Addressing theoretical uncertainties in dark matter direct detection experiments

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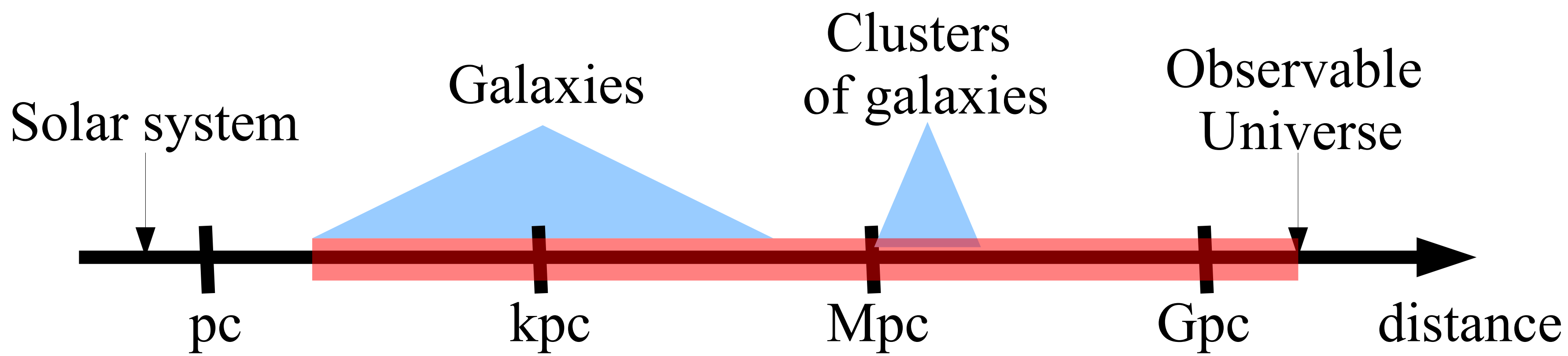
Technische Universität München

Based on works with Andreas Rappelt and Bradley Kavanagh

IPMU
Virtual seminar
June 2020

There is evidence for dark matter in a wide range of distance scales



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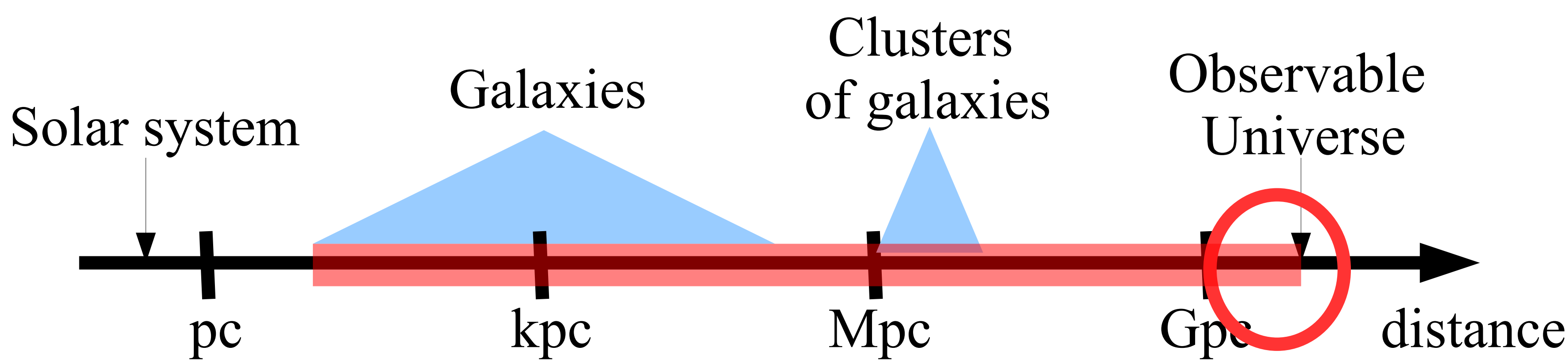

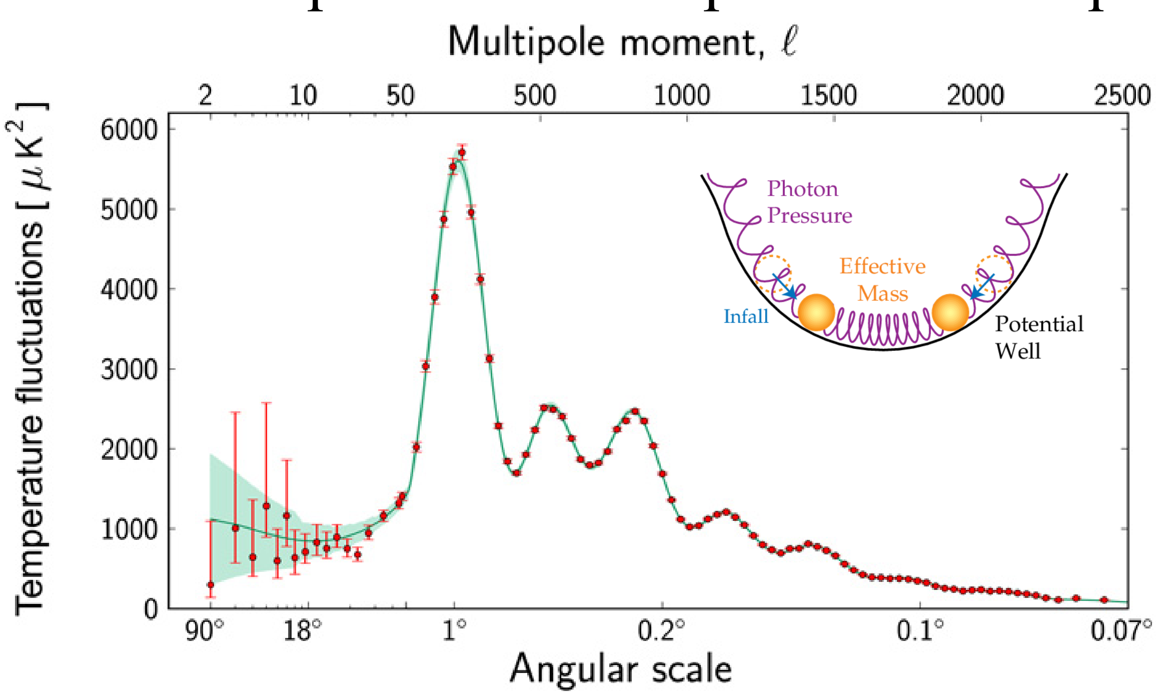

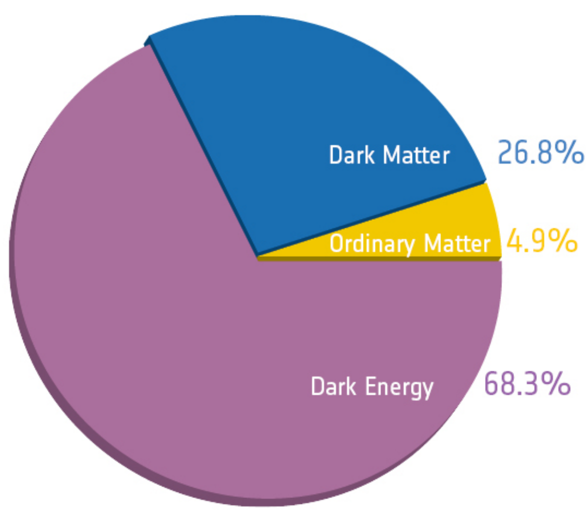

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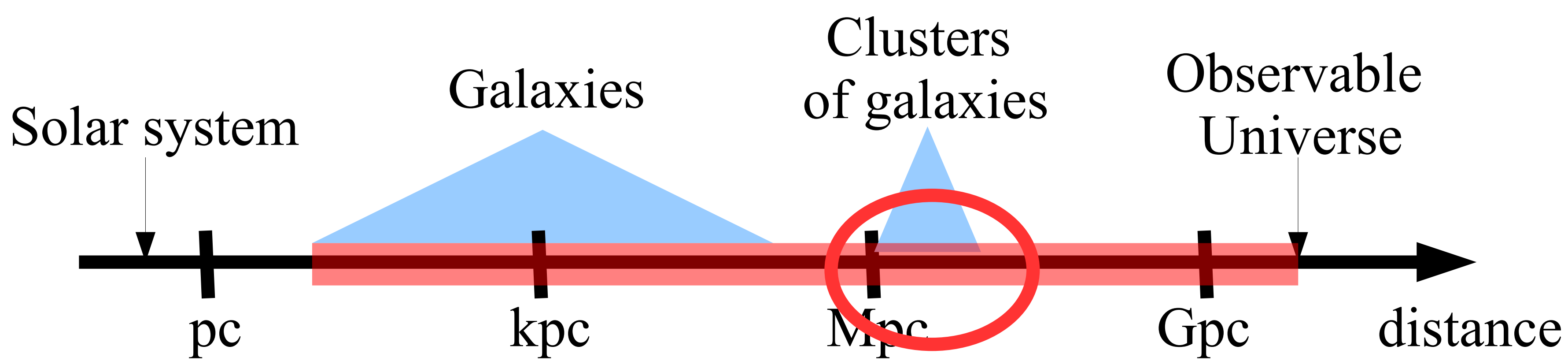

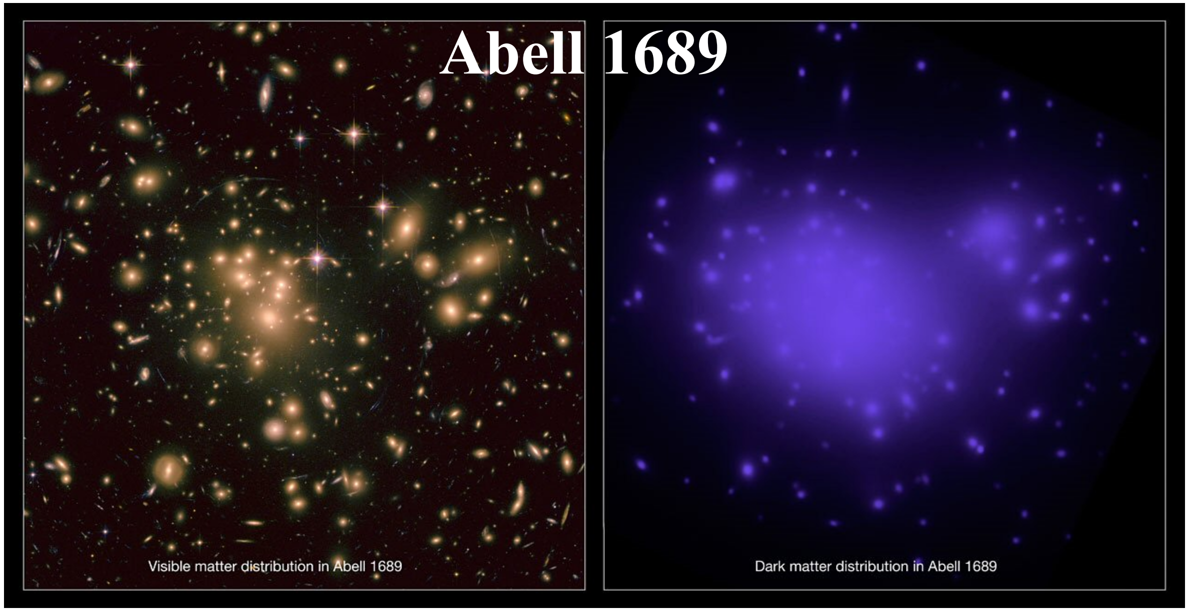

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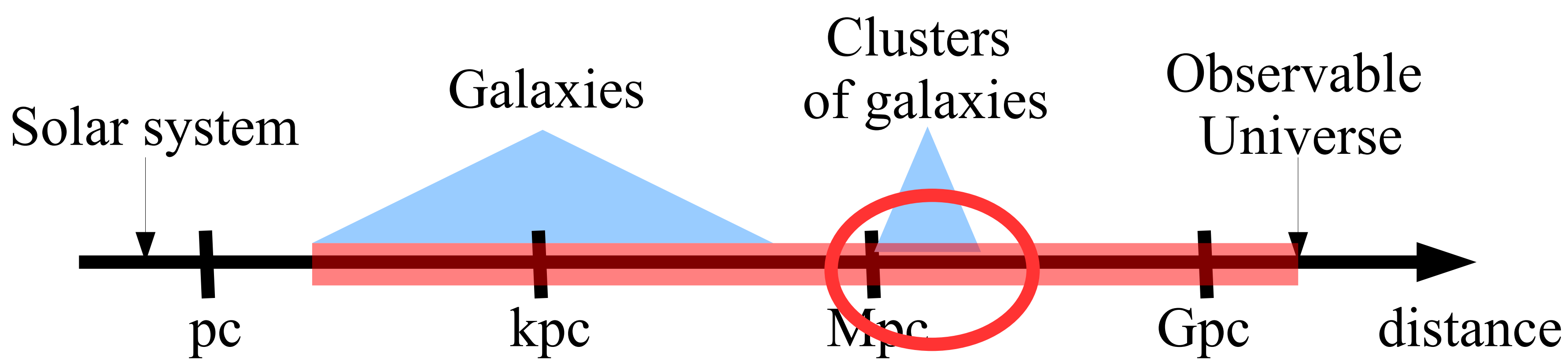

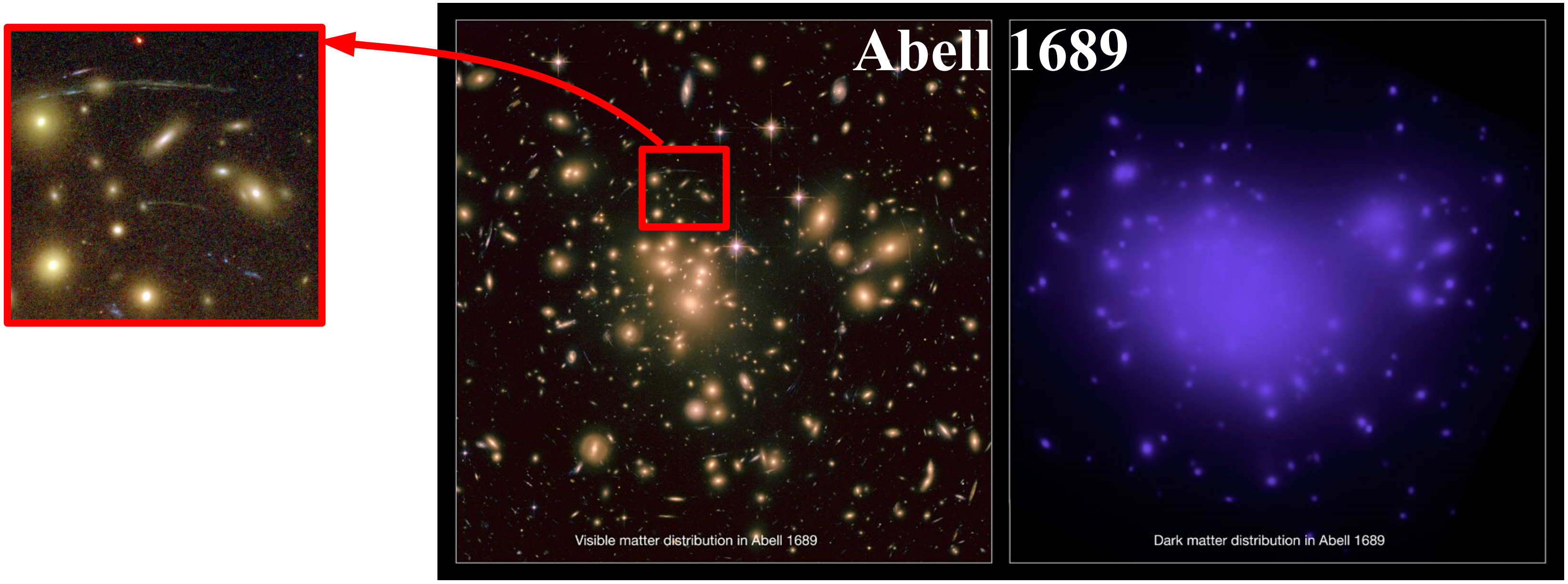

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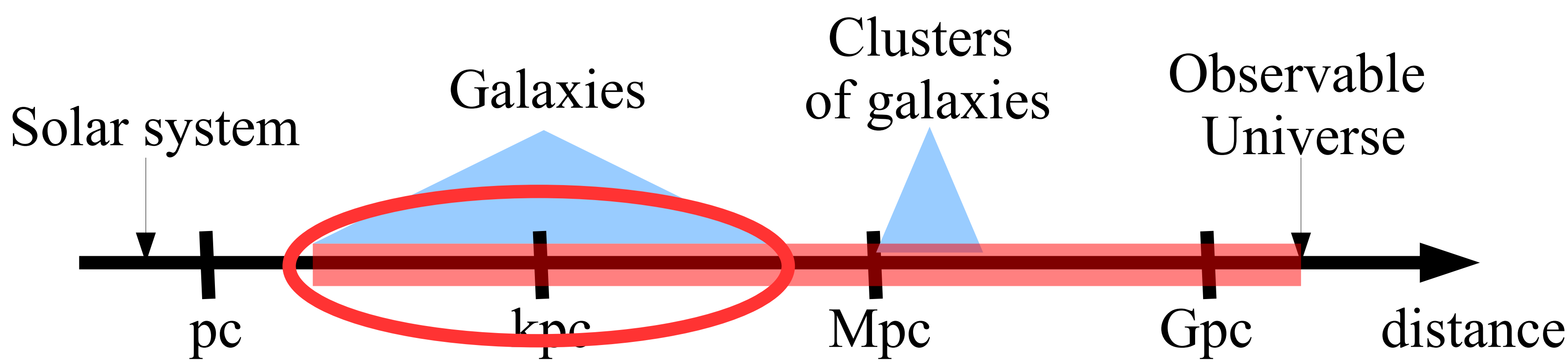

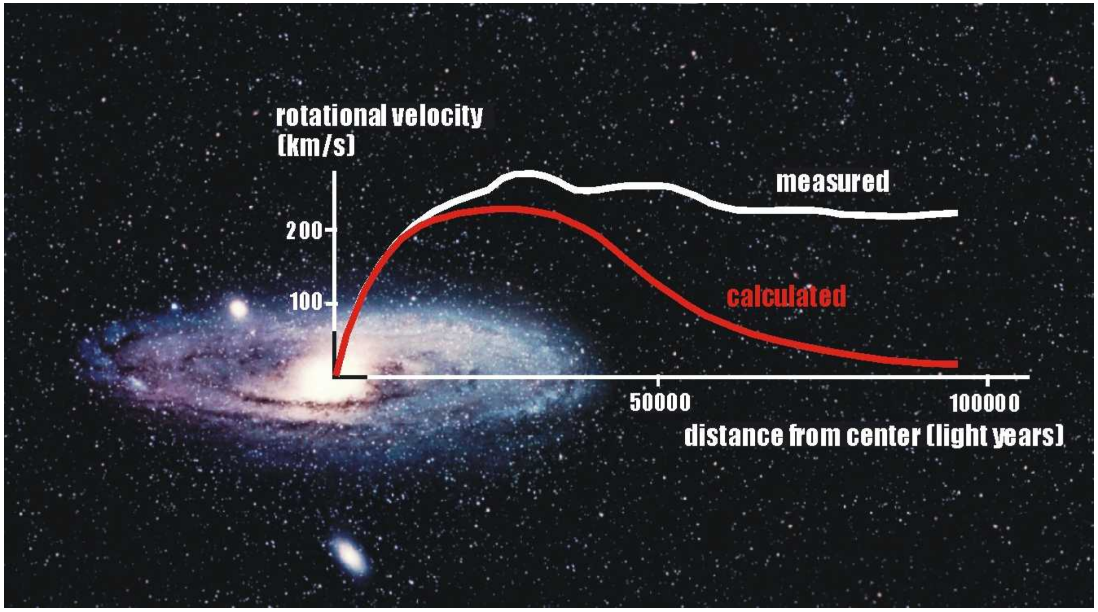

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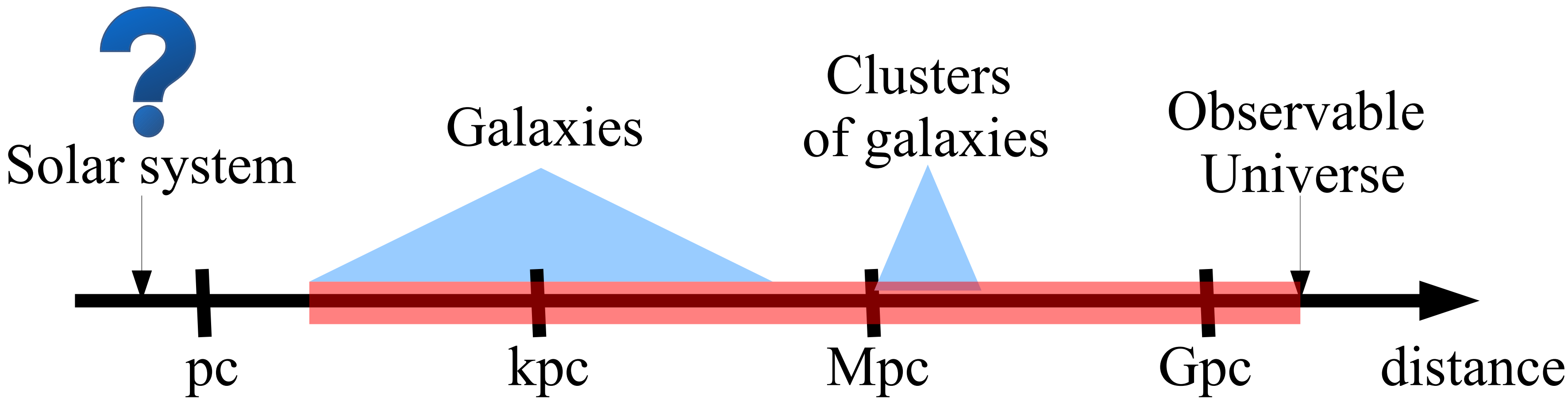

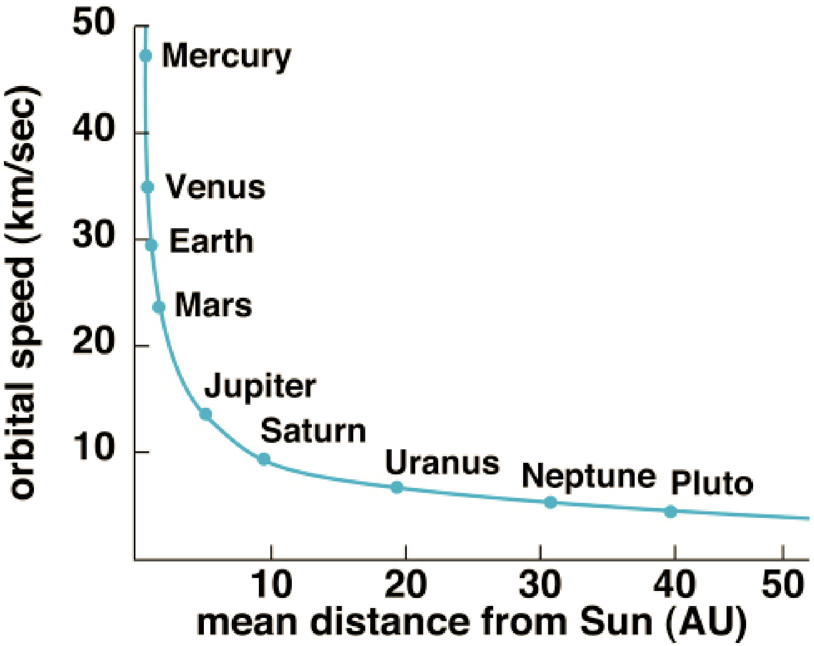

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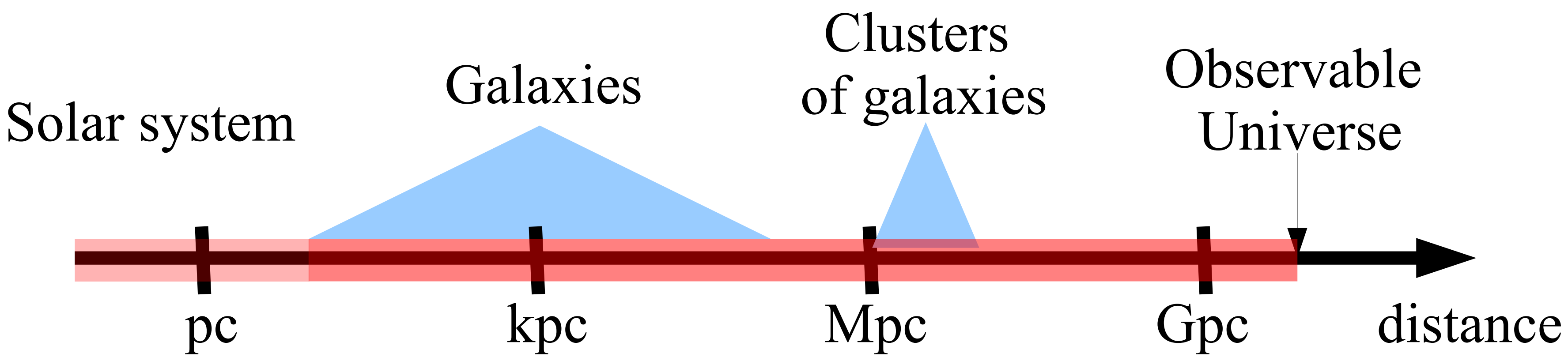
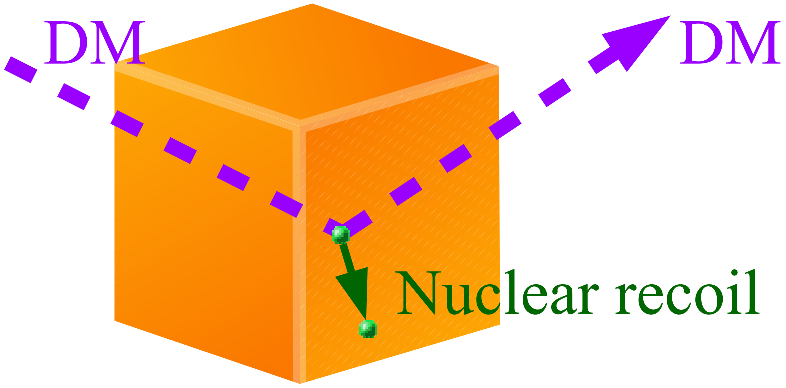


Assumption,
but well motivated

If the DM is made up of WIMPs, the DM population inside the Solar System could be detected

Searching for WIMP DM inside the Solar System



No significant excess detected so far

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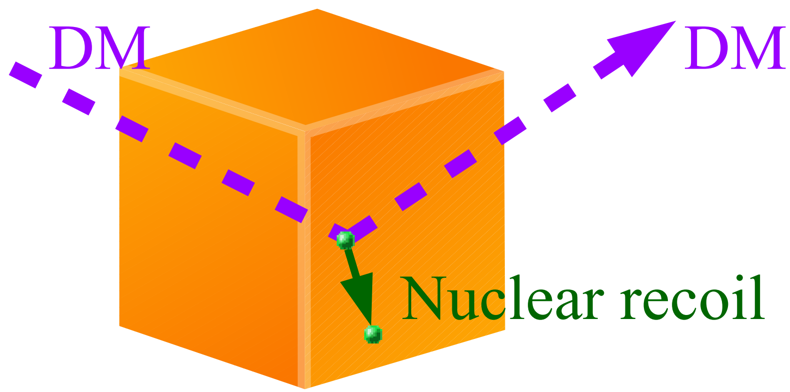

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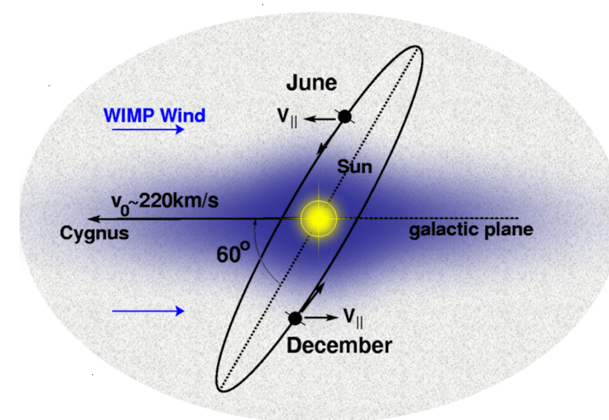

