Quarkonium Transport in Quark-Gluon Plasma: Open Quantum System & Effective Field Theory

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IPMU APEC Seminar

June 17, 2020

XY B.Müller, 1709.03529, 1811.09644

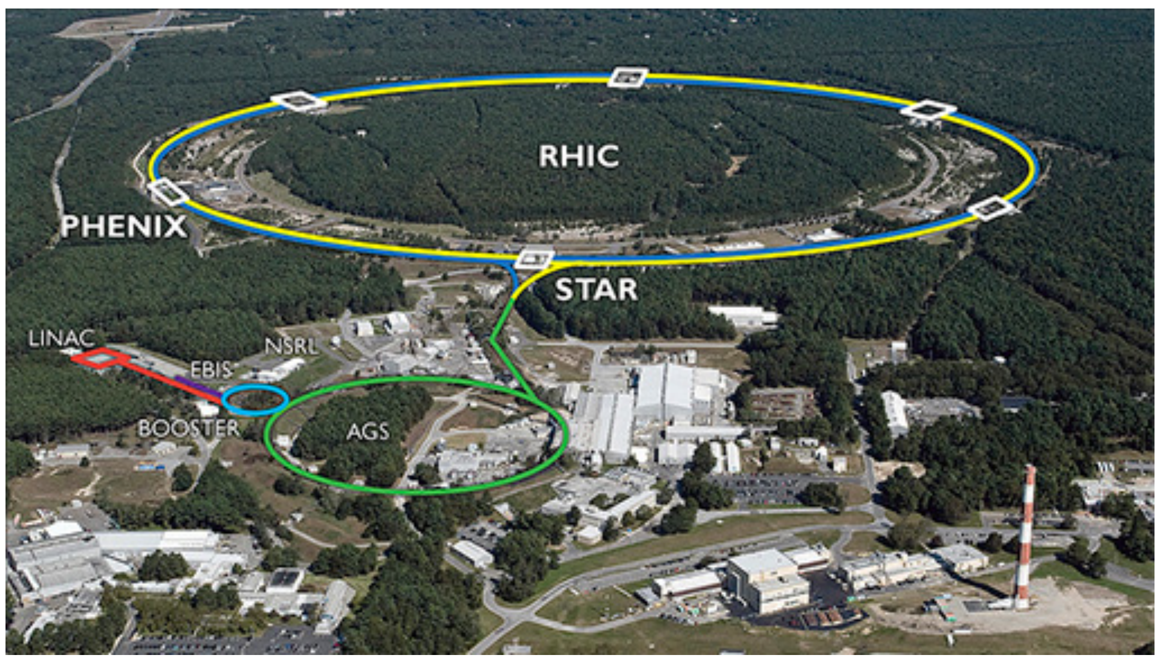
XY T.Mehen, 1811.07027, in preparation

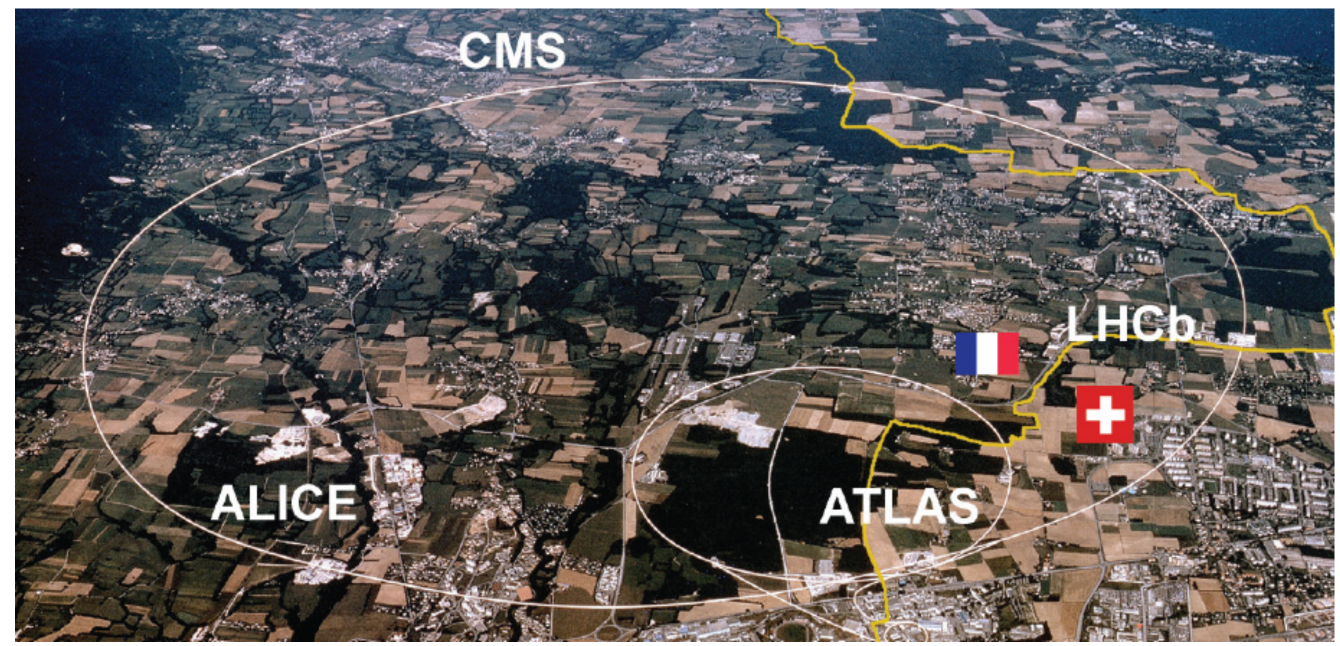
XY W.Ke Y.Xu S.Bass B.Müller, 2004.06746

Heavy Ion Collisions and Quark-Gluon Plasma

- Asymptotic freedom —> deconfined phase of QCD expected at high temperature / density —> quark-gluon plasma (QGP)

- Study QGP: heavy ion collision experiments at RHIC and LHC





- Quark gluon plasma created in Au-Au / Pb-Pb collisions: nearly "perfect" liquid (small viscosity), strongly coupled, temperature ~150-500 MeV, lifetime ~ 10 fm/c

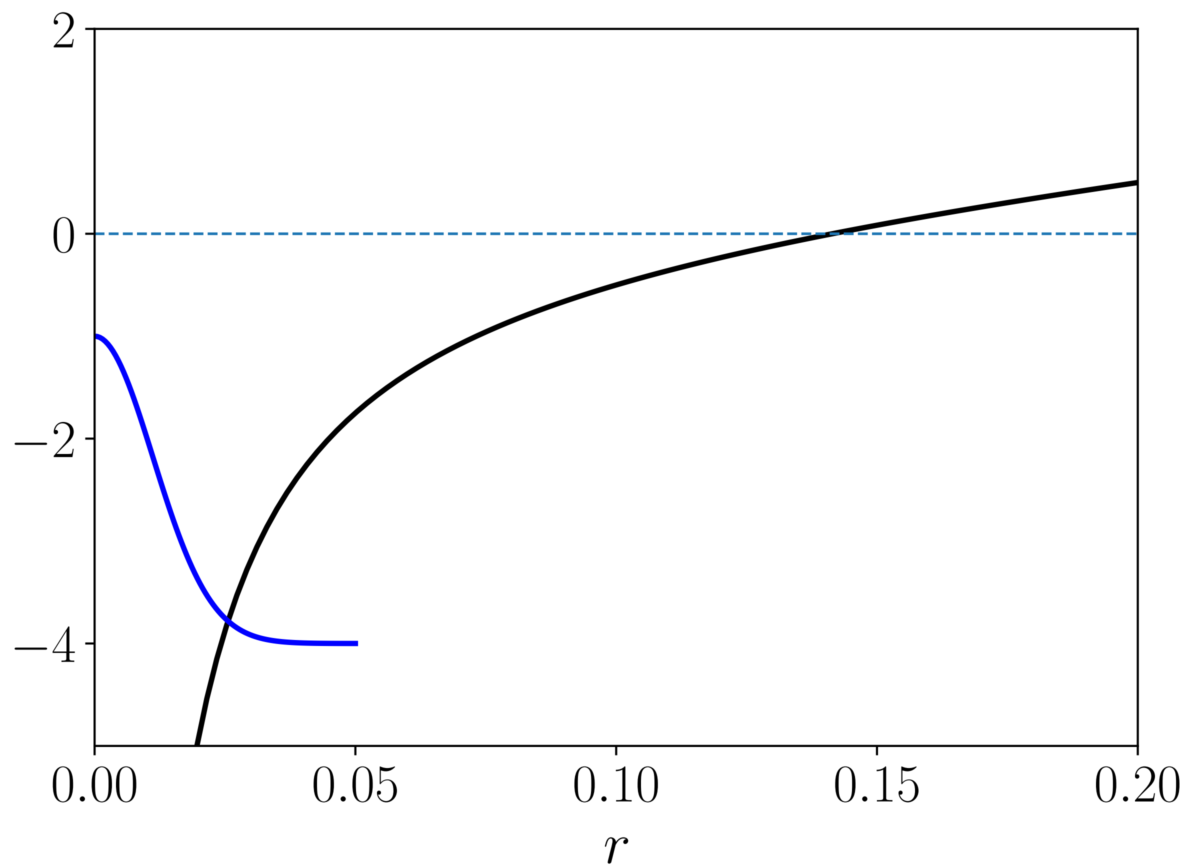
- Hard probes of QGP: jets, heavy quarks; large scale involved

Quarkonium as Probe of Quark-Gluon Plasma

- Quarkonium: bound state of $Q\bar{Q}$, nonrelativistic potential description

- **Static screening**: suppression of color attraction —> melting at high T —> reduced production —> thermometer

$$T = 0 : V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0 :$$ Confining part flattened



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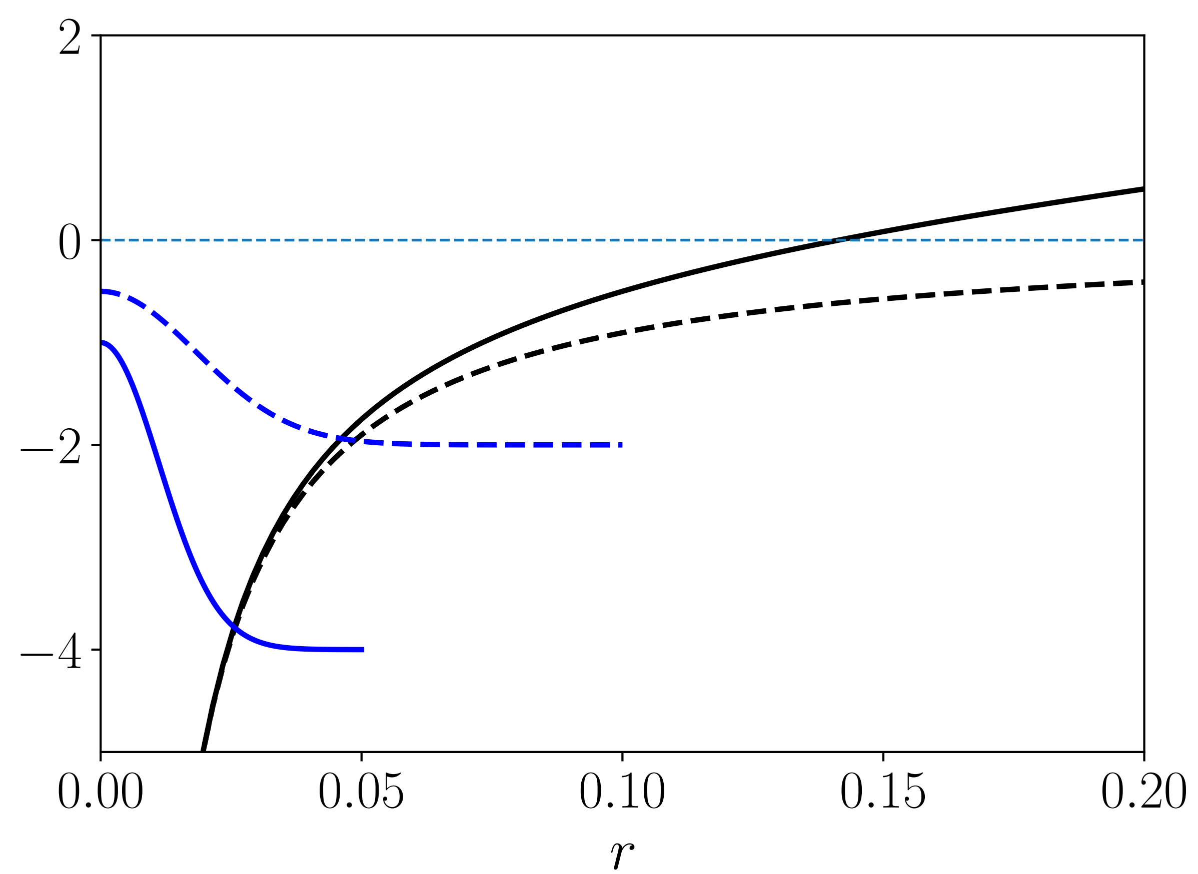

