The physical mass scales of multi-field preheating

Based on:

[OI, E. Sfakianakis, D.G. Wang, A. Achucarro: JCAP 1906, no. 06, 027(2019) [arXiv:1810.02804]

[OI, E. Sfakianakis, D.G. Wang, A. Achucarro: arXiv:2005.00528 (2020)]

Universiteit Leiden

APEC Seminar

21 October 2020

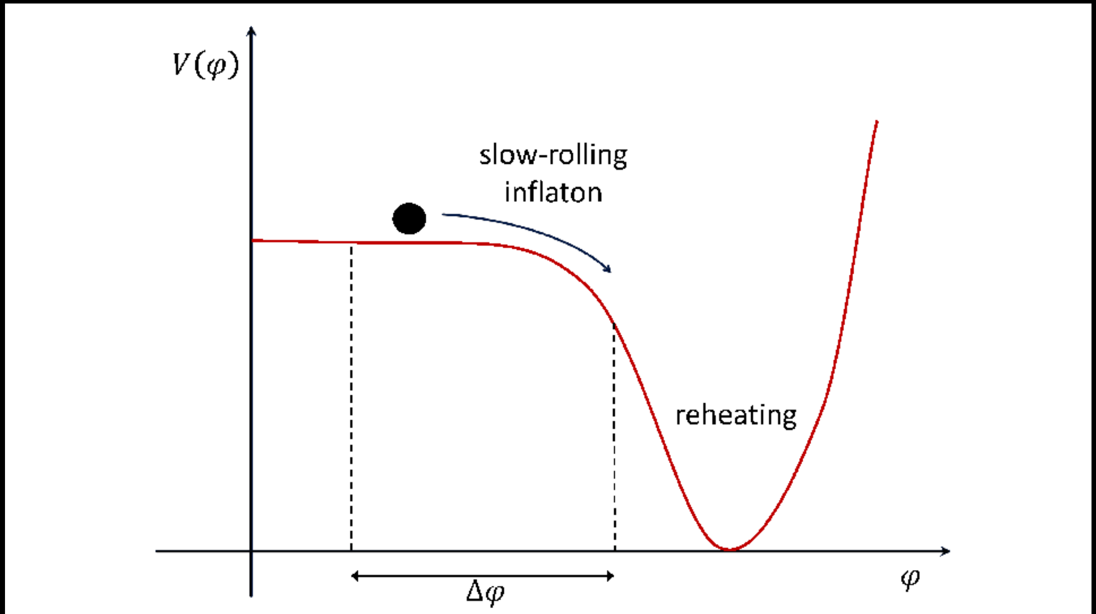
Oksana Iarygina

Outline:

- Why do we need to know the physics of preheating?
- Why multi-field?
- Scaling relations in multi-field alpha-attractors
- Mass scales for preheating

In the beginning, there was (probably) inflation

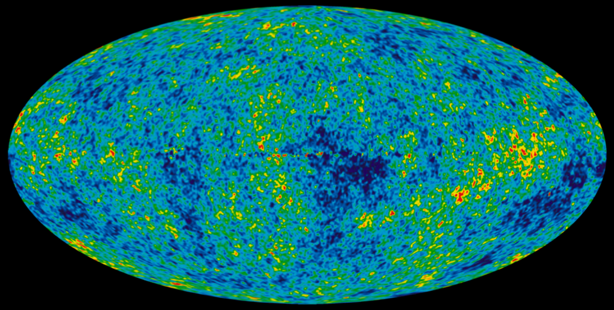
A simple mechanism: scalar field with a flat potential.



- ✓ Solves horizon, flatness problems
- ✓ Explains fluctuations as stretched quantum perturbations → seeds for all structure
- ✓ Predicts a nearly scale invariant spectrum together with Gaussian perturbations

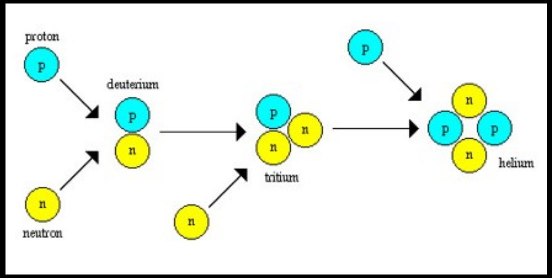
Motivation

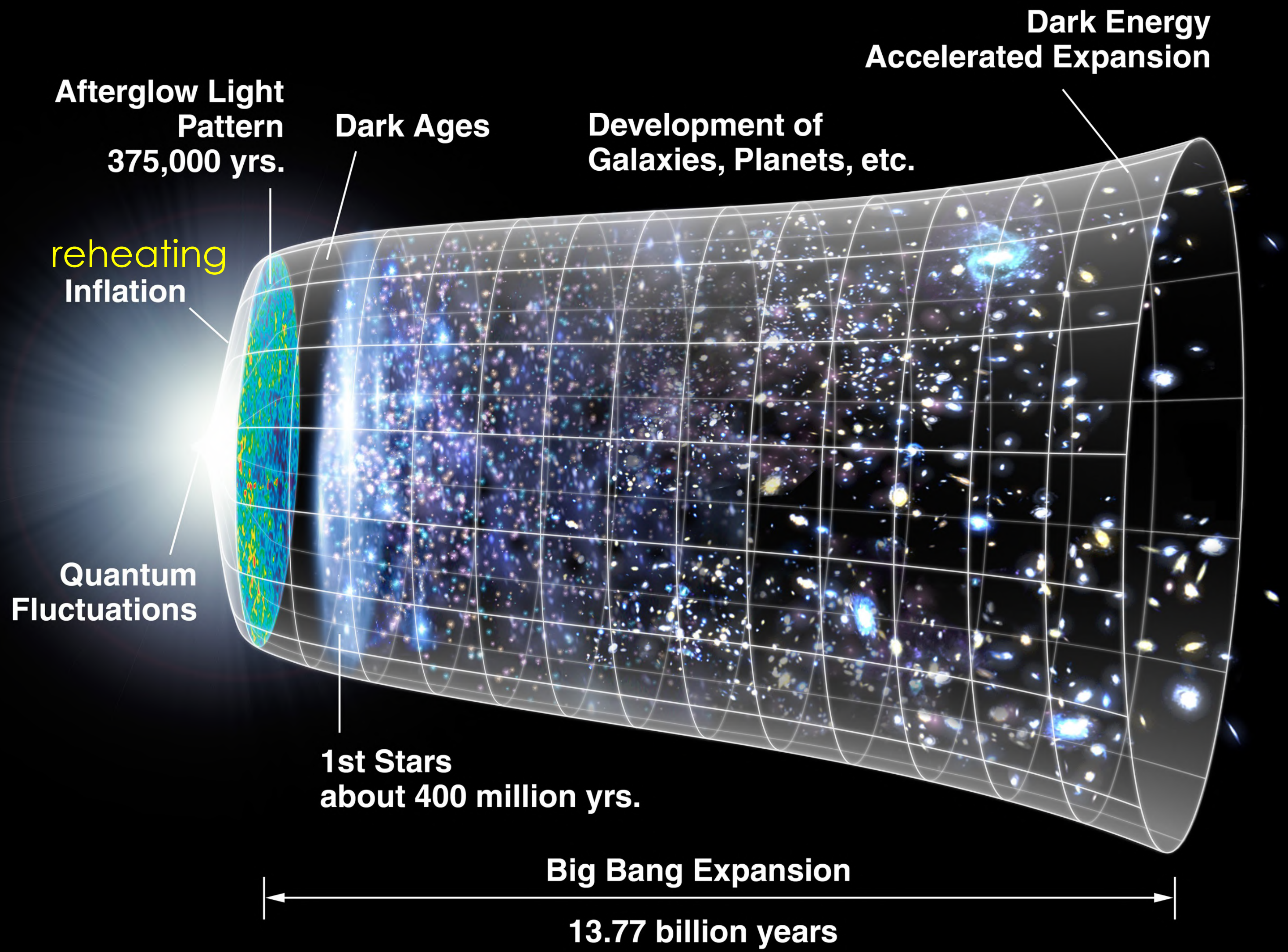
CMB



The reheating era is
poorly explored and constrained

Big Bang nucleosynthesis

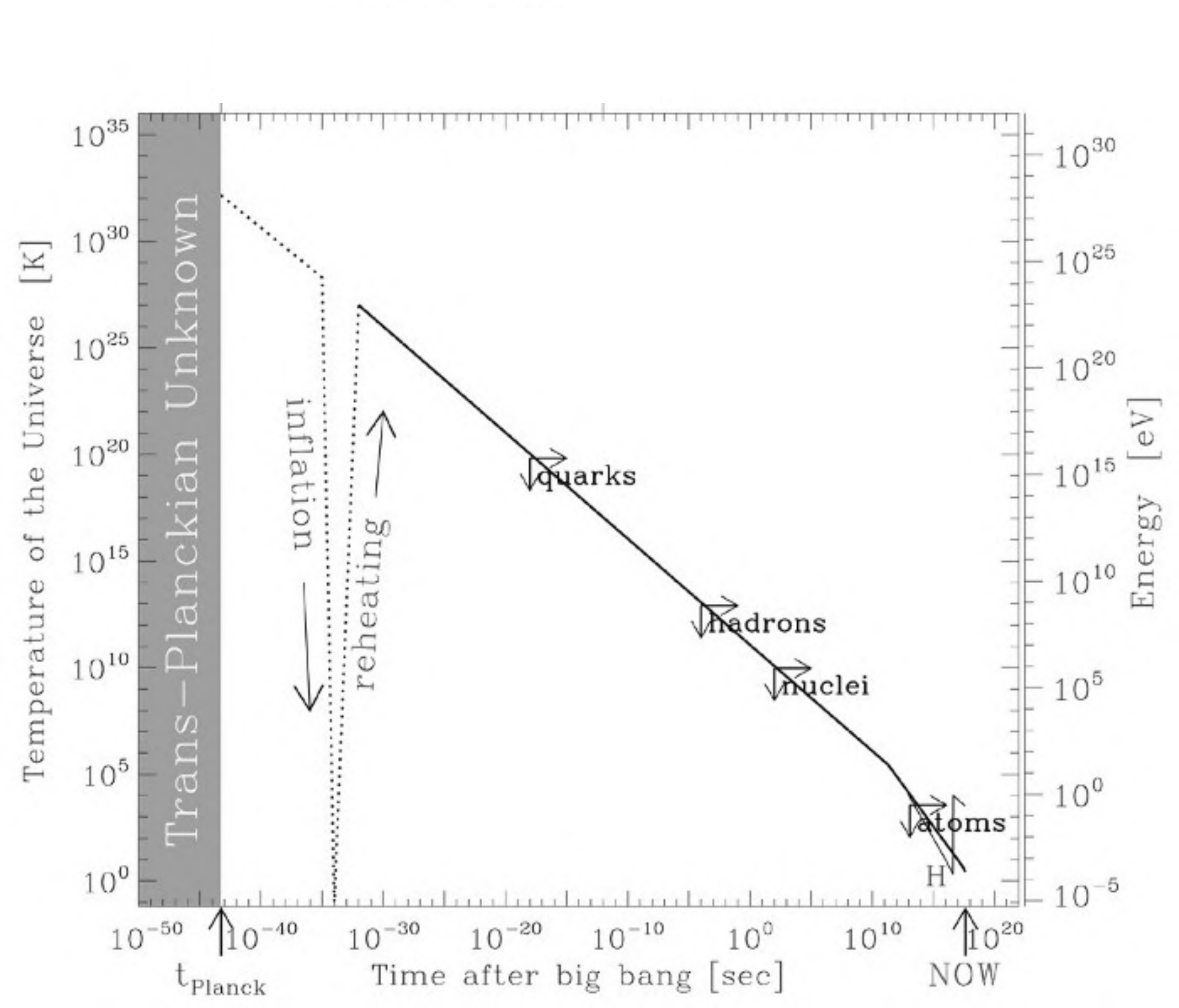




Oksana Iarygina, Leiden University

Why reheating is important?

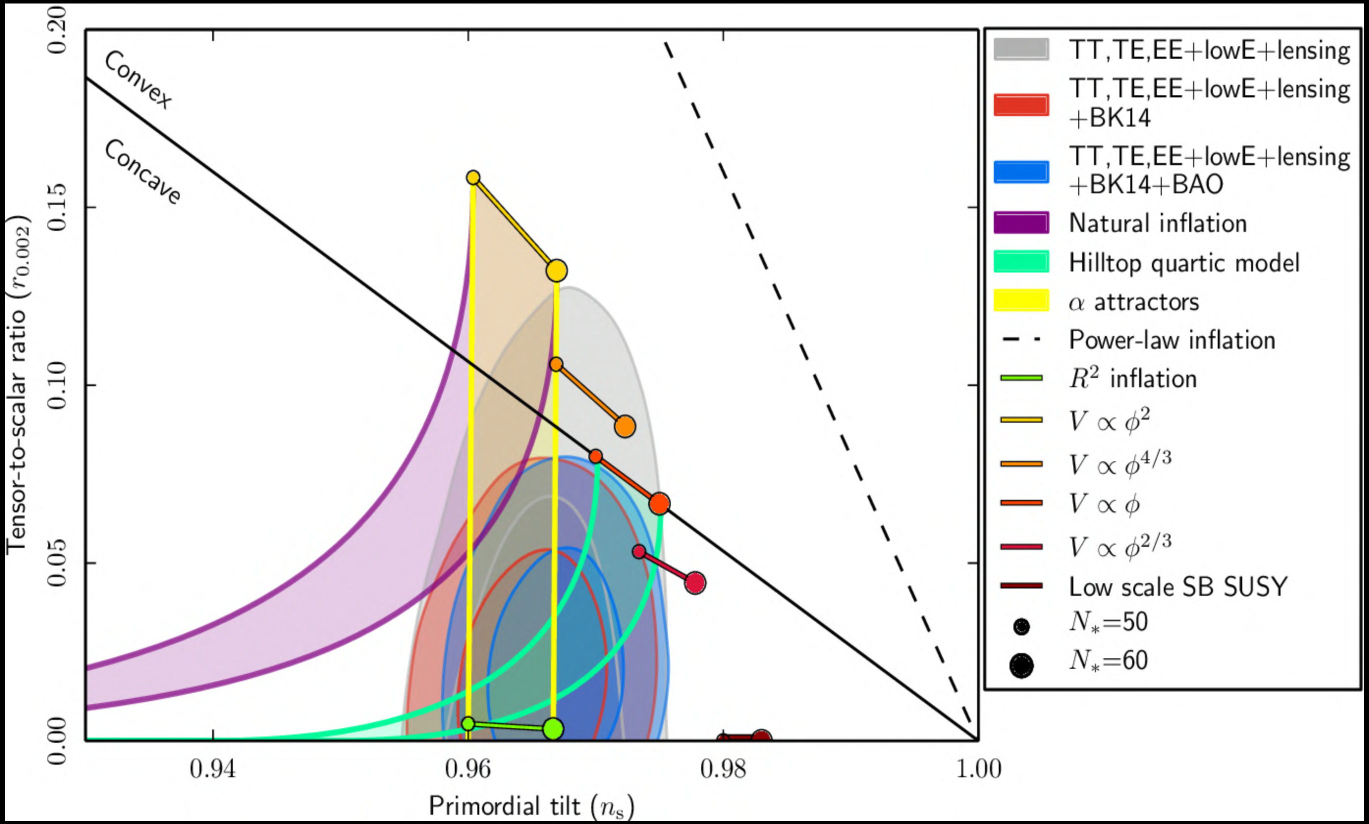
- heats the Universe



[Lineweaver (2003)]

Why reheating is important?

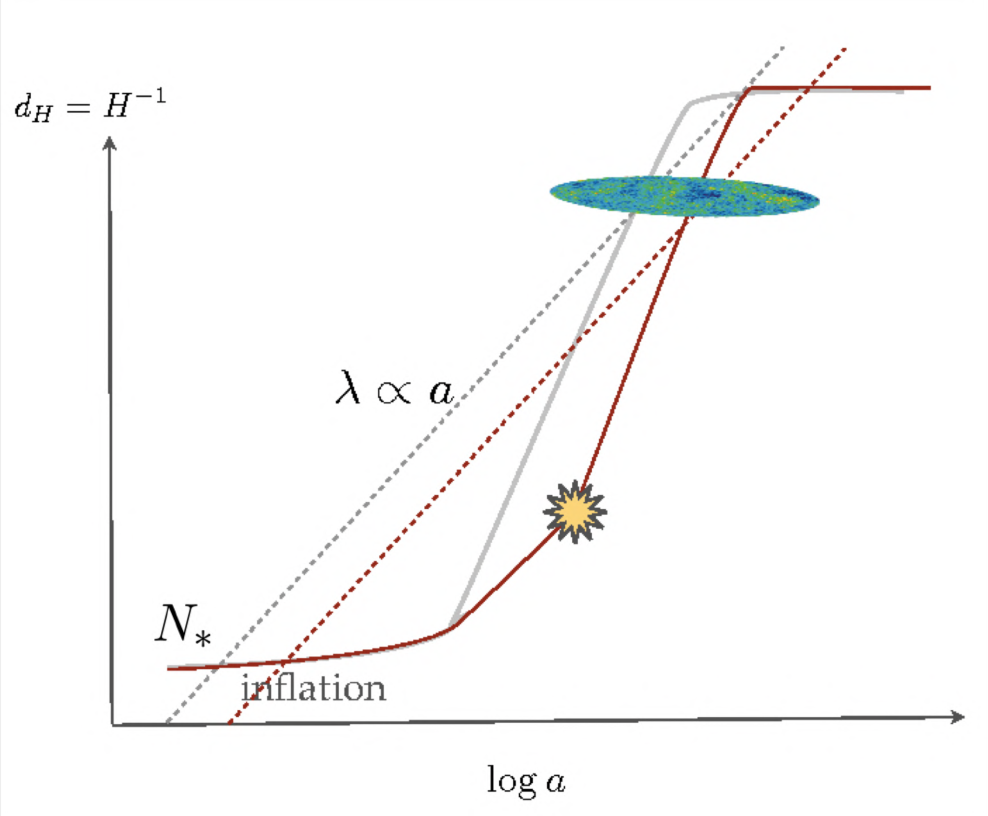
- It is important source of theoretical uncertainty: $50 < N_* < 60$



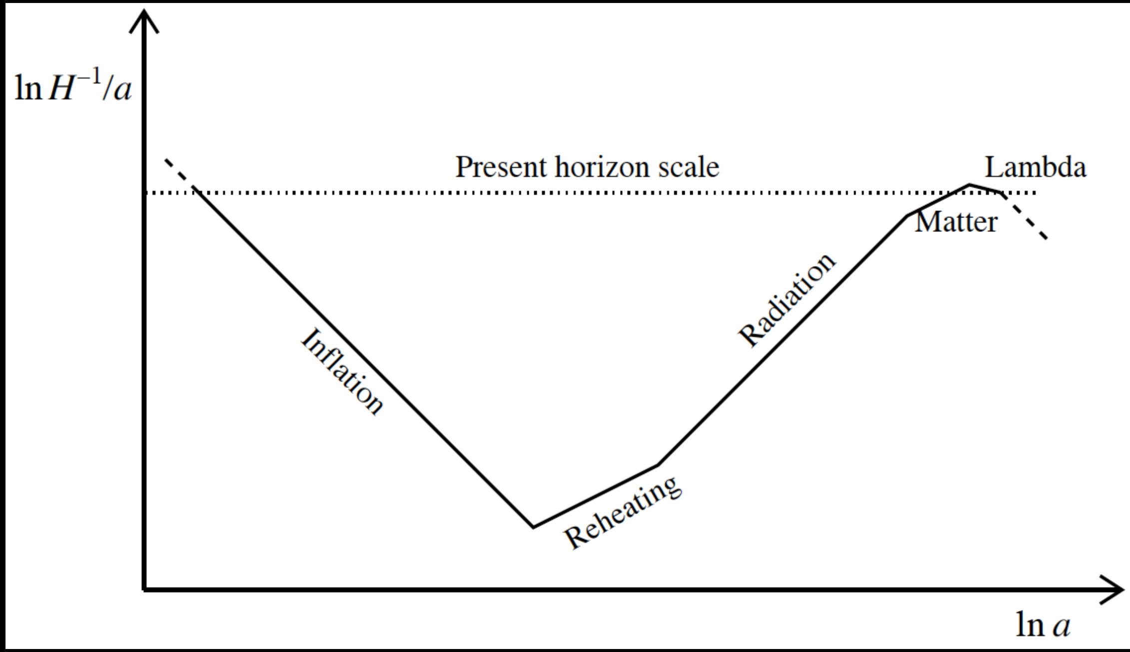
[Planck 2018 results]

Why reheating is important?

