



Existence and stability of rotating black-hole solutions in the hybrid metric-Palatini gravity

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PART I

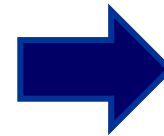
INTRODUCTION

MOTIVATION

If **General Relativity** (GR) is so successful, **why** do we need modified gravity?

GR PROBLEMS:

- No quantum description
- Dark matter and dark energy
- Exotic matter in wormholes
- Existence of singularities



Need for
modified
gravity!

How can we modify gravity?



TWO WAYS

By adding **extra fields** to the action $(\phi, A^\mu, h^{\mu\nu}, \dots)$
By adding **higher-order terms** $f(R, R_{\mu\nu}, R_{\alpha\beta\mu\nu}, \dots)$

Sometimes, the two representations are **connected**, e.g., $f(R)$ with scalar-tensor.

METRIC AND PALATINI

Two **different** variational approaches: **metric** and **Palatini** formalisms

Metric formalism: Γ_{ab}^c is Levi-Civita of g_{ab}
Palatini formalism: Γ_{ab}^c is independent of g_{ab}

Consequences?

General Relativity

$$\boxed{\mathcal{L} = R} \longleftrightarrow \text{Same field equations} \longleftrightarrow \boxed{G_{ab} = 8\pi T_{ab}}$$

f(R) Gravity

$$\boxed{\mathcal{L} = f(R)}$$

Formalism **dependent**

$$f'(R) R_{ab} - \frac{1}{2} g_{ab} f(R) - (\nabla_a \nabla_b - g_{ab} \square) f(R) = 8\pi T_{ab}$$
$$f'(\mathcal{R}(\Gamma)) \mathcal{R}_{ab}(\Gamma) - \frac{1}{2} g_{ab} f(\mathcal{R}(\Gamma)) = 8\pi T_{ab}$$

QUESTION: what happens if we **combine** both formalisms in the same framework?

THE GHMPG THEORY

Action for the **Generalized hybrid metric-Palatini gravity** (GHMPG) theory:

$$S = \frac{1}{2\kappa^2} \int \sqrt{-g} f[R(\Gamma), \mathcal{R}(\hat{\Gamma})] d^4x + S_m.$$

Variables:

$$(g_{ab}, \hat{\Gamma}_{ab}^c)$$

Application of the **variational method** yields the following equations of motion:

Metric

$$\frac{\partial f}{\partial R} R_{ab} + \frac{\partial f}{\partial \mathcal{R}} \mathcal{R}_{ab} - \frac{1}{2} g_{ab} f(R, \mathcal{R}) - (\nabla_a \nabla_b - g_{ab} \square) \frac{\partial f}{\partial R} = \kappa^2 T_{ab},$$

Connection

$$\mathcal{R}_{ab} = R_{ab} + \frac{3}{2f_{\mathcal{R}}^2} \partial_a f_{\mathcal{R}} \partial_b f_{\mathcal{R}} - \frac{1}{f_{\mathcal{R}}} \left(\nabla_a \nabla_b + \frac{1}{2} g_{ab} \square \right) f_{\mathcal{R}},$$

The second equation actually comes from the **conformal relation** between two metrics:

$$\hat{\nabla}_c \left(\sqrt{-g} \frac{\partial f}{\partial \mathcal{R}} g^{ab} \right) = 0$$

The background of the slide features a cosmic image of numerous galaxies in various colors (blue, red, white) against a black space, visible on the left side and partially obscured by a dark blue horizontal bar at the top and bottom. A large, white speech bubble with a dark blue outline is centered on the right side of the slide.