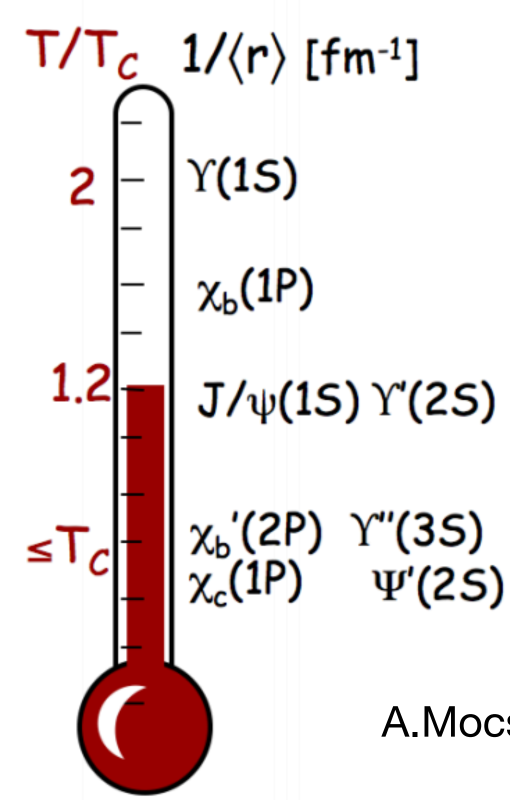

A.Mocsy, 0811.0337

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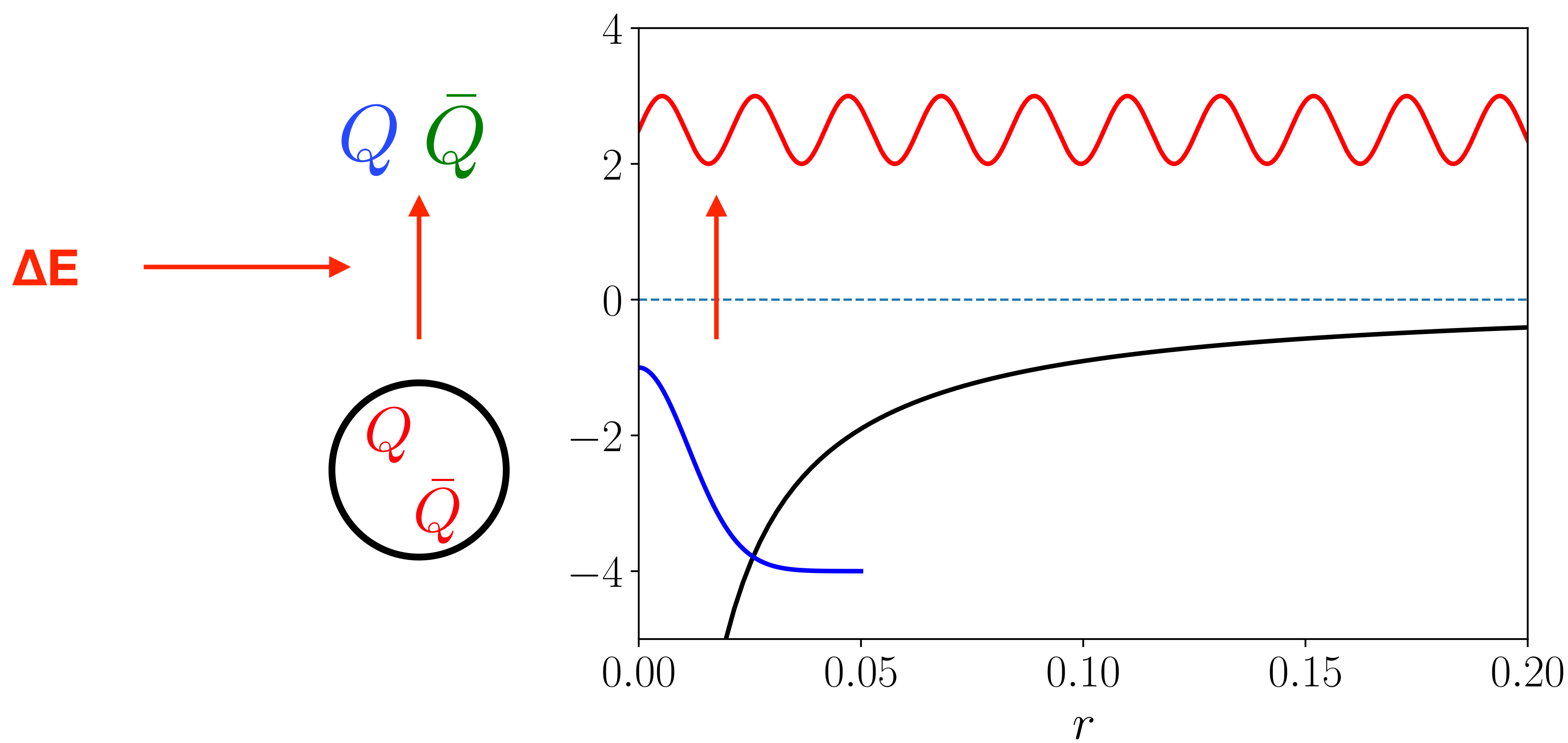
Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer

- **Dynamical screening**: dissociation induced by dynamical process, imaginary potential



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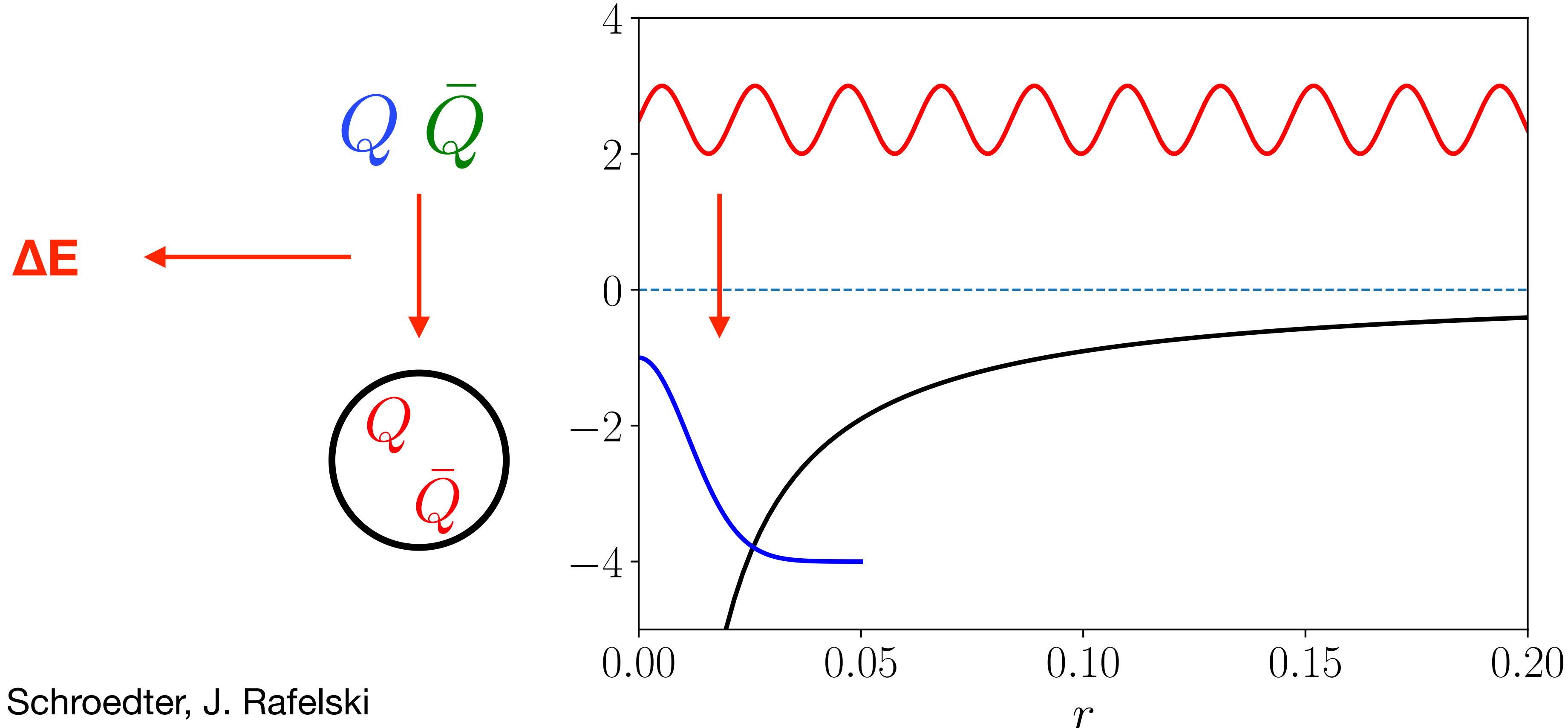
Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer

- Dynamical screening: dissociation induced by dynamical process, imaginary potential

- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T

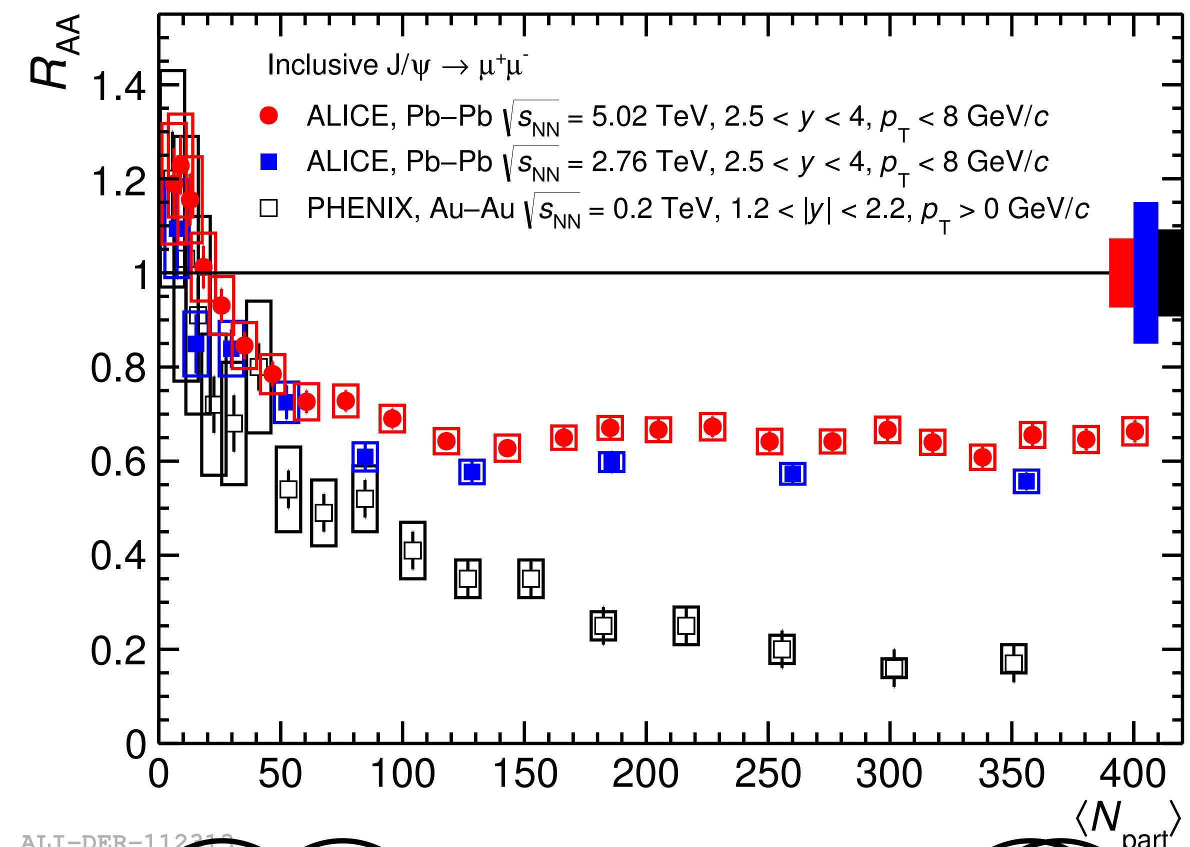


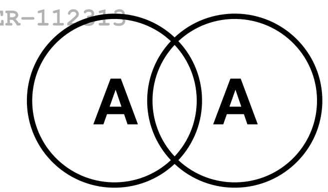
RL. Thews, M. Schroedter, J. Rafelski
Phys.Rev.C 63, 054905 (2001)

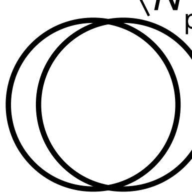
Recombination Crucial for Phenomenology

Recombination needed to explain data on charmonium suppression

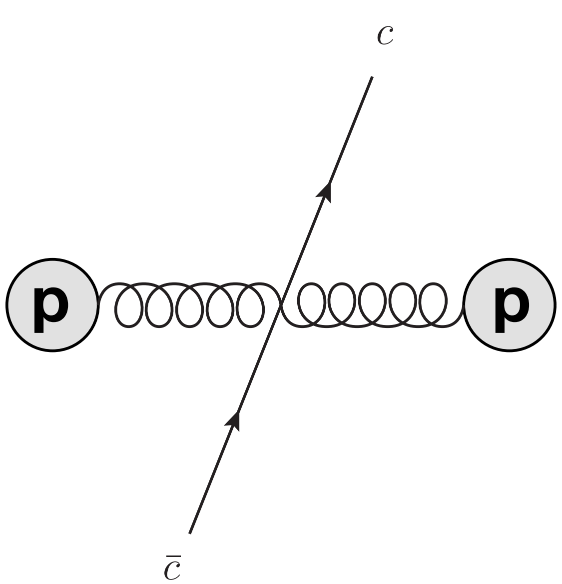
$$R_{\mathrm{AA}} = \frac{\sigma_{\mathrm{AA}}}{N_{\mathrm{coll}}\sigma_{pp}}$$

