O(D, D) completion of the Friedmann equations

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based on

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Outline

1. **Introduction and motivation**

2. **Double Field Theory coupled to matter**
   - Double Field Theory as Stringy Gravity
   - Einstein Double Field Equations

3. **O(D, D)-complete cosmology**
   - Homogeneous and isotropic backgrounds
   - **O**(D, D)-complete Friedmann Equations
   - Solutions
   - de Sitter solutions?

Introduction

General Relativity is a successful theory of gravity.

- Geometry $\Leftrightarrow$ Matter; expressed via Einstein's equations

$$G_{\mu\nu} = 8\pi G_{\mathrm{N}} T_{\mu\nu} \ .$$

- GR accurately describes astrophysical/cosmological phenomena: perihelion precession, gravitational lensing, Friedmann eqns...
- However, some results cannot be explained by GR + visible matter e.g. rotation curves, accelerating expansion, horizon problem, ...

Broadly, two types of solutions to such problems:

1. GR is correct, but there is dark matter, dark energy, inflation, ...
2. Theory of gravity should be modified...

Introduction

String theory predicts its own gravity, the '$\mathbf{O}(D, D)$-completion' of GR.

- In GR, the gravitational field is the spacetime metric $g_{\mu\nu}$
  $\Rightarrow \frac{1}{2}D(D+1)$ off-shell components (in D spacetime dimensions).
- In Stringy Gravity, $\exists$ more gravitational fields: $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$
  $\Rightarrow (D^2+1)$ degrees of freedom coupled independently to matter
  $\Rightarrow$ richer spectrum of gravitational solutions than GR.
- Traditionally in string theory: compactify to $D = 4$;
  fix antisymmetric tensor $B_{\mu\nu} = 0$; fix dilaton $\phi \sim$ constant.
- However, in the most general case, all gravitational components may be dynamical.

Cosmology in Stringy Gravity

Goal of my talk:

To introduce a new framework for cosmology, based on the $\mathbf{O}(D,D)$ *invariant formulation of Stringy Gravity (Double Field Theory).*

A brief introduction to Double Field Theory

In Double Field Theory (DFT) we describe D-dim. physics using $D+D$ coords $x^A = (\tilde{x}_\mu, x^\nu)$, $A = 1, \ldots, 2D$ (Siegel; '93) (Hull, Zwiebach; '09).

Symmetries of DFT:

- $\exists$ an $\mathbf{O}(D,D)$ T-duality gauge symmetry; doubled vector indices are raised and lowered using the $\mathbf{O}(D,D)$-invariant metric: $$\mathcal{J}_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$
- $\exists$ doubled diffeomorphisms acting on vectors ξ^A, etc.
- $\exists$ twofold local Lorentz symmetry: $\mathbf{Spin}(1, D-1) \times \mathbf{Spin}(D-1, 1)$, with local metrics $\eta_{pq} = \mathrm{diag}(-++\cdots+)$, $\bar{\eta}_{\bar{p}\bar{q}} = \mathrm{diag}(+--\cdots-)$.

DFT was originally motivated in $D = 10$ (Hull, Zwiebach; 2009). However, other choices are also possible: later I will focus on $D = 4$.

Aside: DFT can also be defined on 'non-Riemannian' backgrounds $\Rightarrow$ moduli-free compactification (Cho, Morand, Park; 2018).

Section condition

- The doubled coordinates satisfy an equivalence relation

$$x^A \sim x^A + \Delta^A(x) \ ,$$

  where Δ^A is a derivative-index-valued $\mathbf{O}(D, D)$ vector; e.g. $\Delta^A(x) = \mathcal{J}^{AB}\partial_B\Phi(x)$ for some function $\Phi(x)$, where $\partial_A = (\tilde{\partial}^\mu, \partial_\nu)$.
- All fields and functions in DFT should be invariant under this, *i.e.*

$$\Phi(x + \Delta) = \Phi(x) \quad \Longleftrightarrow \quad \Delta^A\partial_A = 0 \ .$$

- This is equivalent to the section condition: $\partial_A\partial^A = 2\,\partial_\mu\tilde{\partial}^\mu = 0$.
- Natural choice: $\tilde{\partial}^\nu = 0$. Thus D coordinates $\{\tilde{x}_\mu\}$ are gauged; gauge orbits $\simeq$ points in D-dim. spacetime $\{x^\nu\}$ (Park; 2013).

