Quarkonium Transport in Quark-Gluon Plasma: Open Quantum System & Effective Field Theory

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IPMU APEC Seminar

June 17, 2020

XY B. Müller, 1709.03529, 1811.09644
XY T. Mehen, 1811.07027, in preparation
XY W. Ke, Y. Xu, S. Bass, B. Müller, 2004.06746
Heavy Ion Collisions and Quark-Gluon Plasma

- Asymptotic freedom $\rightarrow$ deconfined phase of QCD expected at high temperature / density $\rightarrow$ quark-gluon plasma (QGP)

- Study QGP: heavy ion collision experiments at RHIC and LHC

- Quark gluon plasma created in Au-Au / Pb-Pb collisions: nearly “perfect” liquid (small viscosity), strongly coupled, temperature $\sim$150-500 MeV, lifetime $\sim$ 10 fm/c

- Hard probes of QGP: jets, heavy quarks; large scale involved
Quarkonium as Probe of Quark-Gluon Plasma

- Quarkonium: bound state of $Q\bar{Q}$, nonrelativistic potential description

- **Static screening**: suppression of color attraction $\rightarrow$ melting at high $T$ $\rightarrow$ reduced production $\rightarrow$ thermometer

\[ T = 0 : V(r) = -\frac{A}{r} + Br \quad \rightarrow \quad T \neq 0 : \text{Confining part flattened} \]
**Quarkonium as Probe of Quark-Gluon Plasma**

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Quarkonium as Probe of Quark-Gluon Plasma

- **Static screening**: suppression of color attraction $\rightarrow$ melting at high $T$ $\rightarrow$ reduced production $\rightarrow$ thermometer

- **Dynamical screening**: dissociation induced by dynamical process, imaginary potential

![Diagram showing quarkonium states and energy changes](image-url)
Quarkonium as Probe of Quark-Gluon Plasma

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- **Dynamical screening**: dissociation induced by dynamical process, imaginary potential

- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting $T$

RL. Thews, M. Schroedter, J. Rafelski

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Recombination Crucial for Phenomenology

Recombination needed to explain data on charmonium suppression

\[ R_{AA} = \frac{\sigma_{AA}}{N_{\text{coll}} \sigma_{pp}} \]

Enhanced uncorrelated (re)combination

\[ J/\psi \rightarrow \mu^+ \mu^- \]

Inclusive J/\(\psi\) \(\rightarrow\) \(\mu^+ \mu^-\)

- ALICE, Pb-Pb \(\sqrt{s_{\text{TNN}}} = 5.02\) TeV, \(2.5 < y < 4, p_T < 8\) GeV/c
- ALICE, Pb-Pb \(\sqrt{s_{\text{TNN}}} = 2.76\) TeV, \(2.5 < y < 4, p_T < 8\) GeV/c
- PHENIX, Au-Au \(\sqrt{s_{\text{TNN}}} = 0.2\) TeV, \(1.2 < |y| < 2.2, p_T > 0\) GeV/c

\(\langle N_{\text{part}} \rangle\)

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Success of Semiclassical Transport

Evolution of distribution in phase space

\[(\partial_t + \mathbf{v} \cdot \nabla) f(\mathbf{x}, \mathbf{p}, t) = -C^{(-)}(\mathbf{x}, \mathbf{p}, t) + C^{(+)}(\mathbf{x}, \mathbf{p}, t)\]

Why semiclassical transport equation successful? Connection to QCD?

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Compute Recombination from QCD

Evolution of distribution in phase space

\[(\partial_t + v \cdot \nabla) f(x, p, t) = -C^{(-)}(x, p, t) + C^{(+)}(x, p, t)\]

**Dissociation**  **Recombination**

Two screening effects from thermal loops

Real & imaginary parts \(\rightarrow\) static screening & dissociation

Recombination modeled:

- \(\propto f_Q f_{\bar{Q}}\)
- \(\propto f_{J/\psi}^{\text{eq}}\)

Calculate from QCD?

Put screening and recombination into same framework?