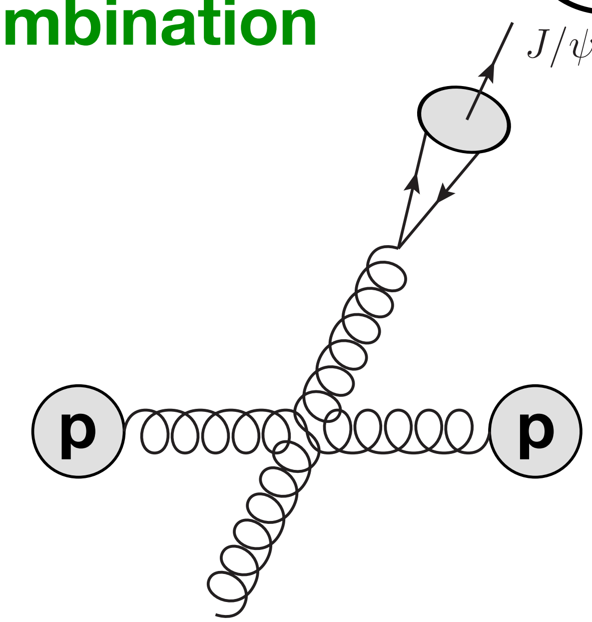


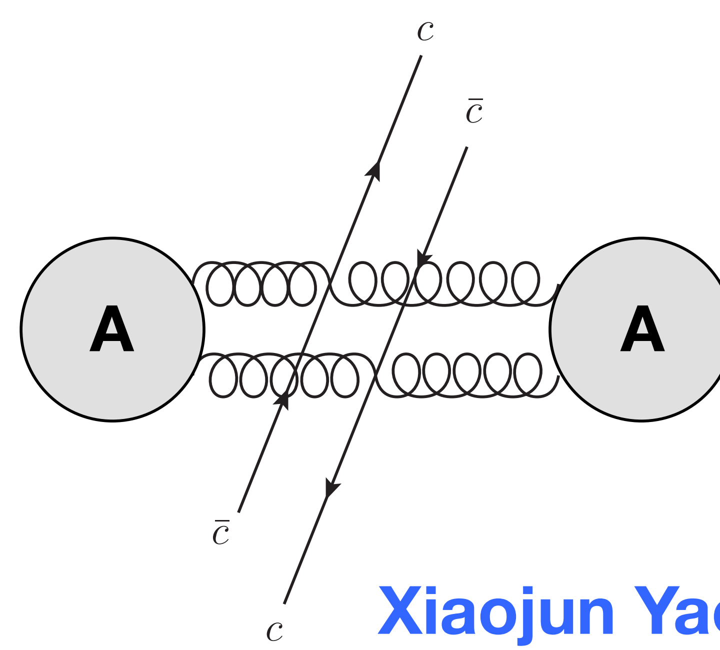




Enhanced uncorrelated (re)combination







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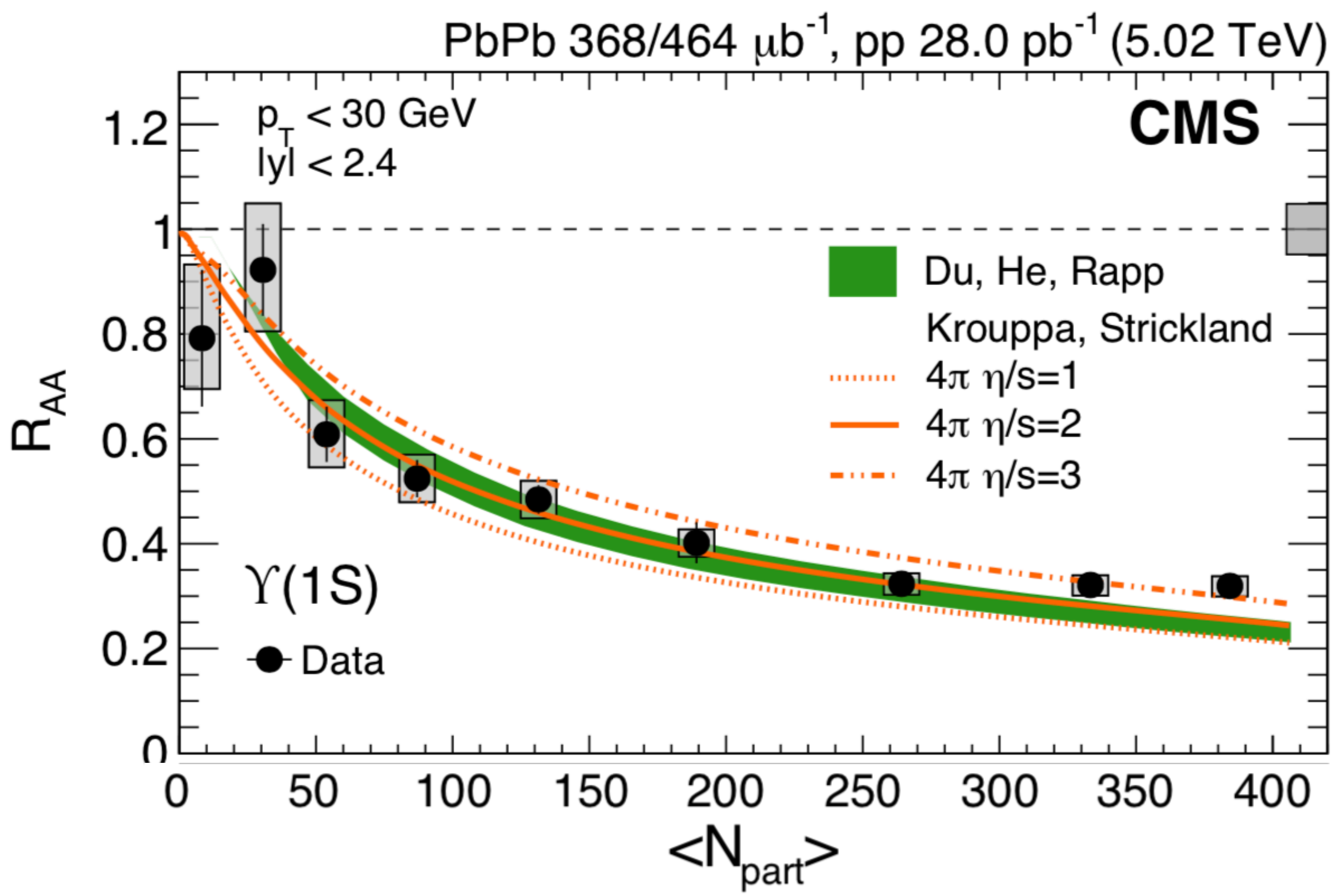
Success of Semiclassical Transport

Evolution of distribution in phase space

$$(\partial_t + \boldsymbol{v}\cdot\nabla)f(\boldsymbol{x},\boldsymbol{p},t) = -C^{(-)}(\boldsymbol{x},\boldsymbol{p},t) + C^{(+)}(\boldsymbol{x},\boldsymbol{p},t)$$

Dissociation   Recombination



X.Du, R.Rapp, M.He,1706.08670

B.Krouppa, M.Strickland,1605.03561

Why semiclassical transport equation successful? Connection to QCD?

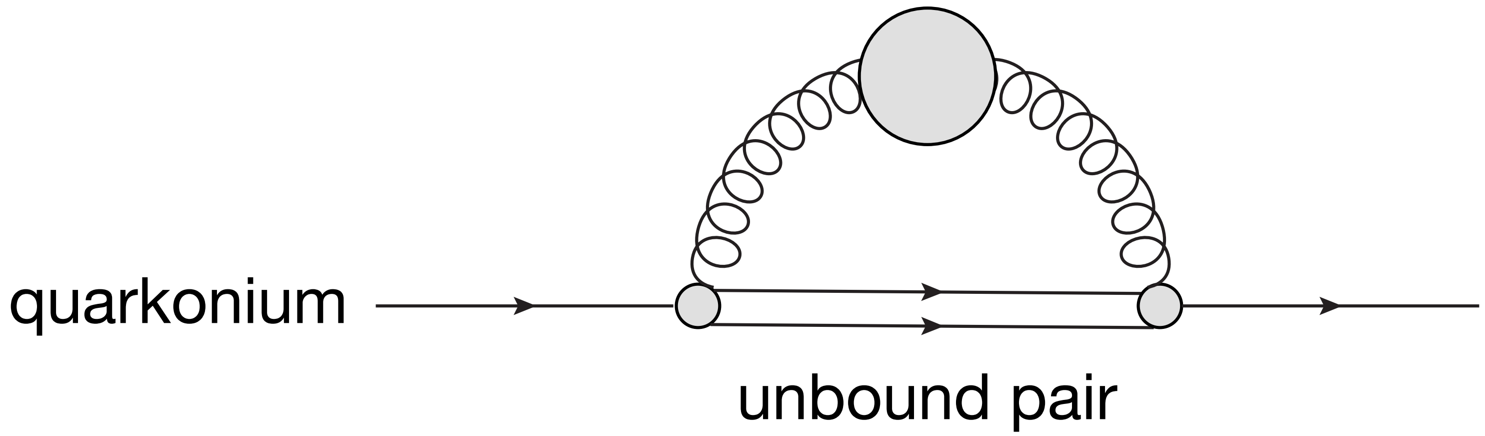
Compute Recombination from QCD

Evolution of distribution in phase space

$$(\partial_t + \boldsymbol{v} \cdot \nabla) f(\boldsymbol{x}, \boldsymbol{p}, t) = -C^{(-)}(\boldsymbol{x}, \boldsymbol{p}, t) + C^{(+)}(\boldsymbol{x}, \boldsymbol{p}, t)$$

Dissociation Recombination

Two screening effects from thermal loops



Real & imaginary parts —> static screening & dissociation

Recombination modeled: $\propto f_Q f_{\bar{Q}}$ / $\propto f^{\mathrm{eq}}_{J/\psi}$ **calculate from QCD?**

Put screening and recombination into same framework?

Importance of correlated recombination?

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Contents

- Derivation of Boltzmann transport equation:
  - Open quantum system
  - Separation of scales, effective field theory
- Phenomenology:
  - Coupled transport equations of open and hidden heavy flavors
  - Impact of correlated recombination on bottomonium production

Open Quantum System

- Total system = subsystem + environment: $H = H_S + H_E + H_I$

$$U(t,0) = \mathcal{T} e^{-i\int_0^t \mathrm{d}t' H_I(t')}$$

$\rho(t=0) = \rho_S \otimes \rho_E$

System & environment

(Heavy quark pairs & QGP)

Unitary evolution

Time reversible

$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$

System & environment

Trace out (integrate out) environment

$\rho_S(t=0)$

System(heavy quark pairs)

Non-unitary

Time irreversible

$\mathrm{Tr}_E\Big[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\Big]$

General Procedure

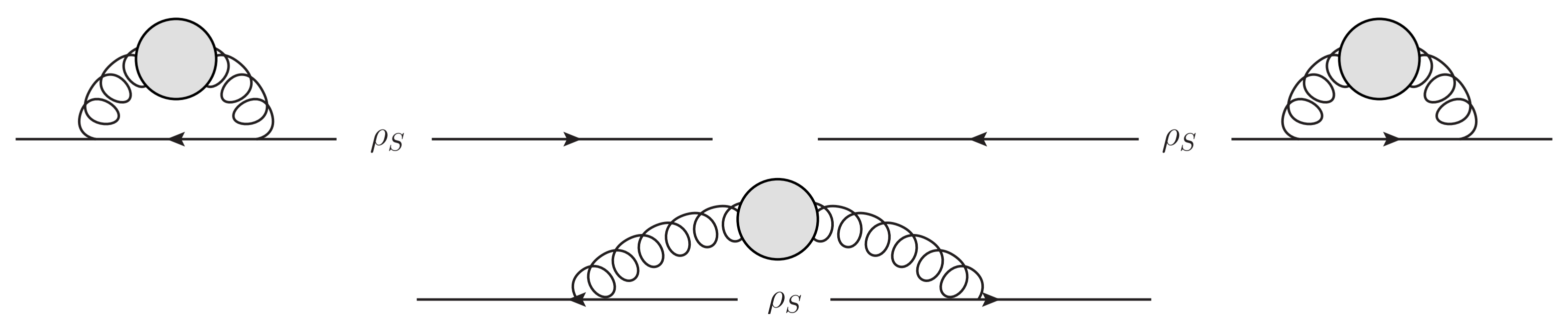
- Assume **weak coupling** between subsystem/environment

$$H = H_S + H_E + \boxed{H_I}$$

- Expand unitary evolution operator (time ordered perturbation theory)

- Trace out environment —> **Lindblad equation**

$$\rho_S(t) = \rho_S(0) - i\Big[tH_S + \sum_{a,b}\sigma_{ab}(t)L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d}\gamma_{ab,cd}(t)\Big(L_{ab}\rho_S(0)L^\dagger_{cd} - \frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S(0)\}\Big)$$



$$H_I = \sum_\alpha O^{(S)}_\alpha \otimes O^{(E)}_\alpha$$

$$\gamma_{ab,cd}(t) \equiv \sum_{\alpha,\beta}\int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1,t_2)\langle a|O^{(S)}_\beta(t_2)|b\rangle\langle c|O^{(S)}_\alpha(t_1)|d\rangle^*$$

$$C_{\alpha\beta}(t_1,t_2) \equiv \mathrm{Tr}_E(O^{(E)}_\alpha(t_1)O^{(E)}_\beta(t_2)\rho_E)$$

$$L_{ab} = |a\rangle\langle b|$$

$|a\rangle$ Eigenstates of H_S

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General Procedure

Lindblad equation:

$$\rho_S(t) = \rho_S(0) - i\Big[tH_S + \sum_{a,b}\sigma_{ab}(t)L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d}\gamma_{ab,cd}\Big(L_{ab}\rho_S(0)L^{\dagger}_{cd} - \frac{1}{2}\{L^{\dagger}_{cd}L_{ab}, \rho_S\}\Big)$$

Markovian approximation (separation of time scales)

Wigner transform (smearing for positivity)

$$f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \langle \boldsymbol{k} + \frac{\boldsymbol{k}'}{2}, nl, 1|\rho_S(t)|\boldsymbol{k} - \frac{\boldsymbol{k}'}{2}, nl, 1\rangle$$

Semiclassical limit

Boltzmann transport equation

$$\frac{\partial}{\partial t} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{(+)}_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{(-)}_{nls}(\boldsymbol{x}, \boldsymbol{k}, t)$$

