Selection Rules and Operator Counting

from Amplitude Basis

presented by Ming-Lei Xiao

arXiv:1902.06752	with T. Ma and J. Shu
arXiv:2001.04481	with M. Jiang, J. Shu and Y.-H. Zheng
arXiv:2002.xxxxx	with H.-L. Li, Z. Ren, J. Shu J.-H. Yu and Y.-H. Zheng

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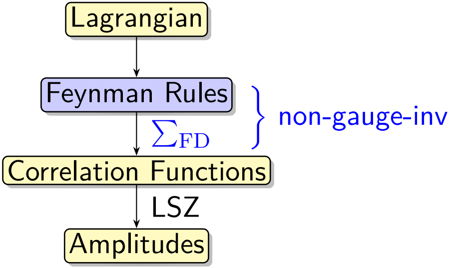
An Alternative Formalism of Scattering Amplitude

The physical observable, S-matrix, is Lorentz invariant, CPT invariant, gauge invariant and unitary.

$$\langle\alpha|S|\beta\rangle = \delta_{\alpha\beta} + i\mathcal{M}_{\alpha\beta}\cdot(2\pi)^4\delta^{(4)}(P_\alpha - P_\beta).$$



Traditional QFT

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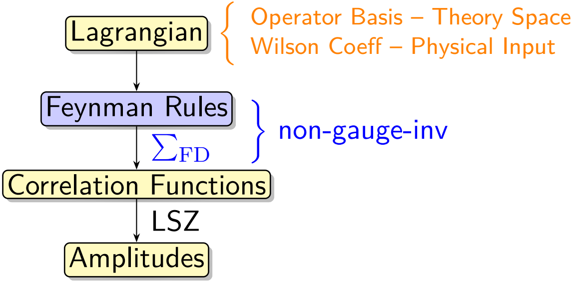

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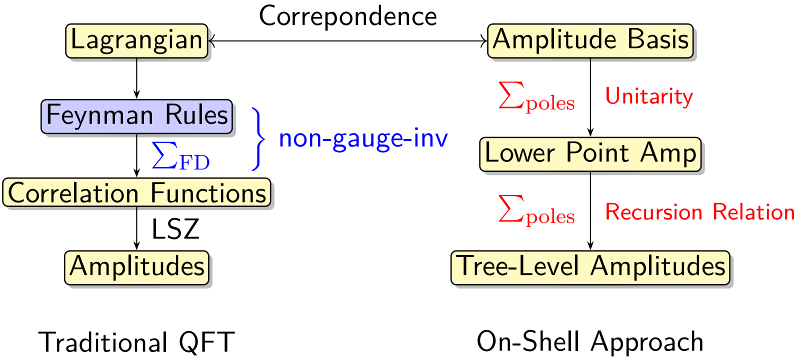

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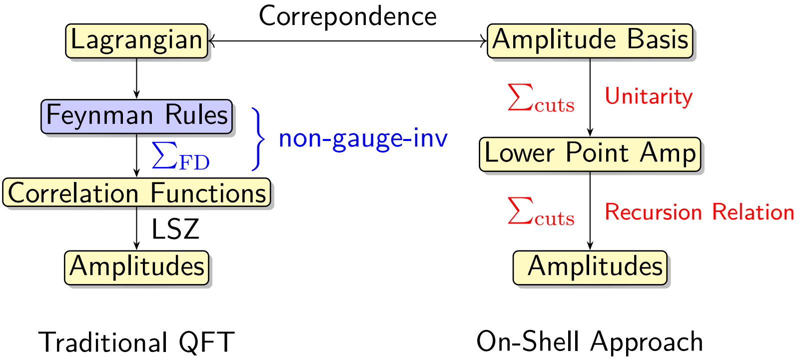

Outline

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Spinor-Helicity Variables

With Clifford algebra, a generic momentum p can be represented by a 2×2 matrix

$$p_{\alpha\dot{\alpha}} \equiv p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda^I_\alpha \tilde{\lambda}_{\dot{\alpha}I}, \quad I = 1, \ldots, \mathrm{rank}(p).$$

These spinor variables constitute a solution $\lambda_{\mathrm{D}} = (\lambda, \tilde{\lambda})^{\mathsf{T}}$ to the Dirac equation $(p_\mu \gamma^\mu - m)\lambda_{\mathrm{D}} = 0$.

