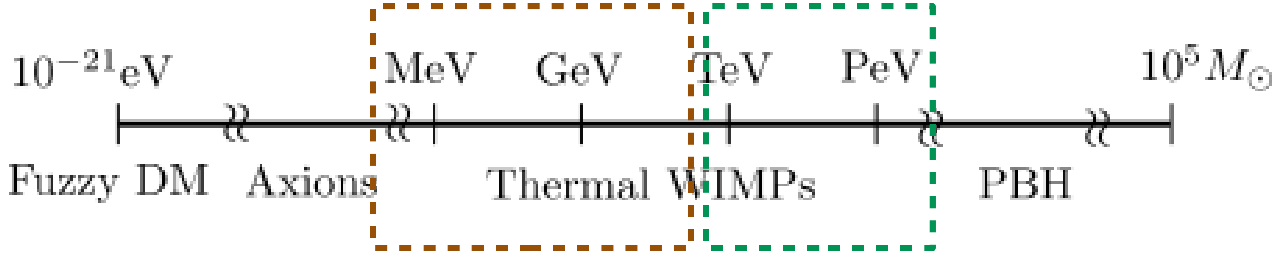
KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

@ New Physics Forum
13/12/19

Understanding Dark Matter Sommerfeld-enhanced annihilation, bound-state formation and decay in the Early Universe

Tobias Binder

Thermally produced dark matter



- One of the **leading DM hypothesis**: Thermal WIMPs.

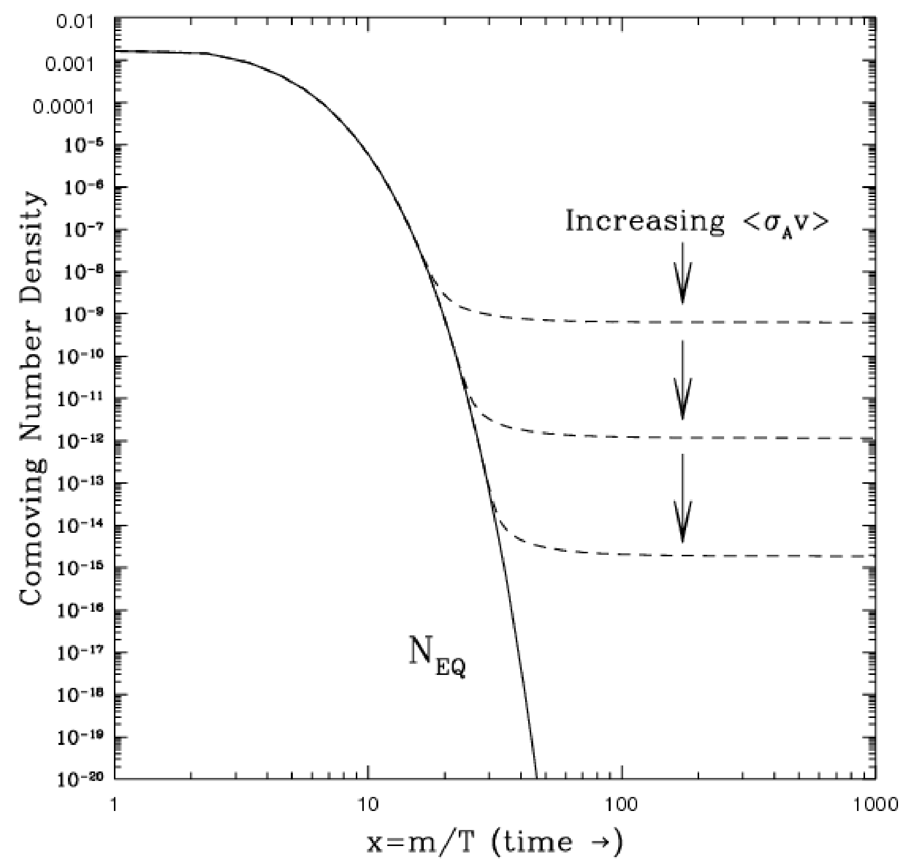
- **Testable** and final relic abundance **independent** of initial conditions.

- Strong constraints on coupling strength rule out many MeV-TeV mass realizations in thermal scenarios.

- TeV-scale and above still remains attractive and much less constrained.

- ***How heavy WIMPs can be?***
  QM effects introduce theoretical uncertainties.

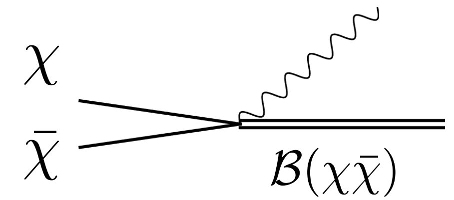
Thermal production mechanism



Quantum mechanical effects

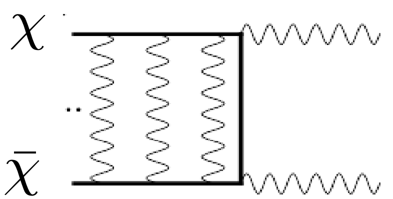
$$m_\phi \lesssim \alpha m_\chi$$

Bound-state formation

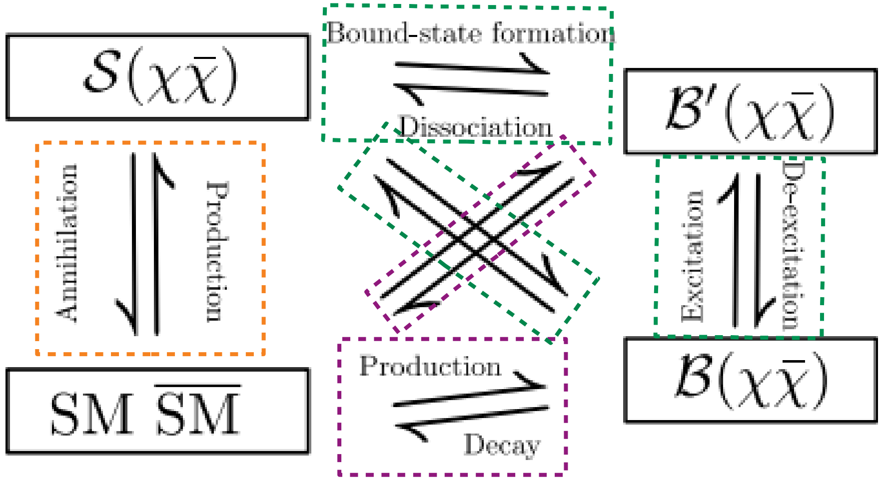


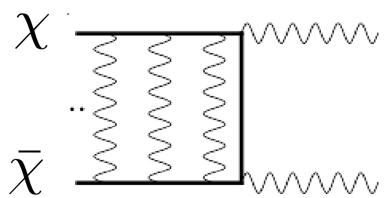
Sommerfeld-enhanced annihilation

[J. Hisano *et al.* ‘03, ‘05, ‘06]



$$(\sigma v) = (\sigma v)_0 \times |\psi(r=0)|^2$$
$$\propto (\sigma v)_0\,(\alpha/v)\,, \text{ for } v \lesssim \alpha.$$