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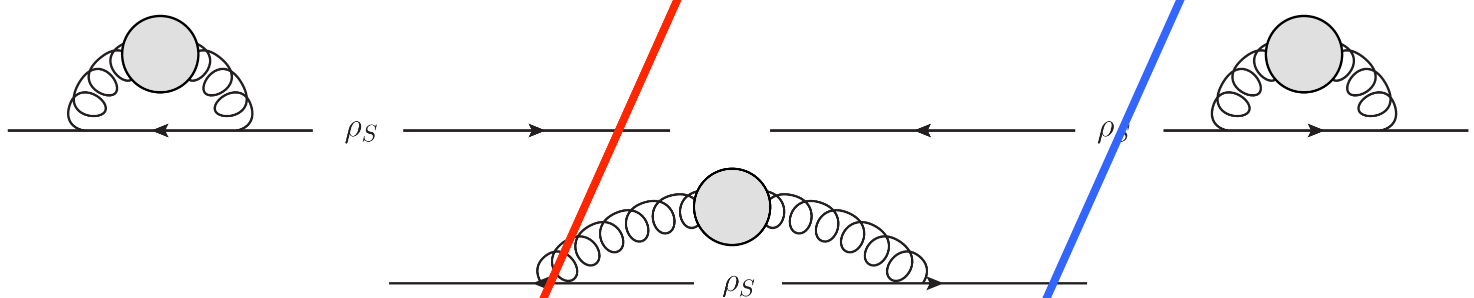
From Open Quantum System to Transport Equation

Lindblad equation:

Correction to Hamiltonian, static screening

$$\rho_S(t) = \rho_S(0) - i\Big[tH_S + \sum_{a,b}\sigma_{ab}(t)L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d}\gamma_{ab,cd}\Big(L_{ab}\rho_S(0)L^{\dagger}_{cd} - \frac{1}{2}\{L^{\dagger}_{cd}L_{ab}, \rho_S\}\Big)$$



Recombination

Dissociation

Boltzmann transport equation

$$\frac{\partial}{\partial t}f_{nls}(\boldsymbol{x},\boldsymbol{k},t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}}f_{nls}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}^{(+)}_{nls}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}^{(-)}_{nls}(\boldsymbol{x},\boldsymbol{k},t)$$

Two Key Assumptions

1. **System interacts weakly with environment ?**

2. **Markovian approximation (no memory effect) ?**

Separation of scales and effective field theory can be used to justify these

Separation of Scales

Separation of scales in vacuum $M \gg Mv \gg Mv^2$

QCD

M — perturbative matching

HQET/NRQCD

Heavy quark physics, A.Manohar, M.Wise

hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

Mv — perturbative / non-perturbative matching

potential NRQCD

$Mv^2 \sim 500\ \mathrm{MeV}$

hep-ph/9907240, N.Brambilla
A.Pineda, J.Soto, A.Vairo

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Inside QGP: thermal scales: T $M \gg Mv \gg Mv^2 \gtrsim T$

pNRQCD in medium

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r\, \mathrm{Tr}\Big(\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - H_o)\mathrm{O} + V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.}) + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots\Big)$$

$$H_s = \frac{(i\boldsymbol{\nabla}_{\mathrm{cm}})^2}{4M} + \boxed{\frac{(i\boldsymbol{\nabla}_{\mathrm{rel}})^2}{M} + V_s^{(0)}} + \frac{V_s^{(1)}}{M} + \frac{V_s^{(2)}}{M^2} + \cdots$$

no hyperfine splitting to lowest order in v

$$|H\rangle \sim |Q\bar{Q}\rangle + |Q\bar{Q}g\rangle + \cdots$$

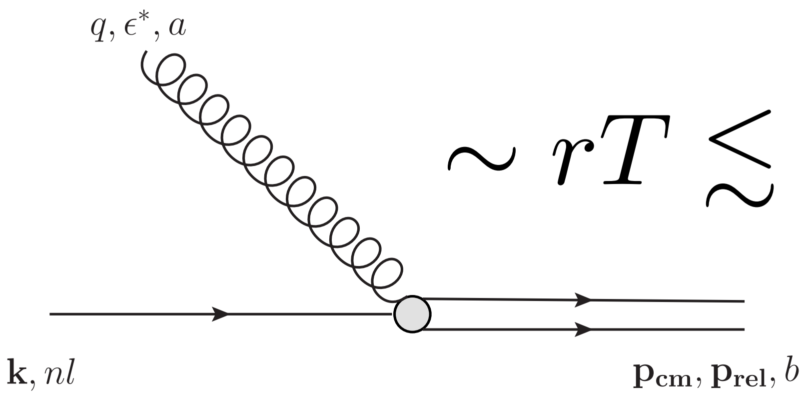
Octet Fock state suppressed in v
Quarkonium = color singlet pair

Weak Coupling & Resummation

Separation of scales $M \gg Mv \gg Mv^2 \gtrsim T$

Dipole interaction

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r\, \mathrm{Tr}\Big(\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{O}^\dagger(iD_0 - H_o)\mathrm{O} + \boxed{V_A(\mathrm{O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.})} + \frac{V_B}{2}\mathrm{O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{O}\} + \cdots\Big)$$



$\sim rT \lesssim v$ **Weak coupling**

$$r \sim \frac{1}{Mv}$$

Arguments breakdown if

(1) large log: Mv —> T, VA has no running at one loop

(2) large pT: medium boosted in rest frame of quarkonium, constrain to low pT

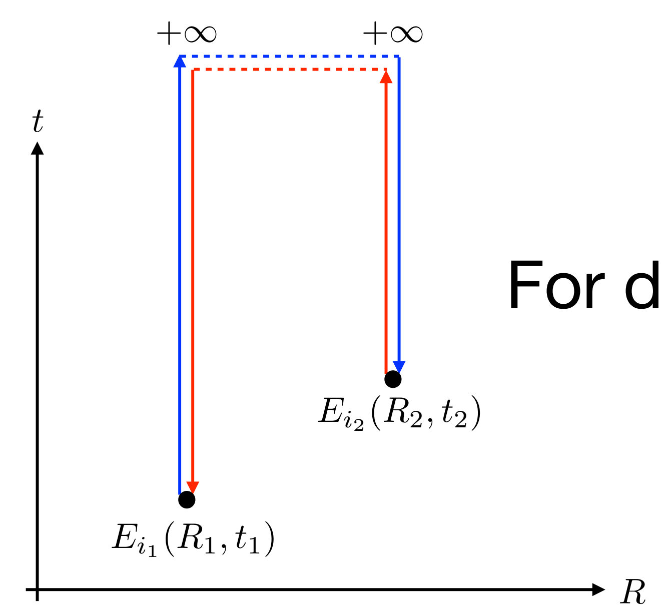
Resum octet-A0 interaction by field redefinition

$$\mathrm{O}(\boldsymbol{R}, \boldsymbol{r}, t) \to W_{[(\boldsymbol{R},t),(\boldsymbol{R},t_L)]}\widetilde{\mathrm{O}}(\boldsymbol{R}, \boldsymbol{r}, t)(W_{[(\boldsymbol{R},t),(\boldsymbol{R},t_R)]})^\dagger$$

$$C_{\alpha\beta}(t_1, t_2) \equiv \mathrm{Tr}_E(O_\alpha^{(E)}(t_1)O_\beta^{(E)}(t_2)\rho_E)$$

$$\downarrow$$

$$\langle WE(\boldsymbol{R}_1, t_1)WE(\boldsymbol{R}_2, t_2)\rangle_T$$



For dissociation

Dissociation $-\gamma_{ab,cd}\frac{1}{2}\{L^{\dagger}_{cd}L_{ab},\rho_S(0)\}$

For $\mathrm{L}^{\dagger}_{\mathrm{cd}}\mathrm{L}_{\mathrm{ab}}\rho_{\mathrm{S}}$: $\gamma_{ab,cd} = \int d^3R_1 \int d^3R_2 \sum_{i_1,i_2,b_1,b_2} \int_0^t dt_1 \int_0^t dt_2\, C_{\boldsymbol{R}_1 i_1 b_1, \boldsymbol{R}_2 i_2 b_2}(t_1,t_2)$

$$\langle \boldsymbol{k}_1, n_1 l_1, 1 | \langle S(\boldsymbol{R}_1,t_1)| r_{i_1} | O^{b_1}(\boldsymbol{R}_1,t_1)\rangle | \boldsymbol{p}_{\mathrm{cm}}, \boldsymbol{p}_{\mathrm{rel}}, a_1\rangle$$

$$\langle \boldsymbol{p}_{\mathrm{cm}}, \boldsymbol{p}_{\mathrm{rel}}, a_1 | \langle O^{b_2}(\boldsymbol{R}_2,t_2)| r_{i_2} | S(\boldsymbol{R}_2,t_2)\rangle | \boldsymbol{k}_3, n_3 l_3, 1\rangle$$

$$\langle \Psi_{\boldsymbol{p}_{\mathrm{rel}}} | r_{i_2} | \psi_{n_3 l_3}\rangle \delta^{a_1 b_2} e^{-i(E_{\boldsymbol{k}_3} t_2 - \boldsymbol{k}_3\cdot\boldsymbol{R}_2)} e^{i(E_{\boldsymbol{p}} t_2 - \boldsymbol{p}_{\mathrm{cm}}\cdot\boldsymbol{R}_2)}$$

Weakly-coupled plasma, LO:

$$\frac{T_F}{N_C} g^2 \delta^{b_1 b_2} \int \frac{d^4q}{(2\pi)^4} e^{iq_0(t_1-t_2) - i\boldsymbol{q}\cdot(\boldsymbol{R}_1-\boldsymbol{R}_2)} (q_0^2 \delta_{i_1 i_2} - q_{i_1} q_{i_2}) n_B(q_0)(2\pi)\mathrm{sign}(q_0)\delta(q_0^2 - \boldsymbol{q}^2)$$

Markovian approximation: $t \longrightarrow \infty$ $\quad \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 e^{i\omega t_1} e^{-i\omega t_2} \xrightarrow{t\to\infty} 2\pi t \delta(\omega)$

E&p conservation

$$tC^{(-)}_{nl} \equiv t \int \frac{d^3p_{\mathrm{cm}}}{(2\pi)^3} \frac{d^3p_{\mathrm{rel}}}{(2\pi)^3} \frac{d^3q}{(2\pi)^3 2q} n_B(q)(2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\mathrm{cm}} + \boldsymbol{q})\delta(E_k - E_p + q)$$

Phase space measure

$$\frac{2}{3} C_F q^2 g^2 |\langle \psi_{nl} | \boldsymbol{r} | \Psi_{\boldsymbol{p}_{\mathrm{rel}}}\rangle|^2 f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t=0)$$

Amplitude squared

Everything Together: Boltzmann Equation

$$\rho_S(t) = \rho_S(0) - it[H_{\text{eff}}, \rho_S(0)] + \sum_{a,b,c,d} \gamma_{ab,cd}\Big(L_{ab}\rho_S(0)L^{\dagger}_{cd} - \frac{1}{2}\{L^{\dagger}_{cd}L_{ab}, \rho_S\}\Big)$$

Wigner transform

$$f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = f_{nl}(\boldsymbol{x}, \boldsymbol{k}, 0) - t\frac{\boldsymbol{k}}{2M}\cdot\nabla f_{nl}(\boldsymbol{x}, \boldsymbol{k}, 0) + tC^{(+)} - tC^{(-)}$$

Dividing by t, set t—> 0 add spin dependence

$$\partial_t f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) = C^{(+)}_{nls} - C^{(-)}_{nls}$$

Not contradictory with t —> ∞

Markovian: environment correlation time << system relaxation time —> coarse-grained

environment correlation time << t << system relaxation time

$$\frac{1}{T} \ll t \ll \frac{1}{v^2 T}$$

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Semiclassical Expansion in Recombination

When evaluating recombination term

$$\int \frac{\mathrm{d}^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}_{\mathrm{cm}}} \left\langle \boldsymbol{p}_{\mathrm{cm}} + \frac{\boldsymbol{k}'}{2}, \boldsymbol{p}_{1\mathrm{rel}} \middle| \rho_S^{(8)}(0) \middle| \boldsymbol{p}_{\mathrm{cm}} - \frac{\boldsymbol{k}'}{2}, \boldsymbol{p}_{2\mathrm{rel}} \right\rangle$$

$$= \int \mathrm{d}^3x_{\mathrm{rel}} e^{-i(\boldsymbol{p}_{1\mathrm{rel}}-\boldsymbol{p}_{2\mathrm{rel}})\cdot\boldsymbol{x}_{\mathrm{rel}}} f_{Q\bar{Q}}^{(8)}\left(\boldsymbol{x}_{\mathrm{cm}}, \boldsymbol{p}_{\mathrm{cm}}, \boldsymbol{x}_{\mathrm{rel}}, \frac{\boldsymbol{p}_{1\mathrm{rel}}+\boldsymbol{p}_{2\mathrm{rel}}}{2}, t=0\right)$$

Classical analog exists for same relative momentum

Gradient expansion

LO = classical

$$f_{Q\bar{Q}}^{(8)}(\boldsymbol{x}_{\mathrm{cm}}, \boldsymbol{p}_{\mathrm{cm}}, \boldsymbol{x}_{\mathrm{rel}}, \frac{\boldsymbol{p}_{1\mathrm{rel}}+\boldsymbol{p}_{2\mathrm{rel}}}{2}, t) = \underline{f_{Q\bar{Q}}^{(8)}(\boldsymbol{x}_{\mathrm{cm}}, \boldsymbol{p}_{\mathrm{cm}}, \boldsymbol{x}_0, \frac{\boldsymbol{p}_{1\mathrm{rel}}+\boldsymbol{p}_{2\mathrm{rel}}}{2}, t)}$$

$$+ \underline{(\boldsymbol{x}_{\mathrm{rel}} - \boldsymbol{x}_0)\cdot\nabla_{\boldsymbol{x}_0} f_{Q\bar{Q}}^{(8)}(\boldsymbol{x}_{\mathrm{cm}}, \boldsymbol{p}_{\mathrm{cm}}, \boldsymbol{x}_0, \frac{\boldsymbol{p}_{1\mathrm{rel}}+\boldsymbol{p}_{2\mathrm{rel}}}{2}, t)} + \cdots$$