- $p_\mu p^\mu = \det(p_{\alpha\dot{\alpha}}) = 0$ for on-shell massless particles:
  - Little group $U(1)$: $\lambda \to t^{-1}\lambda$, $\tilde{\lambda} \to t\tilde{\lambda}$.
  - $\mathcal{A}(\ldots, p_i, h_i, \ldots) \sim \lambda_i^{r_i} \tilde{\lambda}_i^{2h_i + r_i}$.
- $p_\mu p^\mu = \det(p_{\alpha\dot{\alpha}}) = m^2$ for on-shell massive particles:
  - Little group $SU(2)$: $\lambda_I \to (W^{-1})_I^J \lambda_J$, $\tilde{\lambda}^I \to W^I_J \tilde{\lambda}^J$.
  - $\mathcal{A}(\ldots, p_i, s_i, \sigma_i \ldots) \sim \lambda_i^{\{I_1} \cdots \lambda_i^{I_{2s_i}\}}$.
- Denote $\lambda_i^\alpha \lambda_{j\alpha} \equiv \langle ij \rangle$, $\tilde{\lambda}_{i\dot{\alpha}} \tilde{\lambda}_j^{\dot{\alpha}} \equiv [ij]$.

Unitarity and Recursion Relation

The unitarity of S-matrix indicates the following *factorization*:

$$2\mathrm{Im}\mathcal{M}_{\alpha\beta} = \int \mathrm{d}\gamma\, \mathcal{M}_{\alpha\gamma}\mathcal{M}^*_{\beta\gamma},$$

which fixes the analytic structures of $\mathcal{M}$.

- For $N_\gamma = 1$, it provides $\mathrm{Res}_{s=m_\gamma}\mathcal{M}$.
- For $N_\gamma > 1$, it provides $\mathrm{Disc}_{s_*=\sum m_\gamma}\mathcal{M}$.

To fix the rest of $\mathcal{M}$, the usual technique is to adopt complex momenta shift (*i.e.* BCFW arXiv:hep-th/0501052), and requiring the vanishing contribution from the large shift limit.

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On-shell Constructibility: A systematic way of momenta shift without the “big circle” contribution, so that full amplitudes can be constructed solely from the analytic structures.

Amplitude Basis

On-shell Constructibility: A systematic way of momenta shift without the "big circle" contribution, so that full amplitudes can be constructed solely from the analytic structures.

- Examples: Yang-Mills, NLσM, ...
  Counter-examples: Scalar QED, EFT, ...
- The actual logic behind this requirement is to use some *hidden symmetry* (like SUSY, soft theorem) to constrain the form of new ***amplitude basis*** so that the theory is still constructible from the old building blocks.

Amplitude Basis: An independent set of *unfactorizable* amplitudes that determines the theory via unitarity.

Outline

Examples

Among Lorentz invariant functions of N sets of spinor-helicity variables $(\lambda_i, \tilde{\lambda}_i)$, those that do not have poles or branch cuts in Mandelstam variables $s_{\mathcal{I}} = (\sum_{i\in\mathcal{I}} p_i)^2$ are *unfactorizable*, and constitute N-point amplitude basis.

- 3-pt massless amplitudes – special kinematics:
  all $\langle ij\rangle = 0$ or all $[ij] = 0$

$$\mathcal{B}(\psi\psi\phi) \sim \langle 12\rangle, \quad \mathcal{B}(FF\phi) \sim \langle 12\rangle^2,$$
$$\mathcal{B}(F\psi\psi) \sim \langle 12\rangle\langle 13\rangle, \quad \mathcal{B}(FFF) \sim \langle 12\rangle\langle 23\rangle\langle 31\rangle,$$

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Among Lorentz invariant functions of N sets of spinor-helicity variables $(\lambda_i, \tilde{\lambda}_i)$, those that do not have poles or branch cuts in Mandelstam variables $s_{\mathcal{I}} = (\sum_{i\in\mathcal{I}} p_i)^2$ are *unfactorizable*, and constitute N-point amplitude basis.

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  and *spurious poles* (all minimal gauge couplings)

$$\mathcal{B}(F\phi\phi) \sim \frac{\langle 12\rangle\langle 13\rangle}{\langle 23\rangle}, \quad \mathcal{B}(F\psi\bar{\psi}) \sim \frac{\langle 12\rangle^2}{\langle 23\rangle}, \quad \mathcal{B}(FF\bar{F}) \sim \frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}$$

- 4-pt and above massless amplitudes – polynomials of brackets.