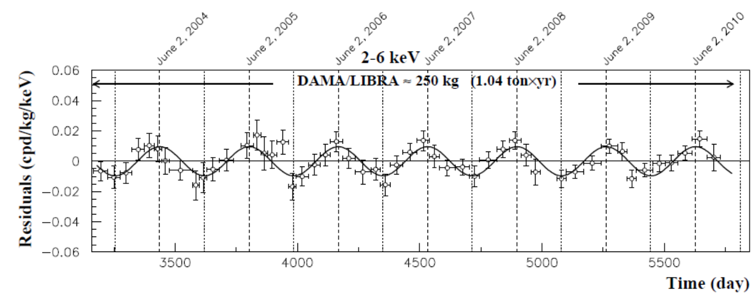

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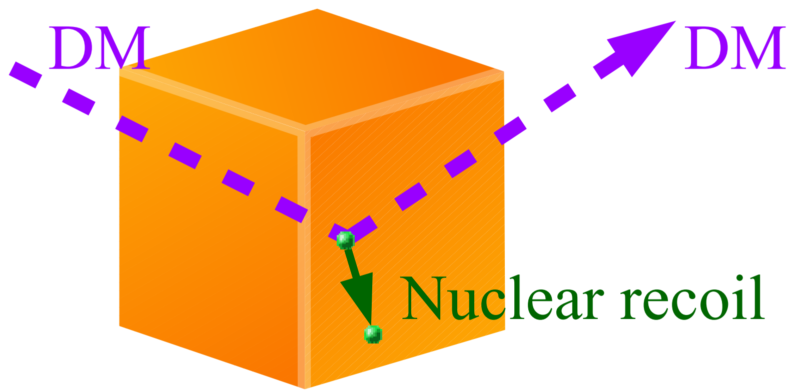

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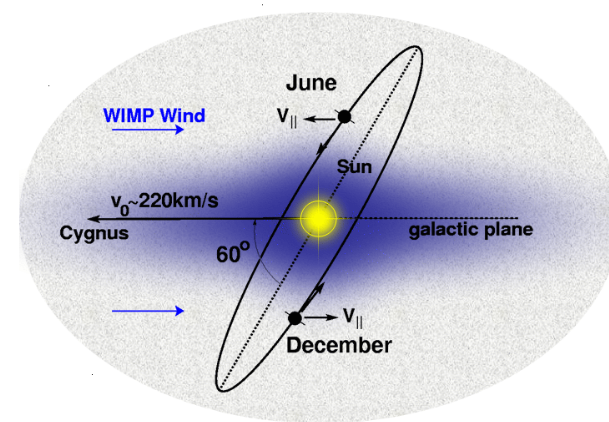

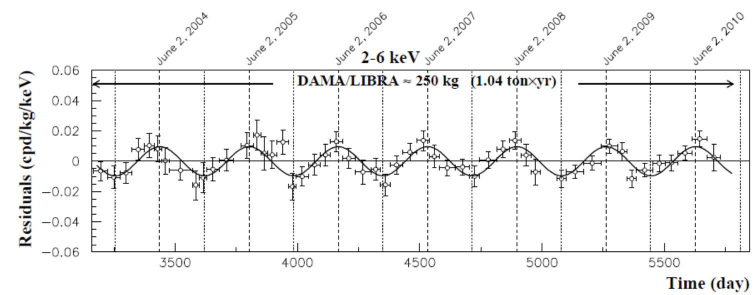

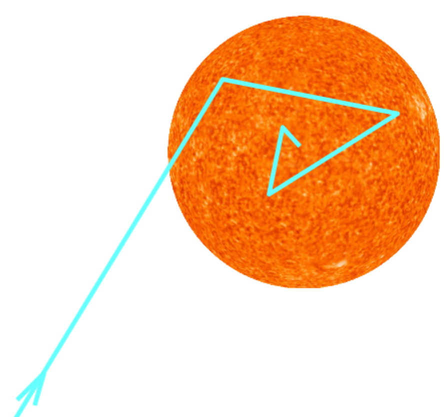

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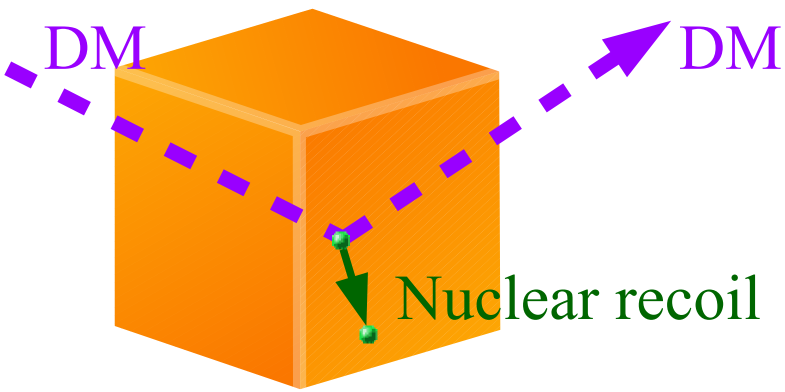

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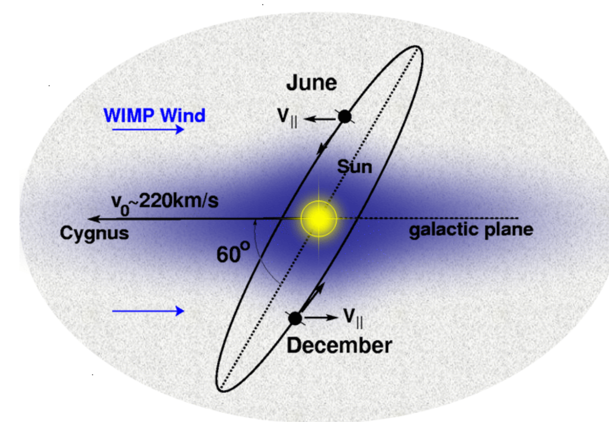

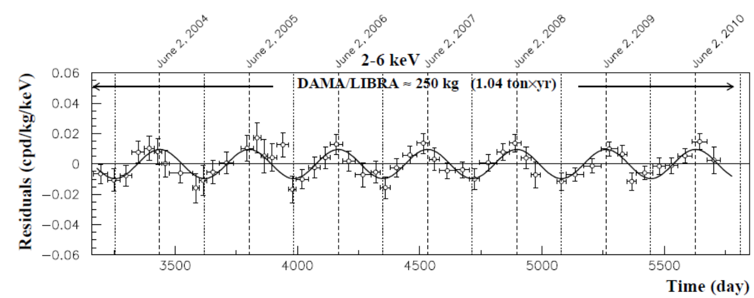

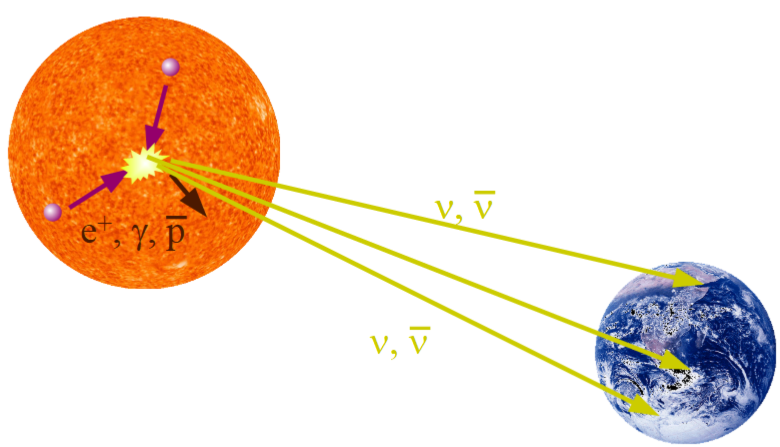

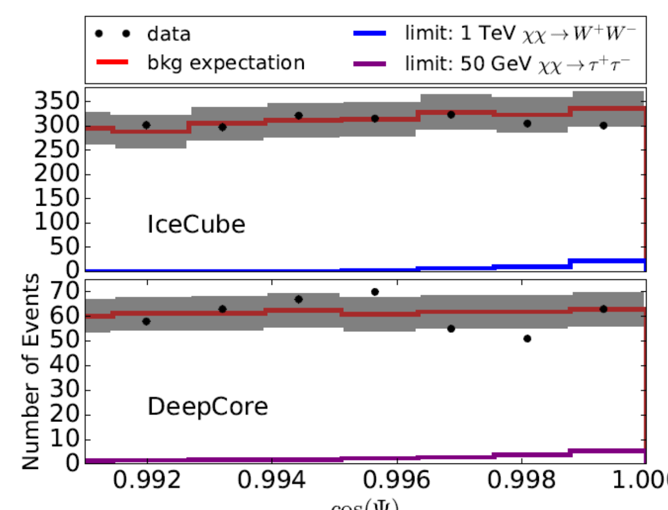

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Theoretical interpretation of the experimental results

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- Differential rate of DM-induced scatterings

$$\frac{dR}{dE_R} = \frac{\rho_{\rm loc}}{m_A m_{\rm DM}} \int_{v \geq v_{\rm min}(E_R)} {\rm d}^3 v \, v \, f(\vec{v} + \vec{v}_{\rm obs}(t)) \, \frac{{\rm d}\sigma}{{\rm d}E_R}$$

- The neutrino flux from annihilations inside the Sun is, under plausible assumptions, determined by the capture rate inside the Sun:

$$C = \int_0^{R_\odot} 4\pi r^2 {\rm d}r \, \frac{\rho_{\rm loc}}{m_{\rm DM}} \int_{v \leq v_{\rm max}^{\rm (Sun)}(r)} {\rm d}^3 v \, \frac{f(\vec{v})}{v} \left(v^2 + [v_{\rm esc}(r)]^2\right) \times \int_{m_{\rm DM} v^2/2}^{2\mu_A^2 \left(v^2 + [v_{\rm esc}(r)]^2\right)/m_A} {\rm d}E_R \, \frac{{\rm d}\sigma}{{\rm d}E_R}$$

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Uncertainties from particle/nuclear physics and from astrophysics

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Theoretical interpretation of the experimental results

Uncertainties from particle/nuclear physics.

- Dark matter mass?

For thermally produced dark matter, $m_{\rm DM}$ = few MeV – 100 TeV

- Differential cross section?

$$\frac{d\sigma}{dE_R} = \frac{m_A}{2\mu_A^2 v^2}\left(\sigma_{\rm SI} F_{\rm SI}^2(E_R) + \sigma_{\rm SD} F_{\rm SD}^2(E_R)\right)$$

Spin-independent and
spin-dependent cross sections
at zero momentum transfer

Nuclear form factors

(In some DM frameworks, other operators may also arise)

Theoretical interpretation of the experimental results

Uncertainties from astrophysics

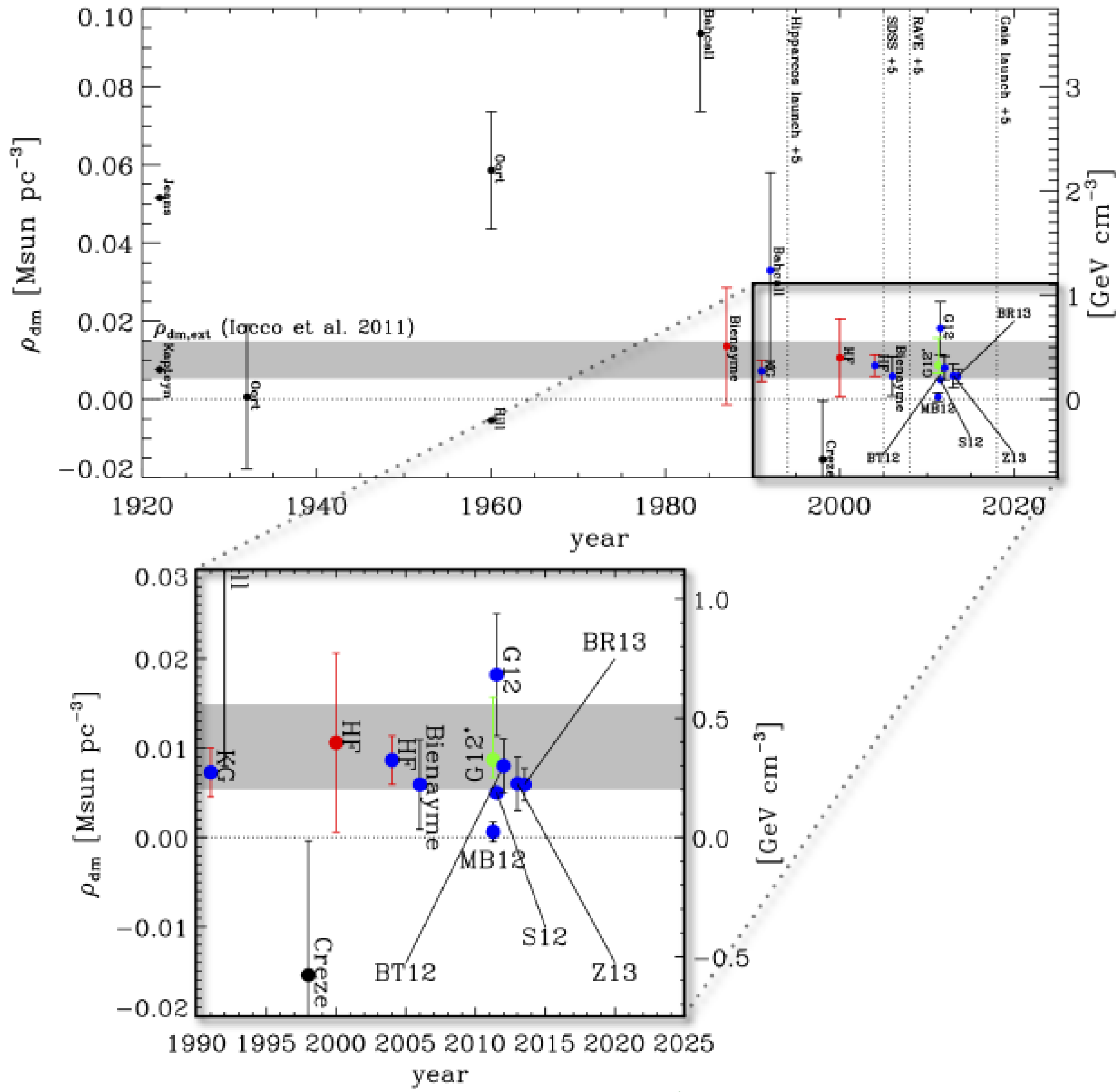
- Local dark matter density?

  - "local measurements":

    From vertical kinematics of stars near (~1 kpc) the Sun

  - "global measurements":

    From extrapolations of $\rho(r)$ determined from rotation curves at large r, to the position of the Solar System.



Read '14

Theoretical interpretation of the experimental results

Uncertainties from astrophysics

- Local dark matter velocity distribution?

  Completely unknown. Rely on theoretical considerations

  - If the density distribution follows a singular isothermal sphere profile, the velocity distribution has a Maxwell-Boltzmann form.

$$\rho(r) \sim \frac{1}{r^2} \longrightarrow f(v) \sim \exp(-v^2/v_0^2)$$

Theoretical interpretation of the experimental results

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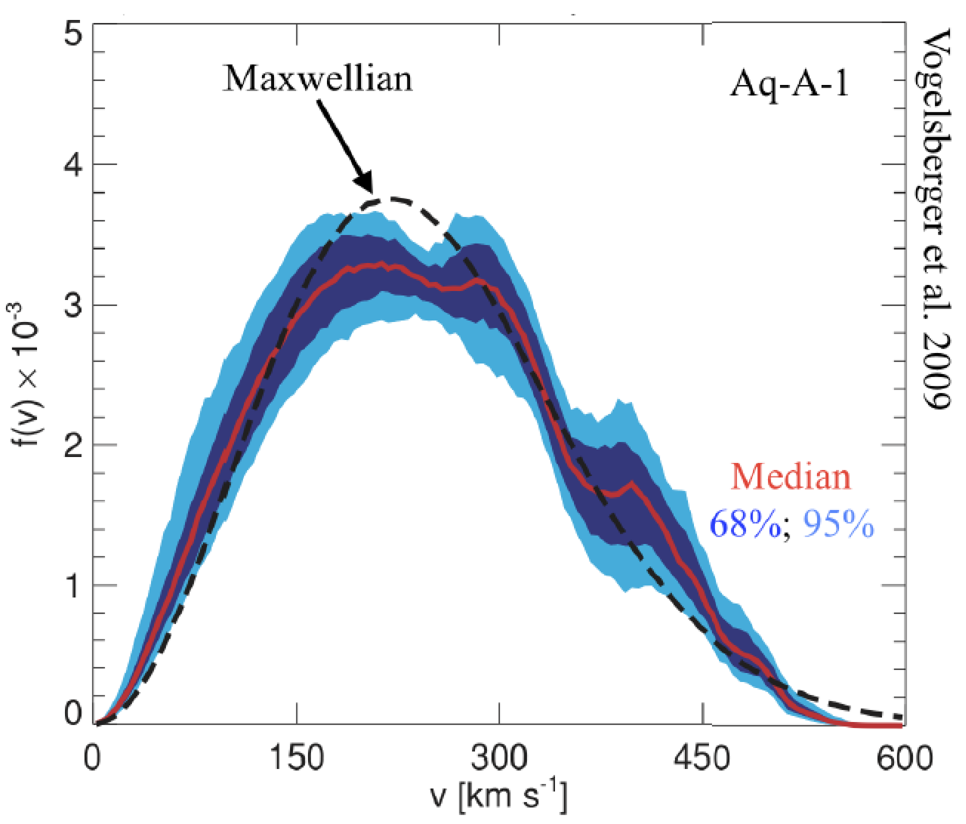
  - If the density distribution follows a singular isothermal sphere profile, the velocity distribution has a Maxwell-Boltzmann form.
  - Dark matter-only simulations. Show deviations from Maxwell-Boltzmann



Theoretical interpretation of the experimental results

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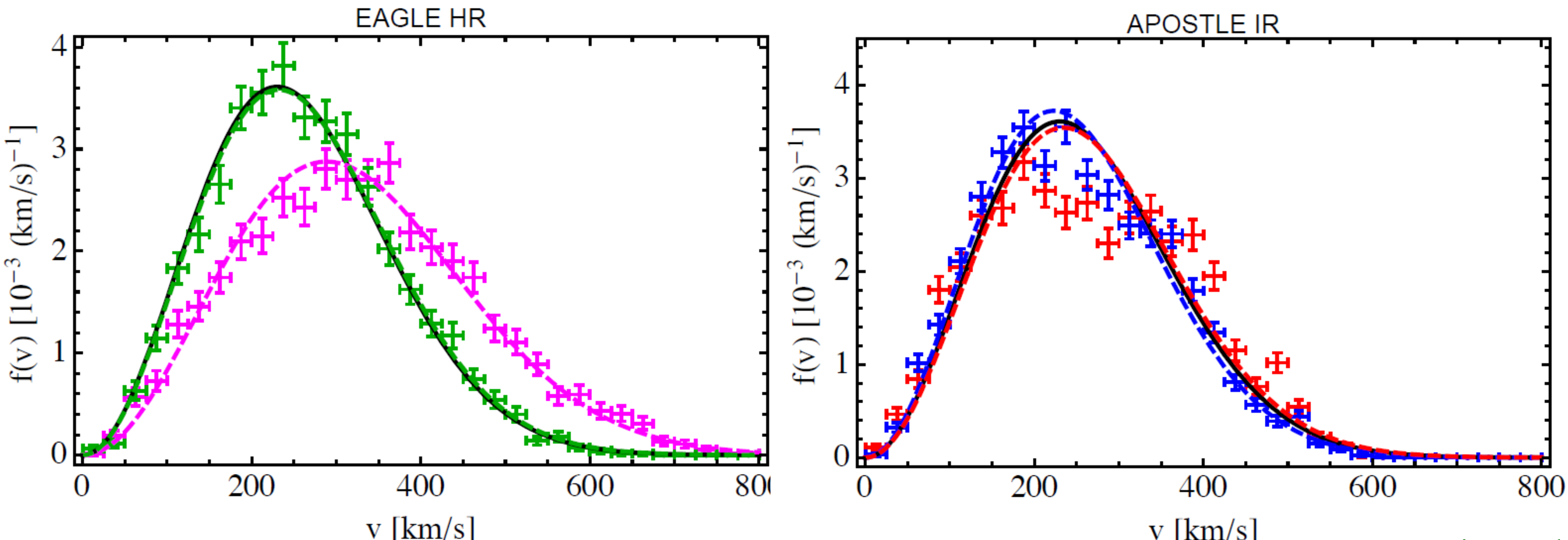
  - If the density distribution follows a singular isothermal sphere profile, the velocity distribution has a Maxwell-Boltzmann form.
  - Dark matter-only simulations. Show deviations from Maxwell-Boltzmann
  - Hydrodynamical simulations (DM+baryons). Inconclusive at the moment.

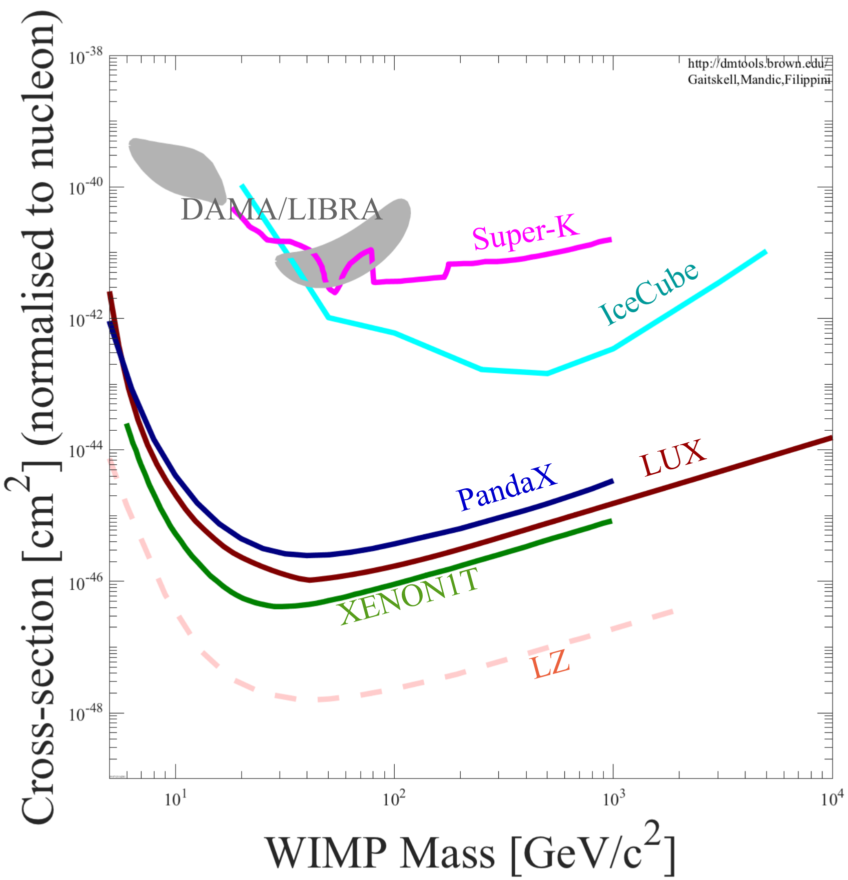


Bozorgnia et al'16

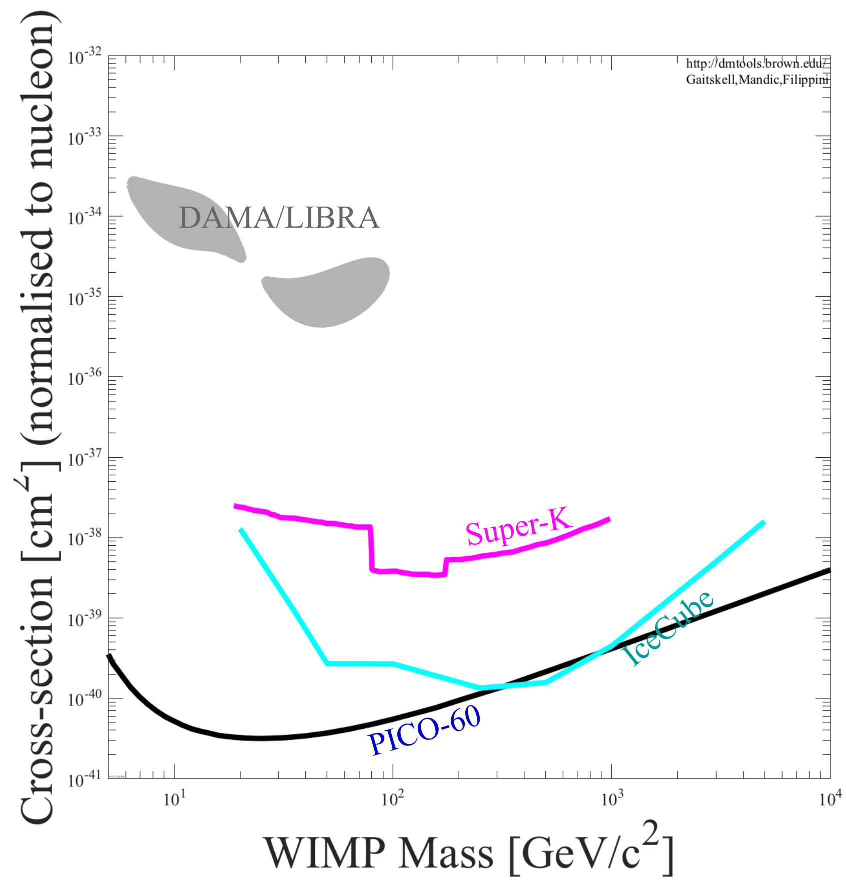
Theoretical interpretation of the experimental results

Common approach: assume SI or SD interaction only, assume ρ_{loc}= 0.3 GeV/cm^3 and assume a Maxwell-Boltzmann velocity distribution

SI



SD



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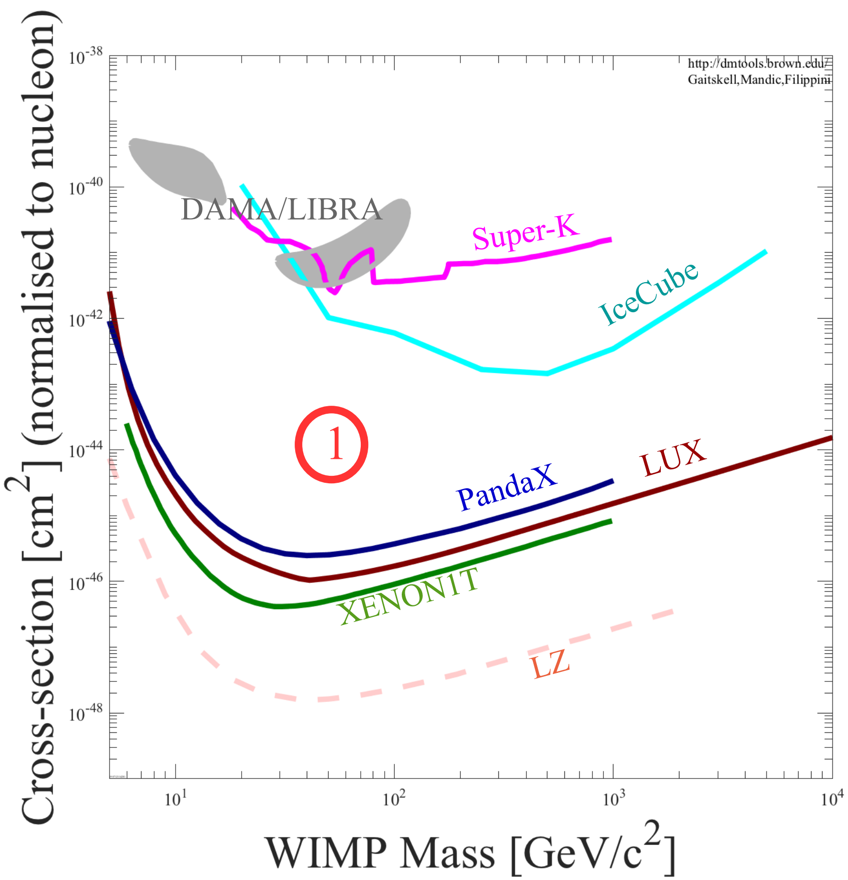

