Stochastic Background of Induced Gravitational Waves: Recent Developments

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- Primordial perturbation, PBHs, and induced GWs
- NANOGrav hints on planet-mass PBHs
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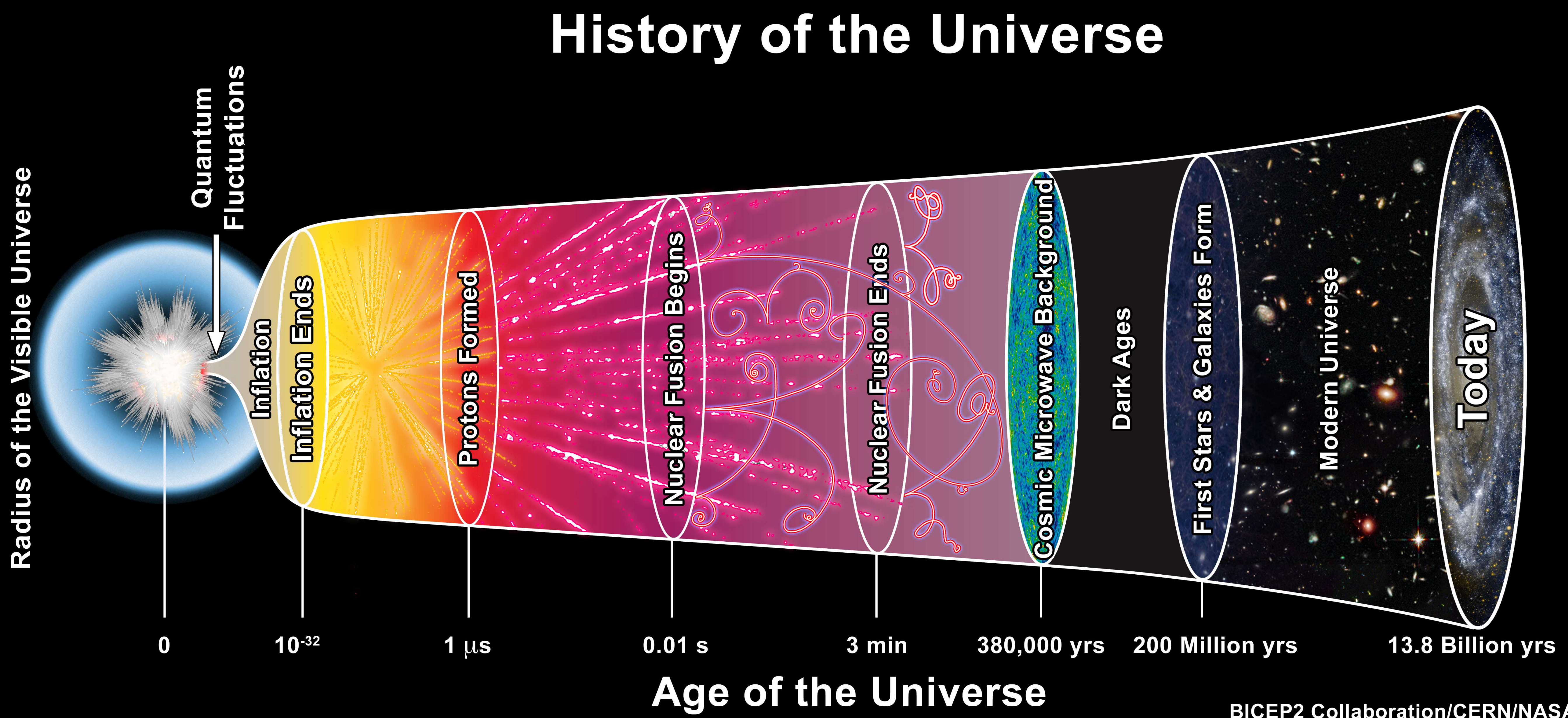
