Model-independent energy budget of gravitational waves from a cosmological first-order phase transition

Jorinde van de Vis

APEC Seminar
18/11/20

F. Giese, T. Konstandin, JvdV, **2004.06995**
JCAP 07 (2020) 07, 057

F. Giese, T. Konstandin, K. Schmitz, JvdV, **2010.09744**

cosmological first-order phase transition

- Sign of physics beyond the standard model
- Generation of the baryon asymmetry?

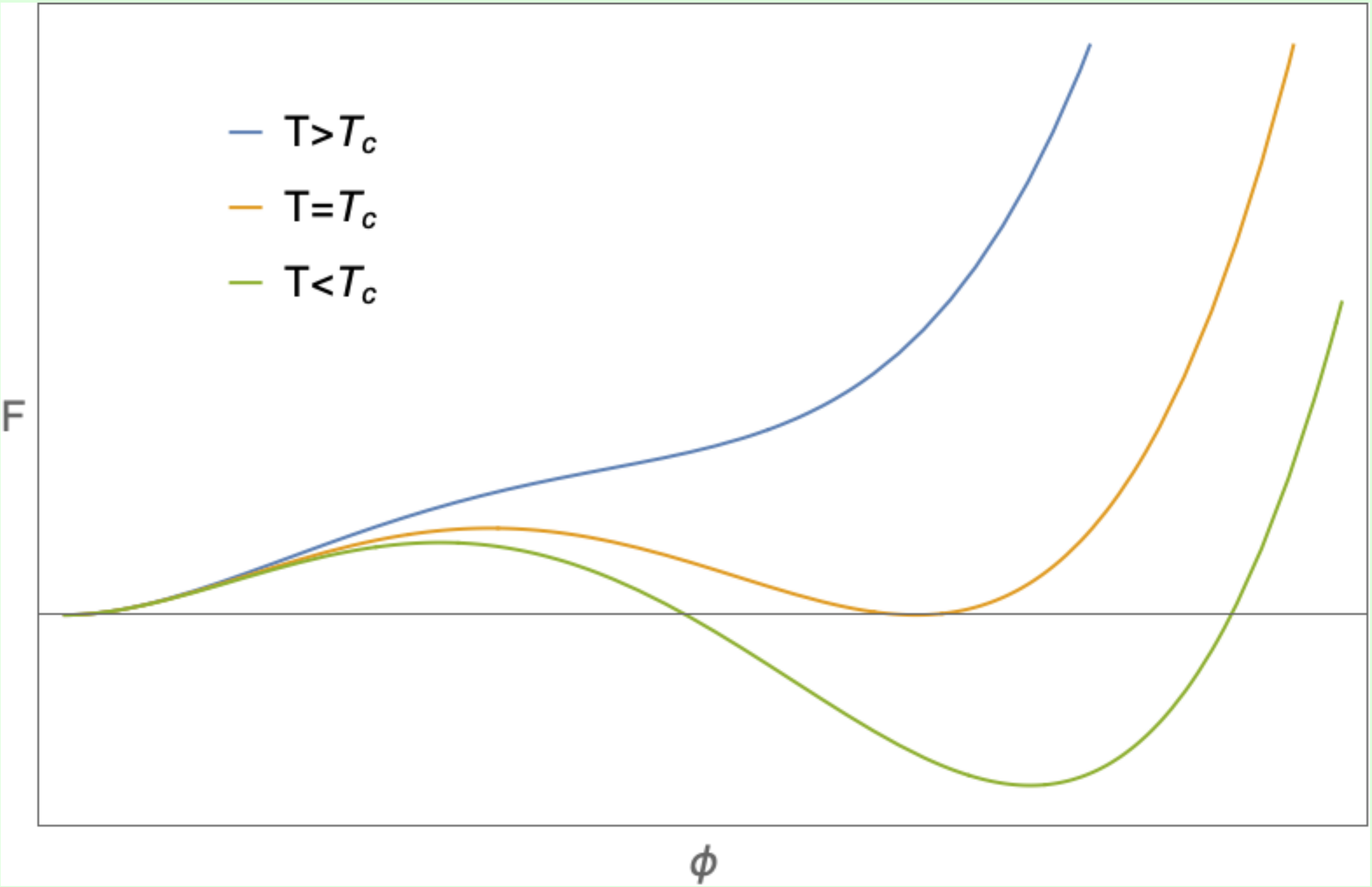
cosmological first-order phase transition

- Generation of the baryon asymmetry?

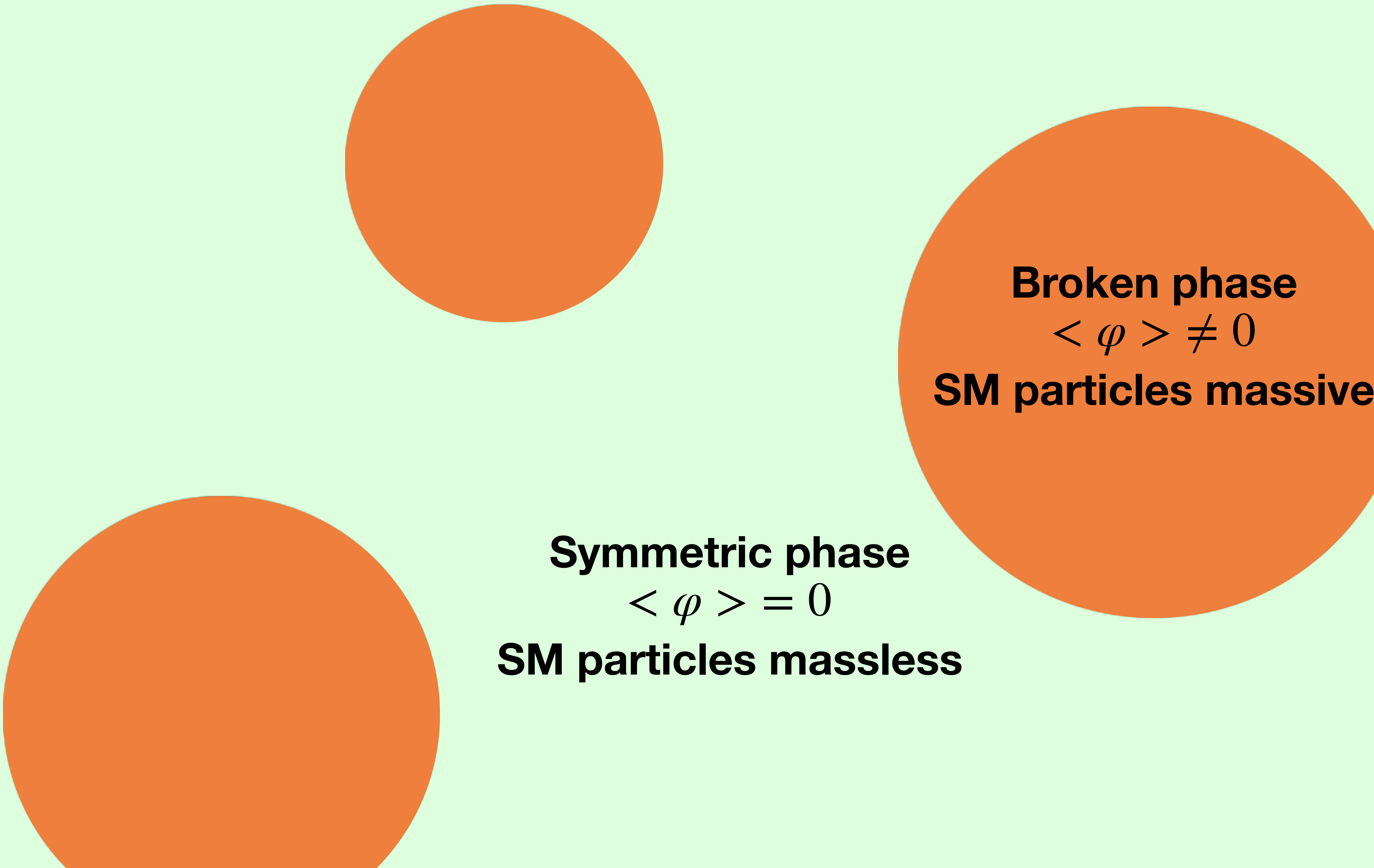
  Sakharov conditions

  - Violation of baryon number
  - C and CP violation
  - Out-of-equilibrium dynamics