Field content of Double Field Theory

- The basic fields of Double Field Theory are: $\{d, \mathcal{H}_{AB}\}$, the DFT dilaton and the symmetric $\mathbf{O}(D, D)$ metric, respectively.
- On Riemannian backgrounds, $\tilde{\partial}^{\mu} = 0$: $\{d, \mathcal{H}_{AB}\} \rightarrow \{g_{\mu\nu}, B_{\mu\nu}, \phi\}$, the closed-string massless sector, e.g. $e^{-2d} \simeq e^{-2\phi}\sqrt{-g}$.
- Define projectors: $P_{AB} = \frac{1}{2}(\mathcal{J}_{AB} + \mathcal{H}_{AB}), \bar{P}_{AB} = \frac{1}{2}(\mathcal{J}_{AB} - \mathcal{H}_{AB})$.
- Square root: $\mathbf{Spin}(1, D{-}1) \times \mathbf{Spin}(D{-}1, 1)$ vielbeins: $\{V_{Ap}, \bar{V}_{A\bar{p}}\}$, where $P_A{}^B = V_{Ap}V^{Bp}$ and $\bar{P}_A{}^B = \bar{V}_{A\bar{p}}\bar{V}^{B\bar{p}}$ (p and $\bar{p}$ local indices).
- 'Semi-covariant' derivatives ∇_A and master derivatives $\mathcal{D}_A$,

$$\nabla_A = \partial_A + \Gamma_A \,, \qquad \mathcal{D}_A = \nabla_A + \Phi_A + \bar{\Phi}_A \,,$$

where Γ_{ABC} are DFT Christoffel symbols and $\{\Phi_{Apq}, \bar{\Phi}_{A\bar{p}\bar{q}}\}$ are DFT spin connections, defined by requiring full compatibility with the DFT fields and vielbeins, respectively.

Curvature

- Semi-covariant Riemann curvature (Jeon, Lee, Park; 2011),

$$S_{ABCD} := \frac{1}{2}\left(R_{ABCD} + R_{CDAB} - \Gamma^{E}{}_{AB}\Gamma_{ECD}\right) .$$

  satisfies symmetry properties and an algebraic Bianchi identity.
- DFT diffeomorphisms generated by a generalized Lie derivative. However, not quite true for diffeomorphisms of objects constructed with ∇_A and $\mathcal{D}_A$ (hence 'semi-covariant').
- The unwanted additional term can be cancelled by contracting with the projectors P and $\bar{P}$ (or V and $\bar{V}$). Thus we can construct the fully covariant DFT Ricci tensor $S_{p\bar{q}}$ and scalar $S_{(0)}$:

$$S_{p\bar{q}} := V^{A}{}_{p}\bar{V}^{B}{}_{\bar{q}}S^{C}{}_{ACB} , \quad S_{(0)} := \left(P^{AC}P^{BD} - \bar{P}^{AC}\bar{P}^{CD}\right) S_{ABCD} .$$

- DFT Ricci scalar gives covariant Lagrangian for Stringy Gravity:

$$S_{(0)} = R + 4\Box\phi - 4\partial_{\mu}\phi\partial^{\mu}\phi - \frac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} , \quad \text{where } H_{\lambda\mu\nu} = \nabla_{[\lambda}B_{\mu\nu]} .$$

Einstein Double Field Equations

Consider Stringy Gravity coupled to matter fields $\{\Upsilon_a\}$. The $\mathbf{O}(D,D)$-covariant action is given over a D-dimensional section Σ by

$$\int_\Sigma e^{-2d}\left[\tfrac{1}{16\pi G}S_{(0)} + L_{\rm matter}(\Upsilon_a)\right] .$$

Note: $\mathbf{O}(D,D)$-invariant coupling $\Rightarrow$ proper distance, geodesic motion, etc. have a natural covariant definition in string (Jordan) frame.