- inefficient preheating can lead to prolonged matter-dominated phase after inflation, changing the time during inflation when the CMB modes exit the horizon



[M. Amin at al (2014)]



[A. Liddle, S. Leach (2003)]

Why reheating is important?

- The duration of reheating shifts CMB predictions thus breaking the degeneracy of inflation models

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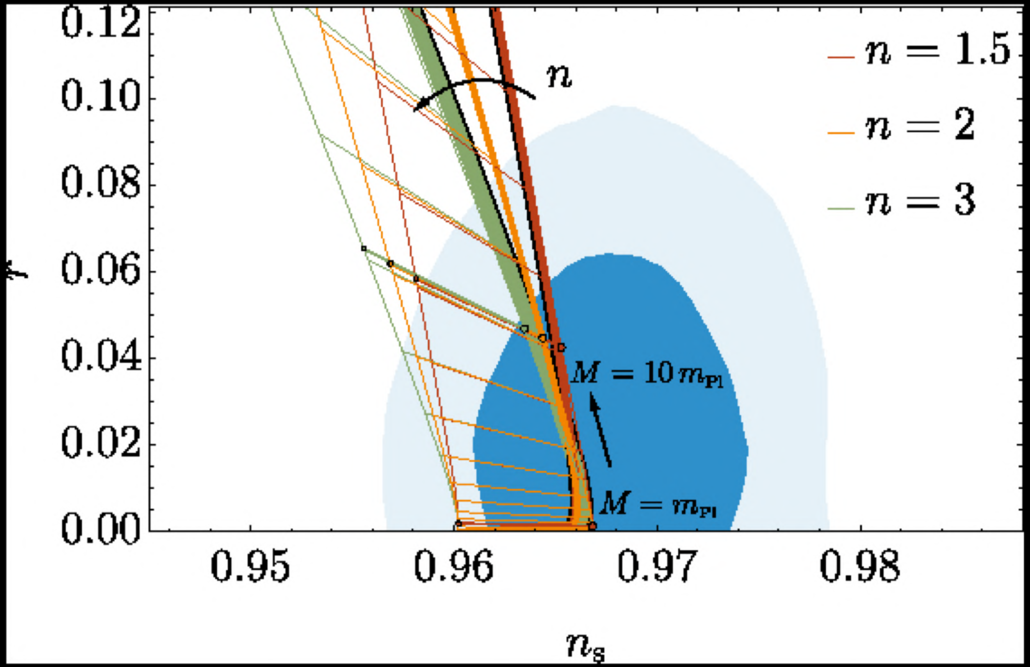

[K. D. Lozanov, M. A. Amin (2017)]

$$\frac{k_*}{a_0 H_0} = e^{-N_*} \frac{a_{\rm end}}{a_{\rm reh}} \frac{a_{\rm reh}}{a_{\rm eq}} \frac{H_*}{H_{\rm eq}} \frac{a_{\rm eq} H_{\rm eq}}{a_0 H_0}$$

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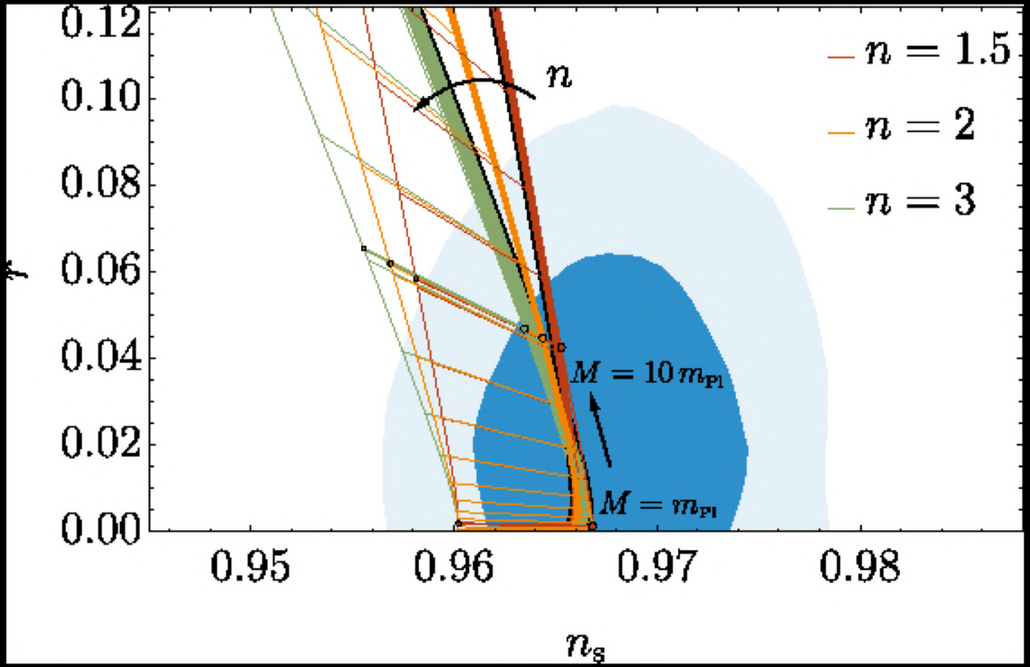

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determined by model of inflation

- Why do we need to know the physics of preheating? ✓
- Why multi-field?
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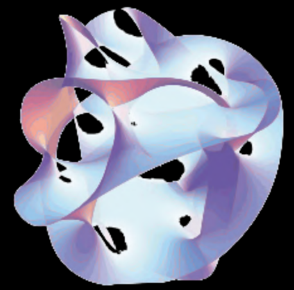
Multi-field inflation

Energy scale of the very early universe could be as high as $10^{15}\,\mathrm{GeV}$.

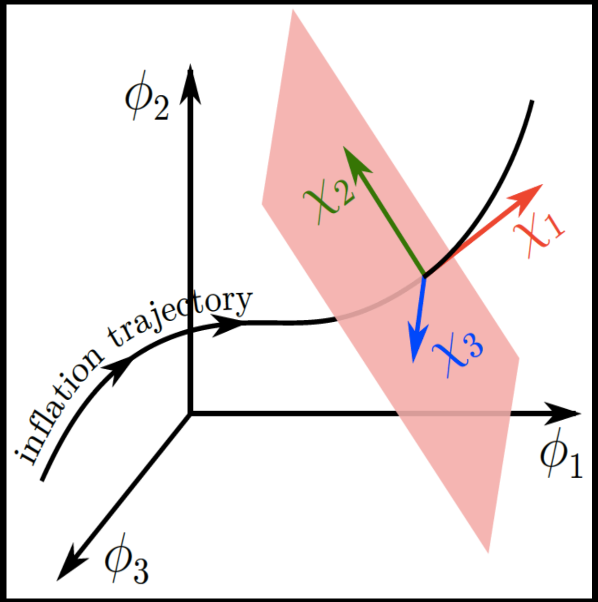


Could contain scalar fields to participate in inflationary dynamics.

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}\mathcal{G}_{\mathcal{IJ}}(\phi^K)g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$$

field-space metric

potential



[figure courtesy Yi Wang]

Two types of perturbations:

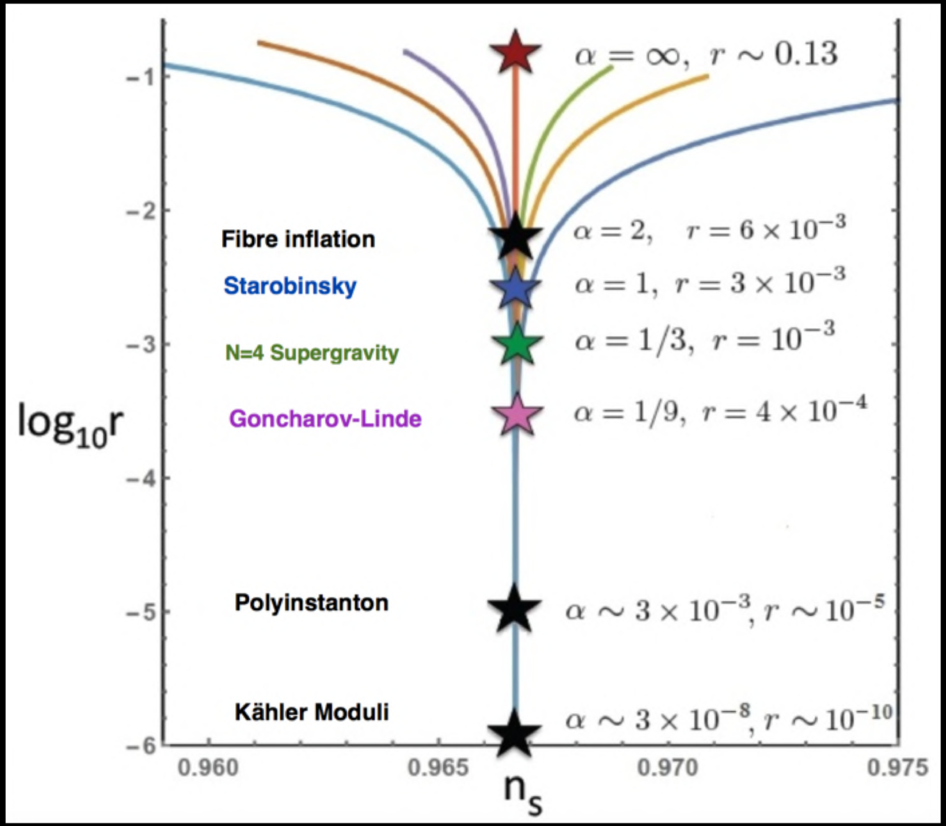
- Adiabatic (curvature)
- Non-Adiabatic (isocurvature)

Derivative interaction/random trajectory turns:
couple the fluctuations and modify their dispersion relations and correlators.

Hyperbolic manifolds from UV completions

[R. Kallosh, A. Linde (2013)]
[S. Ferrara, R. Kallosh, A. Linde and M. Porrati (2013)]
[J. J. M. Carrasco, R. Kallosh, A. Linde and D. Roest (2015)]

- ✓ Supergravity
- ✓ String theory compactification: Fibre inflation
- ✓ …



$$V \approx V_0 \left(1 - 2e^{-\sqrt{2}\phi/\sqrt{3\alpha}} + \dots\right)$$

Flattening of the potential is due to hyperbolic manifolds

$$n_s = 1 - \frac{2}{N_*} \qquad r = \frac{12\alpha}{N_*^2}$$

alpha-attractors provide universal inflationary predictions

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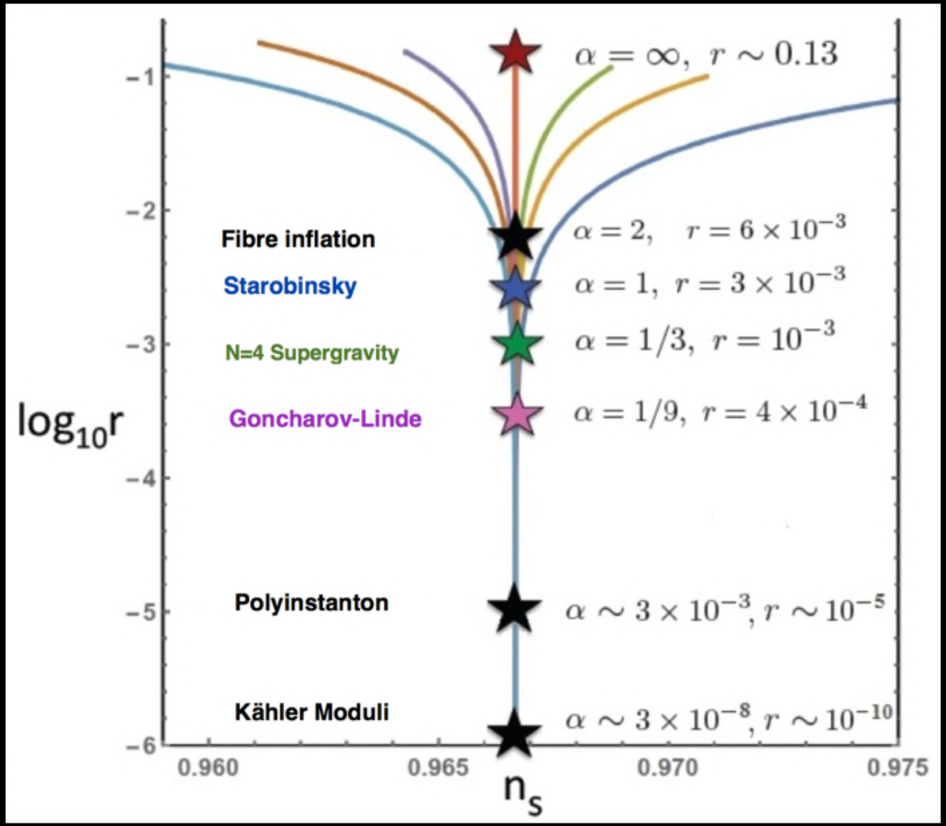

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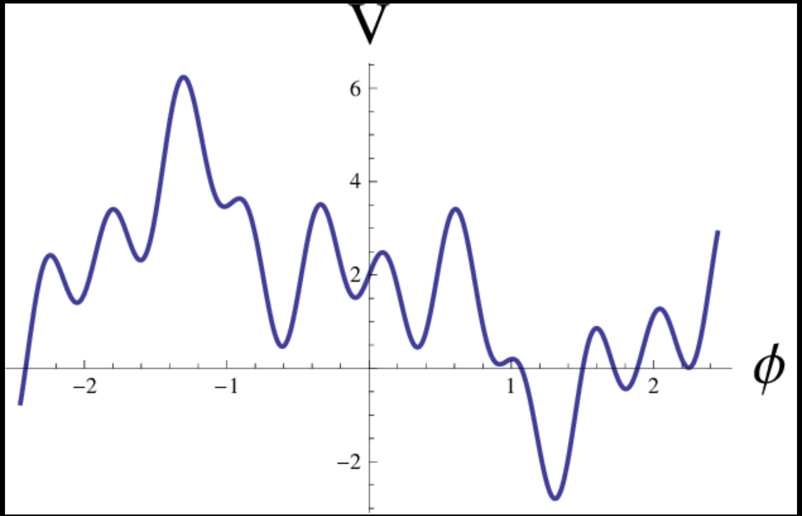
alpha-attractors provide universal inflationary predictions

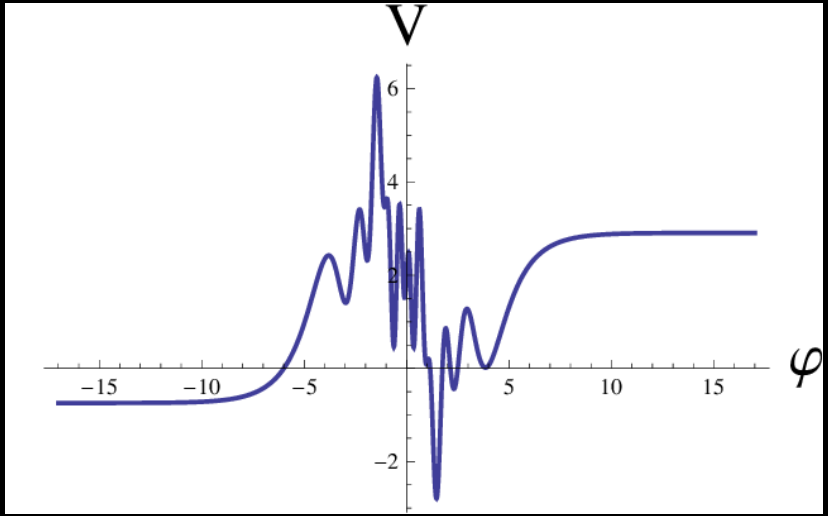
Plateau models of inflation

[R. Kallosh, A. Linde (2013)]

The inflationary plateau appears because of the exponential stretching of the growing branch.

$$V(\phi) \to V\left(\sqrt{6}\tanh\frac{\varphi}{\sqrt{6}}\right)$$



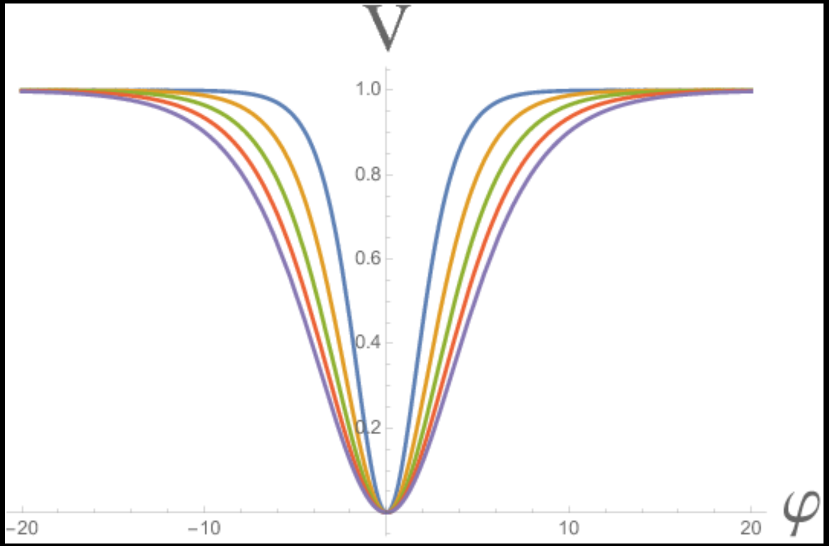


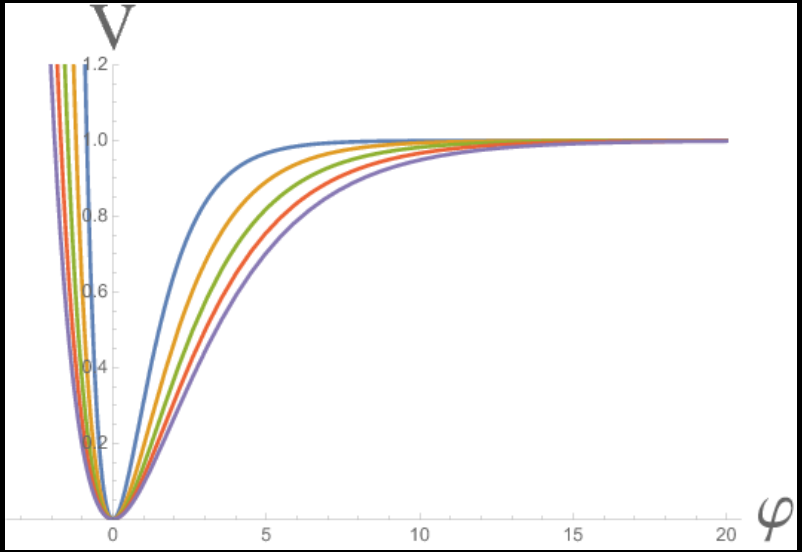
T- and E-models as the prototypical workhorses

