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How to be precise at the Electroweak scale at finite-T

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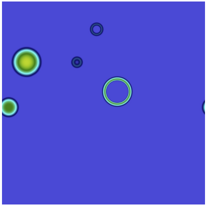
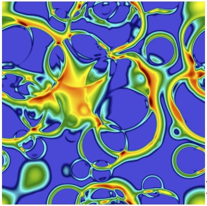
D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions,*

Motivation

The thermal history of electroweak symmetry breaking

- $T_c \sim 100$ GeV
- In the Standard Model EWSB occurs through a smooth crossover. Possible in extensions that it would be first order.
- Study Beyond the Standard Model (BSM) physics near the EW scale in context of electroweak phase transition.
  - Light fields strongly coupled to Higgs
  - Collider targets
- BSM testing pipeline: Collider phenomenology $\rightarrow$
  - Baryogenesis
  - Colliding bubbles $\rightarrow$ Gravitational wave production
- Ensure (improve) quantitative precision at finite-T?



 [1]

[1] by David Weir

Thermal field theory

BSM theories in thermal equilibrium

Preferred choice: First principle lattice methods. Fail at certain regimes:

- Intermediate chemical potential ($\uparrow \mu$).
- Implement chiral fermions on the lattice at finite-T
- Incorporating hierarchy of scales.

Near T_c non-perturbative modes dominate (show later) $\rightarrow$ Lattice.

In a weakly coupled electroweak theory at high-T $\rightarrow$ Analytic methods.

$\Rightarrow$Interplay of both methods. Today: Weak-coupling.

Equilibrium Thermodynamics: Imaginary Time Formalism

Relate density operator to time evolution $\rho(\beta) = e^{-\beta\mathcal{H}} \to \mathcal{U}(t) = e^{-i\mathcal{H}t}$.
Corresponds to path integral over imaginary-time $t \to -i\tau$, $\beta = 1/T$

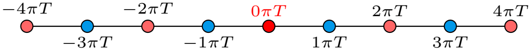
$$\mathcal{Z} = C \int_{\mathrm{b.c.}} \mathcal{D}\phi \, \exp\left[- \int_0^\beta \mathrm{d}\tau \int_{\mathbf{x}} \mathcal{L}_{\mathrm{E}} \right] \, , \quad \phi(0, \mathbf{x}) = \pm\phi(\beta, \mathbf{x}) \, .$$

(Anti-)periodic bosonic(fermionic) fields at boundaries → compactified time direction: $\mathbb{R}^3 \times S^1_\beta$.

Finite-τ and (b.c.) induce a discrete Fourier sum for time component $P = (\omega_n, \mathbf{p})$ with Matsubara frequencies

$$\omega_n = \begin{cases} 2n\pi T & \text{bosonic} \\ (2n+1)\pi T & \text{fermionic} \end{cases}$$

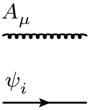
Absent zero mode $\omega_{n=0}$ for fermions:



Differences to zero temperature

$(d+1) \rightarrow d$-dimensional theory with infinite tower of massive modes.

Euclidean free-particle propagator for bosonic(fermionic) fields:



$$\left.\right\} \propto \frac{1}{P^2+m^2} = \frac{1}{\mathbf{p}^2+\omega_n^2+m^2}\ , \qquad P = (\omega_n, \mathbf{p})\ .$$

Same diagrams as in zero temperature QFT, go over to Euclidean space, and substitute Euclidean frequency integrals by sums

$$\int \frac{\mathrm{d}^{d+1}p}{(2\pi)^{d+1}} f(p) \rightarrow T \sum_n \int \frac{\mathrm{d}^d p}{(2\pi)^d} f(\omega_n, \mathbf{p}) = \sum\!\!\!\!\!\!\int_P f(\omega_n, \mathbf{p})\ .$$

- ▹ Ultraviolet (UV) contained at $T=0$
- ▹ Infrared (IR) sensitivity worsened $\rightarrow$ field in reduced spacetime dimension

Multi-scale Hierarchy in hot gauge theories

Evaluate Matsubara sums yielding Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_{\mathrm{B}}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}} |p| \sim g^2/2$.

Theory separates scales rigorously:

$$|p| \sim \begin{cases} \pi T & \textit{hard } \text{scale} \\ gT & \textit{soft } \text{scale} \\ g^2 T/\pi & \textit{ultrasoft } \text{scale} \end{cases}$$

Limit: Confinement-like behaviour in ultrasoft sector $g^2 n_{\mathrm{B}}(g^2 T) \sim \mathcal{O}(1)$. For $m_T \leq g^2 T$ weak expansion breaks down. Light bosons are non-perturbative at finite-T: Linde's IR problem[2].