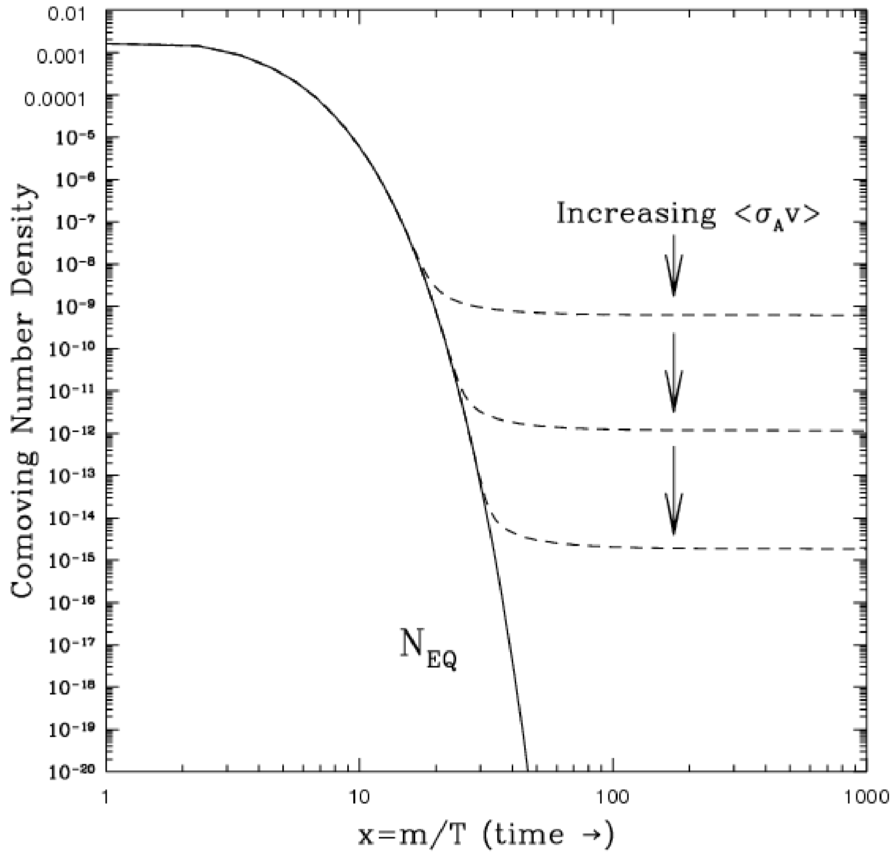


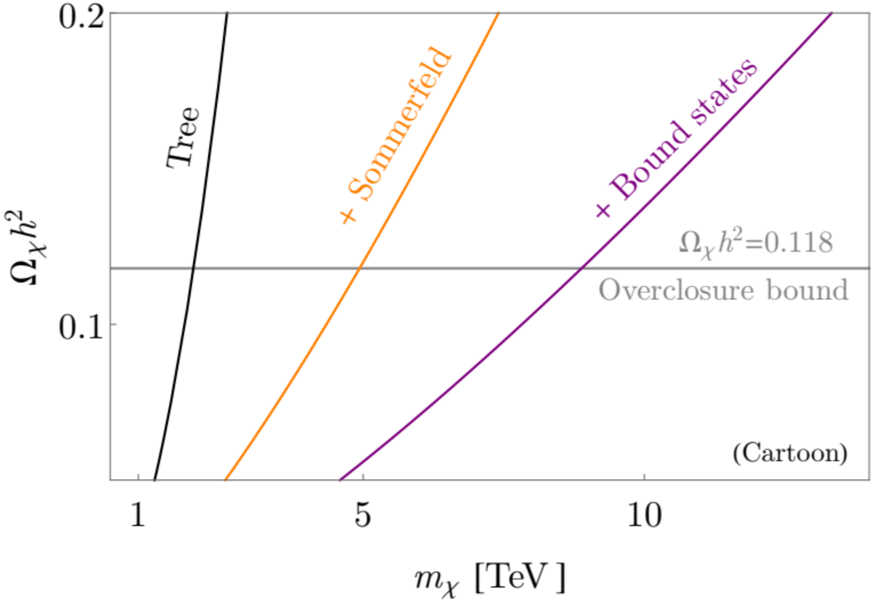


Bound-state decay

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$

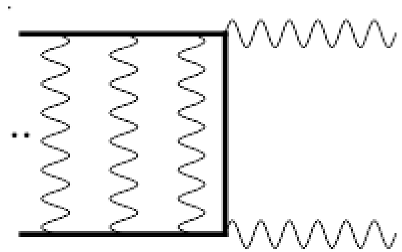
Relic abundance

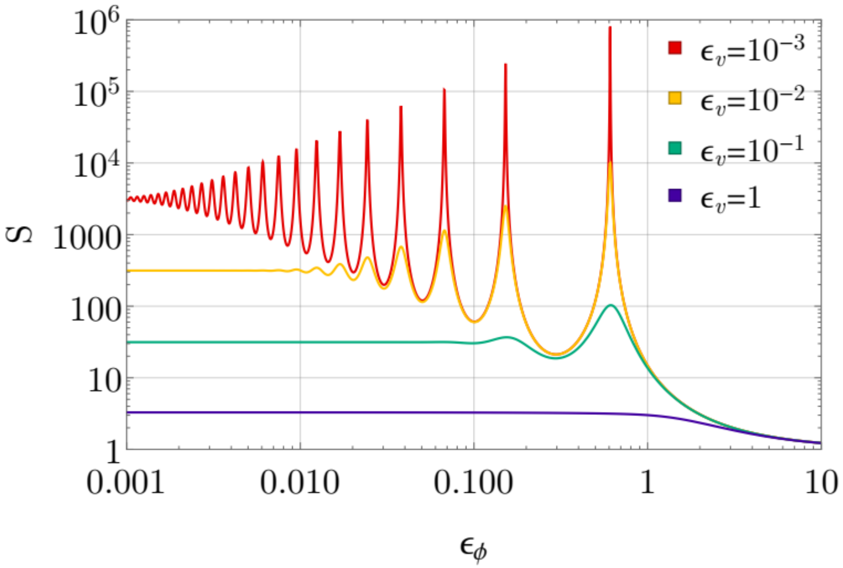




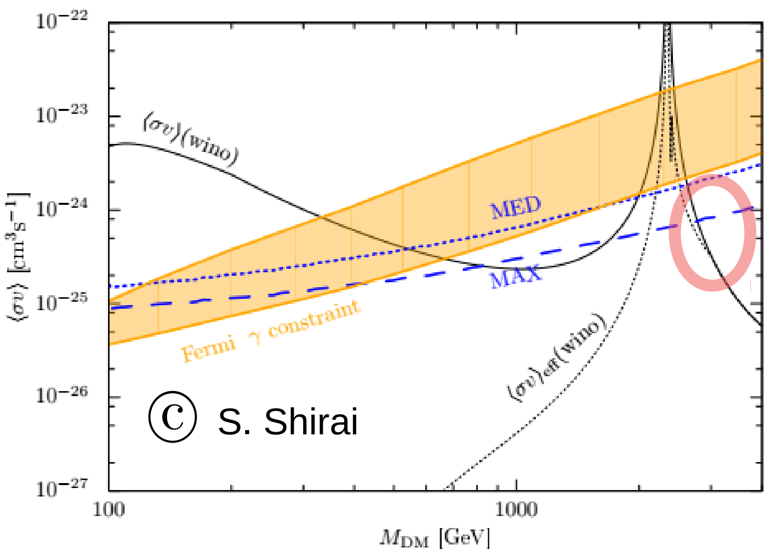
QM effects allow for **larger DM masses.**

Motivation





- Predicted SM flux **sensitive to DM mass** due to Sommerfeld resonances.
- In thermal Wino case: 10% change in the mass would result in 100 % change in the flux!
- For constraining WIMPs reliably, we need to theoretical predict the relic abundance precisely!
- **Vacuum treatment of QM effects in hot and dense early Universe plasma sufficient?**



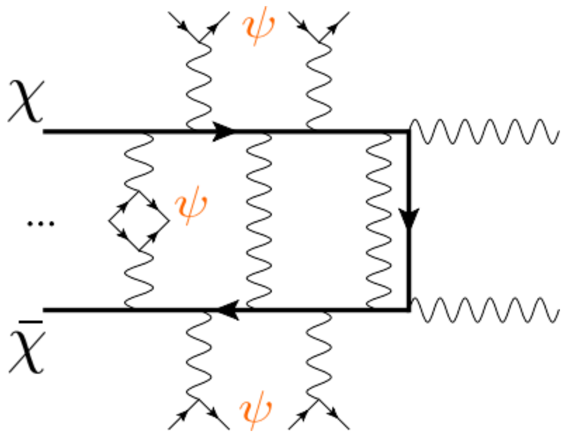
Overview

Toy model:

$$\mathcal{L} \supset g\bar{\chi}\gamma^\mu\chi V_\mu + g\bar{\psi}\gamma^\mu\psi V_\mu$$

Overview

<u>Strong thermal effects:</u>
Screening, energy shift,
large thermal width.



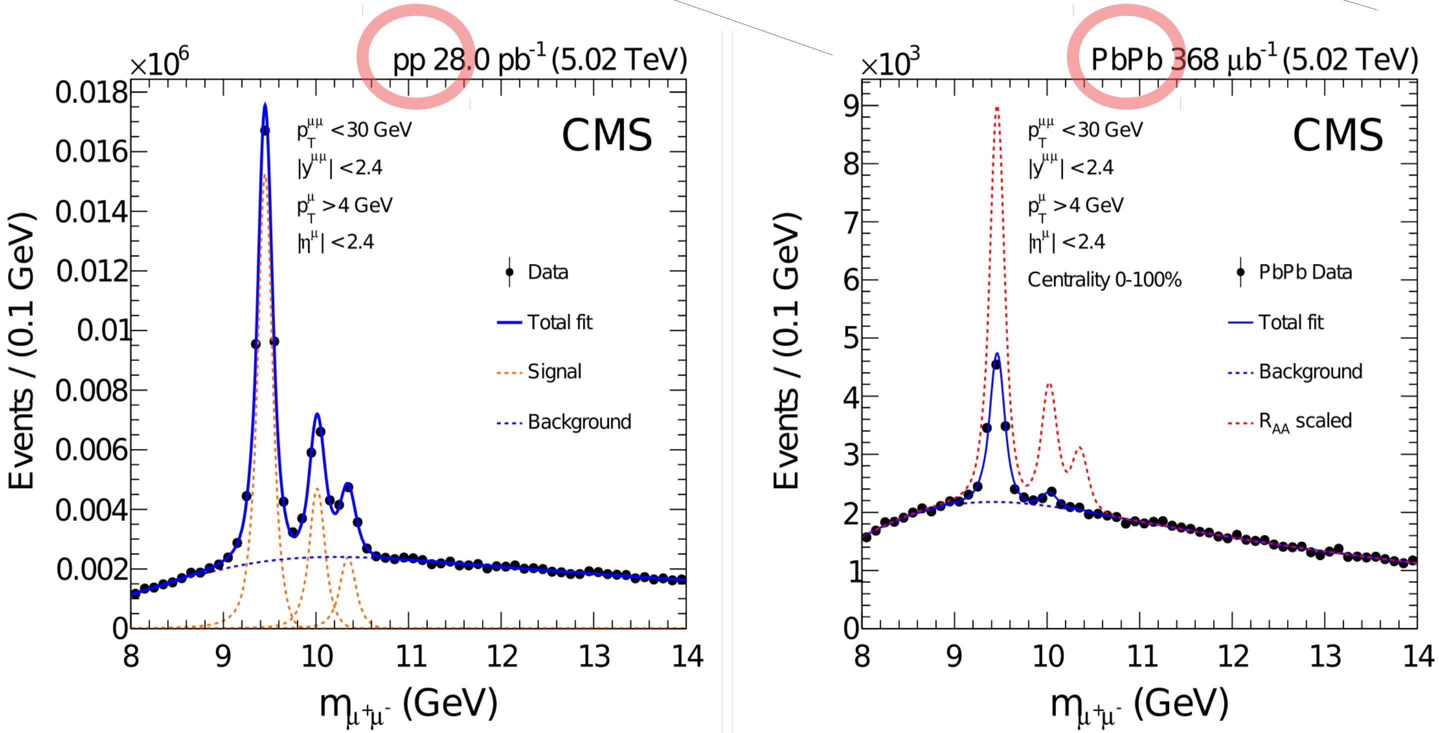
Toy model:

$$\mathcal{L} \supset g\bar{\chi}\gamma^\mu\chi V_\mu + g\bar{\psi}\gamma^\mu\psi V_\mu$$

Overview

Strong thermal effects:
Screening, energy shift,
large thermal width.

$1/\alpha$ $\quad$ $1/\alpha^2$ $\quad$ m_χ/T



CMS collaboration,
Phys.Lett. **B790** (2019) 270-293

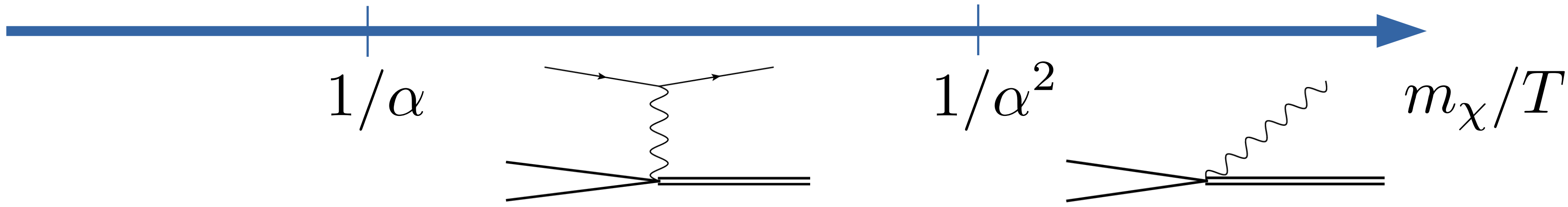
Sequential melting of bound states inside plasma environment observed.

