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The large-charge limit: New Developments

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based on arXiv:1505.01537, 1610.04495, 1707.00710, 1809.06371, 1902.09542, 1905.00026, 1909.02571, 1909.08642, and work in progress with:

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Sh. Chandrasekharan (Duke), S. Favrod (Bern),
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F. Sannino (Odense), M. Watanabe (Weizmann)

Introduction

Strongly coupled physics is notoriously difficult to access.
We do not have small parameters in which to do a perturbative expansion. Our most basic notions of field theory are of a perturbative nature.

Make use of symmetries, look at special limits/ subsectors where things simplify.

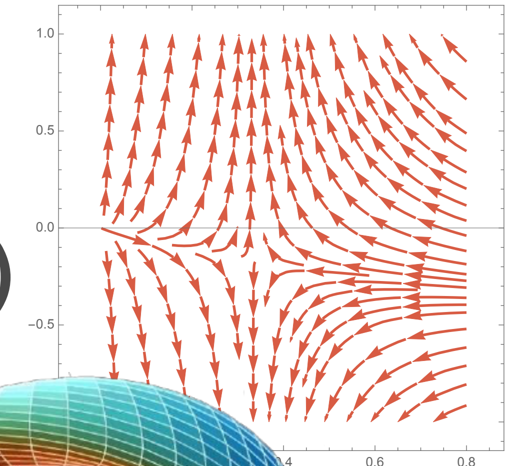
Here: study theories with a global symmetry group.
Hilbert space of the theory can be decomposed into sectors of fixed charge Q under the action of the global symmetry group.
Study subsectors with large charge Q.

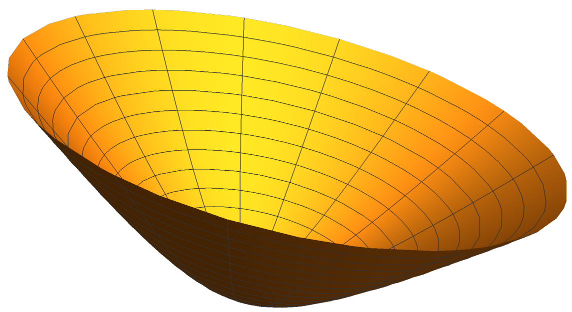
Large charge Q becomes controlling parameter in a perturbative expansion!

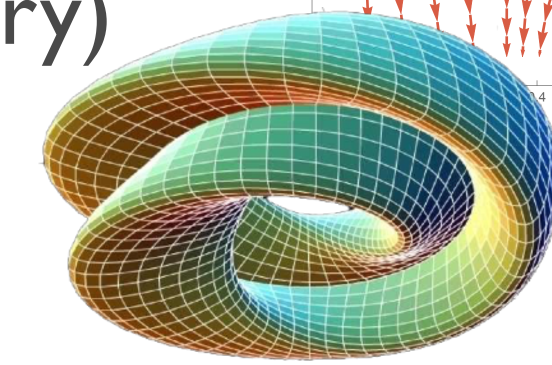
Introduction

CFTs play an important role in theoretical physics:

- fixed points in RG flows
- critical phenomena
- quantum gravity (via AdS/CFT)
- string theory (WS theory)







Conformal field theories (CFTs) do not have any intrinsic scales, most have by naturalness couplings of O(1).

Possibilities: analytic (2d), conformal bootstrap (d>2), lattice calculations, non-perturbative methods…

Prime candidate for the large-charge approach.

(Also: they come with a lot of space-time symmetry that will help us in practice to constrain the eff. action.)

Introduction

The large-charge approach consists of 2 steps:

1. identify the symmetry breaking patterns due to charge fixing for a given order parameter/field
2. write an effective action for the low-energy DOF and compute physical quantities

Step 1: start from the global symmetries of the system and how they act on the order parameter.

Example: in the superfluid transition of 4He, it is known that the system has an O(2) symmetry.
Assume that, just like in the UV, the order parameter is a complex scalar that transforms the same way under O(2).

Introduction

Write down Wilsonian effective action. In general: infinitely many terms - not so useful.
Make self-consistent truncation at large charge:

- Set a cutoff Λ obeying

space dimension

typical scale of the system

$$\frac{1}{L} \ll \Lambda \ll \frac{1}{\ell_Q} = \frac{Q^{1/d}}{L}$$

- write a linear sigma model action for the order parameter. Work at criticality: impose scale invariance of the action, assuming that the fields have vanishing anomalous dimension (at leading order in 1/Q)
- determine the fixed-charge ground state
- compute the quantum fluctuations to verify that they are parametrically small when Q >> 1.

Introduction

In a sector of fixed charge, the classical solution around which the quantum fluctuations are computed will generically break both spacetime (Lorentz) and global symmetries: Goldstone bosons

Step 2: write down EFT encoded by Goldstones. Similar techniques to chiral perturbation theory. Important difference: the symmetry breaking comes from fixing the charge (NOT dynamical).

Use EFT to calculate the CFT data (anomalous dimensions, 3-pt functions).

Wilsonian action has only a handful of terms that are not suppressed by the large charge. Useful!

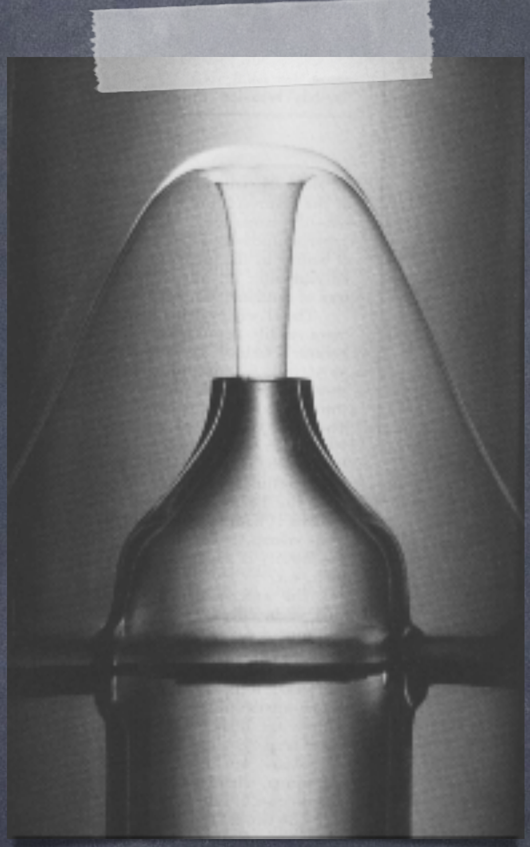
Introduction

Some questions:

- Does it work?
- For what kinds of theories does it work?
- In how many space-time dimensions?
- For what kinds of global symmetries does it work?
- What happens if we fix several charges independently?
- What can we learn via this approach?

Overview

- Introduction
- The O(2) model
  - semi-classical treatment
  - quantum treatment
  - results and lattice comparison
- Beyond O(2)
  - O(2n) vector model
  - an asymptotically safe CFT
  - leaving the conformal point
- Summary/Outlook



The O(2) model

The O(2) model

Consider simple model: O(2) model in (2+1) dimensions

$$\mathcal{L}_{\mathrm{UV}} = \partial_\mu \phi^* \, \partial^\mu \phi - g^2 (\phi^* \phi)^2$$

Flows to Wilson-Fisher fixed point in IR.

Assume that also the IR DOF are encoded by cplx scalar

Global U(1) symmetry: $\varphi_{IR} = a \, e^{ib\chi} \qquad \chi \to \chi + \text{const.}$

Look at scales: put system in box (2-sphere) of scale R

Second scale given by U(1) charge Q: $\rho^{1/2} \sim Q^{1/2}/R$

Study the CFT at the fixed point in a sector with

UV scale

$$\frac{1}{R} \ll \Lambda \ll \frac{Q^{1/2}}{R} \ll g^2$$

cut-off of effective theory

Write Wilsonian action.

The O(2) model

Assume large vev for a: $\Lambda \ll a^2 \ll g^2$

scalar curvature

$$\mathcal{L}_{\mathrm{IR}} = \tfrac{1}{2}\,\partial_\mu a\,\partial^\mu a + \tfrac{1}{2} b^2 a^2\,\partial_\mu \chi\,\partial^\mu \chi - \frac{R}{16} a^2 - \frac{\lambda}{6} a^6 + \text{higher derivative terms}$$

dimensionless constants

Lagrangian is approximately scale-invariant.

φ has approximately mass dimension 1/2 and the action has a potential term $\propto |\varphi|^6$

Do semi-classical analysis: solve classical e.o.m. at fixed Noether charge.

$$\rho = \frac{\delta \mathcal{L}_{\mathrm{IR}}}{\delta \dot{\chi}} = b^2 a^2 \dot{\chi} \qquad Q \sim 4\pi R^2 b\sqrt{\lambda} a^4$$

Classical solution at lowest energy and fixed global charge becomes the vacuum of the quantum theory.

The O(2) model

Classical solution:

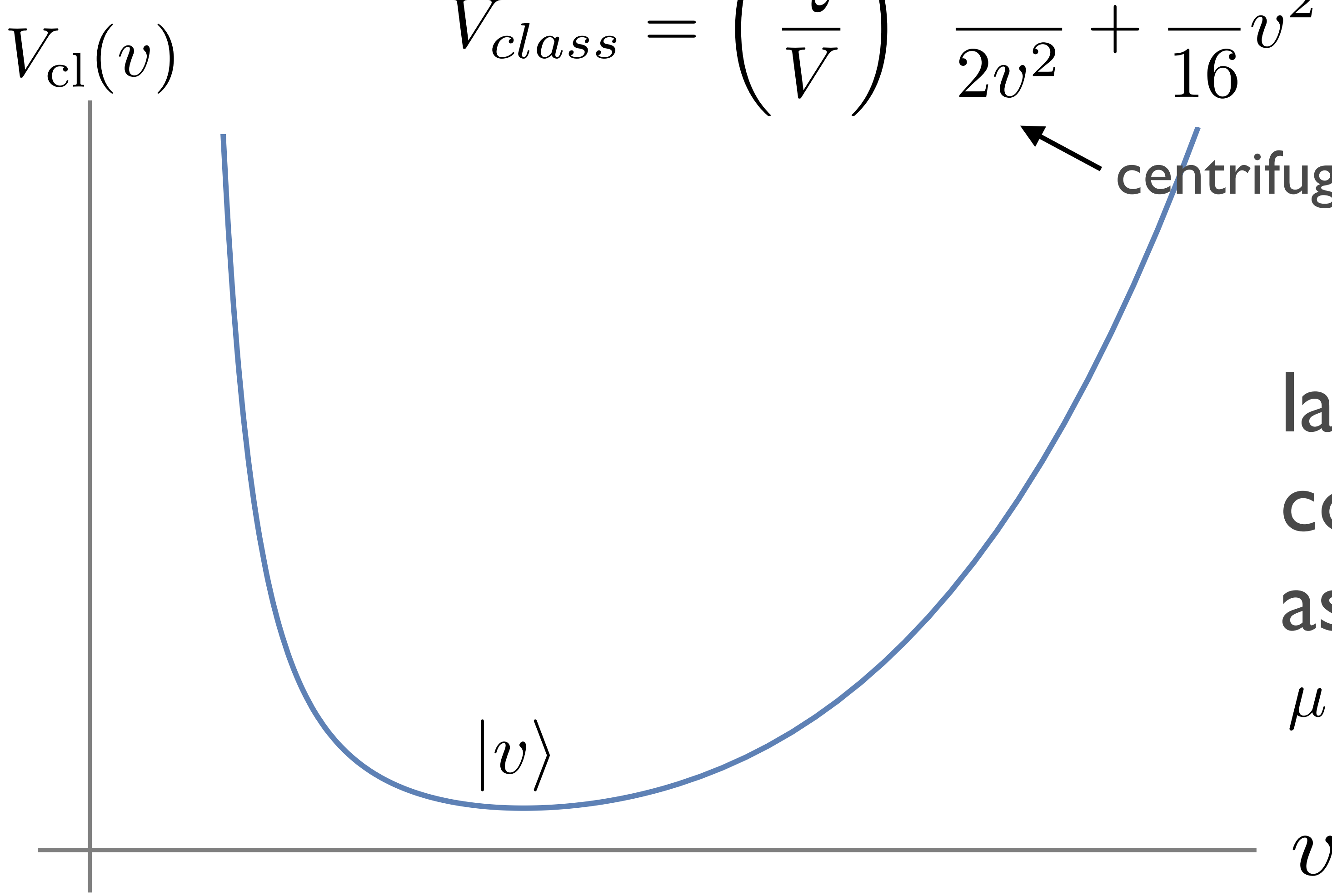
non-const. vev

$$\langle a \rangle = v, \qquad \langle \dot{\chi} \rangle = \mu = \frac{Q}{V \cdot v^2}, \qquad \langle \chi \rangle = \mu\, t$$

Fixed-charge ground state is homogeneous in space.
Determine radial vev v by minimizing the classical potential:

$$V_{class} = \left(\frac{Q}{V}\right)^2 \frac{1}{2v^2} + \frac{R}{16}v^2 + \frac{\lambda}{6}v^6$$

centrifugal term



$$v \sim Q^{1/4}$$

large condensate is compatible with our assumption $a \gg 1$

$\mu \sim \rho^{1/2}$

The O(2) model

Ground state at fixed charge breaks symmetries:

$$SO(1,4)_{\text{spacetime}} \times O(2)_{\text{global}} \xrightarrow{\text{expl.}} SO(3)_{\text{space}} \times D \times O(2)_{\text{global}} \xrightarrow{\text{spont.}} SO(3)_{\text{space}} \times D'$$

$$D' = D - \mu O(2)$$

Quantum story: study the low-energy spectrum

Parametrize fluctuations on top of the classical vacuum

$$a = v + \hat{a} \qquad \chi = \mu t + \frac{\hat{\chi}}{v}$$

Goldstone

massive mode, not relevant
for low-energy spectrum $m \sim \mathcal{O}(\sqrt{Q})$

Go to NLSM: Integrate out a (saddle point for LO).
Dynamics is described by a single Goldstone field χ:

$$\mathcal{L}_{LO} = k_{3/2}(\partial_\mu \chi \, \partial^\mu \chi)^{3/2}$$

can get this purely by
dimensional analysis

The O(2) model

Use dimensional analysis and scale invariance to determine (tree-level) operators in effective action beyond LO (scalar operators of scaling dimension 3, including curvatures of the background metric)

Use ρ-scaling to determine which terms appear:

$$\partial\chi \sim \rho^{1/2}, \quad \partial \ldots \partial\chi \sim \rho^{-1/4}$$

$\mathcal{O}(\rho^{3/2}):$ $\qquad \mathcal{O}_{3/2} = |\,\partial\chi|^3$ ← LO Lagrangian

$\mathcal{O}(\rho^{1/2}):$ $\qquad \mathcal{O}_{1/2} = R|\,\partial\chi| + 2\dfrac{(\partial|\,\partial\chi|)^2}{|\,\partial\chi|}$

conf. inv. combination, negative ρ-scaling

scale-inv. but NOT conformally inv.

For homogeneous solutions, there are no other terms contributing to the effective Lagrangian at non-negative ρ-scaling for d>1.

The O(2) model

Result:

$$\mathcal{L} = k_{3/2}(\partial_\mu\chi\partial^\mu\chi)^{3/2} + k_{1/2}R(\partial_\mu\chi\partial^\mu\chi)^{1/2} + \mathcal{O}(Q^{-1/2})$$

dimensionless parameters

suppressed by inverse powers of Q

To be understood as an expansion around the classical ground state $\mu t + \hat{\chi}$

Expand action to second order in fields:

$$\mathcal{L} = k_{3/2}\mu^3 + k_{1/2}R\mu + (\partial_t\hat{\chi})^2 - \tfrac{1}{2}(\nabla_{S^2}\hat{\chi})^2 + \ldots$$

Compute zeros of inverse propagator and get dispersion relation:

$$\omega_{\vec{p}} = \frac{|\vec{p}|}{\sqrt{2}}$$

dictated by conf. invariance $1/\sqrt{d}$

Spontaneous symmetry breaking

⇒ χ is relativistic Goldstone (type I)

⇒ superfluid phase of O(2) model

The O(2) model

Are also the quantum effects controlled?

All effects except Casimir energy are suppressed (negative ρ-scaling)

Effective theory at large Q:

vacuum + Goldstone + 1/Q-suppressed corrections

Energy of classical ground state at fixed charge:

2 dimensionless parameters (b, λ)

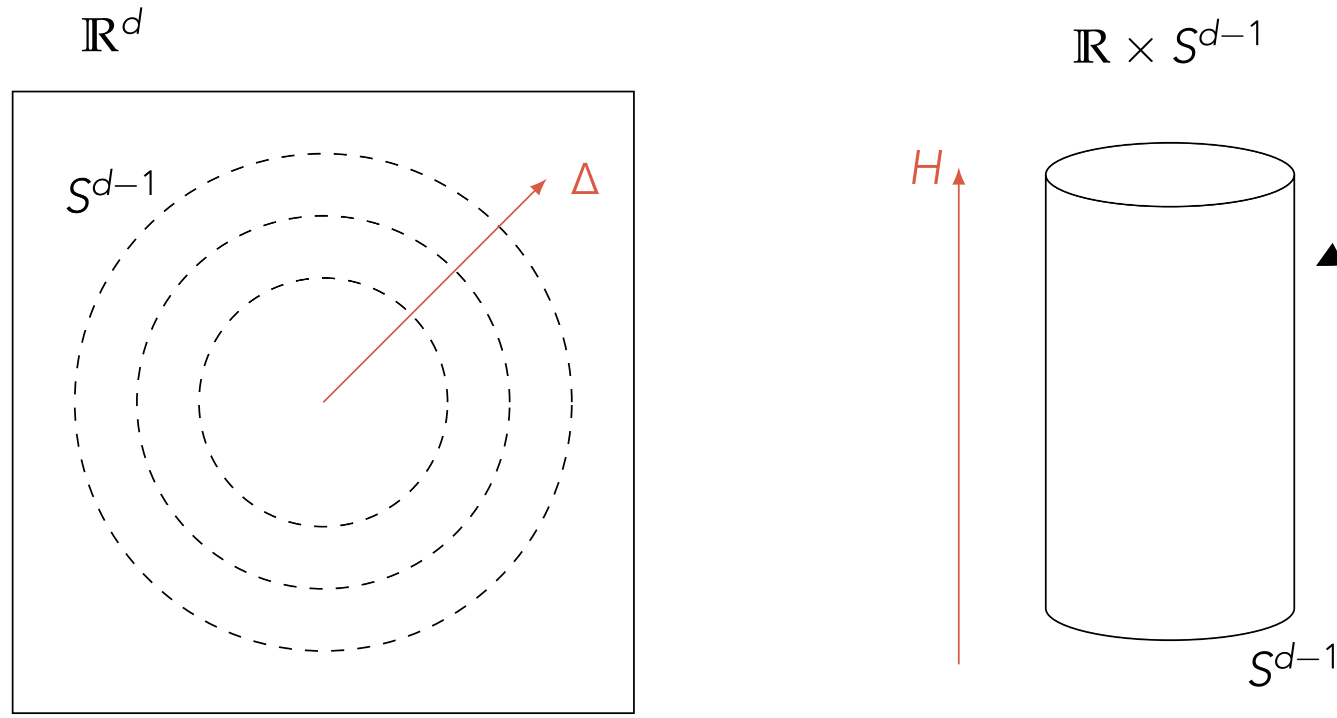
$$E_{\Sigma}(Q) = \frac{c_{3/2}}{\sqrt{V}} Q^{3/2} + \frac{c_{1/2}}{2} R\sqrt{V} Q^{1/2} + \mathcal{O}(Q^{-1/2})$$

dependence on manifold

The O(2) model

Use state-operator correspondence of CFT:

conformal dimension



energy

Conformal dimension of lowest operator of charge Q:

one-loop vacuum energy of Goldstone

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

S. Hellerman, D. Orlando, S. R., M. Watanabe, arXiv:1505.01537 [hep-th]

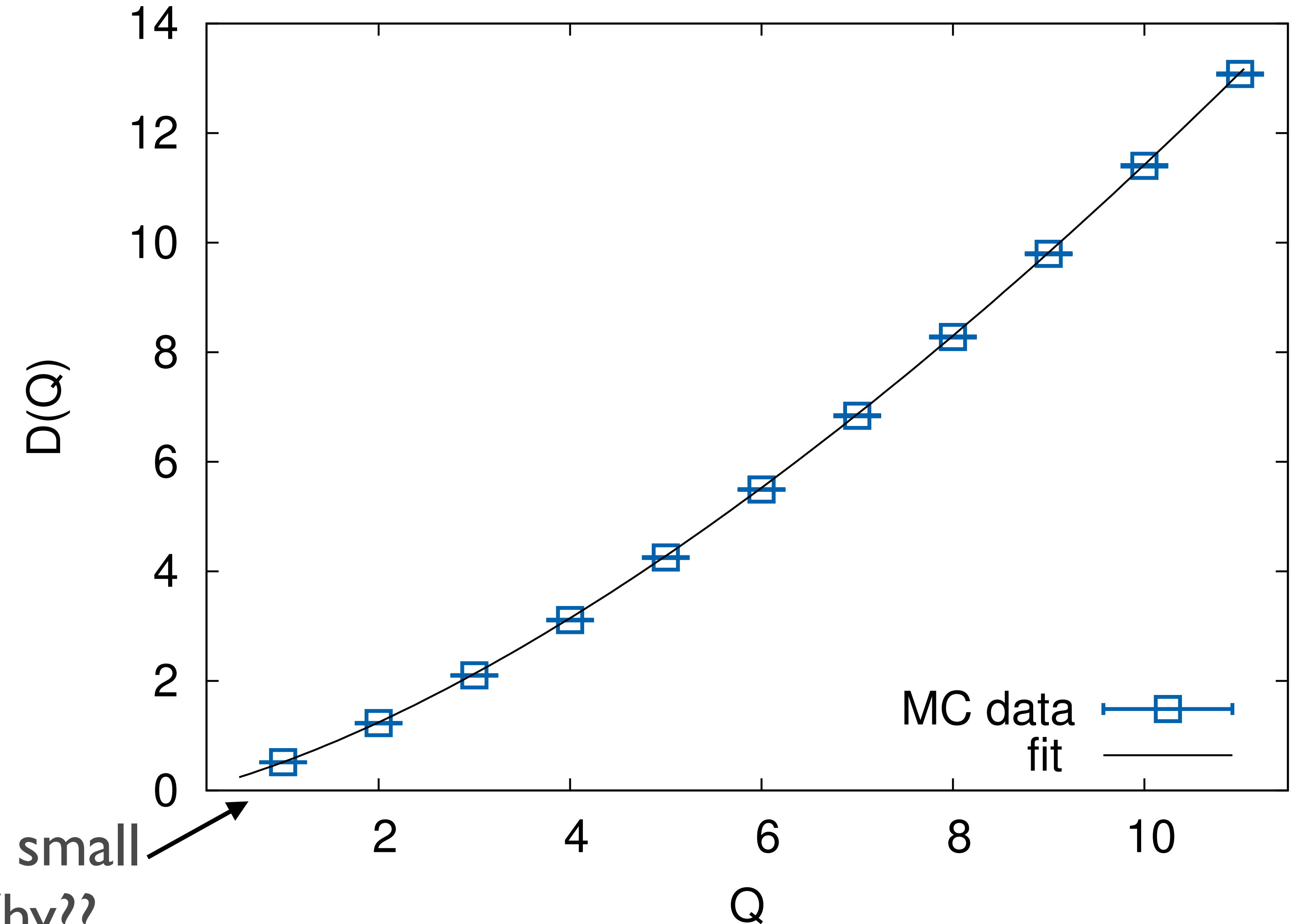
$$E_{\mathrm{VAC}} = \frac{1}{2\sqrt{2}r} \int \frac{\mathrm{d}\omega}{2\pi} \sum_{l=0}^{\infty} (2l+1) \log(\omega^2 + l(l+1)) = \frac{1}{2\sqrt{2}r} \zeta(-1/2|S^2) = -\frac{0.0937\ldots}{r}$$

The O(2) model

Our prediction:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

Independent confirmation from the lattice:



works for small charge. Why??

D. Banerjee, Sh. Chandrasekharan, D. Orlando [hep-th/1707.00711]

Excellent agreement!!

$c_{3/2} = 1.195(10)$

$c_{1/2} = 0.075(10)$

Large-charge expansion works extremely well for O(2). Where else?

Beyond O(2)

Where else can apply the large-charge expansion?

Try out other known CFTs/assume they exist.

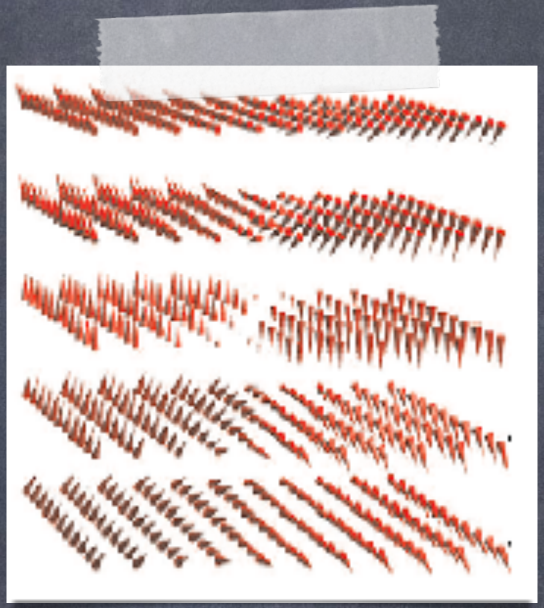
Obvious generalization in 3d: O(2n) vector model
non-Abelian global symmetry group: new effects

SU(N) matrix model in 3d.

Not many examples of (non-susy, non-fermionic) CFTs known in 4d.
Asymptotically safe CFT (UV fixed point)

Superconformal CFTs in 3d and 4d. Cases with moduli space work differently!

Non-relativistic CFTs (Schrödinger symmetry) in 3d, 4d



Beyond O(2):
3d O(2n) vector model

The O(2n) vector model

Generalize to O(2n).

$$L = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{2}\sum_{i=1}^{2n}\left(\frac{1}{8}R\phi^a\phi^a + \frac{\lambda}{12}(\phi^a\phi^a)^3\right), \quad a = 1,\dots,2n \qquad \mathbb{R}_t\times\mathbb{R}^2$$

$U(n)\subset O(2n)$

$$\varphi_1 = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)\,, \qquad \varphi_2 = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4)\,, \qquad \dots,$$

Fix $k \le n$ U(1) charges:

$$\int \mathrm{d}^{d-1}x\, i\,(\dot\varphi_i\varphi_i^* - \dot\varphi_i^*\varphi_i) = \overline{Q}_i = \text{vol.}\times\bar\rho_i$$

Solution for homogeneous ground state:

$$\begin{cases} \varphi_i = \frac{1}{\sqrt{2}}A_i\, e^{i\mu t}, & i = 1,\dots,k\,, \\ \varphi_{k+j} = 0, & j = 1,\dots,n-k\,, \end{cases}$$

same for all fields!

$$A_i^2 = \frac{Q_i}{4\pi\mu}, \qquad \mu = \frac{1}{4}\sqrt{R + \sqrt{R^2 + \frac{2}{\pi^2}\lambda(\sum_i Q_i)^2}}$$

The O(2n) vector model

Fixing k charges explicitly breaks O(2n) to O(2n-2k) x U(k).

We can always rotate $\langle \vec{\varphi} \rangle = \frac{1}{\sqrt{2}}(A_1, \dots, A_k, 0, \dots)$ by a U(k) transformation into $(0, \dots, 0, \sqrt{\frac{A_1^2 + \dots + A_k^2}{2}}, 0, \dots)$

Vacuum breaks symmetry spontaneously to O(2n-2k) x U(k-1).

We also see that all homogeneous states of minimal energy with fixed total charge $(Q_1 + Q_2 + \dots + Q_k)$ are related by an U(k) transformation and have the same energies (and conformal dimensions).

What happens if instead, we choose a configuration with k different chemical potentials that cannot be rotated into the state $(\underbrace{0, \dots, 0}_{k-1}, \frac{v}{\sqrt{2}}, \underbrace{0, \dots, 0}_{n-k})$?

Ground state must be inhomogeneous!

The O(2n) vector model

For quantum description, write effective theory for fluctuations around the ground state.

Expand Lagrangian around the ground state

$$\big(\underbrace{0\,,\,\dots\,,0}_{k-1}\,,\,\tfrac{v}{\sqrt{2}}\,,\underbrace{0\,,\,\dots\,,0}_{n-k}\big)$$

U(1) sector: $\varphi_k = \frac{1}{\sqrt{2}}\, e^{i\mu t + i\hat{\phi}_{2k}/v}\left(v + \hat{\phi}_{2k-1}\right)$ $\qquad \begin{cases} \hat{\phi}_{2k-1} \to \hat{\phi}_{2k-1} \\ \hat{\phi}_{2k} \to \hat{\phi}_{2k} + \theta\,, \end{cases}$

U(k-1) sector: $\varphi_i = e^{i\mu t}\hat{\varphi}_i$ $\qquad \hat{\varphi}_i \mapsto \tilde{U}_i{}^j\hat{\varphi}_j$

Developing to second order in fields:

$$\mathcal{L}^{(2)} = \sum_{i=1}^{k}(\partial_t - i\mu)\varphi_i^*(\partial_t + i\mu)\varphi_i + \sum_{i=k+1}^{n}\dot{\varphi}_i^*\dot{\varphi}_i - \sum_{i=1}^{n}\nabla\varphi_i^*\nabla\varphi_i$$
$$- \sum_{i=1}^{n}\mu^2\varphi_i^*\varphi_i - 2\mu^2\phi_{2k-1}^2$$

Find inverse propagators and dispersion relations.

The O(2n) vector model

We expect dim[U(k)/U(k-1)] = 2k-1 Goldstone d.o.f.

Massless modes:

$$\omega_{nr}^2 = \frac{p^4}{4\mu^2} - \frac{p^6}{8\mu^4} + \mathcal{O}(\mu^{-6}) \qquad k-1 \text{ times}$$

$$\omega_r^2 = \underbrace{\frac{1}{2}}_{\text{“conformal” Goldstone}} p^2 + \frac{p^4}{32\mu^2} + \mathcal{O}(\mu^{-4}) \qquad \text{one time}$$

There are

- 1 relativistic Goldstone $\omega \propto p$
- k-1 non-relativistic Goldstones (count double) $\omega \propto p^2$

Nielsen and Chadha; Murayama and Watanabe

$$1 + 2 \times (k-1) = 2k - 1 = \dim(G/H)$$

Non-relativistic Goldstones have no zero-point energy in flat space and contribute to the conformal dimensions only at higher order. Ground-state energy again determined by a single relativistic Goldstone.

