Infrared modified gravity with dynamical connection

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partially work in progress

Motivaton and Introduction

- Modification of gravity at large distances
  Challenge for theory, motivated in particular by accelerated expansion of the Universe.
  - May accelerated expansion be due to new gravity at cosmological scales rather than due to new form of energy?
- Difficult to construct consistent theory
  - Prototype theory: Fierz–Pauli theory of massive graviton

    Fierz–Pauli action:

$$S = M_{Pl}^2 \int d^4x \sqrt{g} \left\{ L_{EH}(g_{\mu\nu}) - \frac{m_1^2}{4} h_{\mu\nu} h^{\mu\nu} + \frac{m_2^2}{4} (h_\mu^\mu)^2 + \# \cdot h^3 + \dots \right\}$$

with

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$$

  - Danger: explicitly broken gauge invariance
  - How to extract dangerous degrees of freedom?

Stückelberg trick

Arkani-Hamed, Georgi, Schwartz' 03
Creminelli, Nicolis, Papucci, Trincherini' 05
Deffayet, Rombouts' 05; V.R., Tinyakov' 09

- L_{EH} invariant under gauge transformations

$$g_{\mu\nu} \to g_{\mu\nu} + \nabla_\mu \zeta_\nu + \nabla_\nu \zeta_\mu + \nabla_\mu \zeta_\lambda \cdot \nabla_\nu \zeta^\lambda$$

- Introduce new fields $\bar{g}_{\mu\nu}$, ξ_μ and ϕ:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \nabla_\mu(\xi_\nu + \nabla_\nu \phi) + \nabla_\nu(\xi_\mu + \nabla_\nu \phi) + \nabla_\mu(\xi_\lambda + \nabla_\lambda \phi) \cdot \nabla_\nu(\xi^\lambda + \nabla^\lambda \phi)$$

- Gauge invariances:

$$\bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu} + \nabla_\mu \zeta_\nu + \dots\ , \qquad \xi^\mu \to \xi^\mu - \zeta^\mu$$

and

$$\xi^\mu \to \xi^\mu + \nabla^\mu \psi\ , \qquad \phi \to \phi - \psi$$

$$10 + 4 + 1 = 15 \text{ fields}$$

$$4 + 1 = 5 \text{ gauge invariances}$$

$$15 - 2 \times 5 = 5 \text{ propagating degrees of freedom,}$$
right number for massive spin-2 field.

True if action for $\bar{g}_{\mu\nu}$, ξ^μ and ϕ is second order in derivatives

Most dangerous part:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi + \nabla_\mu\nabla_\lambda\phi \cdot \nabla_\nu\nabla^\lambda\phi$$

- Minkowski background:

$$L = -m_1^2(\partial_\mu\partial_\nu\phi)^2 + m_2^2(\partial_\mu^2\phi)^2$$

Fourth order in derivatives $\Longrightarrow$ 6th degree of freedom, ghost,
unless Fierz–Pauli condition imposed:

$$m_1 = m_2 = m$$

- Fierz–Pauli gravity with $m_1 = m_2 = m$, linearized about Minkowski background:

  van Dam–Veltman–Zakharov discontinuity: propagator of massive graviton

$$D_{\mu\nu\lambda\rho} = \frac{1}{p^2 - m^2}\left[\frac{1}{2}\eta_{\mu\lambda}\eta_{\nu\rho} + \frac{1}{2}\eta_{\mu\rho}\eta_{\nu\lambda} - \frac{1}{3}\eta_{\mu\nu}\eta_{\lambda\rho} + (p\text{-dependent terms})\right]$$

  - Limit $m \to 0$ different from propagator of massless graviton,

$$D^{massless}_{\mu\nu\lambda\rho} = \frac{1}{p^2}\left[\frac{1}{2}\eta_{\mu\lambda}\eta_{\nu\rho} + \frac{1}{2}\eta_{\mu\rho}\eta_{\nu\lambda} - \frac{1}{2}\eta_{\mu\nu}\eta_{\lambda\rho} + (p\text{-dependent terms})\right]$$