[2] A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas*, Phys. Lett. B **96** (1980) 289

Resummation

Dynamically generated masses through collective plasma effects

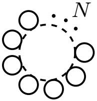
$$m_T = g^n T + m \; .$$

Evaluate Matsubara sums yielding Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_{\rm B}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|} \geq \frac{g^2 T}{m}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\rm F} |p| \sim g^2/2$.

For $m_T \leq g^2 T$ weak expansion breaks down. Light bosons are non-perturbative at finite-T. Cure by thermal resummation of contributions at $m \sim gT$ with most IR sensitivity.



$$\propto \quad g^{2N} \left[{m_T}^{3-2N} T \right] \left[\frac{T^2}{12} \right]^N \propto m^3 T \left[\frac{gT}{m_T} \right]^{2N}$$

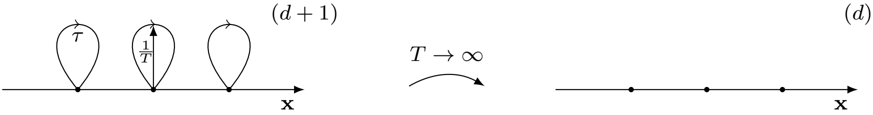
Effective Theory (EFT): Definition

Framework to describe theory with scale hierarchy: Effective Field Theory.

1. Identify soft degrees of freedom.
2. Construct most general low-energy Lagrangian.
3. *Match* Green's functions $\rightarrow$ determine EFT coupling constants.

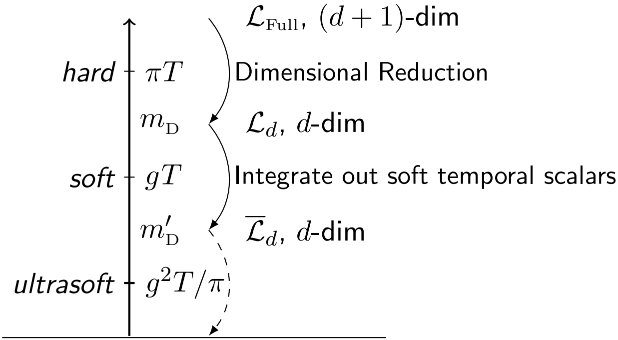
Perturbative and IR safe: Matching in the IR (regime of mutual validity), UV incorporated in EFT coefficients.

Modes with wavelengths $|\mathbf{x}|, |x_0| \gg \beta$ or $\omega_n^2 + m^2 \ll T^2$ *effectively* live in 3-dimensions.



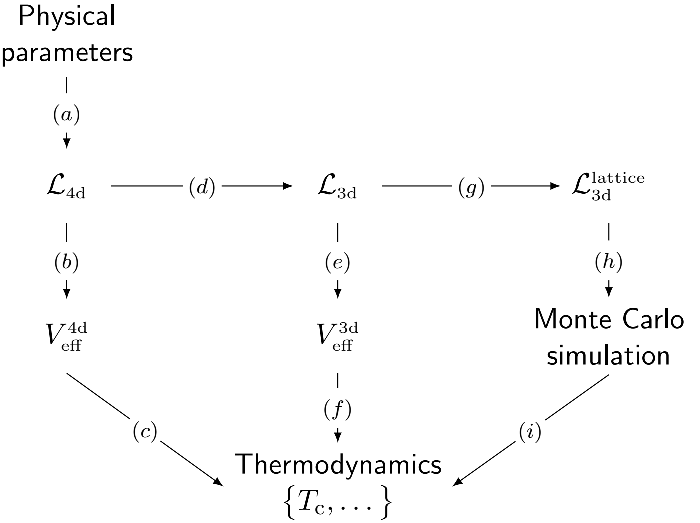
Dimensional Reduction (DR)

Integrate out hard modes perturbatively → EFT for static modes.
Incorporates an all order thermal resummation to by-pass IR problem. Applications for thermodynamics of non-Abelian gauge theories such as (EW) phase transitions[3] and QCD.