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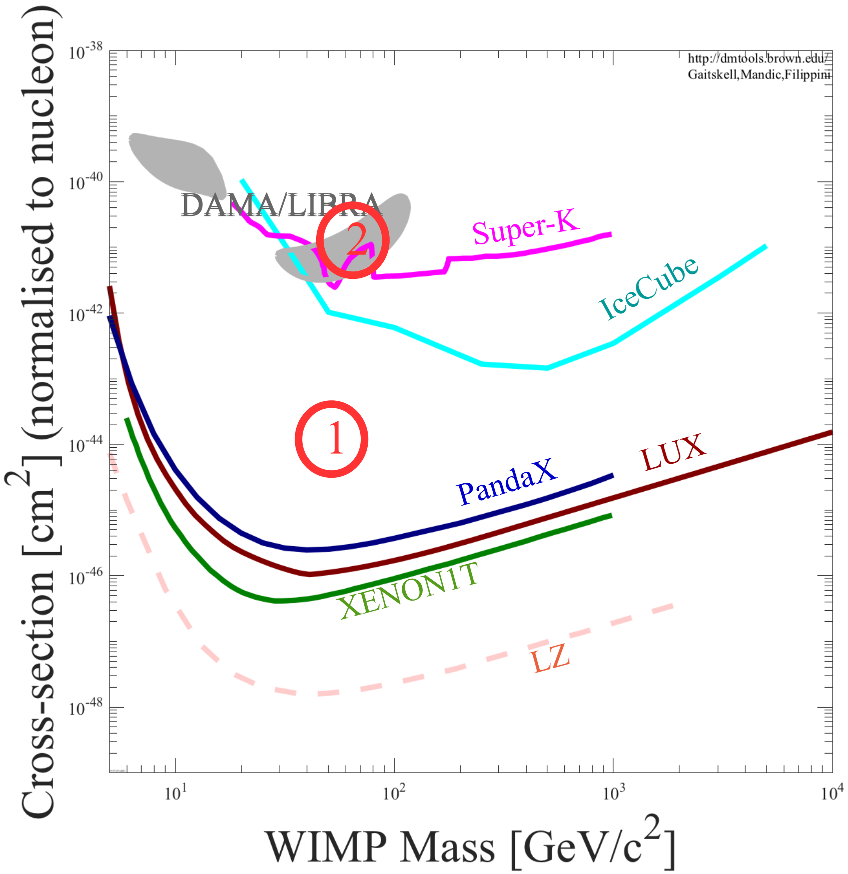

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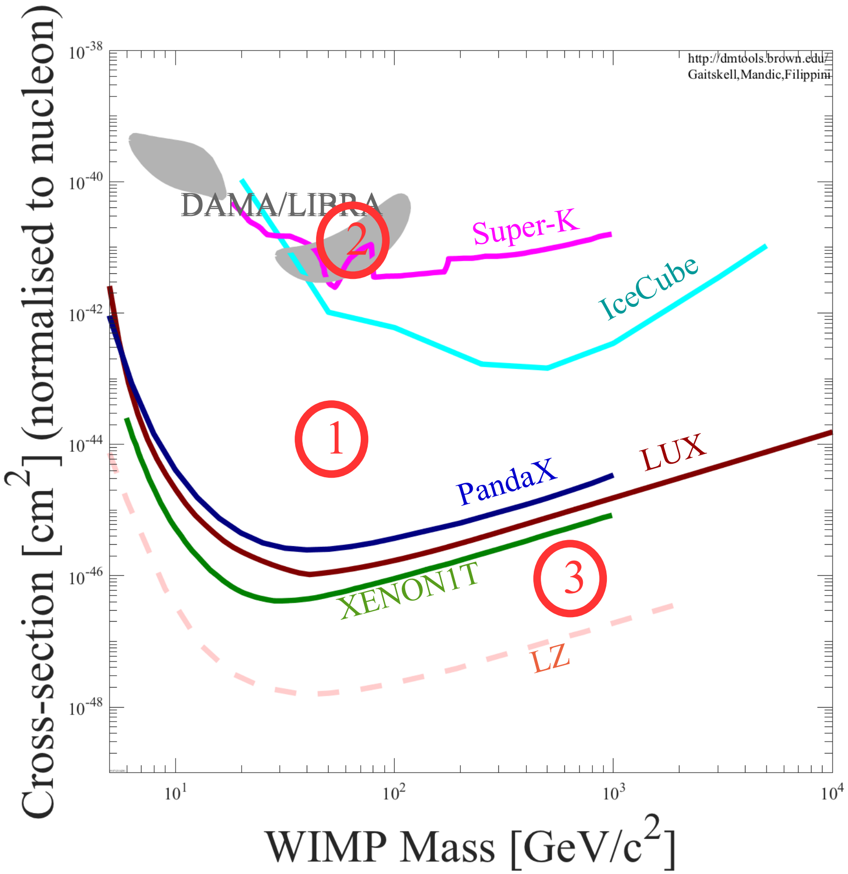

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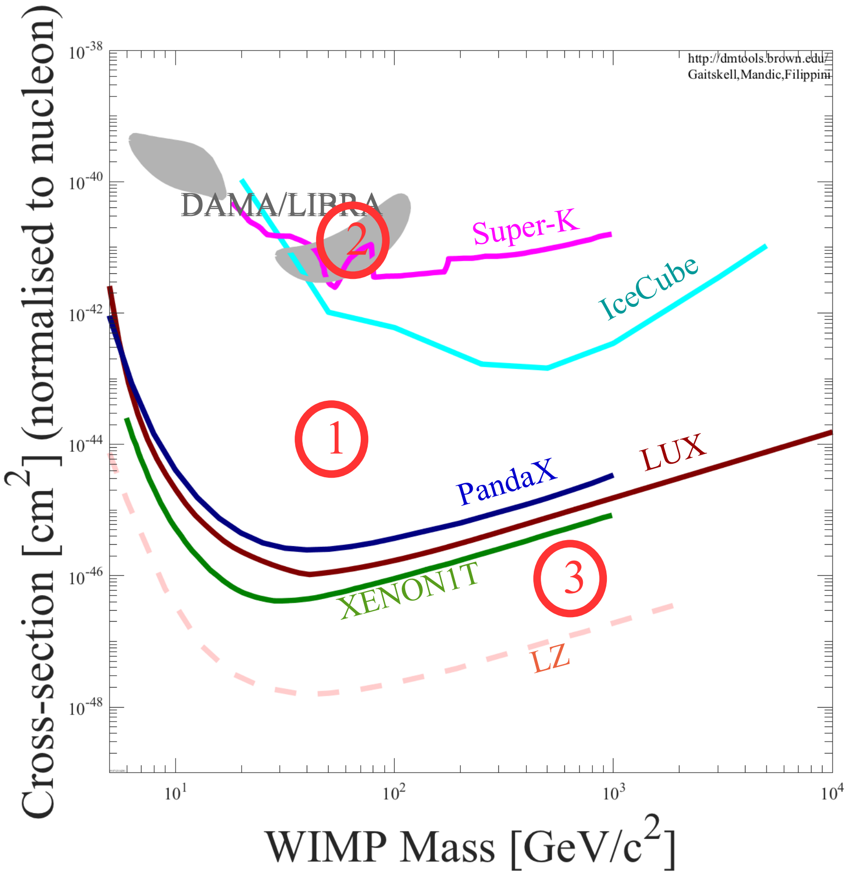

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What is the impact of the astrophysical uncertainties on these conclusions?

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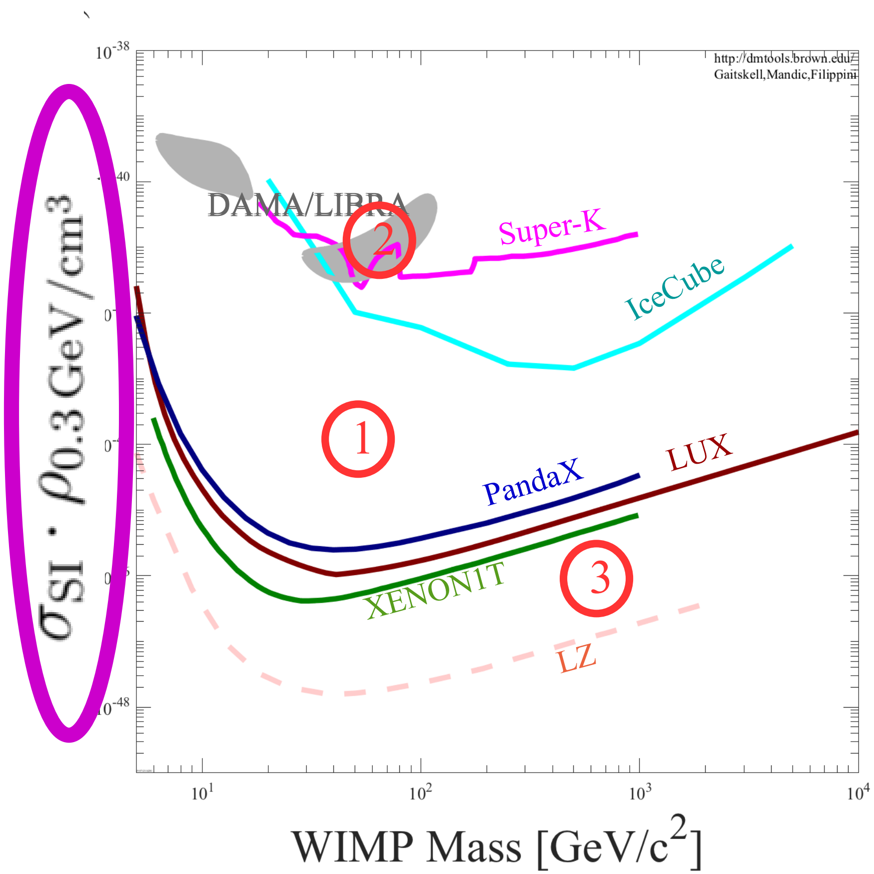

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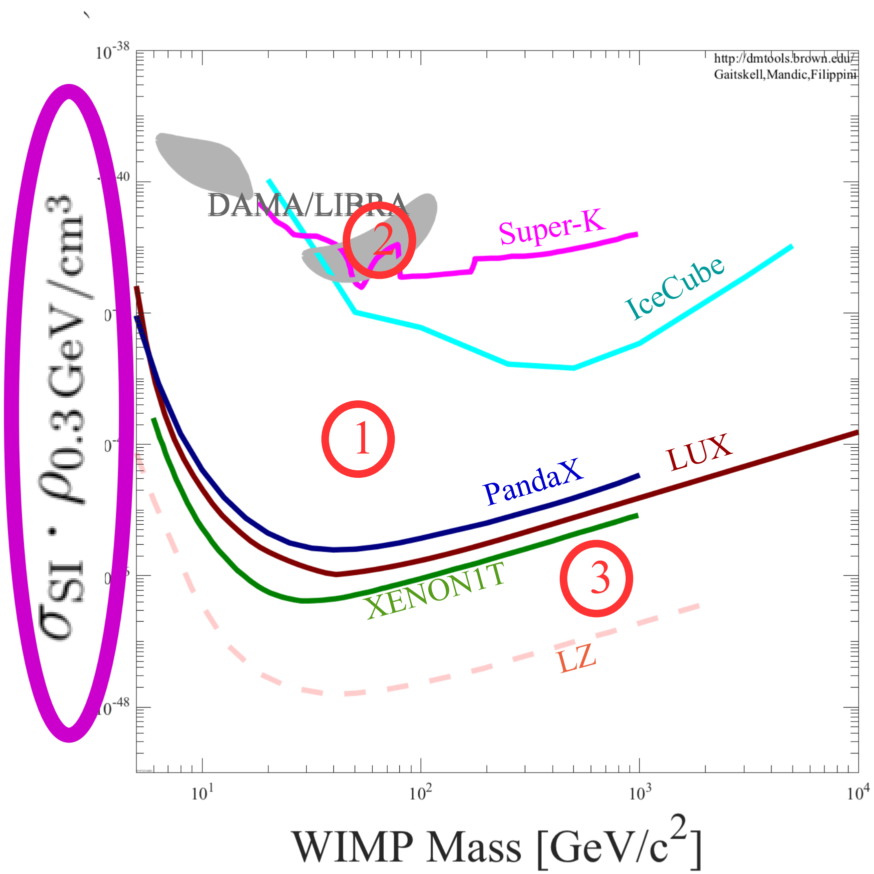

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What is the impact of the astrophysical uncertainties on these conclusions?

Do these conclusions hold for arbitrary velocity distributions?

Addressing astrophysical uncertainties in dark matter detection

Halo-independent approach for DM frameworks

- $(\sigma, m_{\rm DM})$ is ruled out regardless of the velocity distribution if

$$\min_{f(\vec{v})} \left\{ R(\sigma, m_{\rm DM}) \right\} \Big|_{\int f = 1} > R_{\rm u.l.}$$

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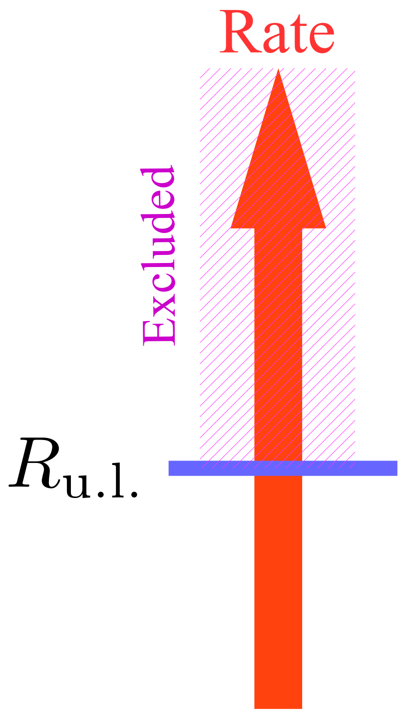

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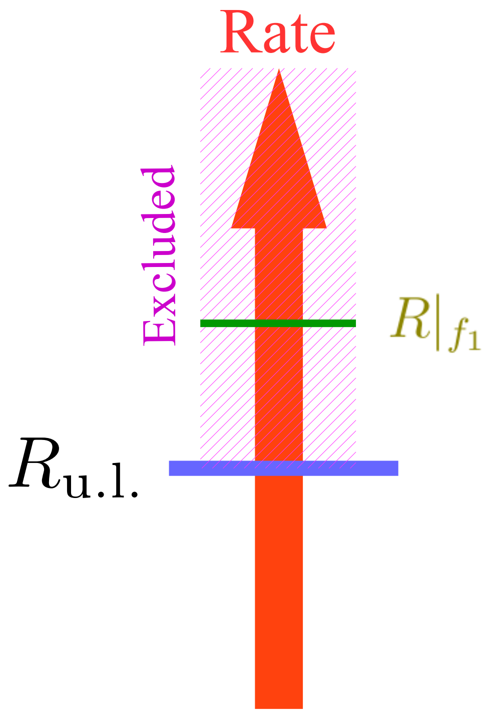

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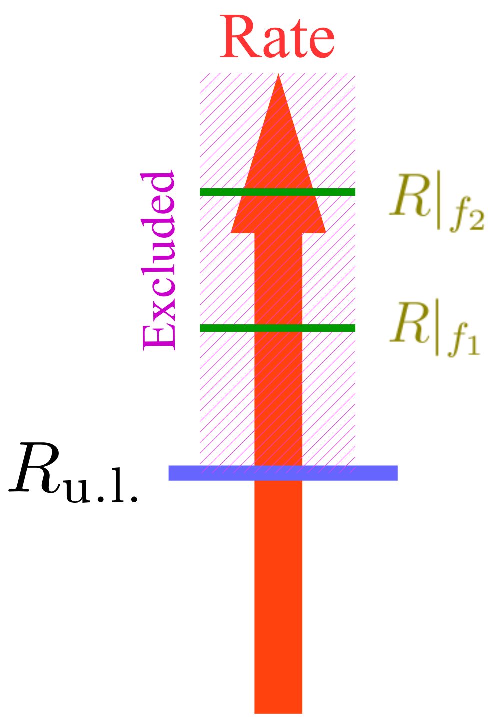

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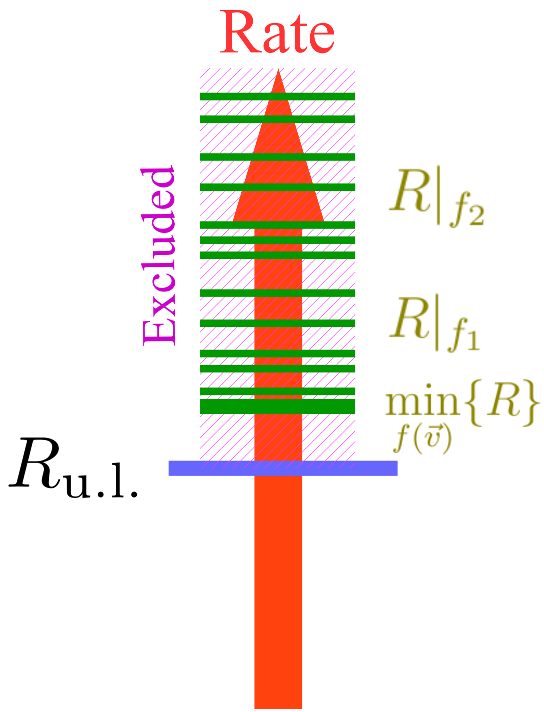

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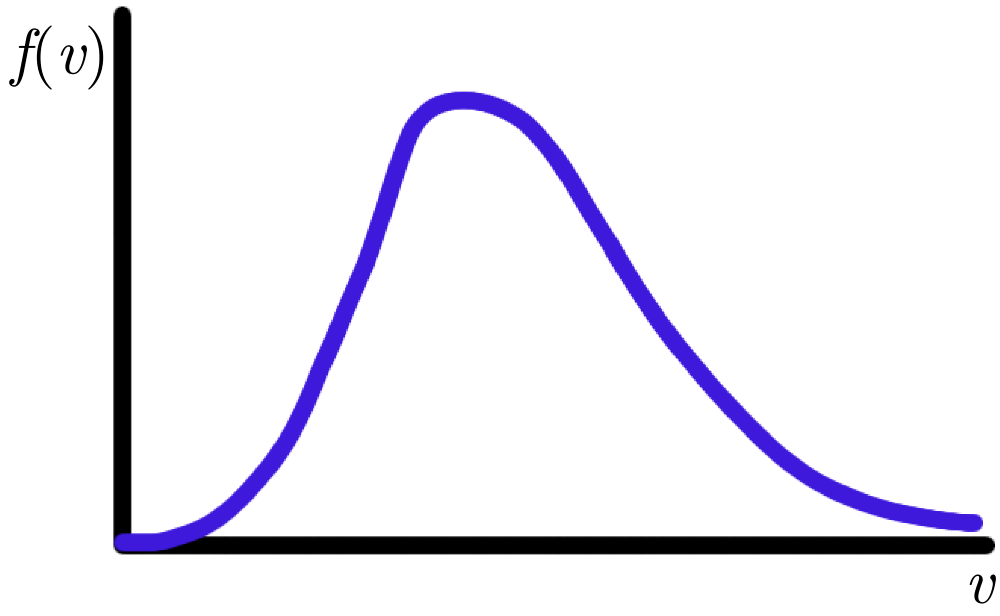
Note: one single direct detection experiment is not sufficient to probe a dark matter model in a totally halo-independent manner



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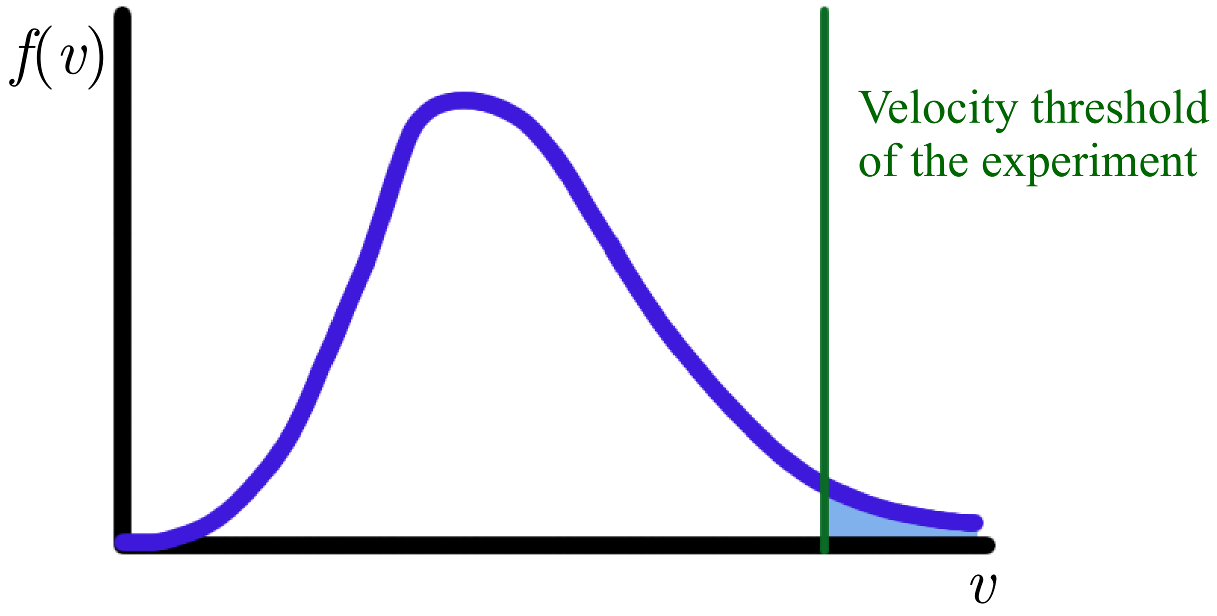

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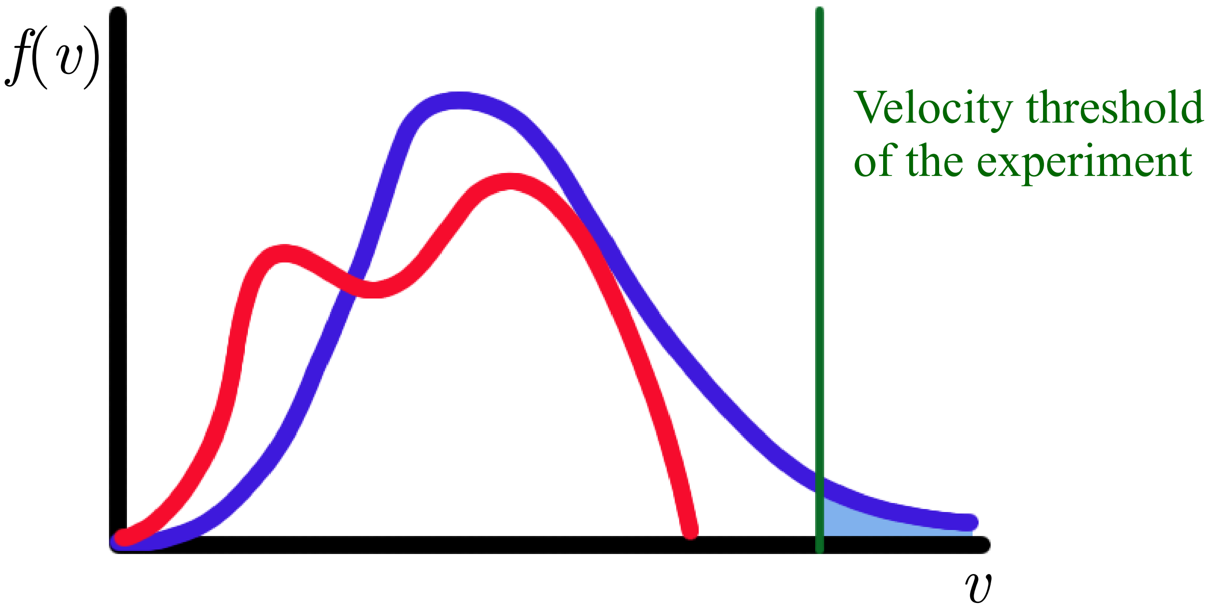

Some velocity distributions will escape detection in the experiment

Halo-independent approach for DM frameworks

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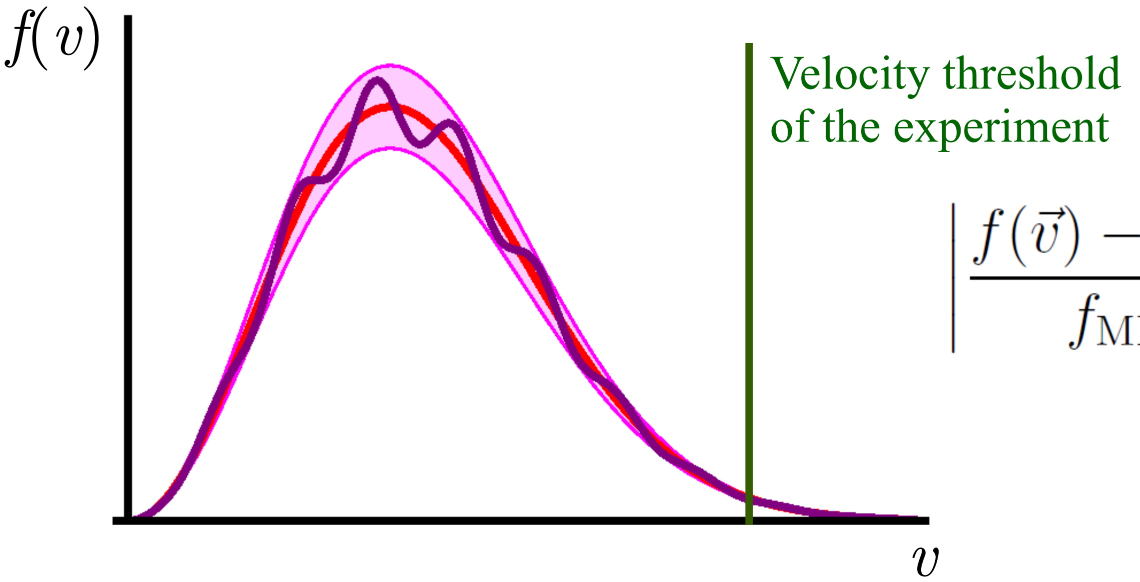

$$\left| \frac{f(\vec{v}) - f_{\mathrm{MB}}(\vec{v})}{f_{\mathrm{MB}}(\vec{v})} \right| \leq \Delta$$

Possibility 1: consider "distortions" of the Maxwell-Boltzmann distribution

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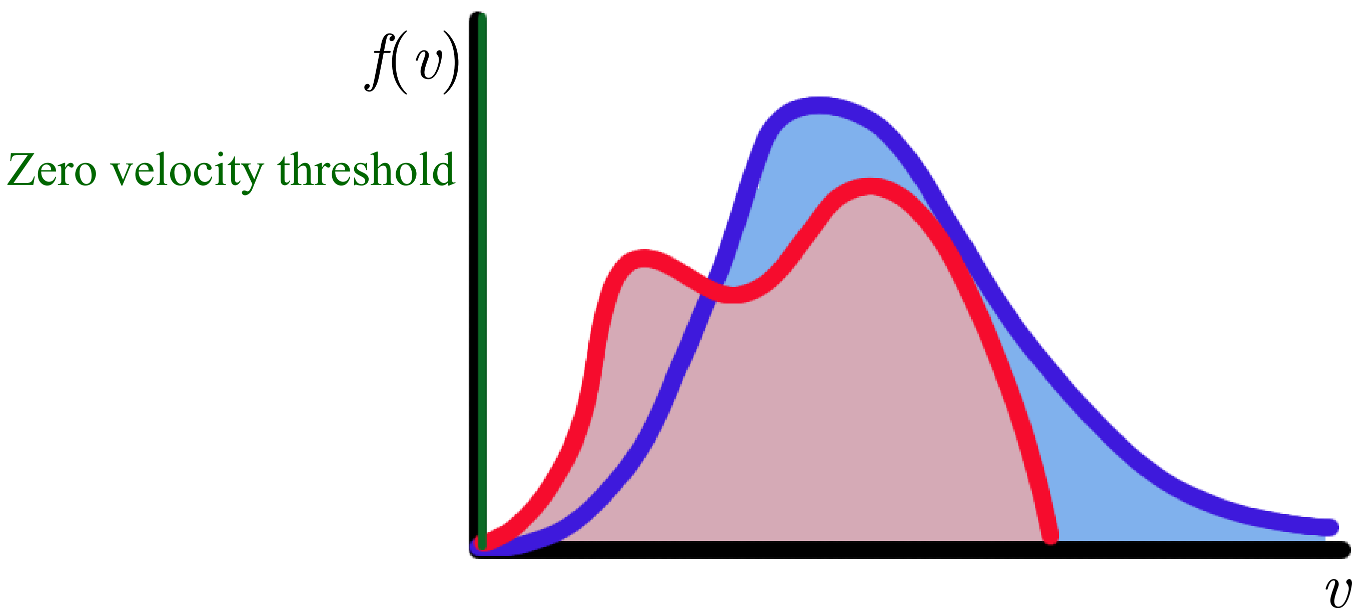

