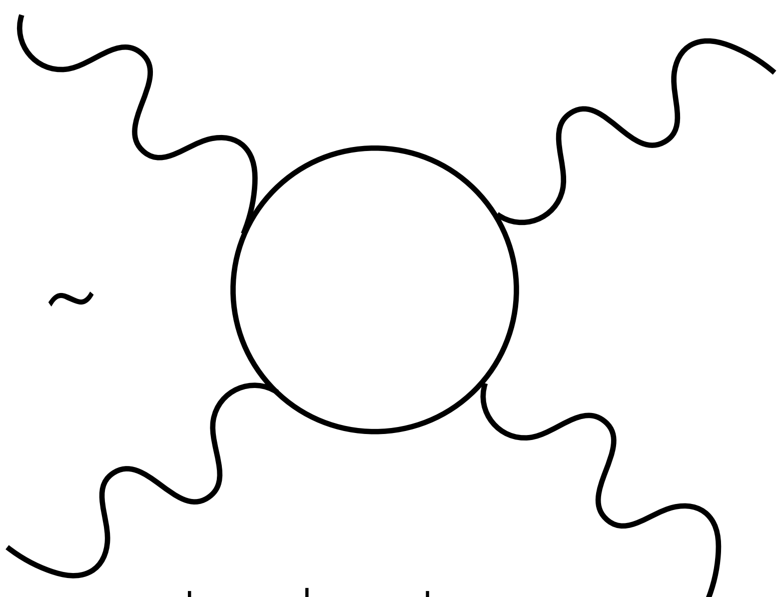
$C > 0$ $\longleftrightarrow$ **bound for z ?**

$$f(z) > 0$$

[Cheung-Remmen '14]

YES!

Take 1-loop EFT obtained by **integrating out charged matter**

HO ~  + … $\longrightarrow$ $C(z)$

at each vertex = qg

$C > 0$ $\Rightarrow$ $\boxed{\text{Conditions on } z}$

YES!

Wait a second!
Did not we say that the WGC, as a Swampland criterium, is a feature dictated by UV Physics ?!

Precisely.

Then how come you can extract something about it from IR Physics ??

There is a caveat!

. . .

Let me explain…

[taken from "Dialogo sopra
i due massimi sistemi del mondo"]

Setup (3d)

Λ

EFT multiple scalar/fermions charged under multiple U(1)'s

$$\Gamma = \int d^3x\sqrt{-g}\left[\frac{M_3}{2}R - \frac{1}{4}\sum_i F_i^2\right] + \Gamma_{s/f} + \underset{\boxed{\textbf{UV}}}{H.O.}$$

$$\Gamma_{\mathrm{s}} = \int d^3x\sqrt{-g}\sum_a\left(-|D_\mu\phi_a|^2 - m_a^2|\phi_a|^2\right)$$
$$\Gamma_{\mathrm{f}} = \int d^3x\sqrt{-g}\sum_a\bar{\psi}_a(-\Gamma^\mu D_\mu - m_a)\psi_a$$

$$H.O. = \sum_{ijkl} c_{ijkl}(F_i\cdot F_j)(F_k\cdot F_l)$$

UV-Physics dof
(kept **generic/unknown**)

Index notation:

i,j,k,l	run over U(1)'s species
a	runs over matter species

Setup (3d)

Λ

EFT multiple scalar/fermions charged under multiple U(1)'s

$$\Gamma = \int d^3x\sqrt{-g}\left[\frac{M_3}{2}R - \frac{1}{4}\sum_i F_i^2\right] + \Gamma_{s/f} + \underbrace{H.O.}_{\boxed{\text{UV}}}$$

$$\Gamma_{\rm s} = \int d^3x\sqrt{-g}\sum_a\left(-|D_\mu\phi_a|^2 - m_a^2|\phi_a|^2\right)$$

$$\Gamma_{\rm f} = \int d^3x\sqrt{-g}\sum_a\bar{\psi}_a(-\Gamma^\mu D_\mu - m_a)\psi_a$$

$$H.O. = \sum_{ijkl} c_{ijkl}(F_i\cdot F_j)(F_k\cdot F_l)$$

UV-Physics dof
(kept **generic/unknown**)

"Elephant in the room"…

Nonetheless, there is a **regime** where we can extract interesting results!