Overview

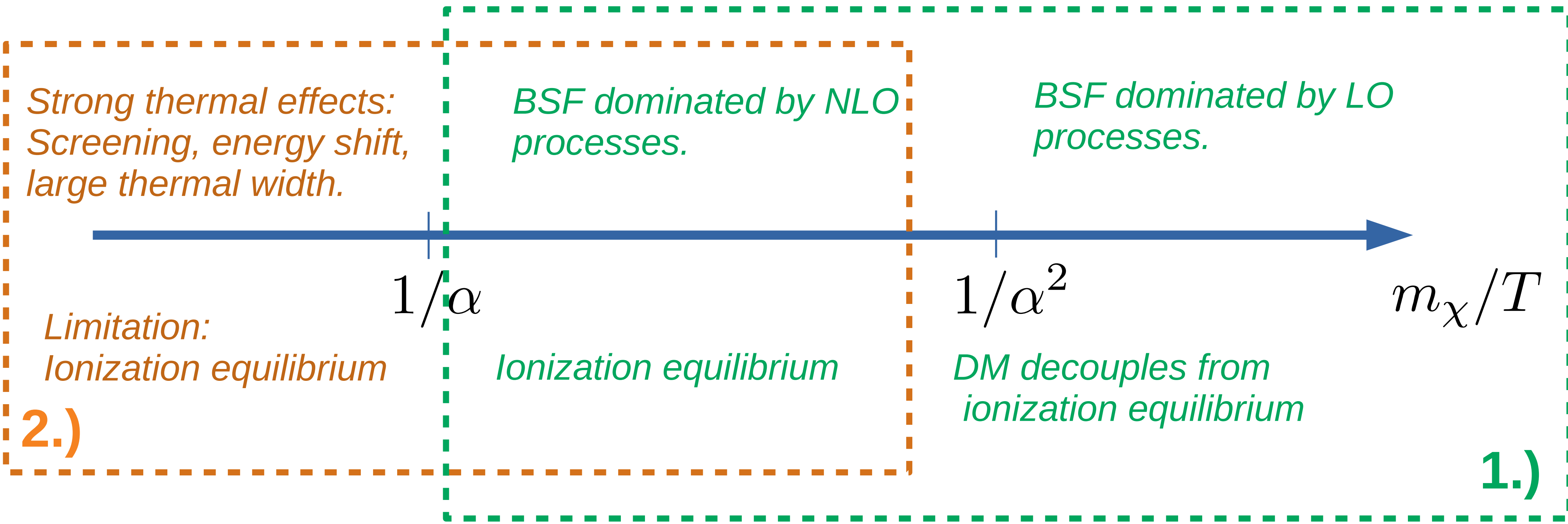
Strong thermal effects: Screening, energy shift, large thermal width.

BSF dominated by NLO processes.

BSF dominated by LO processes.



Overview



Two complementary formalism:

High and intermediate temperature regime

Dark Matter Sommerfeld-enhanced annihilation and bound-state decay at finite temperature

*Physical Review D **98**, 115023 (2018)*

TB, L. Covi, K. Mukaida

Intermediate and low temperature regime

Rapid bound-state formation of Dark Matter in the Early Universe

arXiv:1910.11288 (submitted to PRL)

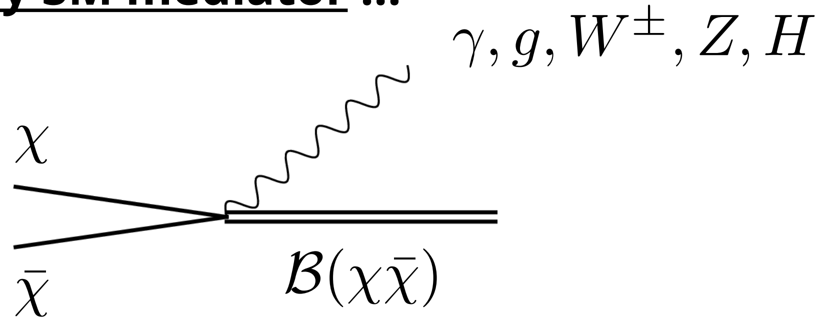
TB, K. Mukaida, K. Petraki

(and also TB, B. Blobel, J. Harz, K. Mukaida soon!)

Rapid bound-state formation of Dark Matter in the Early Universe

Bound-state formation at leading order

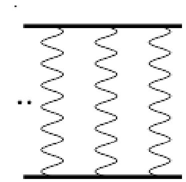
For <u>every SM mediator</u> …



…, possible DM models have been found where QM effects are relevant to include:

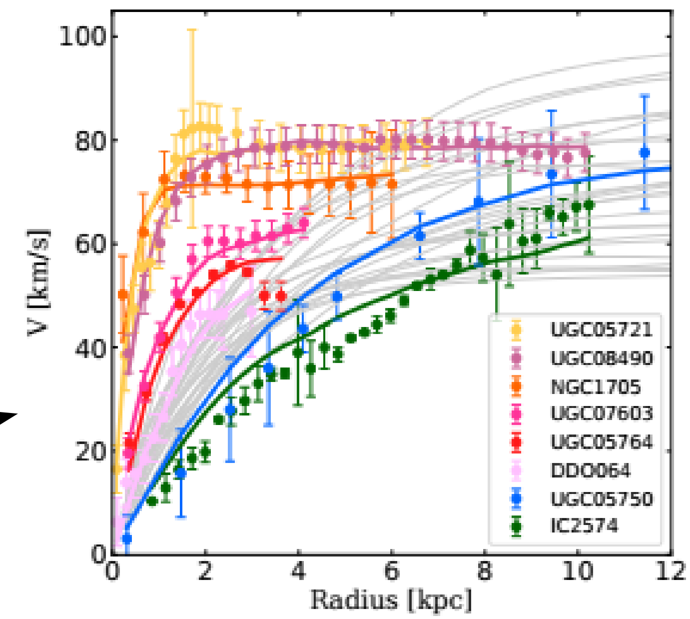
- Minimal DM (includes Wino)
  …, [Cirelli *et al.* '07], [Mitridate *et al.* '17]
- Co-annihilation with color-charged particles
  [J. Ellis *et al.* '16], [Kim&Laine '17],….
  , [Harz&Petraki '18], [S. Biodini *et al.* '19,'19,'19'],…
- Higgs mediated bound states
  [Harz&Petraki '18], [S. Biodini '18],…

Or <u>bottom-up motivated</u> scenarios with <u>exotic mediators</u>:



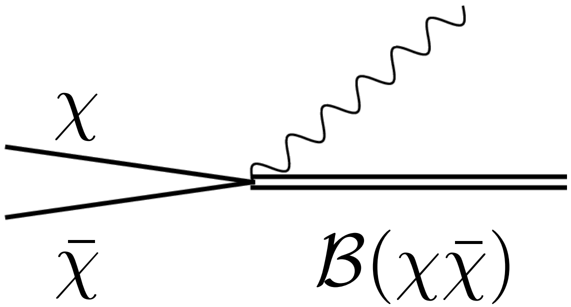
- **S**elf-**I**nteracting **DM** with light mediators
  [J. L. Feng *et al.* '10], [von Harling&Petraki '14], … [many]

SIDM solves Diversity problem:
[Kamada *et al.* '16],…, [Kaplinghat *et al.* '19]

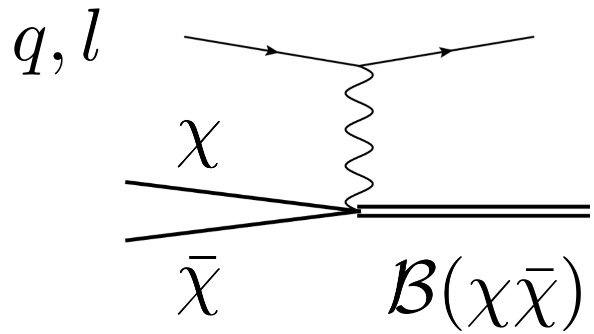


BSF: On-shell or off-shell mediator?

(previous literature)



vs.



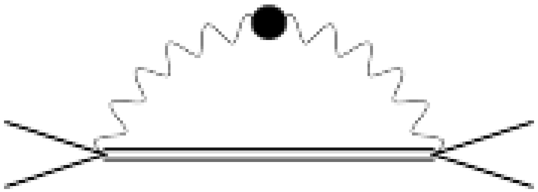
Which process dominates in the Early Universe?

- On-shell emission resembles situation of SM neutral hydrogen recombination in matt. dom. era.
- From heavy quarkionia in quark-gluon plasma, we know that the dominant dissociation process is via parton scattering for temperature larger than the binding energy.
- Detailed balance argument?!

How can we systematically compute higher order BSF processes?

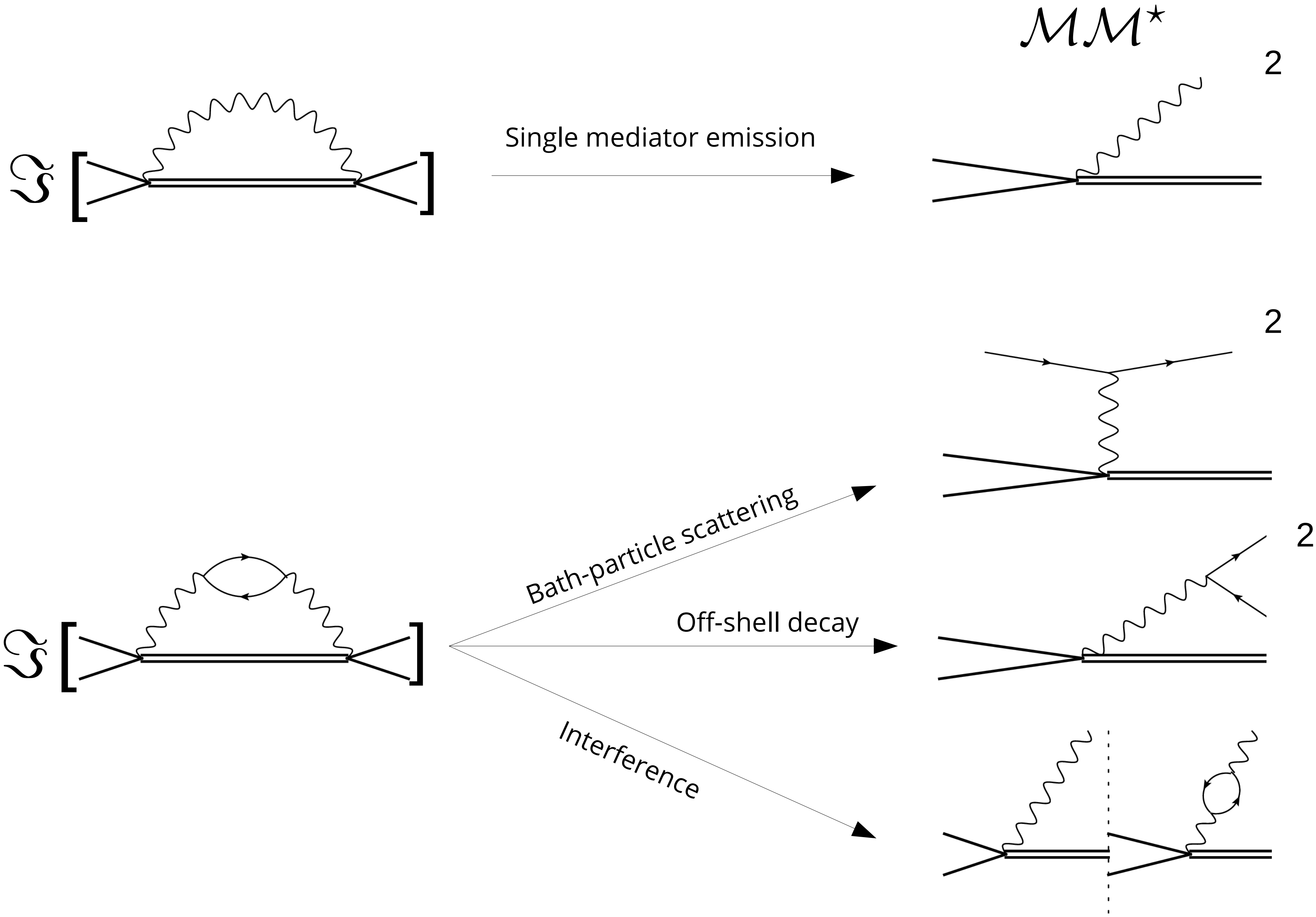
- Boltzmann approach fails for light or massless mediators due to infrared singularities.
- Requires development of a thermal field theory approach.

