Resummation of large radiative effects in indirect dark matter detection

Martin Vollmann Technical University of Munich



Based on Beneke, Broggio, Hasner, Urban, MV arXiv:<u>1903.08702</u> Beneke, Broggio, Hasner, MV arXiv:<u>1805.07367</u>

Outline

- Motivation
- Gamma rays from DM annihilation
- Sommerfeld and Sudakov resummation
- Factorization formulas for the wino model
- Conclusions

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Motivation

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- WIMP DM paradigm is very well motivated and scrutinized
- No discovery so far ⇒ O(1-100GeV) wimp models are subject to stringent constraints

WIMPs not detected so far

XENON collaboration [1805.12562]

Fermi-LAT collaboration [1503.02641]



Maybe the DM is **heavier** than what we would have liked!

- WIMP DM paradigm is very well motivated and scrutinized
- No discovery so far ⇒ O(1-100GeV) wimp models are subject to stringent constraints
 - Turn attention into above-TeV wimps

The wino model (minimal DM)

$$\delta \mathcal{L}_{\text{Wino}} = \frac{1}{2} \bar{\chi} (i \gamma^{\mu} D_{\mu} - m_{\chi}) \chi$$



- Heavy ($\mathcal{O}(1-100\text{TeV})$) DM \rightarrow Indirect detection
- Spectral-line feature in gamma ray spectrum is a smoking-gun signature of WIMP DM annihilation



- Current- and next-generation gamma-ray telescopes will search for such spectral lines
- Particularly promising is the Cherenkov Telescope Array (CTA) with ~1 order of magnitude improved sensitivity w.r.t. current technology





- Annihilation cross section computations for heavy wimps can be intricate (because the hierarchy M_{DM} >> M_{SM})
- Non-perturbative effects such as the Sommerfeld effect play a major role in their determination
- On top of this, large electroweak Sudakov double logarithms invalidate the perturbative expansion and need to be resummed
- In this work we focus on the latter (but also systematically treat the former)

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Gamma rays from dark matter annihilation



$$\Phi(E_{\gamma}) = \frac{1}{8\pi m_{\rm DM}^2} \int_{\Delta\Omega} \mathrm{d}\Omega \int_{\mathrm{l.o.s.}} \mathrm{d}s \rho_{\rm DM}^2(\boldsymbol{r}(s)) \frac{\mathrm{d}}{\mathrm{d}E_{\gamma}} [\sigma v]_{\gamma+X}$$

y rays from dark matter annihilation. Multi-scale problem





 r_s, R_{\odot} $\lambda_{\rm DM} \sim m_{\rm DM}^{-1}$

γ rays from dark matter annihilation. 1st factorization



$$\Phi(E_{\gamma}) = \frac{1}{8\pi m_{\rm DM}^2} \underbrace{\int_{\Delta\Omega} d\Omega \int_{\rm l.o.s.} ds \rho_{\rm DM}^2(\boldsymbol{r}(\boldsymbol{s}))}_{J(\Delta\Omega)} \frac{d}{dE_{\gamma}} [\sigma v]_{\gamma+X}$$
Astrophysical "J" factor
independent of gamma-ray energy

γ rays from dark matter annihilation. 1st factorization



$$\Phi(E_{\gamma}) = \frac{1}{8\pi m_{\rm DM}^2} \underbrace{\int_{\Delta\Omega} \mathrm{d}\Omega \int_{\mathrm{l.o.s.}} \mathrm{d}s \rho_{\rm DM}^2(\boldsymbol{r}(\boldsymbol{s}))}_{J(\Delta\Omega)} \frac{\mathrm{d}}{\mathrm{d}E_{\gamma}} [\sigma v]_{\gamma+X}$$