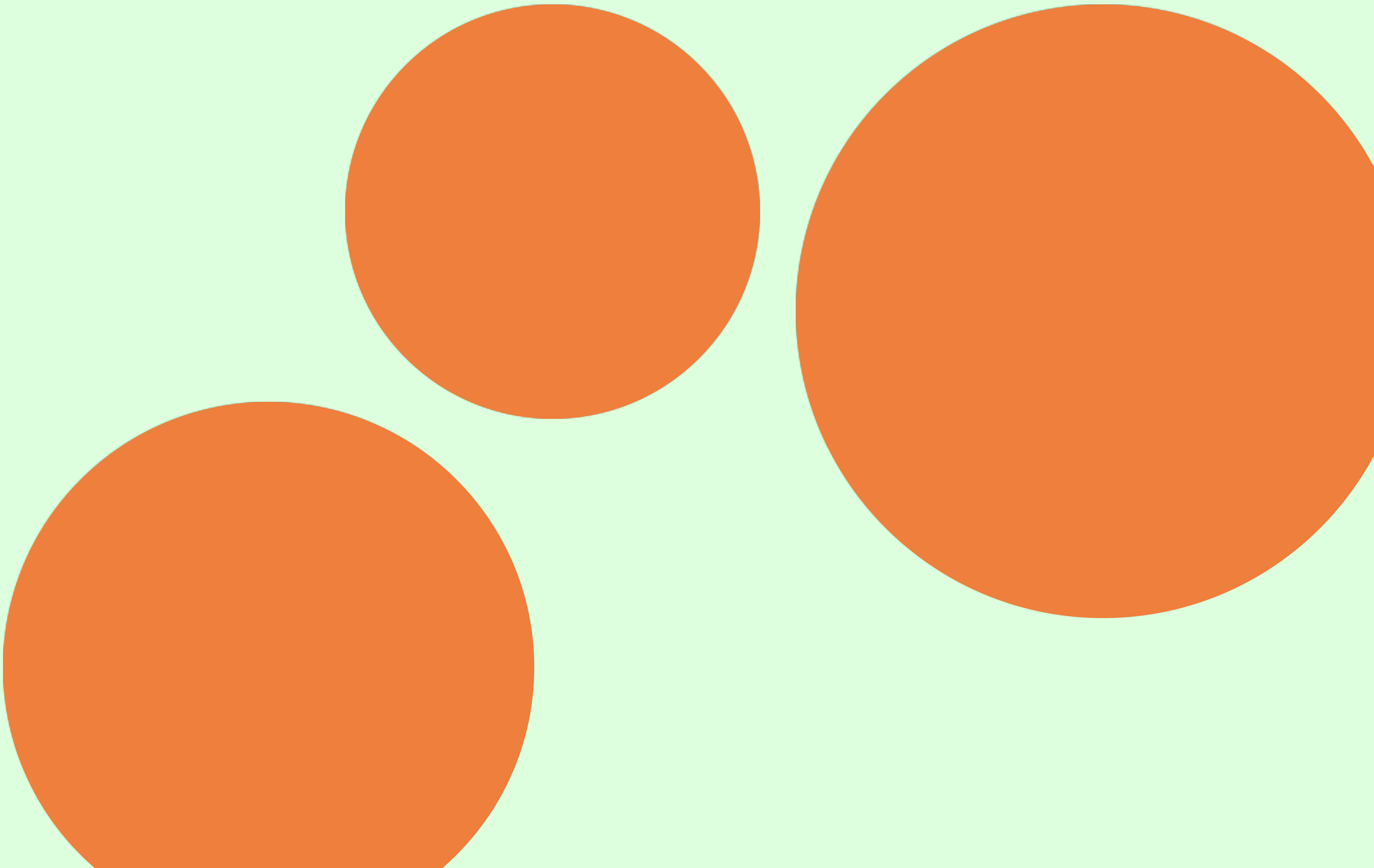
T-dependence of the free energy



Bubbles nucleate…



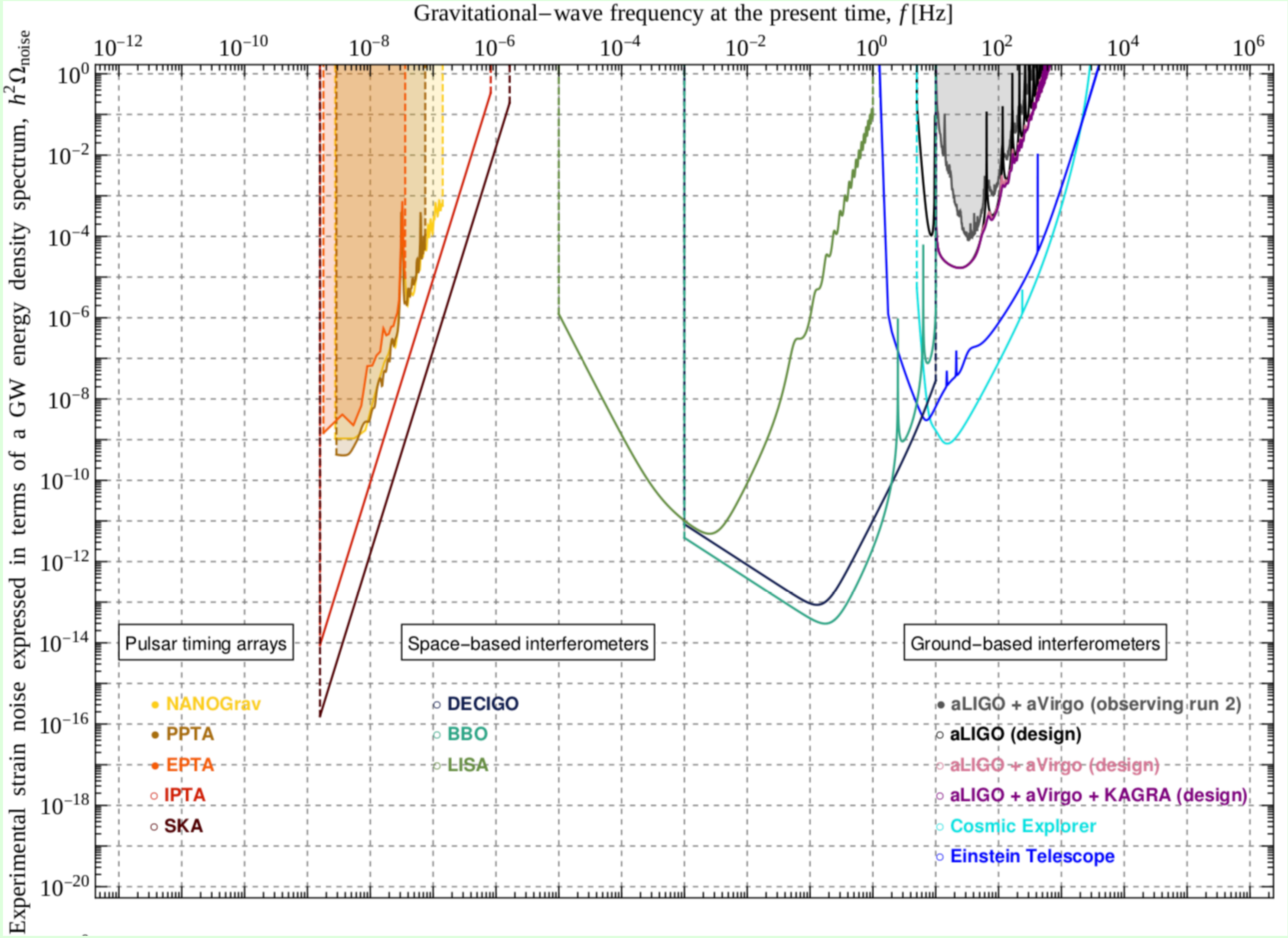
...expand...



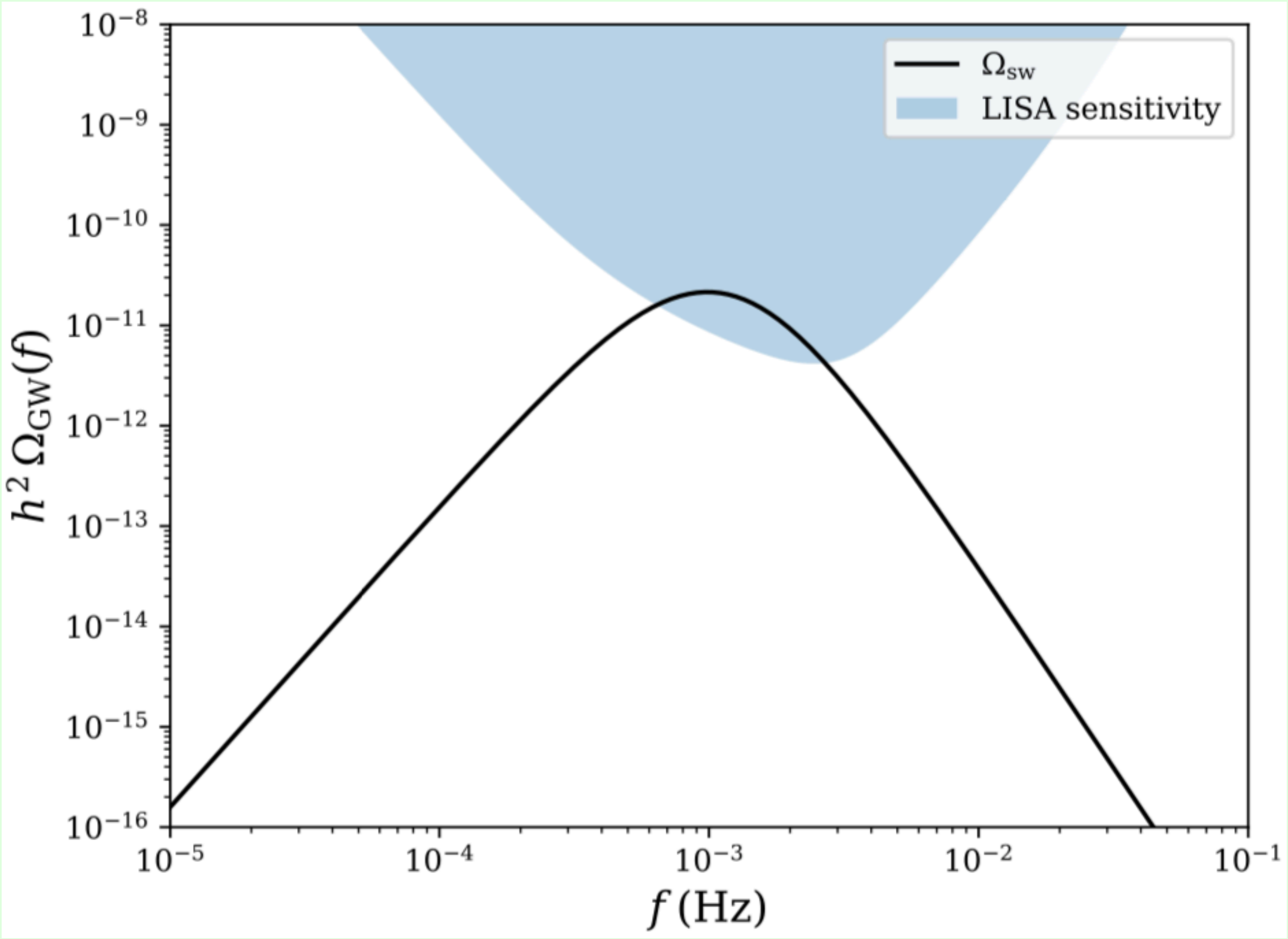
…and collide

Model-independent energy budget of gravitational waves from a cosmological first-order phase transition gravitational waves

- Collision of bubbles can lead to observable gravitational signal

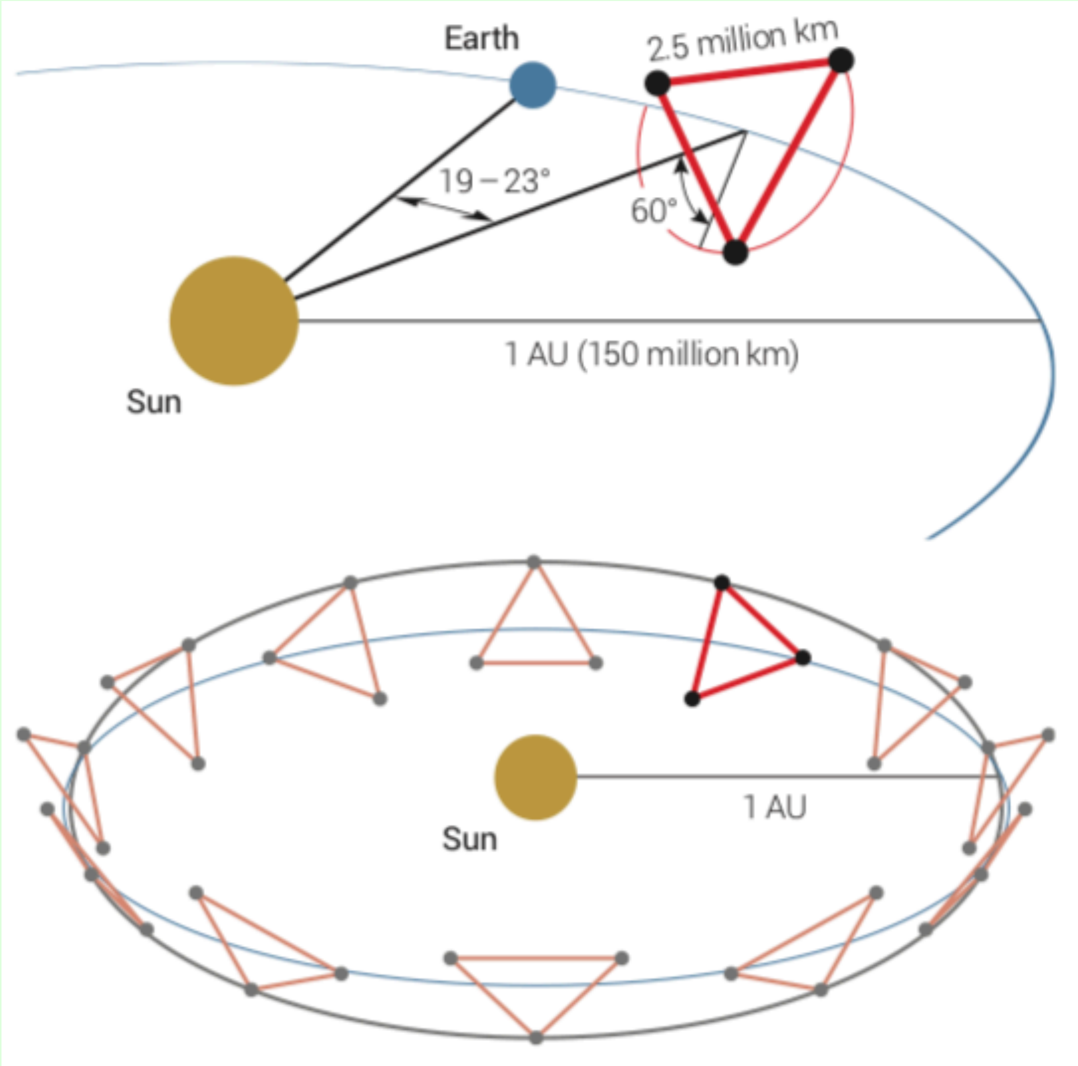


K. Schmitz 2020



LISA Cosmology Working Group 2019

Laser Interferometer Space Antenna (LISA)