Setup (3d)

$\min(m_a)$

Integrating out matter, we obtain

$$\Gamma_1 = \int d^3x \sqrt{-g} \left[\frac{M_3}{2} R - \frac{1}{4} \sum_{i,j} \delta_{ij} F_i \cdot F_j + \sum_{i,j,k,l} C_{ijkl} (F_i \cdot F_j)(F_k \cdot F_l) \right]$$

Setup (3d)

$\min(m_a)$

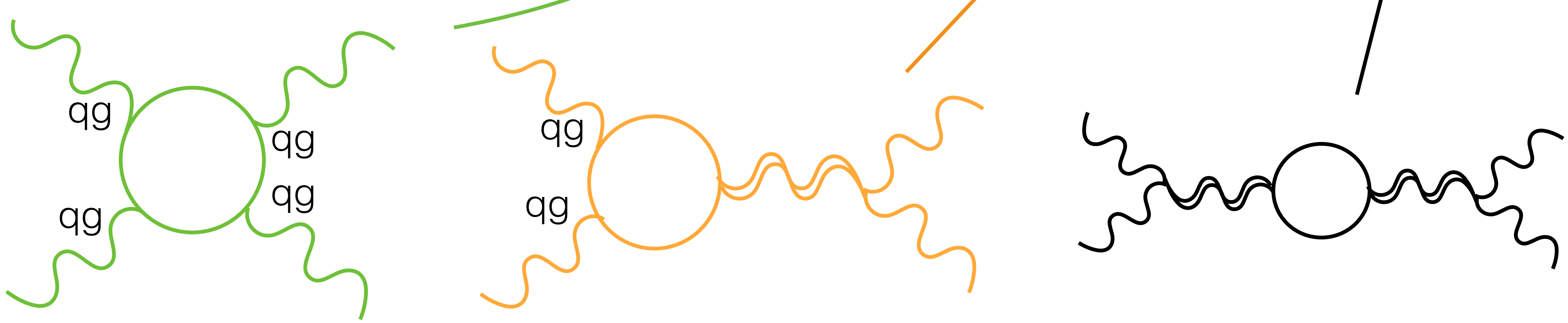
Integrating out matter, we obtain

$$\Gamma_1 = \int d^3x\sqrt{-g}\left[\frac{M_3}{2}R - \frac{1}{4}\sum_{i,j}\delta_{ij}F_i\cdot F_j + \sum_{i,j,k,l}C_{ijkl}(F_i\cdot F_j)(F_k\cdot F_l)\right]$$

$$C^s_{ijkl} = c_{ijkl} + \sum_a \frac{1}{1920\pi|m_a|M_3^2}\left[\frac{7}{8}z_{ai}z_{aj}z_{ak}z_{al} + \frac{3}{2}z_{ai}z_{aj}\delta_{kl} - z_{ai}z_{ak}\delta_{jl} + \frac{1}{2}\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl}\right]$$

UV

$$C^f_{ijkl} = c_{ijkl} + \sum_a \frac{1}{1920\pi|m_a|M_3^2}\left[z_{ai}z_{aj}z_{ak}z_{al} + z_{ai}z_{aj}\delta_{kl} - \frac{3}{2}z_{ai}z_{ak}\delta_{jl} - \frac{1}{2}\delta_{ij}\delta_{kl} + \frac{3}{2}\delta_{ik}\delta_{jl}\right]$$



Implications:

- **single particle**, charged under **single U(1)**

$$C > 0 \begin{cases} \text{scalar:} \quad z^2\left(z^2 + \frac{4}{7}\right) + \mathcal{O}_s(z^0) > 0 \\ \\ \text{fermion:} \quad z^2\left(z^2 - \frac{1}{2}\right) + \mathcal{O}_f(z^0) > 0 \end{cases}$$

Implications:

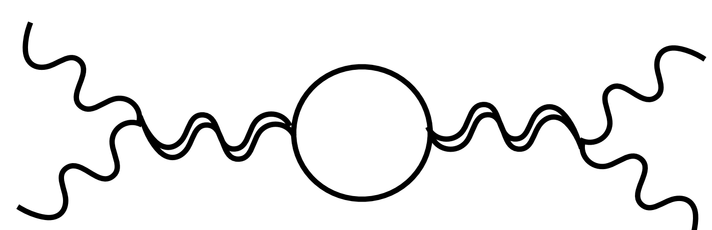
- **single particle**, charged under **single U(1)**

$$C > 0 \begin{cases} \text{scalar:} \quad z^2\left(z^2 + \frac{4}{7}\right) + \mathcal{O}_s(z^0) > 0 \quad \longrightarrow \quad \text{trivial} \\ \\ \text{fermion:} \quad z^2\left(z^2 - \frac{1}{2}\right) + \mathcal{O}_f(z^0) > 0 \quad \longrightarrow \quad z > \frac{1}{\sqrt{2}} \end{cases}$$

WGC!