Generalized bound-state formation cross section



$$\sigma^{\mathrm{BSF}}_{nlm} v_{\mathrm{rel}} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} D^{-+}_{(\mu\nu)}(\Delta E, \mathbf{p}) \mathcal{T}^{(\mu)}_{\mathbf{k},nlm}(\Delta E, \mathbf{p}) \mathcal{T}^{(\nu)\star}_{\mathbf{k},nlm}(\Delta E, \mathbf{p}).$$

Leading and next-to-leading order

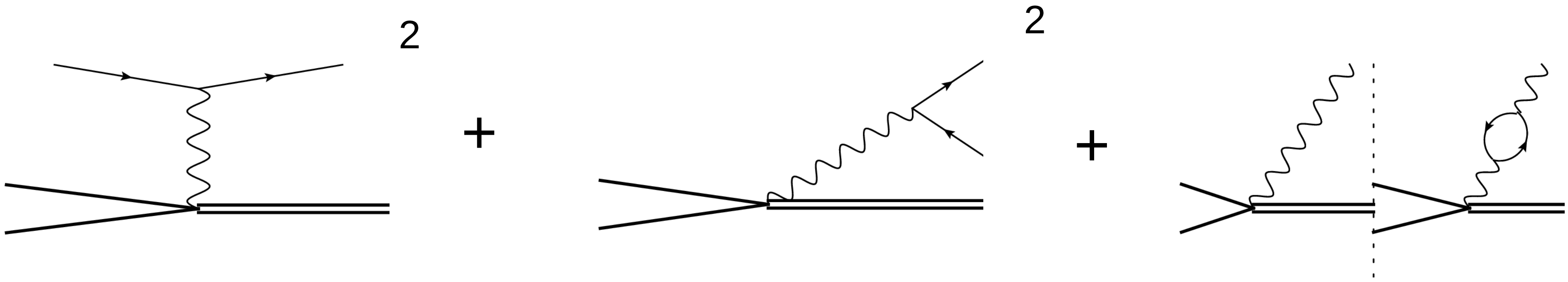


NLO contributions

UV finite,
collinear divergent.

UV finite,
collinear divergent.

Vacuum part UV divergent,
collinear divergent.



= Finite in collinear direction, and UV finite after vacuum renormalization.

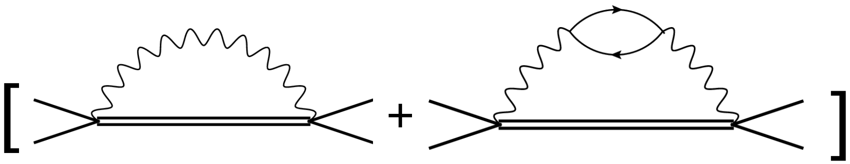
- Provide mathematical proof for cancellation of collinear divergences.
- Holds even for arbitrary phase-space distribution of bath particles, i.e. bath particles do not have to be in thermal equilibrium in order to guarantee finiteness in the forward scattering direction.
- (Bloch-Nordsieck theorem does not help here)

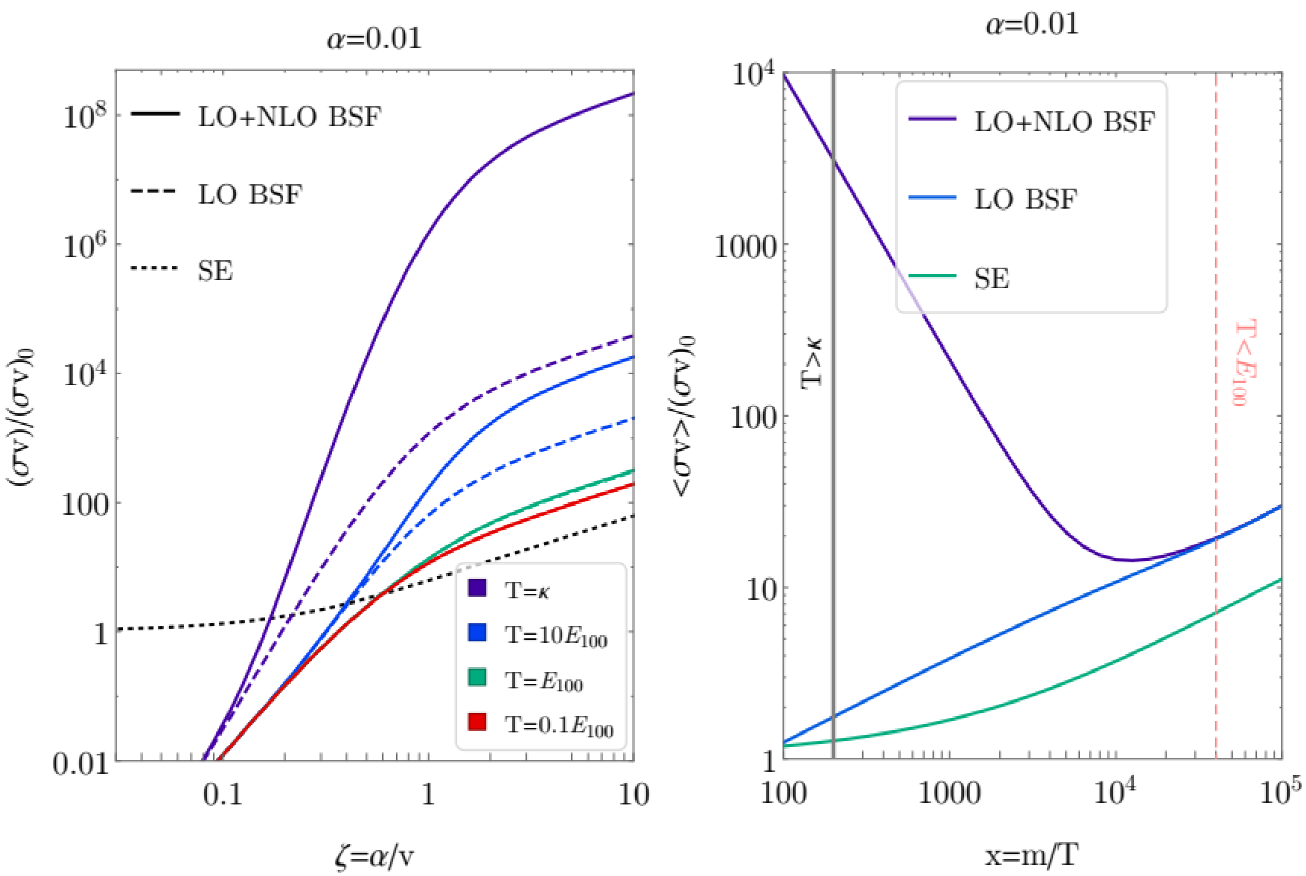
arxiv:**1912.**(in prep.)

Bound-state formation at NLO: massless case

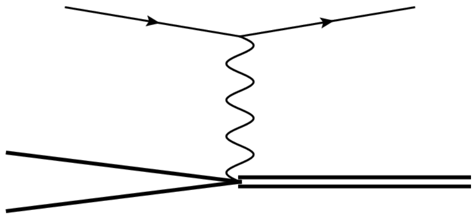
- Interference terms **cancel collinear divergences,** resulting in a finite cross section.
- At high temperature BSF via bath-particle scattering **dominates** over single mediator emission.
- Variation of renormalization scale between DM mass and binding energy doesnt affect plot visually, hence Log-contributions are under control.

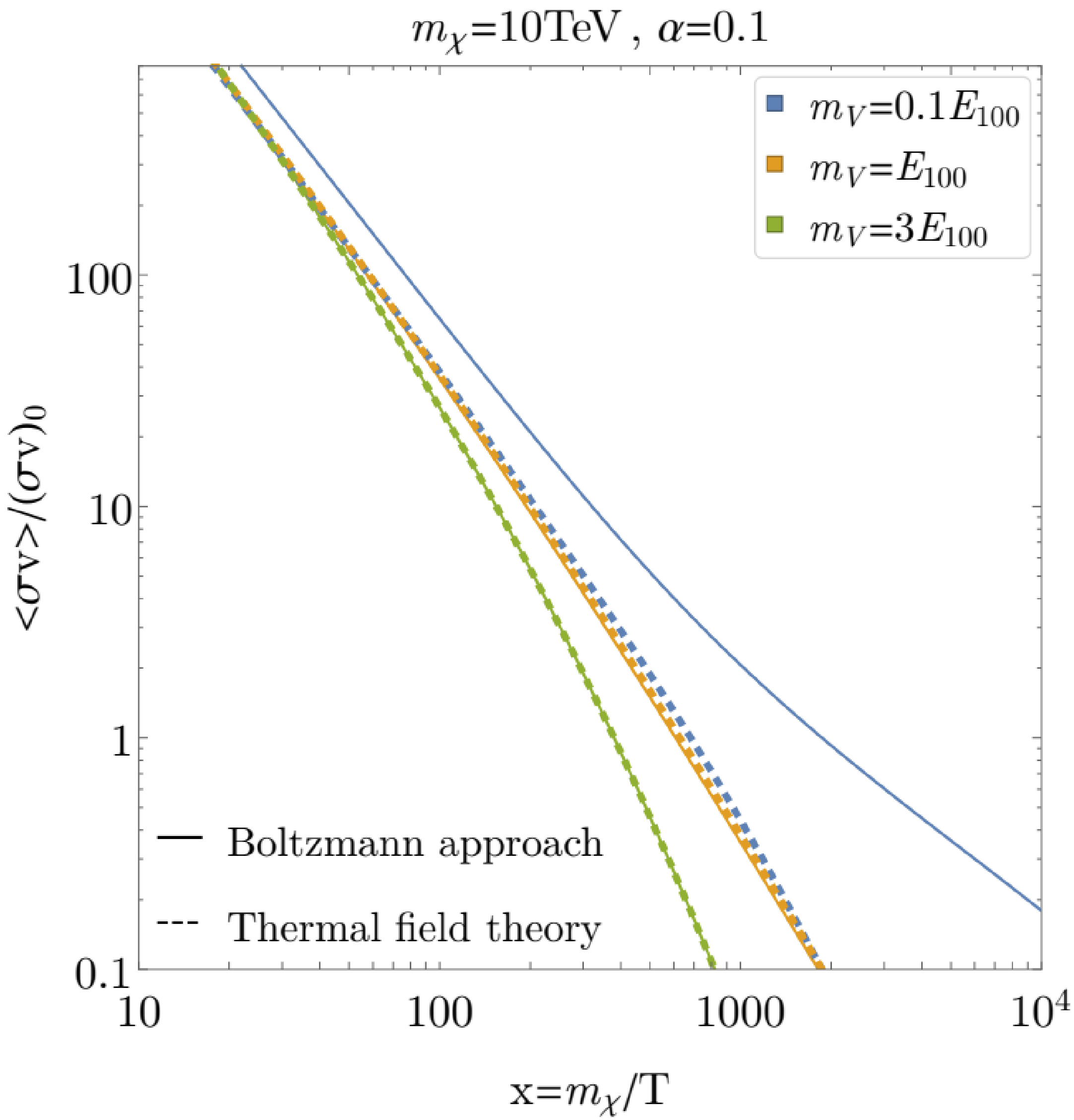
$$(\sigma v)^{\rm NLO}_{\rm BSF} \equiv \Im\Big[$$