The O(2n) vector model

Same formula for anomalous dimensions as for O(2):

n-dependent

universal for O(2n)

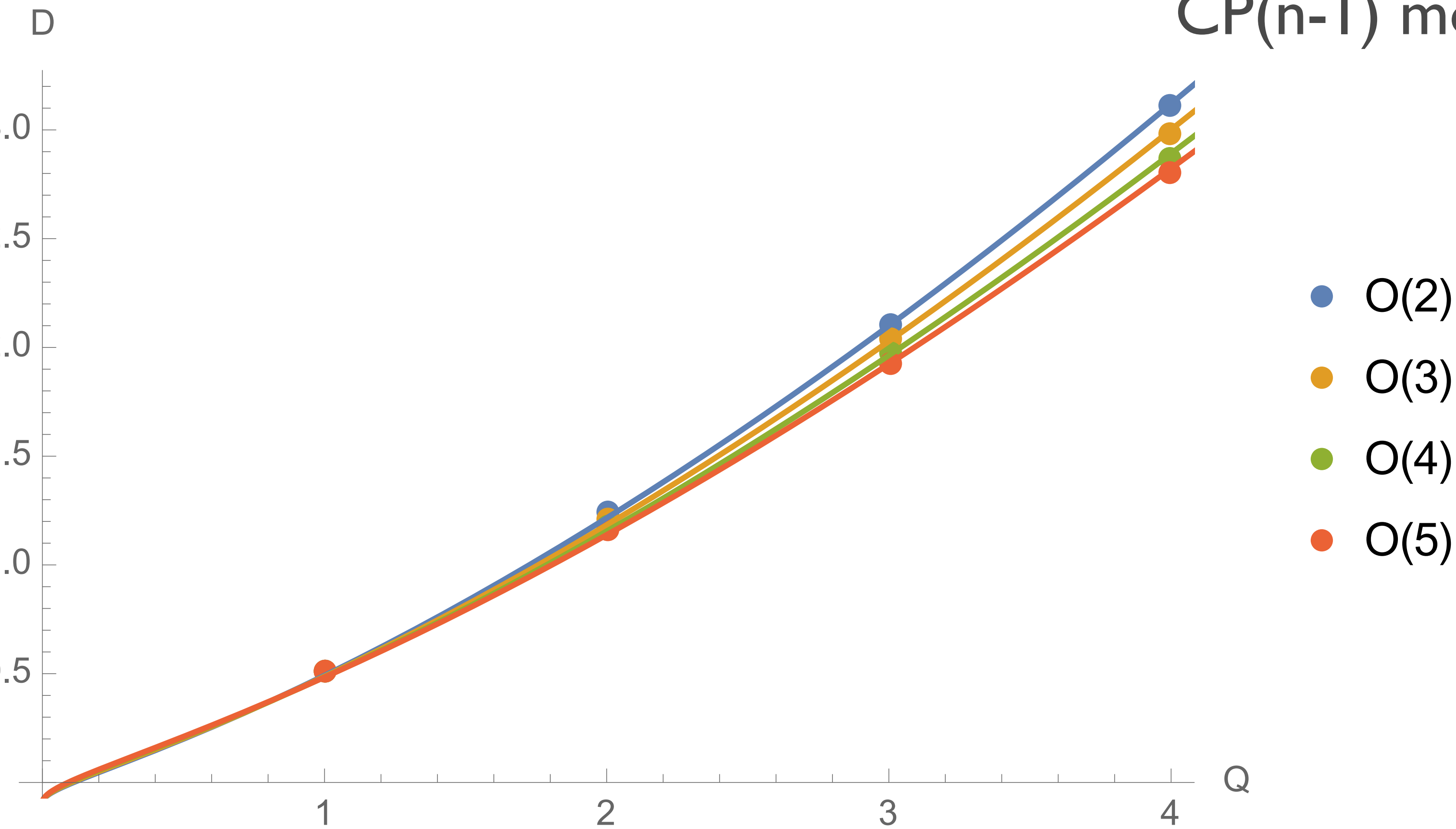
$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

L. Alvarez-Gaume, O. Loukas, D. Orlando and S. R., arXiv:1610.04495 [hep-th]

verified at large n for CP(n-1) model

de la Fuente

Comparison with old lattice data:

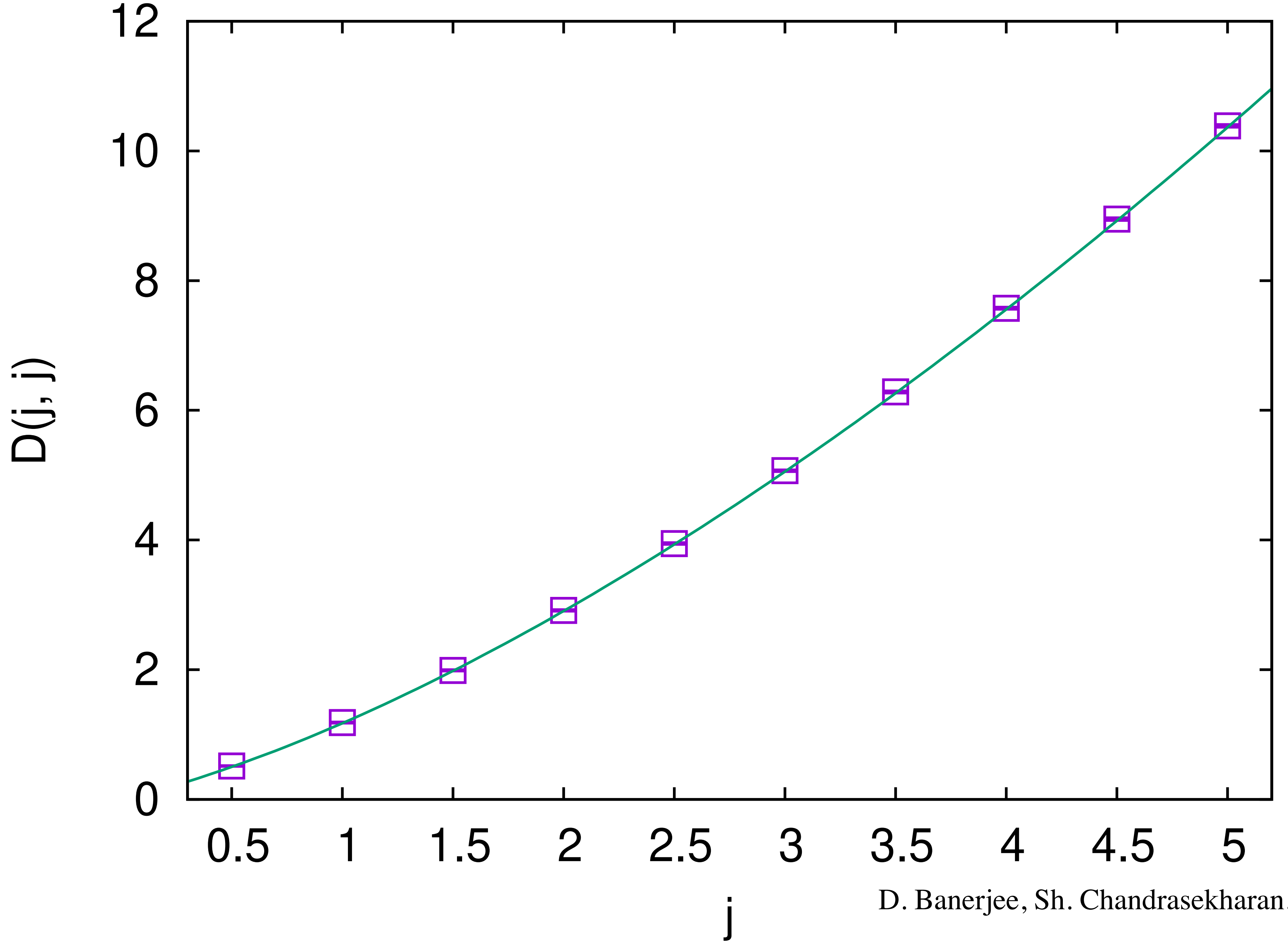


Hasenbusch, Vicari

$c_{3/2}$ decreases, $c_{1/2}$ increases with increasing n

The O(2n) vector model

New lattice data for O(4) model:



$$c_{3/2} = 1.068(4)$$
$$c_{1/2} = 0.083(3)$$

D. Banerjee, Sh. Chandrasekharan, D. Orlando, S.R. 1902.09542

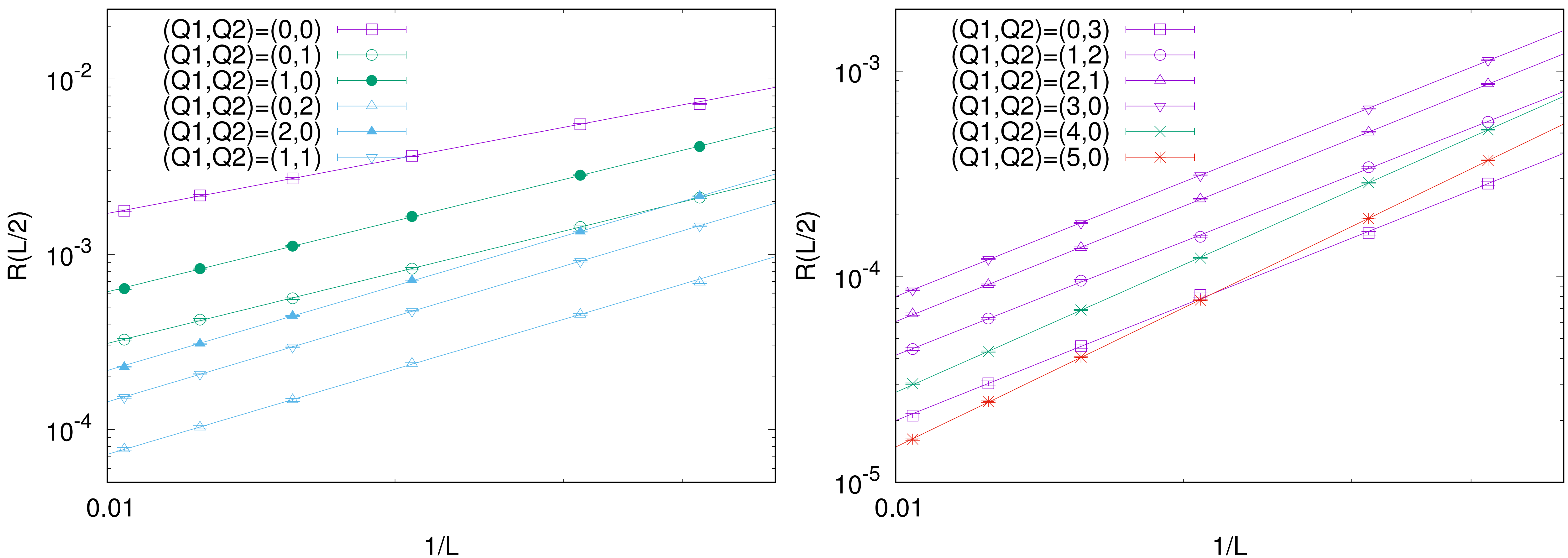
Again excellent agreement with large-Q prediction!

The O(2n) vector model

Only total charge matters for homogeneous case:

Correlation function:

$$C_Q(r) \sim \frac{a(Q)}{|\vec{r}|^{2D(Q)}} \qquad R(L/2) = \frac{C_Q(r=L/2)}{C_{Q-1}(r=L/2)} \qquad R(L) \sim 1/L^{2(D(Q)-D(Q-1))}$$



D. Banerjee, Sh. Chandrasekharan, D. Orlando, S.R. unpublished

Parallel lines in log/log plot: conformal dimensions are the same!