The resulting equations of motion are

$$S_{p\bar{q}} = 8\pi G K_{p\bar{q}} \,, \qquad S_{(0)} = 8\pi G T_{(0)} \,, \qquad \frac{\delta L_{\rm matter}}{\delta\Upsilon_a} \equiv 0 \,.$$

Here the stringy energy-momentum tensor has (D^2+1) components,

$$K_{p\bar{q}} := \frac{1}{2}\left(V_{Ap}\frac{\delta L_{\rm matter}}{\delta\bar{V}_A{}^{\bar{q}}} - \bar{V}_{A\bar{q}}\frac{\delta L_{\rm matter}}{\delta V_A{}^p}\right) \,, \quad T_{(0)} := e^{2d}\times\frac{\delta\left(e^{-2d}L_{\rm matter}\right)}{\delta d} \,.$$

Note: $T_{(0)}$ depends on the Lagrangian density, $\mathcal{L}_{\rm matter} := e^{-2d}L_{\rm matter}$.

Einstein Double Field Equations

Analogously to GR, we can define the stringy Einstein curvature tensor which is covariantly conserved,

$$G_{AB} = 4V_{[A}{}^{p}\bar{V}_{B]}{}^{\bar{q}}S_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{AB}S_{(0)}\ , \qquad \mathcal{D}_A G^{AB} = 0 \qquad \text{(off-shell)}\ .$$

This implies that the energy-momentum tensor can be written similarly,

$$T_{AB} := 4V_{[A}{}^{p}\bar{V}_{B]}{}^{\bar{q}}K_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{AB}T_{(0)}\ , \qquad \mathcal{D}_A T^{AB} \equiv 0 \qquad \text{(on-shell)}\ .$$

Hence the Einstein Double Field Equations can be summarized as

$$G_{AB} = 8\pi G T_{AB}\ .$$

Note: unlike in GR, the DFT Ricci tensor is traceless $\Rightarrow$ the $S_{(0)} \propto T_{(0)}$ part is an essential and independent component of the equations.

Riemannian backgrounds

- Riemannian backgrounds: EDFEs reduce to usual closed-string equations, plus source terms from $K_{\mu\nu} = 2e_\mu{}^p \bar{e}_\nu{}^q K_{p\bar{q}}$ and $T_{(0)}$:

$$\begin{aligned} R_{\mu\nu} + 2\nabla_\mu(\partial_\nu\phi) - \tfrac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} &= 8\pi G K_{(\mu\nu)} \ ; \\ \nabla^\rho\left(e^{-2\phi}H_{\rho\mu\nu}\right) &= 16\pi G e^{-2\phi}K_{[\mu\nu]} \ ; \\ R + 4\Box\phi - 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} &= 8\pi G T_{(0)} \ . \end{aligned}$$

- Asymmetric $K_{\mu\nu}$ possible (e.g. fermions, strings) $\rightarrow$ source for H.
- In addition, the conservation law (on-shell) reduces to

$$\begin{aligned} \nabla^\mu K_{(\mu\nu)} - 2\partial^\mu\phi\, K_{(\mu\nu)} + \tfrac{1}{2}H_\nu{}^{\lambda\mu}K_{[\lambda\mu]} - \tfrac{1}{2}\partial_\nu T_{(0)} &\equiv 0 \ , \\ \nabla^\mu\left(e^{-2\phi}K_{[\mu\nu]}\right) &\equiv 0 \ . \end{aligned}$$

- $D = 4$, spherically symmetric solution: gravity modified at small radius-per-mass, $R/(MG)$ (SA, Cho, Park; 2018).

Homogeneous and isotropic backgrounds

- Consider $D = 4$ solutions which are homogeneous and isotropic.
- These are isometries parametrized by six DFT-Killing vectors: three for rotations, ξ_a^M; three for translations, χ_a^N $(a = 1, 2, 3)$.
- Imposing vanishing of DFT Lie derivatives of the gravitational fields with respect to $\zeta_a^M = (\xi_a^M, \chi_a^M)$ yields DFT-Killing equations.
- On Riemannian backgrounds with the section choice $\tilde{\partial}^\mu = 0$, these reduce to ordinary Lie derivatives acting on $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$:

$$\mathcal{L}_{\zeta_a} g_{\mu\nu} = 0\,, \qquad \mathcal{L}_{\zeta_a} B_{(2)} + \mathrm{d}\tilde{\zeta}_a = 0\,, \qquad \mathcal{L}_{\zeta_a}\phi = 0\,.$$

  Note: the condition on $B_{(2)} = \frac{1}{2} B_{\mu\nu} \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$ implies $\mathcal{L}_{\zeta_a} H_{(3)} = 0$.
- For the matter sector, we impose the cosmological principle on the stringy energy-momentum tensor, which similarly reduces to

$$\mathcal{L}_{\zeta_a} K_{\mu\nu} = 0\,, \qquad \mathcal{L}_{\zeta_a} T_{(0)} = 0\,.$$