Covariant Derivatives

- Except for the case of minimal gauge couplings, each gauge boson contributes two spinors in the amplitude $\lambda_{i\alpha}\lambda_{i\beta} = p_\mu \epsilon^-_\nu \sigma^{\mu\nu}_{\alpha\beta}$ which apparently comes from the field strength operator $F_{\mu\nu}$.
- Does amplitude basis capture the gauge boson from covariant derivatives?

Example: $O^\mu D_\mu \Psi$ where Ψ is charged for gauge boson γ:

$$\mathcal{A}(\mathcal{O}\Psi\gamma) = \langle\mathcal{O}|\mathcal{O}^\mu(-igA_\mu)\Psi|\Psi\gamma\rangle + \langle\mathcal{O}|(\mathcal{O}^\mu\partial_\mu\Psi)(J^\mu_\Psi A_\mu)|\Psi\gamma\rangle$$

The gauge invariant amplitude involves pole of the charged particle, whose residue factorizes due to unitarity

$$\mathcal{A}(\mathcal{O}\Psi\gamma) \to \langle\mathcal{O}|\mathcal{O}^\mu\partial_\mu\Psi|\Psi\rangle \times \langle\Psi|J^\mu_\Psi A_\mu|\Psi\gamma\rangle.$$

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Implication of Basis Correspondence

- Dimension of amplitude basis

$$\dim(\mathcal{O}) = N + \dim(\mathcal{B}).$$

- Basis amplitudes for certain helicity state $\{h_1, \ldots, h_N\}$ at dimension d are represented by a particular set of reduced Semi-Standard Young Tableau (arXiv:1902.06754).
  Ex. Type $\psi^2\phi^3D^2$:

$$\underset{\mathcal{O}_1}{\begin{array}{|c|c|c|}\hline 1&1&2\\\hline 2&3&4\\\hline 5\\\cline{1-1}\end{array}}\quad
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\underset{\mathcal{O}_5}{\begin{array}{|c|c|c|}\hline 1&1&3\\\hline 2&2&5\\\hline 4\\\cline{1-1}\end{array}}\quad
\underset{\mathcal{O}_6}{\begin{array}{|c|c|c|}\hline 1&1&4\\\hline 2&2&5\\\hline 3\\\cline{1-1}\end{array}}$$

- Any selection rule for amplitude basis induces a selection rule for the operator basis!

Outline

Poincaré CG coefficient

- Conventionally we use tensor rep. for multi-particle states in scattering $|\Psi\rangle_\otimes = |p_1 s_1 \sigma_1, \dots, p_N s_N \sigma_N\rangle$.
- We may also use irreducible rep. $|\Psi\rangle_j = |Pj\sigma, a\rangle$.
- Poincaré CG coefficients are the overlap functions

$$_\otimes\langle\Psi|\Psi\rangle_j \equiv C^{Pj\sigma,a}_{p_1 s_1 \sigma_1,\dots,p_N s_N \sigma_N}\delta^{(4)}(P-\sum_i p_i).$$

- Make replacement: $p_i s_i \sigma_i \to (\lambda_i, \tilde{\lambda}_i)$, $Pj\sigma \to (\chi, \tilde{\chi})$, the CG coefficients must take the form of basis amplitudes. Ex. For two-massless-particle state (arXiv:1709.04891)

$$C^{Pj\sigma}_{p_1 h_1; p_2 h_2} \sim [12]^{j+h_1+h_2}\left(\langle 1\chi\rangle^{j-h_1+h_2}\langle 2\chi\rangle^{j+h_1-h_2}\right)^{\{I_1 \dots I_{2j}\}}$$

Partial Wave Amplitude Basis

Partial wave expansion of $N \to M$ amplitudes:

$$\mathcal{A}(\{p_i, \sigma_i, n_i\}^N; \{p'_i, \sigma'_i, n'_i\}^M) \equiv {}_\otimes\langle \Psi_M | \mathcal{M} | \Psi_N \rangle_\otimes$$

$$= \sum_{|\Psi_N\rangle_j, |\Psi_M\rangle_j} {}_\otimes\langle \Psi_M | \Psi_M \rangle_j \times {}_j\langle \Psi_M | \mathcal{M} | \Psi_N \rangle_j \times {}_j\langle \Psi_N | \Psi_N \rangle_\otimes$$