Importance of correlated recombination?
Contents

• Derivation of Boltzmann transport equation:
  • Open quantum system
  • Separation of scales, effective field theory

• Phenomenology:
  • Coupled transport equations of open and hidden heavy flavors
  • Impact of correlated recombination on bottomonium production
**Open Quantum System**

- Total system = subsystem + environment: \( H = H_S + H_E + H_I \)

\[
U(t, 0) = \mathcal{T} e^{-i \int_0^t dt' H_I(t')} \]

\( \rho(t = 0) = \rho_S \otimes \rho_E \)

**System & environment**

*(Heavy quark pairs & QGP)*

**Unitary evolution**

**Time reversible**

\( U(t, 0)(\rho_S \otimes \rho_E)U^\dagger(t, 0) \)

**System & environment**

**Trace out (integrate out) environment**

\( \rho_S(t = 0) \)

**System (heavy quark pairs)**

**Non-unitary**

**Time irreversible**

\( \text{Tr}_E \left[ U(t, 0)(\rho_S \otimes \rho_E)U^\dagger(t, 0) \right] \)

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General Procedure

- Assume **weak coupling** between subsystem/environment
  \[ H = H_S + H_E + H_I \]
- Expand unitary evolution operator (time ordered perturbation theory)
  \[
  \rho_S(t) = \rho_S(0) - i \left[ tH_S + \sum_{a,b} \sigma_{ab}(t)L_{ab}, \rho_S(0) \right] + \sum_{a,b,c,d} \gamma_{ab,cd}(t) \left( L_{ab}\rho_S(0)L_{cd}^\dagger - \frac{1}{2} \{L_{cd}^\dagger L_{ab}, \rho_S(0)\} \right)
  \]
  
  \[ H_I = \sum_{\alpha} O^{(S)}_{\alpha} \otimes O^{(E)}_{\alpha} \]
  
  \[ \gamma_{ab,cd}(t) \equiv \sum_{\alpha,\beta} \int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1, t_2) \langle a|O^{(S)}_{\beta}(t_2)|b\rangle \langle c|O^{(S)}_{\alpha}(t_1)|d\rangle^* \]
  
  \[ C_{\alpha\beta}(t_1, t_2) \equiv \text{Tr}_E(O^{(E)}_{\alpha}(t_1)O^{(E)}_{\beta}(t_2)\rho_E) \]
  
  \[ L_{ab} = |a\rangle \langle b| \] 
  
  \( |a\rangle \) Eigenstates of \( H_S \)
General Procedure

Lindblad equation:

\[
\rho_S(t) = \rho_S(0) - i \left[ tH_S + \sum_{a,b} \sigma_{ab}(t)L_{ab}, \rho_S(0) \right] + \sum_{a,b,c,d} \gamma_{ab,cd} \left( L_{ab}\rho_S(0)L_{cd}^\dagger - \frac{1}{2} \{L_{cd}^\dagger L_{ab}, \rho_S \} \right)
\]

Markovian approximation (separation of time scales)

Wigner transform (smearing for positivity)

\[
f_{nl}(x, k, t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{ik' \cdot x} \langle k' + \frac{k'}{2}, nl, 1 | \rho_S(t) | k - \frac{k'}{2}, nl, 1 \rangle
\]

Semiclassical limit

Boltzmann transport equation

\[
\frac{\partial}{\partial t} f_{nl}(x, k, t) + \mathbf{v} \cdot \nabla_x f_{nl}(x, k, t) = C_{nl}^{(+)}(x, k, t) - C_{nl}^{(-)}(x, k, t)
\]
From Open Quantum System to Transport Equation

**Lindblad equation:**

\[
\rho_S(t) = \rho_S(0) - i \left[ \frac{t}{2} \mathcal{H}_S + \sum_{a,b} \sigma_{ab}(t) L_{ab}, \rho_S(0) \right] + \sum_{a,b,c,d} \gamma_{ab,cd} \left( L_{ab} \rho_S(0) L_{cd}^\dagger - \frac{1}{2} \{ L_{cd}^\dagger L_{ab}, \rho_S \} \right)
\]

**Boltzmann transport equation**

\[
\frac{\partial}{\partial t} f_{nls}(\mathbf{x}, \mathbf{k}, t) + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_{nls}(\mathbf{x}, \mathbf{k}, t) = C_{nls}^{(+)}(\mathbf{x}, \mathbf{k}, t) - C_{nls}^{(-)}(\mathbf{x}, \mathbf{k}, t)
\]