  Helicity-0 graviton couples to T^{μ}_{μ} with gravitational strength even in the limit $m \to 0$ $\Longrightarrow$ phenomenologically unacceptable, unless cured in some way

- Yet another property of Fierz–Pauli gravity with $m_1 = m_2 = m$, linearized about Minkowski background:
  - non-linearity at quite large distances from a source $\Longleftrightarrow$ Vainshtein phenomenon
  - actually useful: appears to cure vDVZ problem

Babichev, Deffayet and Ziour' 09

Disaster: Boulware–Deser ghost in curved backgrounds

Reason:

Removing 6th degree of freedom by fine tuning $m_1 = m_2$ need not be possible in curved backgrounds.

- Fierz–Pauli theory in curved background $g^{(c)}_{\mu\nu}$: in Stückelberg variables

$$g_{\mu\nu} = g^{(c)}_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi + \nabla_\mu \nabla_\lambda \phi \cdot \nabla_\nu \nabla^\lambda \phi + \dots$$

Hence

$$m^2 (g_{\mu\nu} - \eta_{\mu\nu})^2 \quad \Longrightarrow \quad m^2 (\nabla^\mu \nabla_\lambda \phi \cdot \nabla^\nu \nabla^\lambda \phi) \cdot (g^{(c)}_{\mu\nu} - \eta_{\mu\nu})$$

$$m^2 (\eta^{\mu\nu} g_{\mu\nu} - \eta^\mu_\mu)^2 \quad \Longrightarrow \quad m^2 (\nabla_\mu \nabla_\nu \phi)^2 \cdot (\eta^{\lambda\rho} g^{(c)}_{\lambda\rho} - \eta^\lambda_\lambda)$$

No cancellation of fourth-order terms. 6th, Boulware–Deser mode reappears. It is a ghost.

NB: No matter what the background is (Einstein or not; small curvature or not)

Lessons

- Beware of ghosts. Even if absent about flat background, they may reappear in curved backgrounds.

- Probably need some symmetry to ensure the absence of ghosts in both flat and curved backgrounds

- Technicality: Stückelberg formalism convenient

Approaches to infrared modified gravity:

- Scalar–tensor gravities, $f(R)$-gravities

  see review by N. Deruelle at YUKIS' 2010

- Extra dimensions with brane worlds

  Dvali, Gabadadze, Porrati' 01

  de Rham et. al.' 07; Kaloper, Kiley' 07; Kobayashi' 07; etc.

- Lorentz-violating effective Lagrangians

  V.R.' 04; Dubovsky' 04; Dubovsky, Tinyakov, Tkachev' 05

  Berezhiani, Comelli, Nesti, Pilo' 07; etc.

Some proposals work: apparently consistent theories with modified Newton's law at large distances.

NB: Obtaining accelerating Universe in a "natural" way is still difficult

Theories with dynamical connection: old work

Hayashi, Shirafuji' 80

Sezgin, van Nieuwenhuizen' 80

- Theories with both metric and connection as dynamical fields (Poincare gravities).
  - Many have both massive and massless spin-2 fields at linearized level in Minkowski background
  - Tuning of parameters $\Longrightarrow$ no pathologies in Minkowski background (no ghosts, no tachyons)

Suspicious: in curved backgrounds tunings may be ruined $\Longrightarrow$ pathologies may reappear

This is precisely what happens in Fierz–Pauli gravity: Boulware–Deser ghost mode

This work: look into this issue using brute force approach

Outcome: No, theory remains healthy at least in Einstein backgrounds of weak enough curvature