NLO = leading quantum correction

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Importance of Scale Hierarchy

Success of transport equation in quarkonium phenomenology

↕

Separation of scales $M \gg Mv \gg Mv^2 \gtrsim T$

Importance of Scale Hierarchy

Success of transport equation in quarkonium phenomenology

↕

Separation of scales $M \gg Mv \gg Mv^2 \gtrsim T$

What if hierarchy breaks down?

$$M \gg Mv \gg T \gg Mv^2$$

Practically not possible:
v ~ 0.3 for bottomonium

$$M \gg Mv \sim T \gg Mv^2$$

Dipole vertex no longer works

No well-defined bound state

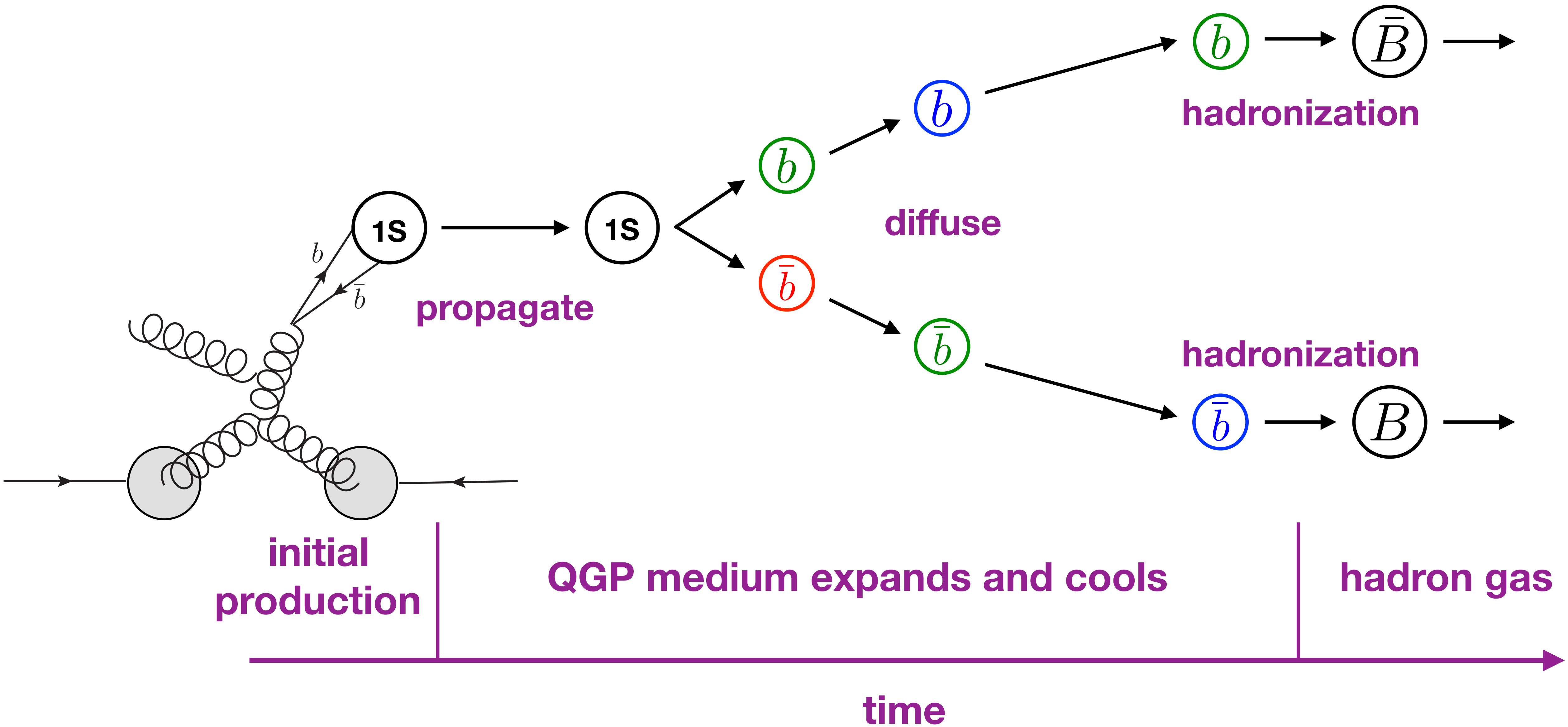
Coupled Transport Equations of Heavy Flavors

open heavy quark antiquark

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q \cdot \nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}} \cdot \nabla_{\boldsymbol{x}_{\bar{Q}}}\right) f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}^+_{Q\bar{Q}} + \mathcal{C}^-_{Q\bar{Q}}$$

each quarkonium state
nl = 1S, 2S,1P etc.

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}^+_{nls} - \mathcal{C}^-_{nls}$$



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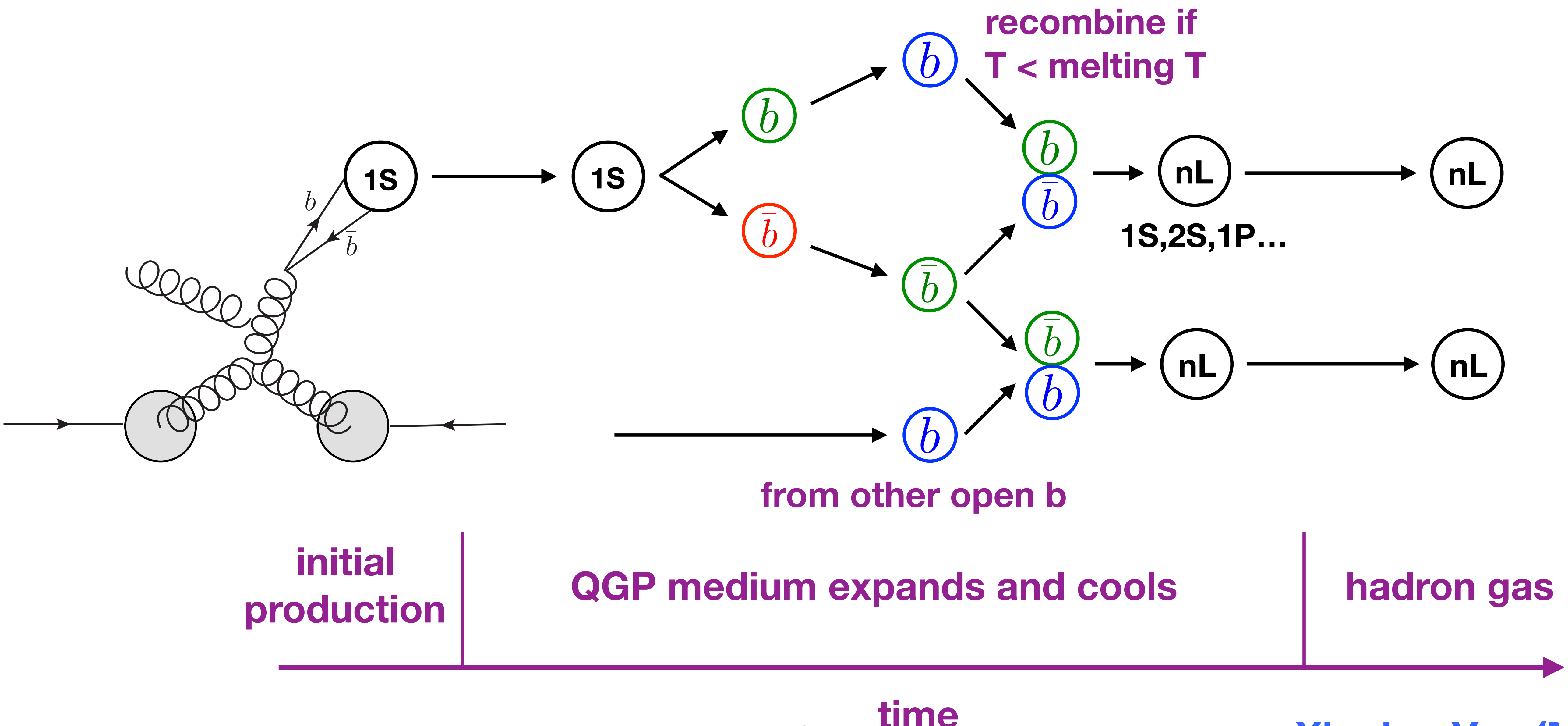

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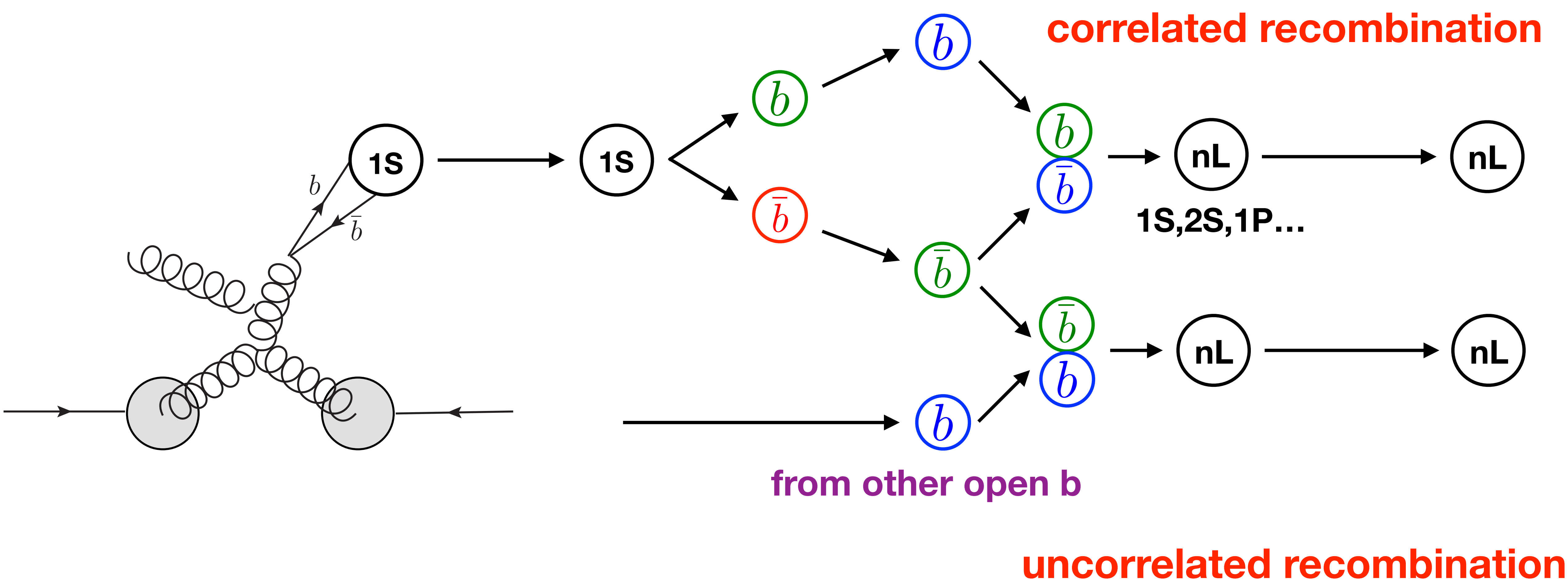
Coupled Transport Equations of Heavy Flavors

open heavy quark antiquark

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q \cdot \nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}} \cdot \nabla_{\boldsymbol{x}_{\bar{Q}}}\right) f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}^{+}_{Q\bar{Q}} + \mathcal{C}^{-}_{Q\bar{Q}}$$

each quarkonium state
nl = 1S, 2S,1P etc.

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}^{+}_{nls} - \mathcal{C}^{-}_{nls}$$



Xiaojun Yao (MIT)

Coupled with Transport of Open Heavy Flavor

heavy quark antiquark

$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}}_Q \cdot \nabla_{\boldsymbol{x}_Q} + \dot{\boldsymbol{x}}_{\bar{Q}} \cdot \nabla_{\boldsymbol{x}_{\bar{Q}}}) f_{Q\bar{Q}}(\boldsymbol{x}_Q, \boldsymbol{p}_Q, \boldsymbol{x}_{\bar{Q}}, \boldsymbol{p}_{\bar{Q}}, t) = \mathcal{C}_{Q\bar{Q}} - \mathcal{C}^{+}_{Q\bar{Q}} + \mathcal{C}^{-}_{Q\bar{Q}}$$

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$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}^{+}_{nls} - \mathcal{C}^{-}_{nls}$$

Can handle both correlated and uncorrelated recombination

$$\mathcal{C}_{Q\bar{Q}} = \mathcal{C}_Q + \mathcal{C}_{\bar{Q}}$$

Each independently interact with medium:
(1) Potential between pair screened
(2) Potential depends on color, average = 0