The O(2n) vector model

Now let’s take the limit $n \to \infty$

Start from first principles, expand path integral around saddle point (no EFT!)

Leading order: theory is solvable and we find the same powers in the large-Q expansion of the anomalous dimension.

Here, Q large means $Q > \frac{n}{4}$

NLO in N: reproduce dispersion relations of Goldstones.

The O(2n) vector model

Find coefficients of the expansion (leading order in N):

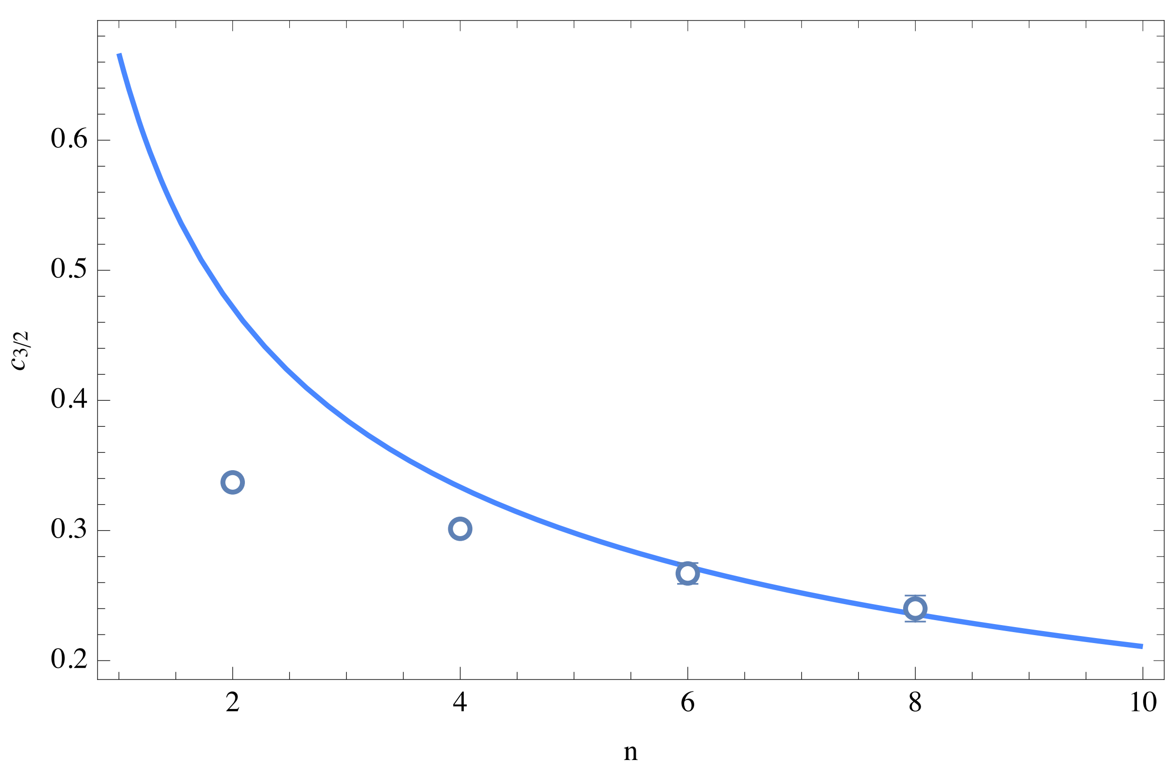
$$c_{3/2} = 4/3\sqrt{\pi/n}$$
$$c_{1/2} = 1/12\sqrt{n/\pi}$$

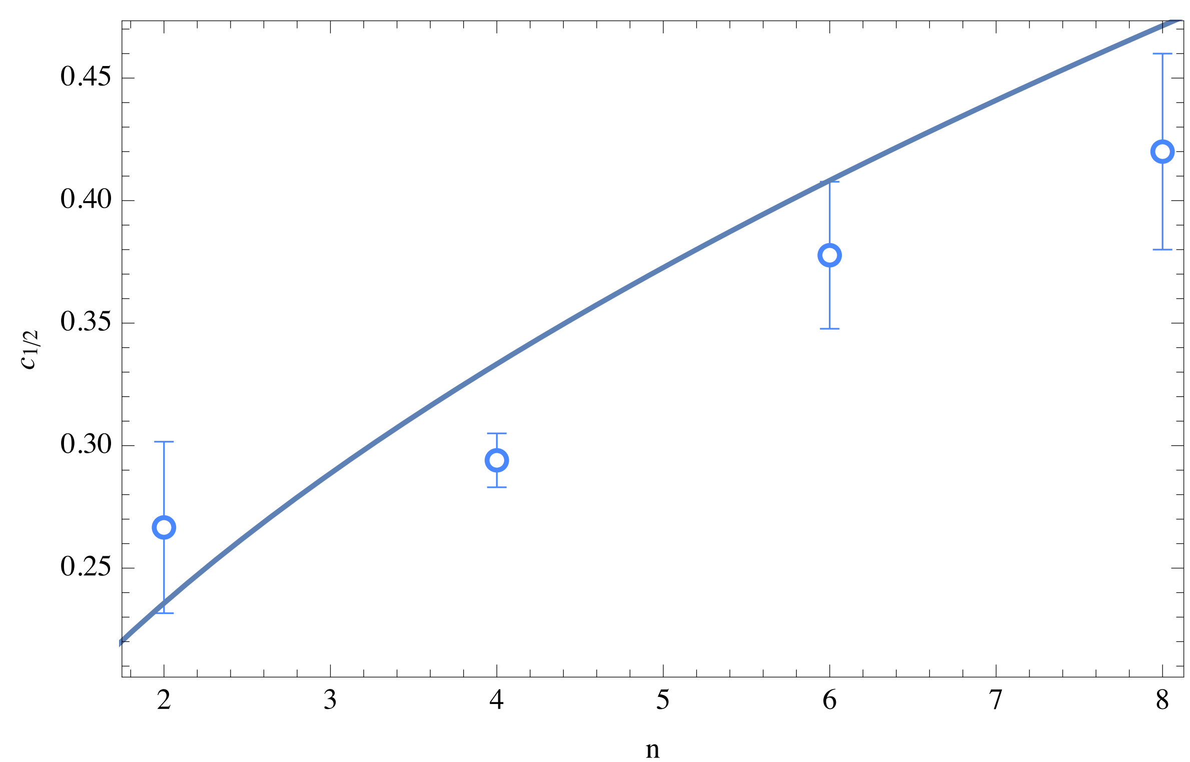
L. Alvarez-Gaume, D. Orlando, S.R. 1909.02571

Within 10% of the lattice measurements for O(4):

$$c_{3/2}^{n=4} = 1.18 \qquad c_{3/2} = 1.068(4)$$
$$c_{1/2}^{n=4} = 0.094 \qquad c_{1/2} = 0.083(3)$$

New lattice data (Chandrasekharan, Singh, unpublished):





The O(2n) vector model

At large n, we now have more control and can also take the limit of $Q/N \ll 1$.

In this limit, the operator of charge Q whose dimension we are calculating is φ^Q.

engineering dimension of φ

$$\frac{\Delta(Q)}{Q} = \frac{1}{2} + \frac{4}{\pi^2}\frac{Q}{N} + \mathcal{O}\left(\frac{Q}{2N}\right)^2$$

one-loop

tree-level

Can be verified by a perturbative (loop) calculation around the zero-charge vacuum (Benvenuti, unpublished)!

The O(2n) vector model

We can also consider the case of $Q \neq 0, \ \ T \neq 0$.

Unbroken phase (transition at T=0).

$$\rho = \frac{Q}{2N}\frac{4\pi}{V}$$

$$\Delta = \frac{2}{3}\rho^{3/2} - \frac{\pi^2}{6\beta^2}\rho^{1/2} - \frac{\zeta(3)}{\beta^3} + \cdots + e^{-\beta\sqrt{\rho}}\left(\frac{\rho^{1/2}}{\beta^2} + \dots\right)$$

contribution of the type II Goldstone (finite temp)

Calculate entropy from low-T expansion of F:

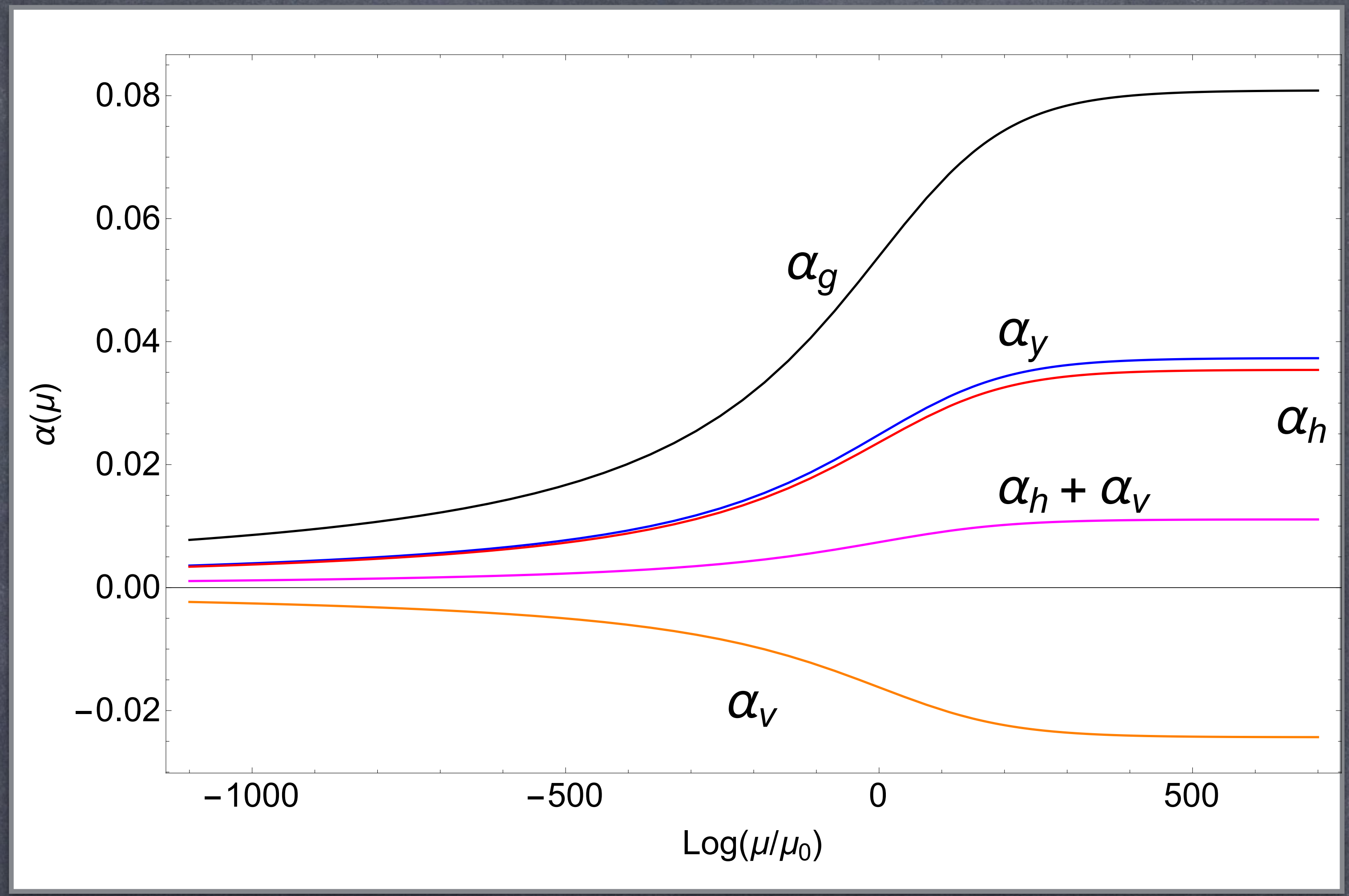
$$\mathcal{S} = \beta^2\frac{\partial F}{\partial \beta} = \frac{\pi}{6}NV\frac{\rho^{1/2}}{\beta} + \frac{3NV\zeta(3)}{2\pi\beta^2} + \cdots - \frac{NV}{2\pi}e^{-\beta\sqrt{\rho}}\left(\rho + 2\frac{\rho^{1/2}}{\beta} + \dots\right).$$