PART II

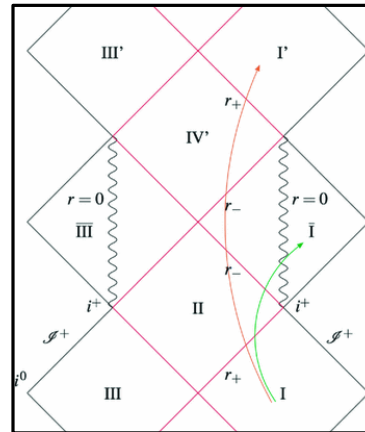
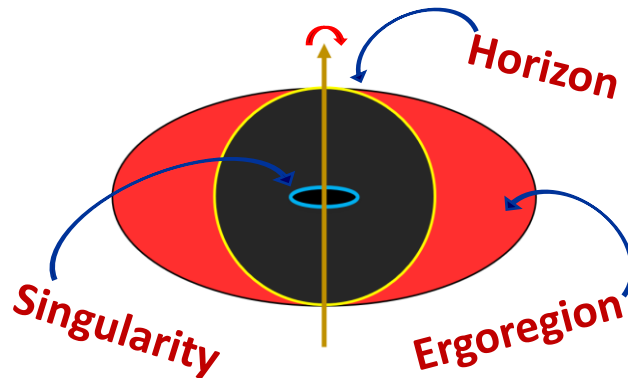
PERTURBATIONS

BLACK-HOLE PERTURBATIONS

A rotating black-hole in vacuum is described by the **Kerr metric**:

$$ds^2 = - \left(1 - \frac{2Mr}{\rho^2} \right) dt^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 - \frac{4Mra \sin^2 \theta}{\rho^2} dt d\phi + \left(r^2 + a^2 + \frac{2Mra^2 \sin^2 \theta}{\rho^2} \right) d\phi^2$$

$$\Delta = r^2 + a^2 - 2Mr, \quad \rho^2 = r^2 + a^2 \cos^2 \theta, \quad a = \frac{J}{M}$$



Time

Metric perturbations

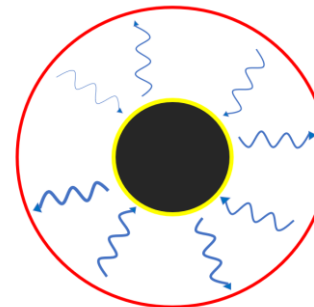
$$g_{ab} = \bar{g}_{ab} + \epsilon \delta g_{ab}$$

Lorentz gauge

$$\nabla_b \delta g^{ab} = \frac{1}{2} \nabla^a \delta g$$

Field equations become **biharmonic** equations for a massive scalar field:

$$(\square - \mu_{\pm}^2) (\square - \mu_{\mp}^2) \delta R = 0$$



Might lead to **superradiant instabilities!**

(Black-hole bombs)

SUPERRADIANCE

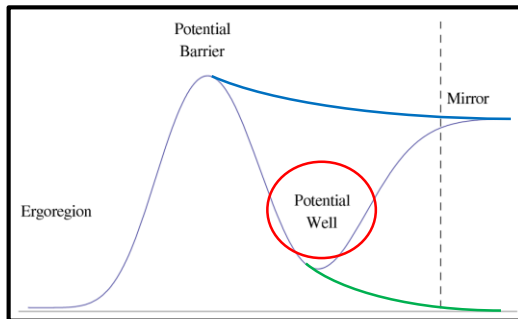
Instabilities are caused by the **confinement** of the superradiant modes:

$$\delta R = \psi(r) S(\theta) \exp(-i\omega t + im\phi)$$

$$\frac{dr}{dr_*} = \frac{\Delta}{r^2 + a^2}, \quad u(r) = \sqrt{r^2 + a^2} \psi(r)$$