[J. J. M. Carrasco, R. Kallosh, A. Linde (2015)]

$$V_T = \alpha\mu^2 \tanh^{2n} \frac{\phi}{\sqrt{6\alpha}}$$

$$V_E = \alpha\mu^2 \left(1 - e^{-\sqrt{2}\phi/\sqrt{3\alpha}}\right)^{2n}$$





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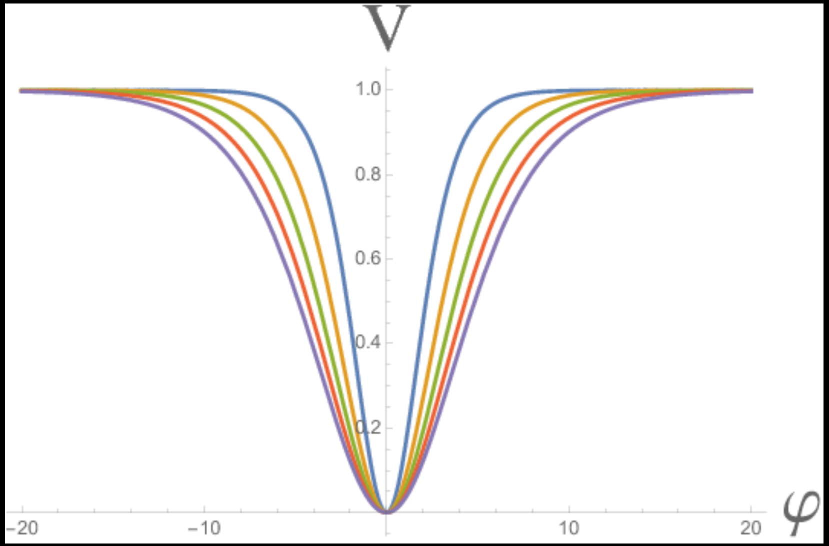

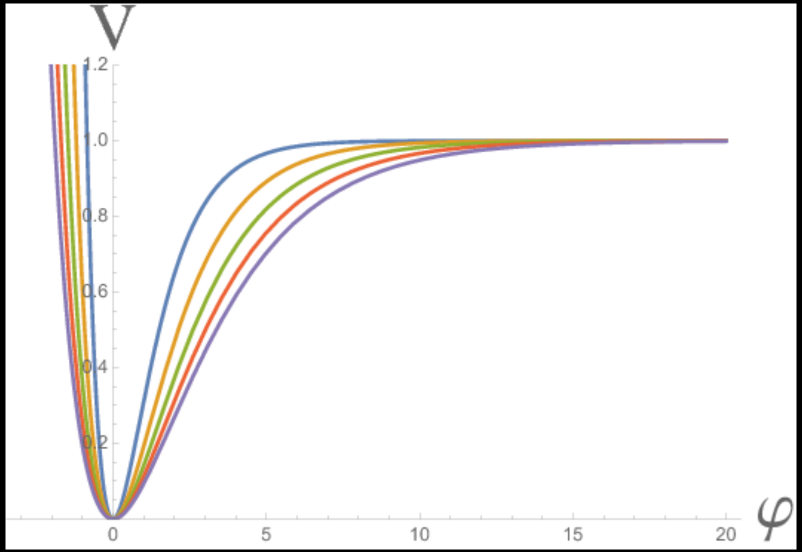

α field-space curvature $^{-1}$

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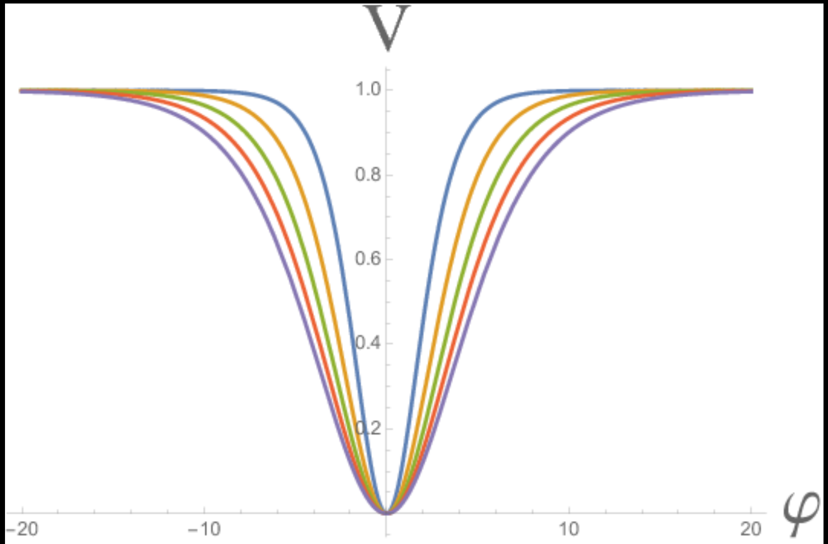

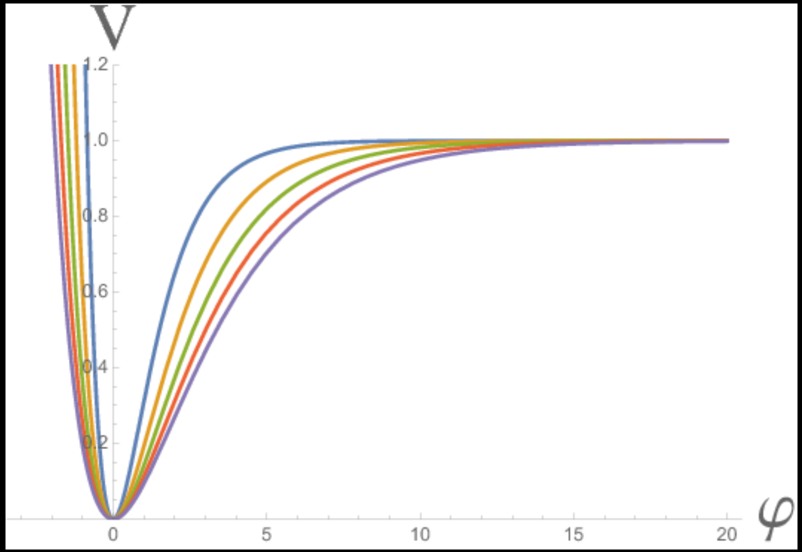

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n potential steepness

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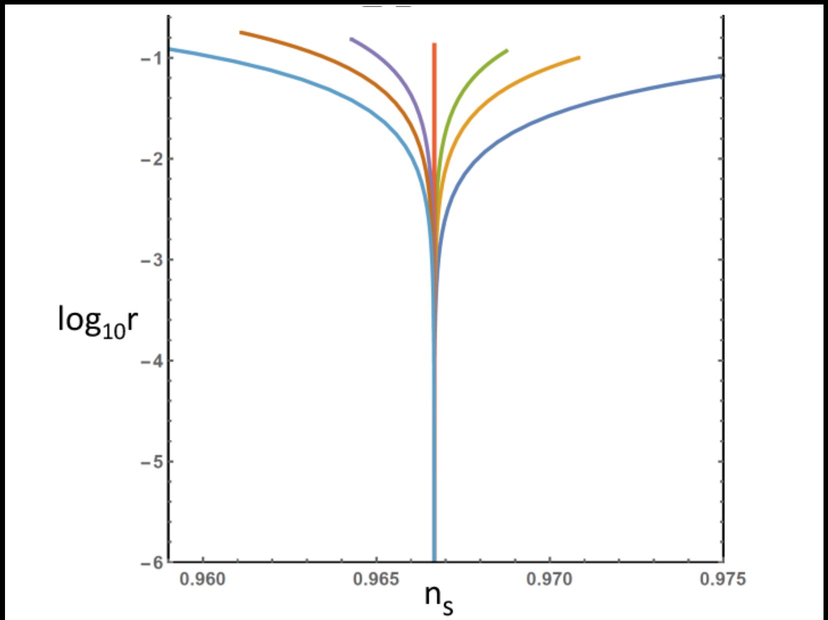

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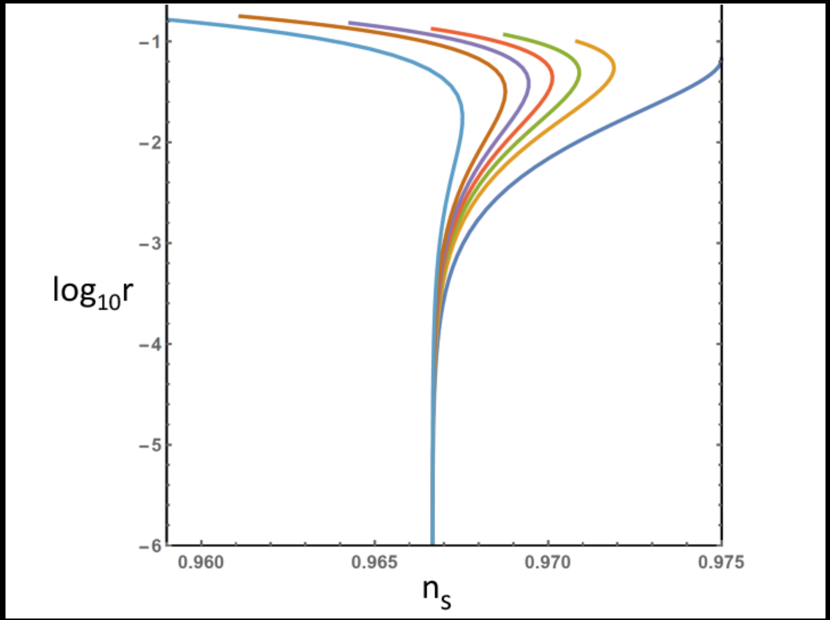
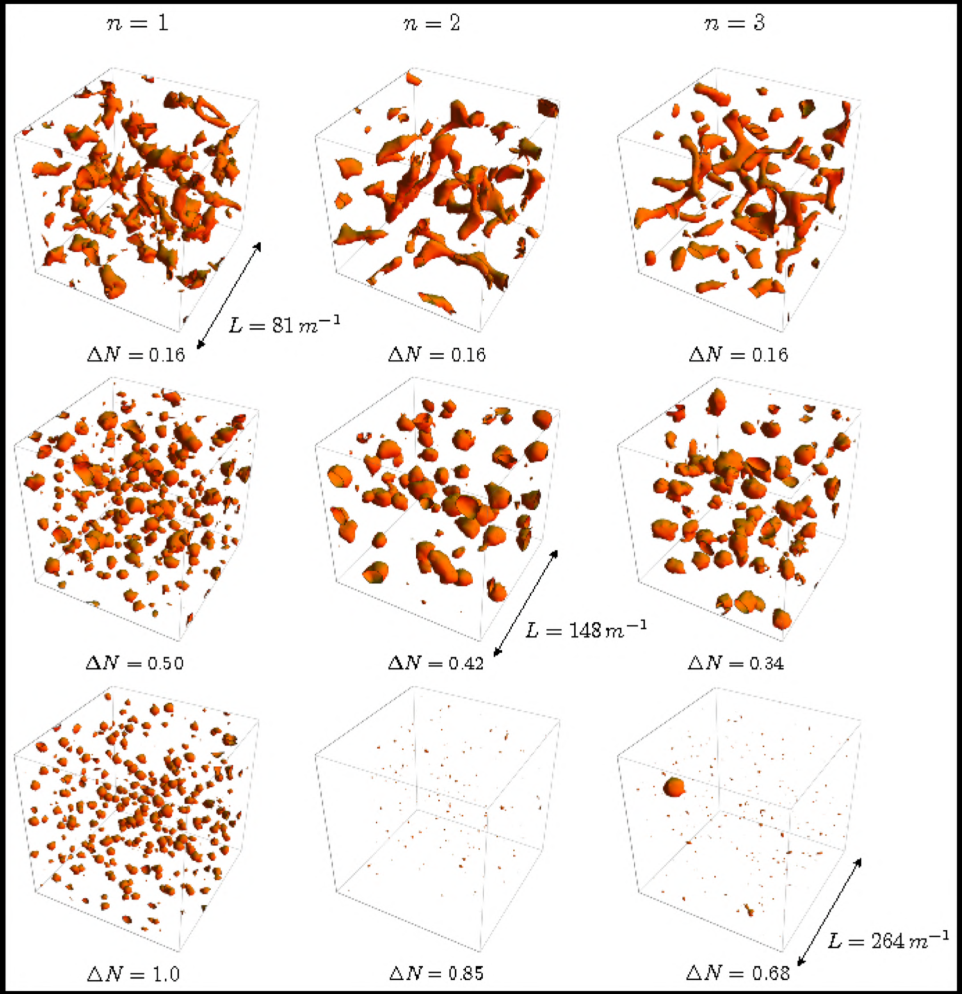
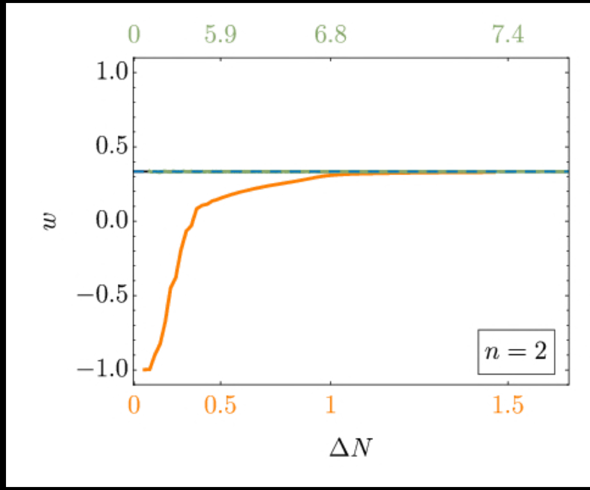


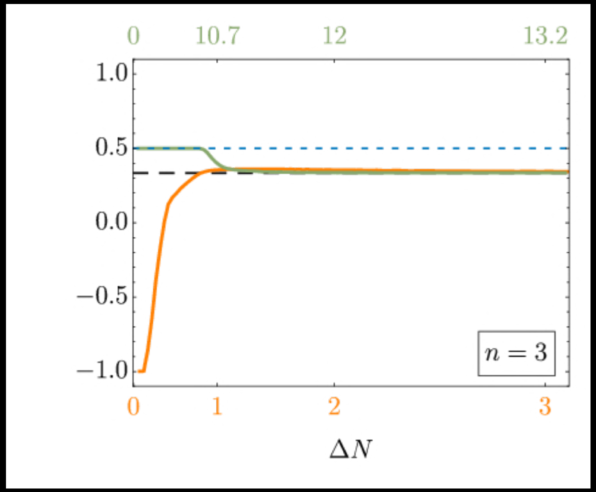
Lattice simulations for single field alpha attractors

[K. D. Lozanov, M. A. Amin (2017)]

efficient preheating through inflaton self-resonance







Alpha-attractors are intrinsically multi-field models

[J. J. M. Carrasco, R. Kallosh, A. Linde (2015)]

N = 1 Supergravity embedding:

the super-potential

$$W_H = \sqrt{\alpha}\mu\, S\, F(Z)$$

T-model

$$F(Z) = Z^n$$

$$\cos(\beta\theta) = \frac{1}{\cosh(\beta\chi)}$$

$$V(\phi,\chi) = \alpha\mu^2 \left(\frac{\cosh(\beta\phi)\cosh(\beta\chi) - 1}{\cosh(\beta\phi)\cosh(\beta\chi) + 1}\right)^n (\cosh(\beta\chi))^{2/\beta^2}$$

$$Z = \tanh\left(\frac{\phi + i\theta}{\sqrt{6\alpha}}\right)$$

E-model

$$F(Z) = \left(\frac{2Z}{Z+1}\right)^n$$

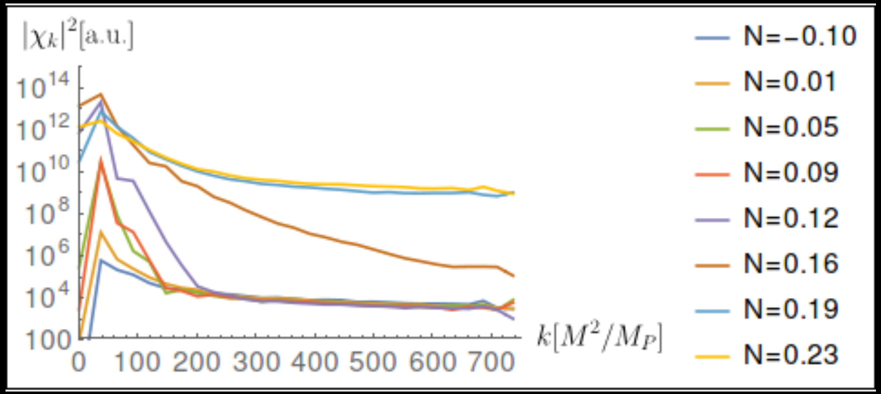
$$\beta = \sqrt{\frac{2}{3\alpha}}$$

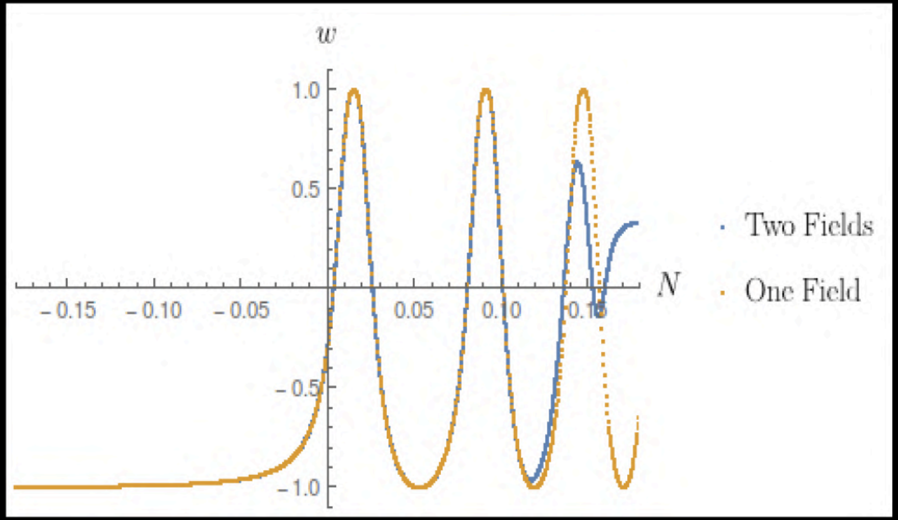
$$V(\phi,\chi) = \alpha\mu^2 \left(1 - \frac{2e^{-\beta\phi}}{\cosh(\beta\chi)} + e^{-2\beta\phi}\right)^n (\cosh(\beta\chi))^{2/\beta^2}$$