contribution of the type II Goldstone (finite temp)

At T=0 consistent with EFT result (S=0).

In the matrix model, this contribution will go like N^2.

Looks like entropy of RN BH (in the right double scaling limit.)



An example in 4d: asymptotically safe CFT

An asymptotically safe CFT

Look for CFTs with bosons in 4D. Start with a QCD-inspired theory with quarks, gluons and scalars:

N_F flavors of fermions

gauge group $SU(N_C)$

$N_F \times N_F$ matrix of cplx scalars

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \mathrm{Tr}\big(\bar{Q}i\not{D}Q\big) + y\,\mathrm{Tr}\big(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\big)$$
$$+ \mathrm{Tr}\big(\partial_\mu H^\dagger \partial^\mu H\big) - u\,\mathrm{Tr}\big(H^\dagger H\big)^2 - v(\mathrm{Tr}\,H^\dagger H)^2 - \frac{R}{6}\mathrm{Tr}\big(H^\dagger H\big)$$

$Q_{L/R} = \frac{1}{2}(1 \pm \gamma_5)Q$

Rescaled couplings

$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \qquad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}, \qquad \alpha_h = \frac{u N_F}{(4\pi)^2}, \qquad \alpha_v = \frac{v N_F^2}{(4\pi)^2}$$

Control parameter $\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$

In the limit $N_F \to \infty,\ N_C \to \infty$ with N_F/N_C fixed: asymptotically safe.

Litim, Sannino

Perturbatively controlled UV fixed point with

$$\alpha_g^* = \frac{26}{57}\epsilon + \ldots \qquad \alpha_y^* = \frac{4}{19}\epsilon + \ldots \qquad \alpha_h^* = \frac{\sqrt{23}-1}{19}\epsilon + \ldots \qquad \alpha_{v1}^* = -0.1373\,\epsilon + \ldots$$

An asymptotically safe CFT

Study this theory at large charge.

$$\mathcal{L} = -\frac{1}{2}\operatorname{Tr}(F^{\mu\nu}F_{\mu\nu}) + \operatorname{Tr}\left(\bar{Q}i\not{D}Q\right) + y\operatorname{Tr}\left(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\right)$$
$$+ \operatorname{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right) - u\operatorname{Tr}\left(H^\dagger H\right)^2 - v(\operatorname{Tr} H^\dagger H)^2 - \frac{R}{6}\operatorname{Tr}\left(H^\dagger H\right)$$

Global symmetry: $SU(N_F)_L \times SU(N_F)_R \times U(1)_B$

New elements compared to vector model:

- H is a matrix field, large non-Abelian global symmetry
- fermions and gluons are present
- 4D, different scalings
- UV fixed point, perturbatively controlled, trustable LSM

Large-charge expansion: focus on scalar sector

An asymptotically safe CFT

Noether currents:

$$J_L = \frac{i}{2}\left(\mathrm{d}H\, H^\dagger - H\,\mathrm{d}H^\dagger\right), \qquad J_R = -\frac{i}{2}\left(H^\dagger\,\mathrm{d}H - \mathrm{d}H^\dagger\, H\right)$$

Corresponding charges:

$$\mathcal{Q}_L = \int \mathrm{d}^3x\, J_L^0, \qquad \mathcal{Q}_R = \int \mathrm{d}^3x\, J_R^0$$

$$\mathrm{spec}(\mathcal{Q}_L) = \{J_1^L, J_2^L, \ldots, J_{N_F}^L\} \qquad \mathrm{spec}(\mathcal{Q}_R) = \{J_1^R, J_2^R, \ldots, J_{N_F}^R\}$$

Ansatz for homogeneous ground state:

Cartan subalgebra

self-adjoint

$$H_0(t) = e^{iM_L t} B e^{-iM_R t}$$

Impose charge conservation:

$$\dot{\mathcal{Q}}_L = -iVe^{iM_L t}\left(\left[M_L^2, BB^\dagger\right] - 2\left[M_L, BM_R B^\dagger\right]\right)e^{-iM_L t} = 0,$$

$$\dot{\mathcal{Q}}_R = iVe^{iM_R t}\left(\left[M_R^2, B^\dagger B\right] - 2\left[M_R, B^\dagger M_L B\right]\right)e^{-iM_R t} = 0$$

M commutes or anti-
comm with B

diagonal

$$\Rightarrow H_0 = e^{2iMt} B$$

An asymptotically safe CFT

We find: $\mathcal{Q}_L = -2VMB^2, \qquad \mathcal{Q}_R = 2VB^2M = -\mathcal{Q}_L$

Simple choice for charges:

$$M = \mu\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right), \qquad B = b\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & \mathbb{1} \end{array}\right) \qquad \mathcal{Q}_L = J\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right)$$

in su(N), traceless

$$J = 2Vb^2\mu$$

EOM on ansatz $H_0 = e^{2iMt}B$:

$$2\mu^2 = (u + vN_F)b^2 + \frac{R}{12}$$

Assume J large, expand in series:

$$\mu = \left(\frac{2\pi^2}{V}\right)^{1/3} \mathcal{J}^{1/3} + \frac{R}{72}\left(\frac{V}{2\pi^2}\right)^{1/3} \mathcal{J}^{-1/3} + \mathcal{O}\left(\mathcal{J}^{-5/3}\right)$$

Natural expansion parameter:

$$\mathcal{J} = J\frac{(u + vN_F)}{8\pi^2} = 2J\frac{\alpha_h + \alpha_v}{N_F} = 2J_{\text{tot}}\frac{\alpha_h + \alpha_v}{N_F^2} \gg 1$$

$$J_{\text{tot}} = JN_F$$

Consistent for $J_{\text{tot}} \gg \dfrac{N_F^2}{\epsilon}$

huge

tiny

An asymptotically safe CFT

Ground-state energy:

not universal

$$E = \frac{3}{2}\frac{N_F^2}{\alpha_h + \alpha_v}\left(\frac{2\pi}{V}\right)^{1/3}\left[\mathcal{J}^{4/3} + \frac{R}{36}\left(\frac{V}{2\pi^2}\right)^{2/3}\mathcal{J}^{2/3} - \frac{1}{144}\left(\frac{R}{6}\right)^2\left(\frac{V}{2\pi^2}\right)^{4/3}\mathcal{J}^0 + \mathcal{O}\left(\mathcal{J}^{-2/3}\right)\right]$$

Specialize to 3-sphere: $E = \frac{3}{2r_0}\frac{N_F^2}{\alpha_h + \alpha_v}\left[\mathcal{J}^{4/3} + \frac{1}{6}\mathcal{J}^{2/3} - \frac{1}{144}\mathcal{J}^0 + \mathcal{O}\left(\mathcal{J}^{-2/3}\right)\right]$

Classical result. What about Goldstone contributions, what about fermions, gluons?

At large charge, the fermions receive large masses and decouple:

kinetic term

Yukawa term

$$m_\psi = (\mu^2 + y^2 b^2)^{1/2} = \left(\frac{2\pi^2}{V}\right)^{1/3}\left(1 + 2\frac{N_F}{N_c}\frac{\alpha_y}{\alpha_h + \alpha_v}\right)^{1/2}\mathcal{J}^{1/3} + \mathcal{O}\left(\mathcal{J}^{-1/3}\right)$$

Below the fermion mass scale, also gluons decouple.

Gap:

$$\Lambda_{YM} = m_\psi \exp\left[-\frac{3}{22\alpha_g(m_\psi)}\right]$$

$\approx \mathcal{O}(\epsilon)$

Low-energy physics described by Goldstones only!

An asymptotically safe CFT

Symmetry-breaking pattern: $H_0 = e^{2iMt} B$

$$SU(N_F) \times SU(N_F) \times U(1) \xrightarrow{\text{exp.}} SU(N_F/2) \times SU(N_F/2) \times U(1)^2 \times SU(N_F)$$
$$\xrightarrow{\text{spont.}} SU(N_F/2) \times SU(N_F/2) \times U(1)^2$$

Expect $\dim(SU(N_F)) = N_F^2 - 1$ Goldstone DoF

Do quadratic expansion of the Lagrangian around the ground state, find dispersion relations.

$\omega = \frac{p^2}{4\mu} + \ldots$ $(N_F/2)^2$ type II Goldstone modes

$\omega = \frac{p}{\sqrt{3}} + \ldots$ conformal Goldstone (type I)

$\omega = \sqrt{\frac{\alpha_h}{3\alpha_h + 2\alpha_v}} p + \ldots$ $N_F^2/2 - 2$ type I Goldstones

causality constraint: $0 < \alpha_h/(3\alpha_h + 2\alpha_v) < 1$

Constraint satisfied at fixed point.

