Existence and stability of rotating black-hole solutions in the hybrid metric-Palatini gravity

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PART I

INTRODUCTION

MOTIVATION

If **General Relativity** (GR) is so successful, **why** do we need modified gravity?

GR PROBLEMS:

- No quantum description
- Dark matter and dark energy
- Exotic matter in wormholes
- Existence of singularities

→ **Need** for modified gravity!

How can we modify gravity?

TWO WAYS

- By adding **extra fields** to the action $(\phi, A^{\mu}, h^{\mu\nu}, \ldots)$
- By adding **higher-order terms** $f\left(R, R_{\mu\nu}, R_{\alpha\beta\mu\nu}, \ldots\right)$

Sometimes, the two representations are **connected**, e.g., f(R) with scalar-tensor.

METRIC AND PALATINI

Two **different** variational approaches: **metric** and **Palatini** formalisms

- **Metric formalism:** Γ^c_{ab} is Levi-Civita of g_{ab}
- **Palatini formalism:** Γ^c_{ab} is independent of g_{ab}

Consequences?

General Relativity

$$\mathcal{L} = R$$

Same field equations

$$G_{ab} = 8\pi T_{ab}$$

f(R) Gravity

$$\mathcal{L} = f(R)$$

Formalism **dependent**

$$f'(R)\, R_{ab} - \frac{1}{2} g_{ab} f(R) - (\nabla_a \nabla_b - g_{ab}\Box)\, f'(R) = 8\pi T_{ab}$$

$$f'(\mathcal{R}(\Gamma))\, \mathcal{R}_{ab}(\Gamma) - \frac{1}{2} g_{ab} f(\mathcal{R}(\Gamma)) = 8\pi T_{ab}$$

QUESTION: what happens if we **combine** both formalisms in the same framework?

THE GHMPG THEORY

Action for the **Generalized hybrid metric-Palatini gravity** (GHMPG) theory:

$$S=\frac{1}{2\kappa^2}\int\sqrt{-g}f[R(\Gamma),\mathcal{R}(\hat{\Gamma})]d^4x+S_m.$$

Variables:

$$\left(g_{ab},\hat{\Gamma}^c_{ab}\right)$$

Application of the **variational method** yields the following equations of motion:

Metric

$$\frac{\partial f}{\partial R}R_{ab}+\frac{\partial f}{\partial \mathcal{R}}\mathcal{R}_{ab}-\frac{1}{2}g_{ab}f\left(R,\mathcal{R}\right)-\left(\nabla_a\nabla_b-g_{ab}\Box\right)\frac{\partial f}{\partial R}=\kappa^2T_{ab},$$

Connection

$$\mathcal{R}_{ab}=R_{ab}+\frac{3}{2f_{\mathcal{R}}^2}\partial_a f_{\mathcal{R}}\partial_b f_{\mathcal{R}}-\frac{1}{f_{\mathcal{R}}}\left(\nabla_a\nabla_b+\frac{1}{2}g_{ab}\Box\right)f_{\mathcal{R}},$$

The second equation actually comes from the **conformal relation** between two metrics:

$$\hat{\nabla}_c\left(\sqrt{-g}\frac{\partial f}{\partial \mathcal{R}}g^{ab}\right)=0$$

PART II

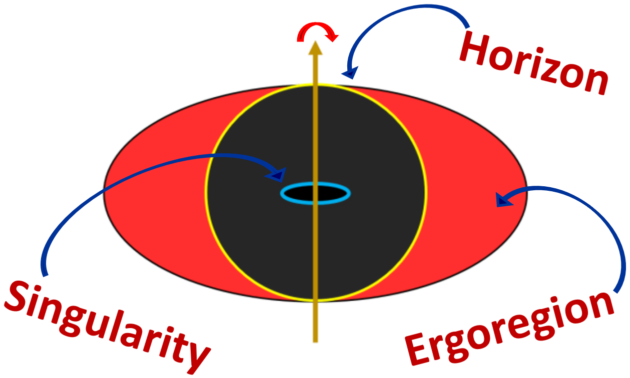
PERTURBATIONS

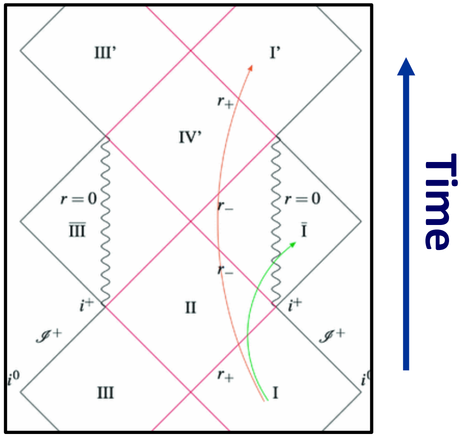
BLACK-HOLE PERTURBATIONS

A rotating black-hole in vacuum is described by the **Kerr metric**:

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 - \frac{4Mra\sin^2\theta}{\rho^2} dt d\phi + \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\rho^2}\right) d\phi^2$$

$$\Delta = r^2 + a^2 - 2Mr, \qquad \rho^2 = r^2 + a^2\cos^2\theta, \qquad a = \frac{J}{M}$$





Metric **perturbations**

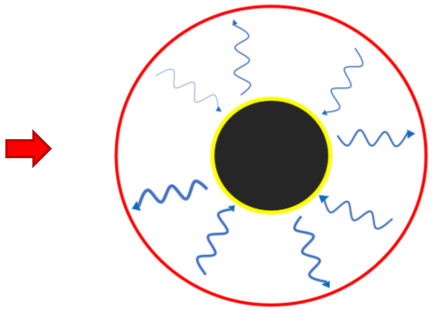
$$g_{ab} = \bar{g}_{ab} + \epsilon \delta g_{ab}$$

Lorentz **gauge**

$$\nabla_b \delta g^{ab} = \frac{1}{2} \nabla^a \delta g$$

Field equations become **biharmonic** equations for a massive scalar field:

$$\left(\Box - \mu_{\pm}^2\right)\left(\Box - \mu_{\mp}^2\right) \delta R = 0$$



Might lead to **superradiant instabilities**!

(**Black-hole bombs**)

SUPERRADIANCE

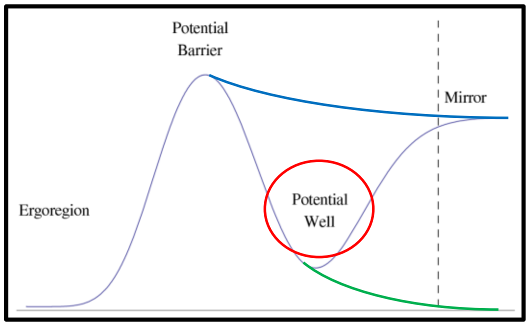
Instabilities are caused by the **confinement** of the superradiant modes:

$$\delta R = \psi(r) S(\theta) \exp(-i\omega t + im\phi)$$

$$\frac{dr}{dr_*} = \frac{\Delta}{r^2 + a^2}, \qquad u(r) = \sqrt{r^2 + a^2}\,\psi(r)$$

Potential barrier:

$$\frac{d^2 u}{dr_*^2} + \left[\omega^2 - V(r)\right] u = 0$$



Problem: a potential well

Solution 1: Massless field

Solution 2: Field too massive

$$\mu^2 = 0$$

$$\mu > \mu_c$$

$$\mu_c = m\Omega\sqrt{1 + \frac{2M}{r_+}}$$

Masses:

$$\mu_\pm = -\frac{1}{2}\left(A \pm \sqrt{A^2 - 4B}\right)$$

only depend on $f(R, \mathcal{R})$

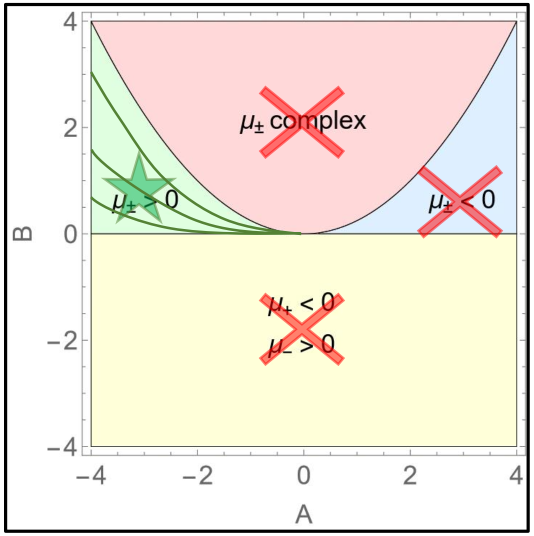
$$A = \frac{f_R f_{\mathcal{R}\mathcal{R}} - 2 f_{\mathcal{R}} f_{R\mathcal{R}} - f_{\mathcal{R}} f_{RR}}{3\left(f_{R\mathcal{R}}^2 - f_{RR} f_{\mathcal{R}\mathcal{R}}\right)}$$

$$B = \frac{f_{\mathcal{R}}\left(f_{\mathcal{R}} + f_R\right)}{9\left(f_{R\mathcal{R}}^2 - f_{RR} f_{\mathcal{R}\mathcal{R}}\right)}$$

QUESTION: what are the forms of the action for which the masses **satisfy** the needed properties?