Metric ansatz

- Solving the DFT–Killing equations ⇒ cosmological ansatz

$$ds^2 = -N(t)^2 \mathrm{d}t^2 + a(t)^2 \left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2 \mathrm{d}\Omega^2 \right] ,$$

$$B_{(2)} = \frac{hr^2}{\sqrt{1-kr^2}} \cos\vartheta \, \mathrm{d}r \wedge \mathrm{d}\varphi , \quad \phi = \phi(t) .$$

- Note: can choose e.g. cosmic gauge where the function $N(t) = 1$; solutions characterized by $a(t)$, $\phi(t)$, and parameters h and k.
- Note: corresponding H-flux is homogeneous and isotropic:

$$H_{(3)} \equiv \mathrm{d}B_{(2)} = \frac{hr^2}{\sqrt{1-kr^2}} \sin(\vartheta) \mathrm{d}r \wedge \mathrm{d}\vartheta \wedge \mathrm{d}\varphi = h \, \mathrm{d}\mathcal{V}_{(3)} .$$

- Homogeneous and isotropic stringy energy-momentum tensor:

$$K^{\mu}{}_{\nu} = \mathrm{diag}(K^t{}_t(t), K^r{}_r(t), \ldots, K^r{}_r(t)) , \quad T_{(0)} = T_{(0)}(t) .$$

Energy density and pressure

- Define energy density and pressure as

$$\rho := \left(-K^{t}{}_{t} + \frac{1}{2} T_{(0)}\right) e^{-2\phi}, \qquad p := \left(K^{r}{}_{r} - \frac{1}{2} T_{(0)}\right) e^{-2\phi} .$$

- Matter action $= \int e^{-2\phi}\sqrt{-g}\, L_{\rm matter}$; $K^{t}{}_{t} = -\pi^{a}\partial_0\Upsilon_a$, $T_0 = -2L_{\rm matter}$.
  $\rho \equiv \mathcal{H} \Rightarrow$ Hamiltonian $\equiv \int \sqrt{-g}\,\rho = \int e^{-2\phi}\sqrt{-g}\,(\pi^{a}\partial_0\Upsilon_a - L_{\rm matter})$.
- Stringy e-m tensor conserved $\Rightarrow$ one non-trivial conservation law:

$$\dot{\rho} + 3H(\rho + p) + \dot{\phi}\, T_{(0)} e^{-2\phi} = 0 ,$$

where the Hubble parameter $H \equiv \frac{\dot{a}}{a}$ (in cosmic gauge); $\dot{\{\}} \equiv \frac{{\rm d}\{\}}{{\rm d}t}$.

O(D, D)-complete Friedmann Equations

In the homogeneous and isotropic case, EDFEs reduce to ($N = 1$)

$$\frac{8\pi G}{3}\rho e^{2\phi} + \frac{h^2}{12a^6} = H^2 - 2\dot{\phi}H + \frac{2}{3}\dot{\phi}^2 + \frac{k}{a^2}\,,$$

$$\frac{4\pi G}{3}(\rho + 3p)e^{2\phi} + \frac{h^2}{6a^6} = -H^2 - \dot{H} + \dot{\phi}H - \frac{2}{3}\dot{\phi}^2 + \ddot{\phi}\,,$$

$$\frac{4\pi G}{3}\left(2\rho e^{2\phi} - T_{(0)}\right) = -H^2 - \dot{H} + \frac{2}{3}\ddot{\phi}$$

→ "**O**(D, D)-complete Friedmann Equations" (OFEs).

- Note: 3 OFEs + 1 conservation law ⇒ 3 independent equations.
- If $\dot{\phi} = \ddot{\phi} = 0$, $h = 0$ ⇒ standard GR cosmology; $T_{(0)}e^{-2\phi} \equiv \rho - 3p$.
- When $h = k = 0$, covariance under **O**(3, 3) spatial T-duality:

Before	a	H	ϕ	ρ	p	$T_{(0)}$
After	a^{-1}	$-H$	$\phi - 3\ln a$	$a^6\rho$	$-a^6\left(p + T_{(0)}e^{-2\phi}\right)$	$T_{(0)}$

Energy conditions

In analogy with GR, we can define various relevant energy conditions (SA, Cho, Park; 2018). In the cosmological framework, these become:

- the *weak energy condition*:

$$\rho+\frac{h^2e^{-2\phi}}{32\pi Ga^6}\geq 0\,,\qquad \rho+p+\frac{h^2e^{-2\phi}}{16\pi Ga^6}>0\,;$$

- the *strong energy condition*:

$$\rho+3p+\frac{h^2e^{-2\phi}}{8\pi Ga^6}\geq 0\,,\qquad \rho+p+\frac{h^2e^{-2\phi}}{16\pi Ga^6}\geq 0\,;$$

- the *positive mass condition* and *pressure condition*, respectively:

$$2\rho-T_{(0)}e^{-2\phi}\geq 0\,;\qquad p+\frac{h^2e^{-2\phi}}{32\pi Ga^6}\geq 0\,.$$

These energy conditions all correspond to terms appearing in the **O**(D, D)-complete Friedmann equations, and their linear combinations.

Generalized perfect fluid

- Consider the conservation equation (in cosmic gauge):

$$\dot{\rho} + 3H(\rho + p) + \dot{\phi}\, T_{(0)}\, e^{-2\phi} = 0 \,.$$

- It is useful to define two equation-of-state parameters,

$$w := \frac{p}{\rho} \,; \qquad \lambda := \frac{T_{(0)}\, e^{-2\phi}}{\rho} \,.$$

- w is the usual parameter corresponding to pressure, while λ is the density rate, at which variation of the dilaton changes the density.
- For constant w and λ ("generalized perfect fluid"), we can solve:

$$\rho = \rho_0 \frac{e^{-\lambda\phi}}{a^{3(1+w)}} \,.$$

Gemeralized perfect fluid

Consider a power-law ansatz, with $h = k = 0$:

$$a = \left(\frac{t}{t_0}\right)^n, \quad e^{\phi} = \left(\frac{t}{t_0}\right)^{-s} \qquad \Rightarrow \qquad H = \frac{n}{t}, \quad \dot{\phi} = -\frac{s}{t}.$$

Solving the OFEs for n and s with generic (constant) w and λ gives

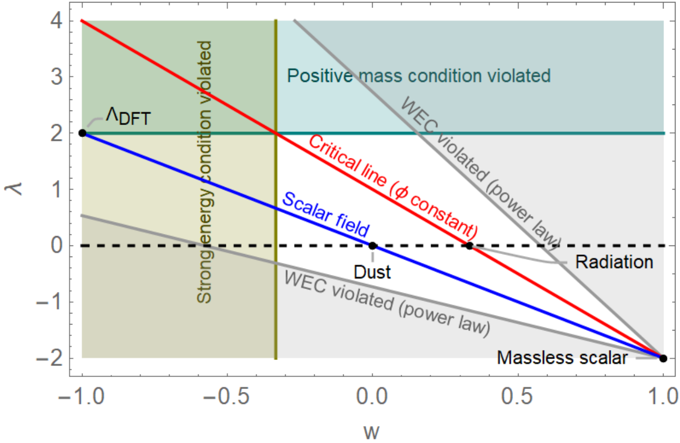
$$n = \frac{2(2w+\lambda)}{2+6w^2+6w\lambda+\lambda^2}, \qquad s = \frac{2(1-3w-\lambda)}{2+6w^2+6w\lambda+\lambda^2},$$

Special cases:

- Constant dilaton, $s = 0$ on the 'critical line', $\lambda = 1 - 3w$
  $\Rightarrow$ recover standard GR cosmology on this line, $T_{(0)} e^{-2\phi} \equiv \rho - 3p$.
- Static universe, $n = 0$ on the line $\lambda = -2w$. Scalar fields also lie on this line (but can have varying w and λ).

Cosmological solutions

Identify various regions and types of matter in the (w, λ)-plane.



Also, pure DFT vacuum: $\rho = 0$ (Copeland, Lahiri, Wands; 1994)

Note: Usual supergravity case is $\lambda = 0 \Rightarrow$ radiation critical, dust is not.

Analytic solutions: critical line

We also wish to find analytic solutions to the OFEs, beyond power law.