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$$= \sum_{\substack{j,\sigma,a,\\ j',\sigma',b}} C^{Pj'\sigma',b}_{\{p'_i s'_i \sigma'_i\}^M} \times \mathcal{M}^j_{ab}(P^2)\delta_{jj'}\delta_{\sigma\sigma'} \times (C^{Pj\sigma,a}_{\{p_i s_i \sigma_i\}^N})^*$$

$$= \sum_{j,a,b} \mathcal{M}^j_{ab}(s) \sum_{\sigma} C^{Pj\sigma,b}_{\{p'_i s'_i \sigma'_i\}^M} (C^{Pj\sigma,a}_{\{p_i s_i \sigma_i\}^N})^*$$

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$$= \sum_{j,a,b} \mathcal{M}^j_{ab}(s)\mathcal{B}^{j,a\to b}_{\{s_i\}^N\to\{s'_i\}^M}.$$

- For $2 \to 2$ we showed: $\mathcal{B}^j_{h_1,h_2\to h_3,h_4} \sim d^j_{h_2-h_1,h_4-h_3}(\theta)$.

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- These are nothing but a special choice of amplitude basis

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- They correspond to the partial wave operator basis $\mathcal{O}^j_{ab}$ in the space of **same type operators.**

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- These are nothing but partial wave amplitude basis.
- They correspond to the partial wave operator basis $\mathcal{O}^j_{ab}$ in the space of **same type operators.**
- New Selection Rules from angular momentum conservation become obvious under this basis!

Angular Momentum: Bridge-Counting Method

By definition

$$\mathcal{B}^{j,a\to b}_{\{h_i\}^N\to\{h'_i\}^M} = \sum_{\sigma} C^{Pj\sigma,b}_{\{p'_ih'_i\}^M}(C^{Pj\sigma,a}_{\{p_ih_i\}^N})^*$$

$$= \left[(f^{j,b}_{h'_1,\ldots,h'_M})^{\{\beta_1,\ldots\beta_{2j}\}}\chi^{I_1}_{\beta_1}\cdots\chi^{I_{2j}}_{\beta_{2j}}\right]\left[(f^{j,a}_{-h_1,\ldots,-h_N})_{\{\alpha_1,\ldots\alpha_{2j}\}}\chi^{\alpha_1}_{I_1}\cdots\chi^{\alpha_{2j}}_{I_{2j}}\right]$$

$$=(-\sqrt{s})^{2j}(f^{j,b}_{h'_1,\ldots,h'_M})^{\{\alpha_1,\ldots\alpha_{2j}\}}(f^{j,a}_{-h_1,\ldots,-h_N})_{\{\alpha_1,\ldots\alpha_{2j}\}}$$

is characterized by $2j$ totally symmetrized $\langle\cdot\rangle$ contractions between initial and final states.

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Example $\mathcal{B}_\pm = \langle 13\rangle\langle 24\rangle \pm \langle 14\rangle\langle 23\rangle$:

- In $\{1,2\}\to\{3,4\}$ channel, $\mathcal{B}_+$ has 2 bridges and $\mathcal{B}_- = \langle 12\rangle\langle 34\rangle$ has 0 bridges.

$$\mathcal{B}^{j=1}_{\psi_1\psi_2\to\psi_3\psi_4} = \mathcal{B}_+, \qquad \mathcal{B}^{j=0}_{\psi_1\psi_2\to\psi_3\psi_4} = \mathcal{B}_-.$$

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- In $\{1,3\}\to\{2,4\}$ channel, $\mathcal{B}_\pm$ are not *eigenfunctions*

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- In $\phi_1\phi_2\to\phi_3\phi_4$ we have $\mathcal{B} = \langle 13\rangle[31] - \langle 14\rangle[41] \sim d^1_{0,0}$

$$\mathcal{B} = -\frac{[12][34]}{2s}(\langle 13\rangle\langle 24\rangle + \langle 14\rangle\langle 23\rangle) \sim \mathcal{B}^{j=1}_{\phi_1\phi_2\to\phi_3\phi_4}.$$