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Two Key Assumptions

1. System interacts weakly with environment?

2. Markovian approximation (no memory effect)?

Separation of scales and effective field theory can be used to justify these
## Separation of Scales

**Separation of scales in vacuum**

\[ M \gg Mv \gg Mv^2 \]

<table>
<thead>
<tr>
<th>QCD</th>
<th>perturbative matching</th>
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<td>( M )</td>
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**HQET/NRQCD**

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**potential NRQCD**

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- Heavy quark physics, A.Manohar, M.Wise
  - hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage
- Separation of scales
- Perturbative matching
- Non-perturbative matching
- Potential NRQCD

- hep-ph/9907240, N.Brambilla
- A.Pineda, J.Soto, A.Vairo

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Separation of Scales

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**Inside QGP: thermal scales: \( T \)**  

\[ M \gg M v \gg M v^2 \gg T \]

**pNRQCD in medium**

\[ \mathcal{L}_{pNRQCD} = \int d^3 r \text{ Tr} \left( S^\dagger \left( i \partial_0 - H_s \right) S + O^\dagger \left( i D_0 - H_o \right) O + V_A (O^\dagger \mathbf{r} \cdot \mathbf{gE} S + \text{h.c.}) + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot \mathbf{gE}, O \} + \cdots \right) \]

\[ H_s = \frac{(i \nabla_{\text{cm}})^2}{4M} + \frac{(i \nabla_{\text{rel}})^2}{M} + V_s^{(0)} + \frac{V_s^{(1)}}{M} + \frac{V_s^{(2)}}{M^2} + \cdots \]

no hyperfine splitting to lowest order in \( v \)

\[ |H\rangle \sim |Q\bar{Q}\rangle + |Q\bar{Q}g\rangle + \cdots \]

Octet Fock state suppressed in \( v \)

Quarkonium = color singlet pair

Heavy quark physics, A.Manohar, M.Wise  
hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

hep-ph/9907240, N.Brambilla  
A.Pineda, J.Soto, A.Vairo
Weak Coupling & Resummation

Separation of scales \[ M \gg Mv \gg Mv^2 \gtrsim T \]

\[ \mathcal{L}_{\text{pNRQCD}} = \int d^3 r \, \text{Tr} \left( S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O + V_A (O^\dagger \cdot gE S + \text{h.c.}) + \frac{V_B}{2} O^\dagger \{ \cdot gE, O \} + \cdots \right) \]

Dipole interaction

Arguments breakdown if

1. large log: \( Mv \to T \), \( VA \) has no running at one loop
2. large \( p_T \): medium boosted in rest frame of quarkonium, constrain to low \( p_T \)

Resum octet-A0 interaction by field redefinition

\[ O(R, r, t) \to W_{[(R, t), (R, t_L)]} \tilde{O}(R, r, t)(W_{[(R, t), (R, t_R)]})^\dagger \]

\[ C_{\alpha \beta}(t_1, t_2) \equiv \text{Tr}_E (O_{\alpha}^{(E)}(t_1) O_{\beta}^{(E)}(t_2) \rho_E) \]

\[ \langle WE(R_1, t_1) WE(R_2, t_2) \rangle_T \]

For dissociation
Dissociation $-\gamma_{ab,cd} \frac{1}{2} \{ L_{cd}^\dagger L_{ab}, \rho_S(0) \}$

For $L_{cd}^\dagger L_{ab} \rho_S$:

$$\gamma_{ab,cd} = \int d^3 R_1 \int d^3 R_2 \sum_{i_1, i_2, b_1, b_2} \int_0^t dt_1 \int_0^t dt_2 \ C^{R_1 i_1 b_1, R_2 i_2 b_2}_{b_1, b_2} \left( t_1, t_2 \right)$$

Weakly-coupled plasma, LO:

$$\langle k_1, n_1 l_1, 1 \rangle \langle S(R_1, t_1) | r_{i_1} | O^{b_1}(R_1, t_1) \rangle \langle p_{cm}, p_{rel}, a_1 \rangle$$

$$\langle p_{cm}, p_{rel}, a_1 \rangle \langle O^{b_2}(R_2, t_2) | r_{i_2} | S(R_2, t_2) \rangle \langle k_3, n_3 l_3, 1 \rangle$$

$$\langle \Psi_p_{rel} | r_{i_2} | \psi_{n_3 l_3} \rangle \delta^{a_1 b_2} e^{-i(E \kappa_3 t_2 - k_3 \cdot R_2)} e^{i(E_p t_2 - p_{cm} \cdot R_2)}$$