LISA Collaboration 2017

- Planned for launch in 2034

3 sources of gravitational waves

- Scalar field contribution
- Sound waves
- Turbulence

Collider signatures of models with a first order phase transition

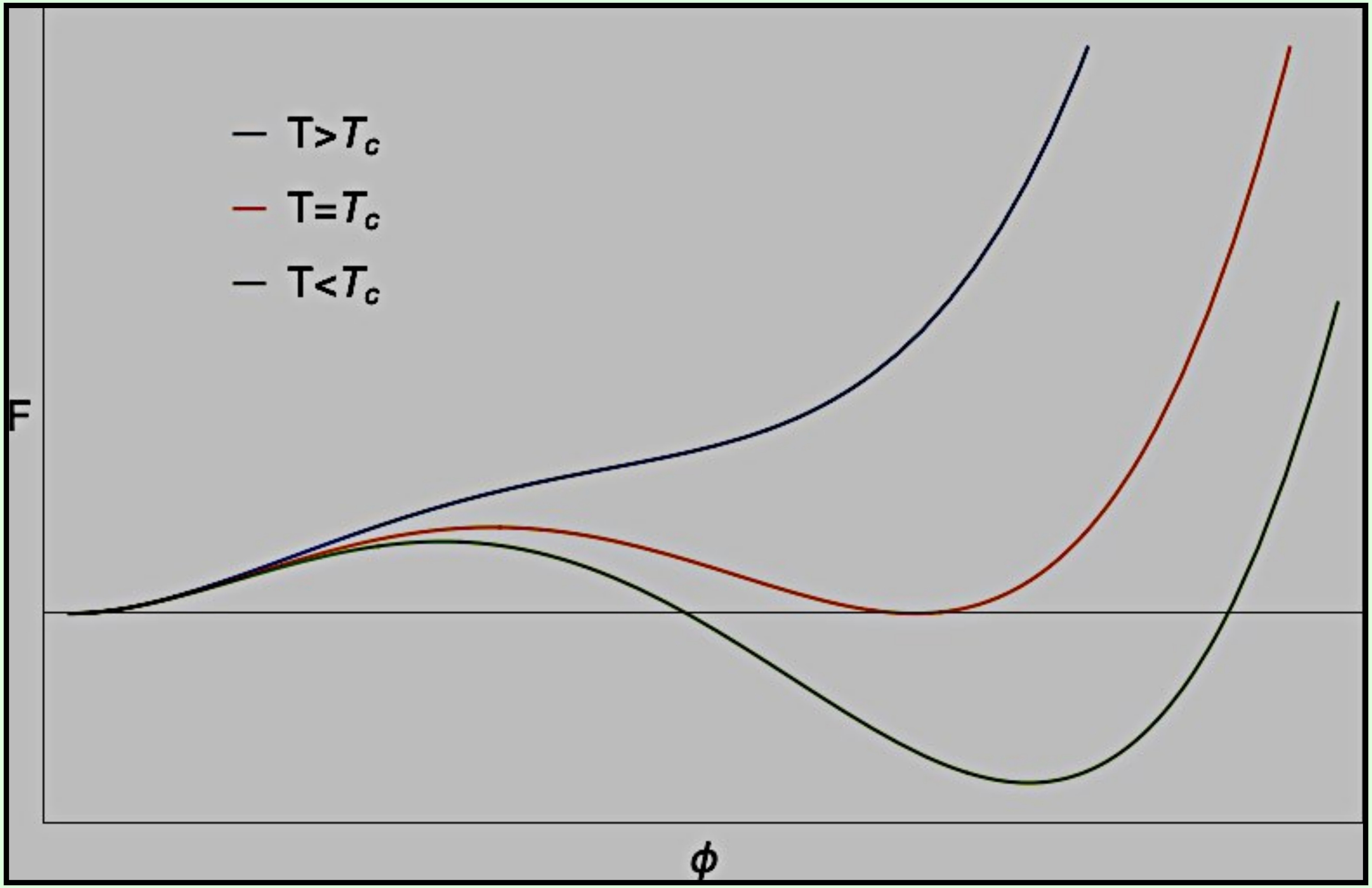
- Exotic Higgs decays
- Mixing (e.g. Higgs-singlet mixing)
- Modification of triple Higgs and Higgs-Z interaction

Nightmare scenarios

$$m > \frac{m_H}{2}, \quad Z_2 -$$ **symmetry**

- Exotic Higgs decays
- Mixing (e.g. Higgs-singlet mixing)
- Modification of triple Higgs and Higgs-Z interaction

Dark phase transition



gravitational waves

- Collision of bubbles can lead to observable gravitational signal
- Complementary to collider search

Model-independent energy budget of gravitational waves from a cosmological first-order phase transition energy budget

How to predict gravitational wave signal in sound waves?

- $\Omega_{tot} = \min\{1, H_* \tau_{\rm sh}\} \, 3F\tilde{\Omega} R_* H_* K^2$

M. Hindmarsh, S. Huber, K. Rummukainen, D.Weir 2017

- F : Redshift
- H_* : Hubble parameter at percolation
- R_* : mean bubble separation

} Particle physics at finite T
Bubble wall velocity

How to predict gravitational wave signal in sound waves?

M. Hindmarsh, S. Huber,
K. Rummukainen, D.Weir 2017

- $\Omega_{tot} = \min\{1, H_* \tau_{\rm sh}\} 3F\tilde{\Omega} R_* H_* K^2$
- $\tilde{\Omega}$: Numerical factor ~ 0.01
- $\tau_{\rm sh}$: Onset of shock formation ~ $\sqrt{K/\Gamma}$
- K : Kinetic energy fraction

Kinetic energy fraction

$$K = \rho_{fl}/e_n$$

- Determined by hydrodynamics of single expanding bubble
- Depends on the phase transition strength and speed of sound in the plasma
- Depends on bubble wall velocity - which is treated as external parameter

Goal

Determine K as a function of the phase transition strength, the speed of sound and the wall speed, without further model-dependence

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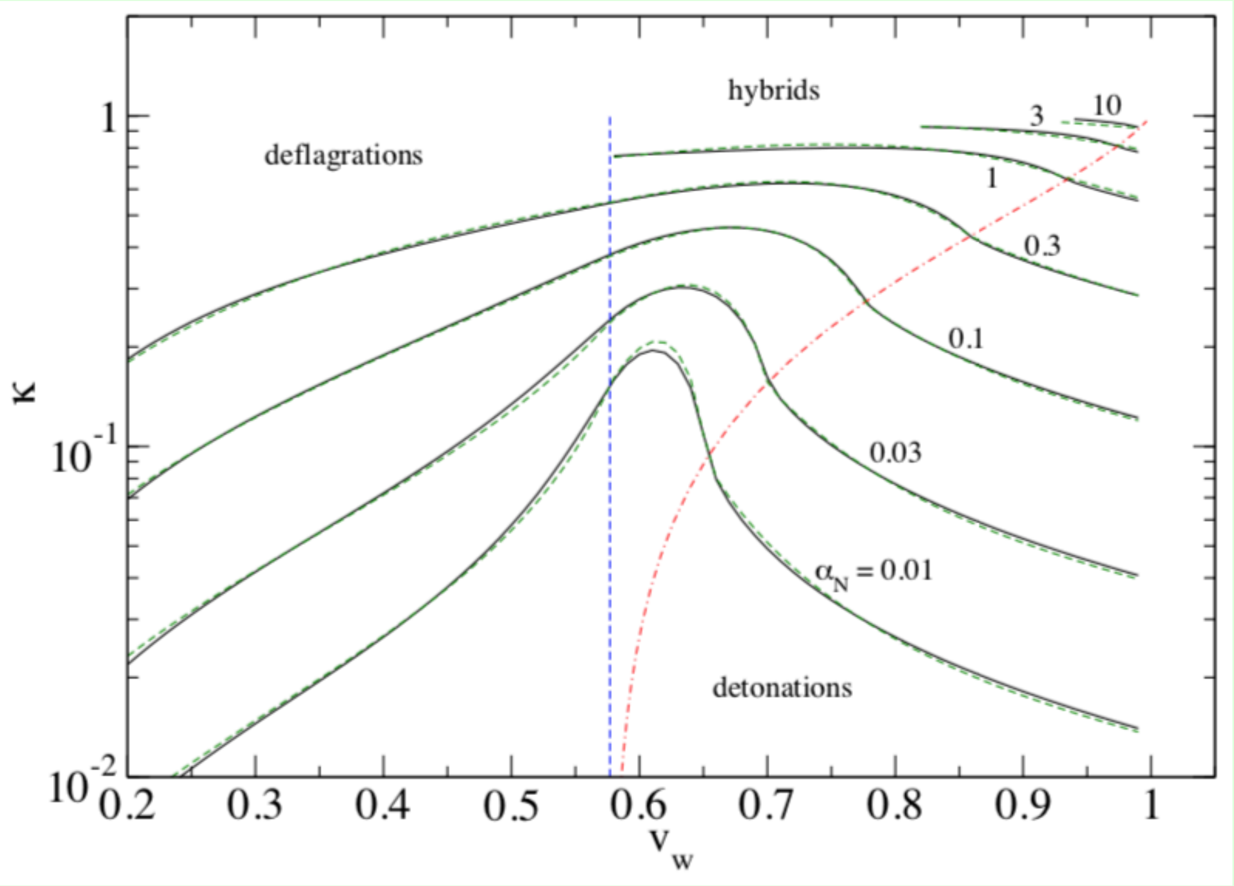
Determine K as a function of the phase transition strength, the speed of sound and the wall speed, without further model-dependence

Result should be reusable without solving the hydrodynamics

Common approach

- Compute phase transition strength α_n and use fit for bag equation of state from **J. Espinosa, T. Konstandin, J. No, G. Servant, 2010**

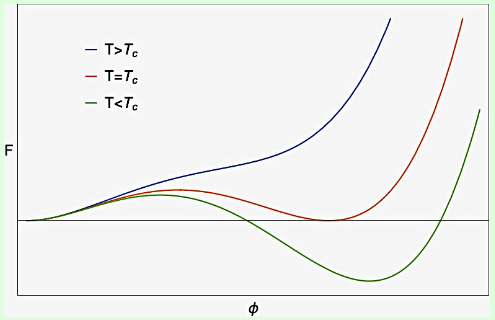
$$\kappa_\epsilon = \frac{\rho_{fl}}{\epsilon}$$



- But how to compute α_n in a different model? And where is the sound speed dependence?