Angular Momentum: Casimir Invariant

A more systematic way to get angular momentum is directly through Poincaré algebra in the helicity-spinor representation

$$J_{\mathcal{I},\alpha\beta} = i\sum_{i\in\mathcal{I}} \left(\lambda_{i\alpha}\partial^i_\beta + \lambda_{i\beta}\partial^i_\alpha\right), \quad \tilde{J}_{\mathcal{I},\dot\alpha\dot\beta} = i\sum_{i\in\mathcal{I}} \left(\tilde\lambda_{i\dot\alpha}\bar\partial^i_{\dot\beta} + \tilde\lambda_{i\dot\beta}\bar\partial^i_{\dot\alpha}\right).$$

We can derive the Casimir invariant W^2

$$W^2_{\mathcal{I}}(\mathcal{B}) = \frac{P^2_{\mathcal{I}}}{8}\left(\mathrm{Tr}\tilde{J}^2_{\mathcal{I}}(\mathcal{B}) + \mathrm{Tr}J^2_{\mathcal{I}}(\mathcal{B})\right) - \frac{1}{4}\mathrm{Tr}\left(P^{\mathsf{T}}_{\mathcal{I}} J_{\mathcal{I}}(\mathcal{B}) P_{\mathcal{I}} \tilde{J}_{\mathcal{I}}(\mathcal{B})\right)$$

whose eigenvalue $-j(j+1)p^2$ gives the covariant angular momentum j in the channel $\mathcal{I} \to \bar{\mathcal{I}}$.

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We can derive the Casimir invariant W^2

$$W^2_{\mathcal{I}}(\mathcal{B}) = \frac{P^2_{\mathcal{I}}}{8}\left(\mathrm{Tr}\tilde{J}^2_{\mathcal{I}}(\mathcal{B}) + \mathrm{Tr}J^2_{\mathcal{I}}(\mathcal{B})\right) - \frac{1}{4}\mathrm{Tr}\left(P^{\mathsf{T}}_{\mathcal{I}} J_{\mathcal{I}}(\mathcal{B}) P_{\mathcal{I}} \tilde{J}_{\mathcal{I}}(\mathcal{B})\right)$$

whose eigenvalue $-j(j+1)p^2$ gives the covariant angular momentum j in the channel $\mathcal{I} \to \bar{\mathcal{I}}$.

- Examples:

$$W^2_{1,2}\mathcal{B}_+ = -2s\mathcal{B}_+, \qquad W^2_{1,2}\mathcal{B}_- = 0,$$
$$W^2_{1,2}(\langle 13\rangle[31] - \langle 14\rangle[41]) = -2s(\langle 13\rangle[31] - \langle 14\rangle[41]).$$

Angular Momentum: Casimir Invariant

A more systematic way to get angular momentum is directly through Poincaré algebra in the helicity-spinor representation

$$J_{\mathcal{I},\alpha\beta} = i\sum_{i\in\mathcal{I}}\left(\lambda_{i\alpha}\partial^i_\beta + \lambda_{i\beta}\partial^i_\alpha\right), \quad \tilde{J}_{\mathcal{I},\dot\alpha\dot\beta} = i\sum_{i\in\mathcal{I}}\left(\tilde\lambda_{i\dot\alpha}\bar\partial^i_{\dot\beta} + \tilde\lambda_{i\dot\beta}\bar\partial^i_{\dot\alpha}\right).$$

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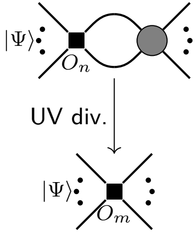
- In general (see appendix in arXiv:2001.04481)
  1. Acting on an initial basis $\mathcal{B}_i$, and expand the result in terms of $\mathcal{B}_i$.
  2. Diagonalize the representation matrix and obtain eigenvalues $j_{\mathcal{I}}$ and corresponding eigenspaces.

New Selection Rule: Renormalization

The anomalous dimension matrix for effective operators $\dot{C}_m = (4\pi)^{-2}\gamma_{mn}C_n$ at one loop is determined by



$$16\pi^2 \mathcal{A}^{\text{1-loop}}_{\text{UV}} = -(\sum_{m,n} \gamma_{mn} C_n \mathcal{B}_m + \mathcal{A}')\frac{1}{\epsilon}$$

In partial wave basis (Poincaré irrep.), the state $|\Psi\rangle$ has to take the particular wave function $f^{j,a}$ that matches with the operator $\mathcal{O}_{n,m}$.