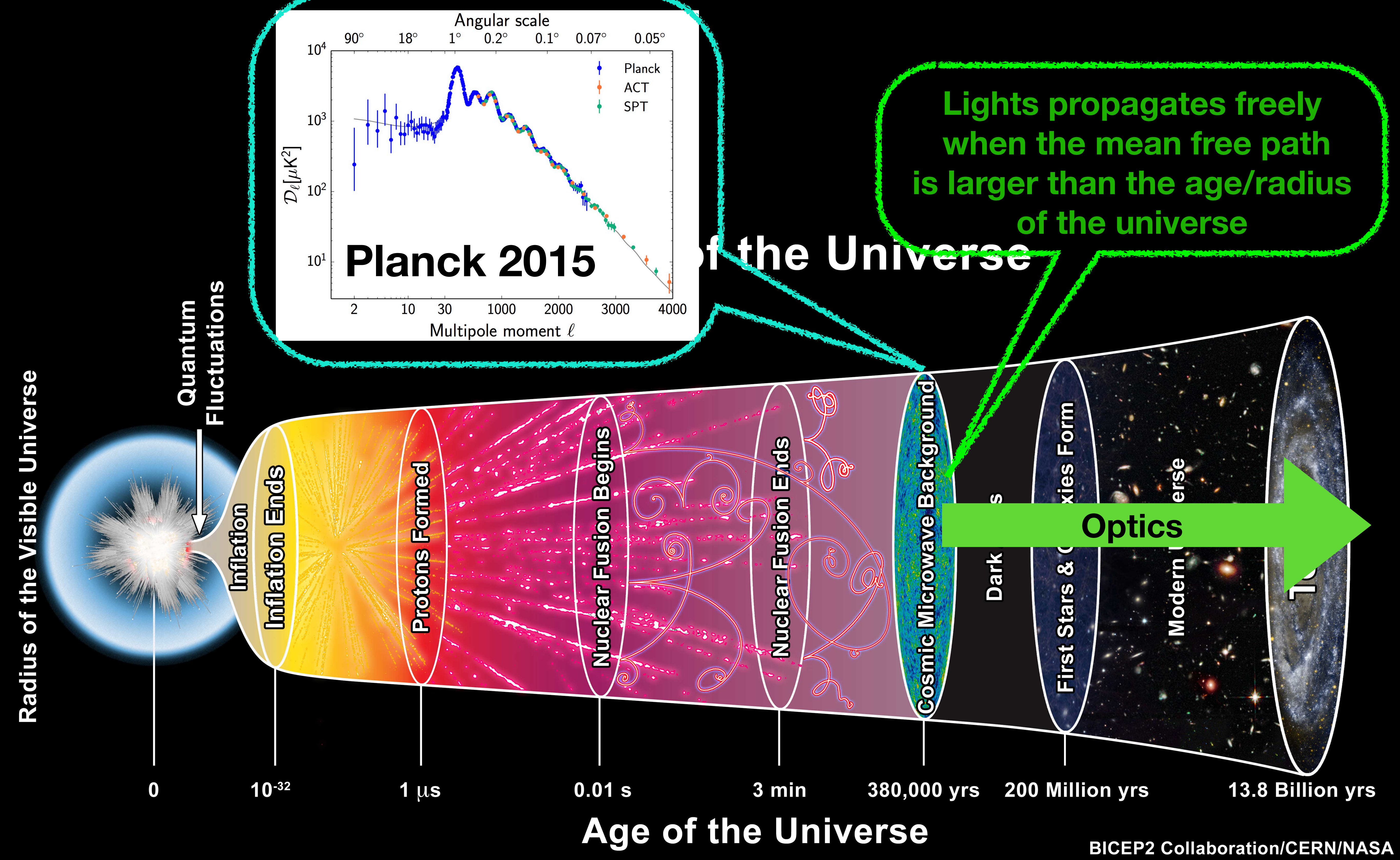
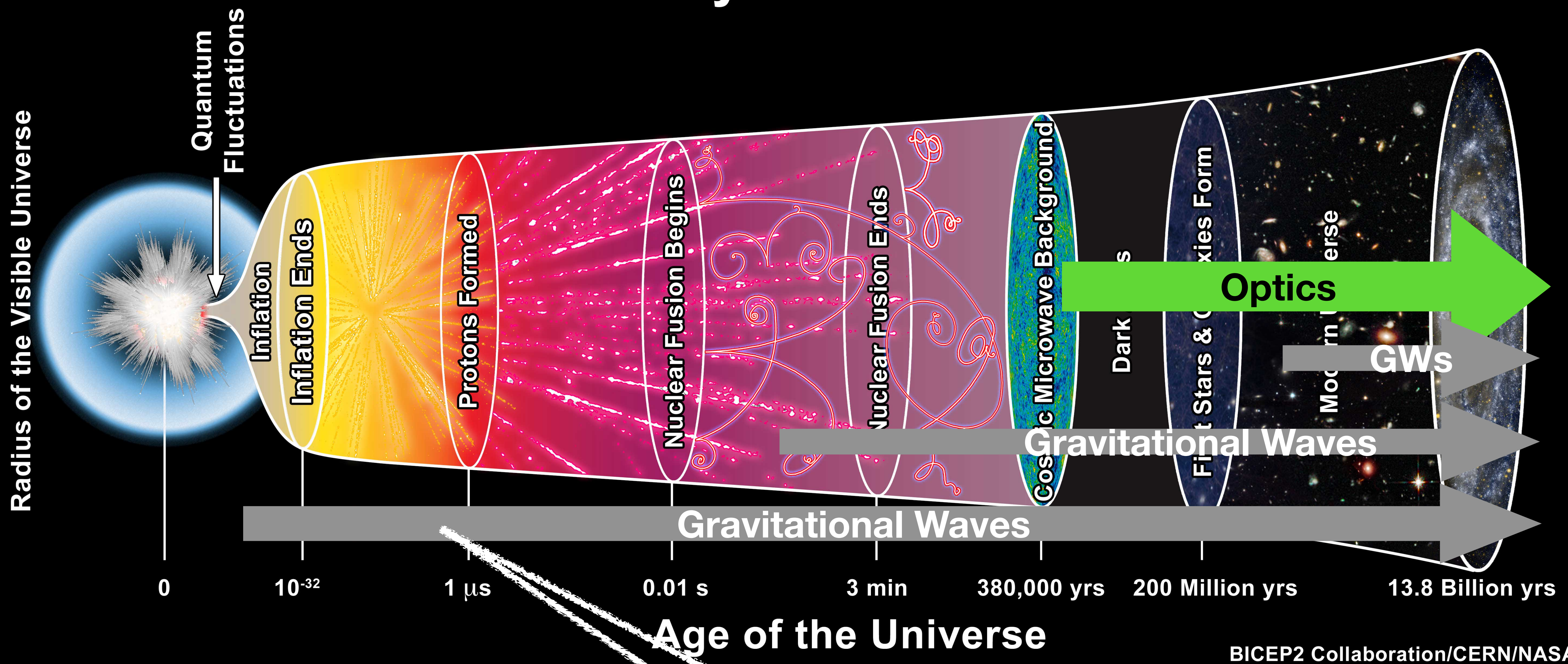


History of the Universe
Radius of the Visible Universe
Quantum Fluctuations
Inflation
Inflation Ends
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Dark Ages