Some hydrodynamics

Hydrodynamics of a single bubble



- Pressure $p = -F$
- Energy density $e = T\frac{\partial p}{\partial T} - p$
- Enthalpy $w = T\frac{\partial p}{\partial T} = e + p$
- Energy-momentum of the fluid $T^{\mu\nu} = u^\mu u^\nu w + \eta^{\mu\nu} p$ (fluid velocity: u^ν)

Hydrodynamics of a single bubble

- Continuity equations $\partial_\mu T^{\mu\nu} = u^\nu \partial_\mu (u^\mu w) + u^\mu w \partial_\mu u^\nu - \partial^\nu p = 0$

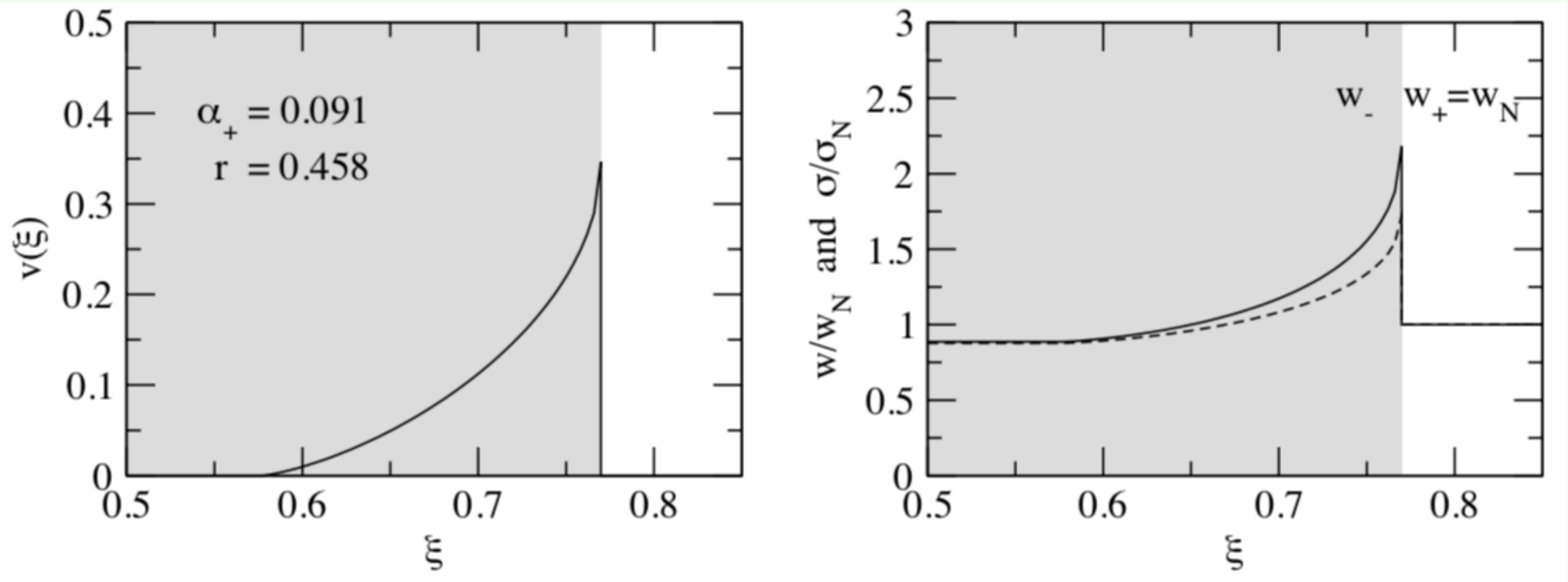
- Project along the flow and perpendicular to the flow and introduce combination $\xi = r/t$ ← time since nucleation

  distance from bubble center ↗

- ξ : velocity of point in wave profile ($\xi_w = R/t$: wall velocity)

- $v(\xi)$: fluid velocity at the point ξ

Velocity and enthalpy profiles



J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

Hydrodynamics of a single bubble

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- Project along the flow and perpendicular to the flow and introduce combination $\xi = r/t$ ⟵ time since nucleation

  distance from bubble center ↗

- $$2\frac{v}{\xi} = \gamma^2 (1 - v\xi) \left[\frac{\mu^2}{c_s^2} - 1 \right] \partial_\xi v$$

- $$\frac{\partial_v w}{w} = \left(\frac{1}{c_s^2} + 1 \right) \gamma^2 \mu$$

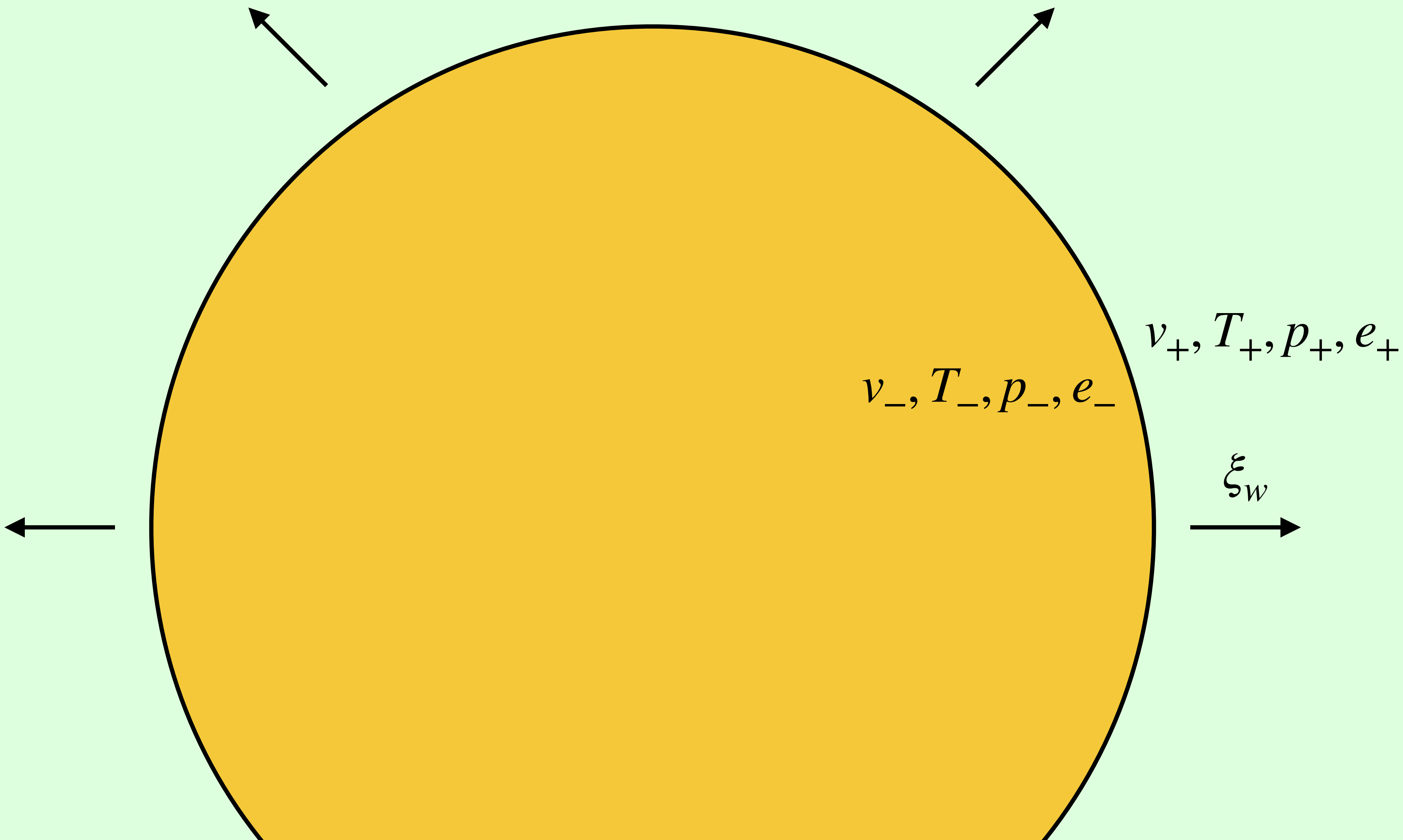
$$\mu(\xi, v) = \frac{\xi - v}{1 - \xi v}$$

$$c_s^2 = \frac{dp/dT}{de/dT}$$

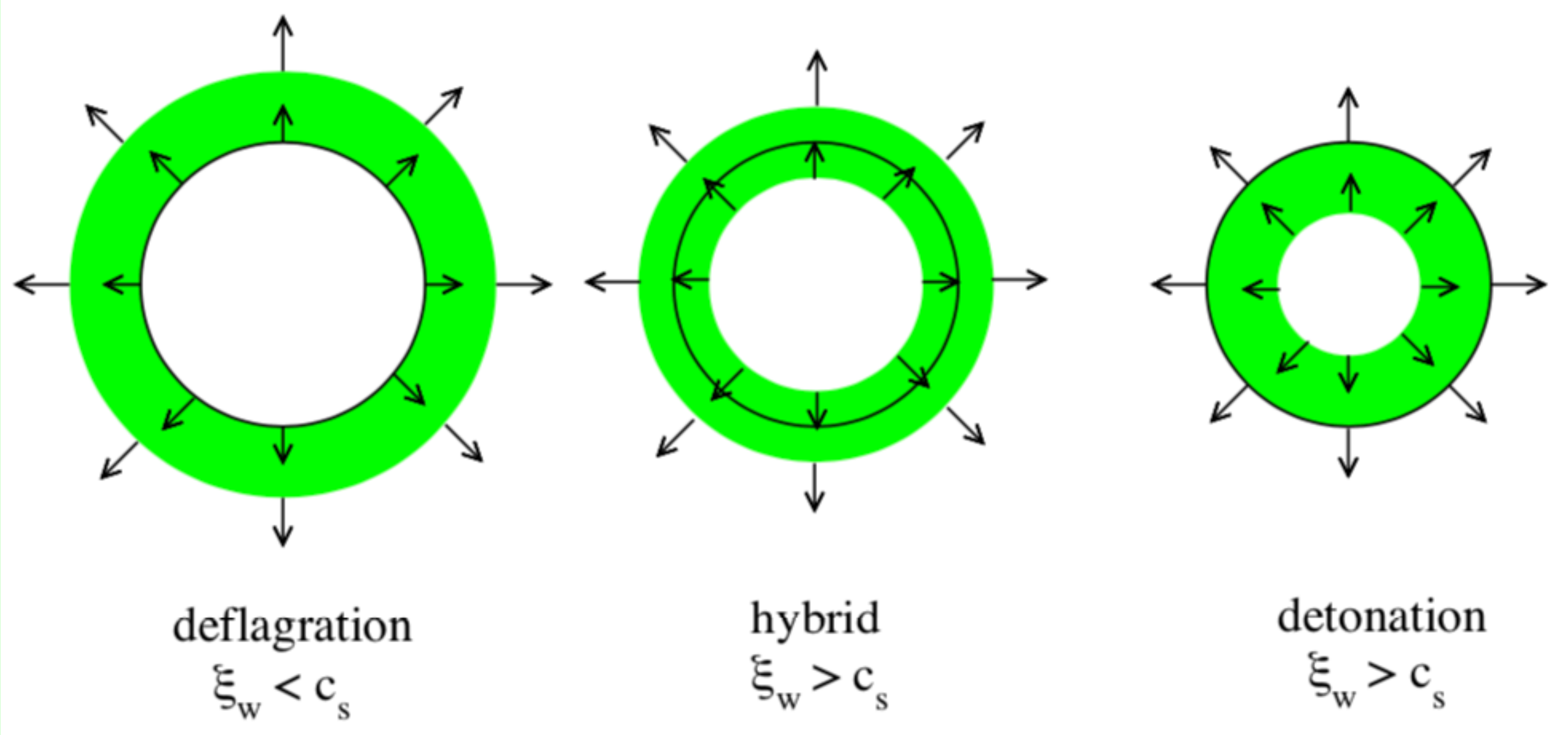
Hydrodynamics of a single bubble

- Matching equations in wall frame:

$$v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-}, \qquad \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-}$$