γ rays from dark matter annihilation. PP term



y rays from dark matter annihilation. Endpoint spectrum



y rays from dark matter annihilation. Endpoint spectrum

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}E_{\gamma}}[\sigma v]_{\gamma+X} &= 2[\sigma v]_{\gamma\gamma}\delta(E_{\gamma} - m_{\mathrm{DM}}) + [\sigma v]_{\gamma Z}\delta(E_{\gamma} - E_{0}^{\gamma Z}) + \\ &+ \frac{\mathrm{d}}{\mathrm{d}E_{\gamma}}[\sigma v]_{\gamma+N \geq 2-\mathrm{bodies}} \end{aligned}$$



y rays from dark matter annihilation. Endpoint spectrum



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Naive computation of $\sigma V_{\gamma\gamma}$



Naive computation of $\sigma V_{\gamma\gamma}$









Sommerfeld effect (Scattering states in 1D QM)

Unitarity: $j_{-}=j_{+}$ \rightarrow $\sigma_{r}+\sigma_{t}=1$ (No absorption)

Sommerfeld effect (Scattering states in 1D QM) $\left(-\frac{1}{m_{\chi}}\frac{d^{2}}{dx^{2}}+V(x)+\frac{i}{2}\sigma_{a}^{(0)}v\delta(x)\right)\psi(x)=E\psi(x)$

Unitarity-violating term $\rightarrow j_+ = j_- + |\psi(0)|^2 \sigma_a v$

$$\sigma_r + \sigma_t + \boldsymbol{\sigma_a} = 1$$

$$\sigma_a = |\psi(\mathbf{0})|^2 \sigma_a^{(0)}$$

Resummed = Sommerfeld factor X QFT cross section (long range physics) X (short range physics)





Sudakov double logarithms



Sudakov-log resummation



Soft-collinear effective theory (SCET). Method of regions



SCET. Momentum regions

$$I_{\text{full}} = \int \frac{\mathrm{d}^{D}q}{(2\pi)^{D}} \frac{1}{(q+k-p_{0})^{2} - m_{\chi}^{2}} \frac{1}{(q+k)^{2}} \frac{1}{q^{2}} \frac{1}{(q-k')^{2}} \Big|_{k^{2},k'^{2} \sim m_{W}^{2} \ll m_{\chi}^{2}}$$

Light-cone
$$q = q_c n + q_{\bar{c}} \bar{n} + q_{\perp} \rightarrow (q_c, q_{\bar{c}}, q_{\perp})$$

Momentum modes

 $q_h \sim m_{\chi}(1, 1, 1)$ $q_{hc} \sim (m_W, m_{\chi}, \sqrt{m_{\chi} m_W})$ $q_c \sim \left(\frac{m_W^2}{m_{\chi}}, m_{\chi}, m_W\right)$

$$q_s \sim m_W(1, 1, 1)$$
$$q_{\bar{h}c} \sim (m_\chi, m_W, \sqrt{m_\chi m_W})$$
$$q_{\bar{c}} \sim \left(m_\chi, \frac{m_W^2}{m_\chi}, m_W\right)$$

SCET. Momentum regions



+ power corrections

SCET. Factorization (narrow resolution)

Interpret each expansion as a Feynman diagram of the SCET



Factorization (after including all diagrams)

SCET-II (narrow resolution)

Interpret each expansion as a Feynman diagram of the SCET


NRDM×SCET for DM annihilation

Integrate out hard modes of the relevant fields but leave hard (anti)collinear and non-relativistic degrees of freedom:

$$\mathcal{L}_{\text{NRDM}\times\text{SCET}} = \mathcal{L}_{\text{NRDM}} + \mathcal{L}_{\text{SCET}} + \frac{1}{2m_{\chi}} \sum_{i=1}^{2} \int \mathrm{d}s \mathrm{d}t \ \hat{C}_{i}(t, s, \mu) \mathcal{O}_{i}$$