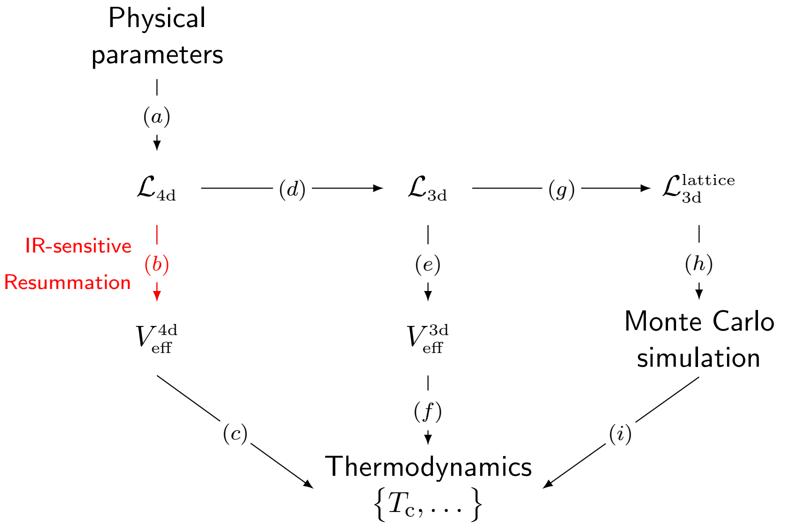


[3] K. Kajantie, M. Laine, K. Rummukainen, and M. Shaposhnikov, *Generic Rules for High Temperature Dimensional Reduction and Their Application to the Standard Model,* Nucl. Phys. B **458** (1995) 90 [hep-ph/9508379], K. Kajantie, M. Laine, K. Rummukainen, and M. Shaposhnikov, *The electroweak phase transition: a non-perturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], T. Brauner, T. V. I. Tenkanen, A. Tranberg, A. Vuorinen, and D. J. Weir, *Dimensional reduction of the Standard Model coupled to a new singlet scalar field,* JHEP **2017** (2016) 7 [1609.06230]