We use “Lido” for open heavy flavor transport

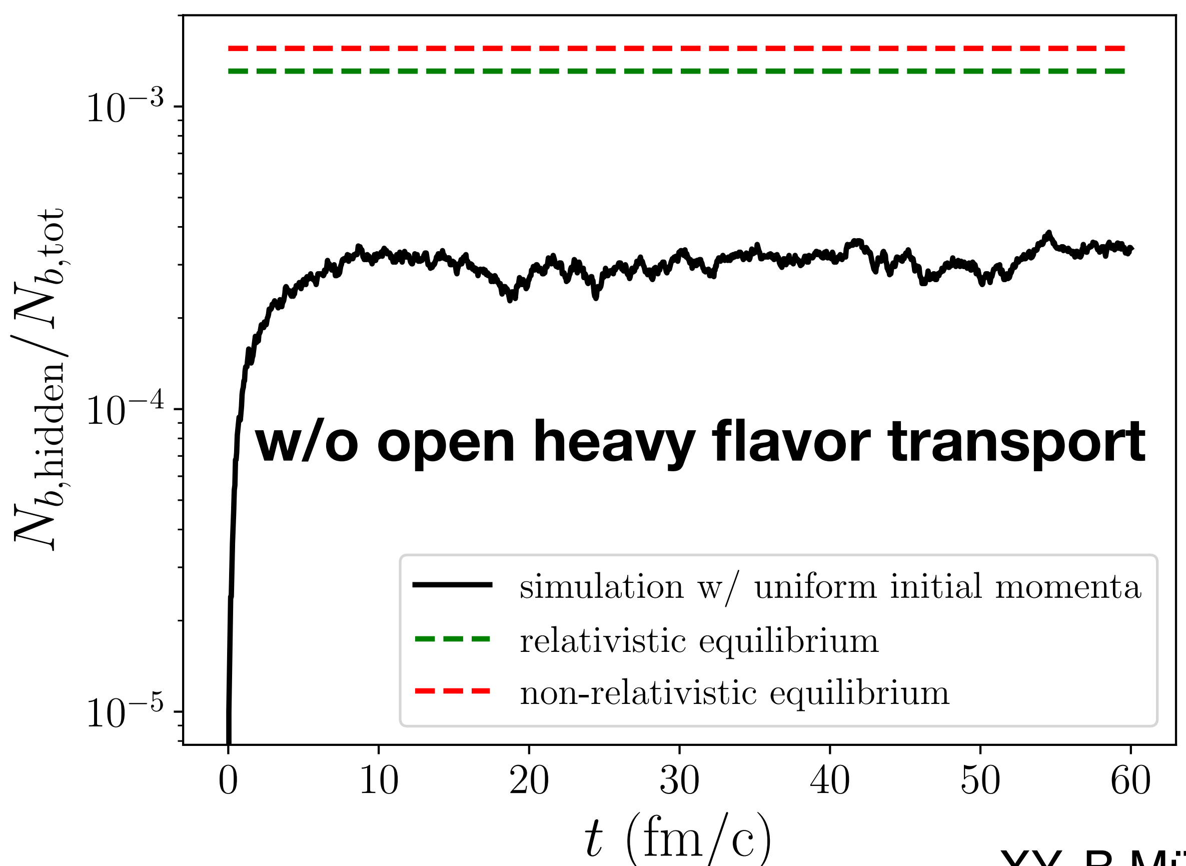
W.Ke, Y.Xu, S.A.Bass, PRC 98, 064901 (2018)

Detailed Balance and Thermalization

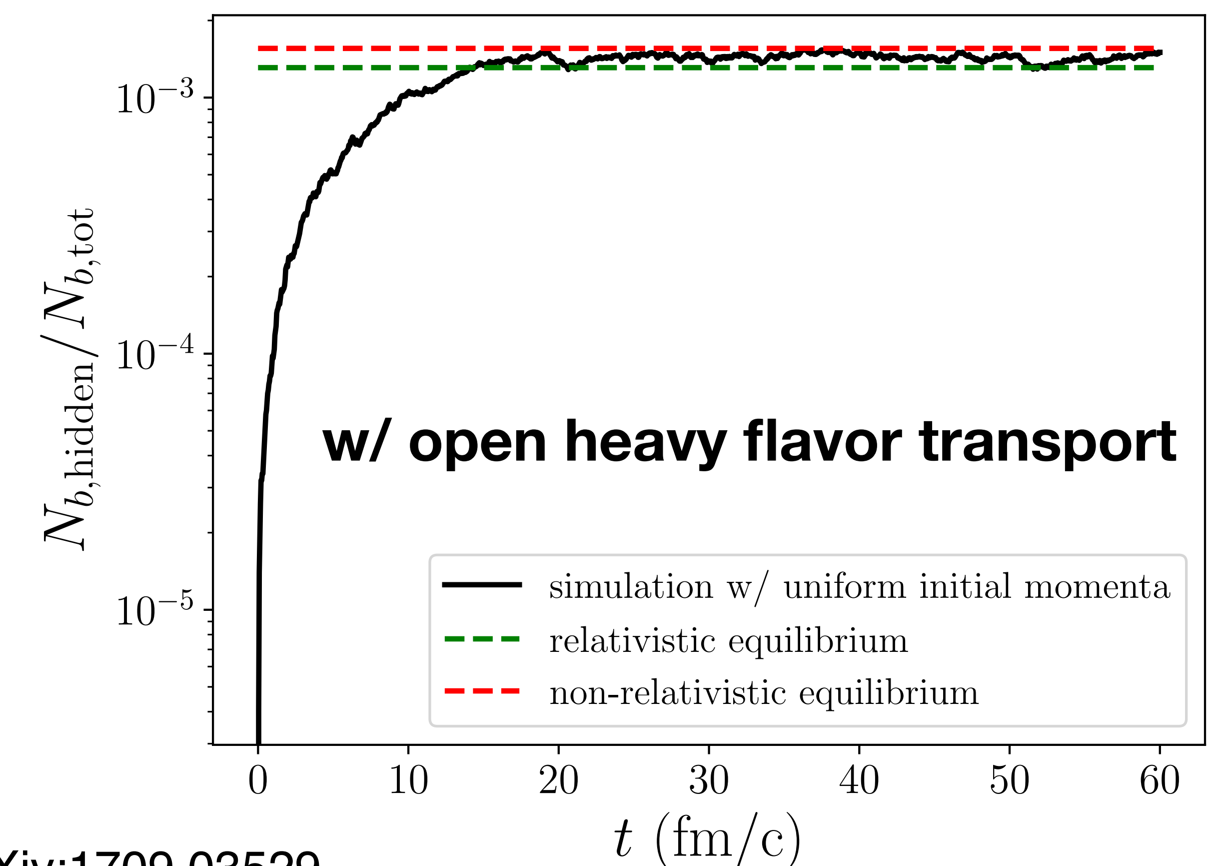
Setup:

- QGP box w/ const T=300 MeV, 1S state & b quarks, total b flavor = 50 (fixed)
- Initial momenta sampled from uniform distributions 0-5 GeV
- Turn on/off open heavy quark transport

Quarkonium percentage v.s. time



Quarkonium percentage v.s. time



XY, B.Müller arXiv:1709.03529

Dissociation-recombination interplay drives to detailed balance

Heavy quark energy gain/loss necessary to drive kinetic equilibrium of quarkonium

Upsilon in 5020 GeV PbPb Collision

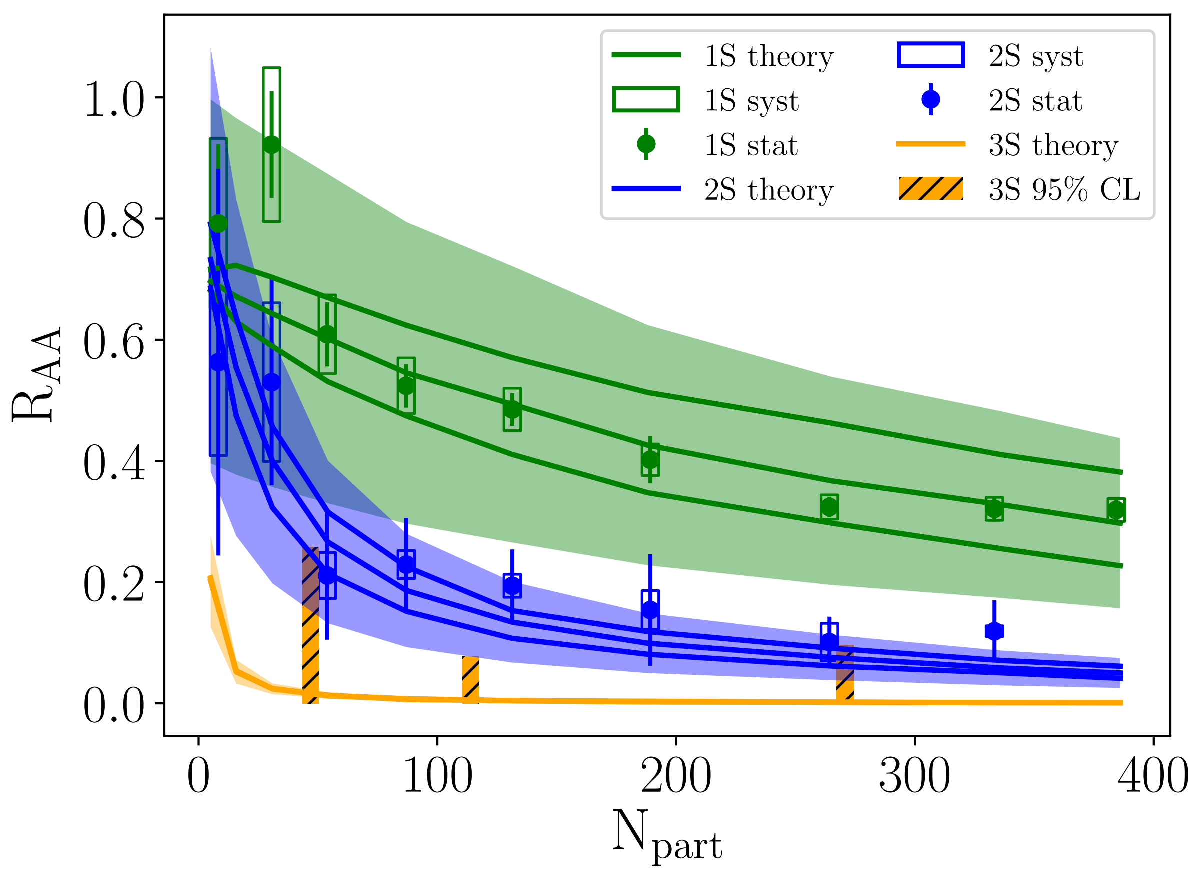
Coulomb potential $\alpha_s^{\text{pot}} = 0.36 \quad \alpha_s = 0.3$ vary by +(-)10%

Pythia + nuclear PDF: EPPS16, uncertainty band

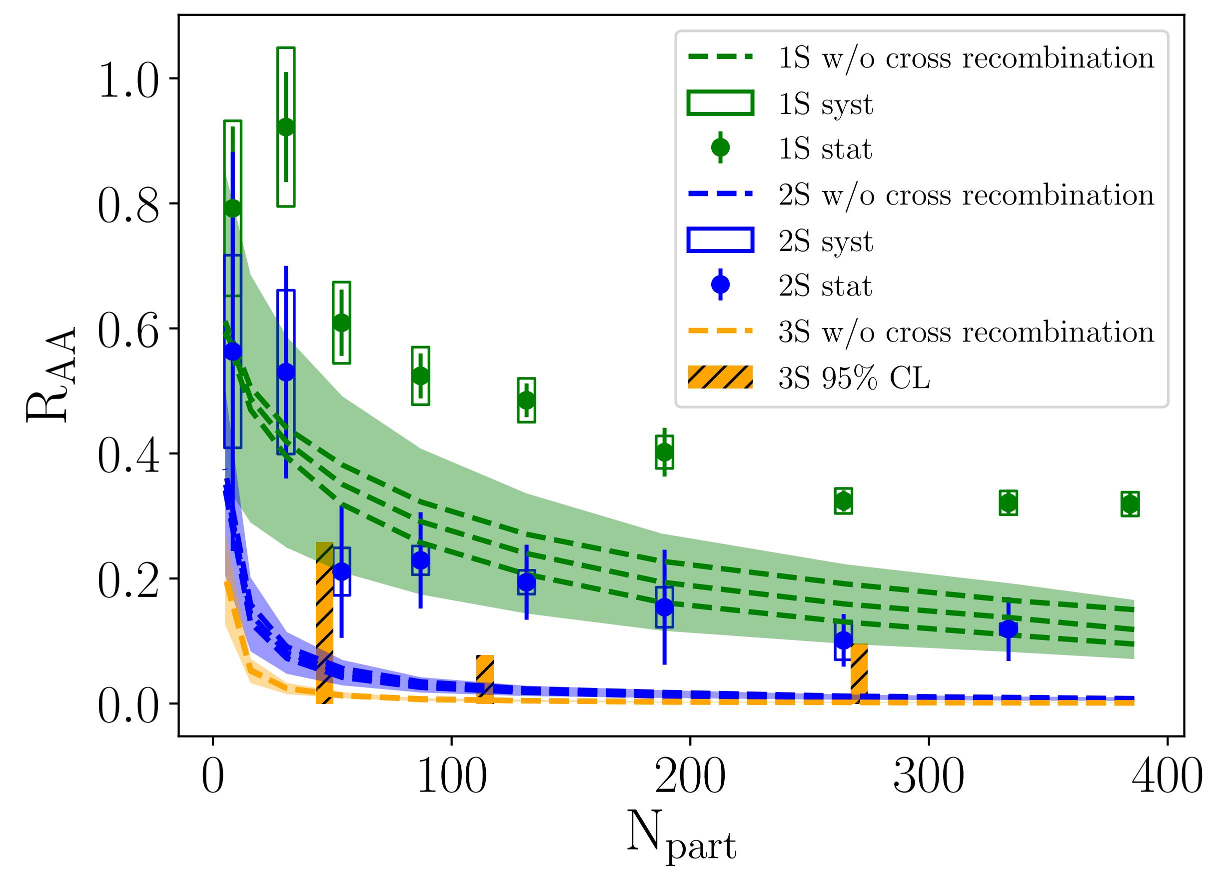
2+1D viscous hydro (calibrated)