Two-dimensional operator basis (for the $\chi\chi \rightarrow \gamma + X$ process) $\mathcal{O}_{1} = \chi_{\mathrm{NR}}^{c\dagger} \chi_{\mathrm{NR}} \varepsilon_{\perp}^{\mu\nu} \mathcal{A}_{\perp \ c, \ \mu}^{C} (sn_{+}) \mathcal{A}_{\perp \ \overline{c}, \ \nu}^{C} (tn_{-})$ $\mathcal{O}_{1} = \chi_{\mathrm{NR}}^{c\dagger} (T_{\mathrm{NR}}^{C} T_{\mathrm{NR}}^{D}) \chi_{\mathrm{NR}} \varepsilon_{\perp}^{\mu\nu} \mathcal{A}_{\perp \ c, \ \mu}^{C} (sn_{+}) \mathcal{A}_{\perp \ \overline{c}, \ \nu}^{D} (tn_{-})$

$$\mathcal{O}_2 = \chi_{\mathrm{NR}}^{c\dagger} \{ T^C, T^D \} \chi_{\mathrm{NR}} \varepsilon_{\perp}^{\mu\nu} \mathcal{A}_{\perp c, \mu}^C(sn_+) \mathcal{A}_{\perp \bar{c}, \nu}^D(tn_-)$$



M. Vollmann — Precise predictions for γ -ray production in dark-matter annihilation

NRDM×SCET for DM annihilation

After several steps one can prove that:



Resummation is achieved by solving

- an appropriate Schrödinger equation
- μ and v renormalization group equations for every piece of the factorization formula

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The wino-like/MDM triplet model

SM + Majorana SU(2) triplet
$$\delta \mathcal{L}_{\text{Wino}} = \frac{1}{2} \bar{\chi} (i \gamma^{\mu} D_{\mu} - m_{\chi}) \chi$$



Q=0 *Majorana* DM Q=1 *Dirac* chargino

- $m_{\chi^{\pm}}$ - $m_{\chi^0} \approx 164 \text{MeV}$
- DM stable through a Z₂ symmetry
- Suitable WIMP for $m_{\chi 0} \lesssim 3$ TeV
- Super-partner of the SU(2) gauge bosons in the MSSM

The wino-like/MDM triplet model



- suppressed direct-detection cross sections (below the so-called neutrino floor)
- too heavy for the LHC

M. Vollmann — Precise predictions for γ -ray production in dark-matter annihilation



$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_2 \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\alpha_2 \frac{e^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2 \frac{e^{-m_Z r}}{r} \end{pmatrix}$$

see e.g. Beneke et al arXiv: <u>1411.6924</u> Hisano arXiv: <u>hep-ph/0412403</u>



1D QM (revisited)











Assumptions on the energy resolutions

The variable $E_{res} = m_{\chi} - E_{\gamma}$ plays a decisive role in the factorization problem

We investigated two situations



$$E_{\rm res} \sim m_W^2 / m_\chi (1805.07367)$$

 $E_{\rm res} \sim m_W (1903.08702)$

See also **Baumgart et al** (1712.07656 and 1808.08956) for the $E_{res} \gg m_W$ case

Energy resolution



Factorization theorem. Exclusive Wino $\chi \chi \rightarrow \gamma + X$ annihilation

$$\frac{d(\sigma v_{\rm rel})}{dE_{\gamma}} = \sum_{I,J} S_{IJ} \Gamma_{IJ}(E_{\gamma})$$

$$\Gamma_{IJ}(E_{\gamma}) = \frac{1}{4} \frac{2}{\pi m_{\chi}} \sum_{i,j=1,2} C_{j}^{*}(\mu_{W}) C_{i}(\mu_{W}) Z_{\gamma}(\mu_{W},\nu_{W})$$
$$\times \int J \left(4m_{\chi}(m_{\chi} - E_{\gamma} - \omega/2), \mu_{W}\right) W_{IJ}^{ij}(\omega, \mu_{W}, \nu_{W})$$

Factorization theorem. Exclusive Wino $\chi \chi \rightarrow \gamma + X$ annihilation

This is highly **non trivial!!** E.g. all (N>2)-body phase-space integrals reduce to the convoluted J⊗W form (modulo power corrections) The SCET formalism is such that proving this result amounts to performing a (in the SCET language) trivial multipole expansion