STABILITY REGIMES

$$\mu_\pm = -\frac{1}{2}\left(A \pm \sqrt{A^2 - 4B}\right)$$



Regions to exclude:

Complex masses lead to **tachyonic instability**

Negative masses **never exceed** critical mass

Acceptable solutions:

(1) $\mu_\pm = 0 \quad \Rightarrow \quad A = B = 0$

(2) $\mu_+ = 0,\ \mu_- > \mu_c \quad \Rightarrow \quad B = 0,\ A < 0$

(3) $\mu_\pm > \mu_c \quad \Rightarrow \quad B \propto A^2$

Particular case:

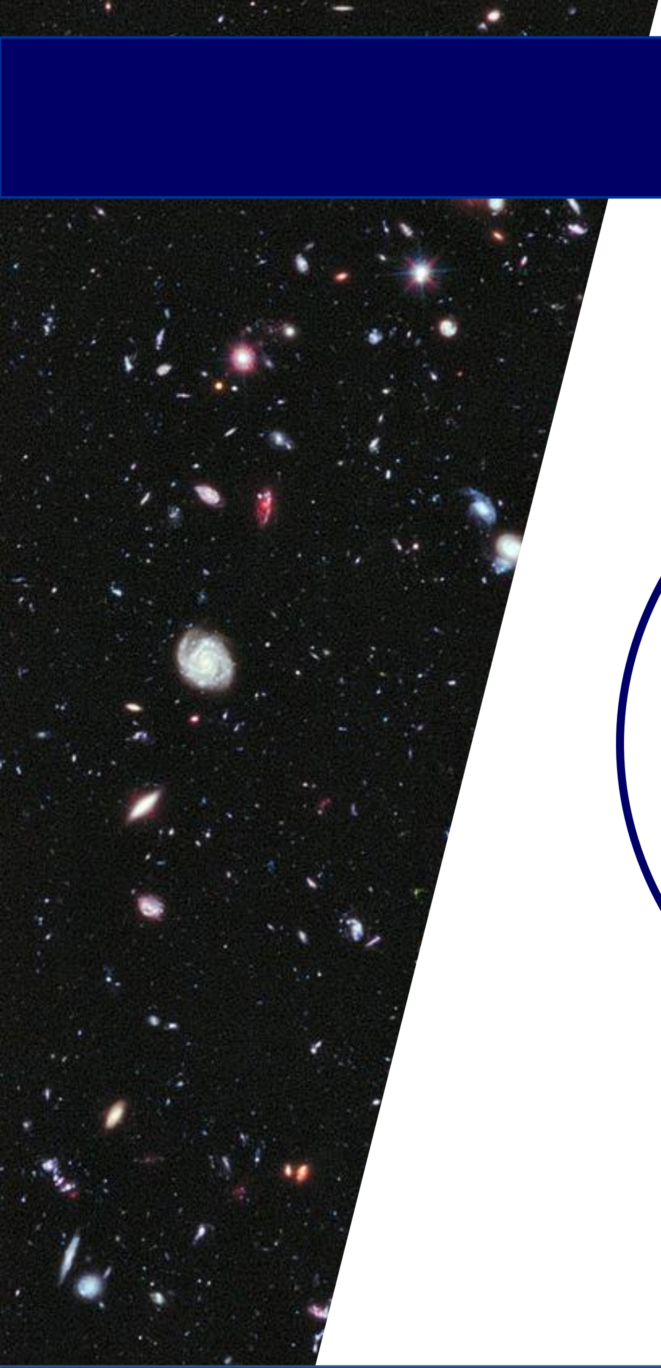
$$\mu_\pm = -\frac{A}{2}\left(1 \mp \sqrt{1 - 4C}\right)$$

Forms of the action for which the solutions are **stable**:

$\mu_\pm^2 = 0 \longleftrightarrow f(R, \mathcal{R}) = (a_1 + a_2 R + a_3 \mathcal{R})(R - \mathcal{R})$

$\mu_+ = 0, \quad \mu_- > \mu_c \longleftrightarrow f(R, \mathcal{R}) = a_1 (R - \mathcal{R}) + a_2 R^2 + a_3 \mathcal{R}^2 + a_4 R\mathcal{R}$

$\mu_\pm > \mu_c \longleftrightarrow f(R, \mathcal{R}) = a_1 R + a_2 \mathcal{R} + a_3 R^2 + a_4 \mathcal{R}^2 + a_5 R\mathcal{R}$



PART III

CONCLUSIONS

CONCLUSIONS

Conclusions

Kerr and Schwarzschild are very general solutions of the hybrid gravity

Lorentz gauge reduces the perturbative analysis to a scalar degree

Particular forms of the action for which the Kerr solution is stable exist

Further work

Drop the Lorentz gauge and study the full set of **tensorial** perturbations

Generalize method to include **Kottler** and **Reissner-Nordstrom** solutions

THANK YOU FOR YOUR ATTENTION!