An asymptotically safe CFT

Goldstones are organized in reps of the unbroken symmetry group:

adjoint

bifundamental

$SU(N_F/2) \times SU(N_F/2)$ representation	$(\mathbf{1},\mathbf{1})$	(adjoint, $\mathbf{1}$)	($\mathbf{1}$, adjoint)	$(\square,\square)$
type	I	I	I	II
DOF	1	$N^2/4-1$	$N^2/4-1$	$2 \times N^2/4$
velocity	$1/\sqrt{3}$	$\sqrt{\frac{\alpha_h}{3\alpha_h+2\alpha_v}}$	$\sqrt{\frac{\alpha_h}{3\alpha_h+2\alpha_v}}$	n/a

Vacuum energy of the type I Goldstones:

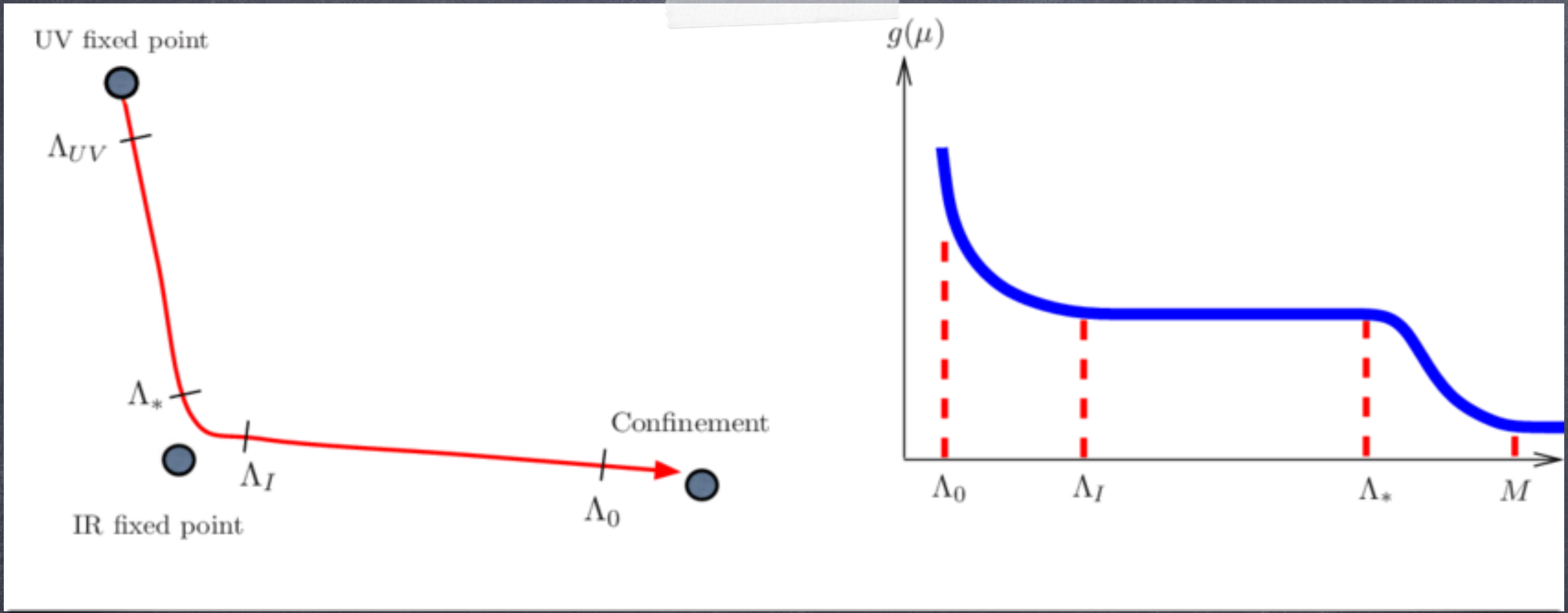
$$\zeta(-1/2|S^3) = -\frac{0.414\ldots}{r_0}$$

$$E_0 = \frac{1}{2}\left(2 \times \left(\frac{N_F^2}{4} - 1\right)\sqrt{\frac{\alpha_h}{3\alpha_h + 2\alpha_v}} + \frac{1}{\sqrt{3}}\right)\zeta(-1/2|M_3).$$

Conformal dimension (via state-operator corr.):

$$\Delta(J) = r_0 E(S^3) = \frac{3}{2}\frac{N_F^2}{\alpha_h + \alpha_v}\left[\mathcal{J}^{4/3} + \frac{1}{6}\mathcal{J}^{2/3} - \frac{1}{144}\mathcal{J}^0 + \mathcal{O}\left(\mathcal{J}^{-2/3}\right)\right]$$
$$-\left(\left(\frac{N_F^2}{2} - 2\right)\sqrt{\frac{\alpha_h}{3\alpha_h + 2\alpha_v}} + \frac{1}{\sqrt{3}}\right) \times 0.212\ldots$$

D.Orlando, S.R., F. Sannino, arXiv:1905.00026



Leaving the conformal point

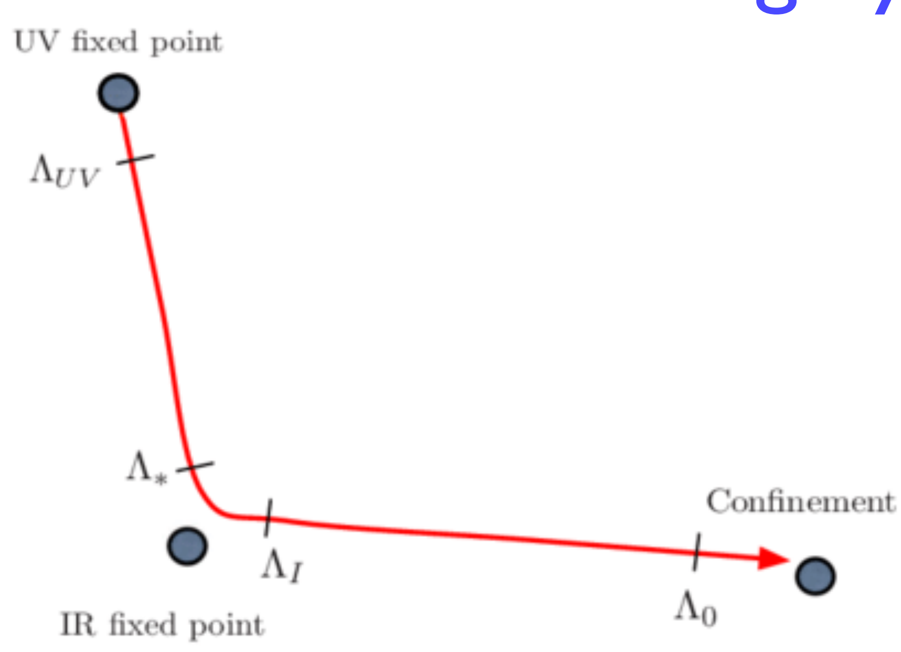
Leaving the conformal point

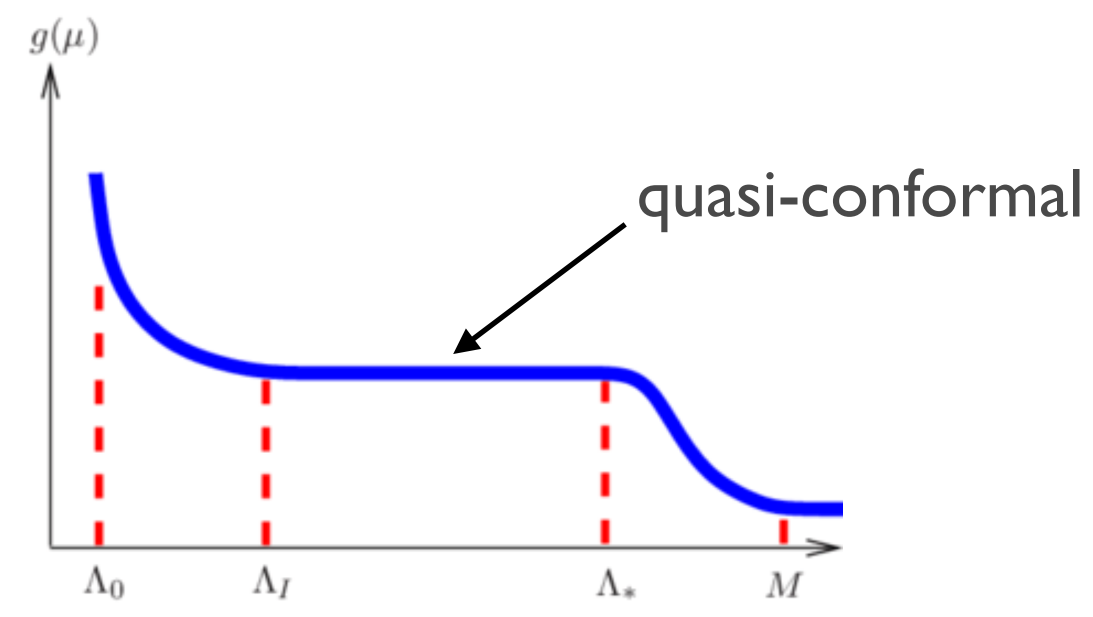
There is no reason why the large-charge approach should not work for general QFTs.

Of course, there are many practical advantages in working at conformality (restricting the form of terms appearing in the eff. action, state/op. correspondence…)

First step: work near enough a conformal point that it still dominates the dynamics.

Possible scenario: walking dynamics





Leaving the conformal point

Consider simple case with a global U(1) at large charge in 4D.

The leading term in the effective action (on torus) is given in terms of the Goldstone,

$$L_{NLSM}[\chi] = k_4(\partial_\mu\chi\,\partial_\mu\chi)^2$$

Class. ground state: $\chi = \mu t \qquad \mu = (4k_4Q)^{1/3}/L$

Start differently: two-derivative EFT for Goldstone:

dim[1] constants

$$L_2[\chi] = \frac{f_\pi^2}{2}\,\partial_\mu\chi\,\partial_\mu\chi - C^4$$

Introduce new field σ to non-linearly realize conformal invariance. σ acts as the massive Goldstone of broken conformal symmetry.

Leaving the conformal point

[f]= -1

Under dilatations: $x \to e^{\alpha} x : \qquad \sigma \to \sigma - \alpha/f$

To non-linearly realize conformal symmetry, dress all operators:

$[\mathcal{O}_k] = k$

$$\mathcal{O}_k \to e^{(k-4)f\sigma} \mathcal{O}_k$$

kin. term

conf. coupling

$$L_{CFT}[\chi, \sigma] = \frac{1}{2} g^{\mu\nu} f_\pi^2 e^{-2\sigma f} \, \partial_\mu \chi \, \partial_\nu \chi - C^4 e^{-4\sigma f} + \frac{1}{2} e^{-2\sigma f} \left(g^{\mu\nu} \, \partial_\mu \sigma \, \partial_\nu \sigma - \frac{\xi R}{f^2} \right) + \mathcal{O}(R^2)$$

Introduce complex field: $\Sigma = \sigma + i f_\pi \chi$

Recast action as

$\varphi = \frac{1}{\sqrt{2} f} e^{-f\Sigma}$ $\qquad u = 4C^4 f^4$

$$L[\varphi] = \partial_\mu \varphi^* \, \partial^\mu \varphi - \xi R \varphi^* \varphi - u(\varphi^* \varphi)^2 + \mathcal{O}(R^2)$$

LSM model action, σ appears as radial mode!