Thermodynamics of electroweak phase transition

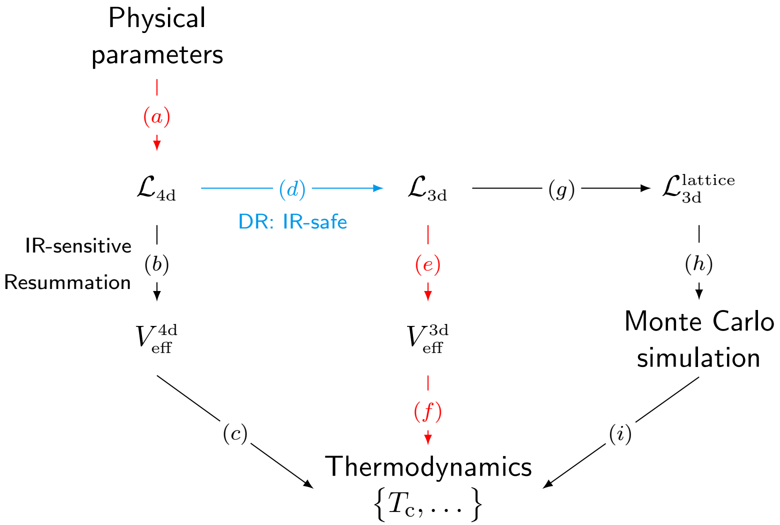


Thermodynamics of electroweak phase transition

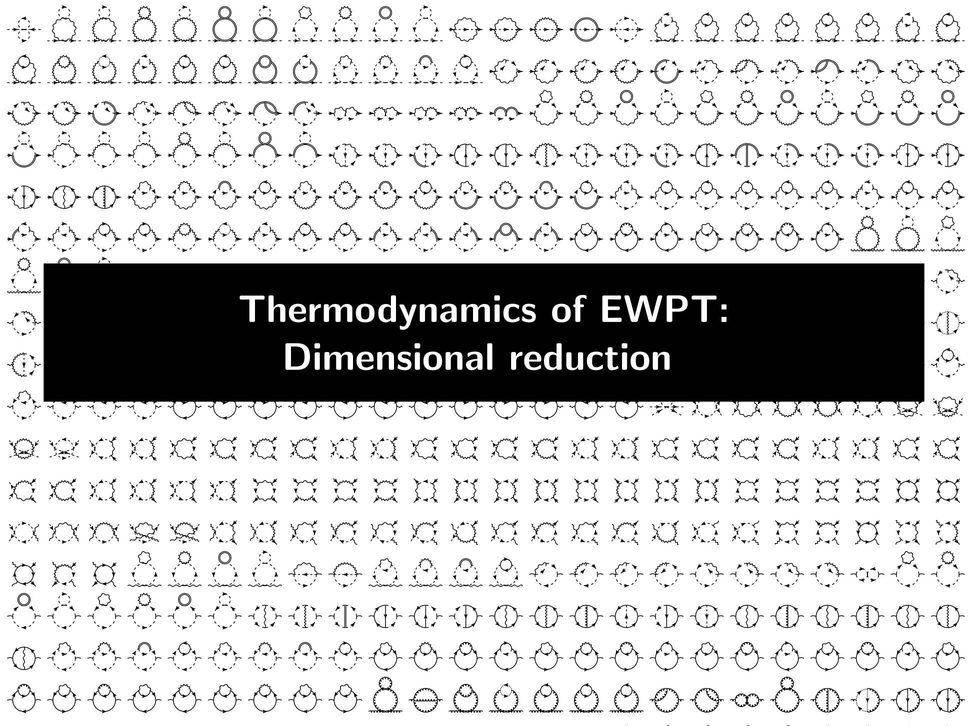


- $4d$ approach: $(a) \to (b) \to (c)$

Thermodynamics of electroweak phase transition



- $4d$ approach: $(a) \to (b) \to (c)$
- Perturbative $3d$ approach: $(a) \to (d) \to (e) \to (f)$



Thermodynamics of EWPT: Dimensional reduction

Dimensionally reduced effective theory for hot SM

SM described by 3-dimensional super-renormalisable theory

$$S_{\rm SM}^{\rm 3d} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}_{\rm SM}^{\rm 3d} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\} .$$

"Electrostatic SM" (E-SM) at high-T developed to study high-T thermodynamics[4].

$$\begin{aligned} V_{\rm 3d}^{\rm soft} &= \mu_{h,3}^2 \phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2 \\ &+ \frac{1}{2}m_{\rm D}^2\, A_0^a A_0^a + \frac{1}{2}m_{\rm D}'^2\, B_0^2 + \frac{1}{2}m_{\rm D}''^2\, C_0^\alpha C_0^\alpha \\ &+ \frac{1}{4}\kappa_1 (A_0^a A_0^a)^2 + \frac{1}{4}\kappa_2 B_0^4 + \frac{1}{4}\kappa_3 A_0^a A_0^a B_0^2 \\ &+ h_1 \phi^\dagger\phi A_0^a A_0^a + h_2 \phi^\dagger\phi B_0^2 + h_3 B_0 \phi^\dagger \mathbf{A}_0 \cdot \boldsymbol{\tau}\phi + h_4 \phi^\dagger\phi C_0^\alpha C_0^\alpha \ . \end{aligned}$$

[4] K. Kajantie, M. Laine, K. Rummukainen, and M. Shaposhnikov, *Generic Rules for High Temperature Dimensional Reduction and Their Application to the Standard Model,* Nucl. Phys. B **458** (1995) 90 [hep-ph/9508379]

Example: Matching of m_{D}

Match static gauge boson 2-point correlator $\Pi_{00}(\mathbf{q}) \equiv \Pi_{\mathrm{E}}(q^2)$

$$\mathbf{q}^2 + \Pi_{\mathrm{E}}(\mathbf{q}^2)\big|_{\mathbf{q}^2=-m_{\mathrm{D}}^2} = 0\;, \quad \text{SM}\;,$$

$$\mathbf{q}^2 + m_{\mathrm{E}}^2 + \Pi_{A_0 A_0}(\mathbf{q}^2)\big|_{\mathbf{q}^2=-m_{\mathrm{D}}^2} = 0\;, \quad \text{E-SM}\;.$$

Taylor-expand soft scales $m_{\mathrm{E}}, q \sim \mathcal{O}(gT)$ using dimensional regularisation

$$\Pi_{\mathrm{E}}(q^2) = \sum_{n=0}^{\infty} q^{2n} \sum_{\ell=1}^{\infty} g_{\mathrm{B}}^{2\ell}\, \Pi_{\mathrm{E}\ell}^{(n)}(0)\;.$$

E-SM: Scaleless integrals vanish in dim. reg.