Markovian approximation:

$$t \rightarrow \infty \quad \int_0^t dt_1 \int_0^t dt_2 e^{i\omega t_1} e^{-i\omega t_2} \rightarrow 2\pi t \delta(\omega)$$

$$tC_{nl}^{(-)} \equiv \ t \int \frac{d^3 p_{cm}}{(2\pi)^3} \frac{d^3 p_{rel}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \frac{N_B(q)(2\pi)^4 \delta^3(k - p_{cm} + q)}{(2\pi)^3 2q} \delta(E_k - E_p + q)$$

Phase space measure

$$\frac{2}{3} C_F q^2 g^2 |\langle \psi_{nl} | r | \Psi_p_{rel} \rangle|^2 f_{nl}(x, k, t = 0)$$

E&P conservation

Amplitude squared
Everything Together: Boltzmann Equation

\[ \rho_S(t) = \rho_S(0) - it[H_{\text{eff}}, \rho_S(0)] + \sum_{a,b,c,d} \gamma_{ab,cd} \left( L_{ab} \rho_S(0) L_{cd}^\dagger - \frac{1}{2} \{ L_{cd}^\dagger L_{ab}, \rho_S \} \right) \]

Wigner transform

\[ f_{nl}(x, k, t) = f_{nl}(x, k, 0) - t \frac{k}{2M} \cdot \nabla f_{nl}(x, k, 0) + tC^{(+) - tC^{(-)}} \]

Dividing by t, set t —> 0  add spin dependence

\[ \partial_t f_{nls}(x, k, t) + \mathbf{v} \cdot \nabla_x f_{nls}(x, k, t) = C_{nls}^{(+) - C_{nls}^{(-)}} \]

Not contradictory with t —> ∞

Markovian: environment correlation time << system relaxation time —> coarse-grained

environment correlation time << t << system relaxation time

\[ \frac{1}{T} \ll t \ll \frac{1}{v^2 T} \]
Semiclassical Expansion in Recombination

When evaluating recombination term

\[
\int \frac{d^3 k'}{(2\pi)^3} e^{i\mathbf{k'} \cdot \mathbf{x}_{\text{cm}}} \left\langle \mathbf{p}_{\text{cm}} + \frac{\mathbf{k}'}{2}, \mathbf{p}_{\text{1rel}} \left| \rho_S^{(8)}(0) \right| \mathbf{p}_{\text{cm}} - \frac{\mathbf{k}'}{2}, \mathbf{p}_{\text{2rel}} \right\rangle
\]

\[
= \int d^3 x_{\text{rel}} e^{-i(\mathbf{p}_{\text{1rel}} - \mathbf{p}_{\text{2rel}}) \cdot \mathbf{x}_{\text{rel}}} f_{\bar{Q}Q}^{(8)}(\mathbf{x}_{\text{cm}}, \mathbf{p}_{\text{cm}}, \mathbf{x}_{\text{rel}}, \frac{\mathbf{p}_{\text{1rel}} + \mathbf{p}_{\text{2rel}}}{2}, t = 0)
\]

Classical analog exists for same relative momentum

**Gradient expansion**

\[
f_{\bar{Q}Q}^{(8)}(\mathbf{x}_{\text{cm}}, \mathbf{p}_{\text{cm}}, \mathbf{x}_{\text{rel}}, \frac{\mathbf{p}_{\text{1rel}} + \mathbf{p}_{\text{2rel}}}{2}, t) = f_{\bar{Q}Q}^{(8)}(\mathbf{x}_{\text{cm}}, \mathbf{p}_{\text{cm}}, \mathbf{x}_0, \frac{\mathbf{p}_{\text{1rel}} + \mathbf{p}_{\text{2rel}}}{2}, t)
\]

\[
+ (\mathbf{x}_{\text{rel}} - \mathbf{x}_0) \cdot \nabla_x f_{\bar{Q}Q}^{(8)}(\mathbf{x}_{\text{cm}}, \mathbf{p}_{\text{cm}}, \mathbf{x}_0, \frac{\mathbf{p}_{\text{1rel}} + \mathbf{p}_{\text{2rel}}}{2}, t) + \cdots
\]

**LO = classical**

**NLO = leading quantum correction**
Importance of Scale Hierarchy

Success of transport equation in quarkonium phenomenology

Separation of scales \( M \gg Mv \gg Mv^2 \gtrsim T \)
Importance of Scale Hierarchy

Success of transport equation in quarkonium phenomenology

Separation of scales \( M \gg Mv \gg Mv^2 \gg T \)

What if hierarchy breaks down?