In the regime where $\mathcal{O}(z^0)$ is negligible

$\mathcal{O}(z^0) \sim$ 

$+$  $\longrightarrow 0$

Implications:

- **more particles**, **multiple U(1)'s**: ➡ Several positivity conditions on

$$C_{ijkl}$$

In some details:

$$\sum_{ijkl} C_{(ij)(kl)} u_i v_j u_k v_l \geq 0$$

to be satisfied for **any** unit vector u, v

Implications:

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In some details: $$\sum_{ijkl} C_{(ij)(kl)} u_i v_j u_k v_l \geq 0$$

to be satisfied for **any** unit vector u, v

E.g. with $U(1)^2$ i=1,2

$$HO = C_{1111}(\partial\phi_1)^4 + C_{2222}(\partial\phi_2)^4 + C_{1212}(\partial\phi_1 \cdot \partial\phi_2)^2 + C_{1122}(\partial\phi_1)^2(\partial\phi_2)^2$$

if we take $u_i = v_i = \delta_i^{1(2)} \longrightarrow C_{1111} \geq 0 \quad (C_{2222} \geq 0)$

Implications:

- **more particles**, **multiple U(1)'s**: ➡ Several positivity conditions on C_{ijkl}

In some details: $$\sum_{ijkl} C_{(ij)(kl)} u_i v_j u_k v_l \geq 0$$

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if we take $u_i = \delta_i^1, \quad v_i = \delta_i^2$ $\longrightarrow$ $C_{1212} \geq 0$

Implications:

- **more particles**, **multiple U(1)'s**: $\Rightarrow$ Several positivity conditions on C_{ijkl}

In some details: $$\sum_{ijkl} C_{(ij)(kl)} u_i v_j u_k v_l \geq 0$$

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E.g. with $U(1)^2$ i=1,2

$$HO = C_{1111}(\partial\phi_1)^4 + C_{2222}(\partial\phi_2)^4 + C_{1212}(\partial\phi_1\cdot\partial\phi_2)^2 + C_{1122}(\partial\phi_1)^2(\partial\phi_2)^2$$

if we take $u_i = \dfrac{\delta_i^1 + \delta_i^2}{\sqrt{2}}$ $v_i = \dfrac{\delta_i^1 - \delta_i^2}{\sqrt{2}}$ $\longrightarrow$ $C_{1111} + C_{2222} - 2C_{1122} \geq 0$

etc…

Implications:

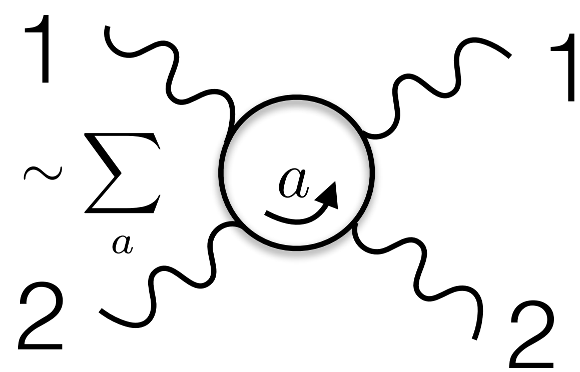
- **more particles**, **multiple U(1)'s**: $\Rightarrow$ Several positivity conditions on

$$C_{ijkl}$$

Strongest positivity conditions are given by **mixed** scatterings

e.g.: **U(1)xU(1)**

$$C_{1212} \sim \sum_a$$



a=matter field ϕ_a, ψ_a

$m_a, \vec{q}_a = (q_{1a}, q_{2a})$

$$C_{(12)(12)} > 0 \qquad \mathcal{O}(z^0) \to 0$$

$$\begin{cases} \text{scalar:} \quad \displaystyle\sum_a \frac{1}{3840\pi|m_a|}\left[z_{1a}^2 z_{2a}^2 - \frac{2}{7}z_{1a}^2 - \frac{2}{7}z_{2a}^2\right] > 0 \\ \text{fermion:} \quad \displaystyle\sum_a \frac{1}{480\pi|m_a|}\left[z_{1a}^2 z_{2a}^2 - \frac{3}{8}z_{1a}^2 - \frac{3}{8}z_{2a}^2\right] > 0 \end{cases}$$

Presence of **bifundamentals** is crucial for IR consistency

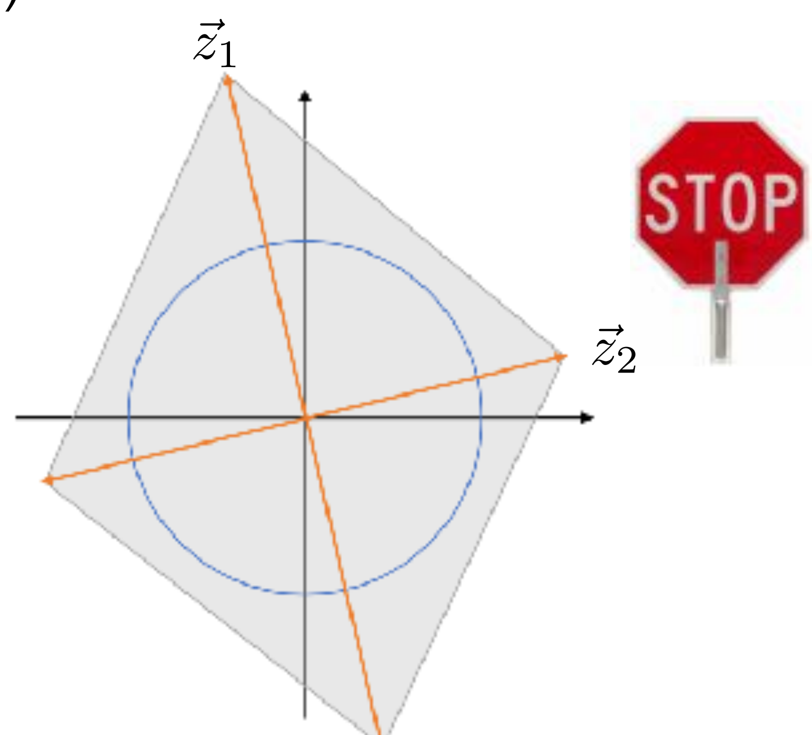
A stronger CHC (3d/4d)