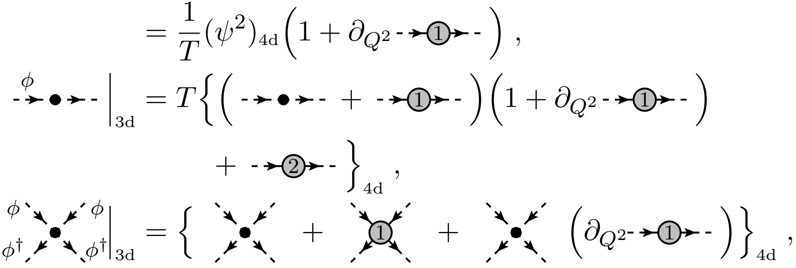
$$\Pi_{A_0 A_0} = 0\;, \quad m_{\mathrm{E}}^2 = m_{\mathrm{D}}^2\;.$$

Solve iteratively up to 2-loop order

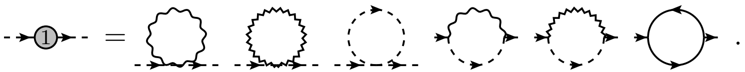
$$m_{\mathrm{E},2\ell}^2 = g^2\, \Pi_{\mathrm{E}1}(0) + g^4 \left[\Pi_{\mathrm{E}2}(0) - \Pi_{\mathrm{E}1}(0)\Pi'_{\mathrm{E}1}(0)\right]\;.$$

EFT setup: Matching correlators

$$(\psi^2)_{\rm 3d} = \frac{1}{T}(\psi^2)_{\rm 4d}\Big(1 + \hat{\Pi}'_{\psi^2}\Big)$$



where



EFT setup: SM $\to$ E-SM $\to$ M-SM

High-T E-SM contains the SM dynamics inside matching coefficients:

$$
\begin{aligned}
m_{\rm D}^2 &= T^2\left[\#g^2 + \#g^4 + \ldots\right] \;, \\
g_3^2 &= T\left[g^2 + \#g^4 + \ldots\right] \;, \\
\lambda_3 &= T\left[\#g^4 + \ldots\right] \;.
\end{aligned}
$$

Second step in DR attains M-SM, an EFT for "Magnetostatic Modes" aka 3-dimensional pure gauge with dynamical Higgs $D_i = \partial_i - i\bar{g}_3 A_i - i\bar{g}B_i$

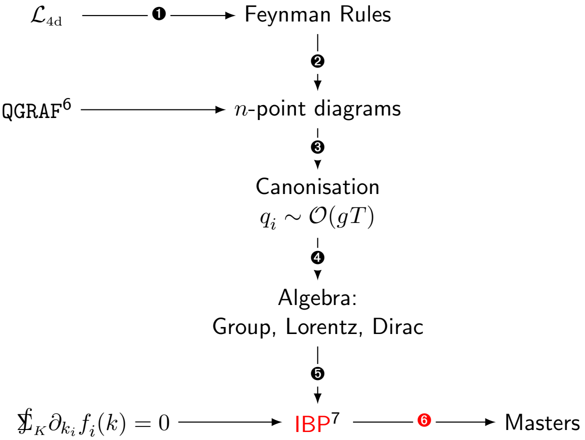
$$
\mathcal{L}_{\rm 3d}^{\rm ultrasoft} \equiv \frac{1}{4}G^a_{ij}G^a_{ij} + \frac{1}{4}F_{ij}F_{ij} + (D_i\phi)^\dagger(D_j\phi) + \bar{\mu}^2_{h,3}\phi^\dagger\phi + \bar{\lambda}_3(\phi^\dagger\phi)^2 \;.
$$

M-SM contains the IR of E-SM with matching coefficients e.g.

$$
\bar{g}_3^2 = g_3^2\left[1 + \#\left(\frac{g_3^2}{m_{\rm D}}\right) + \#\left(\frac{g_3^2}{m_{\rm D}}\right)^2 + \ldots\right] \;.
$$

Taming and automating Dimensional Reduction

In-house state-of-the-art FORM[5] code supplied with model Lagrangian.



[5] B. Ruijl, T. Ueda, and J. Vermaseren, *FORM version 4.2* arXiv (2017) [1707.06453]

[6] P. Nogueira, *Automatic Feynman Graph Generation*, J. Comput. Phys. **105** (1993) 279

[7] S. Laporta, *High-precision calculation of multi-loop Feynman integrals by difference equations*, Int. J. Mod. Phys. A **15** (2001) 5087 [hep-ph/0102033]

Integration-by-parts (IBP[8]) Reduction

Differential operators

$$O_{ij} = \frac{\partial}{\partial k_i} \cdot p_j \ ,$$

acting on integrand yield vanishing surface terms

$$0 = \int_{k_i} \frac{\partial}{\partial k_i} \cdot p_j \, I'(k_1, \ldots, k_{N_k}, q_1, \ldots, q_{N_q}) \ , \quad i = 1, \ldots, N_k \ ,$$

with $p_j \in \{k_1, \ldots, k_{N_k}, q_1, \ldots, q_{N_q}\}$.