Practically not possible: 
\( v \sim 0.3 \) for bottomonium

Dipole vertex no longer works
No well-defined bound state
Coupled Transport Equations of Heavy Flavors

open heavy quark antiquark

\[
\frac{\partial}{\partial t} + \dot{x}_Q \cdot \nabla x_Q + \dot{x}_{\bar{Q}} \cdot \nabla x_{\bar{Q}} \right) f_{Q\bar{Q}}(x_Q, p_Q, x_{\bar{Q}}, p_{\bar{Q}}, t) = C_{Q\bar{Q}} - C_{Q\bar{Q}}^+ + C_{Q\bar{Q}}^-
\]

each quarkonium state
\( nl = 1S, 2S, 1P \) etc.

\[
\frac{\partial}{\partial t} + \dot{x} \cdot \nabla x \right) f_{nls}(x, p, t) = C_{nls}^+ - C_{nls}^-
\]

initial production
QGP medium expands and cools
hadron gas

time

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Coupled Transport Equations of Heavy Flavors

open heavy quark antiquark

\[
\left( \frac{\partial}{\partial t} + \dot{x}_Q \cdot \nabla x_Q + \dot{x}_{\bar{Q}} \cdot \nabla x_{\bar{Q}} \right) f_{Q\bar{Q}}(x_Q, p_Q, x_{\bar{Q}}, p_{\bar{Q}}, t) = C_{Q\bar{Q}} - C_{Q\bar{Q}}^+ + C_{Q\bar{Q}}^- 
\]

each quarkonium state

\( nl = 1S, 2S, 1P \) etc.

\[
\left( \frac{\partial}{\partial t} + \dot{x} \cdot \nabla x \right) f_{nl}(x, p, t) = C_{nl}^+ - C_{nl}^- 
\]

recombine if

\( T < \text{melting } T \)

from other open b

initial production

QGP medium expands and cools

hadron gas

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Coupled Transport Equations of Heavy Flavors

open heavy quark antiquark

\[
\left( \frac{\partial}{\partial t} + \dot{x}_Q \cdot \nabla x_Q + \dot{x}_{\bar{Q}} \cdot \nabla x_{\bar{Q}} \right) f_{Q\bar{Q}}(x_Q, p_Q, x_{\bar{Q}}, p_{\bar{Q}}, t) = C_{Q\bar{Q}} - C_{Q\bar{Q}}^+ + C_{Q\bar{Q}}^- 
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each quarkonium state

nl = 1S, 2S, 1P etc.

\[
\left( \frac{\partial}{\partial t} + \dot{x} \cdot \nabla x \right) f_{nls}(x, p, t) = C_{nls}^+ - C_{nls}^- 
\]

correlated recombination

from other open b

uncorrelated recombination
Coupled with Transport of Open Heavy Flavor

heavy quark antiquark

\[
\left( \frac{\partial}{\partial t} + \dot{x}_Q \cdot \nabla x_Q + \dot{x}_{\bar{Q}} \cdot \nabla x_{\bar{Q}} \right) f_{Q\bar{Q}}(x_Q, p_Q, x_{\bar{Q}}, p_{\bar{Q}}, t) = C_{Q\bar{Q}} - C^+_{Q\bar{Q}} + C^-_{Q\bar{Q}}
\]

each quarkonium state

nl = 1S, 2S, 1P etc.

\[
\left( \frac{\partial}{\partial t} + \dot{x} \cdot \nabla x \right) f_{nls}(x, p, t) = C^+_{nls} - C^-_{nls}
\]

Can handle both correlated and uncorrelated recombination

\[
C_{Q\bar{Q}} = C_Q + C_{\bar{Q}}
\]