Model: field content

- metric $g_{\mu\nu} \Longleftrightarrow$ vierbein e^i_μ
- connection $A_{ij\mu}$ = gauge field of local $O(3,1)$, i.e., Lorentz group of frame rotations $\Longleftrightarrow A_{ijk} \equiv e^\mu_i A_{ij\mu} = -A_{jik}$.
- Covariant quantities:
  - Curvature = Field strength, just like in Yang–Mills:

$$F_{ij\mu\nu} = \partial_\mu A_{ij\nu} - \partial_\nu A_{ij\mu} + A_{ik\mu} A^k{}_{j\nu} - A_{ik\nu} A^k{}_{j\mu}$$

$$F_{ijmn} = e^\mu_m e^\nu_n F_{ij\mu\nu}, \quad \text{frame basis}$$

  - Torsion $\simeq$ (Connection) – (Riemannian connection)
- NB: Hereafter $i, j = 0, 1, 2, 3$ are space-time indices in the frame basis

$$A_{ijk} = \frac{1}{2}\left(T_{ijk} - T_{jik} - T_{kij} + C_{ijk} - C_{jik} - C_{kij}\right)$$

$C_{ijk} = e^{\mu}_{j} e^{\nu}_{k} (\partial_\mu e_{i\nu} - \partial_\nu e_{i\mu}) = -C_{ikj}$: Riemannian connection

$T_{ijk} = -T_{ikj}$: torsion *tensor*.

- Gauge symmetries:
  - Local Lorentz
  - General coordinate

Trick to construct theories with massive fields in gauge-invariant way:

$$L = (F + F^2) + T^2$$

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- One heuristic way to look:

  $F = \partial_\mu A \Longrightarrow F^2 =$ kinetic term for connection A_{ijk}

  $T = A - C = A - \partial_\mu e \qquad \Longrightarrow$

  T^2 = mass term for connection A_{ijk} plus kinetic term for vierbein e^i_μ (plus mixing between the two).

- Another heuristic way to look:

  $A = T + C = T + \partial_\mu e \qquad \Longrightarrow$

  $F^2 =$ higher order kinetic term for vierbein e^i_μ plus kinetic term for torsion T_{ijk} (plus mixing)

  T^2 = mass term for torsion

Signs not guaranteed $\Longrightarrow$
Danger of ghosts and tachyons

- Number of fields: 10 from e^i_μ + 24 from T_{ijk} = 34.

- Number of gauge symmetries: 4 of general coordinate transformations + 6 of local Lorentz = 10

- Generally: number of propagating degrees of freedom:

$$34 - 2 \times 10 = 14$$

  Some of them ghosts

- Parameters fine tuned: less propagating modes, no ghosts about Minkowski background

Hayashi, Shirafuji' 80

Sezgin, van Nieuwenhuizen' 80

In what follows: one of the cases

Model cont'd

- Decomposition of torsion into irreps of $O(3,1)$:

$$T_{ijk} = \frac{2}{3}(t_{ijk} - t_{ikj}) + \frac{1}{3}(\eta_{ij}v_k - \eta_{ik}v_j) + \varepsilon_{ijkl}a^l$$

with $t_{ijk} = t_{jik}$ and

$$t_{ijk} + t_{jki} + t_{kij} = 0, \qquad \eta^{ij}t_{ijk} = 0, \qquad \eta^{ik}t_{ijk} = 0$$

- Fine tuning the parameters: $L = L_{F+F^2} + L_{T^2}$

"Kinetic term":

$$L_{F+F^2} = M^2F + \lambda + c_3F_{ij}F^{ij} + c_4F_{ij}F^{ji} - c_5F^2 + c_6(\varepsilon_{ijkl}F^{ijkl})^2$$

(1): $c_3 + c_4 = 3c_5$, (2): no term $F_{ijkl}F^{ijkl}$

"Mass term":

(3): $L_{T^2} = -\mu^2\left(t_{ijk}t^{ijk} - v_iv^i + \frac{9}{4}a_ia^i\right)$

Parameters:

- M^2, μ^2: dimension $(\text{mass})^2$.
  Assume for the sake of presentation $M \sim \mu \sim M_{Pl}$.
- c_3, c_4, c_5, c_6: dimensionless.
  Assume for the sake of presentation $c_3 \sim c_4 \sim c_5 \sim c_6 \equiv c$
- Signs:

$$c_5 > 0\,, \qquad c_6 > 0\,, \qquad 0 < \mu^2 < \frac{2}{3}M^2$$

This ensures absence of ghosts and tachyons in Minkowski background.