Bottomonium: 1S, 2S, 3S, 1P, 2P

with cross-talk (correlated) recombination

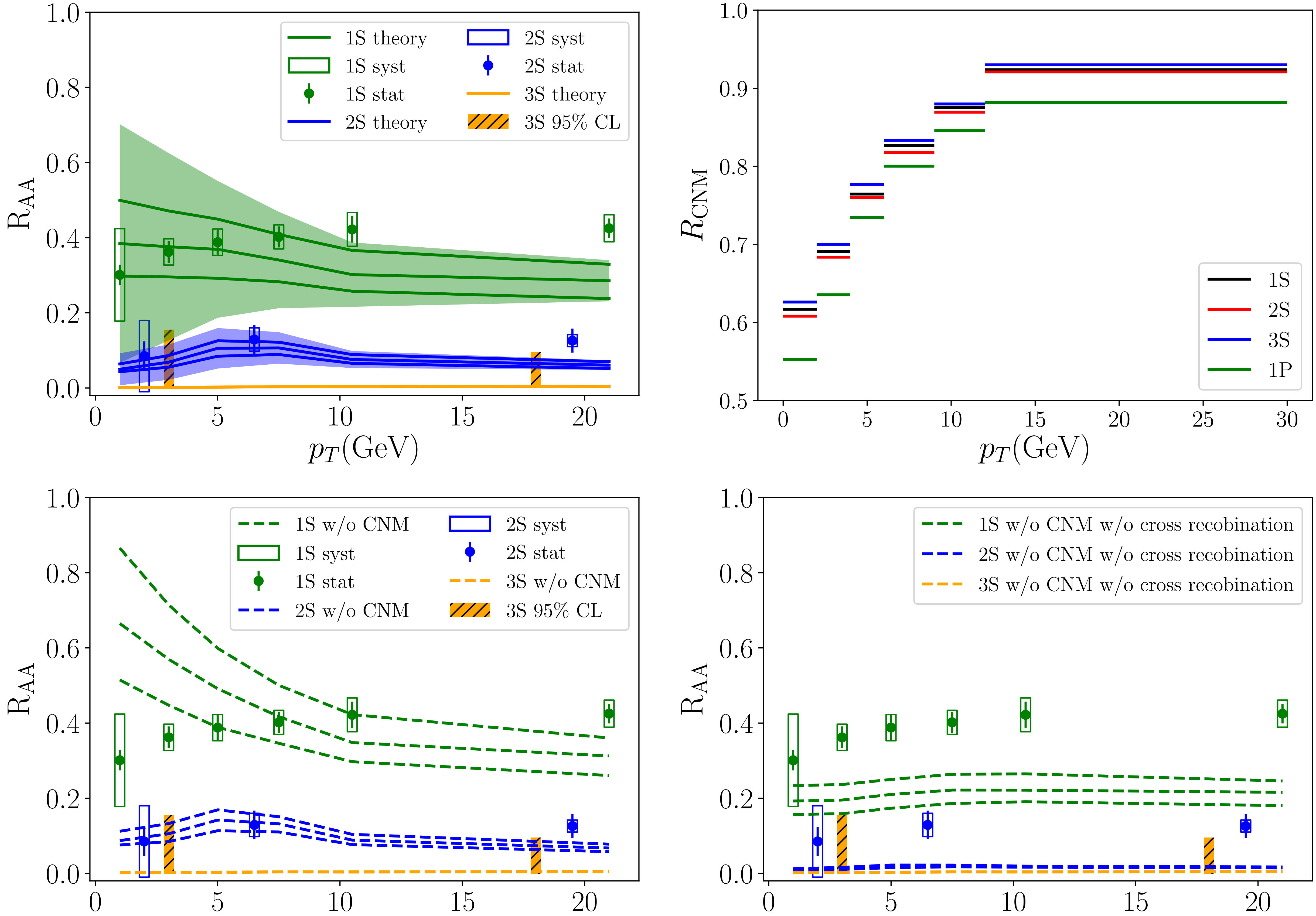


without cross-talk recombination

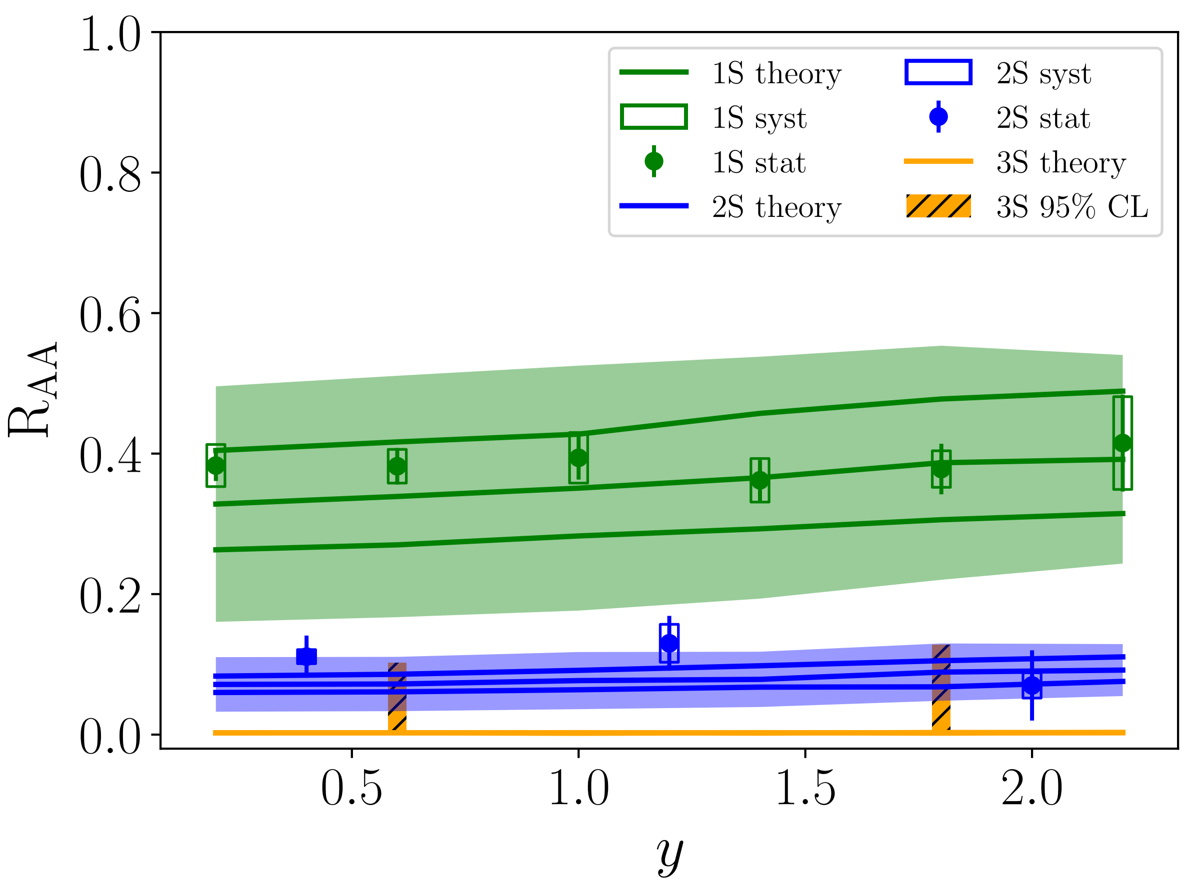


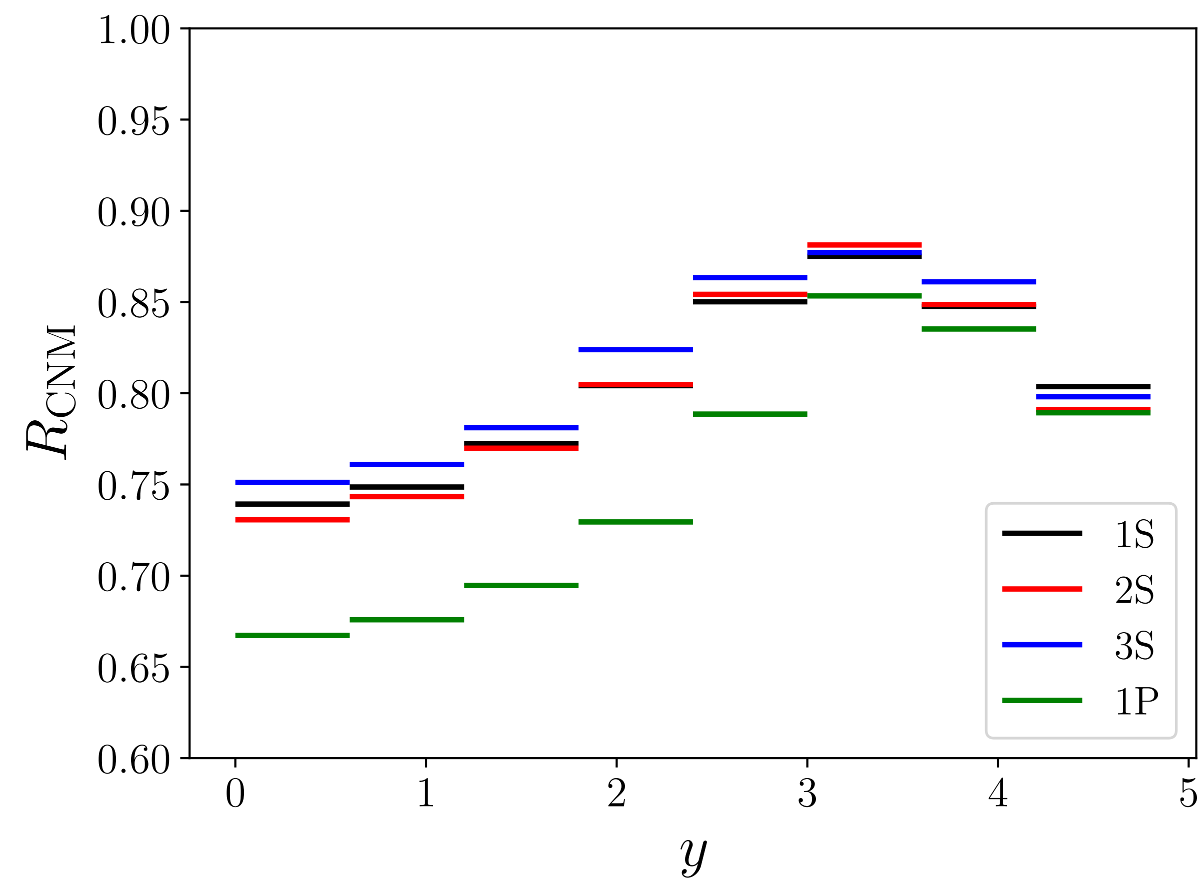
CMS

Upsilon in 5020 GeV PbPb Collision



Upsilon in 5020 GeV PbPb Collision



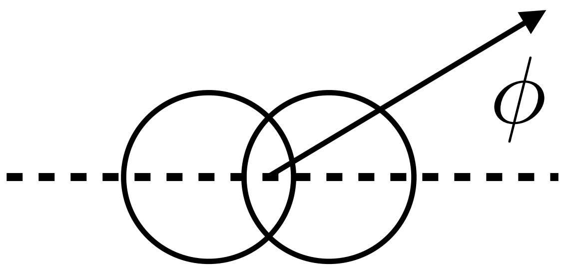


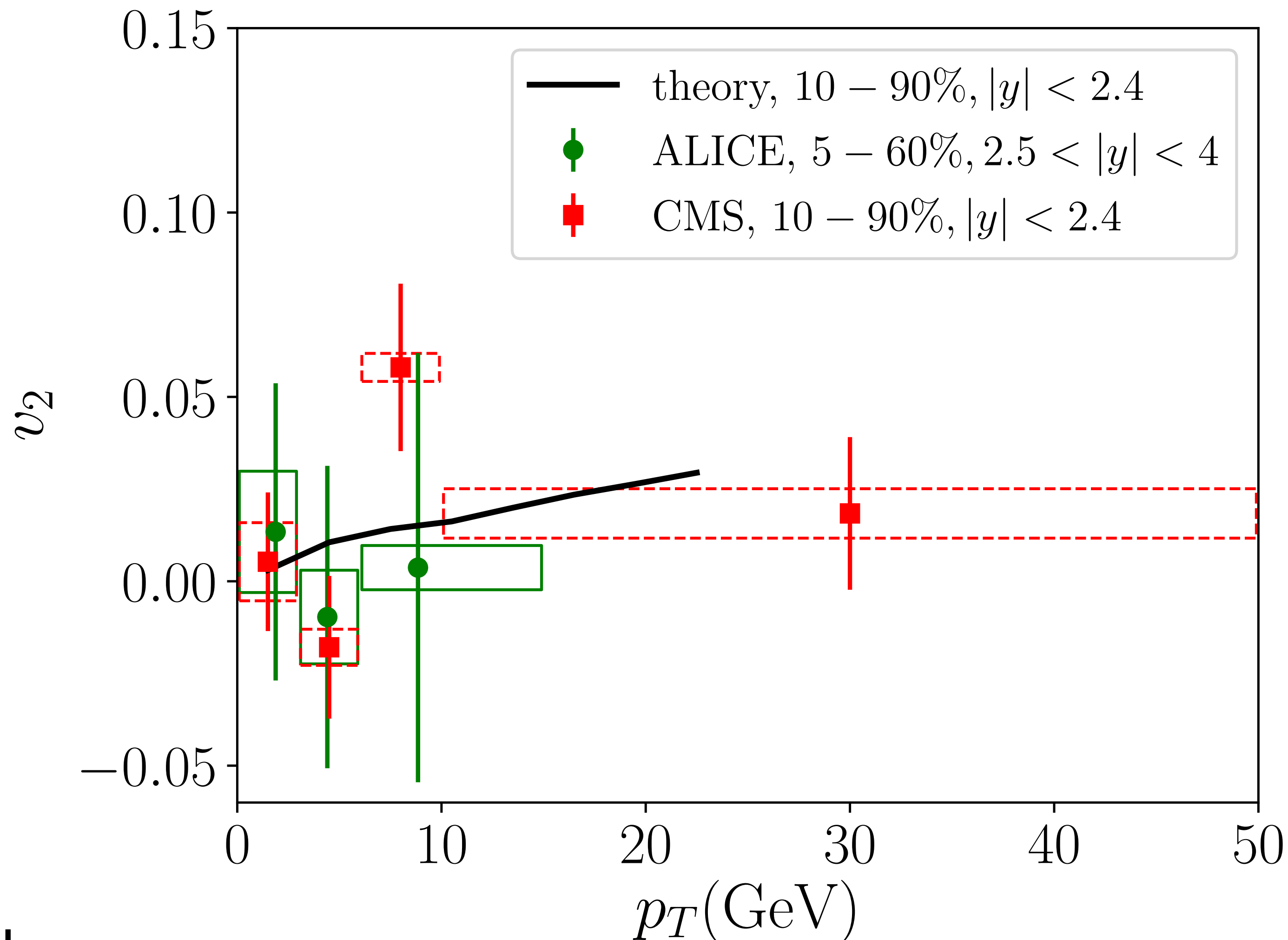
Flat y dependence:

1. medium description is longitudinally boost invariant
2. nPDF mild dependence on y

Upsilon(1S) Azimuthal Anisotropy in 5020 GeV PbPb

$$E\frac{d^3N}{dp^3} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}(1 + 2v_2\cos(2\phi) + \cdots)$$



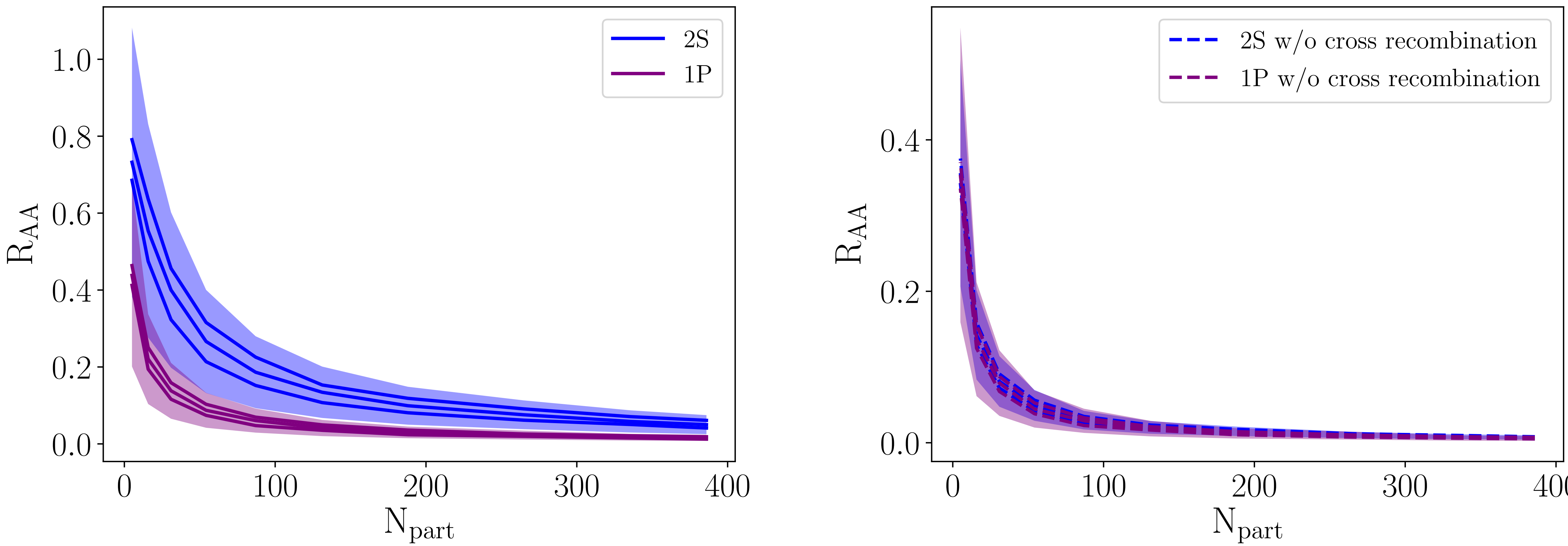


ALICE arXiv:1907.03169
CMS-PAS-HIN-19-002

v2 from:

1. path dependence
2. reaction rates depend on relative velocity between medium and quarkonium
3. correlated recombination: medium interaction after dissociation but before recombination; uncorrelated b-quarks negligible (different for charm)

Experimental Evidence of Correlated Recombination



Dissociation rate of 1P ~ dissociation rate of 2S, due to similar binding energy/size

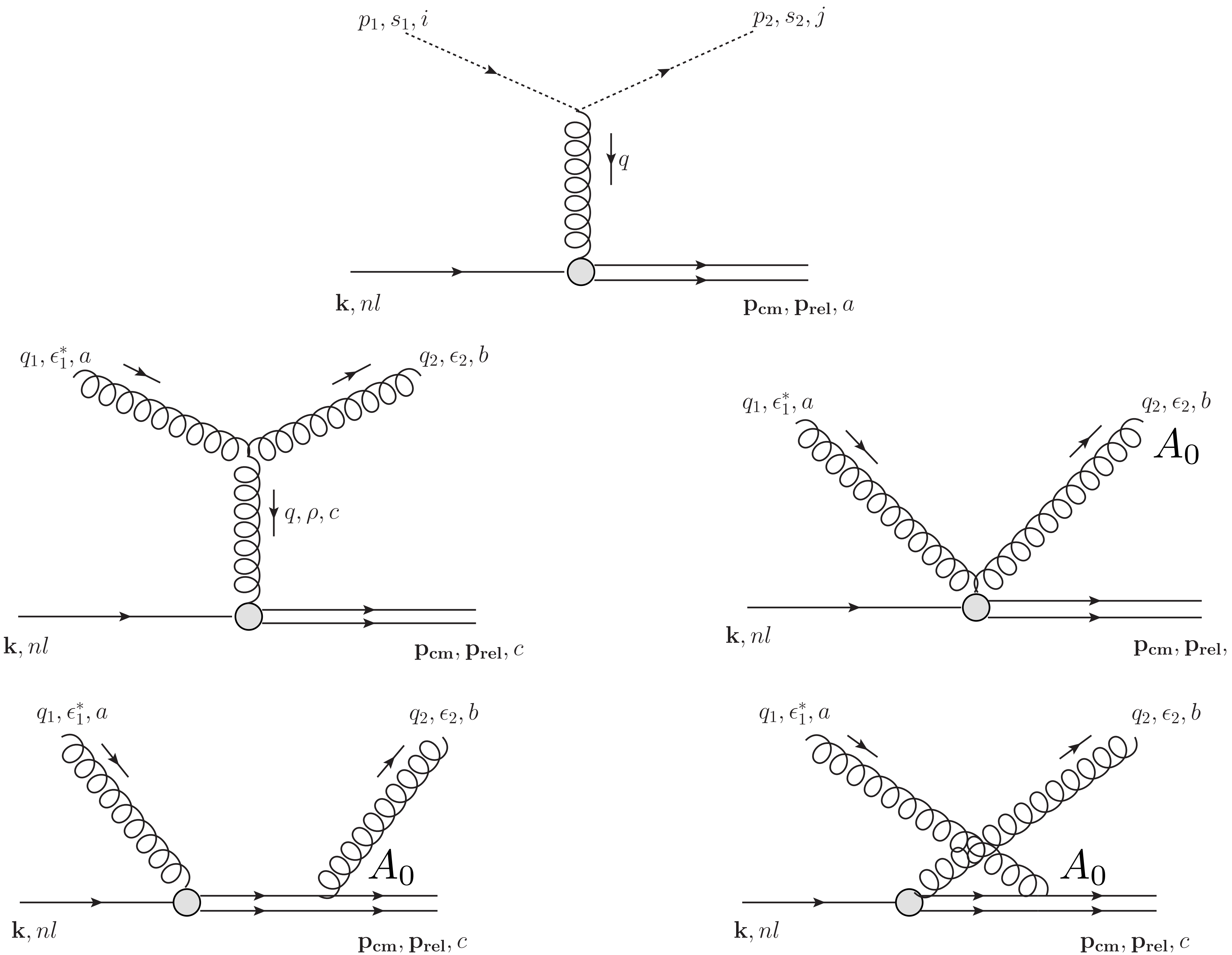
In medium, P(1P—>2S) ~ P(2S—>1P), via dissociation and correlated recombination

But more 1P states produced initially than 2S, so more 2S regenerated than 1P

Conclusion

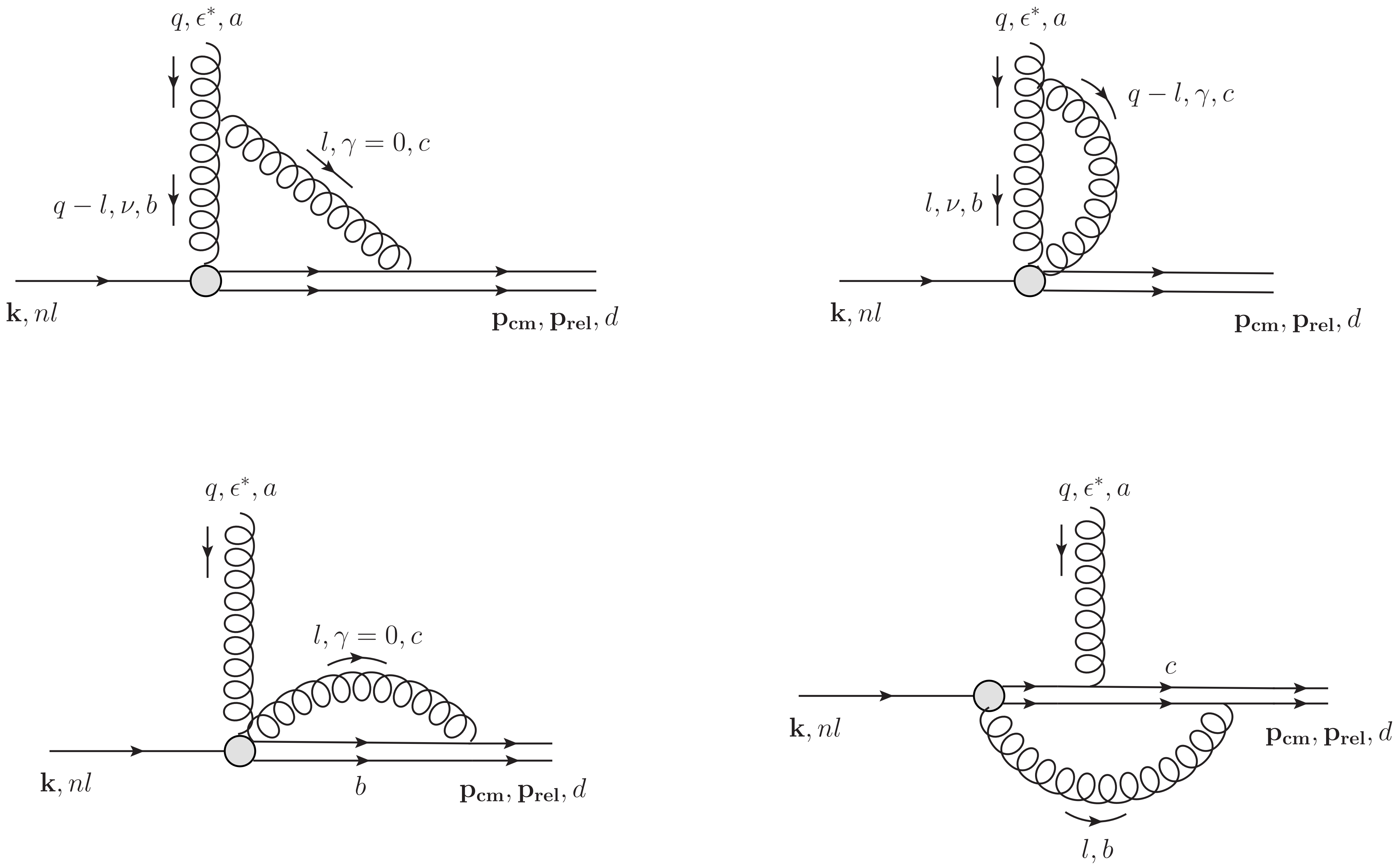
- Quarkonium transport inside QGP:
  - Derivation of Boltzmann equation from open quantum system and effective field theory
  - Hierarchy of scales, nonrelativistic expansion, weak coupling, Markovian limit
- Phenomenological results from coupled transport equations, importance of correlated recombination for bottomonium
- Experimental test of correlated recombination: 1P v.s. 2S

Backup: NLO Contributions



and more …

Backup: Running of Dipole Interaction



$$\frac{0}{\epsilon} + \cdots \qquad \frac{d}{d\mu} V_A(\mu) = 0$$

A. Pineda and J. Soto, Phys. Lett. B 495, 323 (2000)

XY, B.Müller arXiv:1811.09644

Backup: Numerical Implementation

- **Test particle Monte Carlo** $f(\boldsymbol{x}, \boldsymbol{p}, t) = \sum_i \delta^3(\boldsymbol{x} - \boldsymbol{y}_i(t))\delta^3(\boldsymbol{p} - \boldsymbol{k}_i(t))$

- Each time step: read in hydro-cell velocity, temperature; consider diffusion, dissociation, recombination in particle's rest frame and boost back

- If specific process occurs, sample incoming medium particles and outgoing particles from differential rates, conserving energy momentum

- Recombination term contains $f_{Q\bar{Q}}(\boldsymbol{x}_1, \boldsymbol{p}_1, \boldsymbol{x}_2, \boldsymbol{p}_2, t)$

For each HQ, search anti-HQ within a radius, weighted sum

$$f_{Q\bar{Q}}(\boldsymbol{x}_1, \boldsymbol{p}_1, \boldsymbol{x}_2, \boldsymbol{p}_2, t) = \sum_{i,j} \frac{e^{-(\boldsymbol{y}_i - \bar{\boldsymbol{y}}_j)^2/2a_B^2}}{(2\pi a_B)^{3/2}} \delta^3\left(\frac{\boldsymbol{x}_1 + \boldsymbol{x}_2}{2} - \frac{\boldsymbol{y}_i + \bar{\boldsymbol{y}}_j}{2}\right)\delta^3(\boldsymbol{p}_1 - \boldsymbol{k}_i)\delta^3(\boldsymbol{p}_2 - \bar{\boldsymbol{k}}_j)$$

Backup: Nuclear PDF