Possibility 2: Design an experiment with zero velocity threshold

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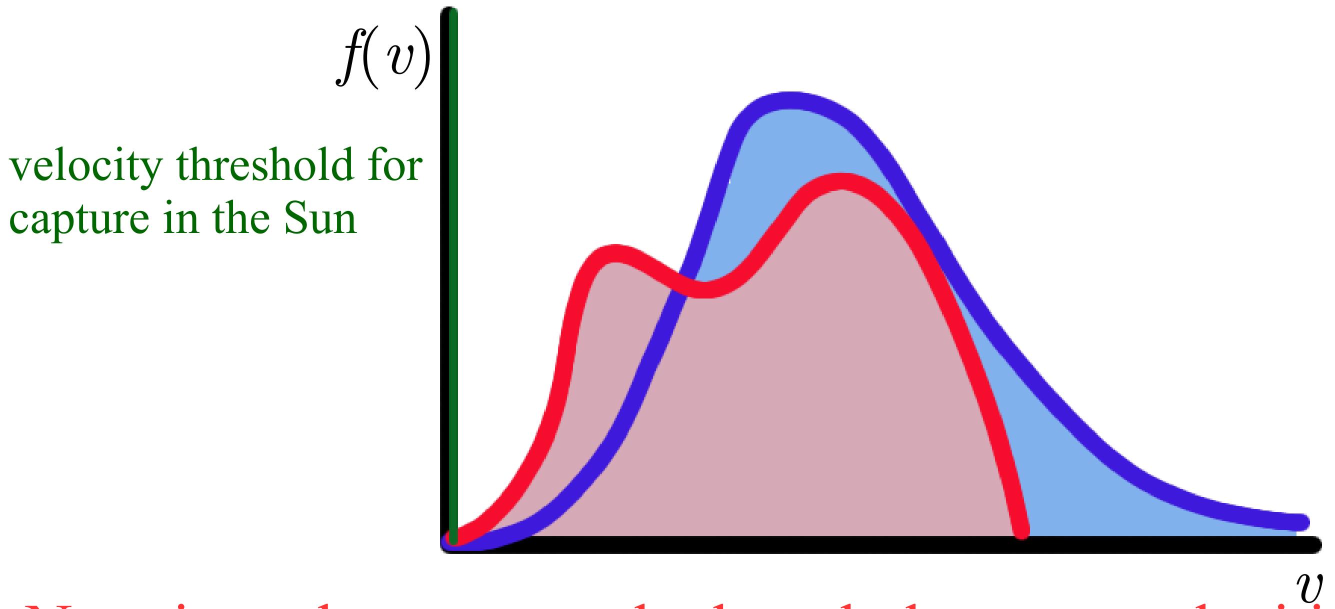

Neutrino telescopes probe low dark matter velocities. In combination with direct detection experiments, one can probe the whole velocity space

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$$\min_{f(\vec{v})} \left\{ R(\sigma, m_{\rm DM}) \right\} \Big|_{\int f = 1} > R_{\rm u.l.}$$

Possibility 1:

$$\min_{f(\vec{v})} R(m_{\rm DM}, \sigma) \Big|_{\substack{\int f = 1 \\ f \text{ within band}}} > R_{\rm u.l.}$$

Possibility 2:

$$\min_{f(\vec{v})} R(m_{\rm DM}, \sigma) \Big|_{\substack{\int f = 1 \\ C < C_{\rm u.l.}}} > R_{\rm u.l.}$$

Optimization problem with constraints

Halo-independent approach for DM frameworks

Technically complicated...

$$R(\sigma, m_{\rm DM}) = \int_{E_{\rm th}}^{\infty} dE_R \frac{\rho_{\rm loc}}{m_A m_{\rm DM}} \int_{v \geq v_{\rm min}(E_R)} {\rm d}^3 v \, v \, f(\vec{v} + \vec{v}_{\rm obs}(t)) \, \frac{{\rm d}\sigma}{{\rm d}E_R}$$

$$C(\sigma, m_{\rm DM}) = \int_0^{R_\odot} 4\pi r^2 {\rm d}r \, \frac{\rho_{\rm loc}}{m_{\rm DM}} \int_{v \leq v_{\rm max}^{\rm (Sun)}(r)} {\rm d}^3 v \, \frac{f(\vec{v})}{v} \left(v^2 + [v_{\rm esc}(r)]^2\right) \times \int_{m_{\rm DM} v^2/2}^{2\mu_A^2 \left(v^2 + [v_{\rm esc}(r)]^2\right)/m_A} {\rm d}E_R \, \frac{{\rm d}\sigma}{{\rm d}E_R}$$

Halo-independent approach for DM frameworks

Express the velocity distribution as a superposition of many many streams:

$$f(\vec{v}) = \sum_{i=1}^{n} c_{\vec{v}_i} \, \delta(\vec{v} - \vec{v}_i)$$

Minimization problem. For given DM mass and cross-section:

$$\text{minimize} \quad R^{(\text{PandaX})}(c_{\vec{v}_1} \ldots, c_{\vec{v}_n}) = \sum_{i=1}^{n} c_{\vec{v}_i} R^{(\text{PandaX})}_{\vec{v}_i} \,,$$

$$\text{subject to} \quad \sum_{i=1}^{n} c_{\vec{v}_i} C^{(\text{NT})}_{\vec{v}_i} \leq C^{(\text{NT})}_{\max} \,,$$

$$\text{and} \quad \sum_{i=1}^{n} c_{\vec{v}_i} = 1 \,,$$

$$\text{and} \quad c_{\vec{v}_i} \geq 0, \quad i = 1 \ldots, n \,,$$

The parameters σ and m_{DM} are excluded in a halo independent manner if :

$$\min \left\{ R^{(\text{PandaX})}(c_{\vec{v}_1} \ldots, c_{\vec{v}_n}) \right\} \Big|_{\text{constraints}} > R^{(\text{PandaX})}_{\max}$$

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$$f(\vec{v}) = \sum_{i=1}^{n} c_{\vec{v}_i}\ \delta(\vec{v} - \vec{v}_i)$$

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$$\text{and} \quad \sum_{i=1}^{n} c_{\vec{v}_i} = 1\ ,$$

$$\text{and} \quad c_{\vec{v}_i} \geq 0, \quad i = 1\dots, n\ ,$$

The objective function and the constraints are linear in the weights of the DM streams

→ Optimize using linear programming techniques.

A tour in linear programming

An automobile company produces cars and trucks. For each car obtains 400€ profit, and for each truck, 700€. What should be the strategy of the company to optimize the weekly profit?

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In real life, the production is subject to constraints

- It takes 4 hours to assemble the engine of a car, and 3 hours for a truck
- It takes 2 hours to paint a car, and 4 hours to paint a truck
- The assembling chain operates 14 hours a day, and the paint workshop operates 10 hours a day, 5 days a week.

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Linear programming problem:

$$
\begin{aligned}
\text{Maximize} \quad & P = 400N_c + 700N_t \\
\text{subject to} \quad & 4N_c + 3N_t \leq 14 \times 5 \\
\text{and} \quad & 2N_c + 4N_t \leq 10 \times 5 \\
\text{and} \quad & N_c \geq 0,\ N_t \geq 0
\end{aligned}
$$

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Linear programming problem:

"Objective function" Maximize $P = 400N_c + 700N_t$

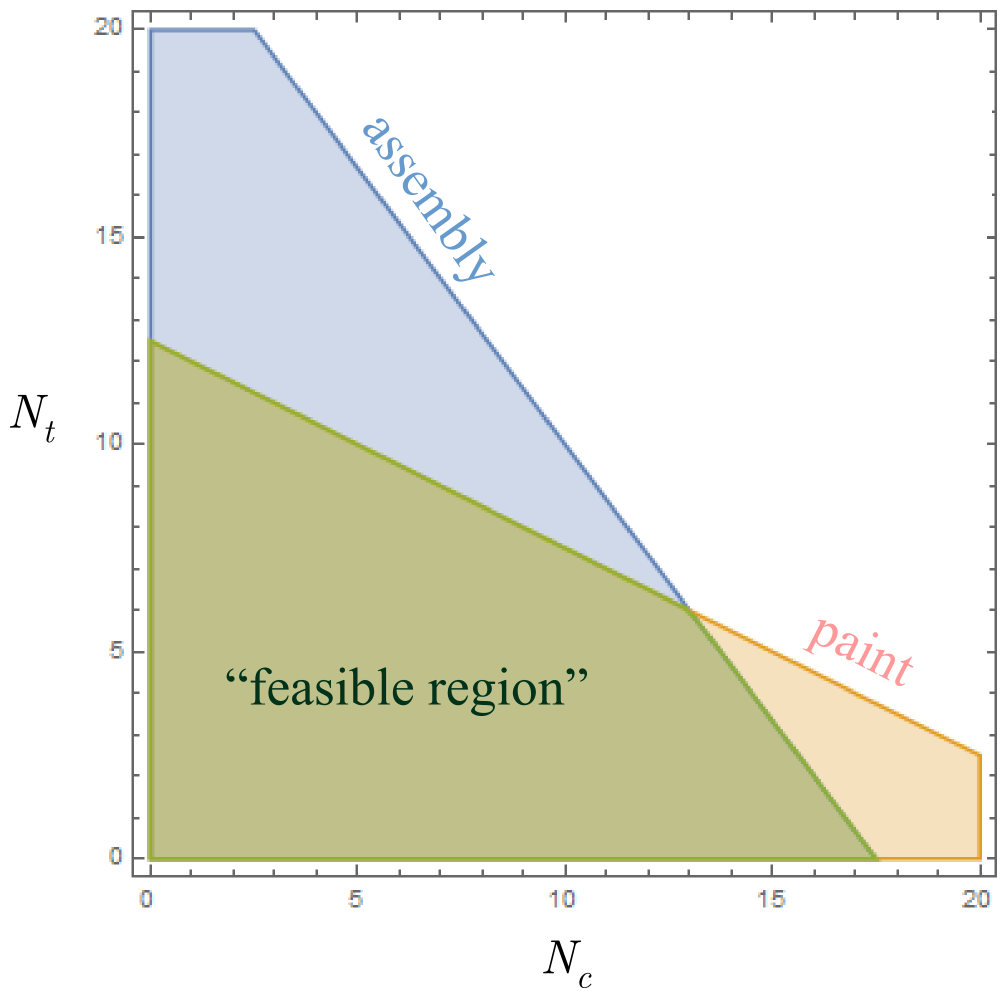
"Constraints"

$$\text{subject to } 4N_c + 3N_t \leq 14 \times 5$$
$$\text{and } 2N_c + 4N_t \leq 10 \times 5$$
$$\text{and } N_c \geq 0, N_t \geq 0$$

"Decision variables" (N_c, N_t)

A tour in linear programming

$$\begin{aligned} \text{Maximize} \quad & P = 400N_c + 700N_t \\ \text{subject to} \quad & 4N_c + 3N_t \leq 14 \times 5 \\ \text{and} \quad & 2N_c + 4N_t \leq 10 \times 5 \\ \text{and} \quad & N_c \geq 0, N_t \geq 0 \end{aligned}$$



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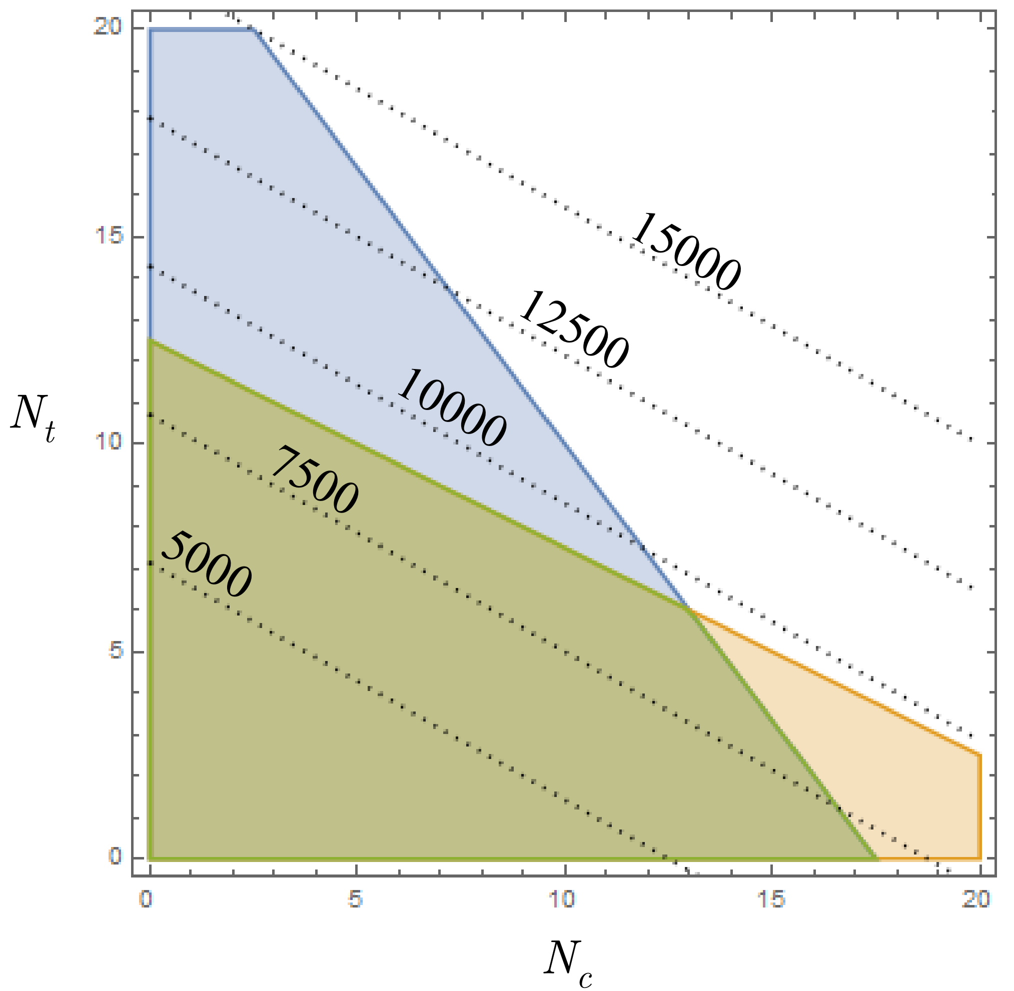

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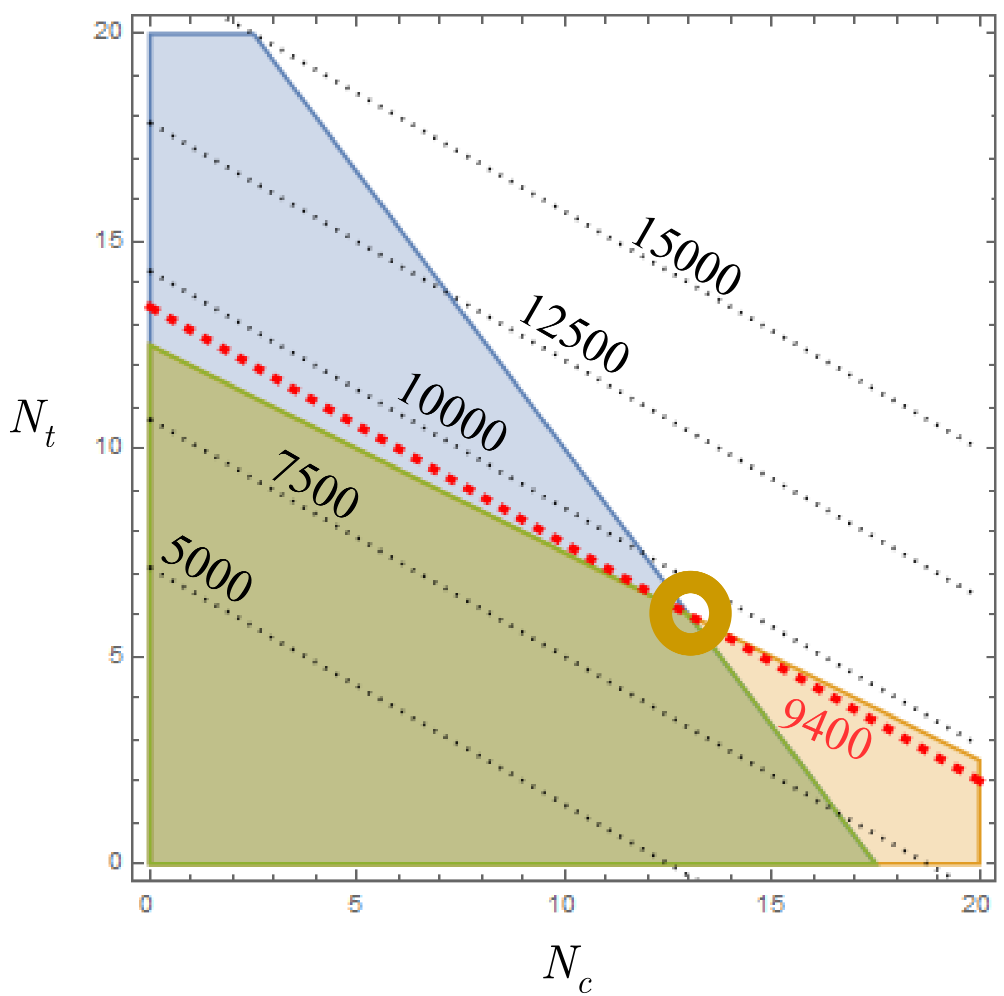
$N_c = 13$

$N_t = 6$

Profit = 9400 €/week



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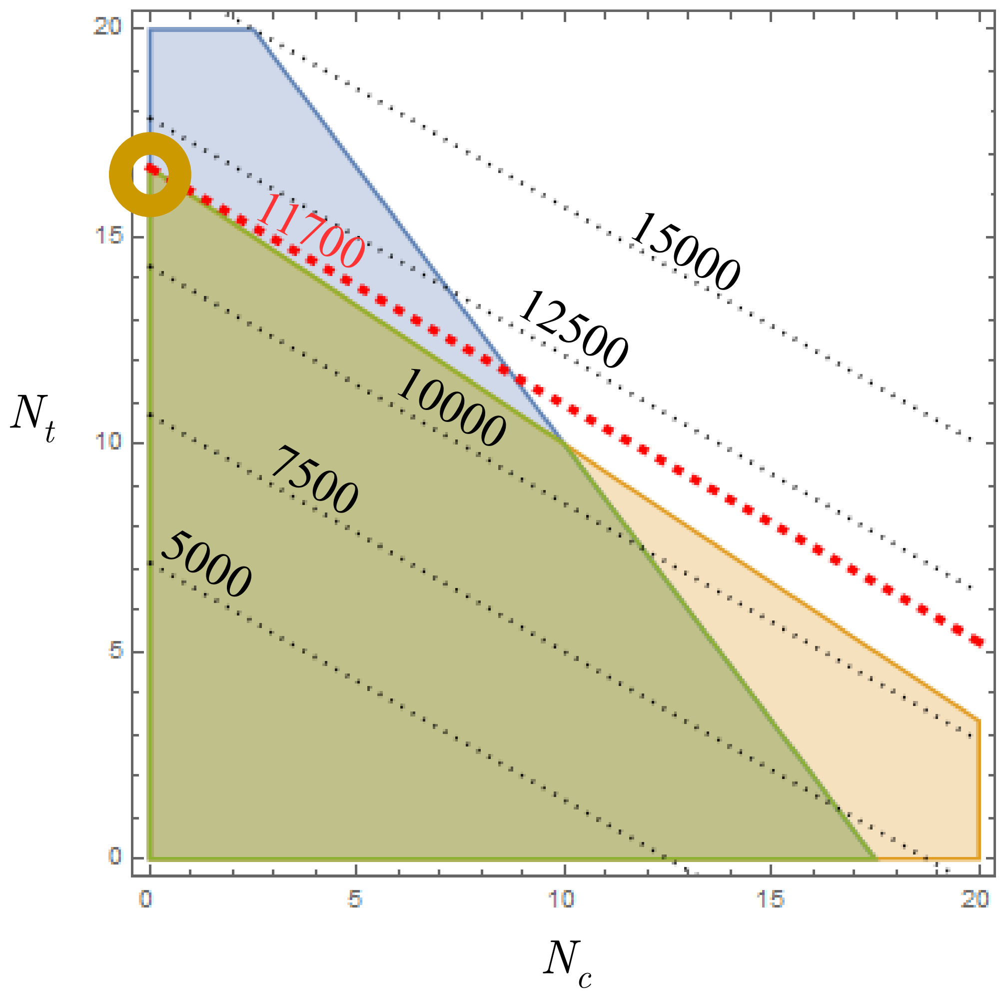
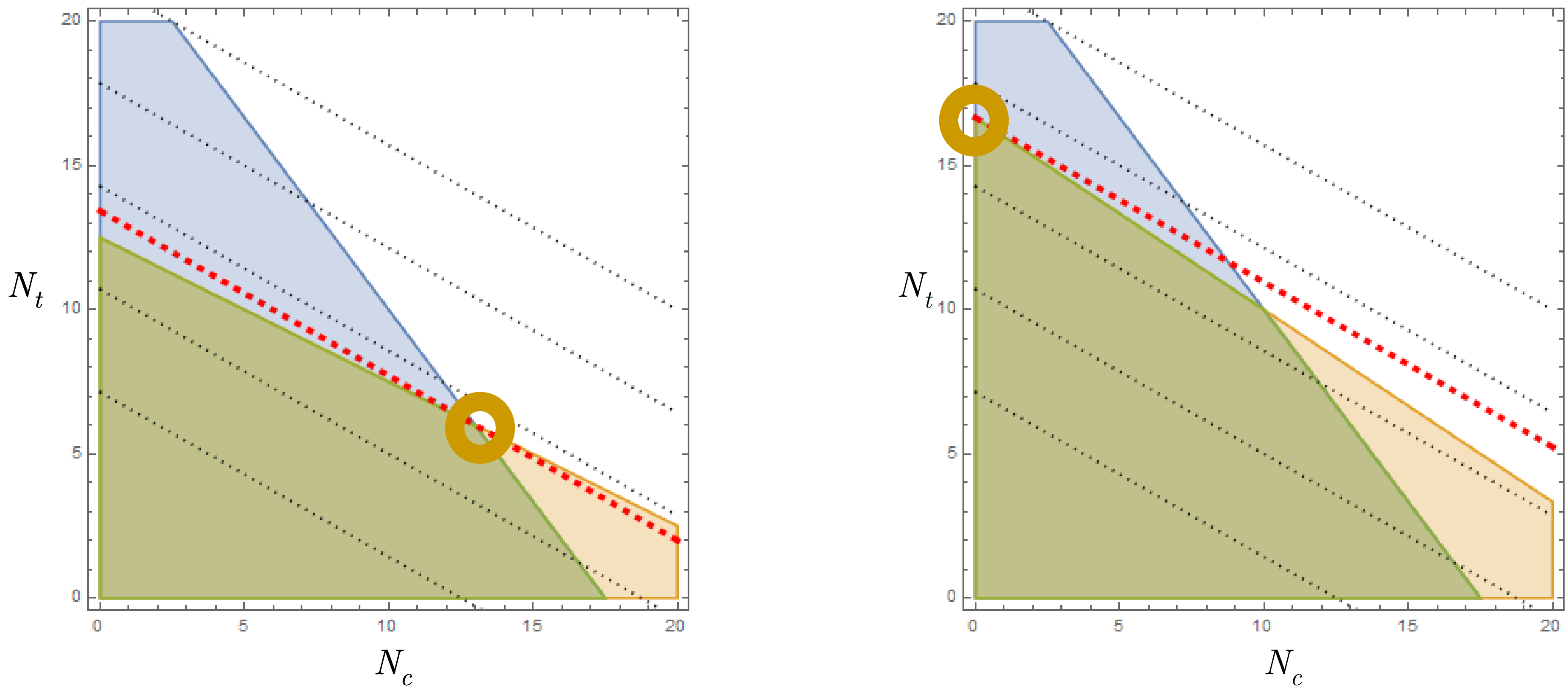
$N_c = 0$

$N_t = 16.7$

Profit = 11700 €/week



A tour in linear programming



Lessons:

1) The solution lies at one of the vertices of the feasible region (polygon)

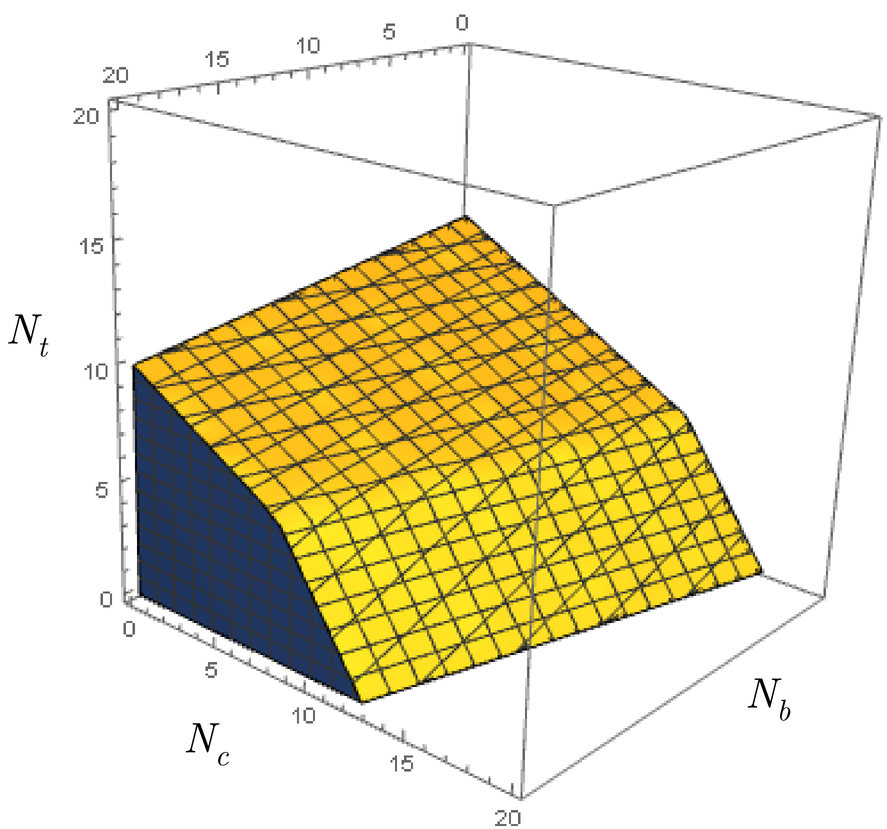
2) For two constraints there are:

- two non-vanishing decision variables, when the two constraints are saturated
- one non-vanishing decision variable, when one of the constraints is not saturated

A tour in linear programming

Suppose that the company also produces motorbikes. The profit is 100€ per motorbike, it takes 1 hour to assemble the engine of the motorbike, and it takes 30 minutes to paint the motorbike.

$$\begin{aligned} \text{Maximize} \quad & P = 400N_c + 700N_t + 100N_b \\ \text{subject to} \quad & 4N_c + N_t + N_b \leq 14 \times 5 \\ \text{and} \quad & 2N_c + 4N_t + 0.5N_b \leq 10 \times 5 \\ \text{and} \quad & N_c \geq 0, N_t \geq 0, N_b \geq 0 \end{aligned}$$



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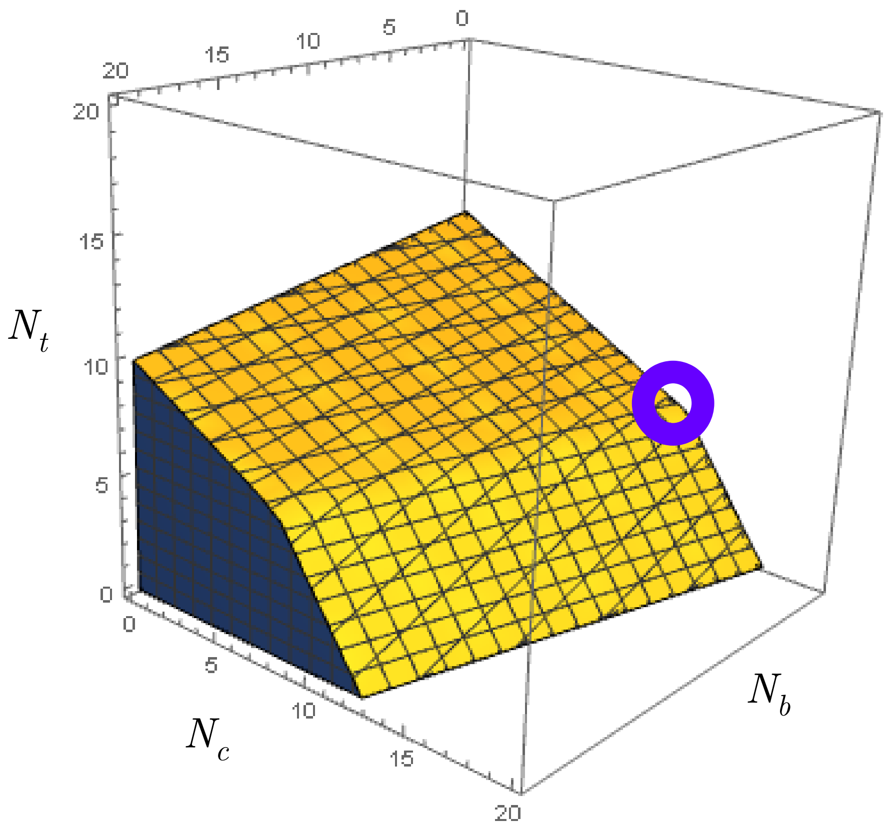


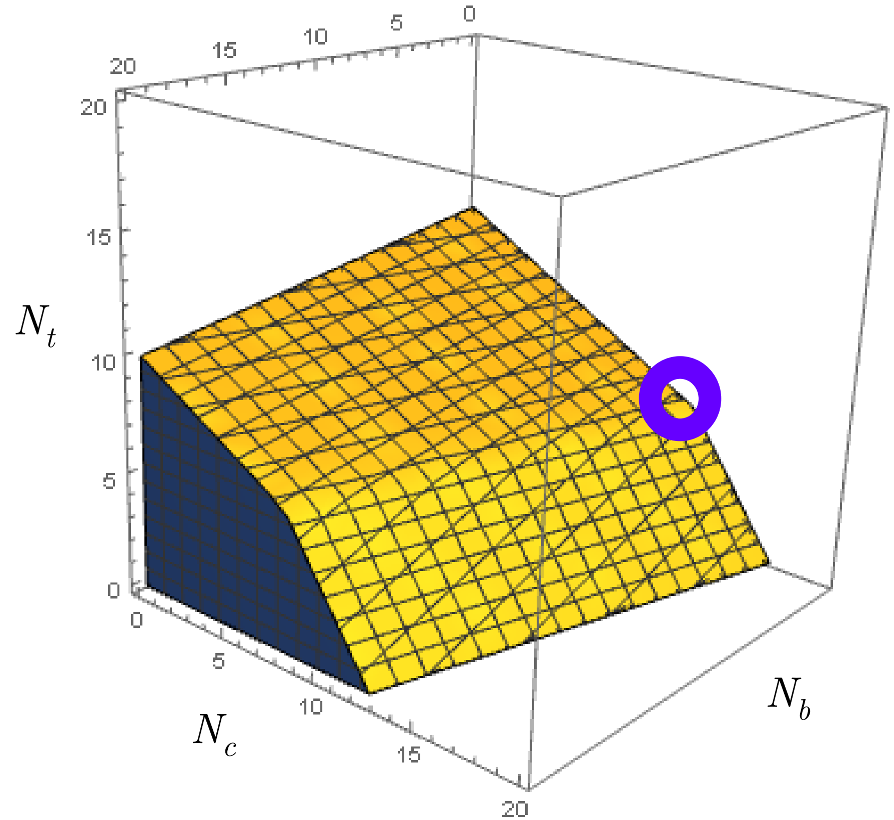
$N_b = 0$

$N_c = 13$

$N_t = 6$

Profit = 9400 €/week

A tour in linear programming



For three decision variables and two constraints, the optimized solution *necessarily* has at least one vanishing decision variable (or, alternatively, at most two non-vanishing decision variables).