- λ: cosmological constant, intoduced for generality: useful for discussing Einstein spaces as backgrounds, not just Ricci flat

Field equations

$$\frac{\delta S}{\delta e^i_\mu}: \qquad M^2 F_{ij} + \{c_i\}(F \cdot F)_{ij} + \mu^2 (\text{torsion})_{ij} - (1/2)\eta_{ij} L = 0$$

$$\frac{\delta S}{\delta A_{ijk}}: \qquad \{c_i\}(DF)_{ijk} + \mu^2 (\text{torsion})_{ijk} = 0$$

NB: generally covariant despite η_{ij}: we are in the frame basis. Consequence of fine tunings: e.o.m.'s involve F_{ijkl} in combinations $F_{ijkl}F^{jk}$ and $\varepsilon^{ijkl}F_{ijkl}$.

Because of this:

- $\lambda = 0 \Longleftrightarrow$ torsion $= 0$, $F_{ij} = R_{ij} = 0$ always a solution.
  - Massless tensor field — graviton from e^i_μ — in Minkowski background.
- $\lambda \neq 0 \Longleftrightarrow$ any Einstein space with zero torsion, $R_{ij} = \Lambda g_{ij}$ also a solution

$$\Lambda = \frac{\lambda}{M^2} \qquad \text{just like in GR.}$$

Spectrum about Minkowski background

Hayashi, Shirafuji' 80

Sezgin, van Nieuwenhuizen' 80

- Massless spin-2 from e^i_μ
- Vector v_i: not a propagating field
- Transverse part of pseudovector a_i: also does not propagate.
- Longitudinal part of a_i: massive pseudoscalar field,

$$m_1^2 \sim \frac{M^2}{c}$$

- Propagating part of t_{ijk} mixed with e^i_μ: massive spin-2 field

$$m^2 = \frac{M^2}{3\mu^2}\frac{2M^2/3-\mu^2}{c_5} \sim \frac{M^2}{c}$$

NB: Number of propagating modes reduced from 14 to 8.
NB: Small mass for large coefficients of F^2 terms

Sources in Minkowski background

- Notation for perturbation of vierbein:

$$e^i_\mu = \delta^i_\mu + h^i_\mu$$

- General source term

$$S_{source} = \int d^4x \left(2h^i_\mu \theta^\mu_i - \frac{1}{2} A_{ij\mu} S^{ij\mu} \right)$$

Gauge invariance $\Longrightarrow$

$$\partial^j \theta_{ij} = 0 \quad \Longrightarrow \quad \partial^j \theta_{(ij)} = -\partial^j \theta_{[ij]}$$

$$\partial_k S^{ijk} = 4\theta^{[ij]}$$

Fields induced by sources:

$$h_{ij} = \frac{1}{M^2}\frac{1}{k^2}\left(\tau_{ij} - \frac{1}{2}\eta_{ij}\tau\right) + \frac{2M^2/3-\mu^2}{\mu^2 M^2}\frac{1}{k^2+m^2}\left(\sigma_{ij} - \frac{1}{3}\eta_{ij}\sigma\right)$$

(up to longitudinal terms), with

$$\tau^{ij} = \theta^{(ij)} - \frac{1}{2}\partial_k S^{k(ij)}$$

$$\sigma^{ij} = \theta^{(ij)} + \frac{\mu^2}{2M^2/3-\mu^2}\frac{1}{2}\partial_k S^{k(ij)}$$

- Sum of massless and massive spin-2 contributions (true also for $S_{ijk} = 0$) $\Longrightarrow$ infrared modified gravity indeed
- Mixing independent of c (strength of F^2 terms); tends to a constant as $m \to 0$.
- vDVZ discontinuity in massive spin-2 sector
- Mixing with torsion $\Longleftrightarrow$ Source for torsion contributes to metric

Curved backgrounds

Perturbations about Einstein, torsion-free backgrounds studied so far. Makes sense: Fierz–Pauli theory shows Boulware–Deser phenomenon even in that case

To make the long story short:

- Massless spin-2 field remains massless (clear from above discussion)
- Longitudinal part of a_i remains healthy massive pseudoscalar.
- The field v_i and transverse part of a_i remain non-dynamical
- Massive spin-2 field becomes massive tensor field.