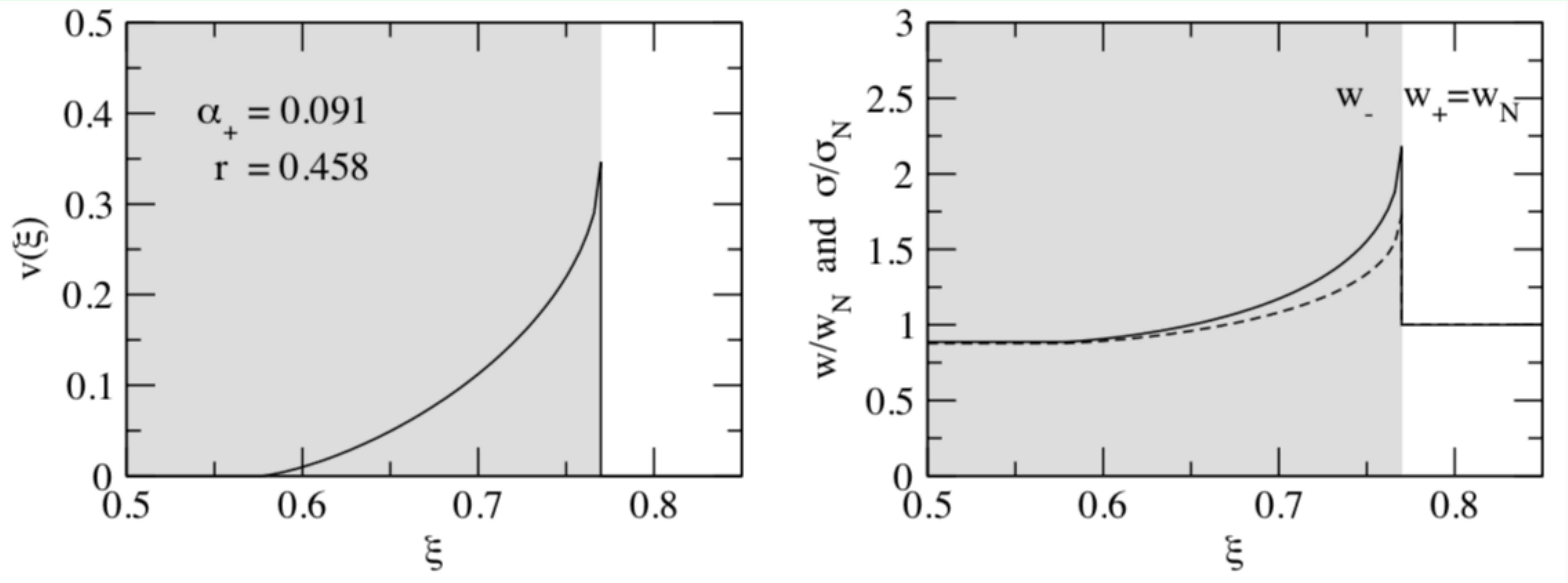


Three types of solutions



J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

Velocity and enthalpy profiles: detonation



J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

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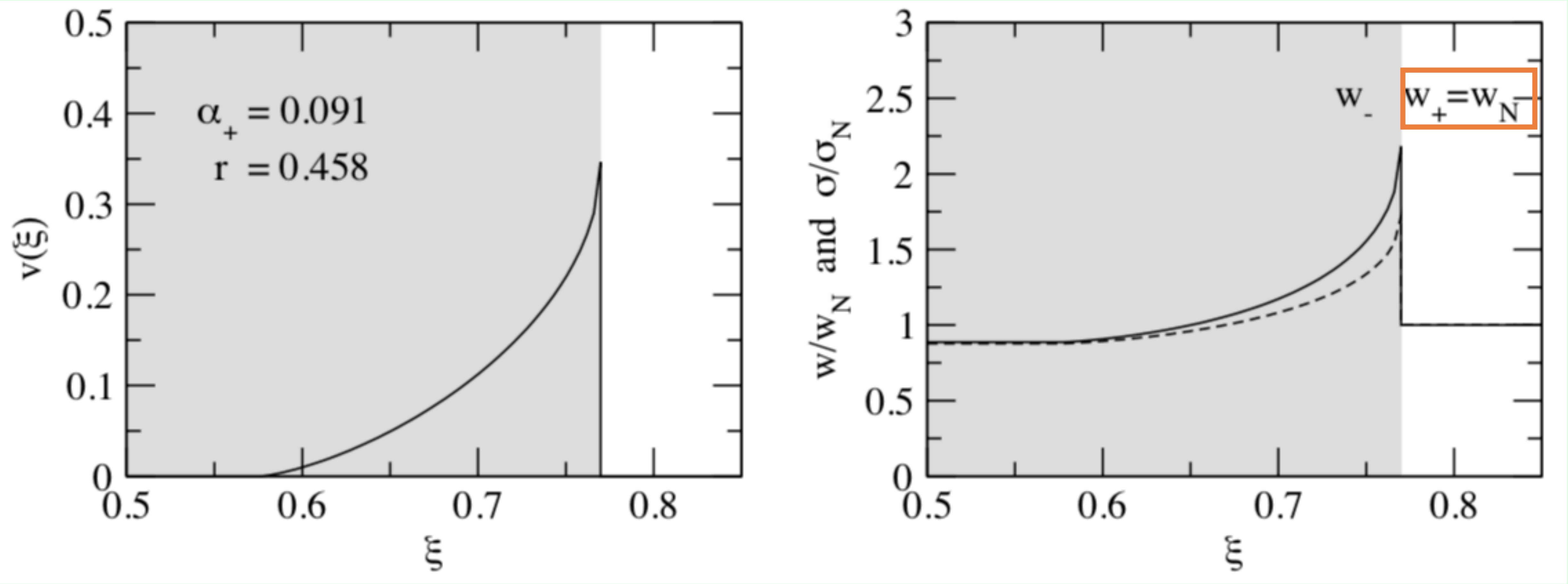
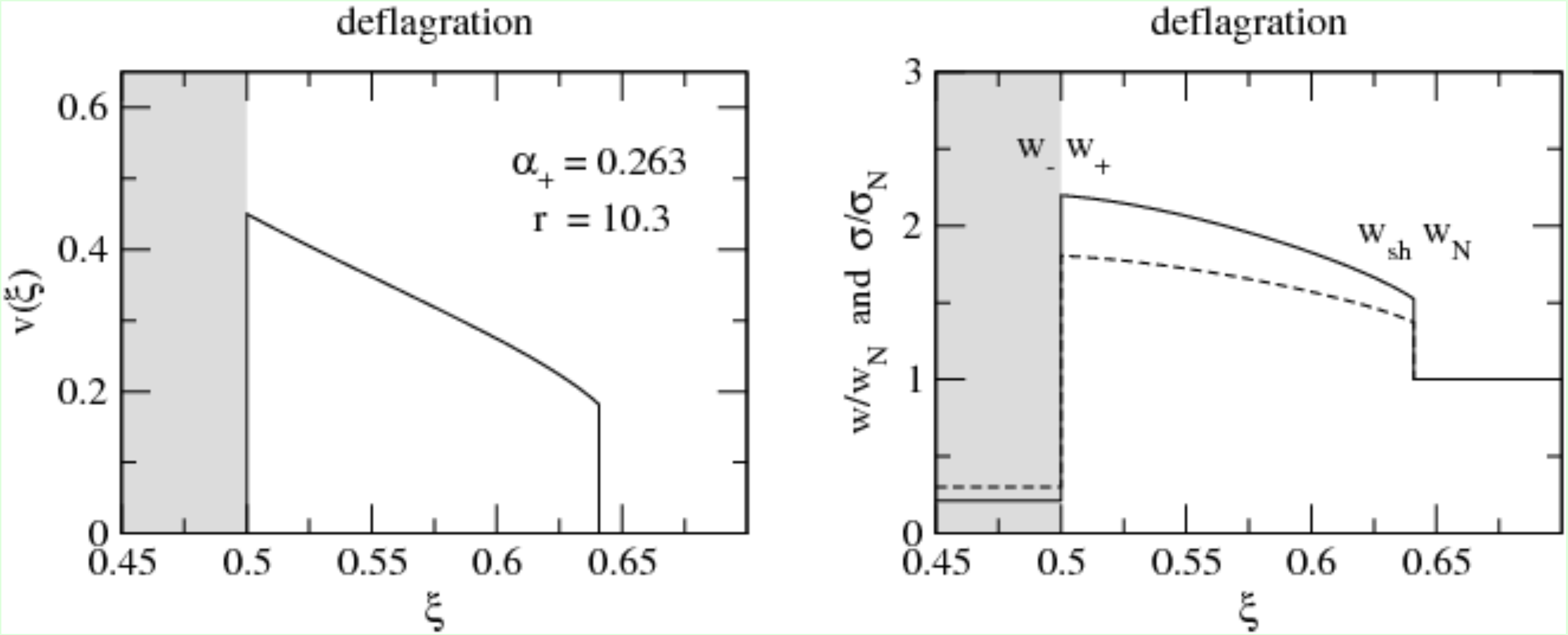


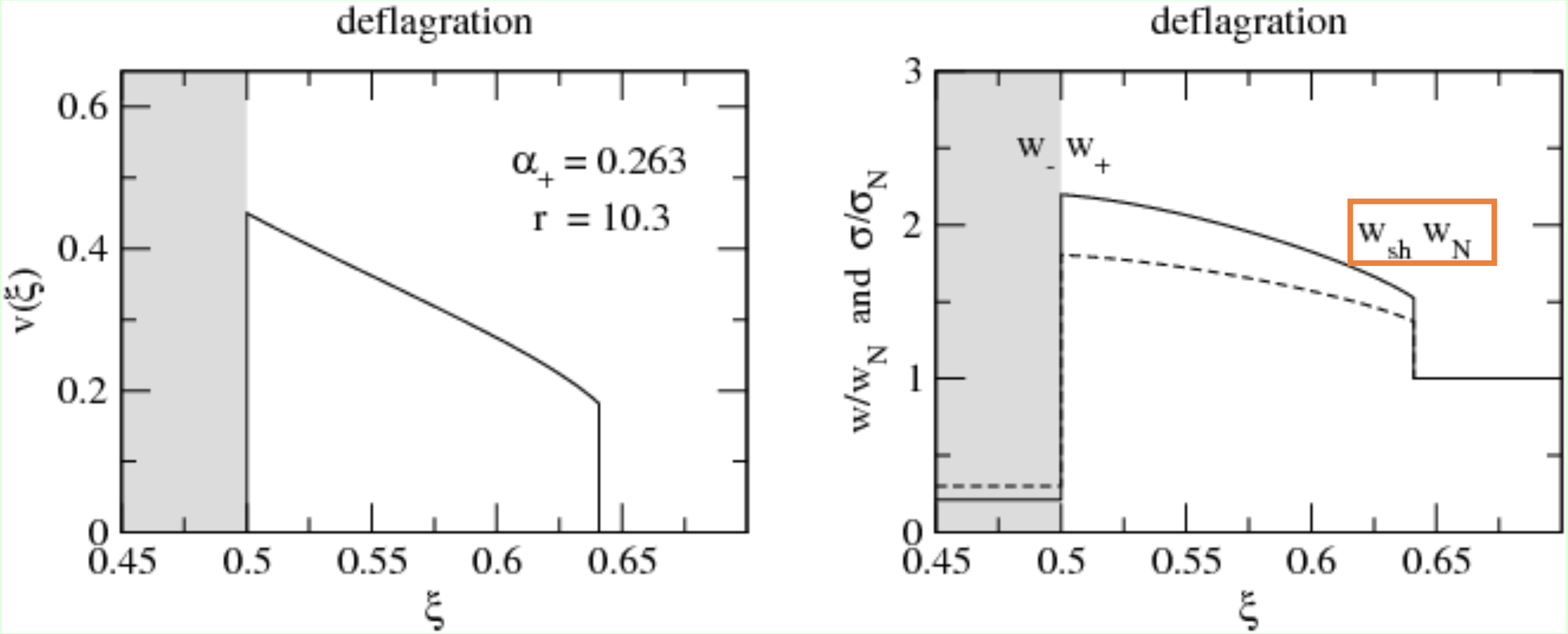
J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

Velocity and enthalpy profiles: deflagration



J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

Velocity and enthalpy profiles: deflagration



J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

Hydrodynamics of a single bubble

- Kinetic energy in the fluid $$\rho_{fl} = \frac{3}{\xi_w^3} \int d\xi \, \xi^2 \, v^2 \, \gamma^2 w$$

- Integration of T^{00} over large volume: $$\rho_{fl} = \frac{3}{\xi_w^3} \int d\xi \xi^2 (e_n - e)$$

- $K = \dfrac{\rho_{fl}}{e_n}$ fraction of energy that is converted into fluid motion ($0 < K < 1$)