Goal: Combine the IBP relations and obtain a suitable reduction.

[8] K. Chetyrkin and F. Tkachov, *Integration by parts: The algorithm to calculate β-functions in 4 loops,* Nucl. Phys. B **192** (1981) 159

IBPs at finite temperature[9]

Extend to Matsubara frequencies and include masses.

ℓ-loop problem = ℓ-scale problem.

Example: Vacuum (0-pt) 1-loop tadpole $Z^{\alpha}_{s;\sigma} = \sum\!\!\!\!\!\!\int_K \frac{k_0^{\alpha}}{[K^2+m^2]^s} \rightarrow$ 1 IBP.

$$
\begin{aligned}
0 &= \sum\!\!\!\!\!\!\!\int_K \partial_k \left\{ k \, \frac{k_0^{\alpha}}{[K^2+m^2]^s} \right\} \\
&= \sum\!\!\!\!\!\!\!\int_K \left\{ d \, \frac{k_0^{\alpha}}{[K^2+m^2]^s} - 2s\mathbf{k}^2 \frac{k_0^{\alpha}}{[K^2+m^2]^{s+1}} \right\} \\
&= (d-2s)\, Z^{\alpha}_{s;\sigma} + 2s\, Z^{\alpha+2}_{s+1;\sigma} + 2s\, m^2\, Z^{\alpha}_{s+1;\sigma} \\
&= (d-2s) + 2s\, \mathbf{1}_+ \mathbf{1}^+ - 2s\, m^2\, \mathbf{1}_+ \, .
\end{aligned}
$$

Particle statistics: $\sigma = 0(1)$. Recover $T = 0 \Rightarrow \mathbf{n}^{\pm} \rightarrow 0$.

[9] M. Nishimura and Y. Schröder, *IBP methods at finite temperature*, JHEP **2012** (2012) 51 [1207.4042]

IBPs at finite temperature: Higher-order

Generates $(\ell^2 + \ell N_q)$-system of multivariate difference equations.
Example: 2-loop

$$\partial_{k_1} k_1 \,, \quad \partial_{k_1} k_2 \,, \quad \partial_{k_1} q \,, \quad \partial_{k_2} k_1 \,, \quad \partial_{k_2} k_2 \,, \quad \partial_{k_2} q \,.$$

$$\begin{aligned}(\partial_{k_1} k_1): \quad 0 = {} & (d - 2s_1 - s_3) + s_3\, \mathbf{3}_+(\mathbf{2}_- - \mathbf{1}_-) + 2s_1\, \mathbf{1}_+(\mathbf{1}^+ + m_1^2) \\ & + s_3\, \mathbf{3}_+(\mathbf{1}^+ - \mathbf{2}^+ + \mathbf{3}^+ + m_1^2 - m_2^2 + m_3^2) \,, \\ \ldots & \end{aligned}$$

Solve systematically $\rightarrow$ Laporta algorithm[10].

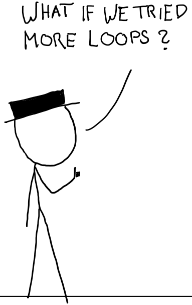
Idea: Establish lexicographic ordering prescription. Determine the most complicated integral of a set of integrals. Solve by Guassian Elimination.

[10] S. Laporta, *High-precision calculation of multi-loop Feynman integrals by difference equations*, Int. J. Mod. Phys. A **15** (2001) 5087 [hep-ph/0102033]

Possible improvements:

- More precise computation of parameters.

Appelquist-Carazzone decoupling theorem[11] breaks down at finite-T[12].