- Why do we need to know the physics of preheating? ✓
- Why multi-field? ✓
- Scaling relations in multi-field alpha-attractors
- Mass scales for preheating

Lattice simulations for two-field alpha attractors

[T. Krajewski, K. Turzynski, M. Wieczorek (2018)]





showed very efficient preheating with the presence of spectator field

Two-field system on a hyperbolic manifold

[OI, E. Sfakianakis, D.G. Wang, A. Achucarro (2020)]
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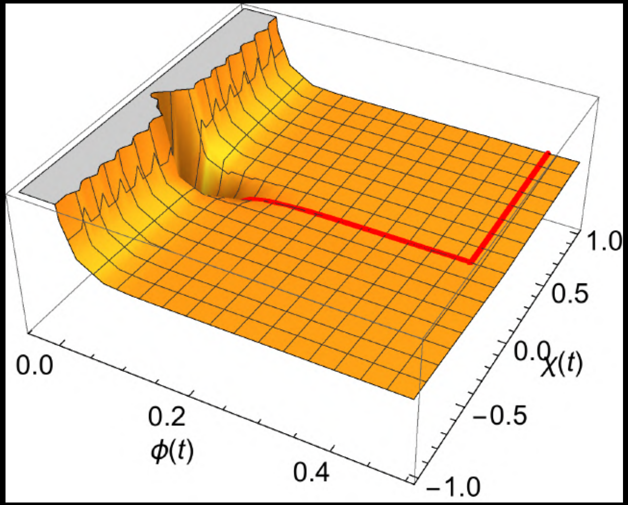
$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu\chi\partial^\mu\chi + e^{2b(\chi)}\partial_\mu\phi\partial^\mu\phi\right) - V(\phi,\chi)$$

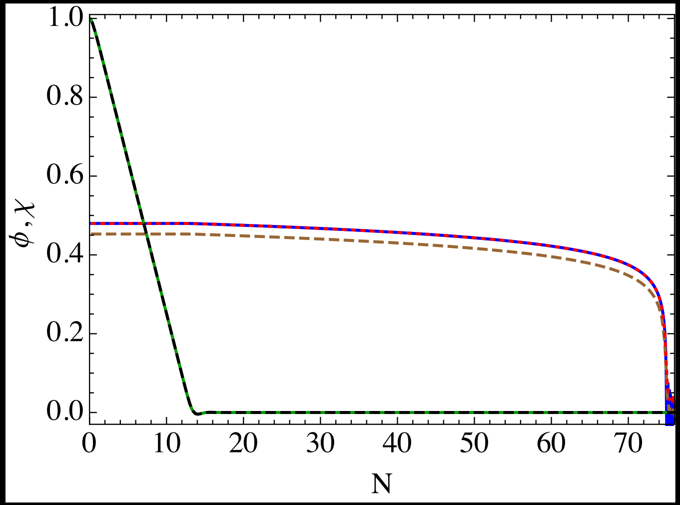
$$b(\chi) = \log\left(\cosh(\beta\chi)\right)$$

$$\beta = \sqrt{2/3\alpha}\ , \quad \mathcal{R} = -\frac{4}{3\alpha}$$

curvature of the field-space

- The two-stage inflation leading to single-field motion at $\chi = 0$.
- The two models during inflation are the same in the multi-field case, up to slow roll corrections.
- Does it hold during preheating?





Two-field system on a hyperbolic manifold

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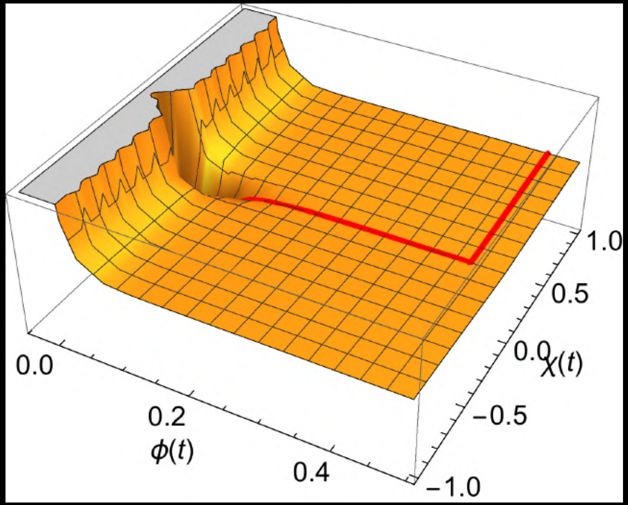

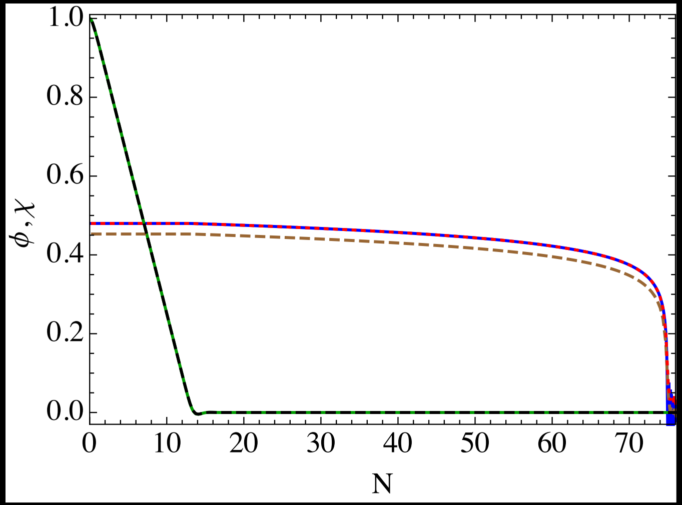

Scaling relations for background quantities

[OI, E. Sfakianakis, D.G. Wang, A. Achucarro (2020)]
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α field-space curvature $^{-1}$
n potential steepness

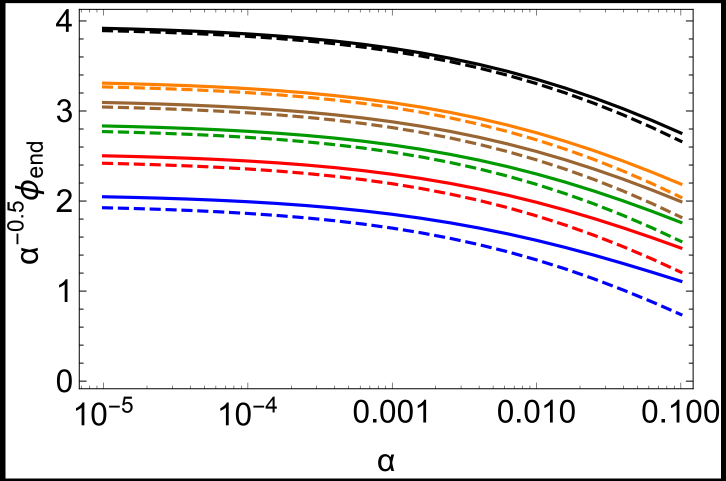
- in the slow-roll approximation and for $\phi \gg \sqrt{\alpha}$:

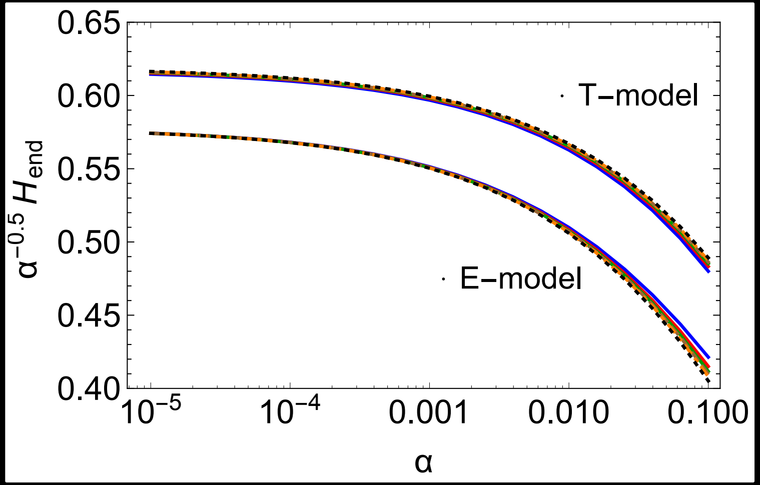
$$3H^2 \simeq \frac{\alpha}{M_{\rm Pl}^2}\mu^2 \qquad \epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{3\alpha}{4N^2} \qquad \phi_T(N) \simeq \phi(N) + \frac{\log(2)}{\beta}$$

$$N = \frac{3\alpha}{4n}e^{\beta\phi} \qquad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \simeq \frac{2}{N} \qquad \beta = \sqrt{2/3\alpha}$$

- the scaling is similar for the E- and T-models, with slightly different pre-factors

$$\phi_{\rm end} = \mathcal{O}(1)\sqrt{\alpha} \qquad H^2_{\rm end} = \mathcal{O}(1)\,\alpha\mu^2$$



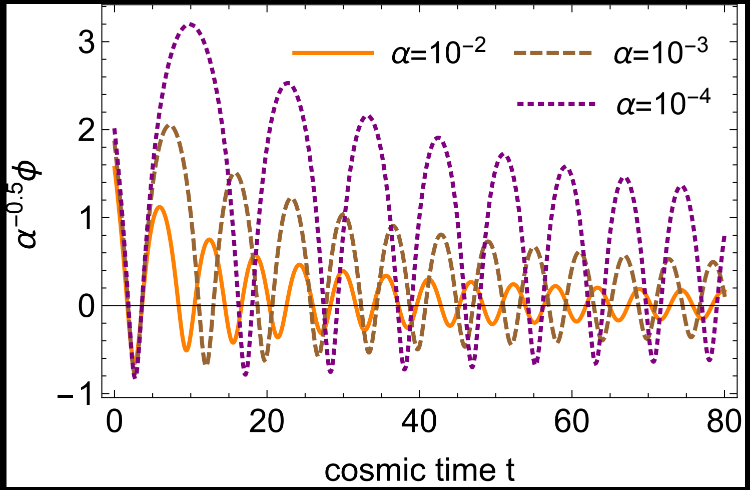


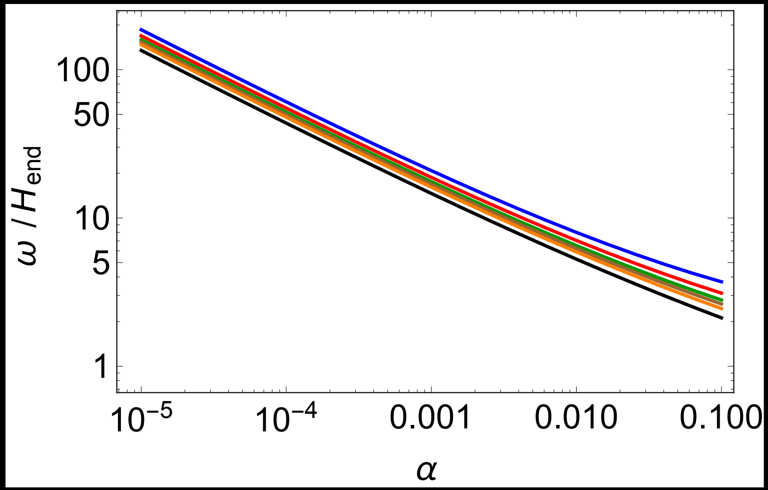
$n = 1, 1.5, 2, 2.5, 3, 5$ (bottom to top)

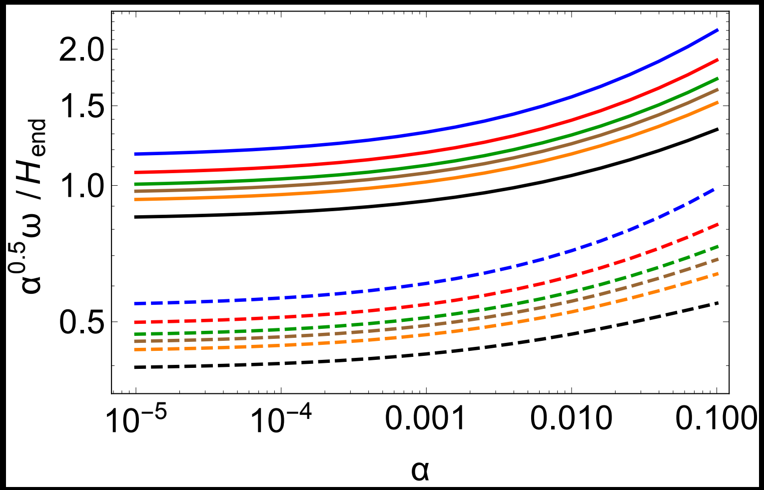
Hierarchy between the frequency of background oscillations and the Hubble scale

- More background oscillations occur per Hubble time for smaller values of alpha
- For small alphas the Hubble scale can be neglected, as it takes a large number of background oscillations for any considerable red-shifting to occur.
- More damping of the background motion per oscillation for the E-model

the scale hierarchy: $\omega \propto H_{\text{end}}/\sqrt{\alpha}$







$n = 1, 1.5, 2, 2.5, 3, 5$ (bottom to top)

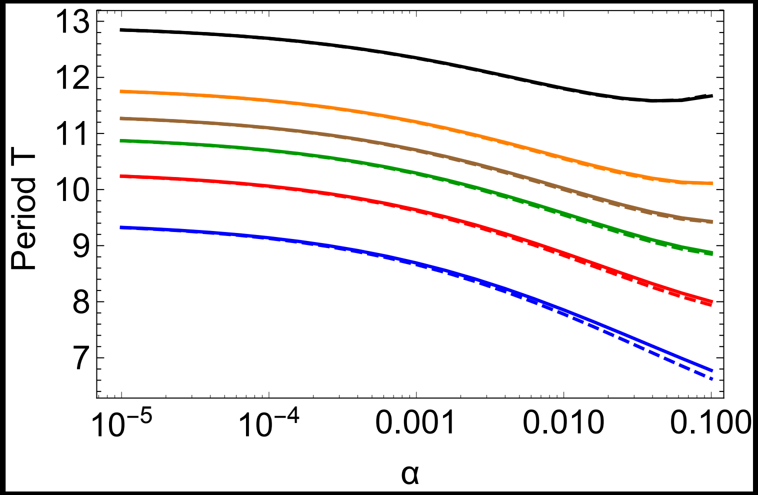
Asymmetric motion of the E-model

- the background motion is asymmetric for the E-model, spending much more time near the plateau and far less time near the steep potential wall.

- the T-model starts "higher up on the plateau" at the end of inflation

$$\boxed{T_T \simeq 2T_E}$$

$$\boxed{\phi_T(N) \simeq \phi(N) + \frac{\log(2)}{\beta}} \qquad \beta = \sqrt{2/3\alpha}$$



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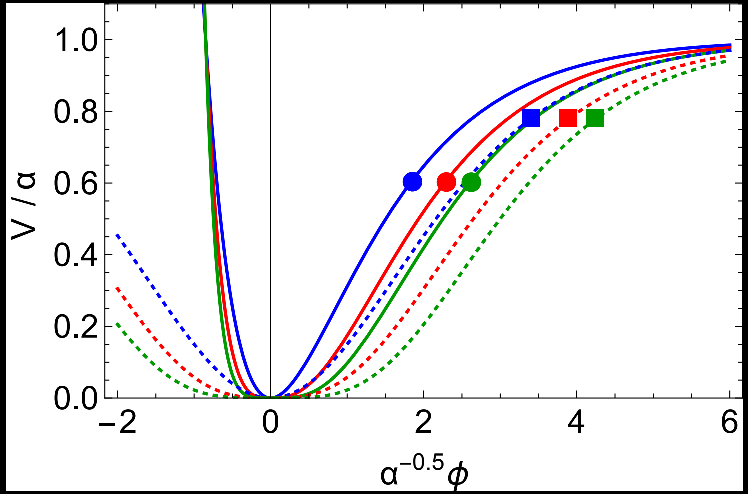

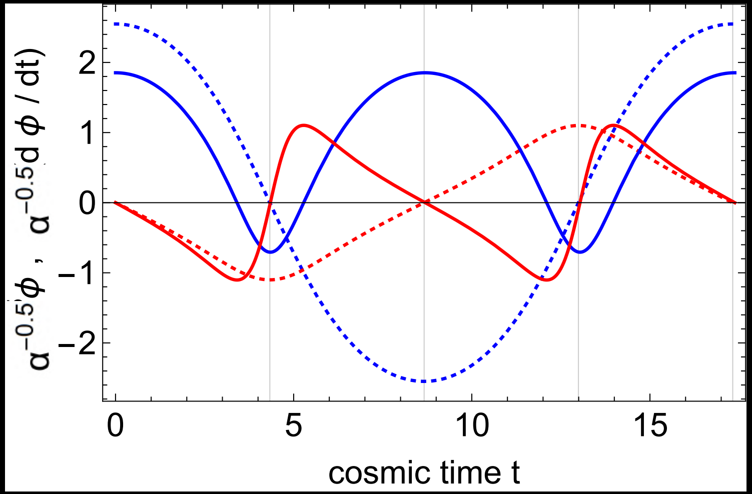

$n = 1 \quad ; \quad \alpha = 10^{-3}$

- Why do we need to know the physics of preheating? ✓
- Why multi-field? ✓
- Scaling relations in multi-field alpha-attractors ✓
- Mass scales for preheating

Covariant formalism to study the evolution of fluctuations

[H. Kodama and M. Sasaki (1984)]
[M. Sasaki, E. D. Stewart (1995)]
[D. Langlois and S. Renaux-Petel (2008)]
[D. I. Kaiser, E. A. Mazenc, and E. I. Sfakianakis (2013)]

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}\mathcal{G}_{\mathcal{IJ}}(\phi^K)g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$$

To study fields at the end of inflation, we consider scalar metric perturbations around a spatially flat FLRW line element

$$ds^2 = -(1+2A)dt^2 + 2a(\partial_i B)dx^i dt + a^2\left[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right]dx^i dx^j$$

$$\phi^I(x^\mu) = \varphi^I(t) + \delta\phi(x^\mu) \qquad Q^I \equiv \delta\phi^I + \frac{\dot\phi^I}{H}\psi$$

The gauge-invariant perturbations obey

$$\mathcal{D}_t^2 Q^I + 3H\mathcal{D}_t Q^I + \left[\frac{k^2}{a^2}\delta^I{}_J + \mathcal{M}^I{}_J\right]Q^J = 0$$

$$\mathcal{M}^I{}_J \equiv \mathcal{G}^{IK}\left(\mathcal{D}_J\mathcal{D}_K V\right) - \mathcal{R}^I{}_{LMJ}\dot\varphi^L\dot\varphi^M - \frac{1}{M_{\rm pl}^2 a^3}\mathcal{D}_t\left(\frac{a^3}{H}\dot\varphi^I\dot\varphi_J\right)$$