The bag equation of state

Bag equation of state

- $p_s = \frac{1}{3} a_+ T^4 - \epsilon$ $\quad e_s = a_+ T^4 + \epsilon$

  $p_b = \frac{1}{3} a_- T^4$ $\quad e_b = a_- T^4$

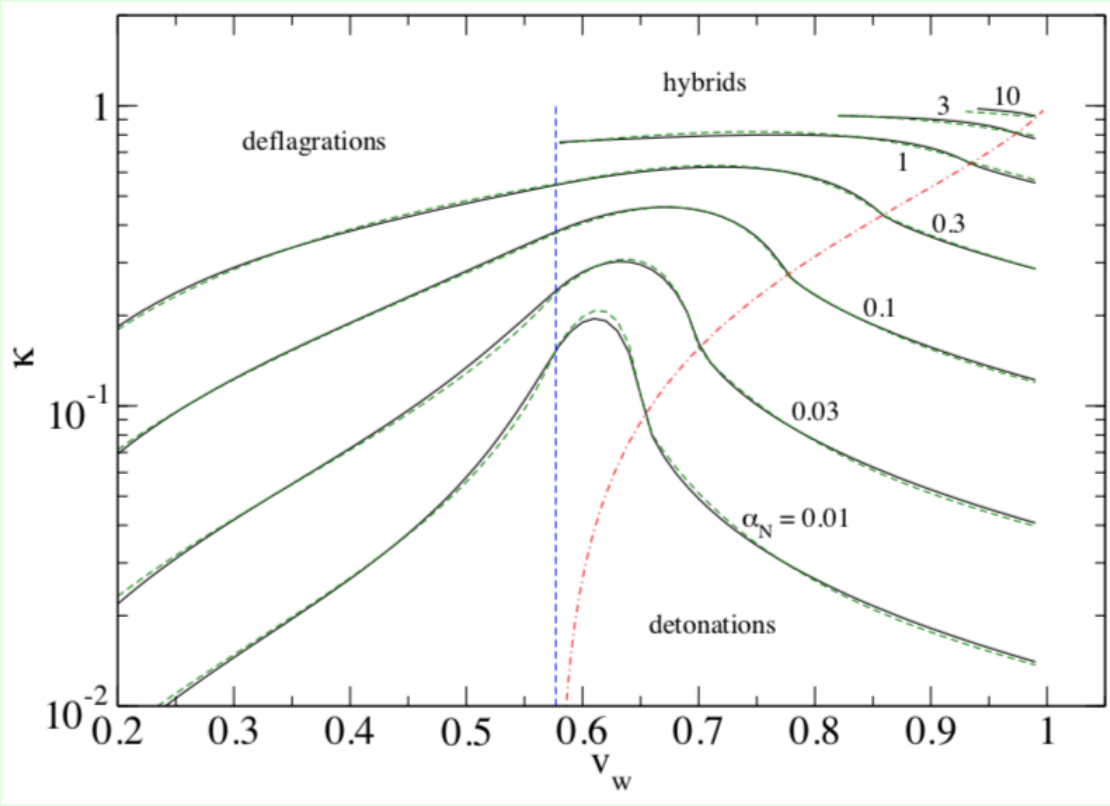
- Bag constant ϵ independent of temperature

- Phase transition strength $\alpha_\epsilon = \frac{4\epsilon}{3w_n}$

- Speed of sound $c_s^2 = \frac{dp/dT}{de/dT} = \frac{1}{3}$

- $K = \frac{\alpha_\epsilon \kappa_\epsilon}{\alpha_\epsilon + 1}$ completely determined by α_ϵ and ξ_w

$\kappa_\epsilon = \frac{\rho_{fl}}{\epsilon}$ ↘



J. Espinosa, T. Konstandin, J. No, G. Servant, 2010

How to use this result for another model?

- Only T_n is known

- $\alpha_p = -\dfrac{4Dp}{3w_s(T_n)}$

$DX(T_n) \equiv X_s(T_n) - X_b(T_n)$

- $\alpha_e = \dfrac{4De}{3w_s(T_n)}$

- $\alpha_\theta = \dfrac{D\theta}{3w_s(T_n)}$

$\theta \equiv e - 3p$

How to use this result for another model?

- Only T_n is known
- $\alpha_p = -\dfrac{4Dp}{3w_s(T_n)}$ $\qquad DX(T_n) \equiv X_s(T_n) - X_b(T_n)$
- $\alpha_e = \dfrac{4De}{3w_s(T_n)}$
- $\alpha_\theta = \dfrac{D\theta}{3w_s(T_n)}$ $\qquad \theta \equiv e - 3p$ ⟵ reduces to α_ϵ
- The speed of sound never enters 🤔

Model-independent hydrodynamics

Model-dependent parts

- Hydrodynamic equations
  - $$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v, \qquad \frac{\partial_v w}{w} = \left(\frac{1}{c_s^2} + 1\right)\gamma^2\mu$$
- Boundary conditions
  - $w(T_n) = w_n$
  - $$\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, \qquad v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)}$$

Capture the model dependence in small number of parameters

- Assume: $c_s^2(T) \sim c_s^2(T_n)$
- Solve for w/w_n instead of w -> obtain $\rho_{\rm fl}/w_n$

Velocity matching

- $$\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, \qquad v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)} \equiv \frac{\Delta p}{\Delta e}$$

- $$\Delta p = \underbrace{p_s(T_+) - p_b(T_+)}_{Dp} + \underbrace{p_b(T_+) - p_b(T_-)}_{\delta p}$$

- We now assume that $T_+ \simeq T_-$ $\longrightarrow$ $\delta p / \delta e \simeq c_s^2$ in broken phase!

- $$\frac{v_+}{v_-} \simeq \frac{(v_+ v_- / c_s^2 - 1) + 3\alpha_{\bar{\theta}}}{(v_+ v_- / c_s^2 - 1) + 3 v_+ v_- \alpha_{\bar{\theta}}}$$, either v_+ or v_- is known

- $$\bar{\theta} \equiv e - \frac{p}{c_s^2} \qquad \alpha_{\bar{\theta}} \equiv \frac{D\bar{\theta}}{3 w_n}$$

Capture the model dependence in small number of parameters

- Model dependent parameters: $\alpha_{\bar{\theta}}, c_{s,\text{broken}}, c_{s,\text{symm}}$ + wall velocity ξ_w

- Can determine $\kappa_{\bar{\theta}} = \dfrac{4\rho_{fl}}{D\bar{\theta}} = \dfrac{4\rho_{\text{fl}}}{3\alpha_{\bar{\theta}} w_n}$

- $K = \dfrac{D\bar{\theta}}{4e_n} \kappa_{\bar{\theta}}$ can now easily be obtained (but depends on the model)

Capture the model dependence in small number of parameters

- Model dependent parameters: $\alpha_{\bar{\theta}}, c_{s,\mathrm{broken}}, c_{s,\mathrm{symm}}$ + wall velocity ξ_w

- Can determine $\kappa_{\bar{\theta}} = \dfrac{4\rho_{fl}}{D\bar{\theta}} = \dfrac{4\rho_{\mathrm{fl}}}{3\alpha_{\bar{\theta}} w_n}$

- $K = \dfrac{D\bar{\theta}}{4e_n}\kappa_{\bar{\theta}}$ can now easily be obtained (but depends on the model)

- Tricky detail: $\alpha_{\bar{\theta}}$ is defined at T_n, which is not T_+ for hybrids and deflagrations -> need shooting algorithm.

Quick recap

- We want to find a model-independent expression/fit for $K = \dfrac{\rho_{fl}}{e_n}$
- We introduced $\bar{\theta} \equiv e - \dfrac{p}{c_s^2}$ $\qquad \alpha_{\bar{\theta}} \equiv \dfrac{D\bar{\theta}}{3w_n}$
- The velocity profile and $\kappa_{\bar{\theta}}$ only depend on $\alpha_{\bar{\theta}}$, $c_{s,\text{broken}}$, $c_{s,\text{symm}}$ and ξ_w
- K can easily be determined from $\kappa_{\bar{\theta}}$

Make the result reusable

Template model with varying speed of sound

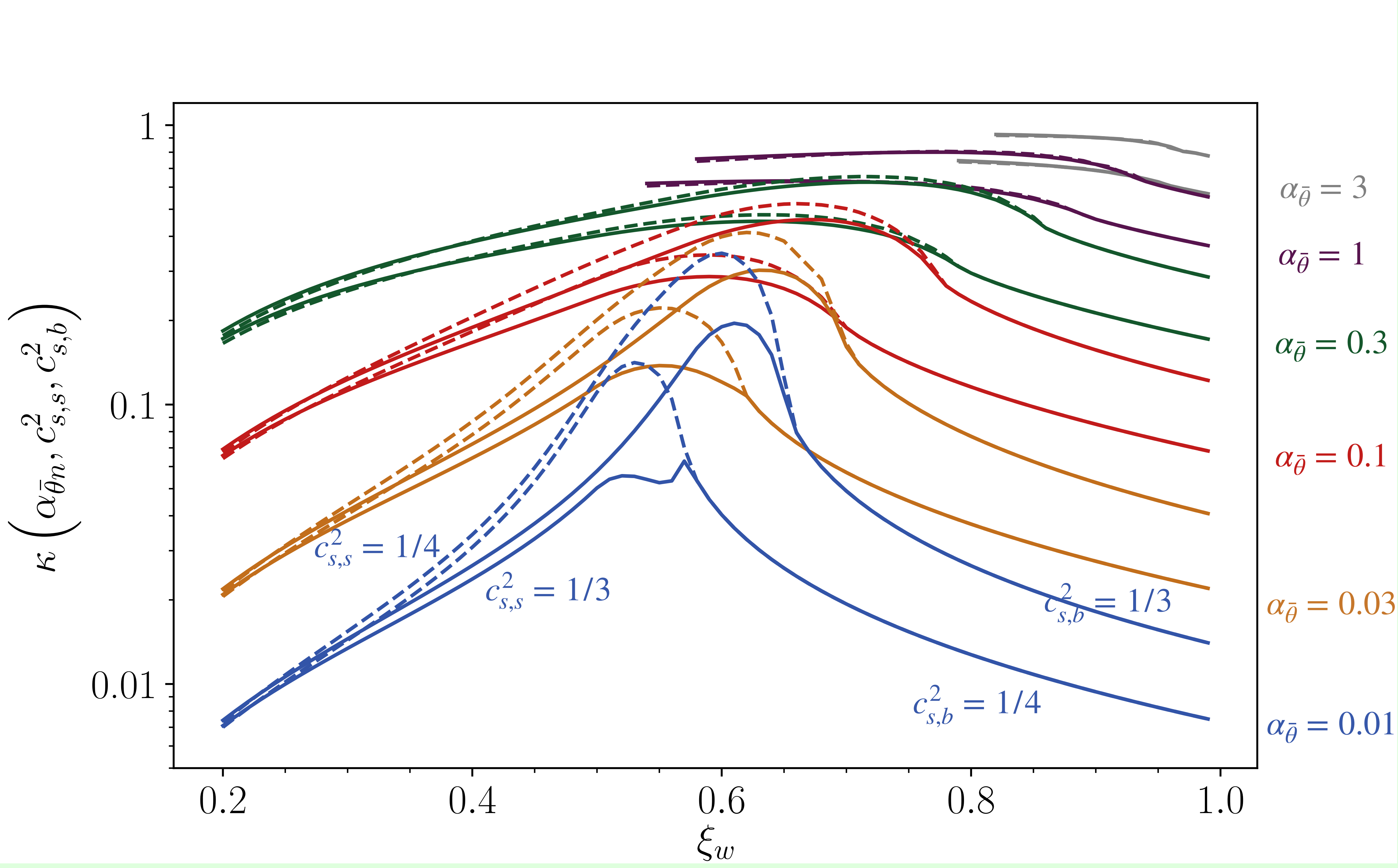
L. Leitao,
A. Megevand, 2015

- $$p_s = \frac{1}{3} a_+ T^\mu - \epsilon \qquad e_s = a_+ T^\mu + \epsilon$$
$$p_b = \frac{1}{3} a_- T^\nu \qquad e_b = \frac{1}{3} a_- (\nu - 1) T^\nu$$
$$\nu = 1 + \frac{1}{c^2_{s,\mathrm{broken}}}, \quad \mu = 1 + \frac{1}{c^2_{s,\mathrm{symm}}}$$

- $$\alpha_{\bar\theta} = \frac{1}{3}\left(1 - \frac{\nu}{\mu} + \frac{3\epsilon\nu}{a_+ \mu T_n^4}\right)$$

NB!

$$\frac{v_+}{v_-} = \frac{v_+ v_- (\nu - 1) - 1 + 3\alpha_{\bar\theta}}{v_+ v_- (\nu - 1) - 1 + 3 v_+ v_- \alpha_{\bar\theta}}$$


$\kappa\left(\alpha_{\bar{\theta}n}, c_{s,s}^2, c_{s,b}^2\right)$
1
0.1
0.01
0.2
0.4
0.6
0.8
1.0
ξ_w
$\alpha_{\bar{\theta}} = 3$
$\alpha_{\bar{\theta}} = 1$
$\alpha_{\bar{\theta}} = 0.3$
$\alpha_{\bar{\theta}} = 0.1$
$\alpha_{\bar{\theta}} = 0.03$
$\alpha_{\bar{\theta}} = 0.01$
$c_{s,s}^2 = 1/4$
$c_{s,s}^2 = 1/3$
$c_{s,b}^2 = 1/3$
$c_{s,b}^2 = 1/4$

How to reuse this result?