BSF via bath-particle scattering: massive case





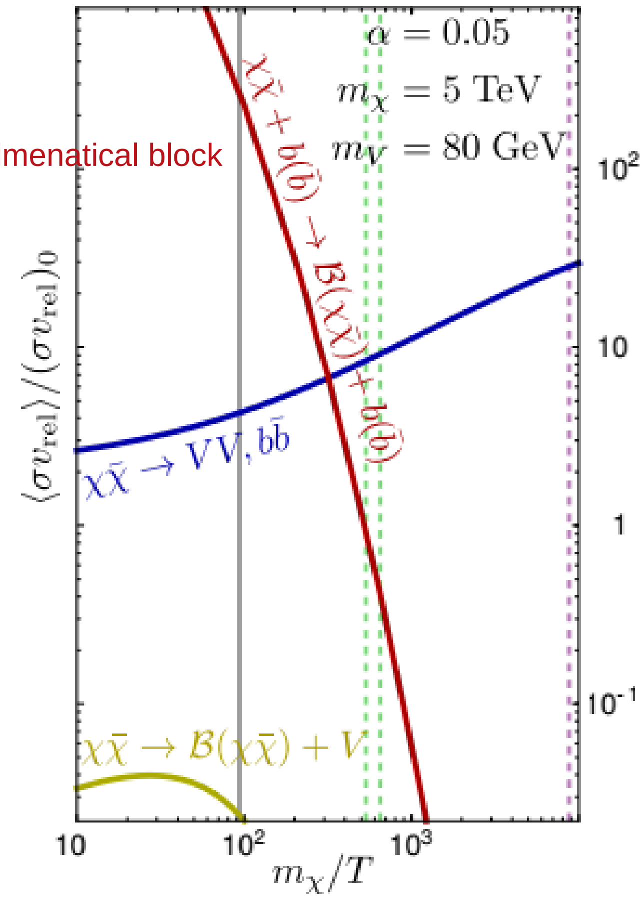
- Interference terms negligible for mediator masses much larger than binding energy. Boltzmann computation ok.

- **Thermal field theory approach required** for mediator masses smaller than or comparable to the binding energy.

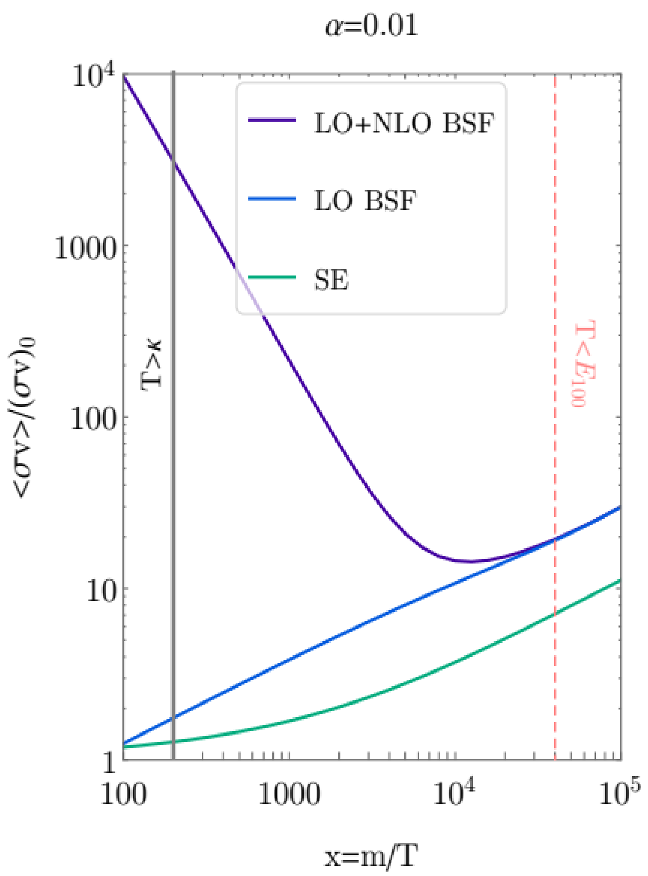
BSF via bath-particle scattering: massive case

Parametrically resembles Wino:

no kimenatical block



Ionization equilibrium



Approx. number density eq. **[von Harling&Petraki '14]**:

$$\dot{n}_s + 3Hn_s = -\left[\langle\sigma v\rangle_{\rm an} + \frac{\Gamma_1 \langle\sigma v\rangle_{\rm BSF}}{\Gamma_1 + \Gamma_{1\to s}}\right]\left(n_s^2 - n_s^{\rm eq} n_s^{\rm eq}\right)$$

$$\Gamma_{1\to s} = \langle\sigma v\rangle_{\rm BSF} \frac{n_s^{\rm eq} n_s^{\rm eq}}{n_1^{\rm eq}}$$

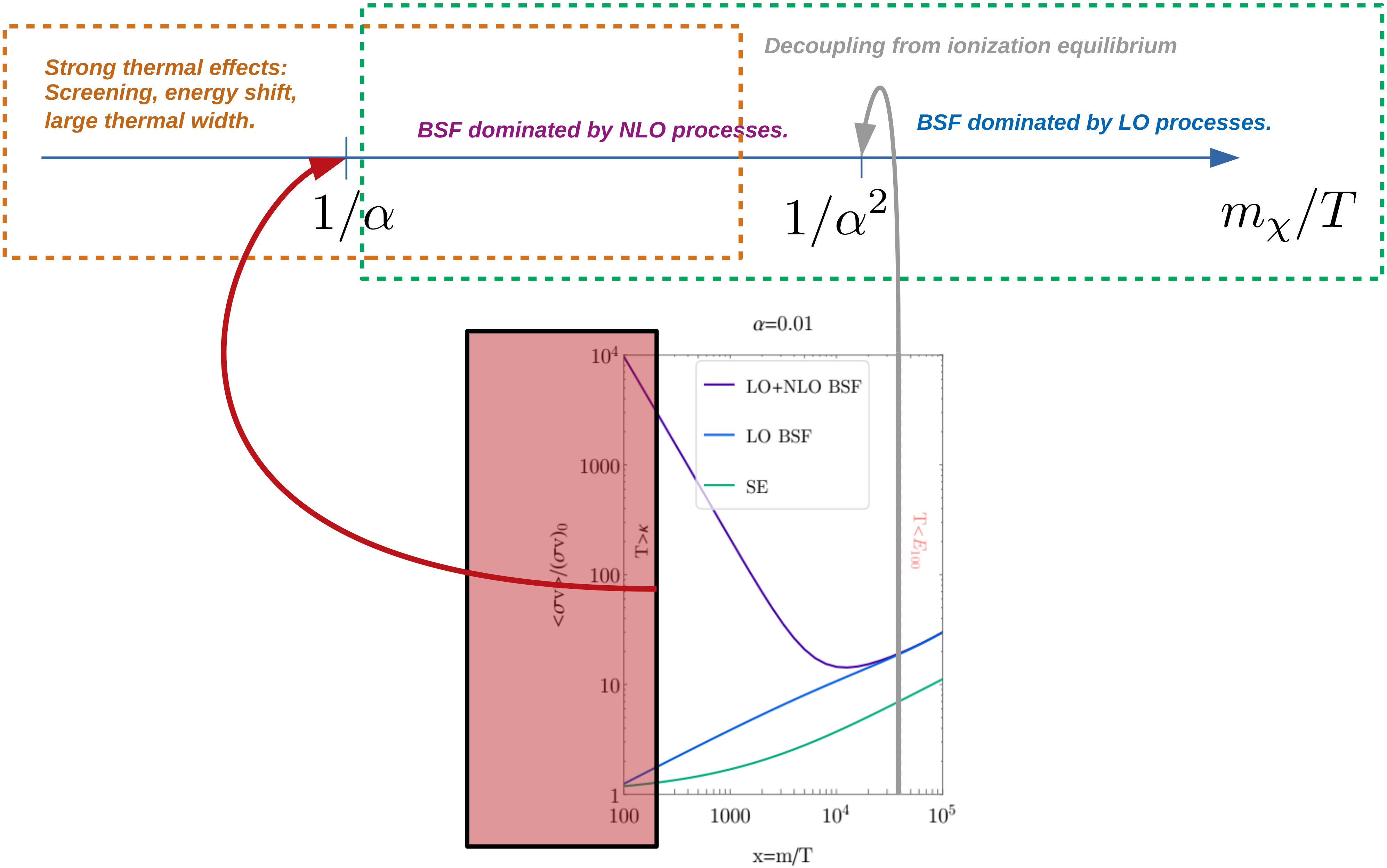
$$\Gamma_1 \ll \Gamma_{1\to s}$$

(Saha-) Ionization equilibrium

$$\dot{n}_s + 3Hn_s = -\left[\langle\sigma v\rangle_{\rm an} + \Gamma_1 \frac{n_1^{\rm eq}}{n_s^{\rm eq} n_s^{\rm eq}}\right]\left(n_s^2 - n_s^{\rm eq} n_s^{\rm eq}\right)$$

Strongly enhanced BSF via bath-particle scattering leads to **ionization equilibrium,** where i) collision term is **independent of BSF cross section and DISS rate**, and ii) **effective depletion cross section takes maximum value** for fixed T.