  Main worry: does this field describe 5 polarizations, or there appears 6th mode — Boulware–Deser?

Convenient variable for massive tensor field

$$u_{ij} = F_{(1)ij} - \frac{1}{6}\eta_{ij}F_{(1)}$$

$F_{(1)ij}$: perturbation of F_{ij}.

Linearized field equation ($\lambda = 0$ for brevity)

$$\nabla^2 u_{ij} - \nabla^k\nabla_i u_{kj} - \nabla^k\nabla_j u_{ki} + \nabla_i\nabla_j u + \eta_{ij}\left(\nabla^k\nabla^l u_{kl} - \nabla^2 u\right)$$
$$- m^2(u_{ij} - \eta_{ij}u) - \frac{2M^2}{3\mu^2}\cdot W_{ilkj}u^{lk} = 0$$

W_{ilkj}: Weyl tensor of background

∇_i: covariant derivative in background metric.

Remarkable properties

$$\nabla^2 u_{ij} - \nabla^k \nabla_i u_{kj} - \nabla^k \nabla_j u_{ki} + \nabla_i \nabla_j u + \eta_{ij} \left(\nabla^k \nabla^l u_{kl} - \nabla^2 u \right)$$
$$- m^2 (u_{ij} - \eta_{ij} u) - \frac{2M^2}{3\mu^2} \cdot W_{ilkj} u^{lk} = 0$$

- Complicated system of equations reduced to single equation
- Covariant under general transformations of background (recall that $i, j = 0, 1, 2, 3$ are frame indices)
- First line: equation for massless tensor field in curved space-time;
  invariance $u_{ij} \to u_{ij} + \nabla_i \zeta_j + \nabla_j \zeta_i$
- Mass term remains of Fierz–Pauli structure
- What about Boulware–Deser?

Stückelberg analysis

$$u_{ij} = \bar{u}_{ij} + \nabla_i \xi_j + \nabla_j \xi_i + \nabla_i \nabla_j \phi$$

- Mass term of Fierz–Pauli structure $\Longleftrightarrow$ higher order terms in the mass part of action for ϕ cancel out
- Weyl term

$$S_W \propto \int d^4x \sqrt{-g}\, W_{iklj} u^{kl} u^{ij} \Longrightarrow \int d^4x \sqrt{-g}\, W_{iklj} \nabla^k \nabla^l \phi \nabla^i \nabla^j \phi$$

Second order upon integration by parts (Bianchi identity for Einstein manifolds implies $\nabla^i W_{ijkl} = 0$; also, $W_{ijkl} = -W_{jikl}$, etc.):

$$S_W \propto \int d^4x \sqrt{-g}\, W_{iklj} W_{iklm} \nabla^j \phi \nabla^m \phi$$

No Boulware–Deser mode (checked by counting constraints)

Healthy theory at least in Einstein backgrounds of small curvature.

A lot to be understood

- Solutions.
  - Is vDVZ problem cured by Vainshtein mechanism? Or one has to fine tune parameters to make massive spin-2 mode weakly interacting with matter?
  - Consistency with tests of GR?
  - Do black holes have tensor hair?
- Further self-consistency checks
  - General backgrounds
  - Strong coupling scale in effective low energy theory
- ...

To conclude

- Poincare gravity with dynamical torsion and tuned couplings has both massless and massive spin-2 modes. Both interact with matter via $T_{\mu\nu}$.

- No pathologies in perturbations about Minkowski and Einstein backgrounds. Good chance that the model is healthy in interesting geometries.

- Many issues have to be clarified