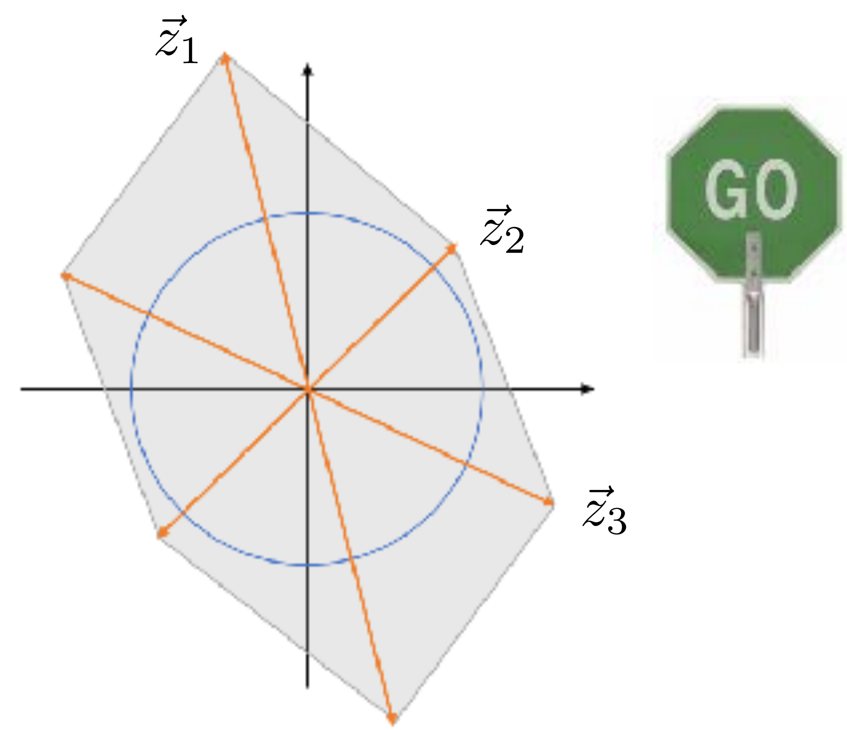
Positivity conditions ⟺

1) CHC is satisfied, but alone is not enough

2) There must exist (at least) a bifundamental particle for any (orthogonal) basis choice for the U(1)'s gauge fields

E.g., U(1)xU(1)





Dimensional reduction

Is the WGC **consistent** under dimensional reduction ?
[B. Heidenreich, M. Reece, T. Rudelius 1509.06374]

D dim: 1 particle, single U(1) $z_0 \geq 1$ WGC ✔

$S^1_{(r)} \downarrow$

D-1 dim: KK tower, U(1)xU(1)$_{KK}$ WGC (CHC) ? **NOT** always!

(problem $r \to 0$ limit)

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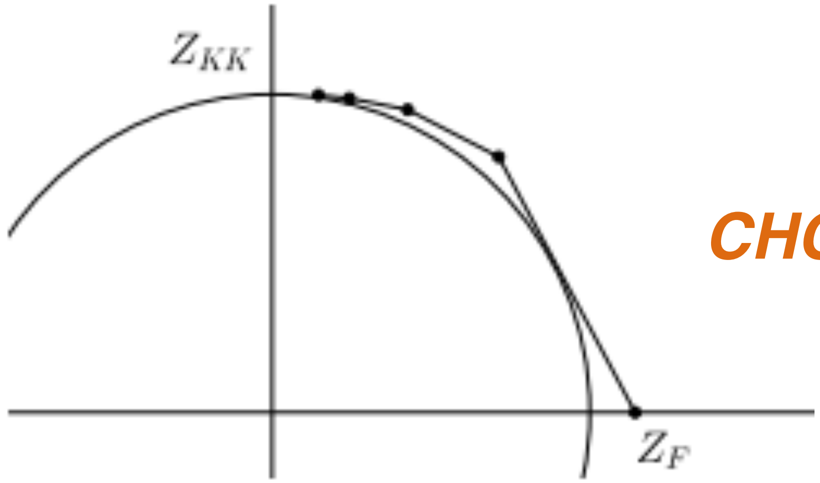

CHC →

$$\boxed{m_0^2 r^2 \geq \frac{1}{4z_0^2(z_0^2-1)}}$$

For any value of z_0 there is some r_{min} below which the CHC is NOT satisfied!