(Three non-vanishing decision variables would correspond to a point singled-out by the intersection of three planes, but we only have two constraints!)

Take-home lessons from linear programming:

1) The solution lies at one of the vertices of the "feasible region"

2) For N constraints, there are between 1 and N non-vanishing decision variables

(when r of the constraints are not saturated, then the optimal solution consists of $N - r$ decision variables)

Upper limit on the scattering cross section from combining PandaX and IceCube/SK.

Express the velocity distribution as a superposition of many many streams:

$$f(\vec{v}) = \sum_{i=1}^{n} c_{\vec{v}_i}\ \delta(\vec{v} - \vec{v}_i)$$

Minimization problem. For given DM mass and cross-section:

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$$\text{and}\quad \sum_{i=1}^{n} c_{\vec{v}_i} = 1\ ,$$

$$\text{and}\quad c_{\vec{v}_i} \geq 0,\quad i = 1\ldots, n\ ,$$

1) The solution lies at one of the vertices of the “feasible region”

2) The optimized velocity distribution contains either one or two streams (depending on the number of constraints that are not saturated).

Generalization

Calculate the maximum/minimum outcome in a direct detection experiment A, given the upper limits on the outcome of p experiments B_α, α= 1..., p , and the lower limits on the outcome of q experiments B_α, α= p+1..., $p+q$ (and the requirement that the velocity distribution is normalized to 1).

$$\text{optimize} \quad F(c_{\vec{v}_1} ..., c_{\vec{v}_n}) = \sum_{i=1}^{n} c_{\vec{v}_i} N_{\vec{v}_i}^{(A)} \ ,$$

$$\text{subject to} \quad \sum_{i=1}^{n} c_{\vec{v}_i} N_{\vec{v}_i}^{(B_\alpha)} \leq N_{\max}^{(B_\alpha)}, \quad \alpha = 1, ..., p \ ,$$

$$\text{and} \quad \sum_{i=1}^{n} c_{\vec{v}_i} N_{\vec{v}_i}^{(B_\alpha)} \geq N_{\min}^{(B_\alpha)}, \quad \alpha = p+1, ..., p+q \ ,$$

$$\text{and} \quad \sum_{i=1}^{n} c_{\vec{v}_i} = 1 \ ,$$

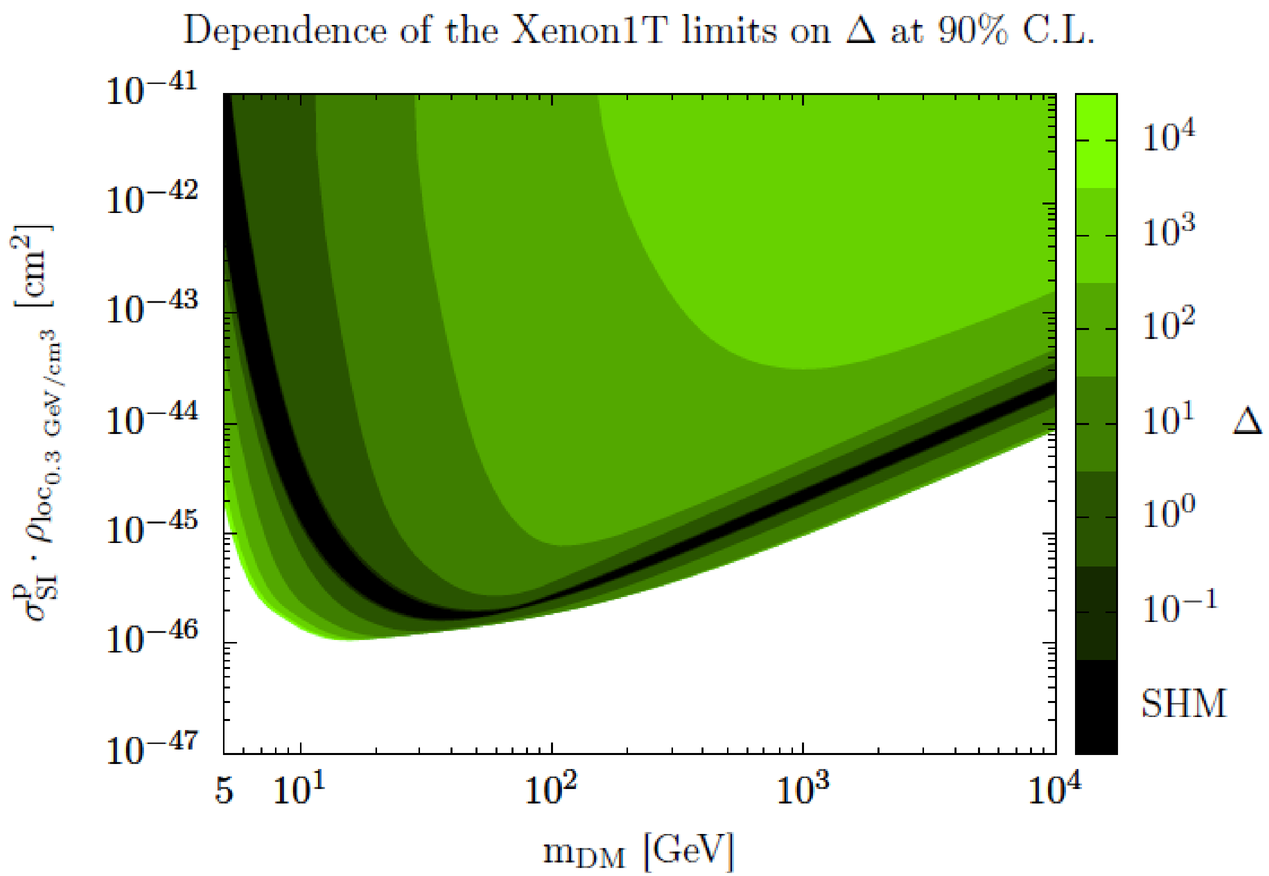
$$\text{and} \quad c_{\vec{v}_i} \geq 0, \quad i = 1..., n \ ,$$

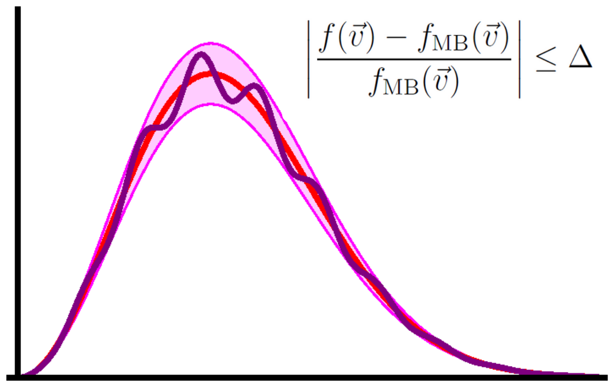
1) The solution lies at one of the vertices of the “feasible region”.
2) The optimized velocity distribution contains between 1 and $p+q+1$ streams (depending on the number of constraints that are not saturated).

Distorting the Maxwell-Boltzmann distribution

Calculate for a given Δ the minimum of the scattering rate among all the velocity distributions within the band. A point in parameter space is excluded if:

$$\min_{f(\vec{v})} R(m_{\rm DM}, \sigma)\Big|_{\substack{\int f=1 \\ f \text{ within band}}} > R_{\rm u.l.}$$





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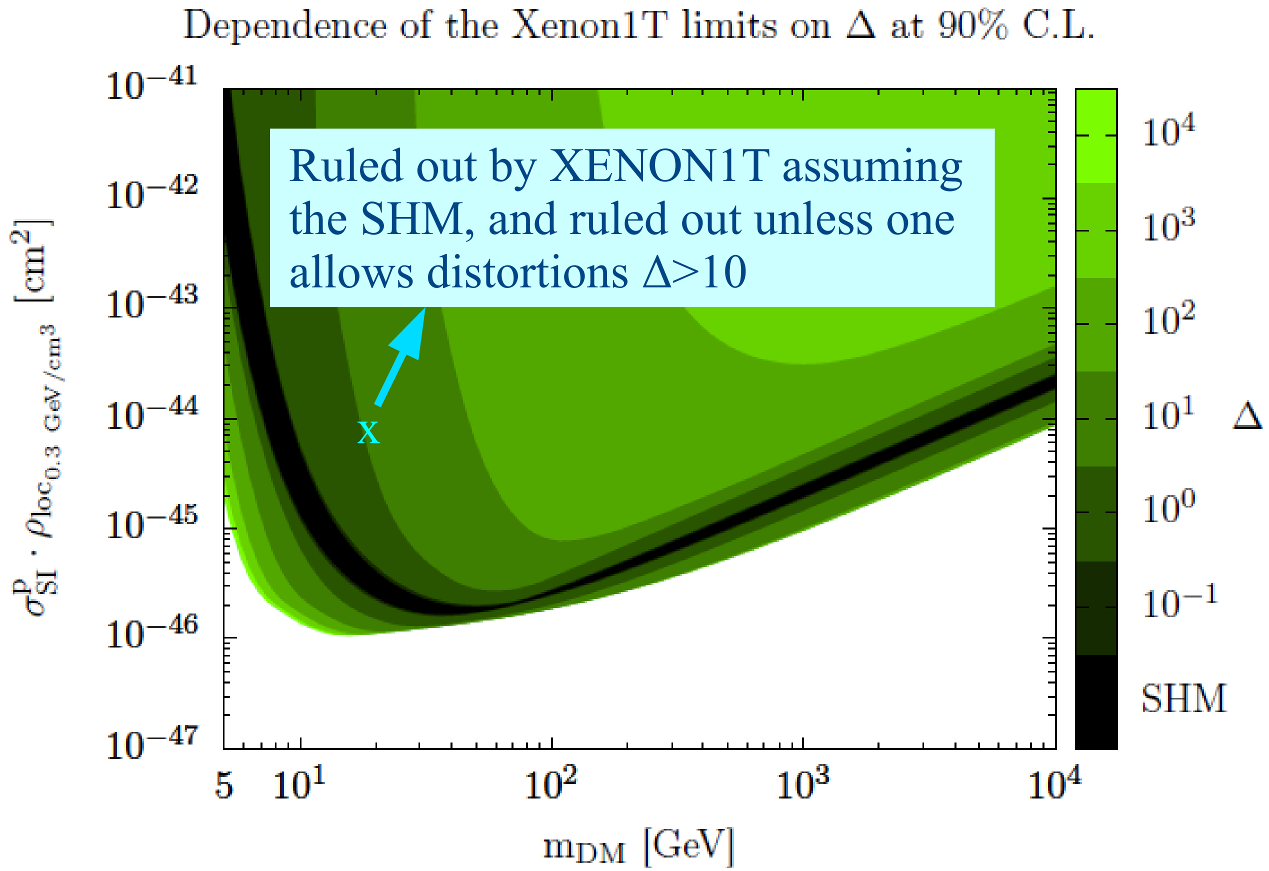

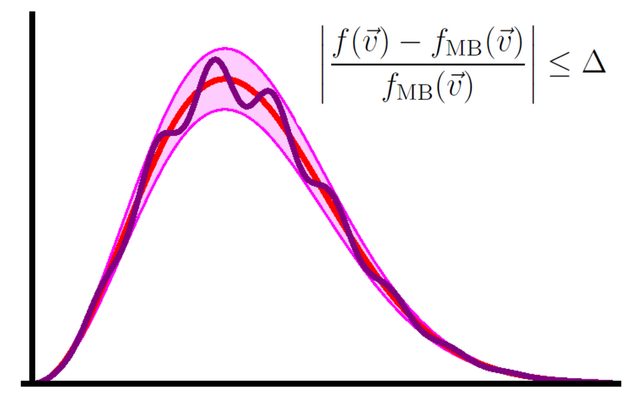

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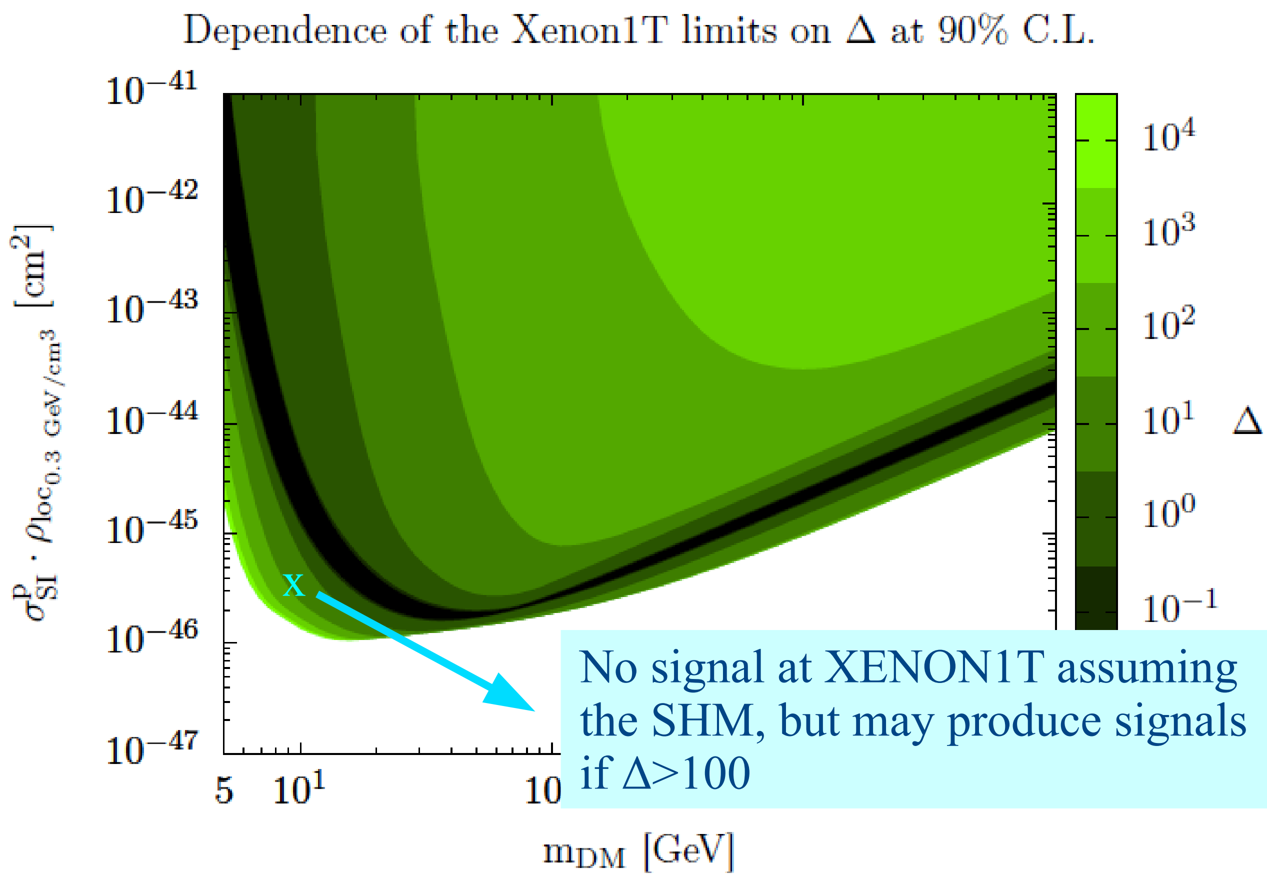
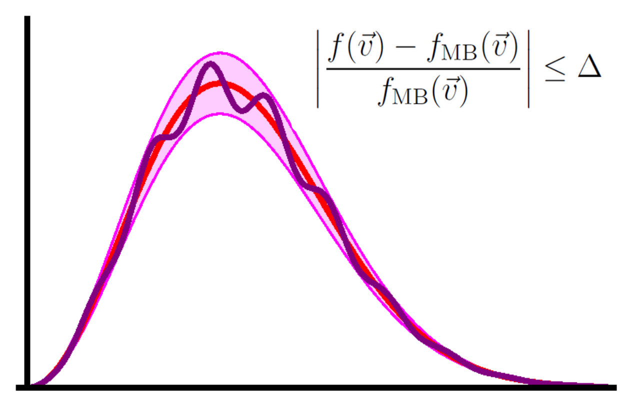


No signal at XENON1T assuming the SHM, but may produce signals if Δ>100



Upper limit on the scattering cross section from combining PandaX and IceCube/SK.

Calculate the minimum of the scattering rate among all the velocity distributions giving a capture rate in agreement with the constraints from neutrino telescopes. A point in parameter space is excluded if:

$$\min_{f(\vec{v})} R(m_{\rm DM}, \sigma)\Big|_{\substack{\int f=1 \\ C<C_{\rm u.l.}}} > R_{\rm u.l.}$$

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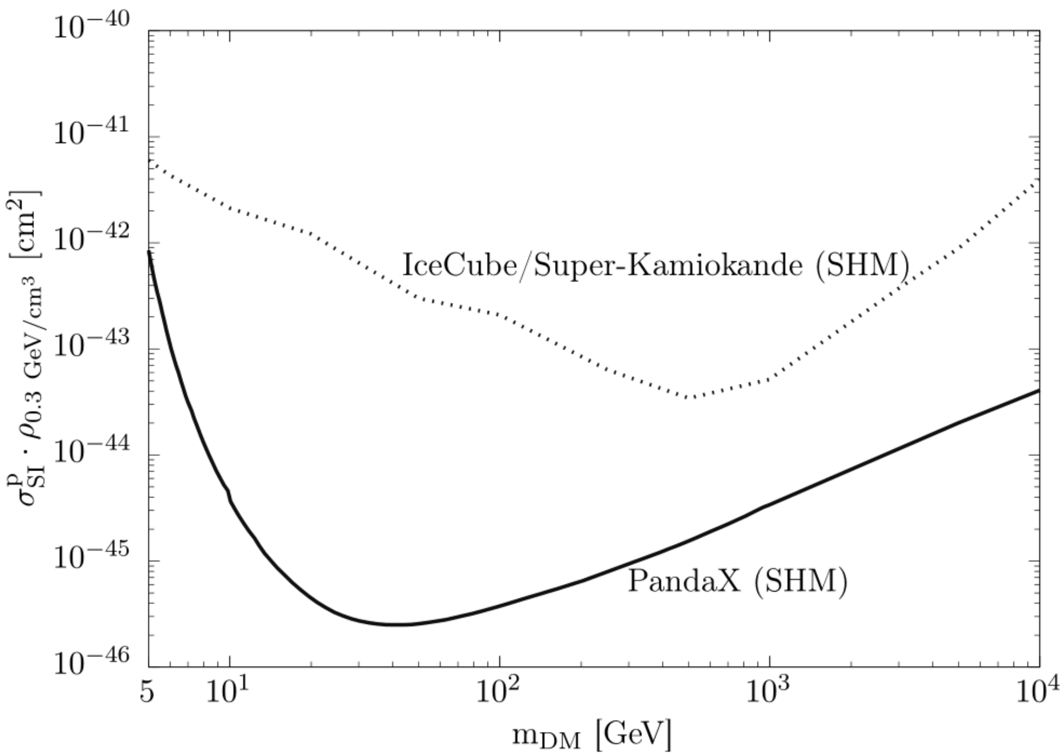

AI, Rappelt'17

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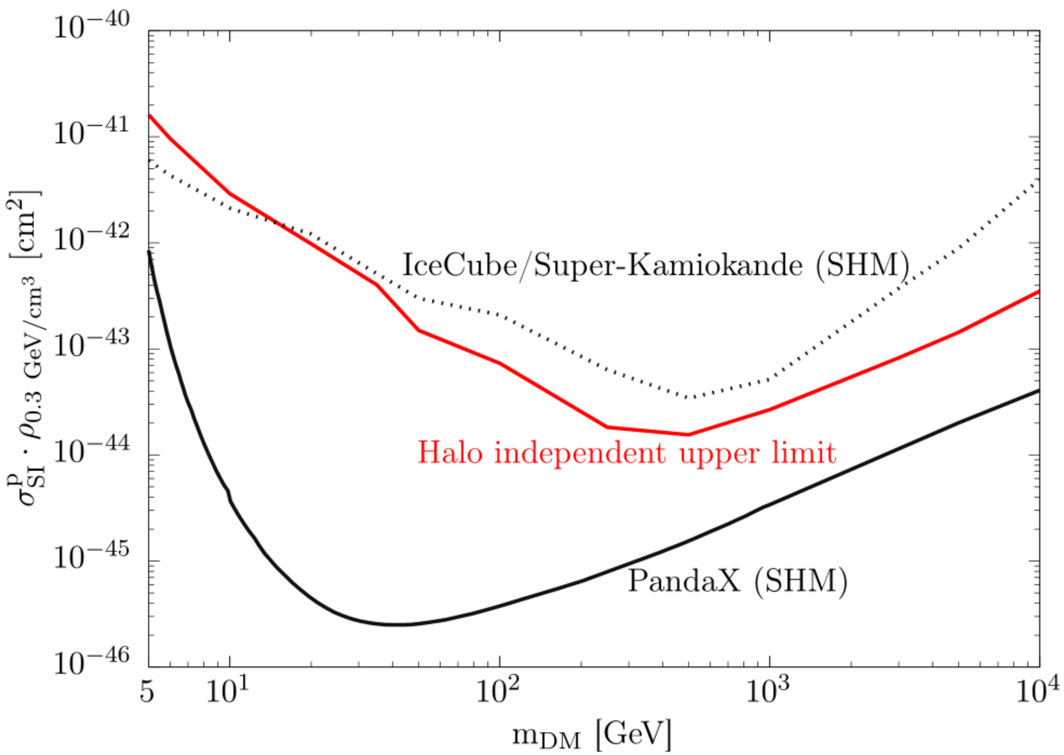

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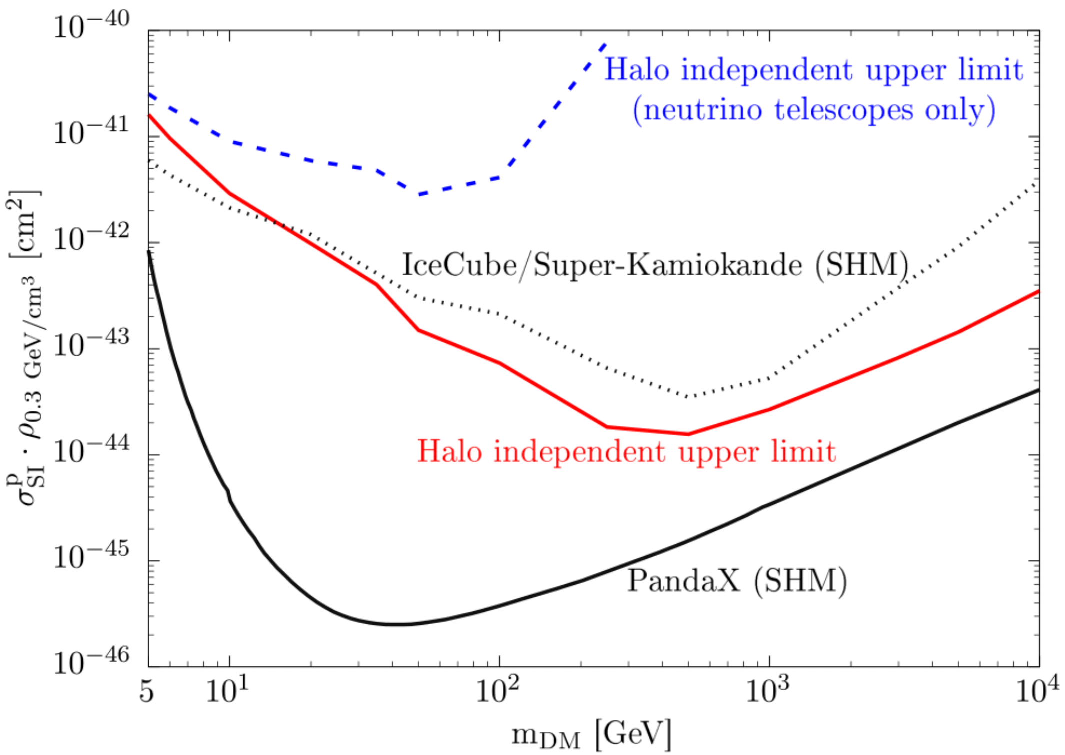