First Stars & Galaxies Form
Modern Universe
Today
0
10^-32
1 μs
0.01 s
3 min
380,000 yrs
200 Million yrs
13.8 Billion yrs
Age of the Universe
BICEP2 Collaboration/CERN/NASA

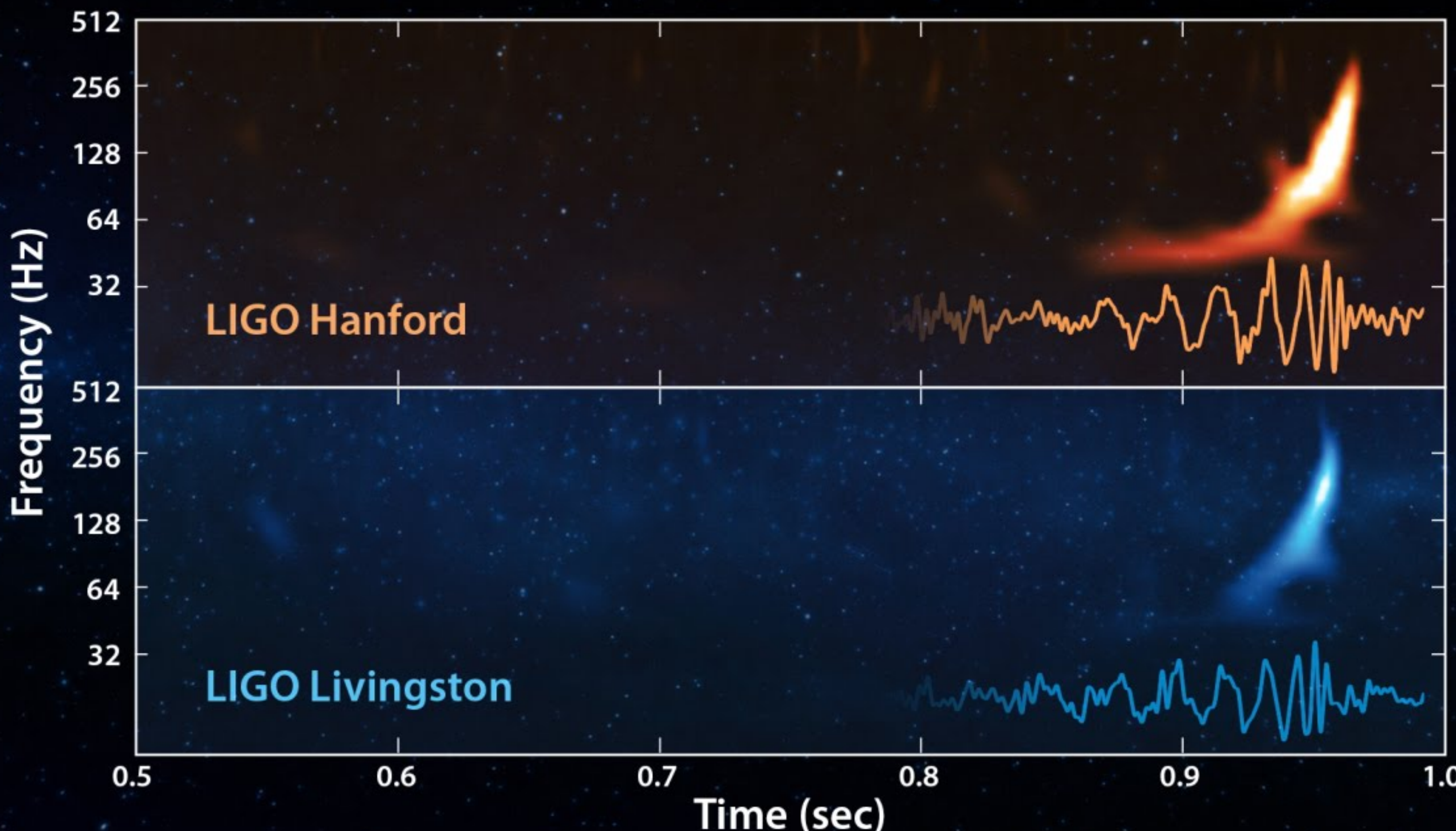

Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
10^4 10^3 10^2 10^1