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D-1 dim: KK tower, U(1)xU(1)$_{KK}$ WGC (CHC) ? **NOT** always!

(problem $r \to 0$ limit)

Solutions ?

- Theory has a cut-off $\Lambda \leq \frac{1}{r_{min}}$ *OR*
- Theory has more particles, all satisfying $z \geq 1$

Dimensional reduction

Is the WGC **consistent** under dimensional reduction ?

[B. Heidenreich, M. Reece, T. Rudelius 1509.06374]

D dim: 1 particle, single U(1) $z_0 \geq 1$ WGC $\checkmark$

$S^1_{(r)} \downarrow$

D-1 dim: KK tower, U(1)xU(1)$_{KK}$ WGC (CHC) ? **NOT** always!

(problem $r \to 0$ limit)

Solution:

A super-extremal particle $z > 1$ should exist for every charge in the charge lattice

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D-1 dim: KK tower, $U(1)xU(1)_{KK}$ WGC (CHC) ? **NOT** always!
(problem $r \to 0$ limit)

Solution:

A super-extremal particle $z > 1$ should exist for every charge in the charge lattice

→ Can we see this by using **positivity bounds** ?

4D EFT Einstein-Maxwell + single scalar/fermion (m,q) $\longrightarrow$ 3D:

Theory of KK states charged under $U(1)^2 = U(1) \times U(1)_{KK}$

$$\vec{Z}_{(n)} = (z_F, z_{KK}) = \left(\frac{q}{\sqrt{m^2 + n^2/r^2}}, \frac{n}{\sqrt{m^2 r^2 + n^2}} \right)$$

In the *small radius* limit

- the lowest mode (n=0): $(z_F, z_{KK}) = (q/m, 0)$

- KK modes (n≠0): $(z_F, z_{KK}) \sim (0, 1)$

$\boxed{\text{Absence of bifundamentals}}$ ➡ IR inconsistency

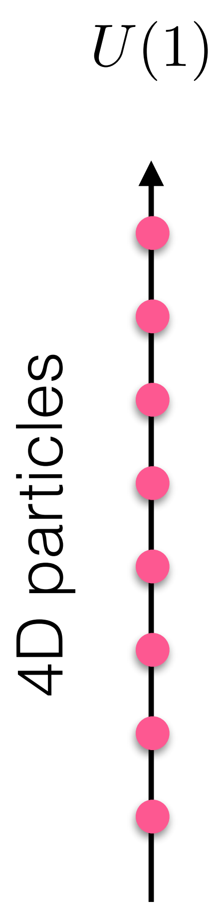
How to recover IR consistency?

- Theory has a cut-off preventing to achieve small radii…
- 4D theory has more particles

Tower-WGC*

(*in absence of fermions)

Replace the 4D field with a **tower** of 4D fields Φ_l charged under U(1) with masses and charges (m_l, q_l) s.t. **bifundamental** contributions (at any r) saves IR cons.