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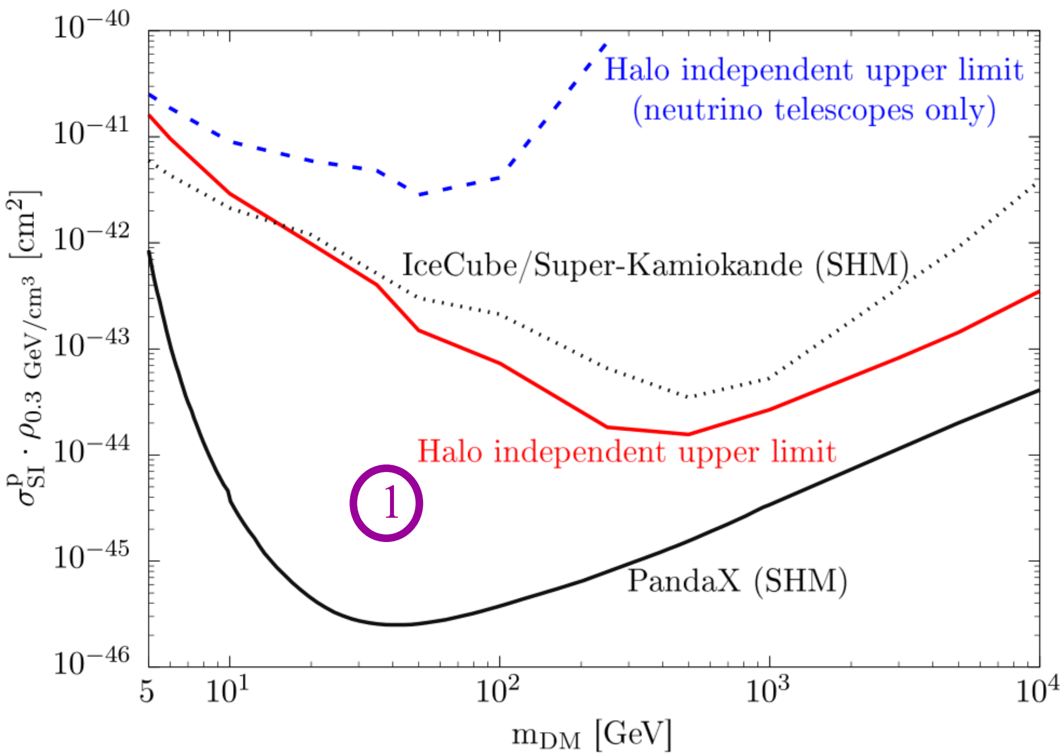

AI, Rappelt'17

(1) is ruled out by PandaX assuming the SHM, but allowed for some velocity distributions

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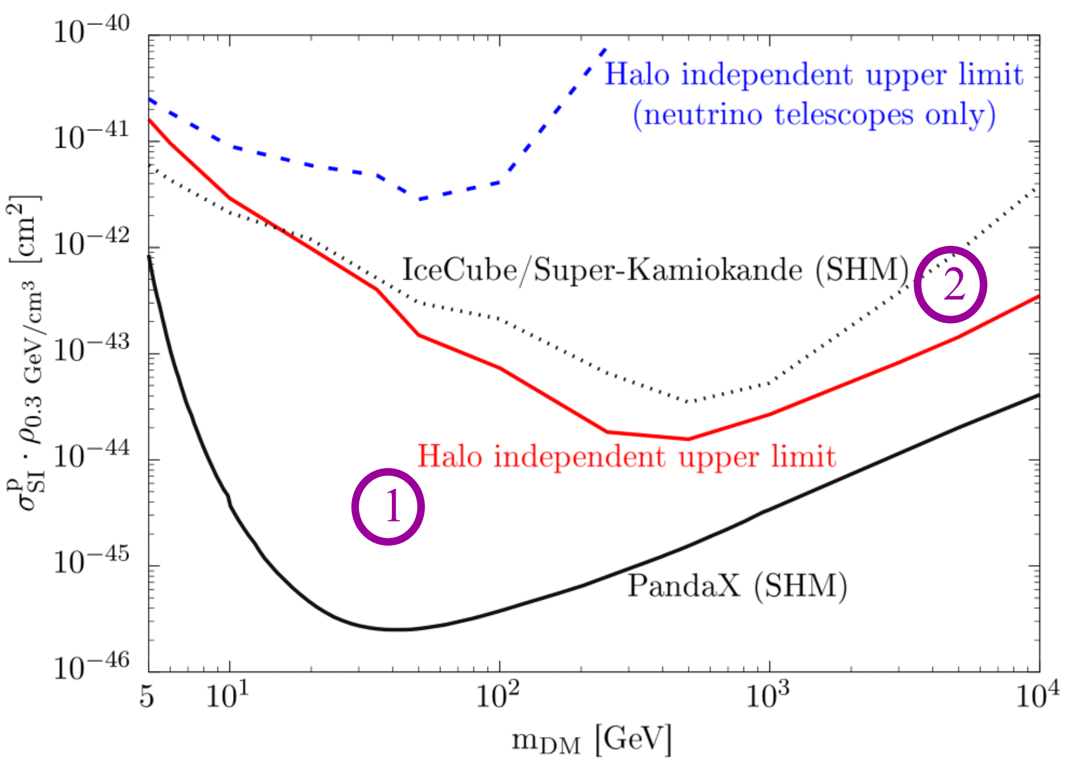

AI, Rappelt'17

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(2) is ruled out from combining PandaX and neutrino telescopes, for *any* velocity distribution.

Upper limit on the scattering cross section from combining PandaX and IceCube/SK.

Calculate the minimum of the scattering rate among all the velocity distributions giving a capture rate in agreement with the constraints from neutrino telescopes. A point in parameter space is excluded if:

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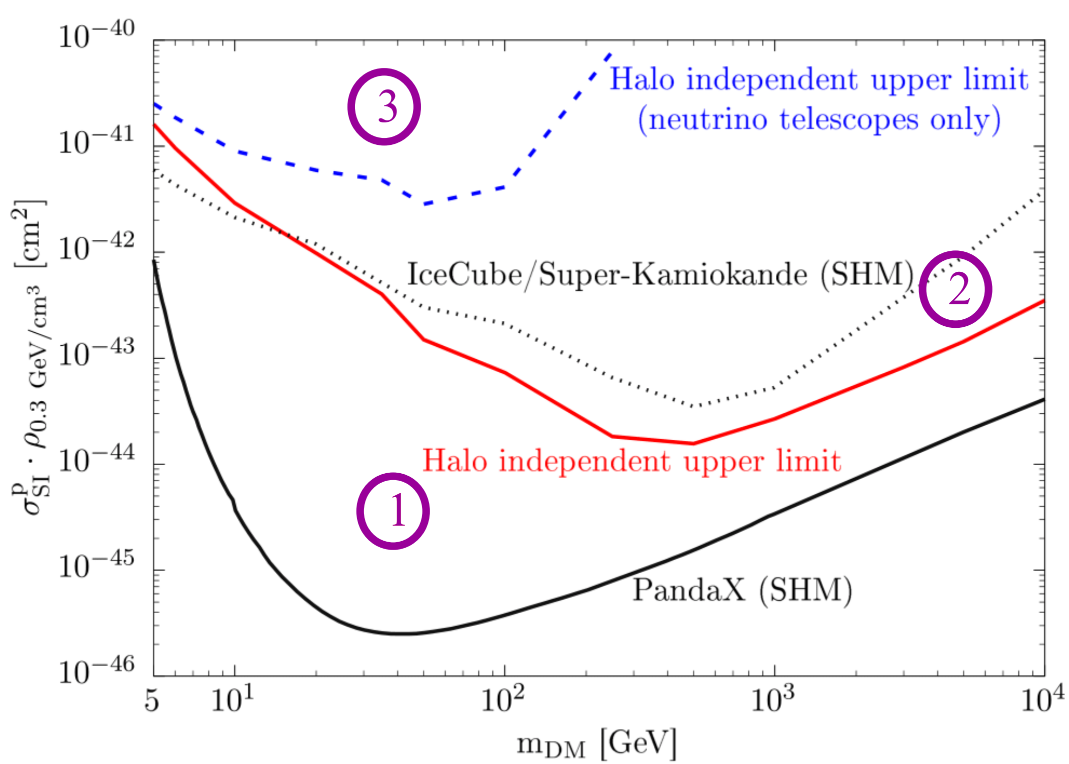


AI, Rappelt'17

(1) is ruled out by PandaX assuming the SHM, but allowed for some velocity distributions

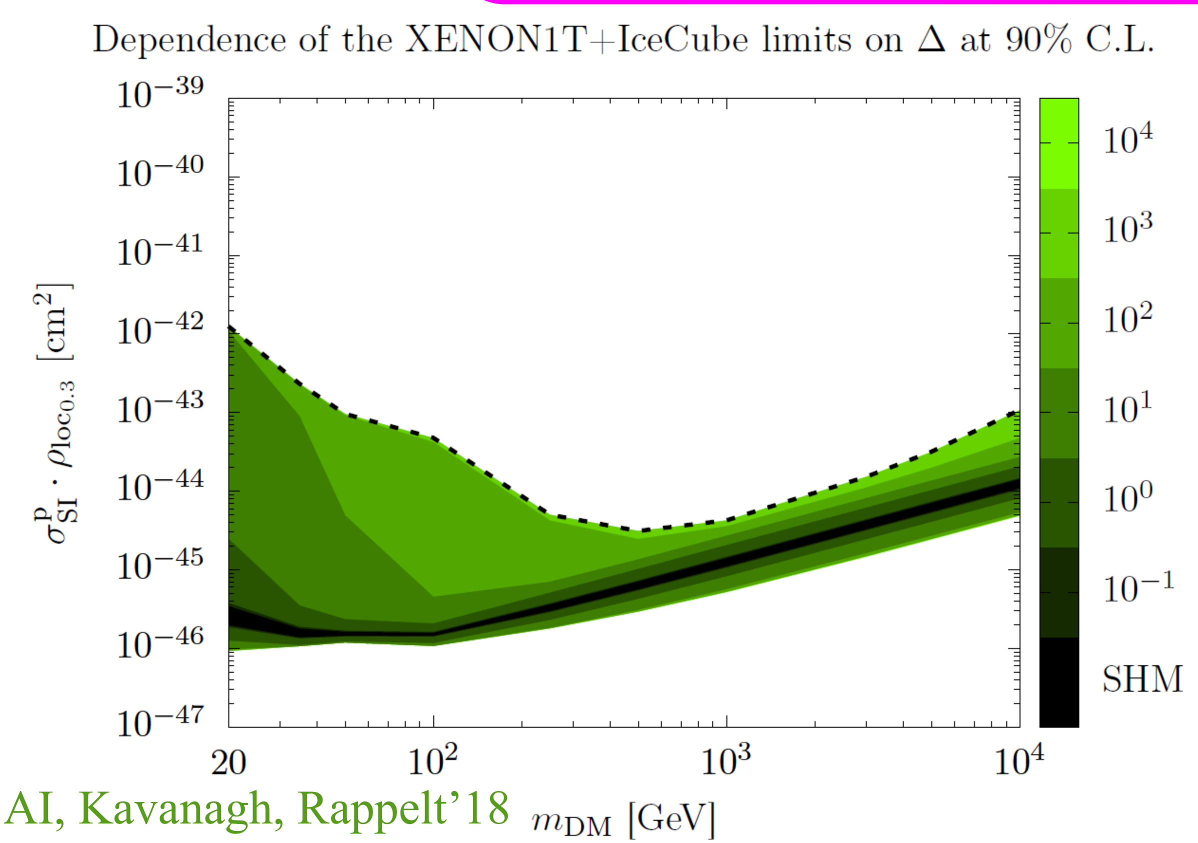
(2) is ruled out from combining PandaX and neutrino telescopes, for *any* velocity distribution.

(3) is ruled out by neutrino telescopes only, for *any* velocity distribution.

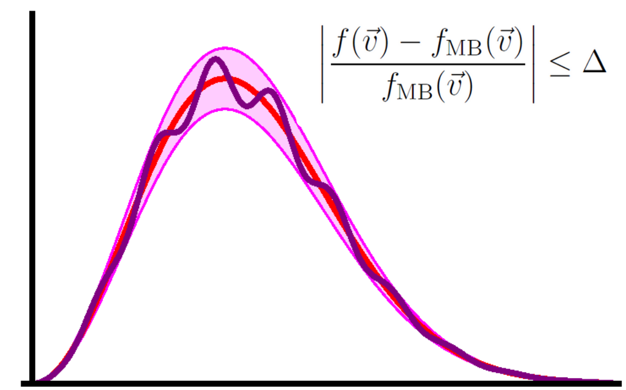
Upper limit on the scattering cross section from combining PandaX and IceCube/SK: extension

Calculate the minimum of the scattering rate among all the velocity distributions within the band of width Δ giving a capture rate in agreement with the constraints from neutrino telescopes. A point in parameter space is excluded if:

$$\min_{f(\vec{v})} R(m_{\rm DM},\sigma)\Big|_{\substack{\int f=1 \\ f\ \text{within band} \\ C<C_{\rm u.l.}}} > R_{\rm u.l.}$$



AI, Kavanagh, Rappelt'18



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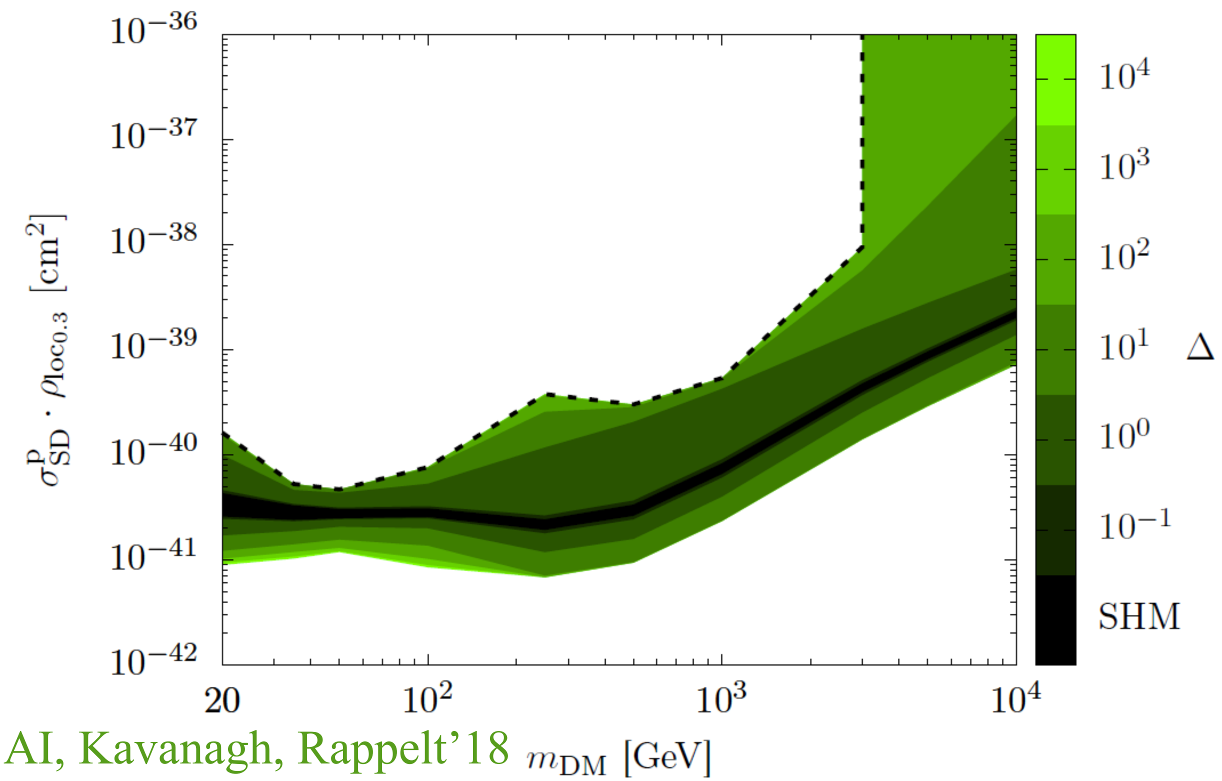
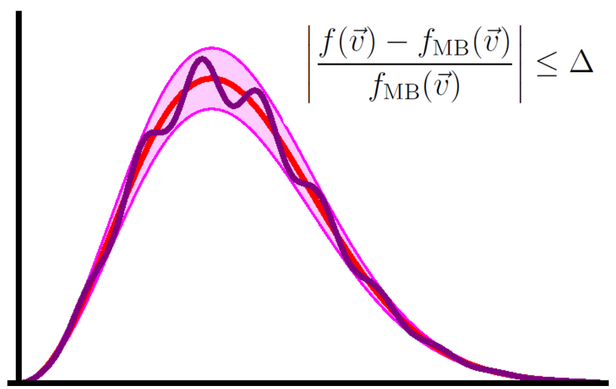
Dependence of the PICO+IceCube limits on Δ at 90% C.L.





AI, Kavanagh, Rappelt'18

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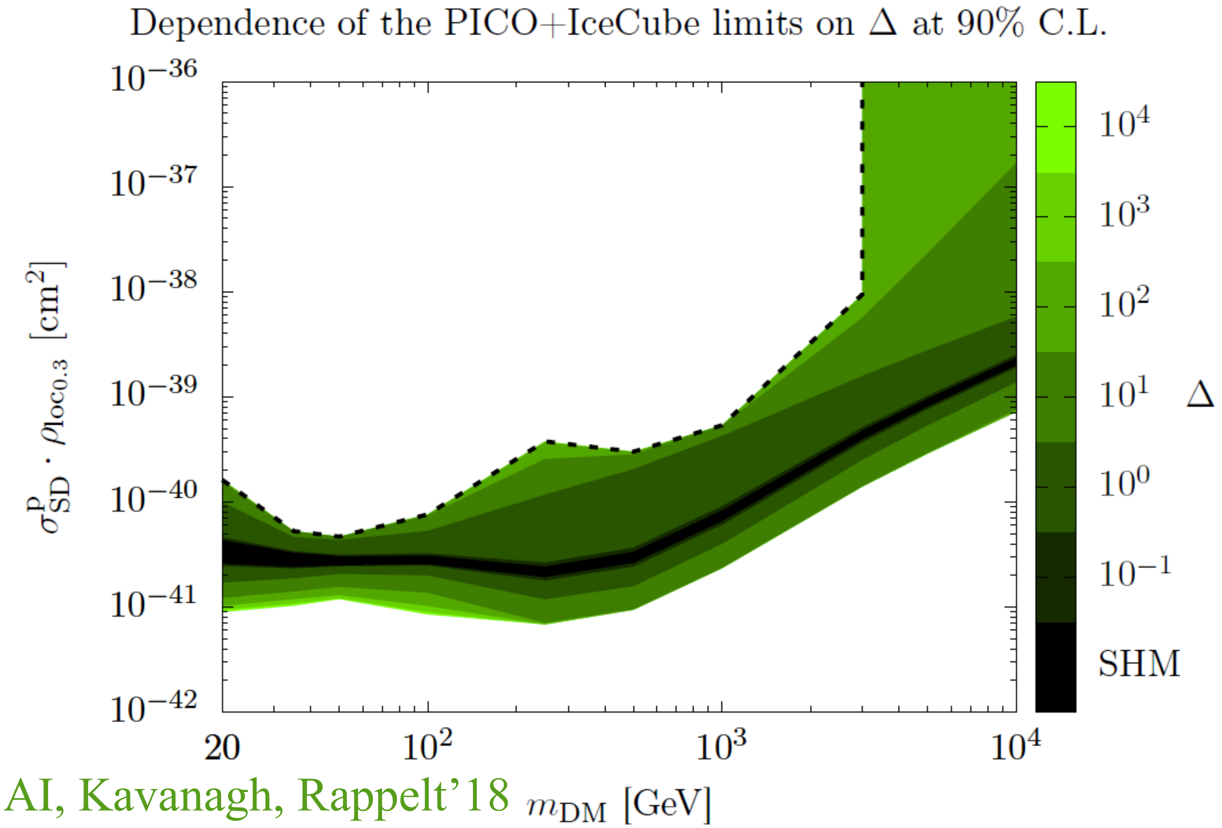

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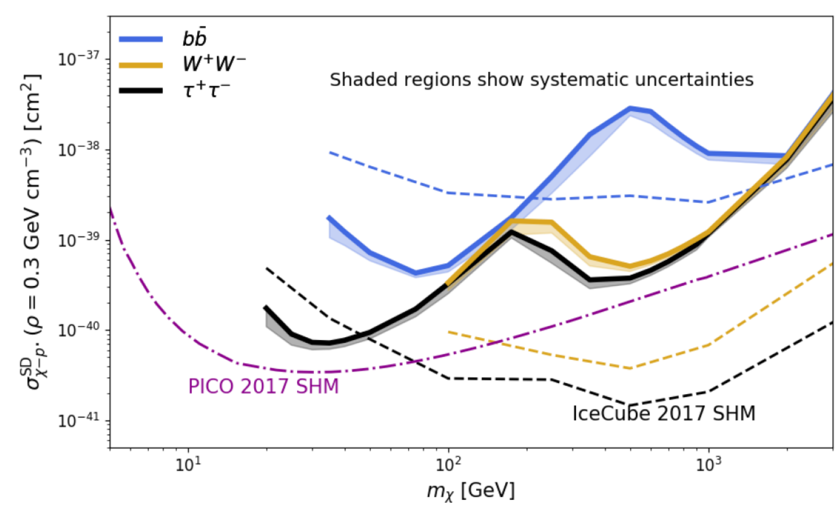


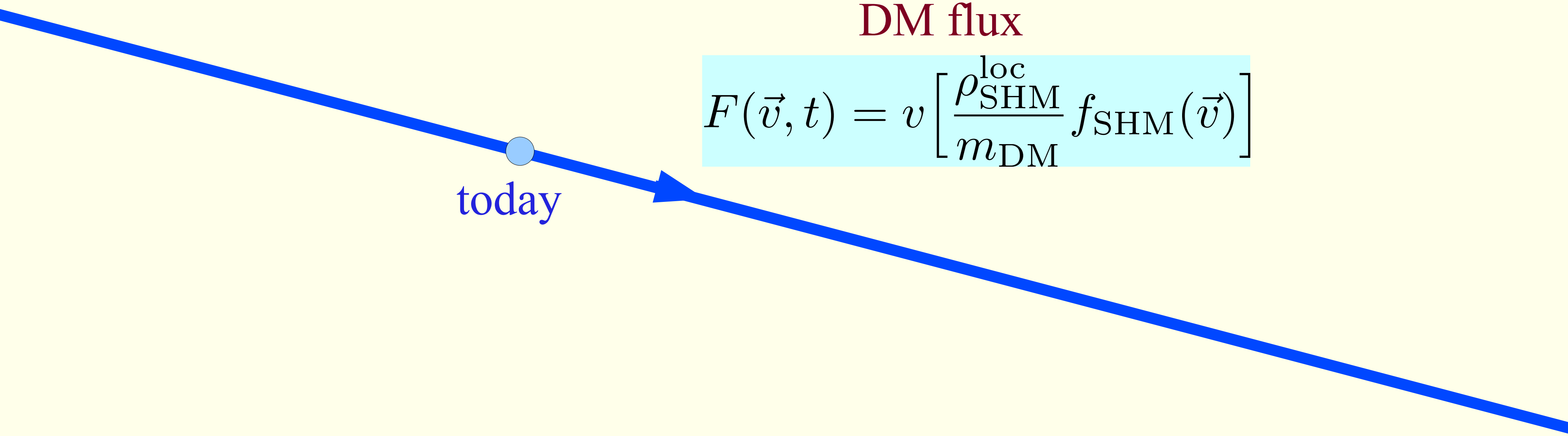
IceCube+PICO coll.'19

A concrete case:

Milky Way sub-halos

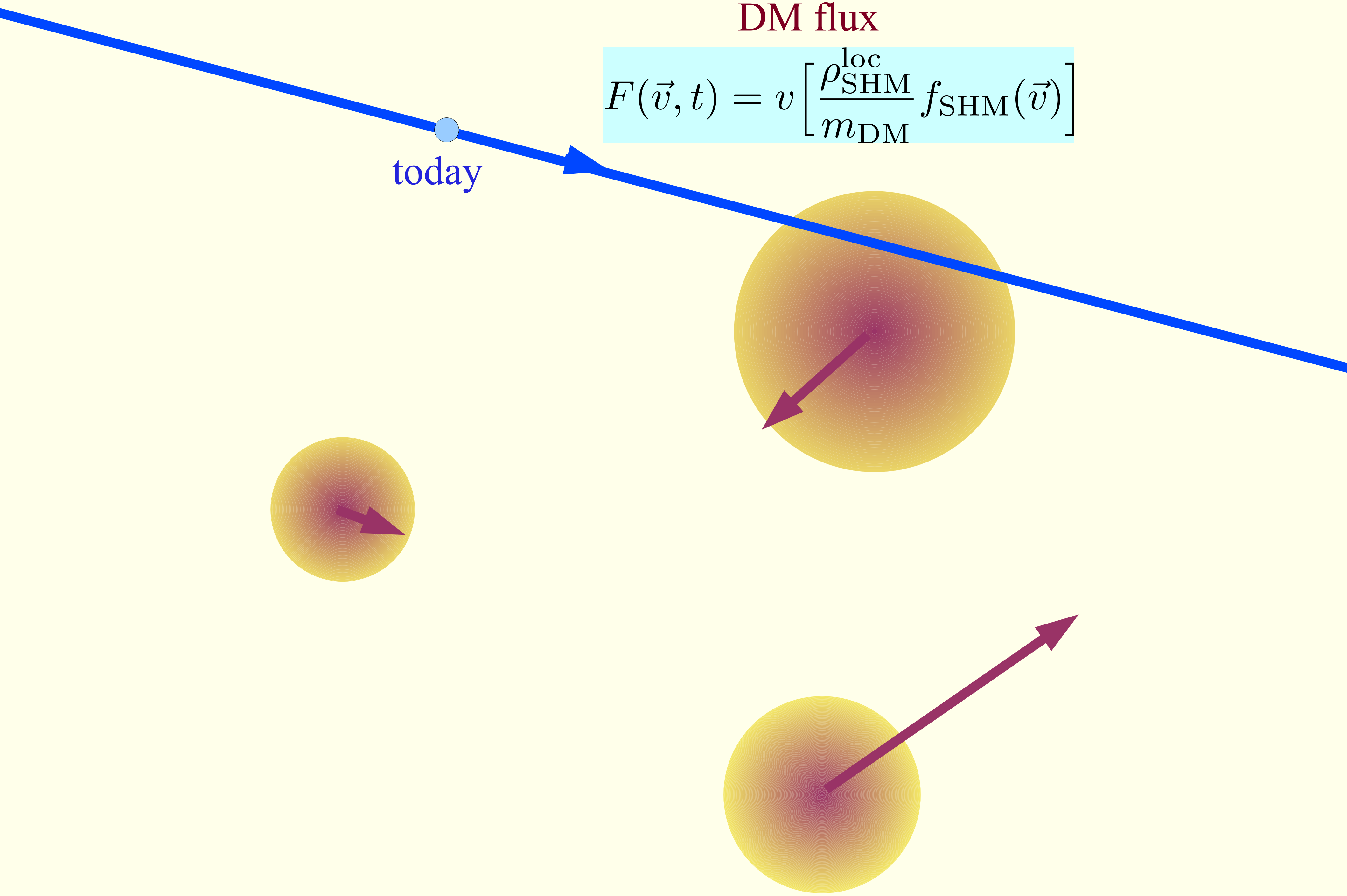
DM flux

$$F(\vec{v},t) = v\left[\frac{\rho^{\rm loc}_{\rm SHM}}{m_{\rm DM}} f_{\rm SHM}(\vec{v})\right]$$