$\mathcal{D}_\ell [\mu K^2]$
Planck
ACT
SPT
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
Planck 2015
Lights propagates freely
when the mean free path
is larger than the age/radius
of the universe
Radius of the Visible Universe
Quantum Fluctuations
Inflation
Inflation Ends
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Optics
0
10^{-32}
1 μs
0.01 s
3 min
380,000 yrs
200 Million yrs
13.8 Billion yrs
Age of the Universe
BICEP2 Collaboration/CERN/NASA

History of the Universe



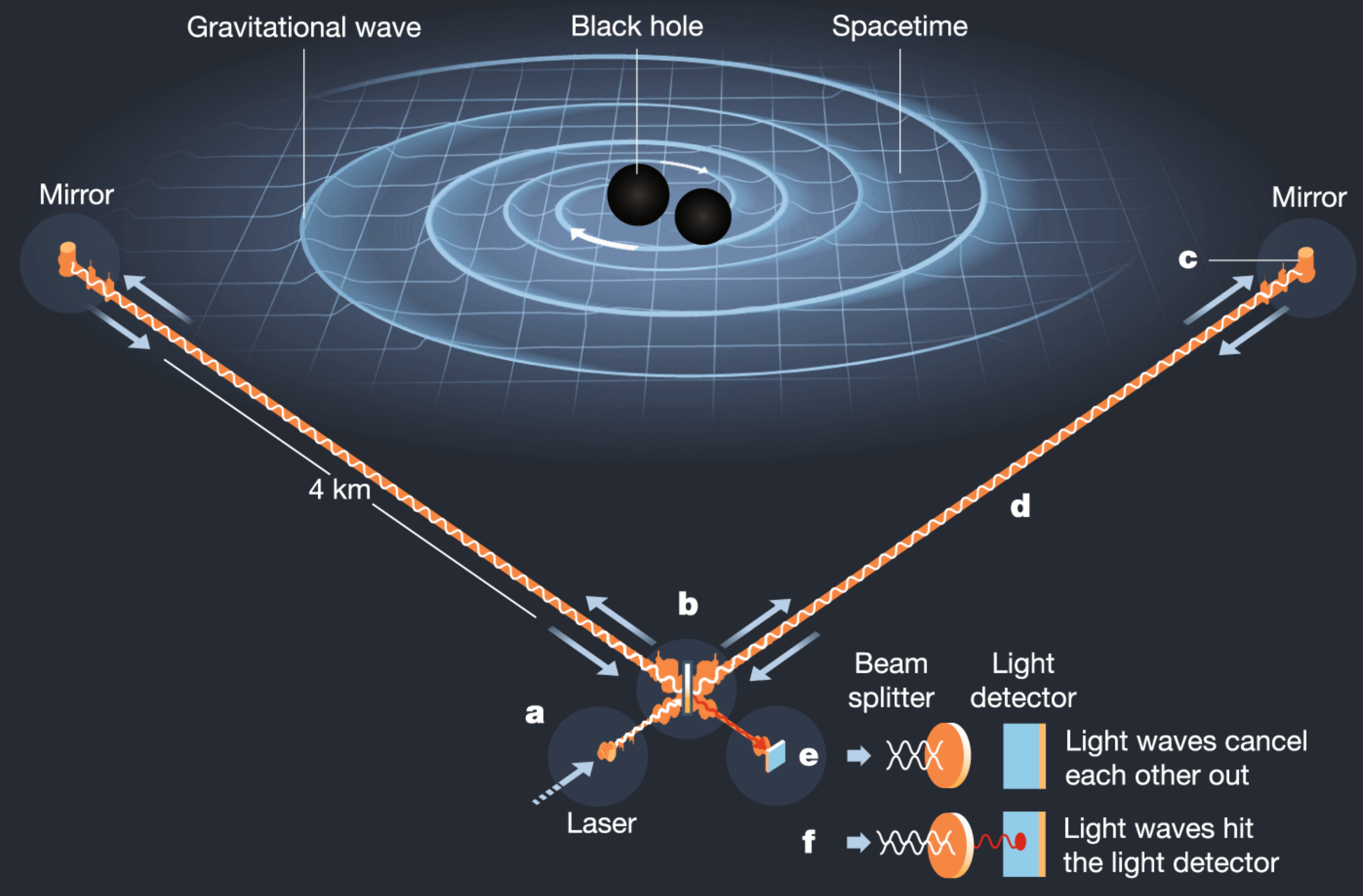
GWs propagates freely all the time!