Covariant formalism to study the evolution of fluctuations

[H. Kodama and M. Sasaki (1984)]
[M. Sasaki, E. D. Stewart (1995)]
[D. Langlois and S. Renaux-Petel (2008)]
[D. I. Kaiser, E. A. Mazenc, and E. I. Sfakianakis (2013)]

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}\mathcal{G}_{\mathcal{IJ}}(\phi^K)g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$$

To study fields at the end of inflation, we consider scalar metric perturbations around a spatially flat FLRW line element

$$ds^2 = -(1+2A)dt^2 + 2a(\partial_i B)dx^i dt + a^2\left[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right]dx^i dx^j$$

$$\phi^I(x^\mu) = \varphi^I(t) + \delta\phi(x^\mu) \qquad\qquad Q^I \equiv \delta\phi^I + \frac{\dot\phi^I}{H}\psi$$

The gauge-invariant perturbations obey

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covariant time derivative in field space

$$\mathcal{M}^I{}_J \equiv \mathcal{G}^{IK}\left(\mathcal{D}_J\mathcal{D}_K V\right) - \mathcal{R}^I{}_{LMJ}\dot\varphi^L\dot\varphi^M - \frac{1}{M_{\rm pl}^2 a^3}\mathcal{D}_t\left(\frac{a^3}{H}\dot\varphi^I\dot\varphi_J\right)$$

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potential + field-space + kinematical effects

Covariant formalism to study the evolution of fluctuations

When the background motion is restricted along the $\chi = 0$ direction

$$\mathcal{G}_{IJ}(\chi = 0) = \delta_{IJ} \qquad \Gamma^{I}_{JK} = 0$$

the field-space structure simplifies and the quantization of the fluctuations proceeds as usual.

$$Q^I(x^\mu) \to X^I(x^\mu)/a(t) \qquad d\eta = dt/a(t)$$

The quadratic action becomes

$$S_2^{(X)} = \int d^3x d\eta \left[-\frac{1}{2}\eta^{\mu\nu}\delta_{IJ}\partial_\mu X^I \partial_\nu X^J - \frac{1}{2}\mathbb{M}_{IJ} X^I X^J \right]$$

$$\mathbb{M}_{IJ} = a^2 \left(\mathcal{M}_{IJ} - \frac{1}{6}\delta_{IJ} R \right)$$

Covariant formalism to study the evolution of fluctuations

$$\hat{X}^I = \int \frac{d^3k}{(2\pi)^{3/2}} \left[u^I(k,\eta)\hat{a}e^{ik\cdot x} + u^{I*}(k,\eta)\hat{a}^\dagger e^{-ik\cdot x} \right]$$

The equations of motion for mode functions with $u^\phi \equiv v$ and $u^\chi \equiv z$ are

$$\partial_\eta^2 v_k + \omega_\phi^2(k,\eta) v_k \simeq 0\,, \quad \omega_\phi(k,\eta)^2 = k^2 + a^2 m_{\mathrm{eff},\phi}^2$$

$$\partial_\eta^2 z_k + \omega_\chi^2(k,\eta) z_k \simeq 0\,, \quad \omega_\chi(k,\eta)^2 = k^2 + a^2 m_{\mathrm{eff},\chi}^2$$

$$\boxed{m_{\mathrm{eff},I}^2 = m_{1,I}^2 + m_{2,I}^2 + m_{3,I}^2 + m_{4,I}^2}$$

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$$m_{1,I}^2 \equiv \mathcal{G}^{IK}\left(\mathcal{D}_I \mathcal{D}_K V\right),$$

$$m_{2,I}^2 \equiv -\mathcal{R}^I{}_{LMI}\dot{\varphi}^L \dot{\varphi}^M,$$

$$m_{3,I}^2 \equiv -\frac{1}{M_{\mathrm{pl}}^2 a^3} \delta^I_K \delta^J_I\, \mathcal{D}_t \left(\frac{a^3}{H}\dot{\varphi}^K \dot{\varphi}_J\right)$$

$$m_{4,I}^2 \equiv -\frac{1}{6} R = (\epsilon - 2) H^2\,.$$

- potential contribution
- geometry of field-space
- kinematical effects
- curvature of space-time

Effective mass terms and scaling for hyperbolic manifolds

$$m_{1,I}^2 \equiv \mathcal{G}^{IK} \left(\mathcal{D}_I \mathcal{D}_K V\right),$$
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$$m_{4,I}^2 \equiv -\frac{1}{6} R = (\epsilon - 2) H^2 .$$

$$m_{1,\phi}^2 = V_{\phi\phi}, \quad m_{1,\chi}^2 = V_{\chi\chi}$$
$$m_{2,\chi}^2 = \frac{1}{2} R \dot{\phi}^2$$
$$m_{3,\phi}^2 = -\frac{1}{M_{\rm Pl}^2 a^3} \mathcal{D}_t \left(\frac{a^3}{H} \dot{\phi}^2 \right)$$
$$m_{4,\phi}^2 = m_{4,\chi}^2 = -\frac{1}{6} R$$

$$m_{\rm eff,\phi}^2 = m_{1,\phi}^2 + m_{2,\phi}^2 + m_{3,\phi}^2 + m_{4,\phi}^2$$
$$m_{\rm eff,\chi}^2 = m_{1,\chi}^2 + m_{2,\chi}^2 + m_{3,\chi}^2 + m_{4,\chi}^2$$

Effective mass terms and scaling for hyperbolic manifolds

$$m^2_{1,I} \equiv \mathcal{G}^{IK}\left(\mathcal{D}_I \mathcal{D}_K V\right),$$

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$$m^2_{1,\phi} = V_{\phi\phi}\,, \quad m^2_{1,\chi} = V_{\chi\chi}$$

$$m^2_{2,\chi} = \frac{1}{2}R\dot{\phi}^2$$

$$m^2_{3,\phi} = -\frac{1}{M_{\rm Pl}^2 a^3}\mathcal{D}_t\left(\frac{a^3}{H}\dot{\phi}^2\right)$$

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$$m^2_{\rm eff,\phi} = m^2_{1,\phi} + \cancel{m^2_{2,\phi}} + m^2_{3,\phi} + m^2_{4,\phi}$$

$$m^2_{\rm eff,\chi} = m^2_{1,\chi} + m^2_{2,\chi} + \cancel{m^2_{3,\chi}} + m^2_{4,\chi}$$

$$m^2_{3,\chi} = 0 = m^2_{2,\phi}$$

Oksana Iarygina, Leiden University

Effective mass terms and scaling for hyperbolic manifolds

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$$m_{3,\chi}^2 = 0 = m_{2,\phi}^2$$

$$m_{3,\phi}^2 \sim \mu^2 \sqrt{\tilde{\alpha}}$$

$$m_{4,\phi}^2 = m_{4,\chi}^2 \sim \mu^2 \tilde{\alpha}$$

vanish for $\alpha \ll 1$

Effective mass terms and scaling for hyperbolic manifolds

$$m^2_{\text{eff},\phi} = m^2_{1,\phi} + m^2_{2,\phi} + m^2_{3,\phi} + m^2_{4,\phi}$$

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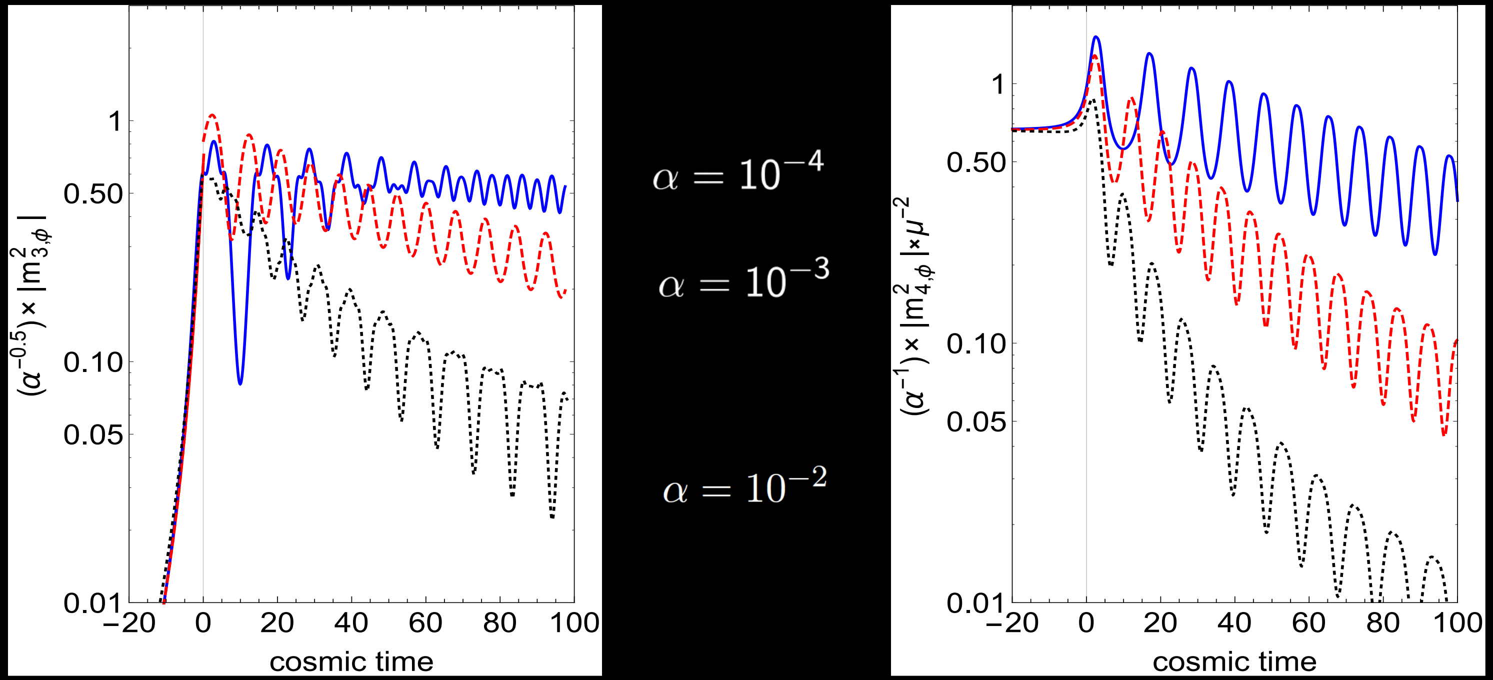
$$m^2_{4,\phi} = m^2_{4,\chi} \sim \mu^2 \tilde{\alpha}$$



Effective mass terms and scaling for hyperbolic manifolds

Our focus: fluctuations $\delta\chi$ can undergo tachyonic excitation, more efficient than parametric amplification and is a truly multi-field phenomenon with a crucial dependence on the field-space geometry.

$$\rho_k^{(\phi)} = \frac{1}{2}\left[|v_k'|^2 + \left(k^2 + a^2 m_{\text{eff},\phi}^2\right)|v_k|^2\right]$$
$$\rho_k^{(\chi)} = \frac{1}{2}\left[|z_k'|^2 + \left(k^2 + a^2 m_{\text{eff},\chi}^2\right)|z_k|^2\right]$$

$$m_{\text{eff},\phi}^2 \simeq V_{\phi\phi}(\chi = 0)$$
$$m_{\text{eff},\chi}^2 \simeq V_{\chi\chi}(\chi = 0) + \frac{1}{2}\mathcal{R}\dot{\phi}^2$$

$$\mathcal{R} = -4/3\alpha$$

the field space Ricci curvature scalar

$$\omega_I{}^2(k,t) = \frac{k^2}{a^2} + m_{\text{eff},I}^2$$

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tachyonic vs parametric resonance

$$\mathcal{R} = -4/3\alpha$$

the field space Ricci curvature scalar

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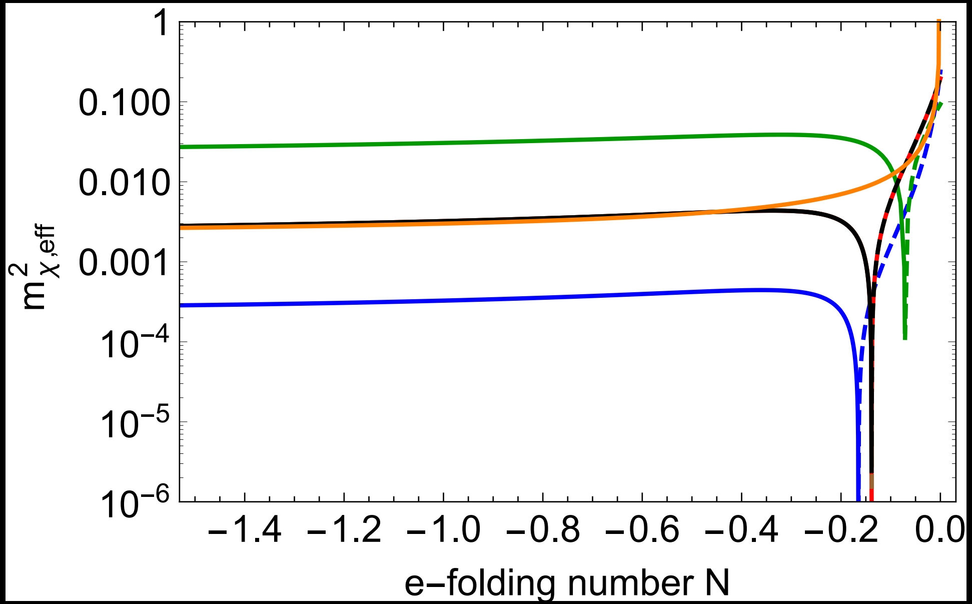
Geometrical destabilization

[S. Renaux-Petel and K. Turzynski (2016)]

Alpha-attractors are safe against geometric destabilization effects until close to the end of inflation.



During inflation the effective super-horizon isocurvature mass in the slow-roll approximation

$$m^2_{\chi,\text{eff}} \simeq \left(2 + \frac{1}{N}\right)\alpha$$

- Inflationary background is safe
- Geometrical destabilization leads to efficient preheating

Effective mass for alpha attractors

$$V_{\chi\chi}(\chi=0) \simeq \frac{4}{3} n e^{-\beta\phi}\left(\left(1-e^{-\beta\phi}\right)^2\right)^{n-1}$$

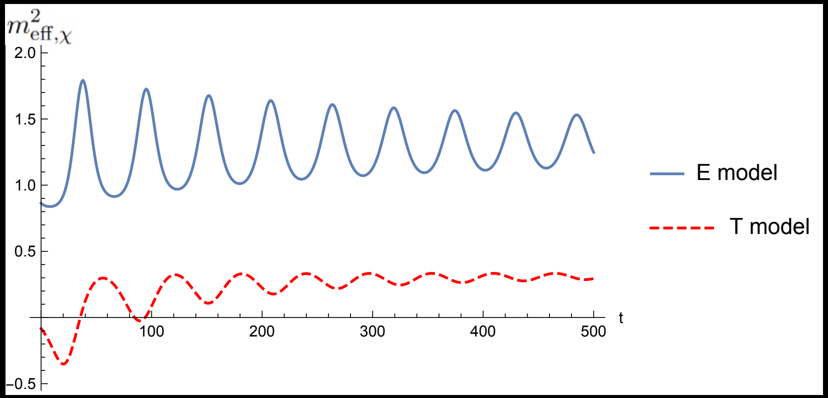
$$m^2_{\text{eff},\chi} \simeq V_{\chi\chi}(\chi=0) + \frac{1}{2}\mathcal{R}\dot{\phi}^2$$

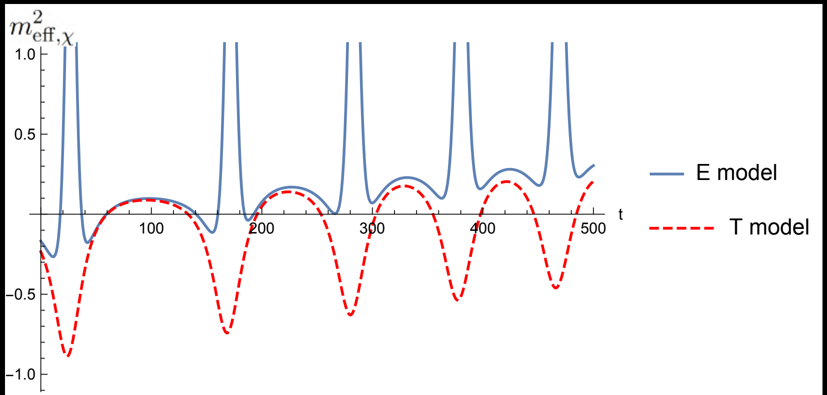
no tachyonic resonance in massive E-model!

potential steepness:

$n = 1$

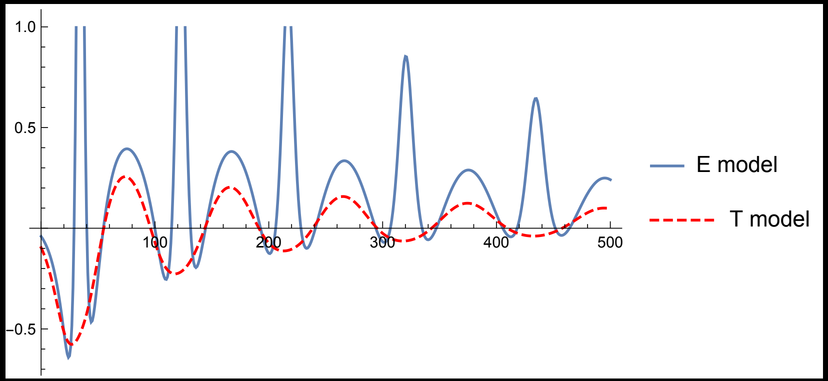
massive case

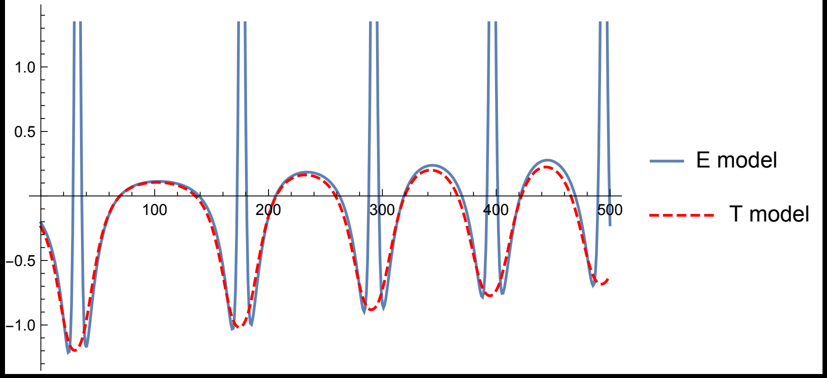




$n > 1$

massless case





$\alpha = 10^{-2}$ $\alpha = 10^{-4}$

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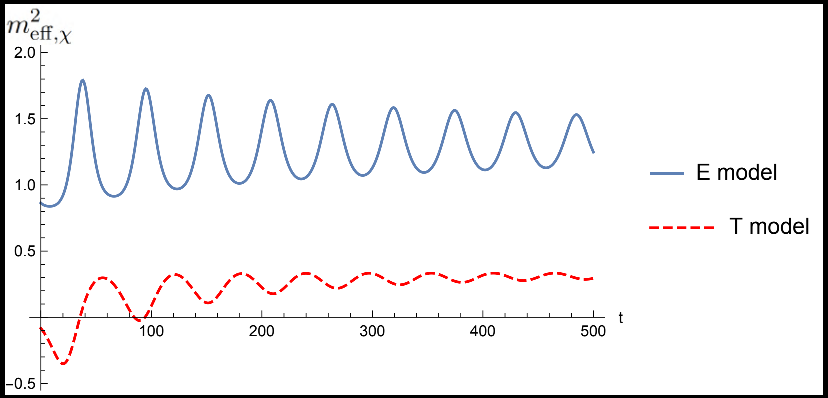
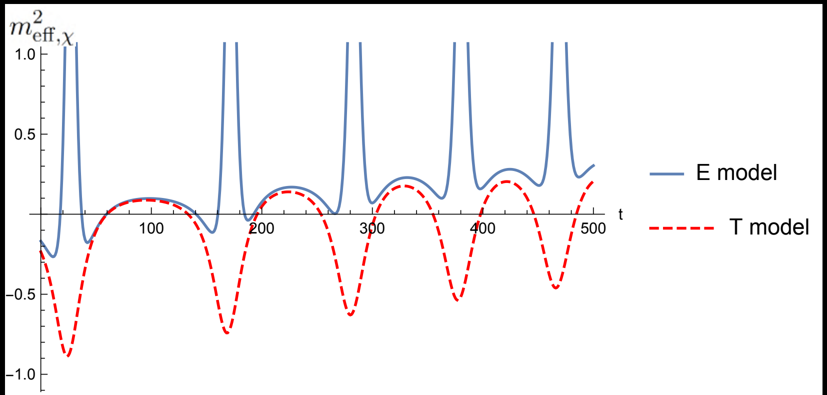
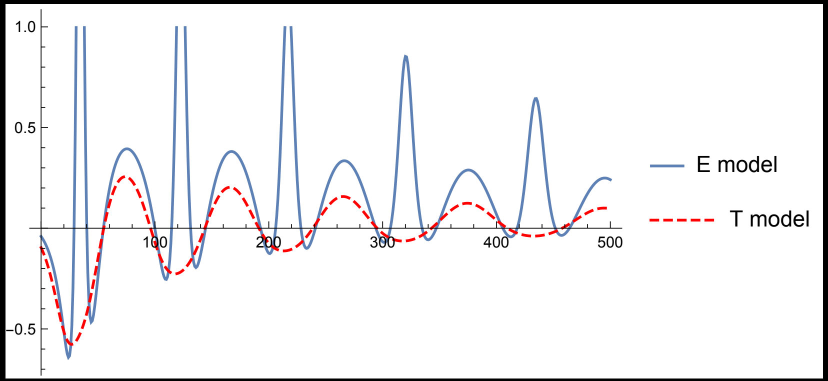
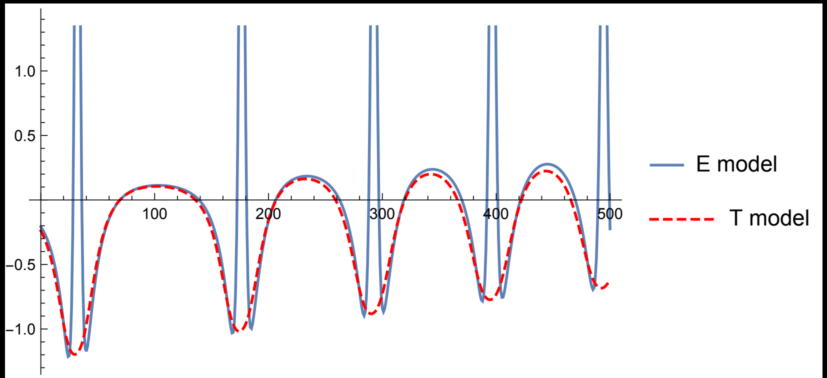




$n > 1$

massless case





$\alpha = 10^{-2}$

$\alpha = 10^{-4}$

Effective mass terms for massive fields

$$m^2_{\text{eff},\phi} \simeq V_{\phi\phi}(\chi = 0)$$
$$m^2_{\text{eff},\chi} \simeq V_{\chi\chi}(\chi = 0) + \frac{1}{2}\mathcal{R}\dot{\phi}^2$$

$$\ddot{u}(t) + [A + 2q\cos(2t)]u(t) = 0$$

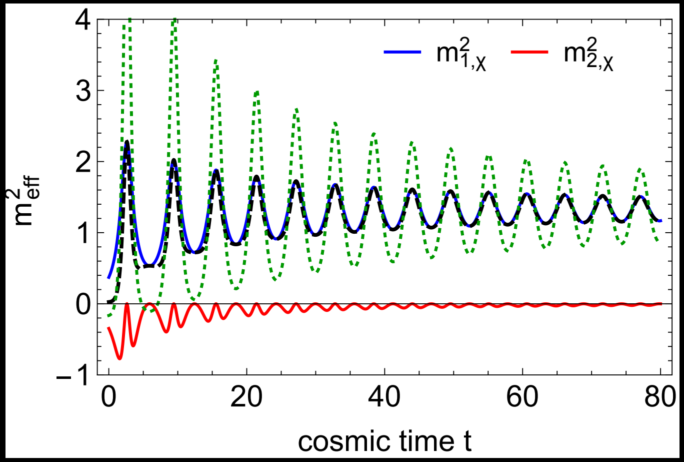
parametric resonance is suppressed for $A \gg q$

E-model

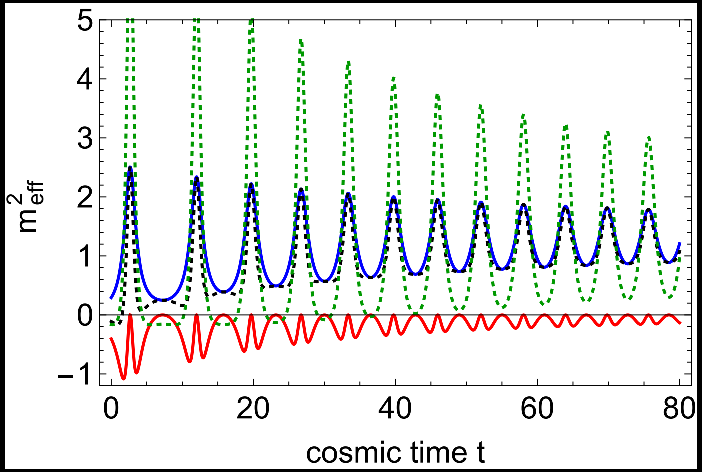
$n = 1$

$$V^{n=1}_{\chi\chi}\big|_{\max,(1)} \lesssim 2.9$$

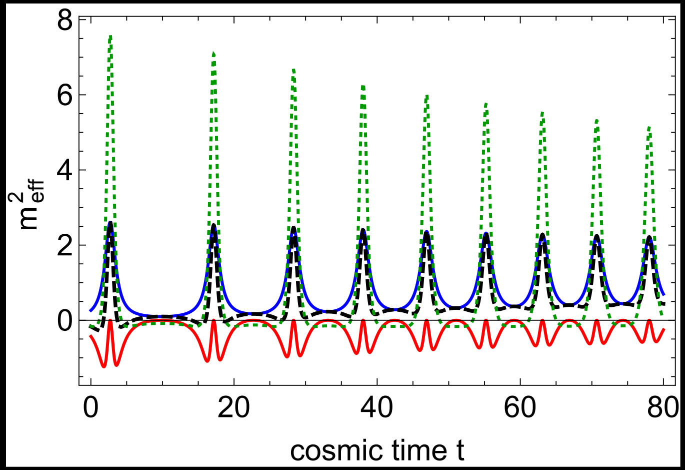
$$\frac{1}{2}\mathcal{R}\dot{\phi}^2 \propto -\frac{1}{\alpha} \times \left(\frac{\sqrt{\alpha}}{\mathcal{O}(1)}\right)^2 = -\mathcal{O}(1)$$



$\alpha = 10^{-2}$



$\alpha = 10^{-3}$



$\alpha = 10^{-4}$

Effective mass terms for massless fields

$$\omega_\chi{}^2(k,t) = \boxed{\frac{k^2}{a^2}} + V_{\chi\chi}(\chi=0) + \frac{1}{2}\mathcal{R}\dot{\phi}^2$$

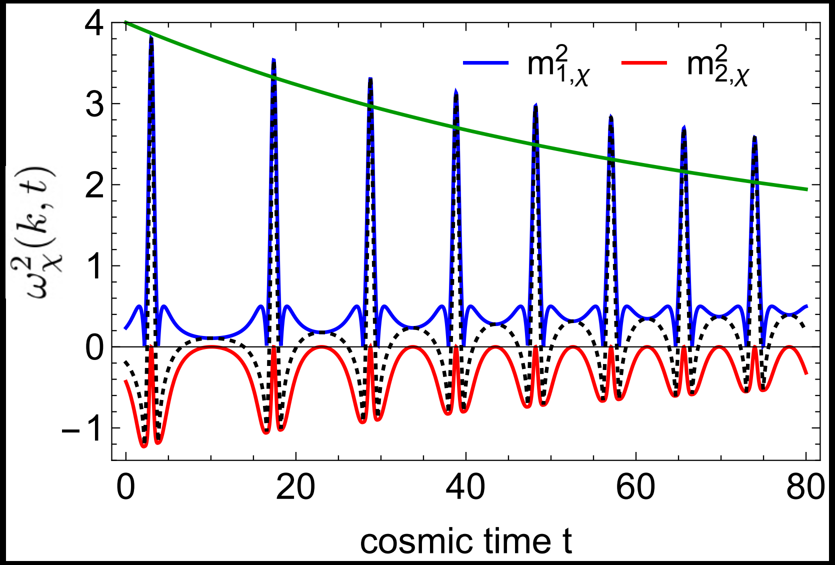
$$V^{\max}_{\chi\chi}(\chi=0) \sim \left(\frac{1}{a}\right)^{\min(n,4)}$$

$$\frac{1}{2}\mathcal{R}\dot{\phi}^2 \sim -\mathcal{O}(1)$$

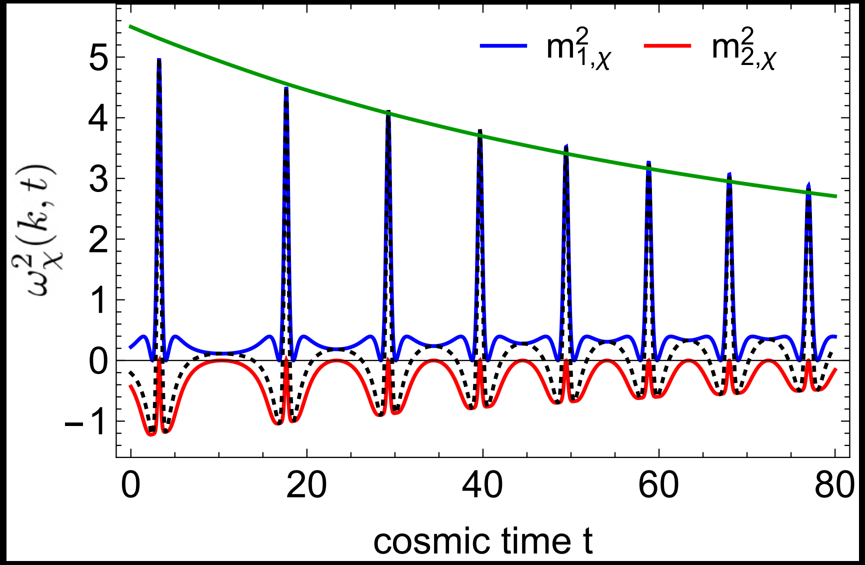
For n < 2 the wavenumber contribution is less important than potential after few oscillations, for n > 2 it dominates over the potential at late times, for sufficiently large wave-numbers.

E-model

$n > 1$



$\alpha = 10^{-4} \quad n = 3/2$



$n = 2$

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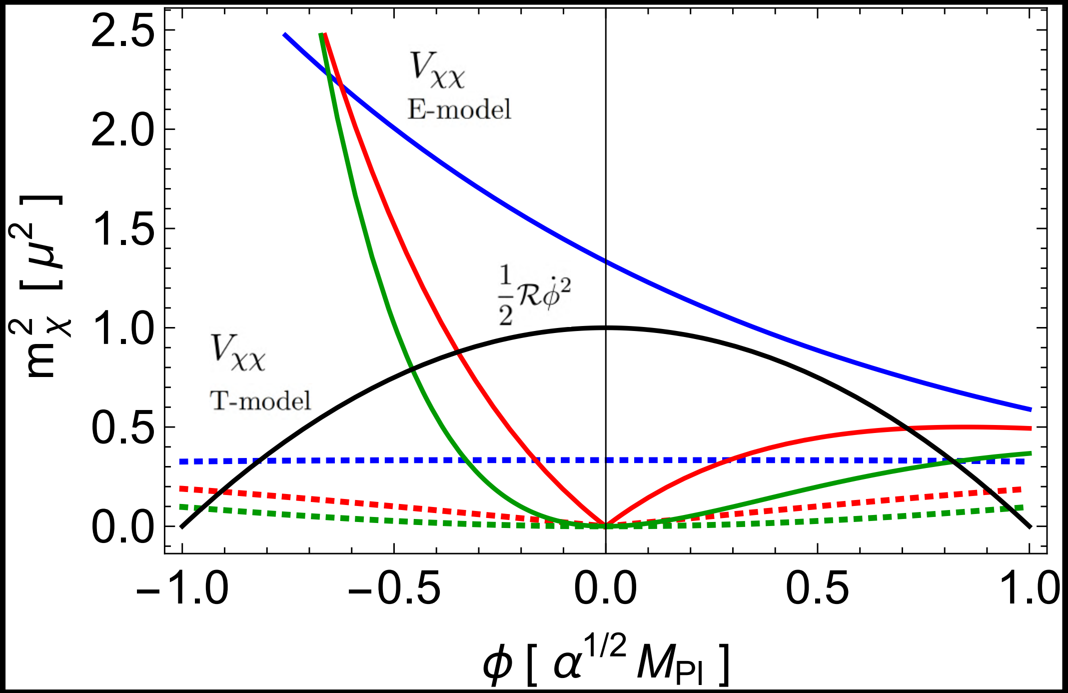
Effective mass terms and scaling for alpha attractor potentials

In the E-model with n = 1 the potential term can dominate over the tachyonic field-space curvature



$n = 1, 3/2, 2$ (blue,red green)

WKB approximation

$$\partial_t^2 \chi_k + \omega_\chi^2(k,t)\, \chi_k = 0$$

$$\Omega_k^2(t) = -\omega_k^2(t)$$

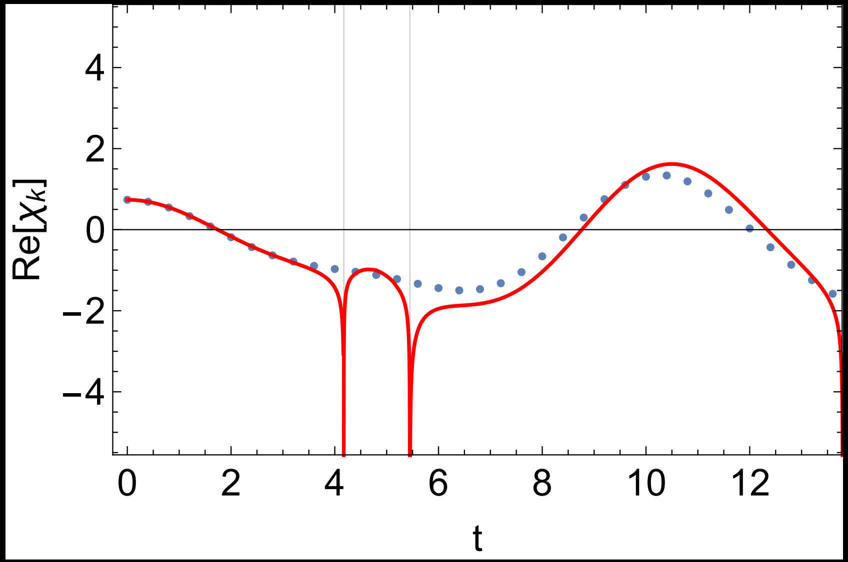
amplification factor
after the first tachyonic region

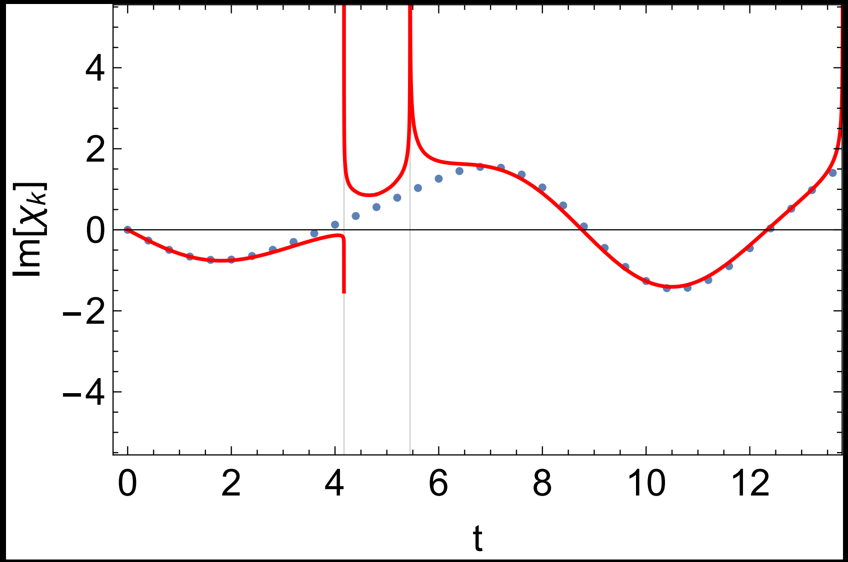
$$A_k = e^{\int_{t_-}^{t_+} \Omega_k(t) dt}$$

$$\chi_k^I = \frac{\alpha^n}{\sqrt{2\omega_k(t)}} e^{-i\int \omega_k(t)dt} + \frac{\beta^n}{\sqrt{2\omega_k(t)}} e^{i\int \omega_k(t)dt}$$

$$\chi_k^{II} = \frac{a^n}{\sqrt{2\Omega_k(t)}} e^{-\int \Omega_k(t)dt} + \frac{b^n}{\sqrt{2\Omega_k(t)}} e^{\int \Omega_k(t)dt}$$

$$\chi_k^{III} = \frac{\alpha^{n+1}}{\sqrt{2\omega_k(t)}} e^{-i\int \omega_k(t)dt} + \frac{\beta^{n+1}}{\sqrt{2k}} e^{i\int \omega_k(t)dt}$$