Compute $\alpha_{\bar{\theta}}$, both c_ss and choose ξ_w

2010.09744

```
import numpy as np
from scipy.integrate import odeint
from scipy.integrate import simps

def mu(a,b):
  return (a-b)/(1.-a*b)

def getwow(a,b):
  return a/(1.-a**2)/b*(1.-b**2)

def getvm(al,vw,cs2b):
  if vw**2<cs2b:
    return (vw,0)
  cc = 1.-3.*al+vw**2*(1./cs2b+3.*al)
  disc = -4.*vw**2/cs2b+cc**2
  if (disc<0.)|(cc<0.):
    return (np.sqrt(cs2b), 1)
  return ((cc+np.sqrt(disc))/2.*cs2b/vw, 2)

def dfdv(xiw, v, cs2):
```

...

```
    Krf*= -wow*getwow(vp,vm)
  else:
    Krf = 0
  return (Ksh + Krf)/al
```

→ $\kappa_{\bar{\theta}}$

$$K = \frac{D\bar{\theta}}{4e_n}\kappa_{\bar{\theta}}$$

Compare different methods to obtain K

Toy model 1: SM-like

- SM-like

$$F(\phi, T) = -\frac{a_+}{3}T^4 + \lambda(\phi^4 - 2E\phi^3 T + \phi^2(E^2T_{\rm cr}^2 + c(T^2 - T_{\rm cr}^2))) + \frac{\lambda}{4}(c - E^2)^2 T_{\rm cr}^4$$

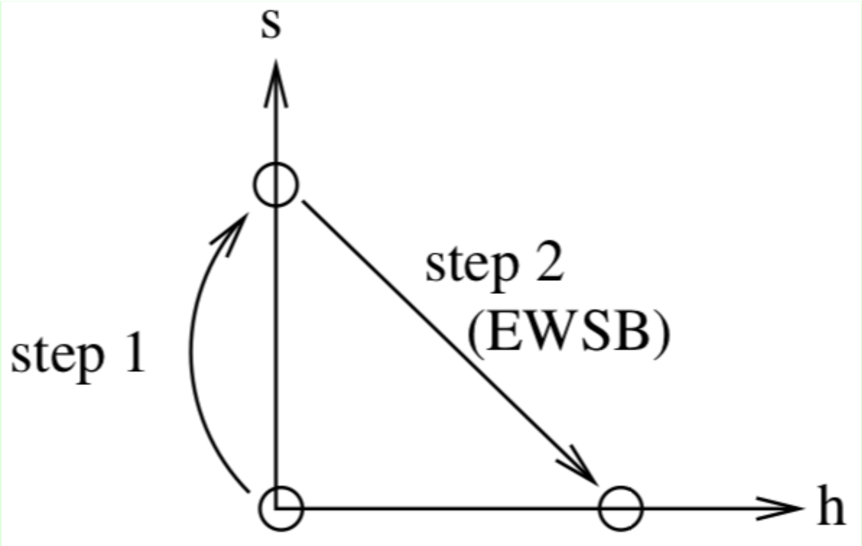
with $p_s = -F(0,T)$ and $p_b = -F(\phi_{min}, T)$

Model	$3\lambda/a_+$	E	d	$T_n/T_{\rm cr}$	$\alpha_{\bar{\theta}n}$	$c_{s,b}^2$
SM_1	10	0.3	0.2	0.9	0.0297	0.326
SM_2	10	0.3	0.2	0.8	0.0498	0.331
SM_3	3	0.3	0.2	0.9	0.00887	0.331
SM_4	3	0.3	0.2	0.8	0.0149	0.333

Toy model 2: Two-step

- Two-step
  $$p_s(T) = \frac{1}{3}a_+T^4 + (b_+ - c_+T^2)^2 - b_-^2\,, \qquad p_b(T) = \frac{1}{3}a_+T^4 + (b_- - c_-T^2)^2 - b_-^2$$

Model	$b_-/(\sqrt{a_+}T_{\rm cr}^2)$	$d_-/\sqrt{a_+}$	$d_+/\sqrt{a_+}$	$T_n/T_{\rm cr}$	$\alpha_{\bar\theta}$	$c_{s,b}^2$	$c_{s,s}^2$
2Step_1	$0.4/\sqrt{3}$	$0.2/\sqrt{3}$	$0.1/\sqrt{3}$	0.9	0.0156	0.311	0.325
2Step_2	$0.4/\sqrt{3}$	$0.2/\sqrt{3}$	$0.1/\sqrt{3}$	0.7	0.0704	0.297	0.320
2Step_3	$0.5/\sqrt{3}$	$0.4/\sqrt{3}$	$0.2/\sqrt{3}$	0.9	0.0254	0.282	0.317
2Step_4	$0.5/\sqrt{3}$	$0.4/\sqrt{3}$	$0.2/\sqrt{3}$	0.7	0.159	0.245	0.306



J. Cline 2019

6 Methods to compute K

M_1	K
M_2	$\left(\frac{D\bar{\theta}}{4e_n}\right)\kappa(\alpha_{\bar{\theta}n}, c_{s,s}, c_{s,b})\vert_{\mu\nu}$
M_3	$\left(\frac{D\theta}{4e_n}\right)\kappa(\alpha_\theta)\vert_{\mathrm{bag}}$
M_4	$\left(\frac{\alpha_\theta}{\alpha_\theta+1}\right)\kappa(\alpha_\theta)\vert_{\mathrm{bag}}$
M_5	$\left(\frac{\alpha_p}{\alpha_p+1}\right)\kappa(\alpha_p)\vert_{\mathrm{bag}}$
M_6	$\left(\frac{\alpha_e}{\alpha_e+1}\right)\kappa(\alpha_e)\vert_{\mathrm{bag}}$

6 Methods to compute K

A realistic model

Two-step revisited

- $V_{\text{tree}}(h,s) = -\frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 - \frac{\mu_s^2}{2}s^2 + \frac{\lambda_s}{4}s^4 + \frac{\lambda_{hs}}{4}h^2s^2 + \Delta V_h$

J

- $V_T(h,s) = \frac{T^4}{2\pi^2}\sum_\alpha N_\alpha \int_0^\infty dx x^2 \log\left[1 \pm e^{-\sqrt{x^2 + M_\alpha^2(h,s)/T^2}}\right] + \frac{T}{12\pi}\sum_{\text{bosons }\alpha} N_\alpha \left[M_\alpha^3(h,s) - M_{T,\alpha}^3(h,s,T)\right]$

↑ **Heavy particles**

Light particles ↓

- $V_{\delta T} = -\frac{\pi^2}{90}g_*' T^4, \qquad g_*' = \frac{345}{4}$

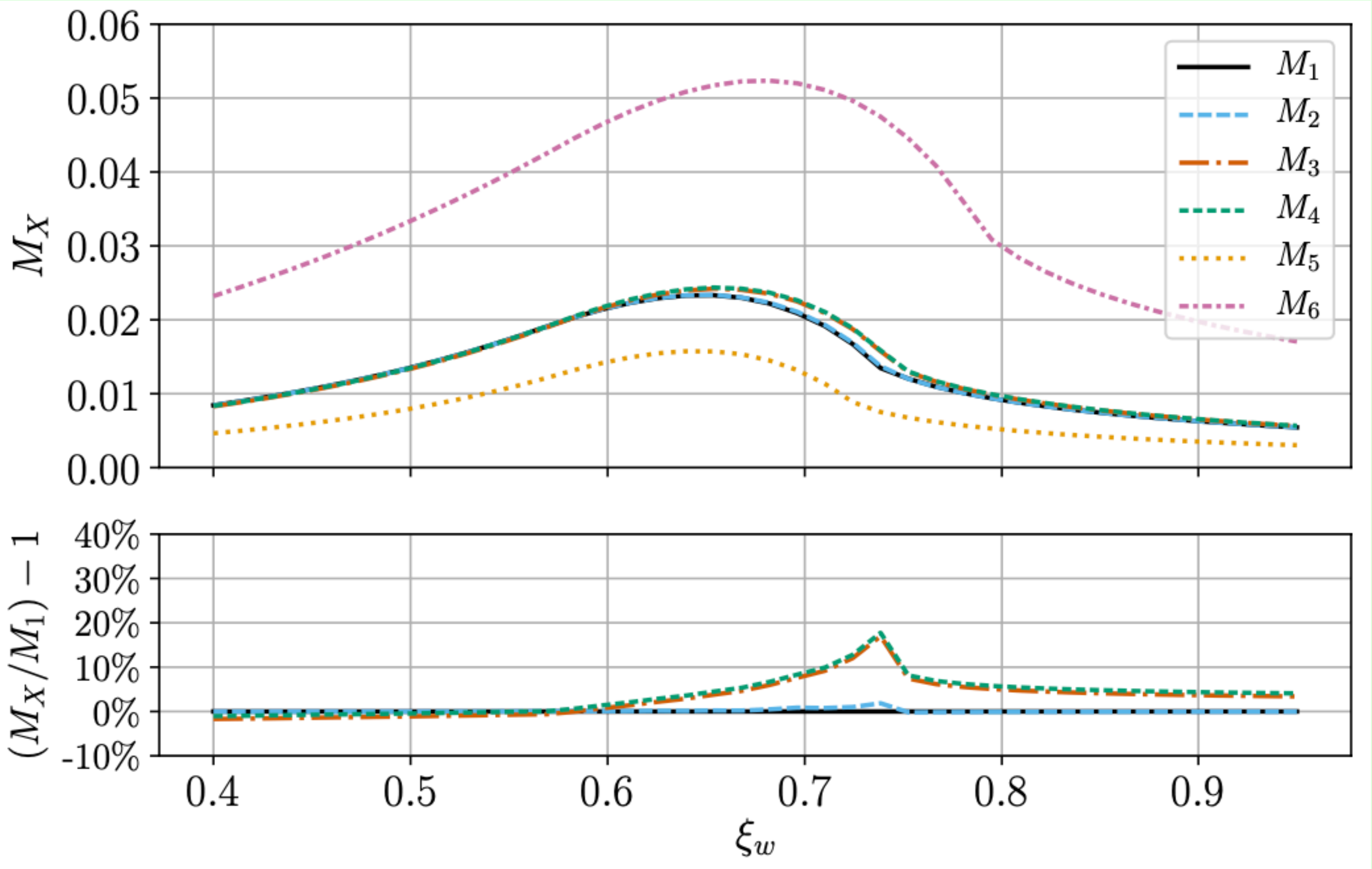
- $V_{\text{eff}} = V_{\text{tree}} + V_{cw} + V_{ct} + V_T + V_{\delta T}$

Two-step revisited

- Find nucleation temperature via $\frac{S_3}{T} \approx 140$

m_s(GeV)	λ_s	λ_{hs}	T_n(GeV)	β/H_*	α_e	$\alpha_{\bar{\theta}n}$	$c_{s,b}^2$	$c_{s,s}^2$
300	1.90	3.50	87.3	288	0.070	0.035	0.324	0.333
250	2.80	2.80	71.1	152	0.126	0.075	0.325	0.334
250	0.40	2.26	98.9	367	0.051	0.022	0.325	0.333
170	2.80	1.80	69.5	335	0.119	0.065	0.324	0.334

Effect on K



- Note that $\Omega_{\text{tot}} \propto K^{3/2}$

$m_s = 170\,\text{GeV}$

Why is the deviation in the sound speed smaller?