Overview


Strong thermal effects:
Screening, energy shift,
large thermal width.
BSF dominated by NLO processes.
Decoupling from ionization equilibrium
BSF dominated by LO processes.
$1/\alpha$
$1/\alpha^2$
m_χ/T
α=0.01
LO+NLO BSF
LO BSF
SE
10^4
1000
100
10
1
$<\sigma v>/(\sigma v)_0$
T>κ
$T<E_{100}$
100
1000
10^4
10^5
x=m/T

Dark Matter Sommerfeld-enhanced annihilation and bound-state decay at finite temperature

Generalized number density equation

$$\dot{n} + 3Hn = -2(\sigma v)_0 G_4^{++--}\Big|_{\text{eq.}} \left[\left(\alpha n / n_\chi^{\text{eq}}\right)^2 - 1 \right]$$

Consistency check

$V_{\text{eff}}(r,T) = 0$ →

$$\dot{n} + 3Hn = -(\sigma v)_0 \left[n^2 - n_{\text{eq}}^2 \right]$$

Lee-Weinberg equation ✓

$\lim_{T \to 0} V_{\text{eff}}(r,T)$ →

$$\dot{n} + 3Hn = -\left(\langle (\sigma v)_0 S \rangle + \sum_i \Gamma_i R_i \right) \left[(\alpha n)^2 - (n_s^{\text{eq}})^2 \right]$$

BEs in ionization equilibrium ✓

full $V_{\text{eff}}(r,T)$, $n \sim n_{\text{eq}}$ →

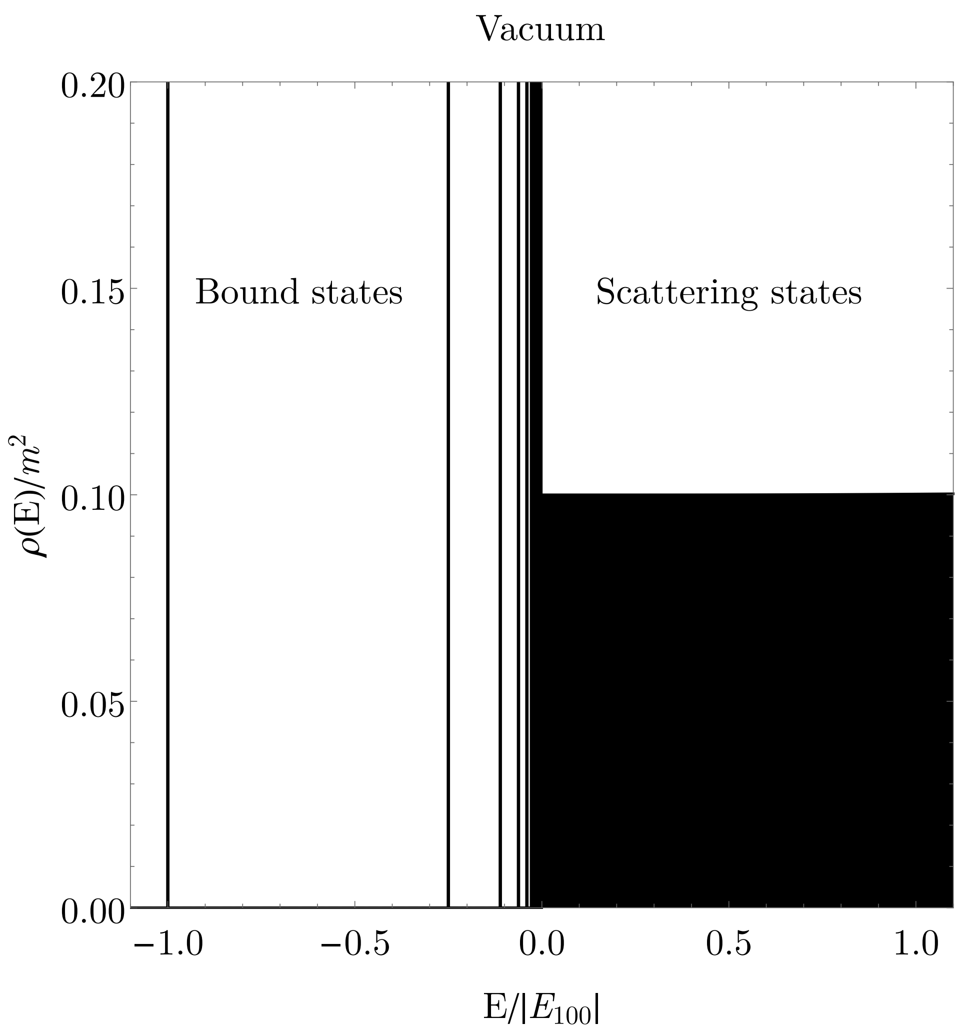
$$\dot{n} + 3Hn = -\Gamma_{\text{chem}} \left[n - n_{\text{eq}} \right]$$

consistent with Langevin approach

in linear regime close to chem. equil. ✓

Spectral function in vacuum

$$G_4^{++--}(x,x,x,x) = e^{-2\beta M} \int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} \rho(\mathbf{0},\mathbf{0};E).$$



$$\left[\frac{\nabla_{\mathbf{r}}^2}{M} + E + i\epsilon - V_{\text{eff}}(\mathbf{r},T)\right] G^R(\mathbf{r},\mathbf{r}';E) = 2i\delta(\mathbf{r}-\mathbf{r}')$$

$$\dot{n} + 3Hn = -(\sigma v)_0 \left[n^2 - n_{\text{eq}}^2\right]$$

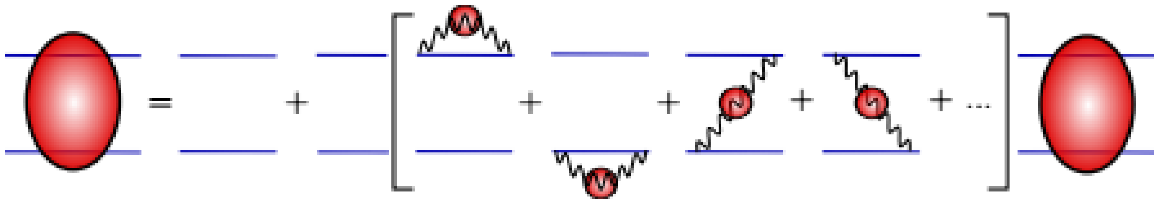
Lee-Weinberg equation ✓

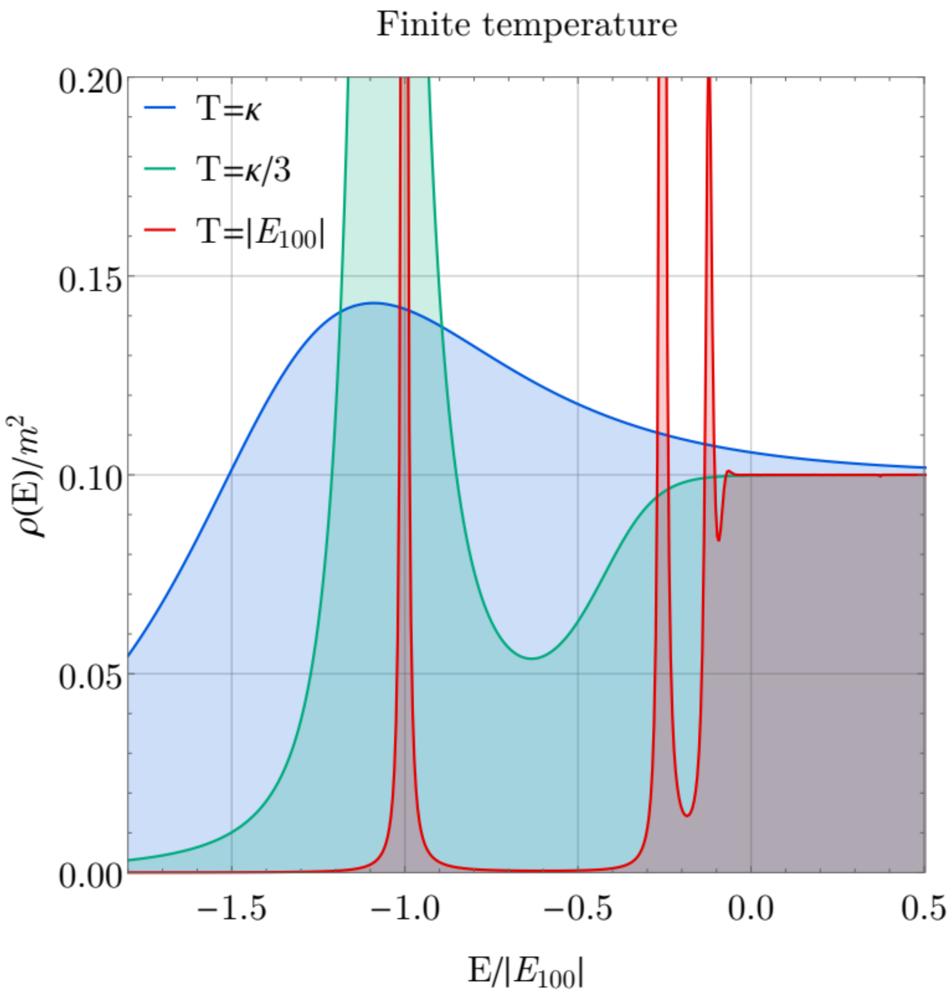
$$\dot{n} + 3Hn = -\left(\langle(\sigma v)_0 S\rangle + \sum_i \Gamma_i R_i\right)\left[(\alpha n)^2 - (n_s^{\text{eq}})^2\right]$$

BEs in ionization equilibrium ✓

Spectral function at finite temperature

$$G_4^{++--}(x,x,x,x) = e^{-2\beta M}\int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E}\rho(\mathbf{0},\mathbf{0};E).$$



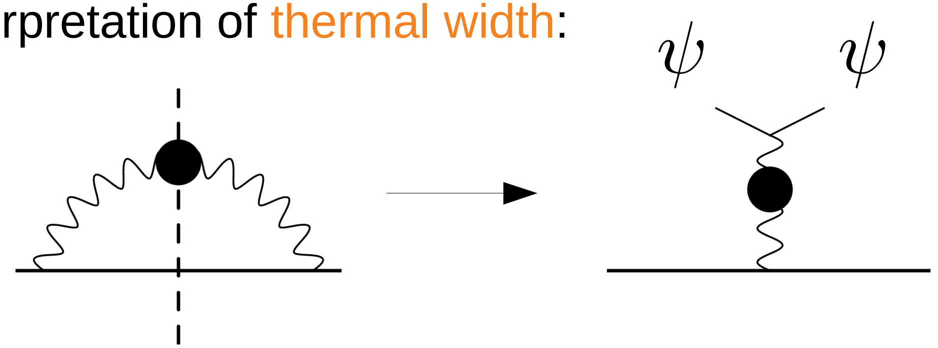


$$\left[\frac{\nabla_{\mathbf{r}}^2}{M} + E + i\epsilon - V_{\mathrm{eff}}(\mathbf{r},T)\right] G^R(\mathbf{r},\mathbf{r}';E) = 2i\delta(\mathbf{r}-\mathbf{r}')$$

$$V_{\mathrm{eff}}(\mathbf{r},T) = -\alpha_\chi m_D - \frac{\alpha_\chi}{r}e^{-m_D r} - i\alpha_\chi T\phi(m_D r)$$