$$\gamma_{mn} \neq 0 \quad \Rightarrow \quad (j,a)_m = (j',b)_n$$

New Selection Rule: Renormalization

$\vert\Psi\rangle$	$j=0$	$j=1/2$	$j=1$
F^+F^+	$F^2\phi^2(2,6)$		
$F^+\psi^+$		$F\psi^2\phi(2,6)$	
$F^+\phi$			$F\psi^2\phi(2,6)$ $F^2\phi^2(2,6)$
$\psi^+\psi^+$	$\psi^4(2,6)$ $\psi^2\bar\psi^2(4,4)$ $\psi^2\phi^3(4,6)$		$\psi^4(2,6)$ $F\psi^2\phi(2,6)$
$\psi^+\psi^-$			$\psi\bar\psi\phi^2D(4,4)$
$\psi^+\phi$		$\psi^2\phi^3(4,6)$ $F\psi^2\phi(2,6)$ $\psi\bar\psi\phi^2D(4,4)$	
$\phi\phi$	$\phi^4D^2(4,4)$ $\psi^2\phi^3(4,6)$ $\phi^6(6,6)$		$\phi^4D^2(4,4)$ $\psi\bar\psi\phi^2D(4,4)$

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$\psi^+\psi^+$	$\bar{L}e\bar{Q}u$ $\bar{L}eH\vert H\vert^2$		$\bar{L}\tau^i e\bar{Q}\tau^i u$ $W^i_{\mu\nu}\bar{L}\tau^i\sigma^{\mu\nu}eH$
$\psi^+\psi^-$			$\psi\bar{\psi}\phi^2D(4,4)$
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$\phi\phi$	$H^4D^2(?)$ $\bar{L}eH\lvert H\rvert^2$		$H^4D^2(?)$ $\mathcal{O}^1_{Hl},\mathcal{O}^3_{Hl}$

$$\mathcal{O}^1_{Hl}=(\bar{L}\gamma^\mu L)(iH^\dagger\overleftrightarrow{D}_\mu H),\quad \mathcal{O}^3_{Hl}=(\bar{L}\tau^i\gamma^\mu L)(iH^\dagger\tau^i\overleftrightarrow{D}_\mu H)$$

New Selection Rule: Renormalization

Partial wave basis is usually different from "permutation basis" in case of identical particles.

$$\mathcal{L}_{\rm SMEFT} \supset C_{HD}|H^\dagger D_\mu H|^2 + C_{H\Box}(H^\dagger H)\Box(H^\dagger H),$$

$$\mathcal{A}(H_1, H_2^\dagger, H_3, H_4^\dagger) = [3C_{H\Box}T^{I=0} + (C_{HD} - C_{H\Box})T^{I=1}](\mathcal{B}^{j=0}_{12\to34} = s_{12})$$
$$+ [(2C_{H\Box} + 2C_{HD})T^{I=0} + 2C_{H\Box}T^{I=1}](\mathcal{B}^{j=1}_{12\to34} = s_{13} - s_{14}).$$

With conservation of both angular momentum j and isospin I, we must have

$$\dot{C}^1_{Hl} \propto C_{HD} + C_{H\Box}, \quad \dot{C}^3_{Hl} \propto C_{H\Box}.$$

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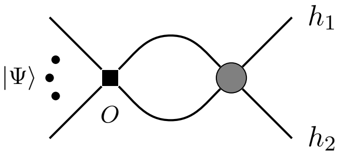
  1. Since $l \perp \hat{z}$ for two particles, $j \geq j_z = s_z = \Delta h = |h_1 - h_2|$.
  2. When the two particles on RHS are identical, j is even.



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$$\mathcal{O}^{j=3/2,a=1} = -2\mathcal{O}_1 + \mathcal{O}_2 + 3\mathcal{O}_4 + \mathcal{O}_5,$$
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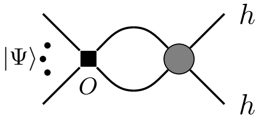
More fun with the degenerate space in arXiv:2001.04481.