- Useful parametrization: 'Einstein-conformal' gauge, $N = a =: be^{\phi}$
  (Recall that the 'lapse function' $N(t)$ rescales the time coordinate.)
- (OFE2 − OFE3) in Einstein-conformal gauge:

$$4\pi G b^2 e^{4\phi} \rho \, (3w + \lambda - 1) = \phi'' + \frac{2b'\phi'}{b} - \frac{h^2}{2b^4} e^{-4\phi} ,$$

  where $'$ denotes differentiation w.r.t. 'Einstein-conformal' time η.
- Critical line, $\lambda = 1 - 3w$: LHS $= 0 \Rightarrow$ can integrate explicitly, giving

$$(b^2\phi')^2 + \frac{h^2}{4} e^{-4\phi} \sim \text{constant} \geq 0 \, .$$

- This resembles the total energy carried by the dilaton and H-flux: on the critical line it is conserved.
- Special case: constant $= 0 \Rightarrow \phi' = h = 0 \Rightarrow$ GR cosmology.

Example: DFT scalar & cosmological constant

DFT action for a canonical scalar Φ ($D = 4$, Riemannian background):

$$S_\Phi = \int d^4x\sqrt{-g}e^{-2\phi}\Big(-\frac{1}{2}g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi - V(\Phi)\Big)\,.$$

Homogeneous and isotropic ansatz $\Rightarrow K_r{}^r = 0$, so

$$\rho e^{2\phi} = \tfrac{1}{2}\dot\Phi^2 + V(\Phi)\,, \quad pe^{2\phi} = -\tfrac{1}{2}T_{(0)} = \tfrac{1}{2}\dot\Phi^2 - V(\Phi) \quad \Rightarrow \quad \lambda = -2w\,.$$

There are two special cases which are analytically solvable:

1. DFT cosmological constant: $\dot\Phi = 0$, $V(\Phi) = \Lambda_{\rm DFT}$; $(w,\lambda) = (-1,2)$ (Note: effective dilaton potential, $V_{\rm eff}(\phi) = e^{-2\phi}\Lambda_{\rm DFT}$). Solution:

$$e^{2\phi(t)} = C_\phi\frac{\tanh^{\pm\sqrt{3}}\left(m(t-t_0)\right)}{\sinh\left(2m(t-t_0)\right)}\,, \quad a^2(t) = a_0^2\tanh^{\pm\sqrt{\frac{4}{3}}}\left(m(t-t_0)\right)\,,$$

for $\pm(t-t_0) > 0$, where $m \equiv \sqrt{\Lambda_{\rm DFT}/2}$ (+ sign: (Mueller; 1990)).

Example: DFT scalar & cosmological constant

2. Massless scalar (vanishing potential): $V(\Phi) = 0$; $(w, \lambda) = (1, -2)$, the special point where the scalar line and critical line intersect. Solve in Einstein-conformal gauge ⇒ conformal scale factor

$$b^2 = \frac{C_1 \tau}{1 + k\tau^2}\,, \quad \text{where} \quad \tau = \begin{cases} \tan(\eta - \eta_0) & \text{for} \quad k = 1\,, \\ \eta - \eta_0 & \text{for} \quad k = 0\,, \\ \tanh(\eta - \eta_0) & \text{for} \quad k = -1\,. \end{cases}$$

with C_1 an integration constant. Dilaton and (cosmic) scale factor:

$$e^{2\phi} = \left(\frac{\tau}{\tau_*}\right)^{\pm \frac{h_o}{C_1}} + \frac{1}{4}\frac{h^2}{h_o^2}\left(\frac{\tau}{\tau_*}\right)^{\mp \frac{h_o}{C_1}} \; ; \qquad a^2 = b^2 e^{2\phi}\,.$$

Scalar:

$$\Phi = \Phi_0 \pm \sqrt{\frac{1}{16\pi G}\left(3 - \frac{h_o^2}{C_1^2}\right)}\,\ln\tau\,,$$

which is real for $|h_o/C_1| \leq \sqrt{3}$ (where equality ⇒ DFT vacuum).

Example: radiation with H-flux and freezing dilaton

Another example: $\exists$ an analytic solution for radiation ($w = 1/3$, $\lambda = 0$), in the presence of non-vanishing spatial curvature and H-flux, in which the dilaton is frozen at late times.