Jet functions are **universal**: they can be used in several contexts

For invariant masses of the order of the EW breaking scale, computed the neutral components of the EW jet function for the first time

$$\Gamma_{IJ}(E_{\gamma}) = \frac{1}{4} \frac{2}{\pi m_{\chi}} \sum_{i,j=1,2} C_{j}^{*}(\mu_{W}) C_{i}(\mu_{W}) Z_{\gamma}(\mu_{W},\nu_{W})$$
$$\times \int J \left(4m_{\chi}(m_{\chi} - E_{\gamma} - \omega/2), \mu_{W}\right) W_{IJ}^{ij}(\omega, \mu_{W}, \nu_{W})$$













Not an obvious result

Can be understood by expanding our factorization formulas at fixed orders



$$[\sigma v]_{IJ}(E_{\gamma}) = \frac{2\pi \hat{\alpha}_2(\mu) \hat{s}_W(\mu)}{\sqrt{2}^{n_{\rm id}} m_{\chi}^2} \sum_{n=0}^{\infty} \sum_{m=0}^{2n} c_{IJ}^{(n,m)}(E_{\gamma},\mu) \left(\frac{\hat{\alpha}_2(\mu)}{\pi}\right)^n \ln^m \frac{4m_{\chi}^2}{m_W^2}$$

$$[\sigma v]_{IJ}(E_{\gamma}) = \frac{2\pi\hat{\alpha}_{2}(\mu)\hat{s}_{W}(\mu)}{\sqrt{2}^{n_{\rm id}}m_{\chi}^{2}} \sum_{n=0}^{\infty} \sum_{m=0}^{2n} c_{IJ}^{(n,m)}(E_{\gamma},\mu) \left(\frac{\hat{\alpha}_{2}(\mu)}{\pi}\right)^{n} \ln^{m} \frac{4m_{\chi}^{2}}{m_{W}^{2}}$$

Factorization-theorem dependent

• Coefficients are (by definition)

of $\mathcal{O}(1)$ but dependent on E_{χ}

- When evaluated outside the range of validity of the Fact. Th. they can become large
 - (and contribute to the (m+1) term instead)



$$[\sigma v]_{(+-)(+-)}^{\text{nrw(NLO)}} = \frac{2\pi\hat{\alpha}_2^2\hat{s}_W^2}{m_\chi^2} \left[1 + \frac{\hat{\alpha}_2(\mu)}{\pi} \left(-1\ln^2\frac{4m_\chi^2}{m_W^2} + 1\ln\frac{4m_\chi^2}{m_W^2} \right) + c_0(E_\gamma) \right]$$

$$[\sigma v]_{(+-)(+-)}^{\text{int(NLO)}} = \frac{2\pi\hat{\alpha}_2^2\hat{s}_W^2}{m_\chi^2} \left[1 + \frac{\hat{\alpha}_2(\mu)}{\pi} \left(-\frac{3}{4}\ln^2\frac{4m_\chi^2}{m_W^2} + \left(\frac{29}{48} + \ln\frac{2E_\gamma^{\text{res}}}{m_W} \right) \ln\frac{4m_\chi^2}{m_W^2} \right) + \tilde{c}_0(E_\gamma) \right]$$

$$c_0(E_{\gamma}) \sim 4 \ln^2 \frac{4m_{\chi} E_{\gamma}^{\text{res}}}{m_W^2} - \frac{19}{6} \ln \frac{4m_{\chi} E_{\gamma}^{\text{res}}}{m_W^2} + \text{const.}$$

 $\mathcal{O}(1)$ for $E_{\chi}^{\text{res}} \sim m_W^2/m_{\chi}$ but parametrically large for $E_{\chi}^{\text{res}} \sim m_W$ After reshuffling the logs you get