WKB (red) before, during and after a tachyonic transition

Floquet charts

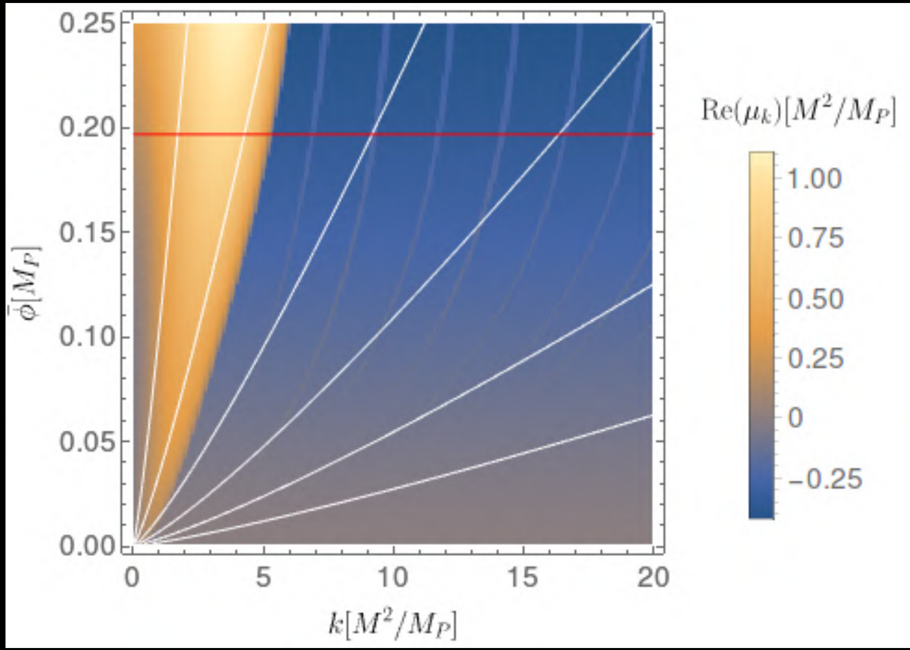
[T. Krajewski, K. Turzyński, M. Wieczorek (2018)]

$$\partial_t^2 \chi_k + \omega_\chi^2(k,t)\,\chi_k = 0$$

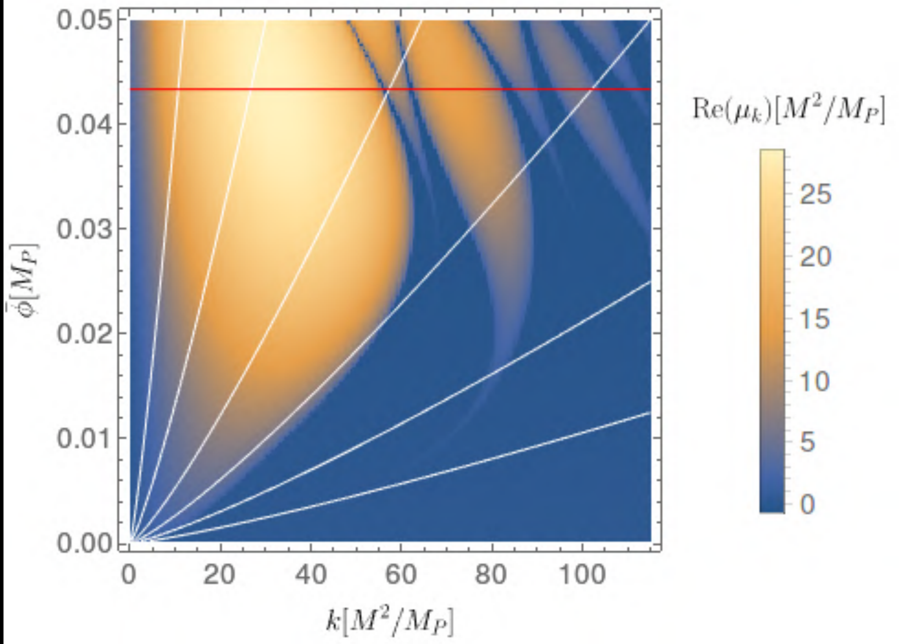
$$\chi_k(t) \sim e^{\mu_k t} P(t)$$

where $P(t)$ is a periodic function

The resonance structure looks very different!



$\alpha = 10^{-2}$



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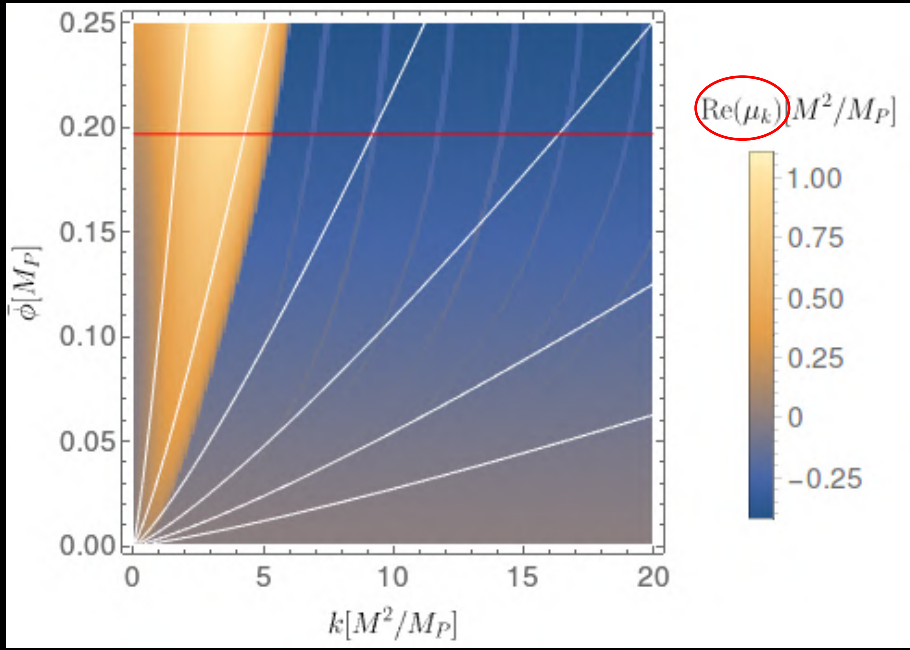

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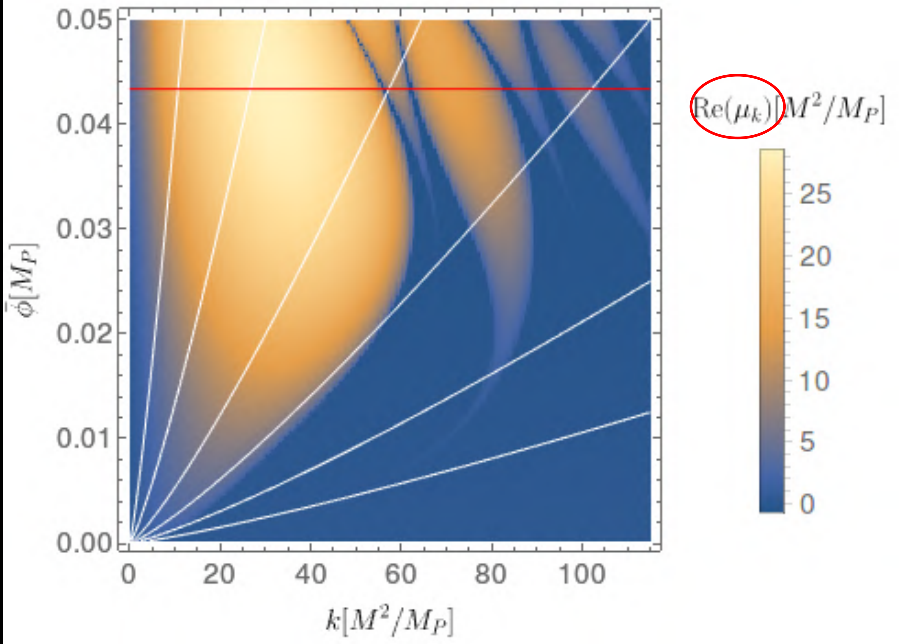
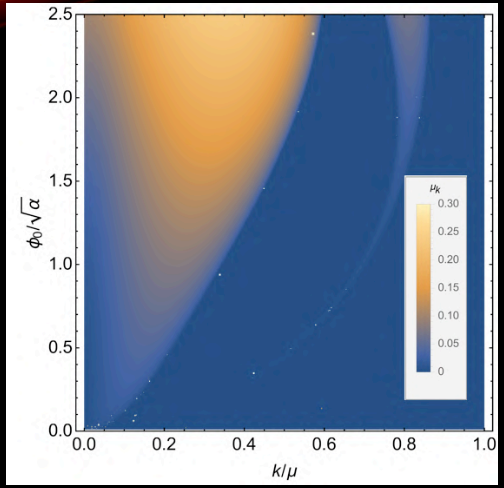


$\alpha = 10^{-4}$

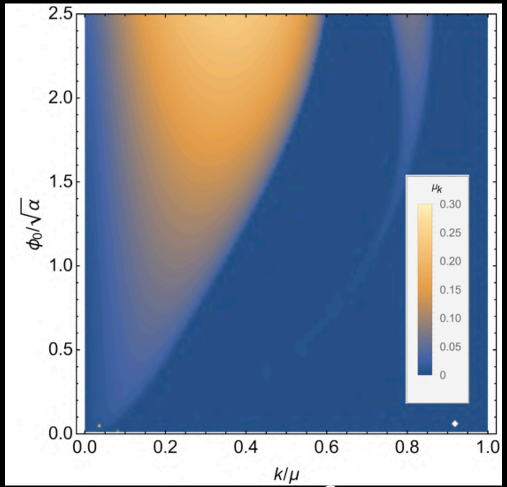
Floquet charts for symmetric potentials

With a proper rescaling

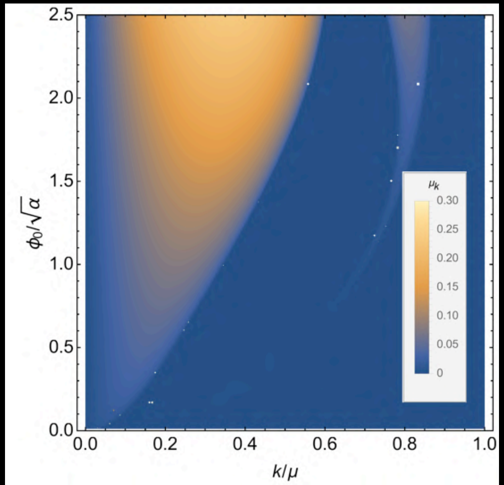
$\phi_0 \sim \sqrt{\alpha}$



$\alpha = 10^{-2}$



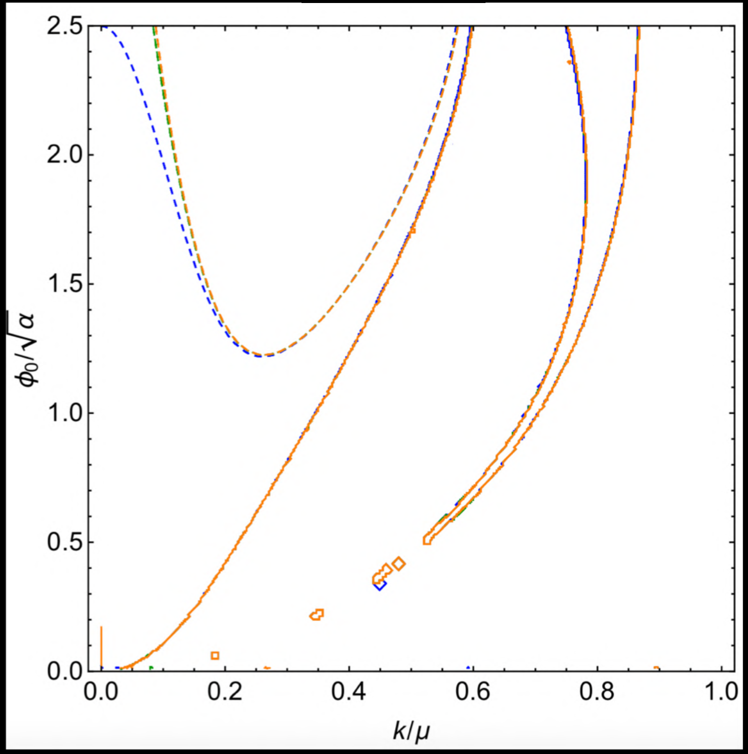
$\alpha = 10^{-3}$



$\alpha = 10^{-4}$

emerges a unifying picture

"master diagram"



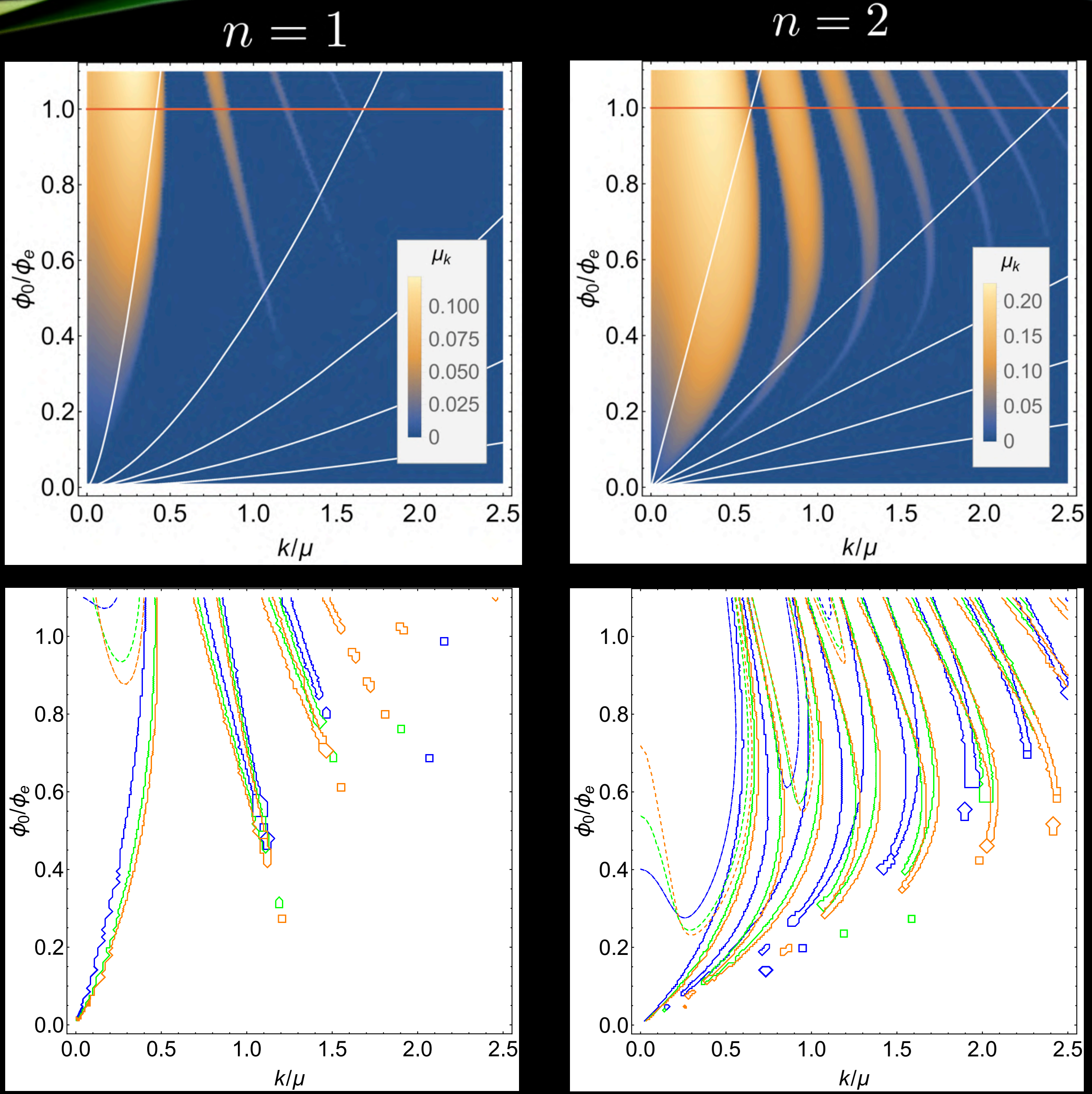
the resonance structure is identical, regardless of the exact value of the field-space curvature

Floquet charts E-model

The E-model has a richer resonance structure during (p)reheating, due to competing mass scales

$$m_{\mathrm{eff},\chi}^2 \simeq V_{\chi\chi}(\chi = 0) + \frac{1}{2}\mathcal{R}\dot{\phi}^2$$

$$\boxed{\phi_{\mathrm{end}} = \mathcal{O}(1)\sqrt{\alpha}}$$

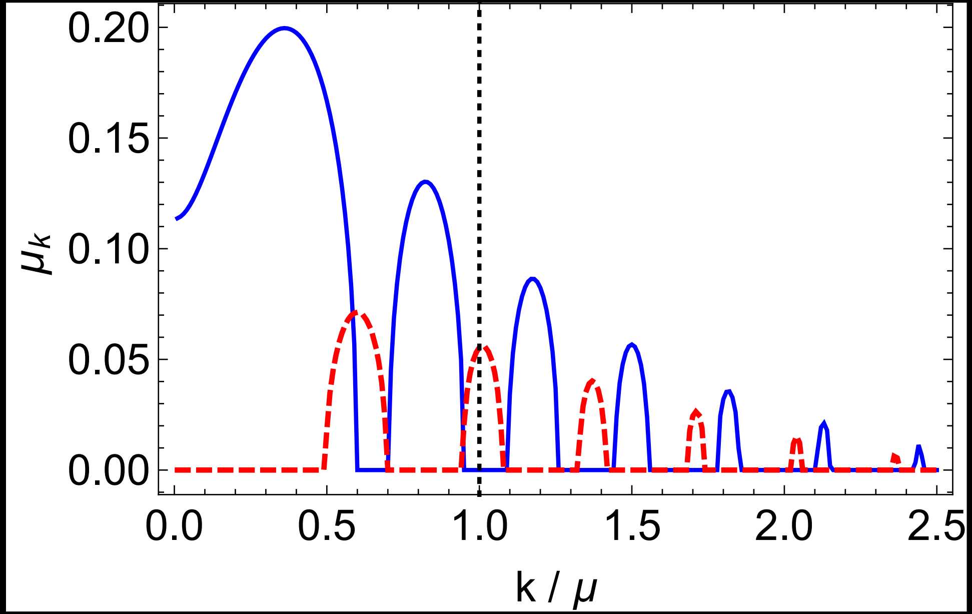


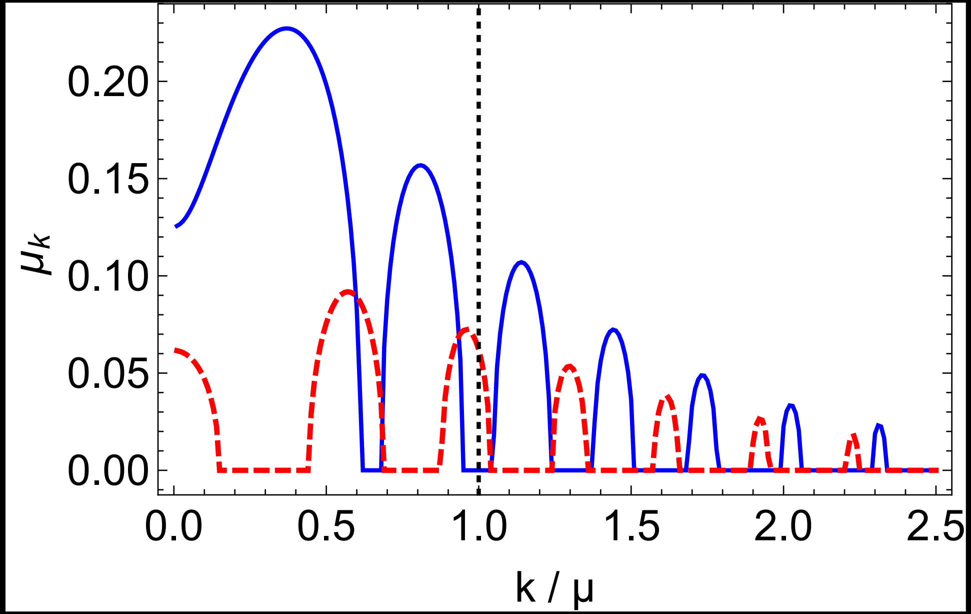
Full floquet diagram vs potential contribution