[11] T. Appelquist and J. Carazzone, *Infrared singularities and massive fields,* Phys. Rev. D **11** (1975) 2856

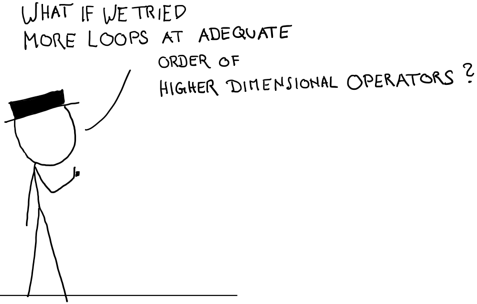
[12] N. Landsman, *Limitations to dimensional reduction at high temperature,* Nucl. Phys. B **322** (1989) 498

Possible improvements:

- More precise computation of parameters.
- Inclusion of higher-dimensional operators.

Appelquist-Carazzone decoupling theorem[11] breaks down at finite-T[12].

These are related!



[11] T. Appelquist and J. Carazzone, *Infrared singularities and massive fields*, Phys. Rev. D **11** (1975) 2856

[12] N. Landsman, *Limitations to dimensional reduction at high temperature*, Nucl. Phys. B **322** (1989) 498

Dimension-six operators in E-SM

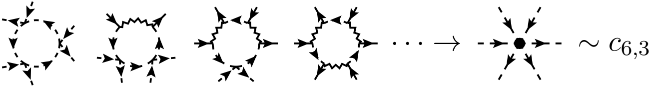
1-loop sum-integral yields finite contributions

$$\sum\!\!\!\!\!\!\!\int_P' \frac{1}{P^6} = \frac{\zeta_3}{128\pi^4 T^2}\left[1+\mathcal{O}(\epsilon)\right] .$$

Augment $3d$ Lagrangian with dim-6 (and higher) operators[13]:

$$\delta\mathcal{L}^{\rm SM}_{\rm 3d} = c_{6,3}(\phi^\dagger\phi)^3 + c_{8,3}(\phi^\dagger\phi)^4 + c_{10,3}(\phi^\dagger\phi)^5 + \dots \;.$$

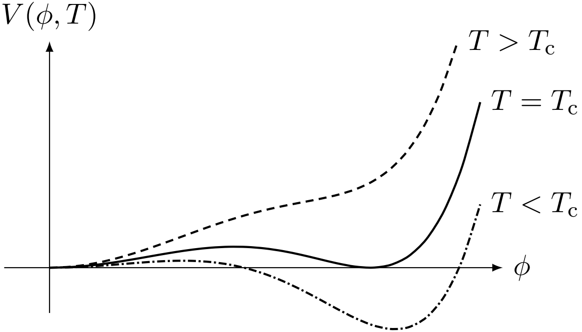
$\mathcal{L}^{\rm SM}_{\rm 3d}$ is non-super-renormalisable. Determine coefficients $c_{i,3}(d)$ in d-dimensions and evaluate 6-point vertices at one-loop order in hot SM $\rightarrow$ uniqueness.



[13] W. Buchmüller and D. Wyler, *Effective lagrangian analysis of new interactions and flavour conservation,* Nucl. Phys. B **268** (1986) 621, B. Grzadkowski, M. Iskrzyński, M. Misiak, and J. Rosiek, *Dimension-six terms in the Standard Model Lagrangian,* JHEP **2010** (2010) 85 [1008.4884]

Improving accuracy of EWPT: Effective potential

The effective potential in perturbation theory



$$V_{\text{eff}} \simeq \frac{1}{2}(-\mu^2 + \gamma T^2)\phi^2 + \frac{1}{2}\lambda\phi^4 + \#\phi^3 + \dots \ .$$

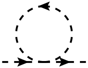
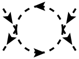
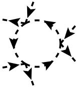
With $\gamma \sim g^n$ close to critical temperature T_c fulfill

$$(-\mu^2 + \gamma T^2) \sim 0 \times (gT)^2 + (gT^2)^2 \ .$$

The thermal effective potential at LO

$$V_{\text{eff}} = V_{\text{eff,tree}} + V_{\text{eff},1\ell}\ .$$

At 1-loop sum over n-point functions at $Q_i = 0$ external momenta

$$V_{\text{eff},1\ell} =$$  $$+\frac{1}{2}$$  $$+\frac{1}{3}$$  $$+\ \dots\Big|_{Q_i=0}$$