Leaving the conformal point

Fixed-charge ground state:

$$\chi = \mu t, \qquad \sigma = \frac{1}{f}\log(v),$$

$$\mu = 4c_{4/3}\Lambda_Q/3, \qquad v = 2f_\pi\sqrt{c_{4/3}/3}/\Lambda_Q,$$

$$c_{4/3} = 3(C/(2f_\pi))^{4/3}, \qquad \Lambda_Q = Q^{1/3}/L$$

Expanding the fields around this vacuum, we find (as expected) a massless and a massive mode (which decouples in the EFT) ⇒ go back to NLSM

Can use it to explicitly break conformal invariance: add a (small!) mass for σ.

$$L_m[\chi,\sigma] = L_{CFT}[\chi,\sigma] - U_m(\sigma)$$

$$U_m(\sigma) = \frac{m_\sigma^2}{16f^2}(e^{-4\sigma f} + 4\sigma f - 1)$$

Energy-momentum tensor no longer traceless:

$$T^\mu{}_\mu = \frac{m_\sigma^2}{f}\sigma$$

Leaving the conformal point

What is the signature of this mass term at large charge?

Action admits same type of fixed-charge ground state solution.

Energy:
$$E = c_{4/3}\frac{Q^{4/3}}{L} - \frac{m_\sigma^2 L^3}{12 f^2}\log(Q) + c_0$$

Dispersion relations of the two modes:

$$\omega = \frac{1}{\sqrt{3}}\left(1 + \frac{m_\sigma^2}{9 c_{4/3} f^2 \Lambda_Q^4}\right) p,$$
$$\omega = b c_{4/3}\sqrt{\frac{32}{3}}\Lambda_Q + \frac{5}{8\sqrt{6} b c_{4/3}\Lambda_Q}\left(1 - \frac{m_\sigma^2}{20 c_{4/3} f^2 \Lambda_Q^4}\right) p^2.$$

Near the conformal point, physics is still governed by fixed point. Makes sense to study conformal dimension.

Leaving the conformal point

Calculate 2-point fn on the cylinder and map it to flat space via Weyl-rescaling:

$$\langle \mathcal{O}_Q(t_0, \mathbf{n}_0) \mathcal{O}_{-Q}(t_1, \mathbf{n}_1) \rangle_{\text{cyl}} = \int \mathcal{D}\chi \mathcal{D}\sigma \exp[Q \log(\varphi(t_0, \mathbf{n}_0) \bar{\varphi}(t_1, \mathbf{n}_1)) - \int \mathrm{d}t \mathrm{d}\Omega L_m[\chi, \sigma]]$$

Large Q: integral is dominated by saddle point,

$$\chi = i\mu t, \qquad \sigma = \text{const.}$$

$$\langle \mathcal{O}_Q(t_0, \mathbf{n}_0) \mathcal{O}_{-Q}(t_1, \mathbf{n}_1) \rangle_{\text{cyl}} \approx e^{-E_{\text{cyl}} |t_1 - t_0|}$$

$$r_0 E_{\text{cyl}} = \frac{c_{4/3}}{(4\pi^2)^{1/3}} Q^{4/3} + c_{2/3} Q^{2/3} + c_0 - \frac{\pi^2 m_\sigma^2 r_0^4}{3 f^2} \log Q + \dots$$

$$c_{2/3} = (\pi / (f_\pi \Lambda^2))^{2/3} / (2 f^2)$$

Map to flat space:

$$\langle \mathcal{O}_Q(t_0, \mathbf{n}_0) \mathcal{O}_{-Q}(t_1, \mathbf{n}_1) \rangle_{\text{flat}} = \frac{c_Q}{|x|^{\Delta^* + r_0 E_{\text{cyl}}}} = \frac{c_Q}{|x|^{2\Delta}}$$

$$\Delta = \Delta^* \left(1 - \frac{m_\sigma^2}{24 c_{4/3} f^2 \Lambda_Q^4} \log Q + \dots \right)$$

D.Orlando, S.R., F. Sannino, arXiv:1909.08642

Summary

Summary

We studied various CFTs in sectors of large global charge

Concrete examples where a (strongly-coupled) CFT simplifies in a special sector.

- O(2N) model in 3d: in the limit of large U(1) charge Q, we computed the conformal dimensions in a controlled perturbative expansion:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

  - Excellent agreement with lattice results for O(2), O(4)
- Can be applied beyond vector model: SU(N) matrix models, SCFT

Summary

- Asymptotically safe CFT in 4d (scalars, fermions and gauge fields). Controllable UV fixed point.
  - fermions and gluons decouple
  - large-charge expansion for scalar sector
  - interesting Goldstone spectrum
- near-conformal/walking dynamics:
  - radial mode can be reinterpreted as dilaton of spontaneously broken conformal symmetry.
  - Explicitly break conformality by adding mass term for dilaton.
  - log(Q)-term appears in ground state energy: signature of massive dilaton

Summary

Some questions:

- Does it work?
  - For all the examples, we tried, yes! Confirmation from lattice data (O(2) and O(4))
- For what kinds of theories does it work?
  - (S)CFTs and non-relativistic CFTs
- In how many space-time dimensions?
  - $d>1$ space dimensions
- For what kinds of global symmetries does it work?
  - we checked U(1), O(2n) vector models, SU(N) matrix models

Summary

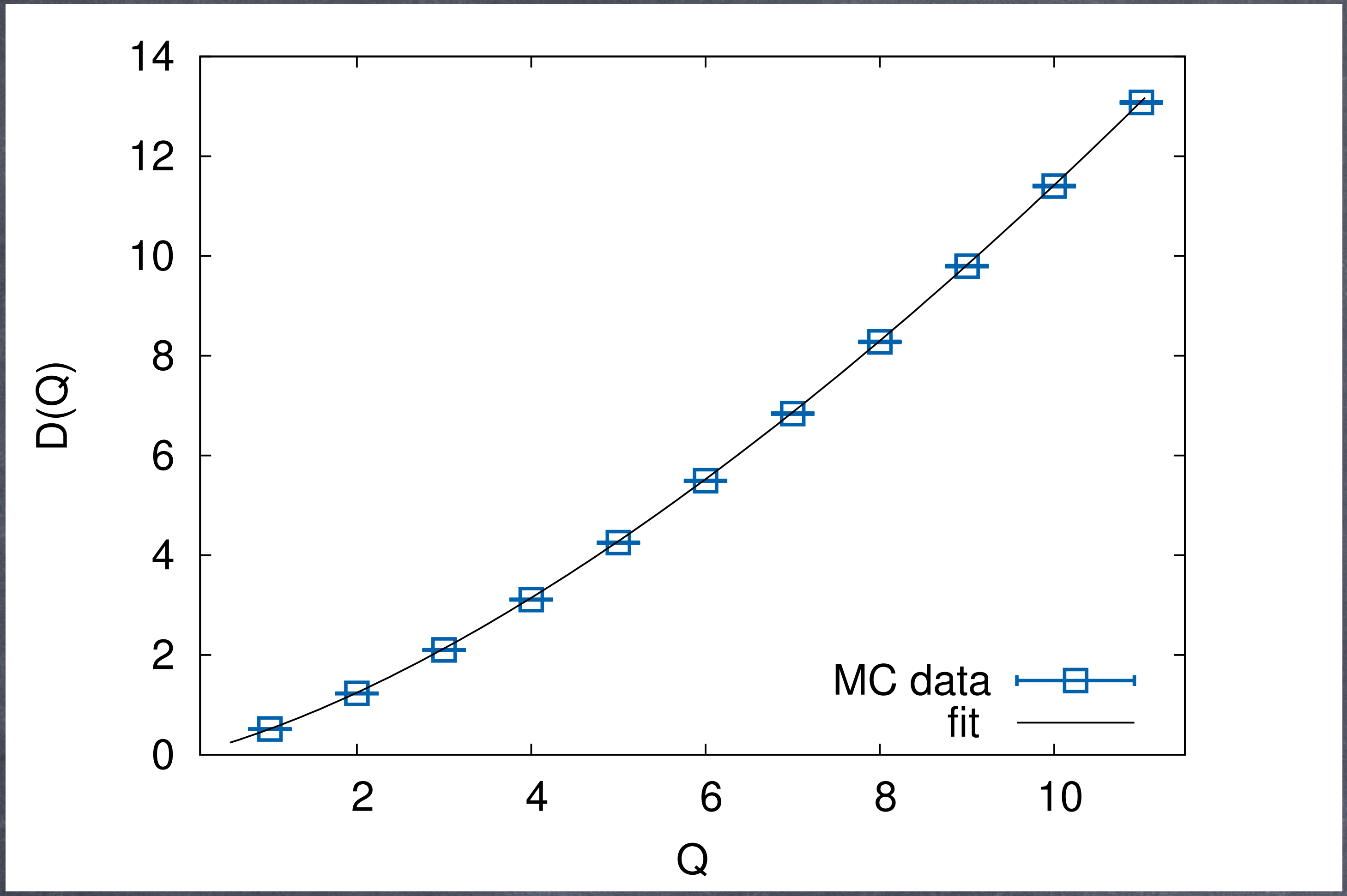
- What happens if we fix several charges?
  - k charges with same chemical potential: homogeneous solution with type I and type II Goldstones.
  - different chemical potentials: inhomogeneous solutions
- What can we learn via this approach?
  - calculate CFT data of <u>strongly coupled</u> CFTs at large charge!

Further directions

- Further study of supersymmetric models at large R-charge (higher-dim. moduli spaces) Hellerman, Maeda, Orlando, Reffert, Watanabe
- Connection to holography (gravity duals) Loukas, Orlando, Reffert, Sarkar
- Operators with spin; connection to large-spin results Cuomo, de la Fuente, Monin, Pirtskhalava, Rattazzi; Cuomo
- Understanding dualities semi-classically at large charge
- Use/check large-charge results in conformal bootstrap Jafferis and Zhiboedov
- Further lattice simulations: inhomogeneous sector, general O(N) Chandrasekharan et al.

Further directions

- Chern-Simons matter theories @large charge Watanabe
- 4-ε expansion @large charge Arias-Tamargo, Rodriguez-Gomez, Russo; Badel, Cuomo, Monin, Rattazzi; Watanabe
- strongly coupled CFTs in 4d at IR fixed point
- non-conformal case Orlando, Reffert, Sannino; Dodelson, Hellerman, Yamazaki
- Fishnet CFTs (non-unitary)
- Study fermionic theories. Can large-charge approach be used for QCD (e.g. large baryon number)?



Thank you for your attention!