Each independently interact with medium:

1. Potential between pair screened
2. Potential depends on color, average = 0

We use “Lido” for open heavy flavor transport

Detailed Balance and Thermalization

Setup:
- QGP box w/ const T=300 MeV, 1S state & b quarks, total b flavor = 50 (fixed)
- Initial momenta sampled from uniform distributions 0-5 GeV
- Turn on/off open heavy quark transport

Quarkonium percentage v.s. time

![Graph](image1.png)

w/o open heavy flavor transport

Quarkonium percentage v.s. time

![Graph](image2.png)

w/ open heavy flavor transport

Dissociation-recombination interplay drives to detailed balance

Heavy quark energy gain/loss necessary to drive kinetic equilibrium of quarkonium

XY, B. Müller arXiv:1709.03529
Upsilon in 5020 GeV PbPb Collision

Coulomb potential $\alpha_s^{\text{pot}} = 0.36$ $\alpha_s = 0.3$ vary by $\pm 10\%$

Pythia + nuclear PDF: EPPS16, uncertainty band

2+1D viscous hydro (calibrated)

Bottomonium: 1S, 2S, 3S, 1P, 2P

with cross-talk (correlated) recombination

without cross-talk recombination
Upsilon in 5020 GeV PbPb Collision

Flat y dependence:
1. medium description is longitudinally boost invariant
2. nPDF mild dependence on y
Upsilon(1S) Azimuthal Anisotropy in 5020 GeV PbPb

\[ E \frac{d^3 N}{dp^3} = \frac{1}{2\pi} \frac{d^2 N}{p_T dp_T dy} (1 + 2v_2 \cos(2\phi) + \cdots) \]

\[ v_2 \text{ from:} \]
1. path dependence
2. reaction rates depend on relative velocity between medium and quarkonium
3. correlated recombination: medium interaction after dissociation but before recombination; uncorrelated b-quarks negligible (different for charm)
Experimental Evidence of Correlated Recombination

Dissociation rate of 1P ~ dissociation rate of 2S, due to similar binding energy/size

In medium, $P(1P \rightarrow 2S) \sim P(2S \rightarrow 1P)$, via dissociation and correlated recombination

But more 1P states produced initially than 2S, so more 2S regenerated than 1P
Conclusion

• Quarkonium transport inside QGP:
  • Derivation of Boltzmann equation from open quantum system and effective field theory
  • Hierarchy of scales, nonrelativistic expansion, weak coupling, Markovian limit
  • Phenomenological results from coupled transport equations, importance of correlated recombination for bottomonium
  • Experimental test of correlated recombination: 1P v.s. 2S
Backup: NLO Contributions

\[ p_1, s_1, i \]

\[ p_2, s_2, j \]

\[ q \]

\[ k, nl \]

\[ p_{cm}, p_{rel}, a \]

\[ q_1, \epsilon_1^a, a \]

\[ q_2, \epsilon_2^b, b \]

\[ q, \rho, c \]

\[ A_0 \]

and more ...
Backup: Running of Dipole Interaction

\[ 0 - \epsilon + \cdots \]

\[ \frac{d}{d\mu} V_A(\mu) = 0 \]


XY, B. Müller arXiv:1811.09644
Test particle Monte Carlo

\[ f(x, p, t) = \sum_i \delta^3(x - y_i(t)) \delta^3(p - k_i(t)) \]

Each time step: read in hydro-cell velocity, temperature; consider diffusion, dissociation, recombination in particle’s rest frame and boost back

If specific process occurs, sample incoming medium particles and outgoing particles from differential rates, conserving energy momentum

Recombination term contains

\[ f_{Q\bar{Q}}(x_1, p_1, x_2, p_2, t) \]

For each HQ, search anti-HQ within a radius, weighted sum

\[
f_{Q\bar{Q}}(x_1, p_1, x_2, p_2, t) = \sum_{i,j} \frac{e^{-(y_i - \bar{y}_j)^2/2a_B^2}}{(2\pi a_B)^{3/2}} \delta^3\left(\frac{x_1 + x_2}{2} - \frac{y_i + \bar{y}_j}{2}\right) \delta^3(p_1 - k_i) \delta^3(p_2 - \bar{k}_j)\]

Backup: Numerical Implementation