 $[\sigma v]_{(+-)(+-)}^{\operatorname{nrw}(\operatorname{NLO})} = [\sigma v]_{(+-)(+-)}^{\operatorname{int}(\operatorname{NLO})} + \mathcal{O}(\alpha_2^3) \text{ Not even } \alpha_2^3 \times \log \text{ terms!!}$

Further applications of our fixed-order expansions







Effect of the Sudakov resumation



Further technicalities addressed

Photon jet function (by definition) sensitive to the lowest scales in the SM

In particular, large QCD effects on the wave-renormalization of the photon field are tackled by using dispersion-relation methods

 γ

Z-pole singularity can be cured by Dyson-resumming the Z propagator



Summary

Sudakov is the new Sommerfeld for indirect DM detection

Flux
$$\approx \frac{1}{8\pi m_{\chi}^2 c^4} \times J \times S_{(+-)(+-)} e^{-\frac{\alpha_2}{\pi} \frac{3}{4} \ln^2 \frac{4m_{\chi}^2}{m_W^2}} \frac{2\pi \hbar^2 \alpha_2^2 \sin^2 \theta_W}{m_{\chi}^2 c} \delta(E_{\gamma} - m_{\chi})$$

FRESH RESULTS!! Higgsino DM

Higgsino DM in a nutshell:

- Minimal DM pseudo-Dirac doublet
- Hypercharge $\neq 0$
 - $\delta \mathcal{L}_{\text{Higgsino}} = \bar{\chi} (i \gamma^{\mu} D_{\mu} m_{\chi}) \chi + \mathcal{L}_{\text{dim}-5}$
- EWSB \rightarrow 2 Majorana χ_1^0, χ_2^0 and 1 chargino χ^+
- Thermal production hypothesis:

 $m_{\chi} \approx 1 \,\mathrm{TeV}$

FRESH RESULTS!! Higgsino DM

Factorization: Sommerfeld matrix $I, J = (\chi_1^0 \chi_1^0), (\chi_2^0 \chi_2^0) \text{ or } (\chi + \chi -)$ $\frac{d(\sigma v_{\rm rel})}{dE_{\gamma}} = \sum_{I = I} S_{IJ} \Gamma_{IJ}(E_{\gamma})$ $\Gamma_{IJ}(E_{\gamma}) = \frac{1}{4} \frac{2}{\pi m_{\chi}} \sum_{i,j} C_j^*(\mu) C_i(\mu) Z_{\gamma}^{WY}(\mu,\nu)$ $\times \int \mathrm{d}\omega \left(J^{\mathrm{SU}(2)} (2m_{\chi} (2E_{\mathrm{res}}^{\gamma} - \omega), \mu) W_{IJ, \boldsymbol{W}\boldsymbol{Y}}^{\mathrm{SU}(2)\,ij}(\omega, \mu, \nu) \right)$ $+J^{\mathrm{U}(1)}(2m_{\chi}(2E_{\mathrm{res}}^{\gamma}-\omega),\mu)W^{\mathrm{U}(1)\,ij}_{IJ,WY}(\omega,\mu,\nu)\right)$

$\chi \chi \rightarrow \gamma + X$ cross sections ($E_{res} \sim m_W$) for Higgsino DM



$\chi \chi \rightarrow \gamma + X$ ratio plots ($E_{res} = m_W$) for Higgsino DM



$\chi \chi \rightarrow \gamma + X$ ratio plots ($E_{res} = m_W$) for Higgsino DM


Conclusions

- Heavy WIMP region will be probed by indirect detection observations in the near future. Theory input is urgent!
- Tackled the technically/conceptually involved problem of correctly
 predicting cross sections that are relevant for spectral multi-TeV γ-ray
 line searches in two regimes: *narrow* and *intermediate* energy resolution
- Employed these methods on **wino** and **Higgsino** DM
 - Reduced theoretical uncertainties down to the permille level for the (intermediate) energy resolutions of the CTA at the interesting mass range of 1-10 TeV
 - Observed a remarkable matching of the two factorization formulas in the "transition" region (narrow <-> intermediate resolutions)