Potential barrier:

$$\frac{d^2 u}{dr_*^2} + [\omega^2 - V(r)] u = 0$$



Problem: a potential well

Solution 1: Massless field

Solution 2: Field too massive

$$\mu^2 = 0$$

$$\mu > \mu_c$$

$$\mu_c = m\Omega \sqrt{1 + \frac{2M}{r_+}}$$

Masses:

$$\mu_{\pm} = -\frac{1}{2} \left(A \pm \sqrt{A^2 - 4B} \right)$$

only depend on
 $f(R, \mathcal{R})$

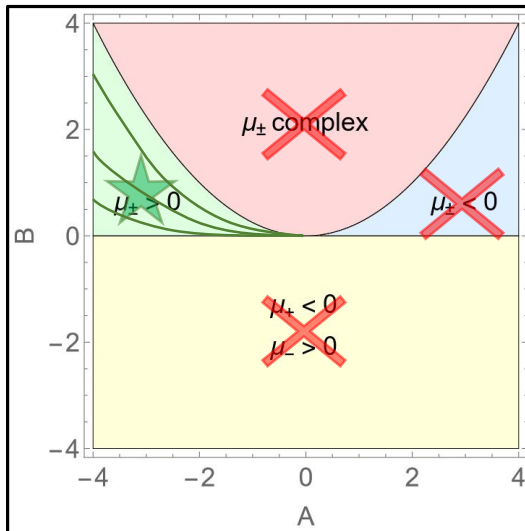
$$A = \frac{f_R f_{\mathcal{R}\mathcal{R}} - 2f_{\mathcal{R}} f_{\mathcal{R}R} - f_{\mathcal{R}} f_{RR}}{3(f_{\mathcal{R}R}^2 - f_{RR} f_{\mathcal{R}\mathcal{R}})}$$

$$B = \frac{f_{\mathcal{R}} (f_{\mathcal{R}} + f_R)}{9(f_{\mathcal{R}R}^2 - f_{RR} f_{\mathcal{R}\mathcal{R}})}$$

QUESTION: what are the forms of the action for which the masses **satisfy** the needed properties?

STABILITY REGIMES

$$\mu_{\pm} = -\frac{1}{2} \left(A \pm \sqrt{A^2 - 4B} \right)$$



Regions to exclude:

Complex masses lead to **tachyonic instability**

Negative masses **never exceed** critical mass

Acceptable solutions:

$$(1) \quad \mu_{\pm} = 0 \quad \Rightarrow \quad A = B = 0$$

$$(2) \quad \mu_+ = 0, \quad \mu_- > \mu_c \quad \Rightarrow \quad B = 0, \quad A < 0$$

$$(3) \quad \mu_{\pm} > \mu_c \quad \Rightarrow \quad B \propto A^2$$

Particular case:

$$\mu_{\pm} = -\frac{A}{2} \left(1 \mp \sqrt{1 - 4C} \right)$$

Forms of the action for which the solutions are **stable**:

$$\mu_{\pm}^2 = 0 \quad \longleftrightarrow$$

$$f(R, \mathcal{R}) = (a_1 + a_2 R + a_3 \mathcal{R})(R - \mathcal{R})$$

$$\mu_+ = 0, \quad \mu_- > \mu_c \quad \longleftrightarrow$$

$$f(R, \mathcal{R}) = a_1(R - \mathcal{R}) + a_2 R^2 + a_3 \mathcal{R}^2 + a_4 R\mathcal{R}$$

$$\mu_{\pm} > \mu_c \quad \longleftrightarrow$$

$$f(R, \mathcal{R}) = a_1 R + a_2 \mathcal{R} + a_3 R^2 + a_4 \mathcal{R}^2 + a_5 R\mathcal{R}$$

The slide features a cosmic background image showing a dense field of galaxies in various colors (blue, red, white) against a black space. A dark blue horizontal bar is at the top, and a white diagonal band runs from the top left towards the bottom right. A large blue-outlined speech bubble is centered on the right side of the slide.

PART III

CONCLUSIONS

CONCLUSIONS

Conclusions

Kerr and Schwarzschild are very general solutions of the hybrid gravity
Lorentz gauge reduces the perturbative analysis to a scalar degree
Particular forms of the action for which the Kerr solution is stable exist

Further work

Drop the Lorentz gauge and study the full set of **tensorial** perturbations
Generalize method to include **Kottler** and **Reissner-Nordstrom** solutions

THANK YOU FOR YOUR ATTENTION!