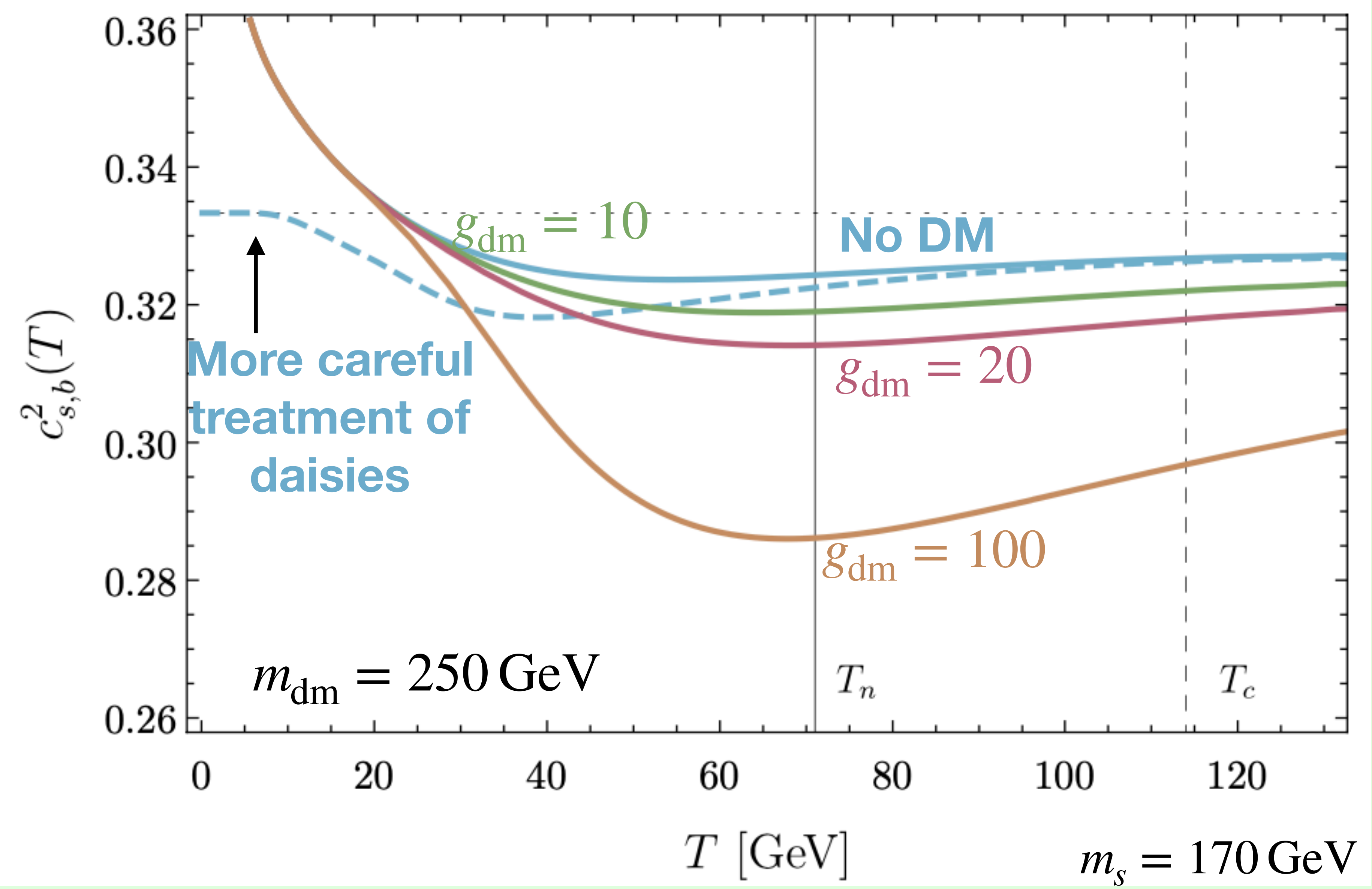
- High-temperature expansion

- $$V_T(h,s) = \frac{T^4}{2\pi^2}\sum_\alpha N_\alpha \int_0^\infty dx\, x^2 \log\left[1 \pm e^{-\sqrt{x^2 + M_\alpha^2(h,s)/T^2}}\right]$$

$$\sim \sum_{\text{bosons}\,\alpha} N_\alpha \left(-\frac{\pi^2 T^4}{90} + \underline{\frac{M_\alpha^2 T^2}{24} - \frac{M_\alpha^3 T}{12\pi}}\right) + \sum_{\text{fermions}\,\alpha} N_\alpha \left(\frac{7\pi^2 T^4}{720} - \underline{\frac{M_\alpha^2 T^2}{48}}\right)$$

- Daisy term: $\frac{M_\alpha^3 T}{12\pi} \to \frac{M_{\alpha,T}^3 T}{12\pi} \propto T^4$

Adding dark matter



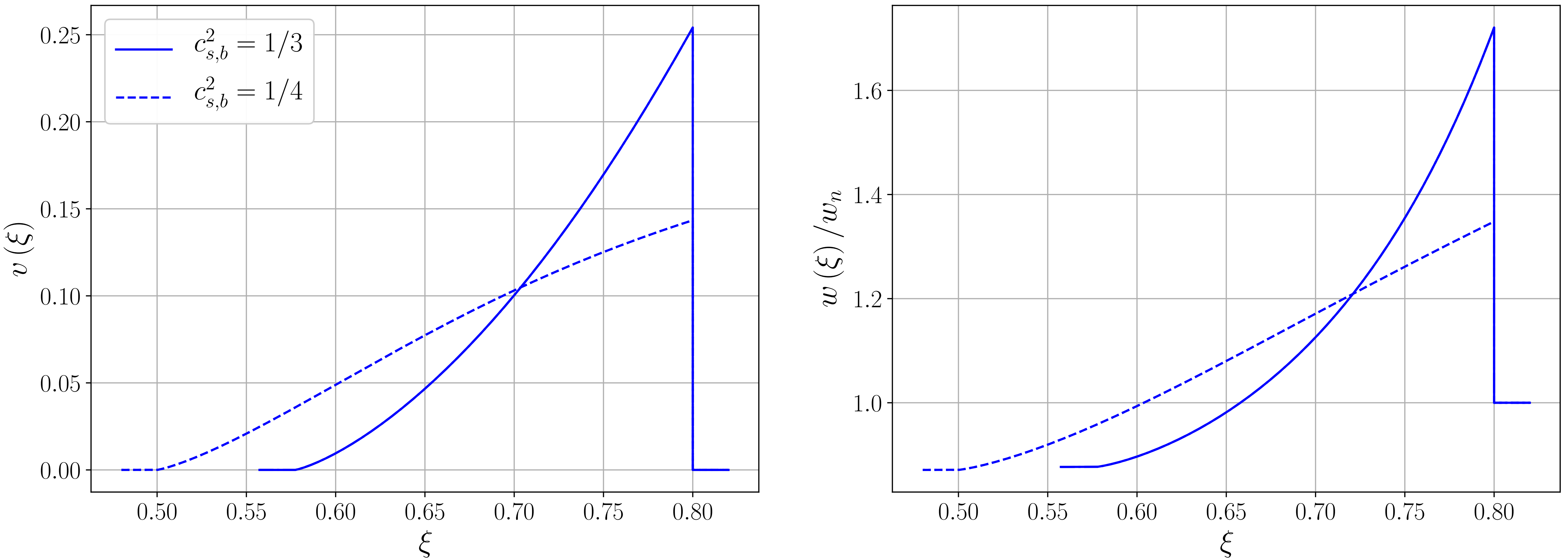
- Adding $g_{\rm dm} = 20$, leads to a deviation between M2 and M3 of ~ 35%

Summary

- Gravitational waves are a promising test of cosmological phase transitions
- $\alpha_{\bar{\theta}}$ and speeds of sound : model-independent parameterization of the hydrodynamics
- Matching onto the template model gives most accurate approximation of K, compared to other methods in the literature
- Sound speed deviation becomes stronger if additional fermions/weakly interacting particles are present

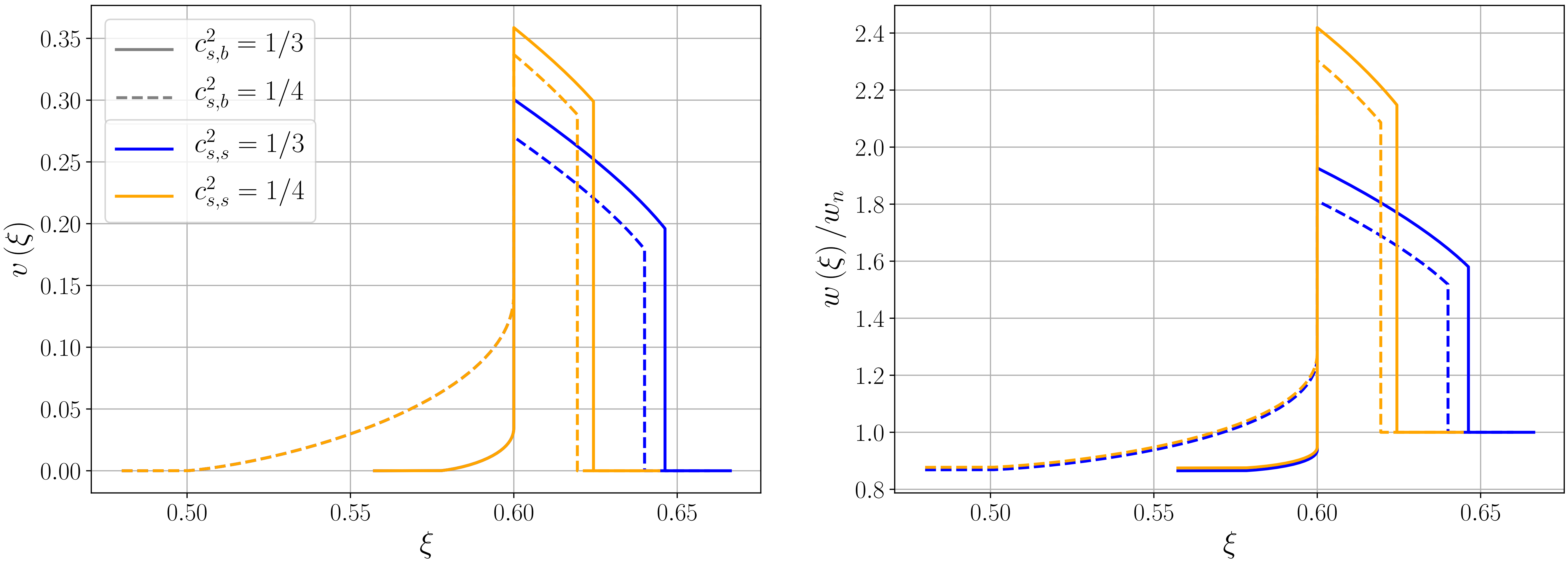
Backup

Detonation



$c_{s,b}^2 = 1/3$
$c_{s,b}^2 = 1/4$
0.25
0.20
0.15
0.10
0.05
0.00
$v(\xi)$
0.50
0.55
0.60
0.65
0.70
0.75
0.80
ξ
1.6
1.4
1.2
1.0
$w(\xi)/w_n$
0.50
0.55
0.60
0.65
0.70
0.75
0.80
ξ

Hybrid



$c_{s,b}^2 = 1/3$
$c_{s,b}^2 = 1/4$
$c_{s,s}^2 = 1/3$
$c_{s,s}^2 = 1/4$
$v(\xi)$
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.50
0.55
0.60
0.65
ξ
$w(\xi)/w_n$
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.50
0.55
0.60
0.65
ξ

Deflagration