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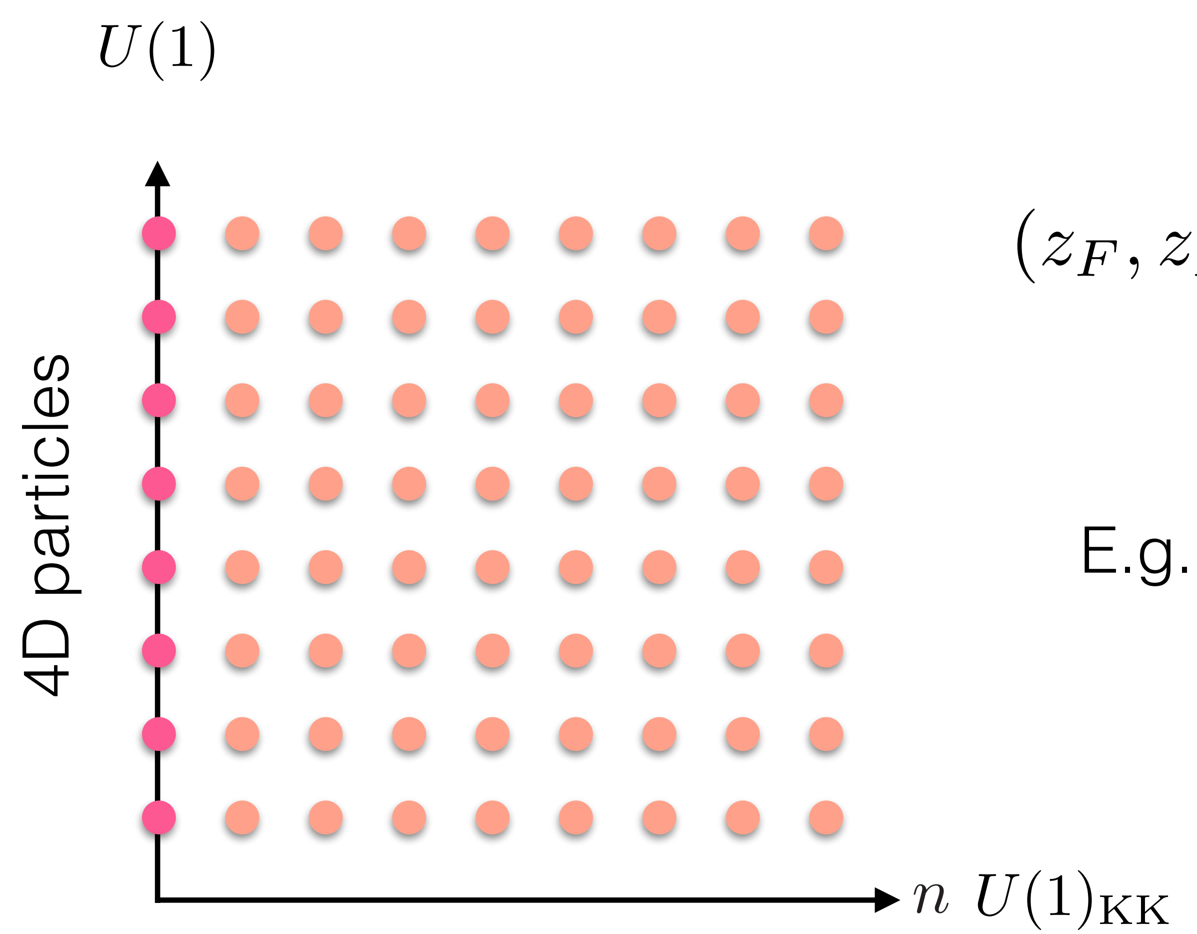
Replace the 4D field with a **tower** of 4D fields Φ_l charged under U(1) with masses and charges (m_l, q_l) s.t. **bifundamental** contributions (at any r) saves IR cons.



$$(z_F, z_{KK}) = \left(\frac{q_l}{\sqrt{m_l^2 + n^2/r^2}}, \frac{n}{\sqrt{m_l^2 r^2 + n^2}} \right)$$

E.g.: $m_l = \sqrt{m^2 + l^2\mu^2}$ $\qquad q_l = (l+1)q$

(depending on μ)

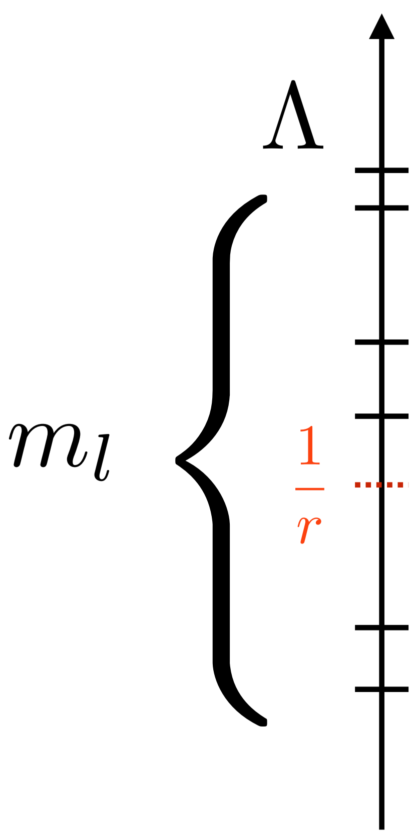
there may be bifundamentals at small radii: IR-OK

Many other possibilities…(check case-by-case)

Tower-WGC

Replace the 4D field with a **tower** of 4D fields Φ_l charged under U(1) with masses and charges (m_l, q_l) s.t. bifundamental contributions (at any r) saves IR cons.

Conditions:



- There must exist particles with mass near the cut-off $m_l \lesssim \Lambda$
- Such particles must have $z_l \gtrsim \mathcal{O}(1)$
- In case the lightest particle has mass $m \ll r^{-1}, \Lambda$
  then the number of particles in the tower is *at least* of order $(mr)^{-1}$

TWGC is *weaker* than the LWGC

no counterexample
+ reminiscent of swampland distance conjecture!!!

Summary & Outlook

Important “take-away’s”

Swampland/Landscape:

- Not all good-looking EFT’s can be UV-completed in a theory of QG (as String Theory)

- pheno/cosmo model building: better check the EFT you are using is not in the Swampland

Swampland criteria: SDC, WGC, etc…

Weak Gravity Conjecture:

- EFT U(1) coupled to gravity requires existence of super-extremal particle (or CHC for multiple U(1)’s)

IR consistency:

- Causality, analyticity and unitarity constrain effective interactions

Summary & Outlook

Results & future directions

Relation IR consistency-WGC?