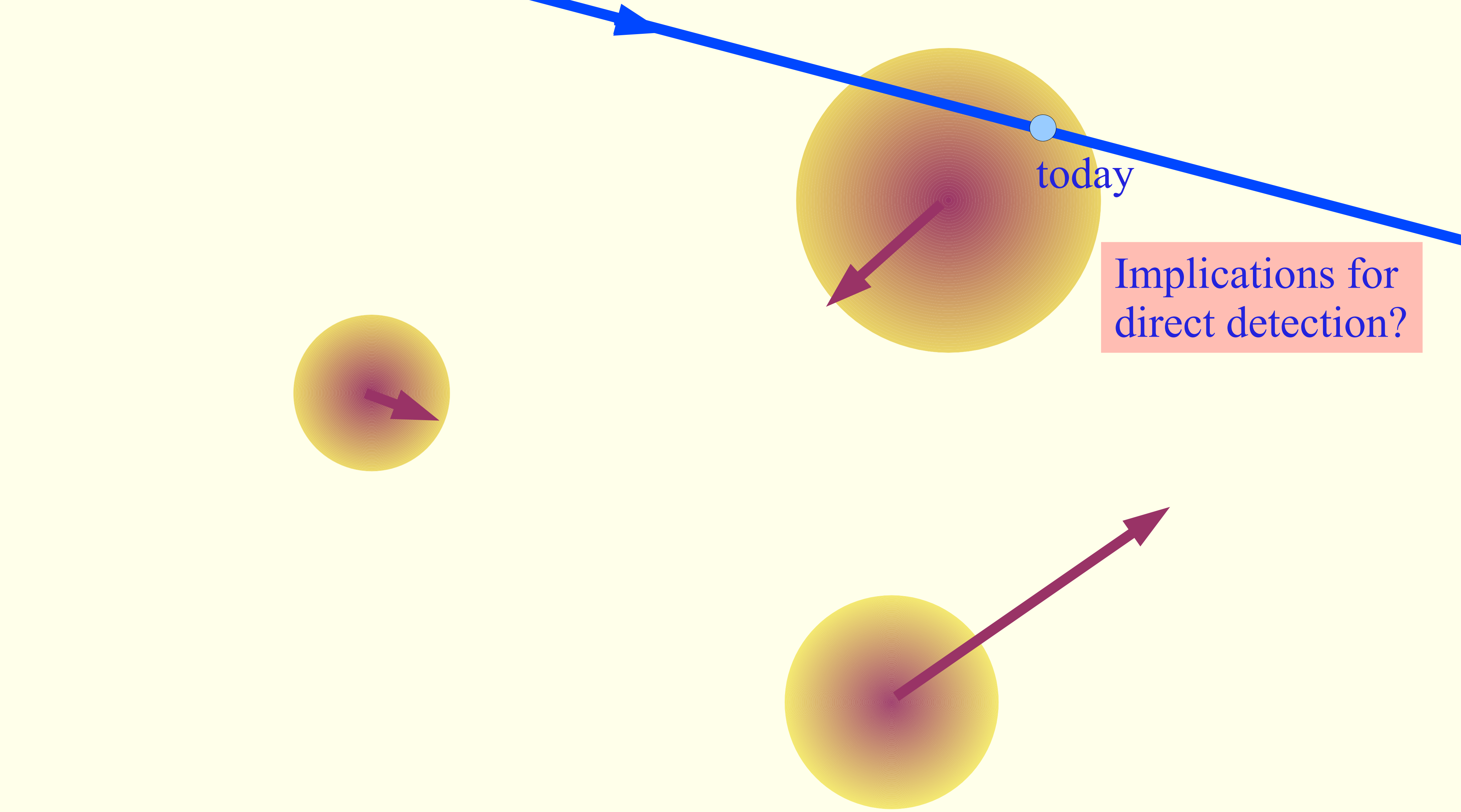
DM flux

$$F(\vec{v},t) = v\left[\frac{\rho^{\rm loc}_{\rm SHM}}{m_{\rm DM}} f_{\rm SHM}(\vec{v})\right]$$



Time dependent DM flux

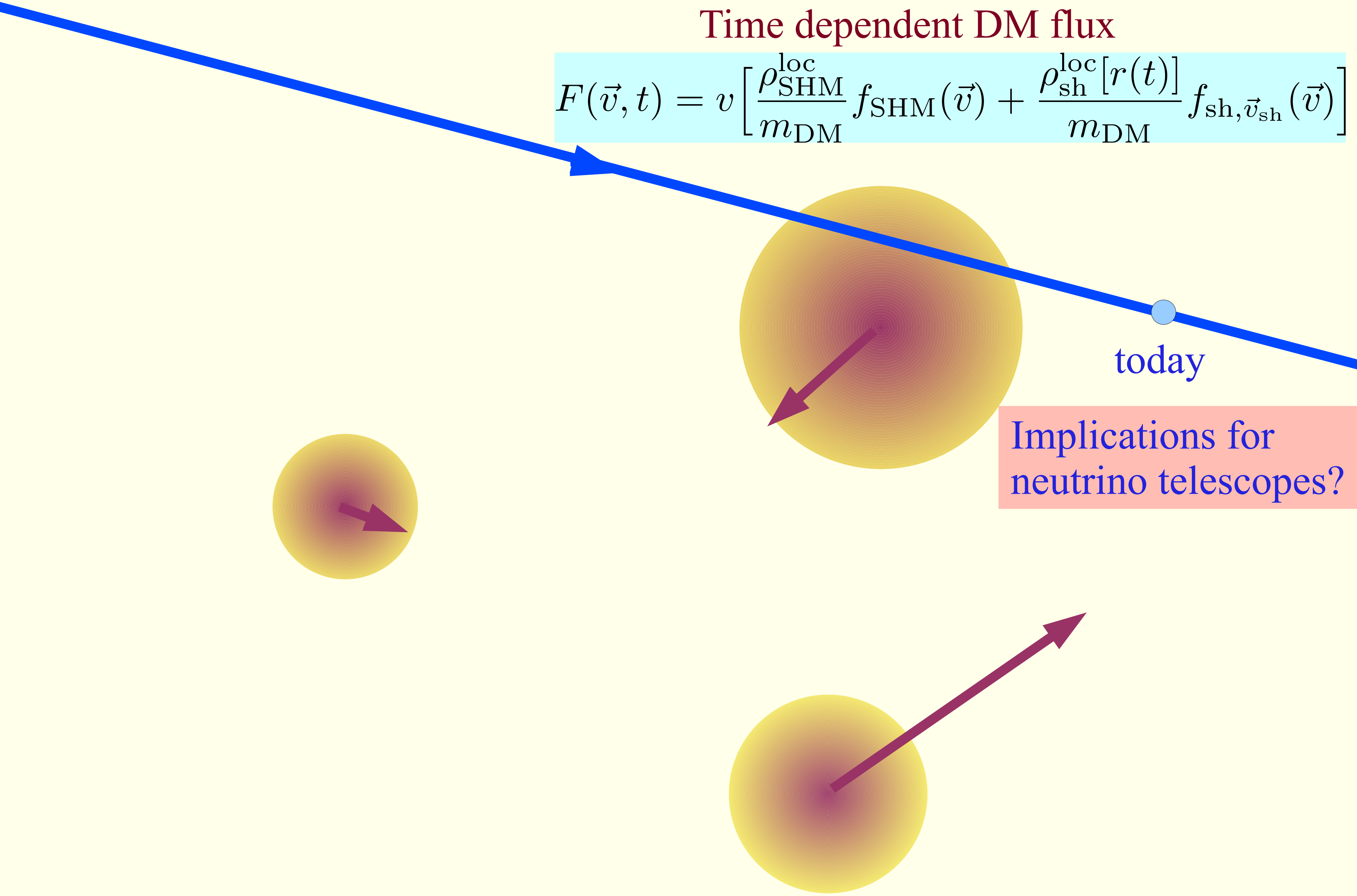
$$F(\vec{v},t) = v\left[\frac{\rho^{\rm loc}_{\rm SHM}}{m_{\rm DM}} f_{\rm SHM}(\vec{v}) + \frac{\rho^{\rm loc}_{\rm sh}[r(t)]}{m_{\rm DM}} f_{{\rm sh},\vec{v}_{\rm sh}}(\vec{v})\right]$$



Implications for direct detection?

Time dependent DM flux

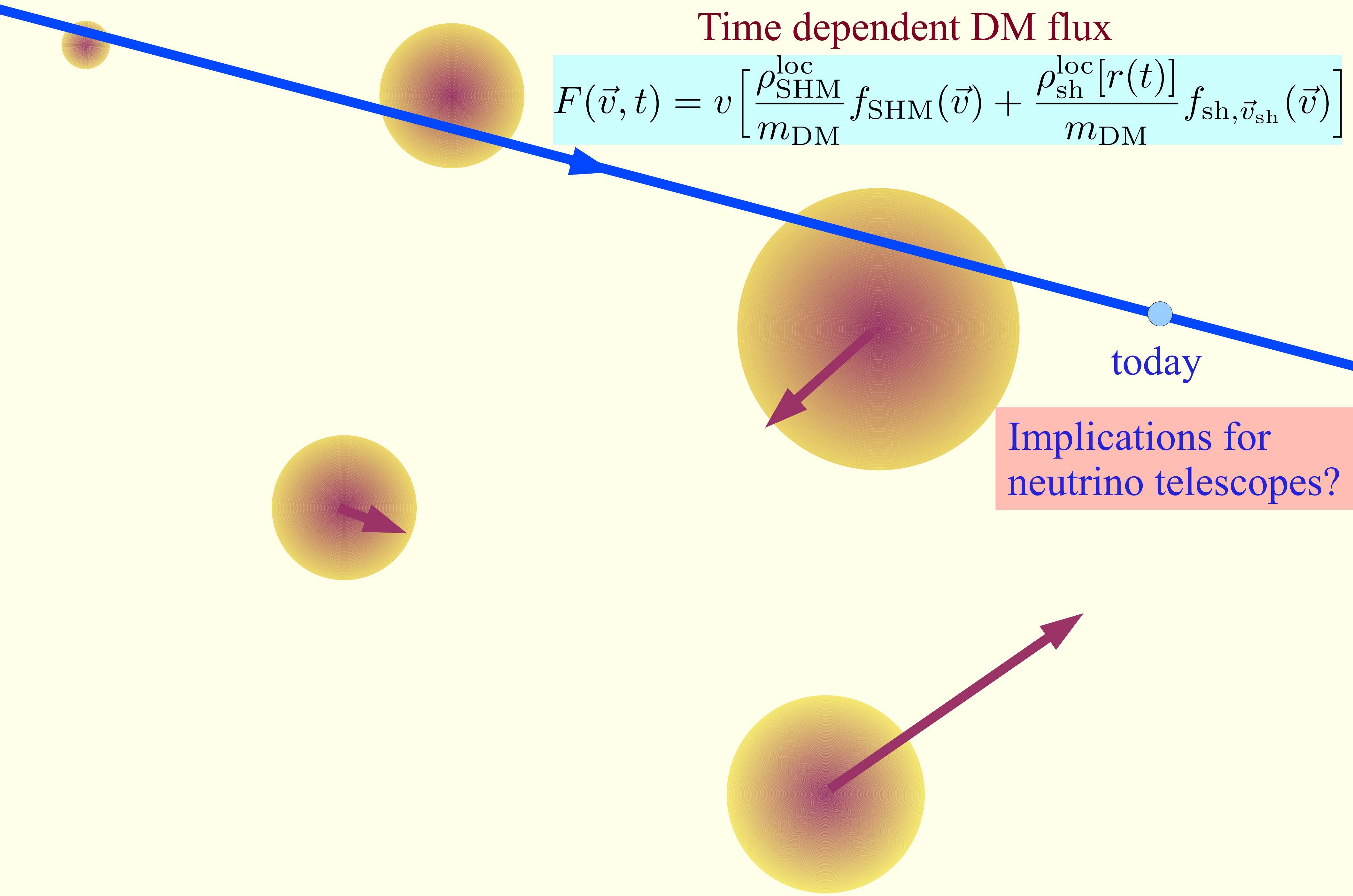
$$F(\vec{v},t) = v\left[\frac{\rho_{\rm SHM}^{\rm loc}}{m_{\rm DM}} f_{\rm SHM}(\vec{v}) + \frac{\rho_{\rm sh}^{\rm loc}[r(t)]}{m_{\rm DM}} f_{{\rm sh},\vec{v}_{\rm sh}}(\vec{v})\right]$$



Implications for neutrino telescopes?

Time dependent DM flux

$$F(\vec{v},t) = v\left[\frac{\rho_{\rm SHM}^{\rm loc}}{m_{\rm DM}} f_{\rm SHM}(\vec{v}) + \frac{\rho_{\rm sh}^{\rm loc}[r(t)]}{m_{\rm DM}} f_{{\rm sh},\vec{v}_{\rm sh}}(\vec{v})\right]$$

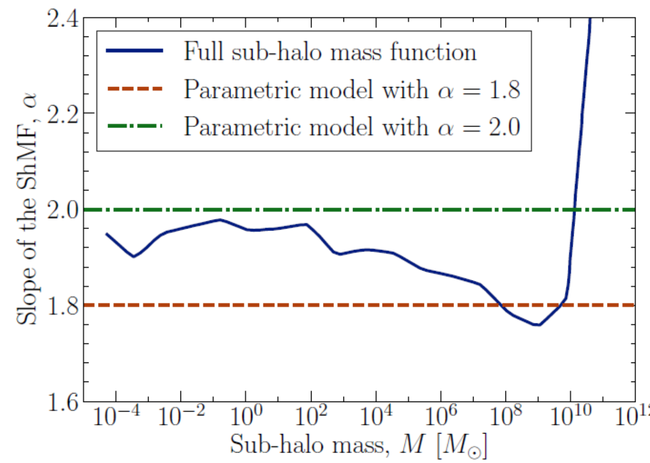


Implications for neutrino telescopes?

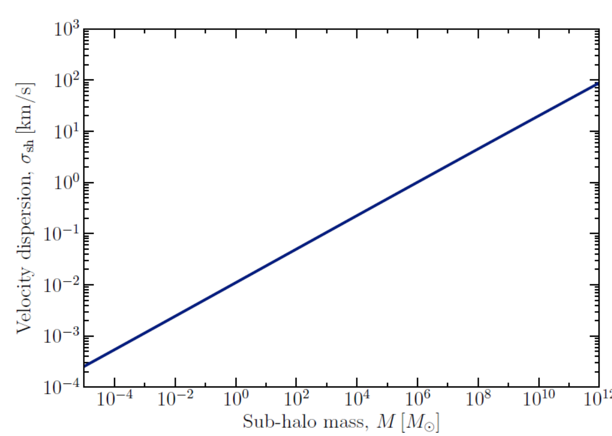
Impact of sub-halos in local DM searches

Assume:

- Sub-halos spatially distributed following an Einasto profile.
- Velocity distribution of sub-halos following Maxwell-Boltzmann.
- Sub-halo mass function from Hiroshima, Ando, and Ishiyama'18



- Internal density profile described by a truncated NFW profile.
- Concentration parameter following a log-normal distribution
- Internal velocity distribution described by a MB distribution

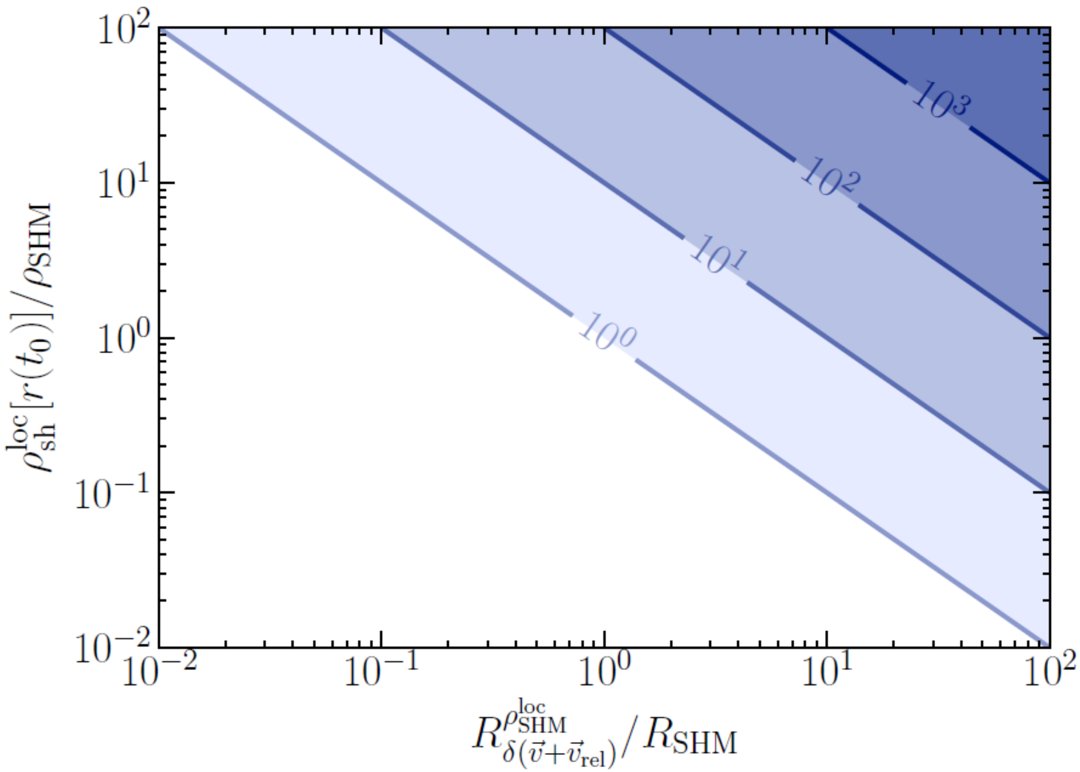


Impact of sub-halos in direct detection experiments

$$R = \sum_i \int_0^\infty \mathrm{d}E_R\, \epsilon_i(E_R) \frac{\xi_i}{m_{A_i}} \int_{v \geq v_{\min,i}^{(E_R)}} \mathrm{d}^3 v F(\vec{v} + \vec{v}_{\rm Earth}, t_0)\, \frac{\mathrm{d}\sigma_i}{\mathrm{d}E_R}(v, E_R)\,.$$

Increment in the rate with respect to the SHM:

$$\mathcal{I}_R \equiv \frac{R}{R_{\rm SHM}} - 1 \simeq \frac{\rho_{\rm sh}^{\rm loc}[r(t_0)]}{\rho_{\rm SHM}} \frac{R^{\rho_{\rm SHM}^{\rm loc}}_{\delta(\vec{v}+\vec{v}_{\rm sh}+\vec{v}_\odot+\vec{v}_{\rm Earth})}}{R_{\rm SHM}}$$



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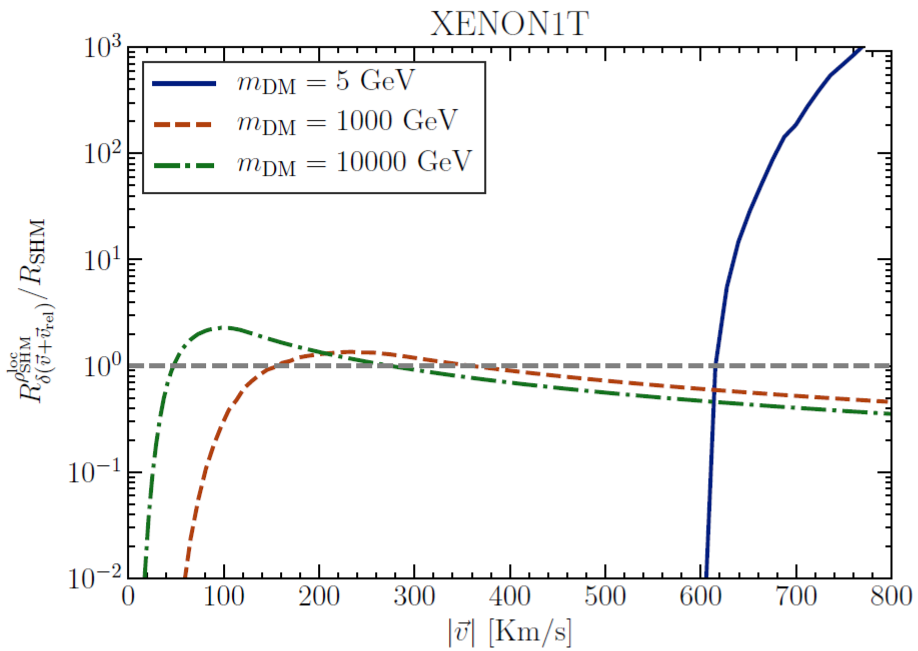

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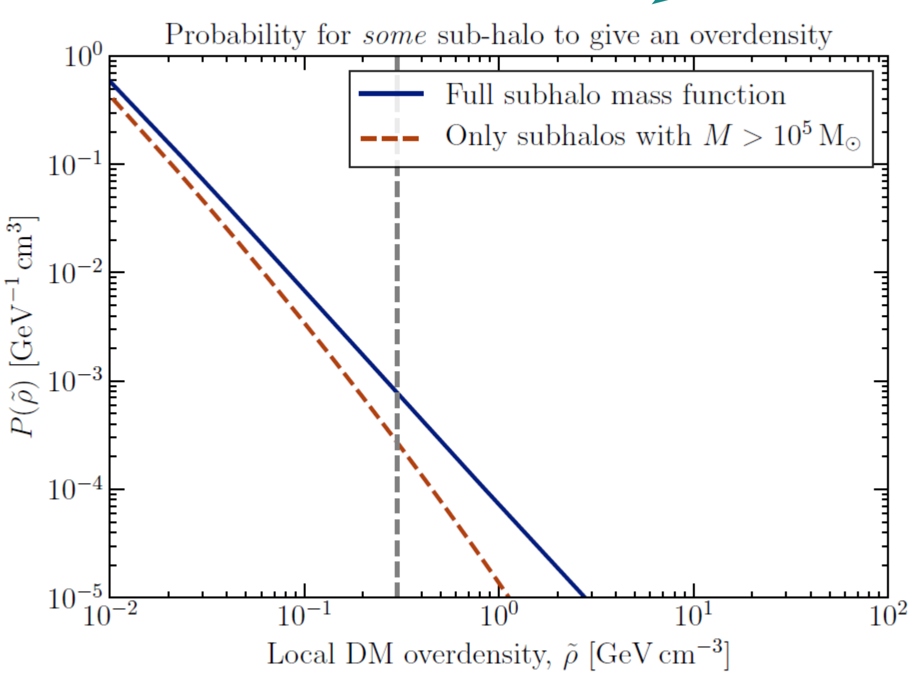

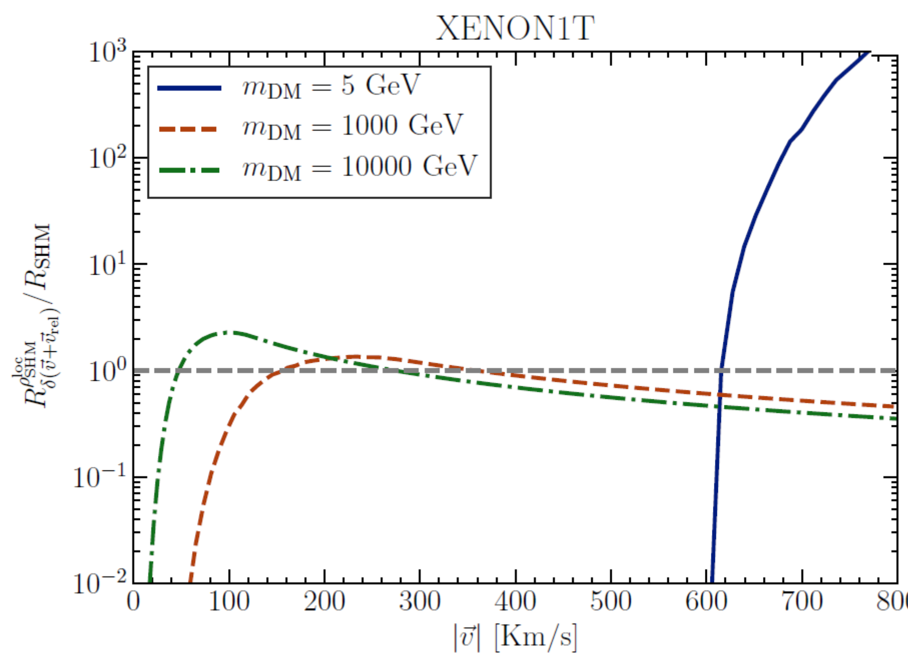

Impact of sub-halos in direct detection experiments

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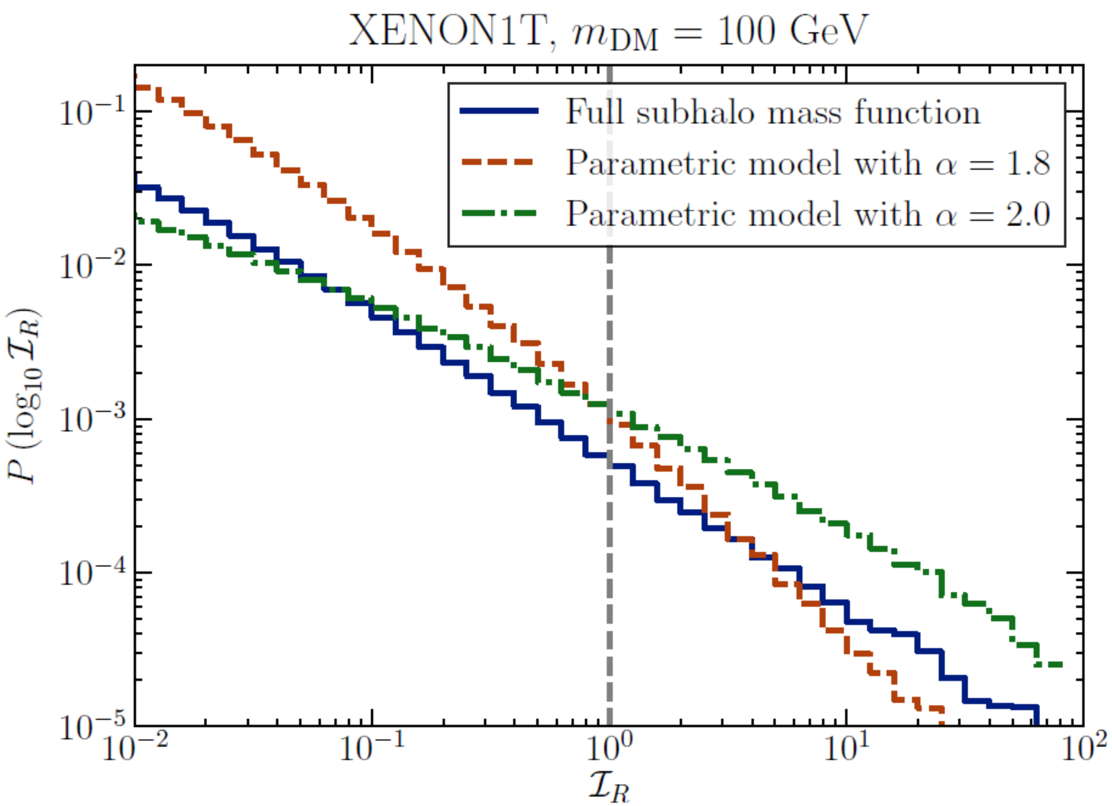
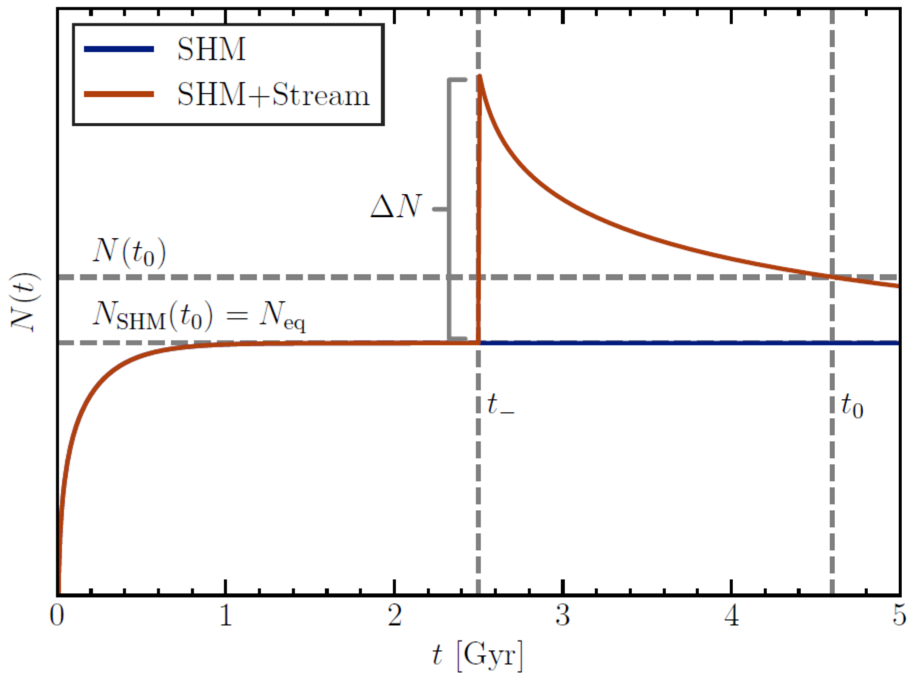