$$m^2_{\text{eff},\chi} \simeq V_{\chi\chi}(\chi = 0) + \frac{1}{2}\mathcal{R}\dot{\phi}^2$$

tachyonic part strongly enhances modes with $k \lesssim \mu$

the high-k resonance bands are mostly controlled by the potential





$n = 3/2, 2$

very similar for $k > \mu$ and differ greatly for $k \lesssim \mu$

Preheating efficiency

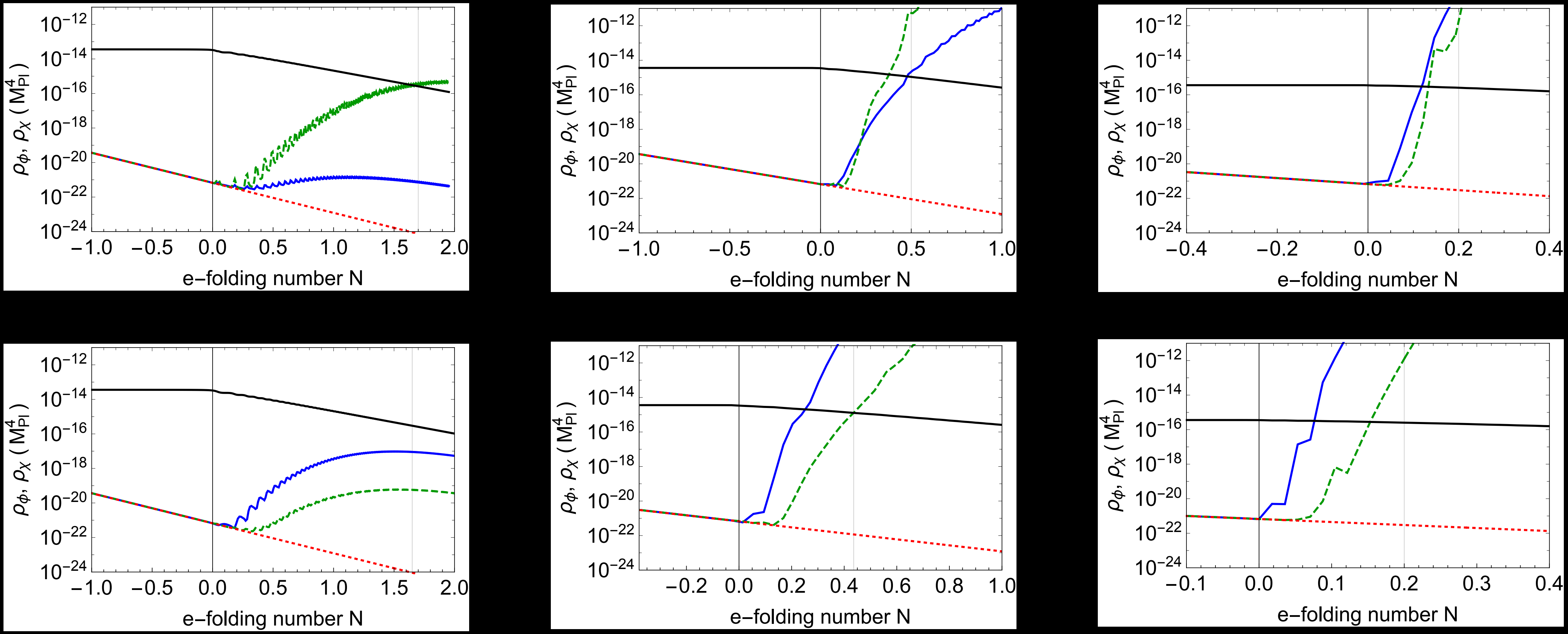
The E-model can preheat through ϕ resonance, when the T-model cannot.

$n = 1$

$\alpha = 10^{-3}$ $\alpha = 10^{-4}$ $\alpha = 10^{-5}$

E-model

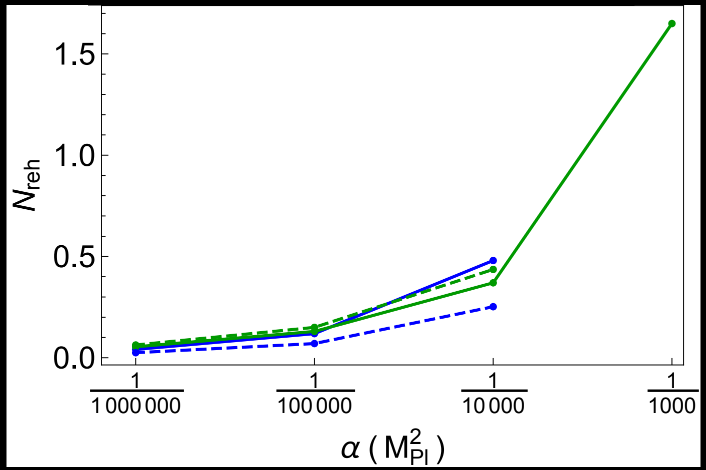
T-model



Energy density in ϕ and χ fluctuations (green-dashed and blue)

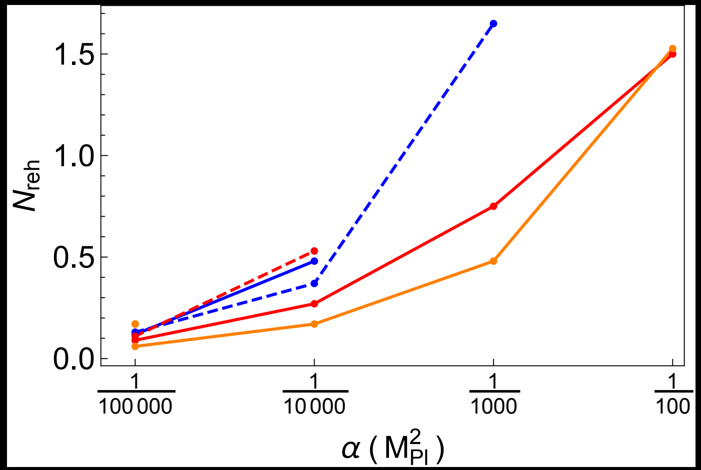
E-T preheating efficiency

ϕ (green) and χ (blue) fluctuations



T-model (dashed), E-model (solid) $n = 1$

ϕ (dashed) and χ (solid) modes



$n = 1, 3/2, 6$ (blue, red, orange)

Preheating for massive fields $n = 1$

- The T-model: tachyonic resonance of the χ field for $\alpha \lesssim 10^{-4}$
- The E-model: self-resonance of the ϕ field for $\alpha \approx 10^{-3}$, while the T-model does not preheat!

For massless fields and steeper potentials $n > 1$

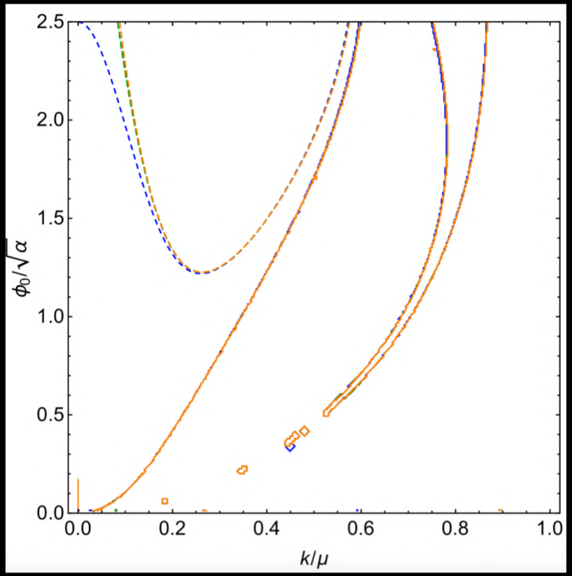
- tachyonic resonance of a spectator χ field, starting at $\alpha \approx 10^{-3}$

For alphas $\alpha \lesssim 10^{-4}$ preheating is practically instantaneous for any n .

The sum up of the scaling results for hyperbolic manifolds

For $\boxed{\alpha \ll 1}$:

- The amplification factor during each tachyonic regime is approximately the same
- There are $\mathcal{O}\left(1/\sqrt{\alpha}\right)$ oscillations per Hubble time
- We can ignore the slow red-shifting of the background



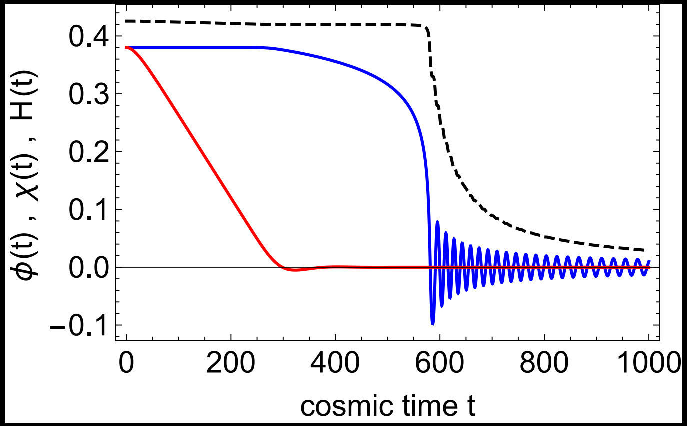
↓

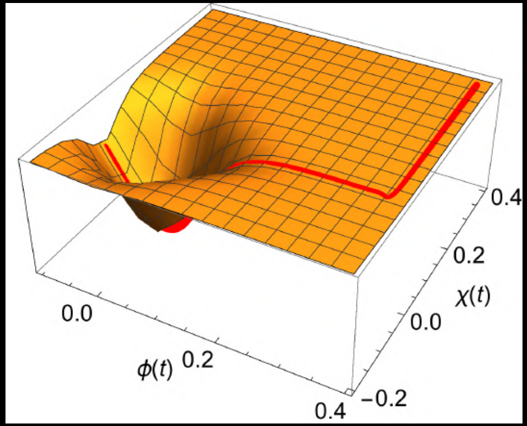
The amplification per Hubble time grows as $\mathcal{O}\left(1/\sqrt{\alpha}\right)$

The Floquet chart is "universal" and can be scaled between different values of alpha.

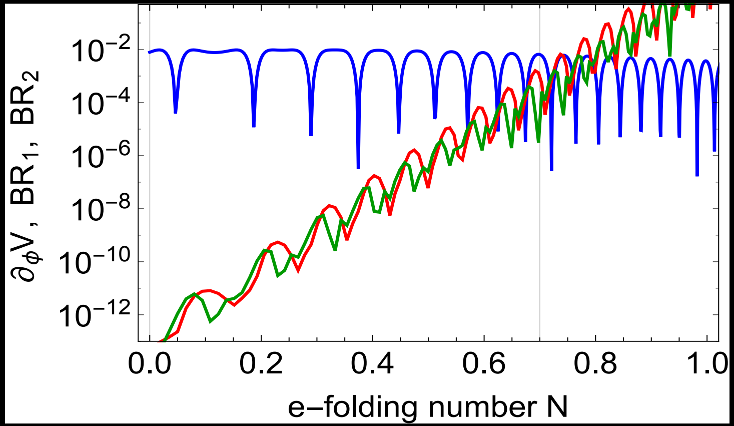
Outlook

- Inflation along spectator direction and turning around horizon crossing can have observational consequences





- Non-linear effects and backreaction?



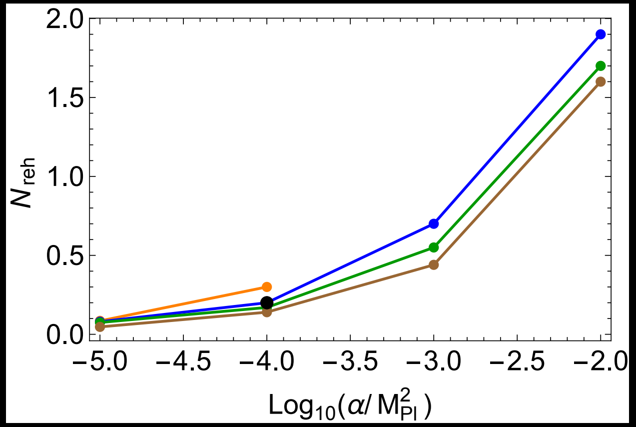
$|V_{,\phi}|$ (blue)

$BR_1 \equiv |\beta^2 \langle \chi \dot{\chi} \rangle \dot{\phi}|$ (green)

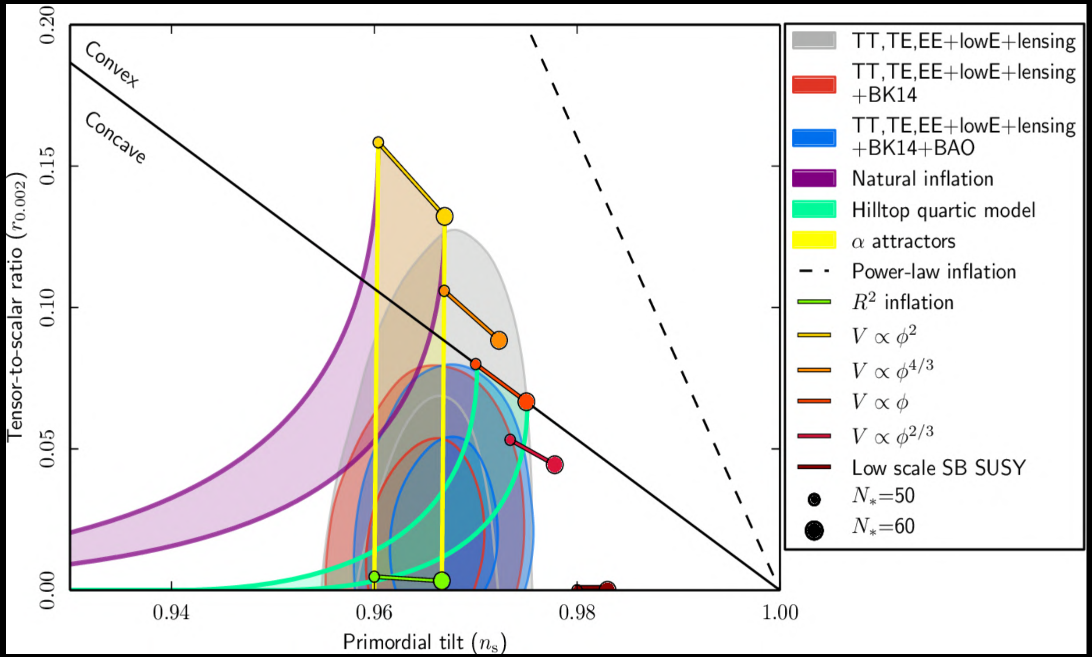
$BR_2 \equiv \Delta V \langle \chi^2 \rangle |V_{,\phi}|$ (red)

$\tilde{\alpha} = 0.001$ and $n = 3/2$

Multi-field preheating reduces theoretical uncertainties



Single-field simulations are unable to capture the most important time-scales, which control the tachyonic growth of the spectator field.



Effective field theory of preheating leads to reducing of error bars of the $n_s - r$ plot.

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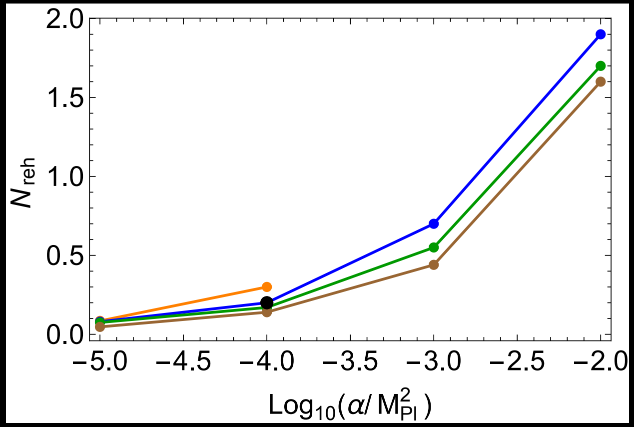

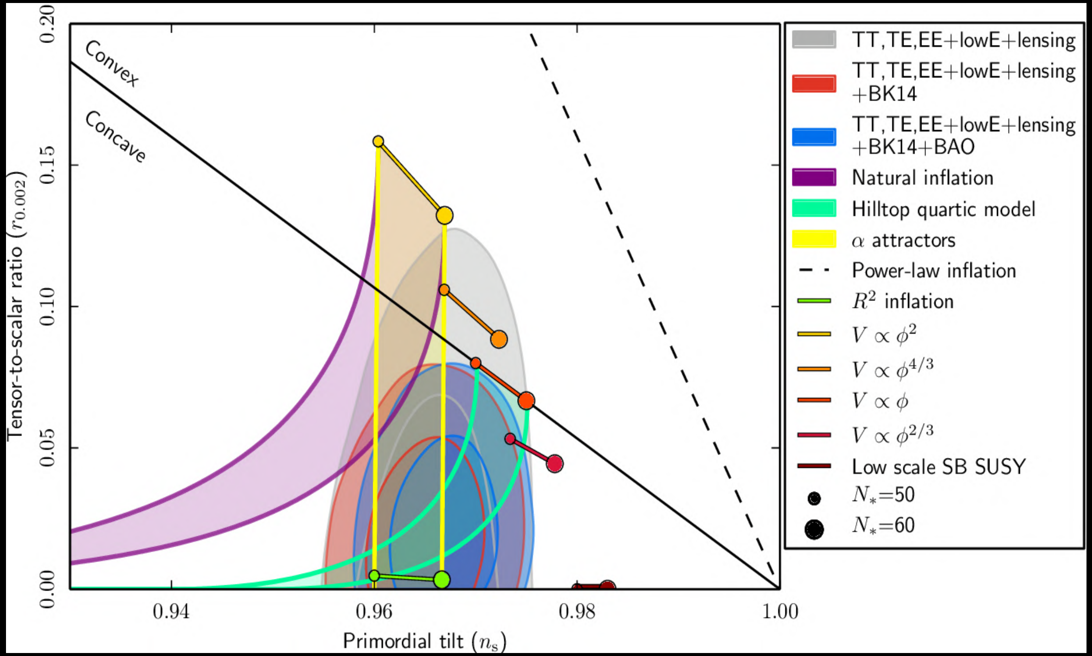

Thank you!

Effective field theory of preheating leads to reducing of error bars of the $n_s - r$ plot.