- clear connection when **UV contribution** to the EFT in a certain regime. Possibly, compute UV contributions and check…

- existence of **bifundamentals** is crucial in case of multiple U(1)'s:

  # in general: yields a stronger CHC

  # under KK reduction: implies the necessity of a tower of particles in the parent D-dimensional theory **(TWGC)**

  # **TWGC** is weaker than LWGC and agrees with literature

- Extension of IR consistency arguments to check axionic version of WGC (*in progress w/ Huang, Noumi, Ooguri, Shiu)*

- Relation to SDC, *Emergence*…?

ありがとう
ございます

Multiple U(1)'s in 3d (similarly in 4d)

The 1-loop U(1)^N+gravity EFT is obtained by integrating out matter fields in

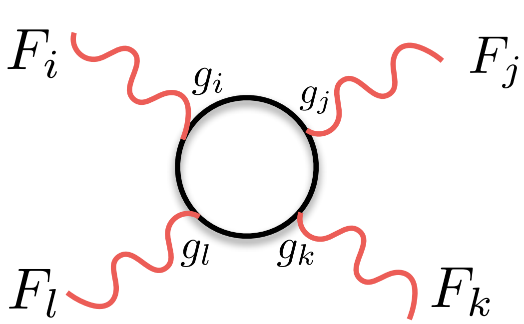
$$\Gamma = \int d^3x \sqrt{-g}\left[\frac{M_3}{2}R - \frac{1}{4}\sum_i F_i^2\right] + \Gamma_{s/f} + H.O.$$

$$H.O. \sim \mathcal{O}(R^2) + \mathcal{O}(RF^2) + \mathcal{O}(F^4)$$

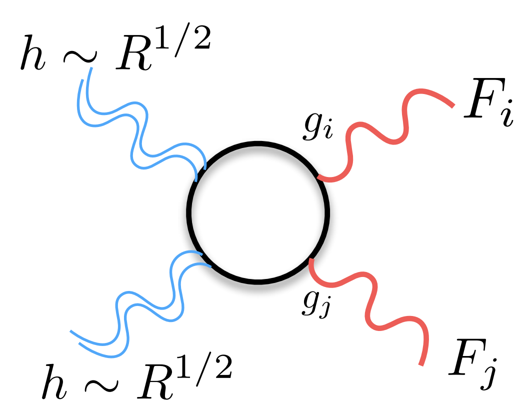
via the heat kernel method…

One obtains the following HO 4-derivatives ops (to be added to H.O.)

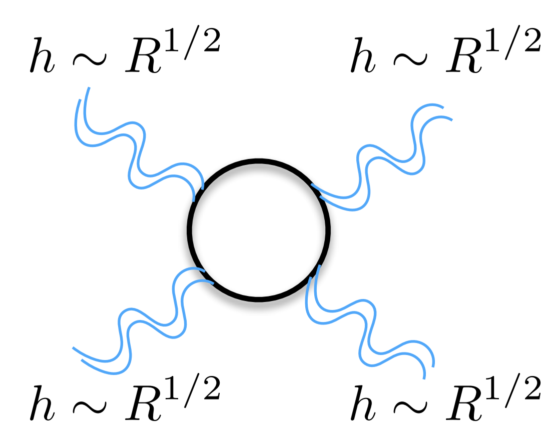
$\mathcal{F}^4$



$\mathcal{R}\mathcal{F}^2$



$\mathcal{R}^2$



Multiple U(1)'s in 3d (similarly in 4d)

Eventually, by field redefinitions, we can recast all the HO operators as

$$\mathcal{F}^4, \mathcal{R}\mathcal{F}^2, \mathcal{R}^2 \rightsquigarrow (\mathcal{F}^2)^2$$

$(F^2)^2$	RF^2	R^2
F^4	$R_{\mu\nu}F^{\mu\alpha}F^{\nu}{}_{\alpha}$	$R_{\mu\nu}R^{\mu\nu}$
	$R^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$	$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$

TL E.E. $R, R_{\mu\nu} \sim F^2, F_{\mu\alpha}F_{\nu}{}^{\alpha}$

vanishing Weyl $R_{\mu\nu\rho\sigma}(R, R_{\mu\nu})$

3d identities $F^4 \sim (F^2)^2$

So finally one has:

$$\Gamma_1 = \int d^3x\sqrt{-g}\left[\frac{M_3}{2}R - \frac{1}{4}\sum_{i,j}\delta_{ij}F_i \cdot F_j + \sum_{i,j,k,l}C_{ijkl}(F_i \cdot F_j)(F_k \cdot F_l)\right]$$

$$C^s_{ijkl} = c_{ijkl} + \sum_a \frac{1}{1920\pi|m_a|M_3^2}\left[\frac{7}{8}z_{ai}z_{aj}z_{ak}z_{al} + \frac{3}{2}z_{ai}z_{aj}\delta_{kl} - z_{ai}z_{ak}\delta_{jl} + \frac{1}{2}\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl}\right]$$