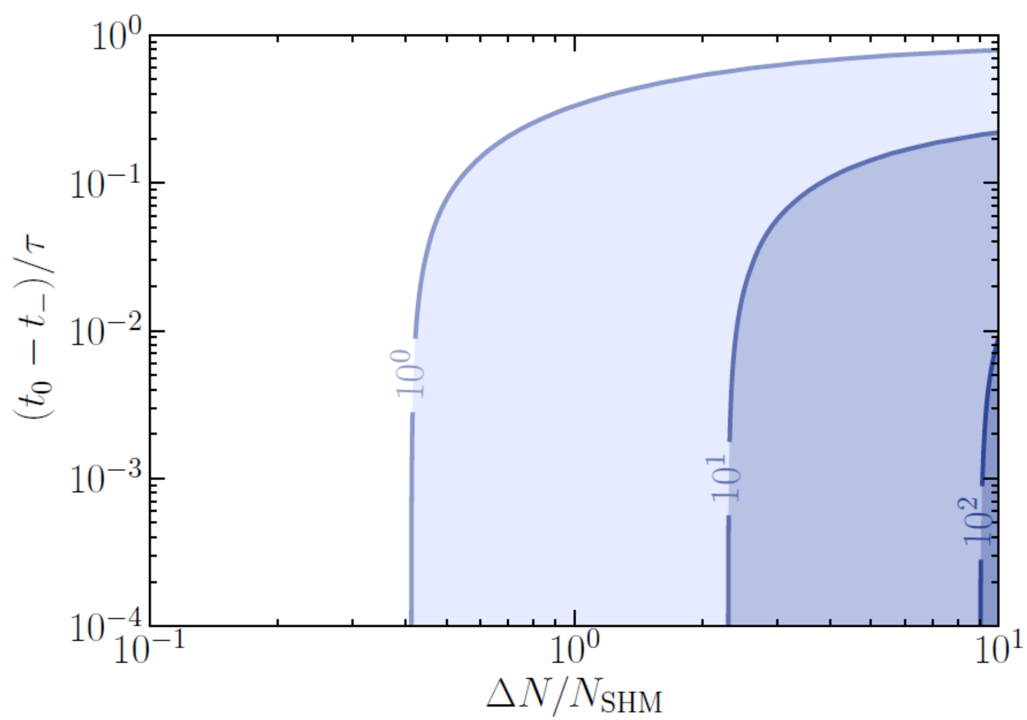
Impact of sub-halos in the neutrino flux from the Sun



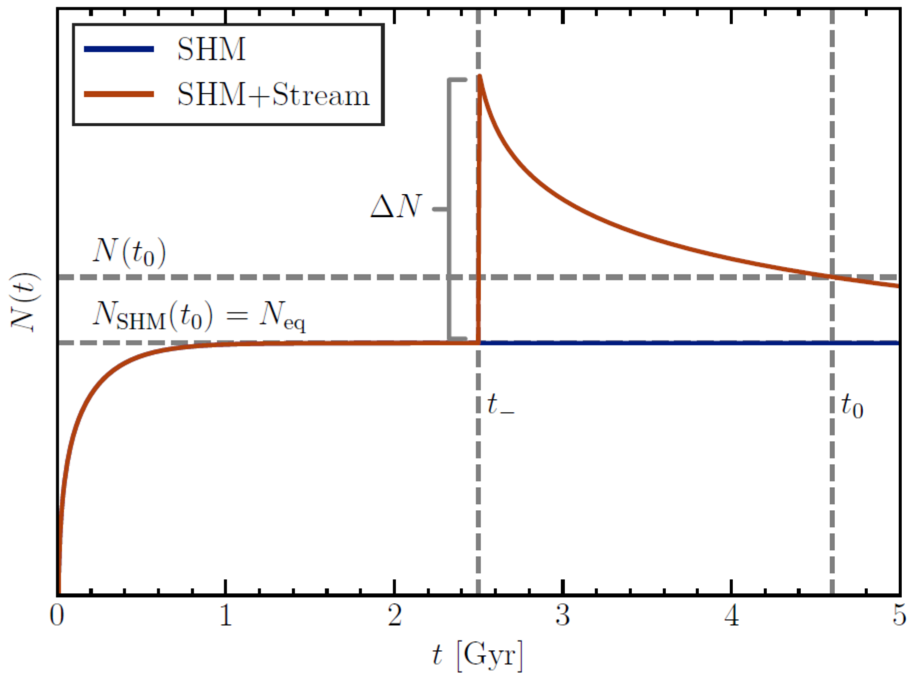
The increment in the number of DM particles captured depends on:

$$\frac{t_0 - t}{\tau}$$

$$\Delta N \simeq N_{\rm eq} \frac{C^{\rho_{\rm SHM}}_{\delta(\vec{v}+\vec{v}_{\rm sh}+\vec{v}_\odot)}}{C_{\rm SHM}} \frac{\langle \rho^{\rm loc}_{\rm sh}[r(t)]\rangle}{\rho_{\rm SHM}} \frac{\Delta t}{\tau}$$



Impact of sub-halos in the neutrino flux from the Sun



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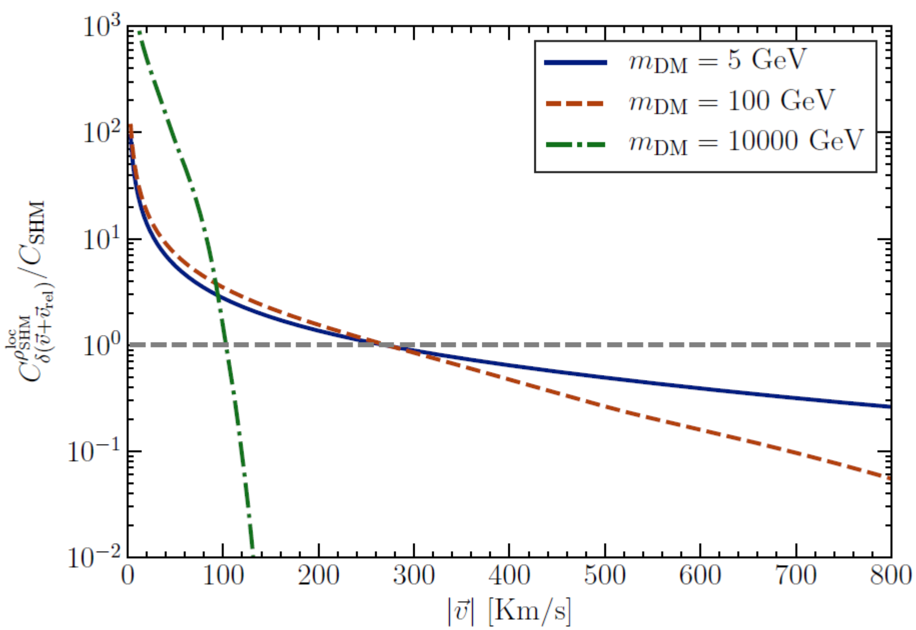
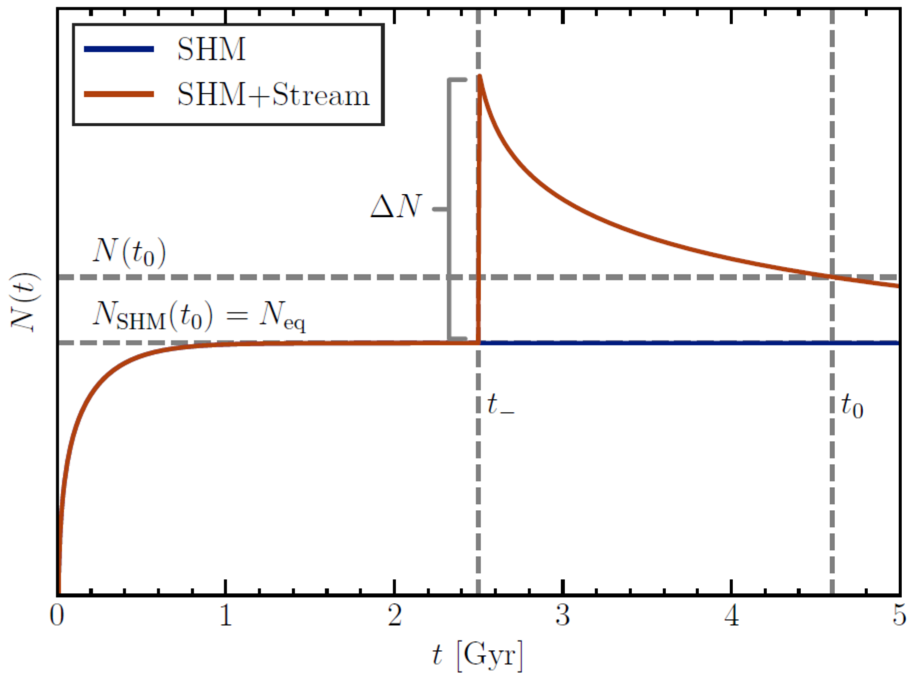


Impact of sub-halos in the neutrino flux from the Sun



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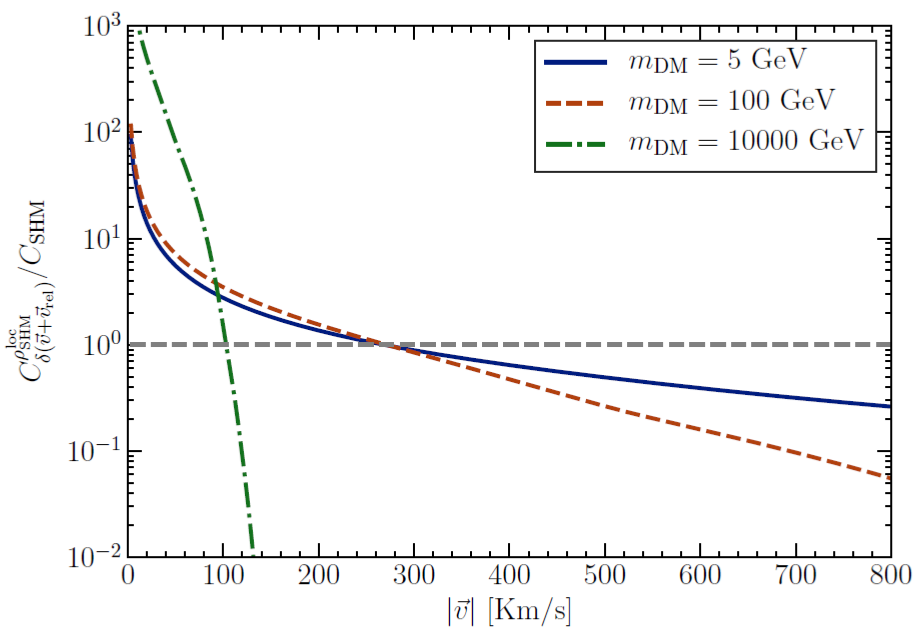
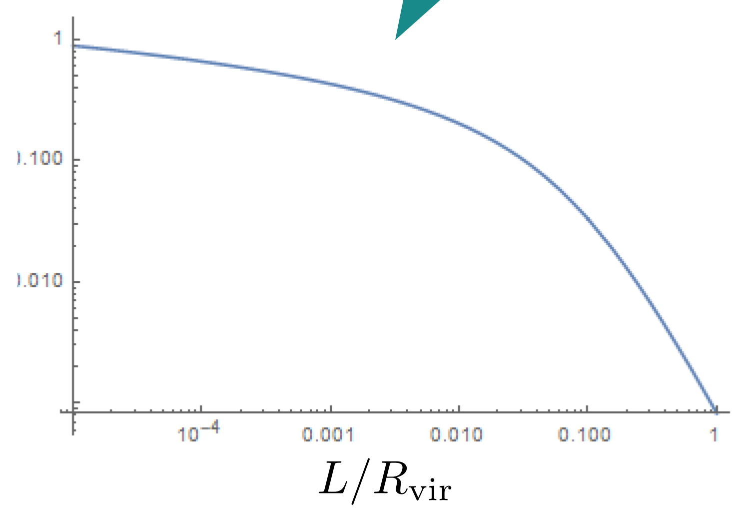
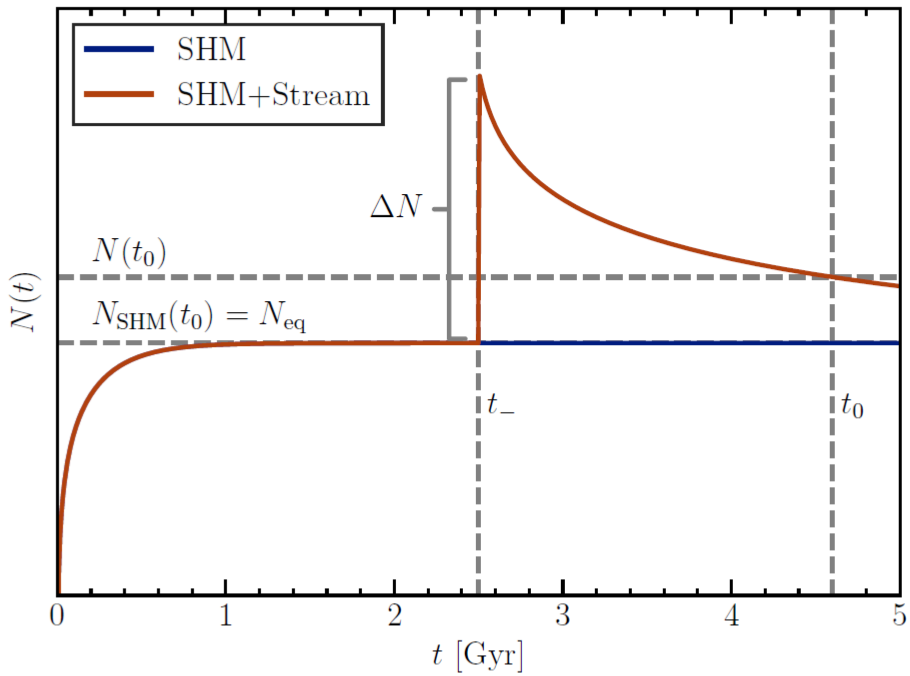




Impact of sub-halos in the neutrino flux from the Sun

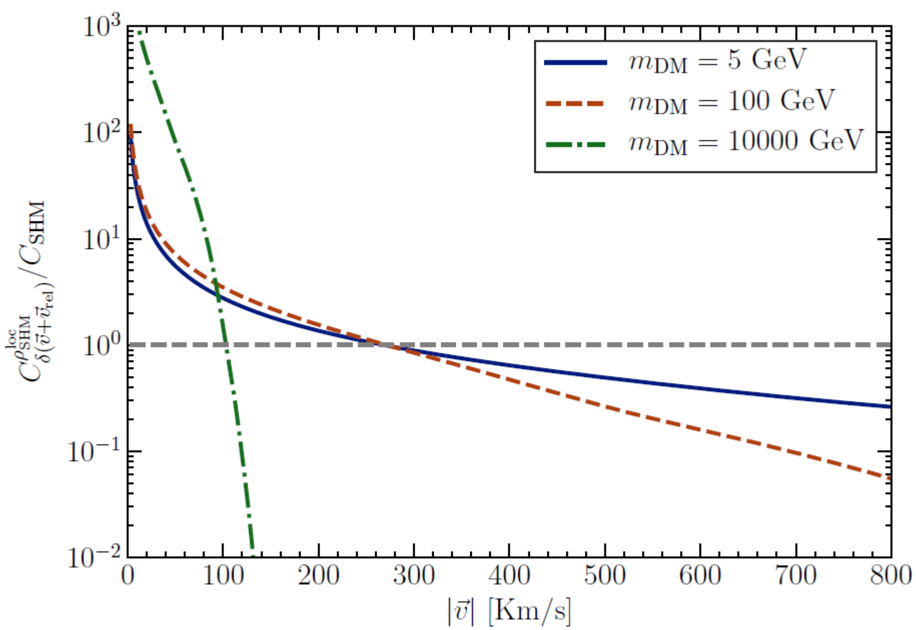


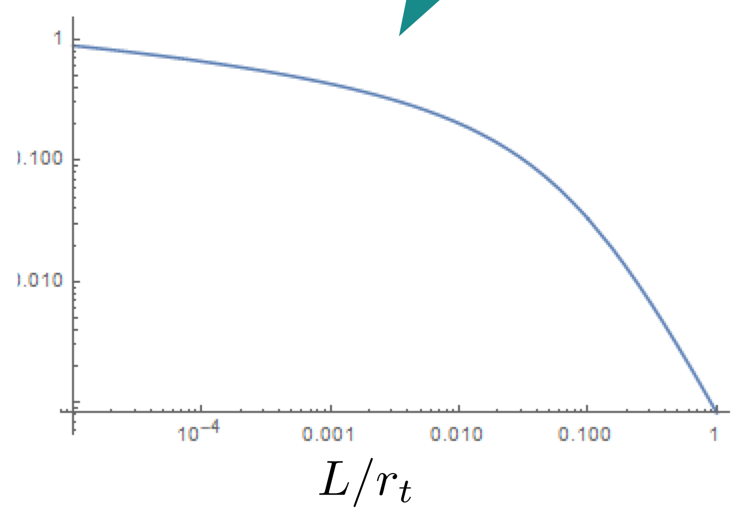
The increment in the number of DM particles captured depends on:

$$\frac{t_0 - t}{\tau}$$

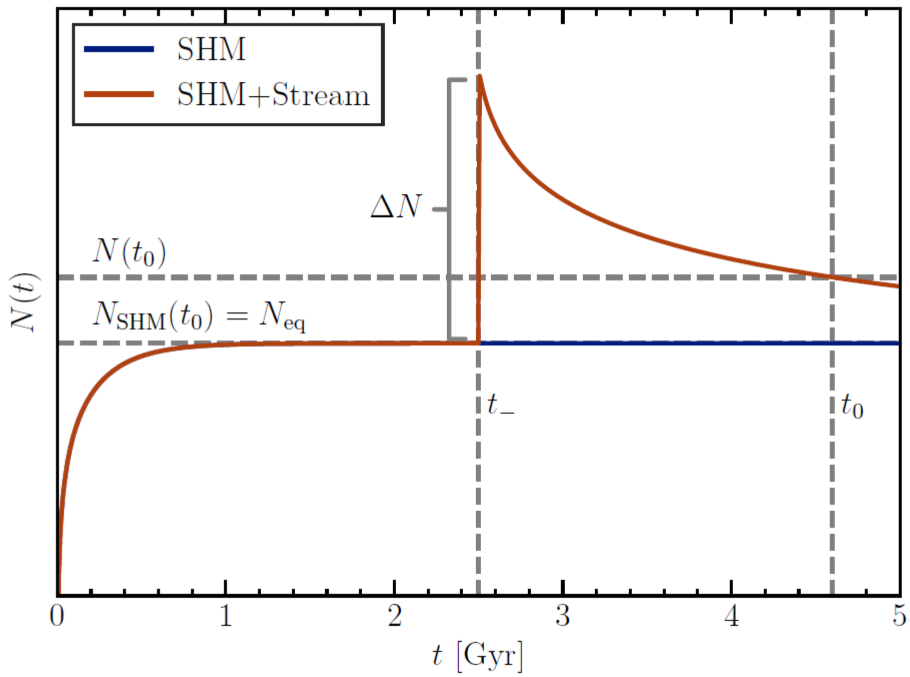
$$\Delta t = 2\sqrt{r_t^2 - L^2}/v_{\rm rel}$$

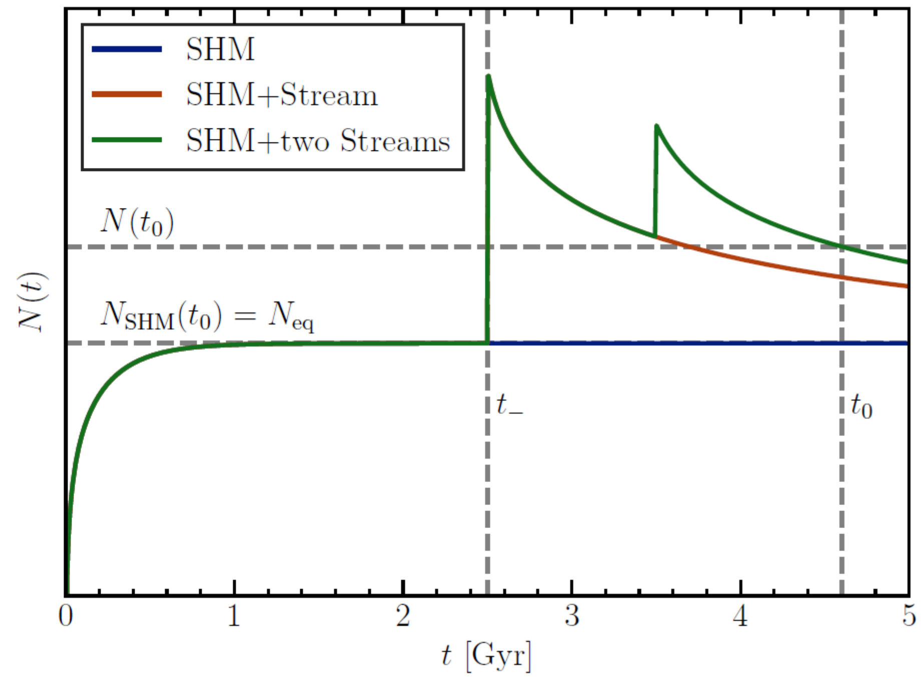
$$\Delta N \simeq N_{\rm eq} \frac{C^{\rho_{\rm SHM}}_{\delta(\vec{v}+\vec{v}_{\rm sh}+\vec{v}_\odot)}}{C_{\rm SHM}} \frac{\rho^{\rm loc}_{\rm sh}[r(t)]}{\rho_{\rm SHM}} \frac{\Delta t}{\tau}$$



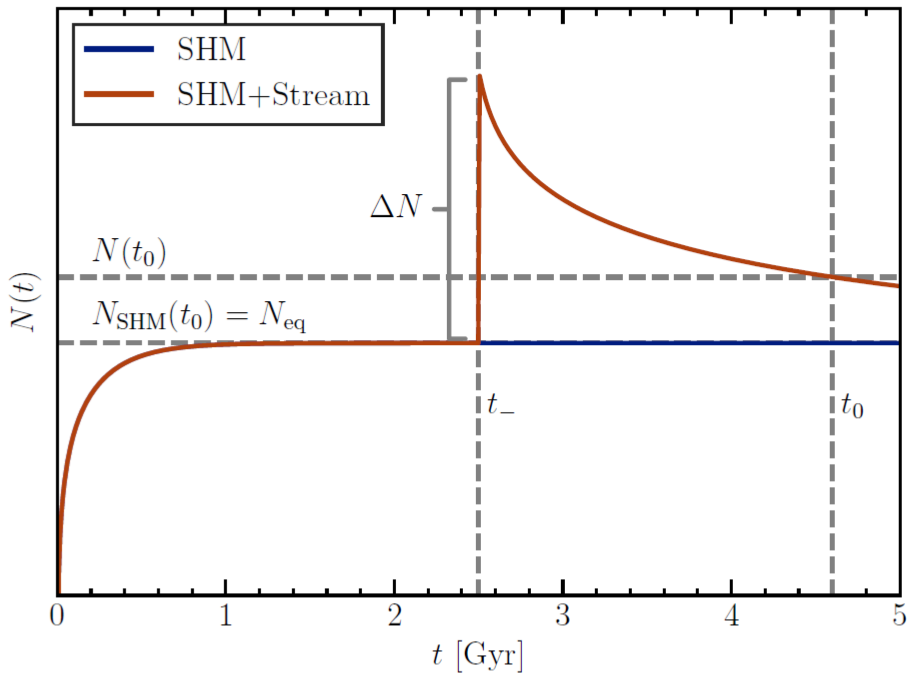


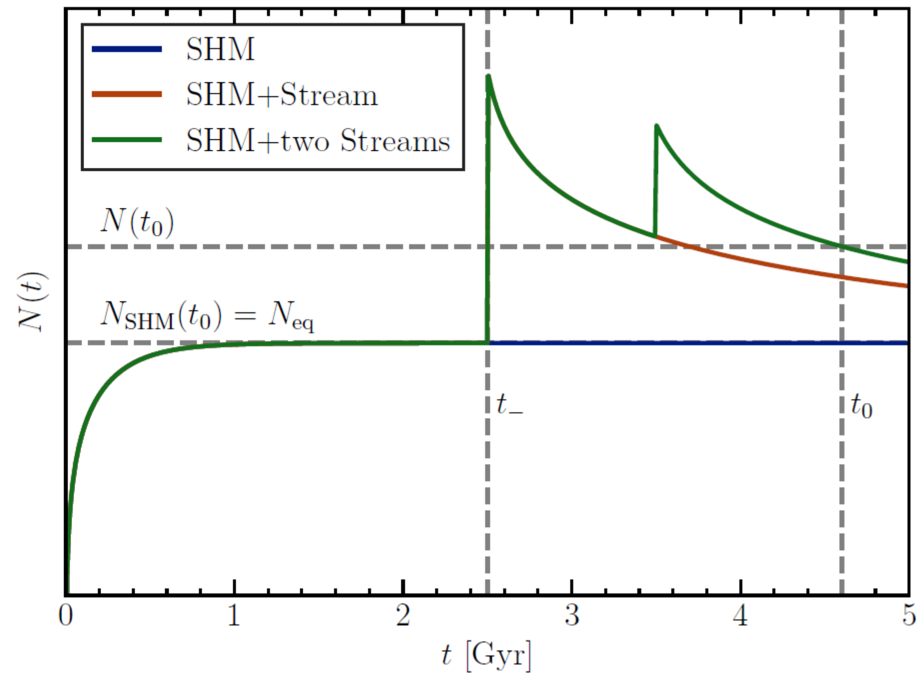
Impact of sub-halos in the neutrino flux from the Sun



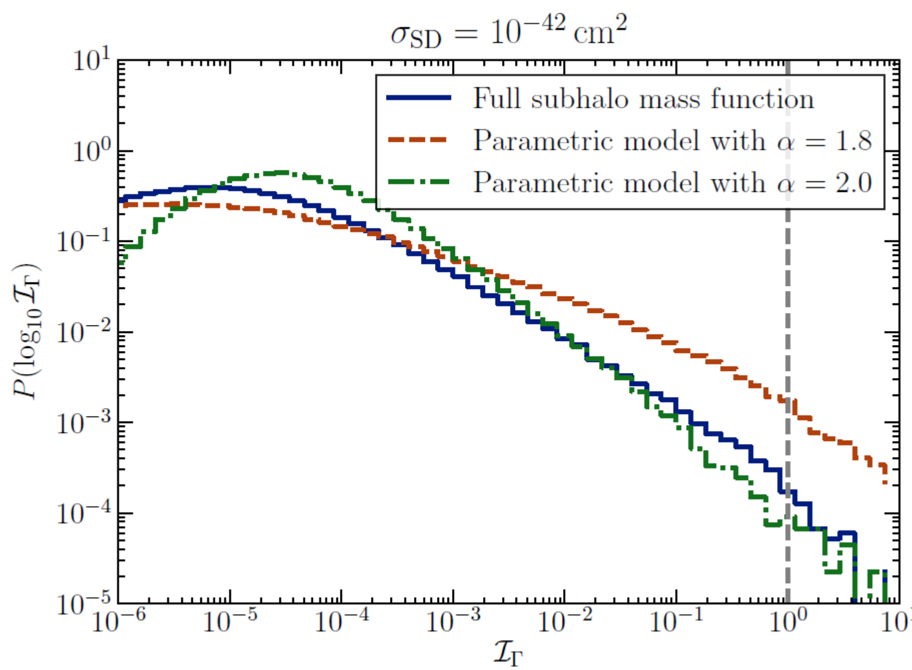


Impact of sub-halos in the neutrino flux from the Sun





$$\mathcal{I}_\Gamma = \frac{\Gamma(t_0)}{\Gamma_{\rm SHM}(t_0)} - 1$$



Conclusions

- The interpretation of any experiment probing the dark matter distribution inside the Solar System is subject to our ignorance of the local dark matter density and velocity distribution.

- We have developed a method to bracket the uncertainties in the velocity distribution when interpreting the results from direct searches, due to distortions in the Maxwell-Boltzmann distribution and/or by exploiting the synergy with dark matter searches in the Sun.

- Sub-halos in our Galaxy may induce a time-dependent DM flux at the Solar System. There is a probability of ~1 per mil of changing by an O(1) factor the signal rate at a direct detection experiment or at a neutrino telescope.