New Selection Rule: Vanishing Loops B

The permutation symmetry of two particle state:

$$C^{Pj\sigma}_{p_1h_1;p_2h_2} \sim [12]^{j+h_1+h_2}\left(\langle 1\chi\rangle^{j-h_1+h_2}\langle 2\chi\rangle^{j+h_1-h_2}\right)^{\{I_1\ldots I_{2j}\}}$$

$$\overset{h_1=h_2}{\longrightarrow} [12]^{j+2h}\left(\langle 1\chi\rangle^{j}\langle 2\chi\rangle^{j}\right)^{\{I_1\ldots I_{2j}\}}$$



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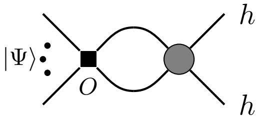

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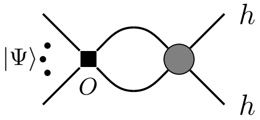

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- Generalized when we consider full permutation symmetry involving group factors (next part!).

Outline

Constraints from Spin Statistics

- Without spin-statistics constraint, the counting of Lorentz structures and other group structures are independent.
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- Without spin-statistics constraint, the counting of Lorentz structures and other group structures are independent.
- Since rSSYT already solves the former, it is straightforward to count and list all amplitude basis and effective operators.
- However with spin-statistics and identical particles, we shall take inner-product-decomposition of permutation rep. from all structures to select the non-vanishing ones. (arXiv:1907.12584)

	QQQ	L
$SU(3)_C$	$[1^3]$	$[1]$
$SU(2)_L$	$[2,1]$	$[1]$
$SU(2)_l$	$[2,1]$	$[1]$
$SU(2)_r$	$[3]$	$[1]$
Grassmann	$[1^3]$	$[1]$
Total symmetry	$[1^3]^2 \times [2,1]^2 \times [3] = [3] + [2,1] + [1^3]$	$[1]^5 = [1]$

Permutation Basis

- To get amplitude basis with a certain irrep. λ of S_n group, we use Young symmetrizer $\mathcal{Y}^{\lambda}_{\mu} = \prod_{\text{row}} P \prod_{\text{col}} Q$.
  Example: $\mathcal{Y}_{\tiny\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array}} = [1+(12)][1+(34)][1-(13)][1-(24)]$.
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     1. Sort particles as $h_1 \le h_2 \le \cdots \le h_N$ and eliminate all p_1.
     2. Eliminate p_2 in $\langle 1|p_2|3]$ and $\langle 3|p_2|1]$.
     3. Eliminate p_3 in $\langle 1|p_3|2]$, $\langle 2|p_3|p_1]$ and $p_2 \cdot p_3$.

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  2. Use Schouten identity to make both smaller and larger numbers in brackets sorted in ascending order at the same time

$$[14][23] \to [13][24] - [12][34]$$

Outline

Summary

- On-shell formalism is a promising program in the study of scattering amplitudes.
- Amplitude Basis is the essential building block for on-shell EFT.
- Various choices of amplitude basis are convenient for different situations:
  – the partial wave basis is good for tracing conserved angular momentum;
  – the permutation basis is better when identical particles are involved.
- Ambiguity of splitting a generic amplitudes into the factorizable part and unfactorizable part is the key problem of amplitude basis: both challenge and opportunity!

Thank You

Fine

The $\mathcal{A}'$ problem

- From our selection rule for renormalization, $\mathcal{O}_H = (H^\dagger H)^3$ with $j = 0$ in any channel should **NOT** be renormalized by operators of type $\bar{\psi}\psi H^\dagger H D$ which have $j = 1$ in $\{H^\dagger, H\}$ channel

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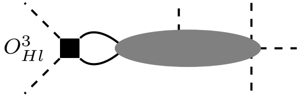

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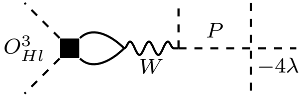

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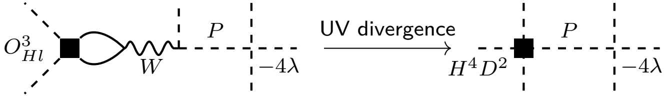

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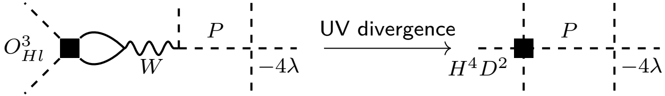

- $j = 1$ and $I = 1$ fixes the form of left sub-amplitude $\mathcal{B}^{j=1,I=1}$.
- The Warsaw basis suggests the decomposition

$$\mathcal{B}^{j=1,I=1} = \frac{3}{4}\mathcal{B}_{H\Box} - \frac{1}{2}P^2.$$

so that the second term induces a "local" UV divergence corresponding to $\dot{C}_H$.