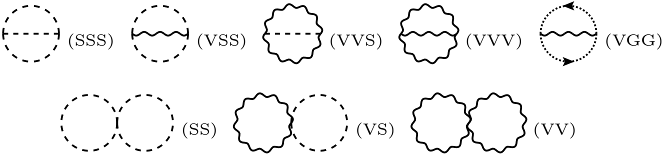
$$= \frac{1}{2}\sum\!\!\!\!\!\!\!\int_P \ln\left(P^2 + m^2\right)$$

$$V_{\text{eff},1\ell} = \underbrace{\frac{1}{2}\int_P \ln(P^2+m^2)}_{\equiv V_{\text{CW}}(m)} \underbrace{- \int_p T\ln\Big(1 \mp n_{\text{B/F}}(E_p, T)\Big)}_{V_{T,b/f}\left(\frac{m^2}{T^2}\right)}$$

$$= \underbrace{\frac{T}{2}\int_p \ln(p^2+m^2)}_{\equiv T V_{\text{soft}}(m)} + \underbrace{\frac{1}{2}\sum\!\!\!\!\!\!\!\int_{P/\{P\}}{}' \ln(P^2+m^2)}_{\equiv V_{\text{hard}}(m)}\ .$$

The effective potential at NLO

Computation up to 2-loop V_{eff}[14] straightforward with vacuum integrals in $3d$ theory:



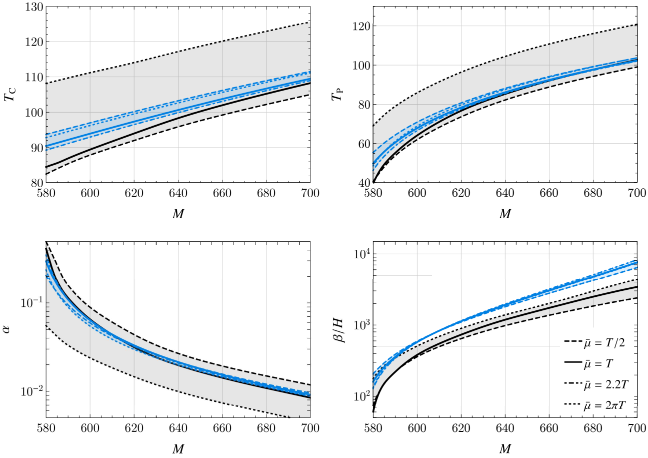
[14] K. Farakos, K. Kajantie, K. Rummukainen, and M. Shaposhnikov, *3D Physics and the Electroweak Phase Transition: Perturbation Theory,* Nucl. Phys. B **425** (1994) 67 [hep-ph/9404201], M. Laine, *The 2-loop effective potential of the 3d SU(2)-Higgs model in a general covariant gauge,* Phys. Lett. B **335** (1994) 173 [hep-ph/9406268], L. Niemi, M. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, *Thermodynamics of a two-step electroweak phase transition,*

Increasing accuracy to $\mathcal{O}(g^4)$

Also include dim-6 operator in full SM → "SMEFT"

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{M^2}(\phi^\dagger\phi)^3$$

Dependence on $\bar{\mu}$ in the 3d approach and the 4d approach



Scale dependence at finite-T

Zero temperature

$$V_{\text{eff}}(\phi,\bar{\mu}) = \underbrace{V_{\text{eff,tree}}}_{\mathcal{O}(g^2)} + \underbrace{V_{\text{CW},1\ell}}_{\mathcal{O}(g^4)} \,.$$

Changes at finite-temperature

$$V^{\text{res.}}_{\text{eff}}(\phi,T,\bar{\mu}) = \underbrace{V_{\text{tree}}}_{\mathcal{O}(g^2)} + \underbrace{V^{\text{res.}}_{\text{soft}}}_{\mathcal{O}(g^3)} + \underbrace{V_{\text{hard}}}_{\mathcal{O}(T^2g^2)+\mathcal{O}(g^4)} \,,$$

need 2-loop contributions to thermal masses. Automatically included in dimensionally reduced $3d$-approach.

Conclusions

- ▷ Thermodynamic quantities from BSM theories essential for cosmology and gravitational wave production
  - □ Numerically on the lattice around $T_c \sim 100$ GeV.
  - □ Practical approach: Effective Theories.

- ▷ Dimensionally reduced 3-dim theories permit
  - □ Automatic all-order resummation at high-T
  - □ Analytic treatment of fermions, lattice treatment for 3-dim theory,
  - □ Systematic higher-loop/operator improvement.
  - □ Automation: Multi-loop sports.
  - □ Universality
  - □ Description of the phase transition[15]

☆ Neither $3d$- nor $4d$-perturbative approaches "solve" the IR problem $\rightarrow$ Lattice (much more feasible now)

[15] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *Theoretical uncertainties for cosmological first-order phase transitions,*