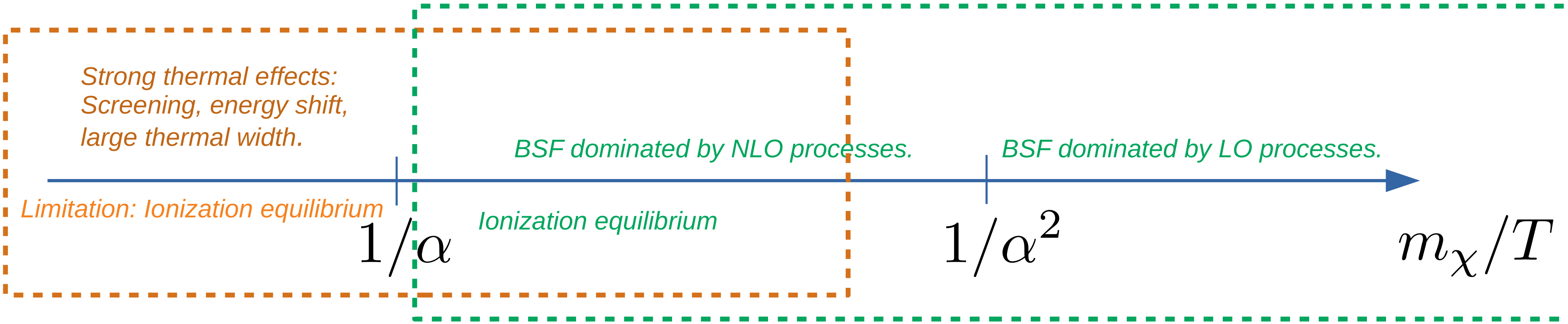
consistent with **[M. Laine *et al.* '07]**

Interpretation of thermal width:

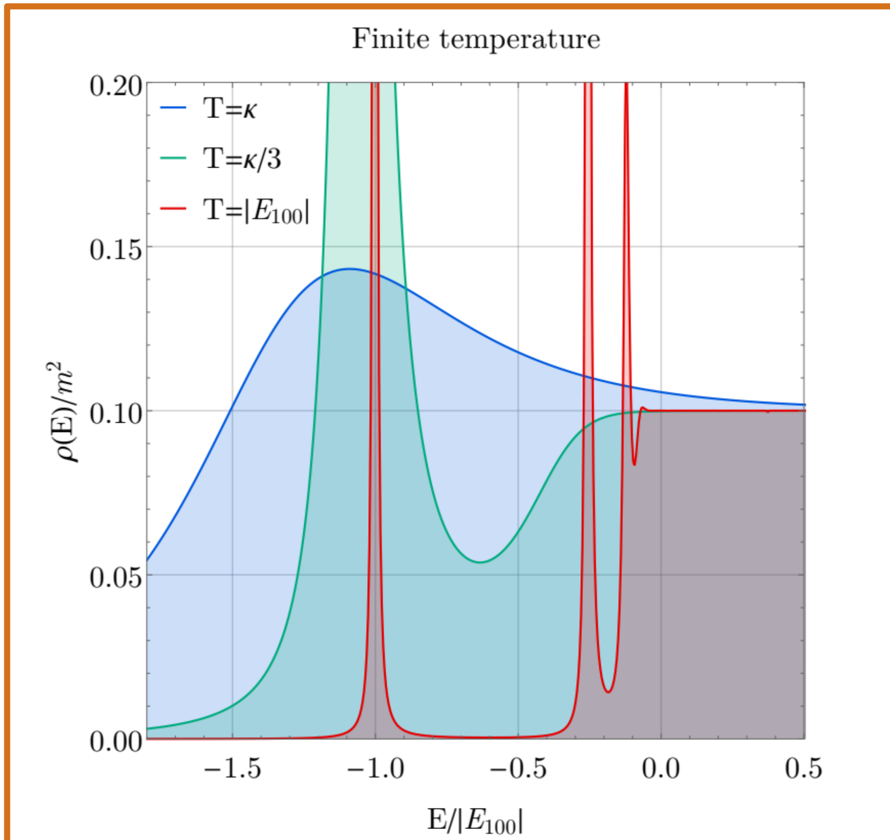


Soft scattering mixes bound and scattering states

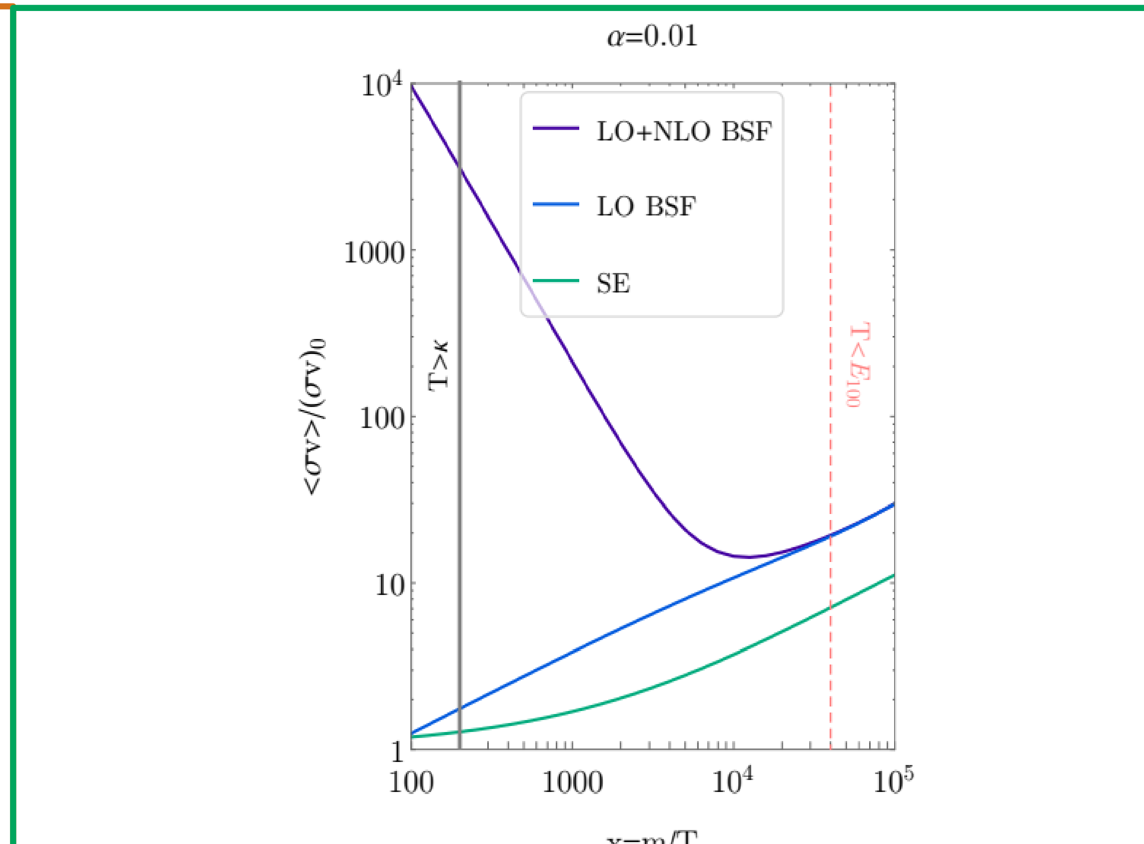
Summary



Strong thermal effects:
Screening, energy shift,
large thermal width.
Limitation: Ionization equilibrium
BSF dominated by NLO processes.
Ionization equilibrium
BSF dominated by LO processes.
$1/\alpha$
$1/\alpha^2$
m_χ/T



Finite temperature
$T=\kappa$
$T=\kappa/3$
$T=|E_{100}|$
$\rho(E)/m^2$
0.20
0.15
0.10
0.05
0.00
−1.5
−1.0
−0.5
0.0
0.5
$E/|E_{100}|$



α=0.01
LO+NLO BSF
LO BSF
SE
$T>\kappa$
$T<E_{100}$
$<\sigma v>/(\sigma v)_0$
10^4
1000
100
10
1
100
1000
10^4
10^5
x=m/T

Conclusion

Formal achievements:

- Non-equilibrium QFT analysis shows that **collinear divergences cancel**, even for arbitrary phase-space distributions of bath particles.
- We achieved more **complete description** of the DM freeze-out, ranging from **melting effects** of bound states at high T down to far below the **decoupling from ionization equilibrium.**

Phenomenological results and their implications:

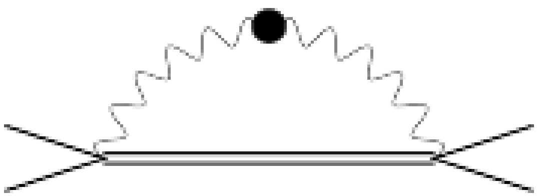
- **Previous literature** considered BSF via **on-shell mediator emission** only.
- For temperature larger binding energy, we find that the **dominant BSF channel** is **via bath-particle scattering**.
- Statement expected to be true also for non-abelian gauge or Yukawa theories.
- **Consequently, DM mass could be heavier than previously expected.**

(Eventually informs indirect searches and construction of future colliders)

Ready to (re-)analyse multi-TeV scale thermal relics!

Thank you!

Generalized bound-state formation cross section



$$\sigma^{\rm BSF}_{nlm} v_{\rm rel} = \int \frac{{\rm d}^3 p}{(2\pi)^3} D^{-+}_{\mu\nu}(\Delta E, \mathbf{p}) \sum_{\rm spins} \mathcal{T}^{\mu}_{\mathbf{k},nlm}(\Delta E, \mathbf{p}) \mathcal{T}^{\nu\star}_{\mathbf{k},nlm}(\Delta E, \mathbf{p}).$$

arxiv:**1911.**(in prep.)

$$\mathcal{L} \supset g\bar{\chi}\gamma^{\mu}\chi V_{\mu} + g\bar{\psi}\gamma^{\mu}\psi V_{\mu}$$

arxiv:**1910.11288**

Interacting two-point correlation fct.:

$$D^{-+}_{\mu\nu}(x,y) \equiv \langle V_{\mu}(x) V_{\nu}(y) \rangle$$

$$\langle ... \rangle = {\rm Tr}[e^{-H_{\rm env}/T} ...]$$

Kubo-Martin-Schwinger relation:

$$D^{-+}_{\mu\nu}(\Delta E, \mathbf{p}) = [1 + f^{\rm eq}_V(\Delta E)]\, D^{\rho}_{\mu\nu}(\Delta E, \mathbf{p})$$

$$D^{\rho}_{\mu\nu} = 2\Im \left[i D^{R}_{\mu\nu} \right]$$

$$D^{R}_{\mu\nu} = D^{R,0}_{\mu\nu} + D^{R,0}_{\mu\alpha} \Pi^{\alpha\beta}_{R} D^{R,0}_{\beta\nu} + \ldots$$

S-B transition matrix elements:

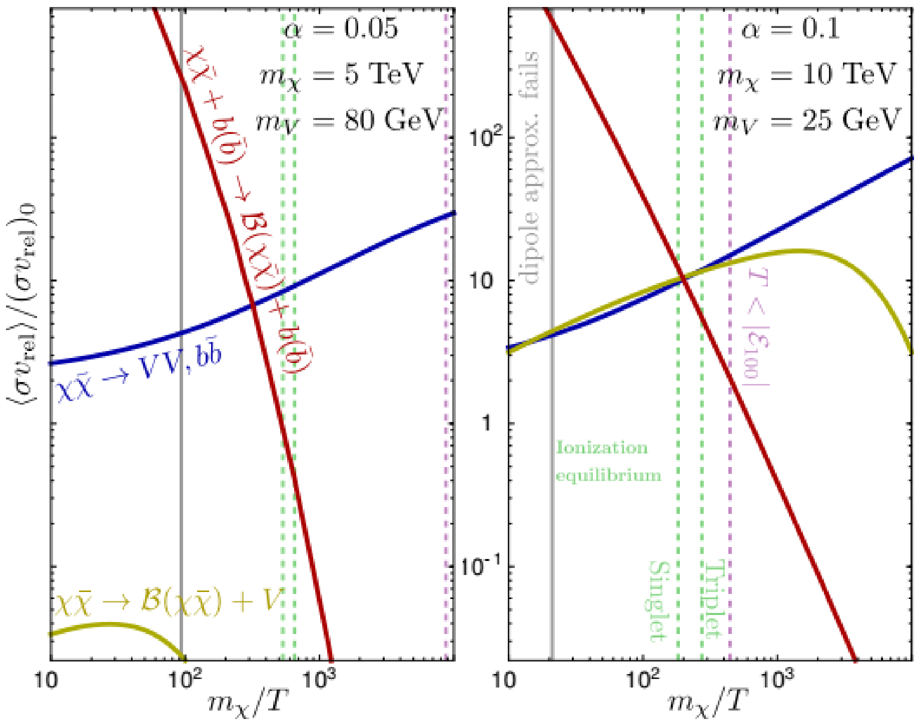
$$\mathcal{T}^{\mu}_{\mathbf{k},nlm}(P) \equiv (g_{\chi} g_{\bar{\chi}} 4 m^2_{\chi} 2M)^{-1/2} \mathcal{M}^{\mu}_{\mathbf{k},nlm}\Big|^{\rm NR}_{\rm dip}$$

$$\delta^4 \mathcal{M}^{\mu}_{\mathbf{k},nlm} = \int {\rm d}^4 x \; e^{iPx} \left\langle \mathcal{B}_{nlm} \right| g\bar{\chi}(x)\gamma^{\mu}\chi(x) \left| \mathcal{S}_{\mathbf{k}} \right\rangle$$

Well developed, see, e.g., Kallias works.

Implications of strongly enhanced BSF

arxiv:**1910.11288**



Approx. number density eq. **[von Harling&Petraki '14]**:

$$\dot{n}_s + 3Hn_s = -\left[\langle \sigma v \rangle_{\rm an} + \frac{\Gamma_1 \langle \sigma v \rangle_{\rm BSF}}{\Gamma_1 + \Gamma_{1\to s}}\right]\left(n_s^2 - n_s^{\rm eq} n_s^{\rm eq}\right)$$

$$\Gamma_{1\to s} = \langle \sigma v \rangle_{\rm BSF} \frac{n_s^{\rm eq} n_s^{\rm eq}}{n_1^{\rm eq}}$$

$$\Gamma_1 \ll \Gamma_{1\to s}$$

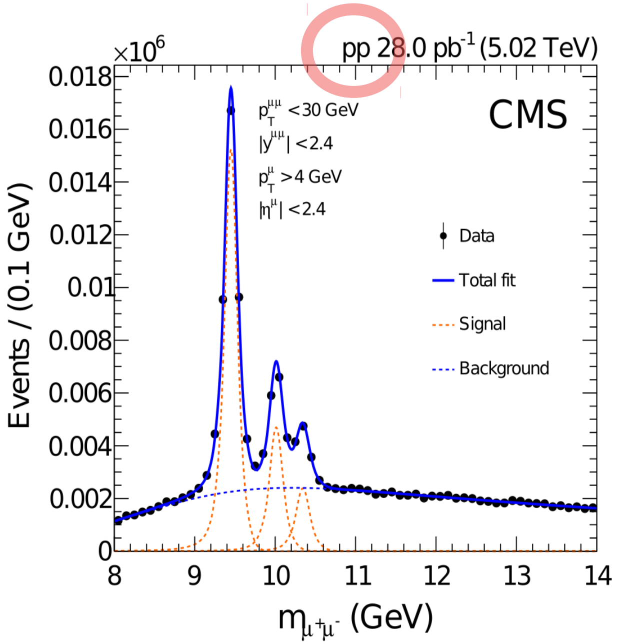
(Saha-) Ionization equilibrium [TB, Covi, Mukaida '18]:

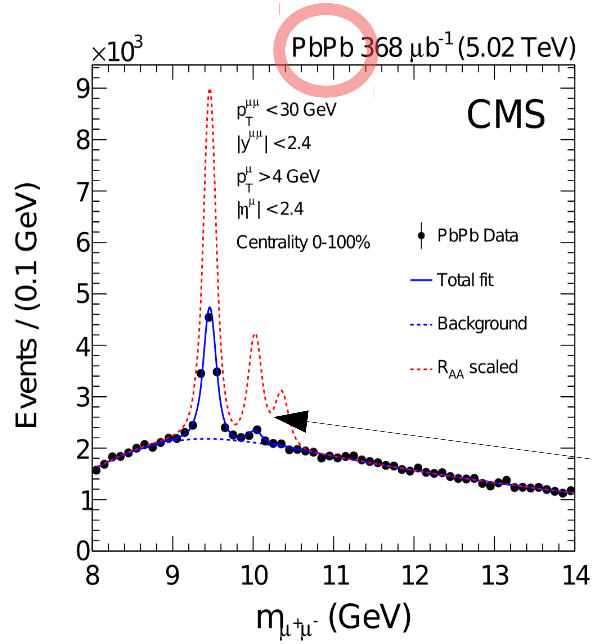
$$\dot{n}_s + 3Hn_s = -\left[\langle \sigma v \rangle_{\rm an} + \Gamma_1 \frac{n_1^{\rm eq}}{n_s^{\rm eq} n_s^{\rm eq}}\right]\left(n_s^2 - n_s^{\rm eq} n_s^{\rm eq}\right)$$

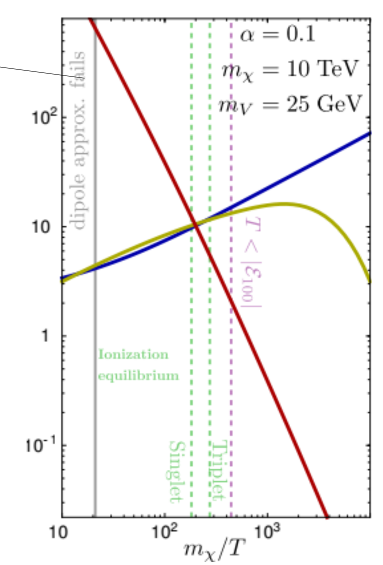
Strongly enhanced BSF via bath-particle scattering leads to **ionization equilibrium,** where i) collision term is **independent of BSF cross section and DISS rate**, and ii) **effective depletion cross section takes maximum value** for fixed T.

Limitation

CMS collaboration, Phys.Lett. **B790** (2019) 270-293

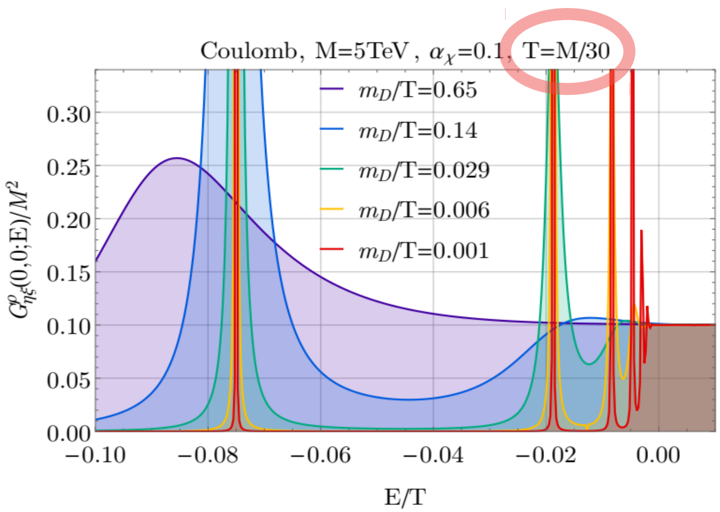


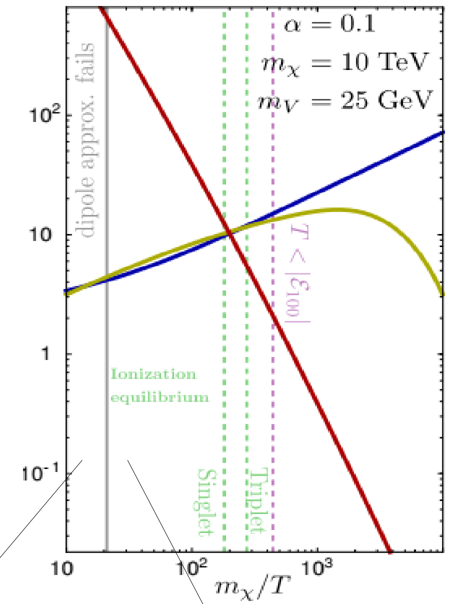




Melting of bound states inside plasma environment observed.

Complete picture





[TB, Covi, Mukaida '18]

$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\rm eq}\left[e^{\beta 2\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}\big|_{\rm eq} = e^{-2\beta M}\int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta\mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} G^\rho_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

arxiv:**1911.**(in prep.)

arxiv:**1910.11288**

$$\sigma^{\rm BSF}_{nlm} v_{\rm rel} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} D^{-+}_{\mu\nu}(\Delta E, \mathbf{p}) \sum_{\rm spins} \mathcal{T}^{\mu}_{\mathbf{k},nlm}(\Delta E,\mathbf{p}) \mathcal{T}^{\nu\star}_{\mathbf{k},nlm}(\Delta E,\mathbf{p}).$$

Backup

1 BS+R.h.s of BS equation vanishes

$$\dot{n}_s + 3Hn_s = -\left[\langle\sigma v\rangle_{\rm an} + \frac{\Gamma_1 \langle\sigma v\rangle_{\rm BSF}}{\Gamma_1 + \Gamma_{1\to s}}\right](n_s^2 - n_s^{\rm eq} n_s^{\rm eq})$$

Coupled BEs:

$$\dot{n}_s + 3Hn_s = -\langle\sigma v\rangle_{\rm an}\left[n_s^2 - (n_s^{\rm eq})^2\right] - \sum_i \langle\sigma^i v\rangle_{\rm BSF}\left[n_s^2 - n_i (n_s^{\rm eq})^2 / n_i^{\rm eq}\right],$$

$$\dot{n}_i + 3Hn_i = -\Gamma_i\left[n_i - n_i^{\rm eq}\right] + \langle\sigma^i v\rangle_{\rm BSF}\left[n_s^2 - n_i (n_s^{\rm eq})^2 / n_i^{\rm eq}\right] - \sum_j \Gamma_{i\to j}\left[n_i - n_j n_i^{\rm eq}/n_j^{\rm eq}\right]$$

$\Gamma_1 \ll \Gamma_{1\to s}$

Radiative processes much faster than total number violating processes -> ionization equilibrium

[TB, Covi, Mukaida '18]

$$\dot{n} + 3Hn = -\left[\langle\sigma v\rangle_{\rm an} + \sum_i \Gamma_i \frac{n_i^{\rm eq}}{n_s^{\rm eq} n_s^{\rm eq}}\right](\alpha^2 n^2 - n_s^{\rm eq} n_s^{\rm eq})$$