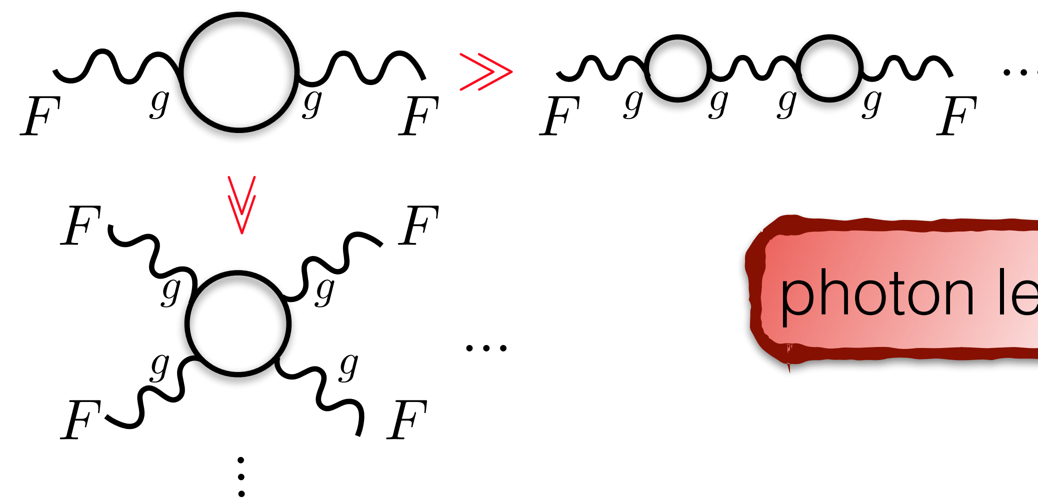
$$C^f_{ijkl} = c_{ijkl} + \sum_a \frac{1}{1920\pi|m_a|M_3^2}\left[z_{ai}z_{aj}z_{ak}z_{al} + z_{ai}z_{aj}\delta_{kl} - \frac{3}{2}z_{ai}z_{ak}\delta_{jl} - \frac{1}{2}\delta_{ij}\delta_{kl} + \frac{3}{2}\delta_{ik}\delta_{jl}\right]$$

<u>Multiple U(1)'s in 3d</u> (similarly in 4d)

Validity of 4-derivatives 1-loop expansion

1 higher loops suppressed $\quad \alpha \equiv \dfrac{qg}{\sqrt{m}} \ll 1$

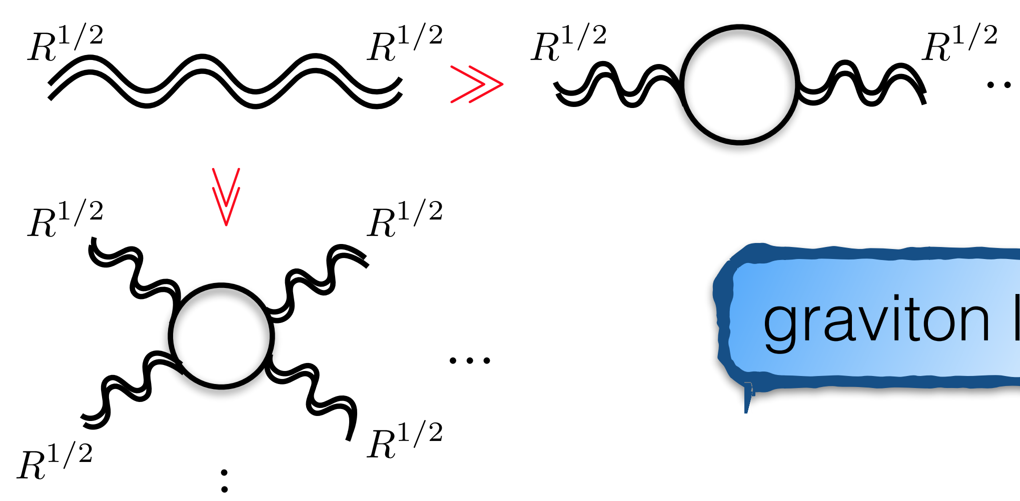
2 higher points suppressed $\quad \dfrac{qgF}{m^2} \ll 1$ weak field



photon legs

3 higher loops suppressed $\quad \beta \equiv \dfrac{m}{M_p} \ll 1$

4 higher points suppressed $\quad \dfrac{R}{m^2} \ll 1$ weak field



graviton legs

$$z = \frac{qg\sqrt{M_p}}{m} = \frac{\alpha}{\sqrt{\beta}}$$

$$\alpha \le z \le \frac{1}{\sqrt{\beta}} \quad \Rightarrow \quad z \gtreqless \mathcal{O}(1)$$

self-consistent