- Conformal scale factor (Note: string frame scale factor, $a = e^{\phi} b$):

$$b^2 = \frac{\tau(C_1 + \Omega_{\rm rad} H_0^2 \tau)}{1 + k\tau^2}\,; \quad \tau = \begin{cases} \tan(\eta - \eta_0) & \text{for} \quad k = 1\,, \\ \eta - \eta_0 & \text{for} \quad k = 0\,, \\ \tanh(\eta - \eta_0) & \text{for} \quad k = -1\,. \end{cases}$$

- The dilaton profile is (c.f. $h = 0$: Copeland, Lahiri, Wands; 1994)

$$e^{2\phi} = \left(\frac{C_1 \tau}{\tau_* \left(C_1 + \Omega_{\rm rad} H_0^2 \tau\right)} \right)^{\pm\sqrt{3}} + \frac{1}{12}\frac{h^2}{C_1^2} \left(\frac{C_1 \tau}{\tau_* \left(C_1 + \Omega_{\rm rad} H_0^2 \tau\right)} \right)^{\mp\sqrt{3}}\,,$$

which converges to a constant as $\eta \to \infty$ (for $k \in \{0, -1\}$).

de Sitter solutions?

It is worthwhile to test whether de Sitter solutions are natural in $\mathbf{O}(D, D)$ cosmology. Hence we considered the possibility of solutions with exponential scale factors, in both string and Einstein frames.

For simplicity, we focused on cases where $\lambda = -2w$, which includes both a DFT cosmological constant and scalar fields with arbitrary potential (e.g. $L_{\text{scalar}} = T - V = p_{\text{scalar}}$; $T_{(0),\text{scalar}} = -2L_{\text{scalar}}$). Also to make the connection with de Sitter we set $k = 0$.

String frame: $a = e^{Ht}$, $N = 1$, $\lambda = -2w$

- From the OFEs, the energy density is given by

$$8\pi G e^{2\phi}\rho + \frac{h^2}{4}e^{-6Ht} = -\frac{3}{2}H^2 + \frac{h^4}{8H^2}e^{-12Ht}$$

$\Rightarrow$ negative at late times $\Rightarrow$ weak energy condition violated.

de Sitter solutions?

Einstein frame: $b = e^{H_{\mathrm{E}} t}$ (recall $a = e^{\phi} b$), $N = e^{\phi}$, $\lambda = -2w$

- Define Hubble parameter in Einstein frame: $H_{\mathrm{E}} = \frac{\dot{b}}{b}$ ($=$ constant).
- In this case we find, in particular (e.g. $k = 0$ case),

$$4\pi G \left(\rho_{\mathrm{E}} + p_{\mathrm{E}} \right) := 4\pi G e^{4\phi} \left(\rho + p \right) = -\dot{\phi}^2 - \frac{h^2}{4} e^{-6H_{\mathrm{E}} t - 4\phi} \, .$$

- RHS $\leq 0 \Rightarrow$ strong and weak energy conditions violated.

In both cases, weak energy condition violated; scalars tachyonic.

More generally: $(w, \lambda) = (-1, 4) \Rightarrow$ GR cosmological constant!? Hence de Sitter solves the OFEs... but this case does not correspond to any known $\mathbf{O}(D, D)$-covariant Lagrangian. Would need $\rho = K_t{}^t e^{-2\phi}$, but this is minus the kinetic energy, so expect $\rho < 0 \Rightarrow$ WEC violated.

Summary

- Stringy Gravity (Double Field Theory) coupled to matter satisfies the Einstein Double Field Equations, $G_{AB} = 8\pi G T_{AB}$.
- From the Einstein Double Field Equations on homogeneous and isotropic Riemannian backgrounds, we derived the **O**(D, D)-complete Friedmann equations.
- We found various solutions, including a radiation solution with non-vanishing H-flux and frozen dilaton at late times.
- de Sitter solution for a DFT Λ/scalar violates the weak energy condition in both string and Einstein frames. Another GR-like dS solution exists but no DFT origin: is de Sitter an artefact of GR?
- New **O**(D, D)-covariant framework for cosmology.

Future directions

We have only scratched the surface of $\mathbf{O}(D, D)$-complete cosmology. Many issues yet to be addressed, such as:

- inflation, or in general, generating (almost) scale-invariant curvature perturbations which match observations of the CMB;
- frame dependence: is string frame or Einstein frame correct? (In DFT, point particles follow geodesics in string frame...) Our universe is 13.8 billion years old... but in which frame?
- Maxwell fields couple to the dilaton... consequences?
- consistency with local measurements of G_{N}... slowly-varying dilaton at late times? (quintessence?)
- and many more...

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ありがとうございました。