LIGO detection of GWs



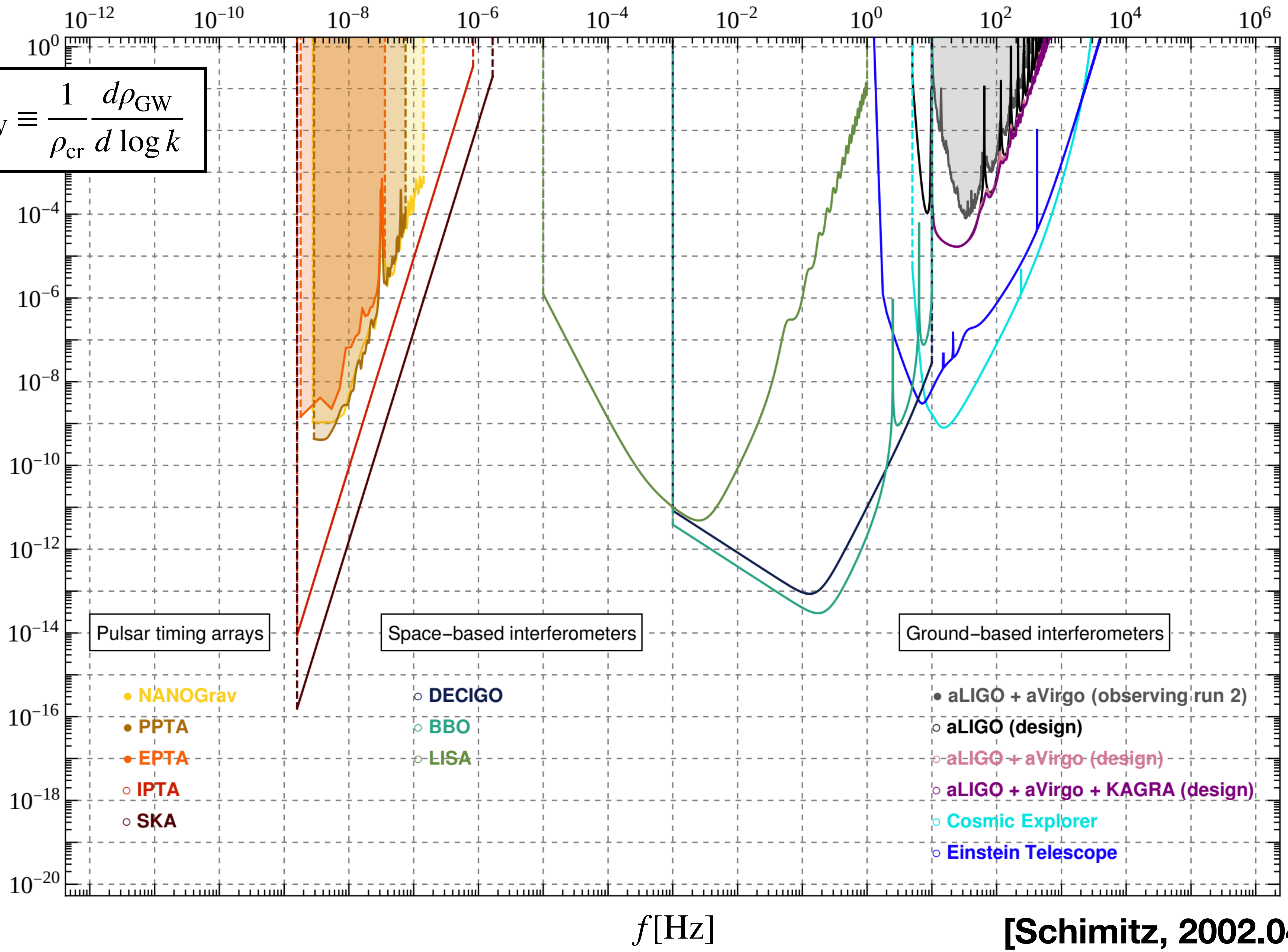
ligo.org

Detection of GWs



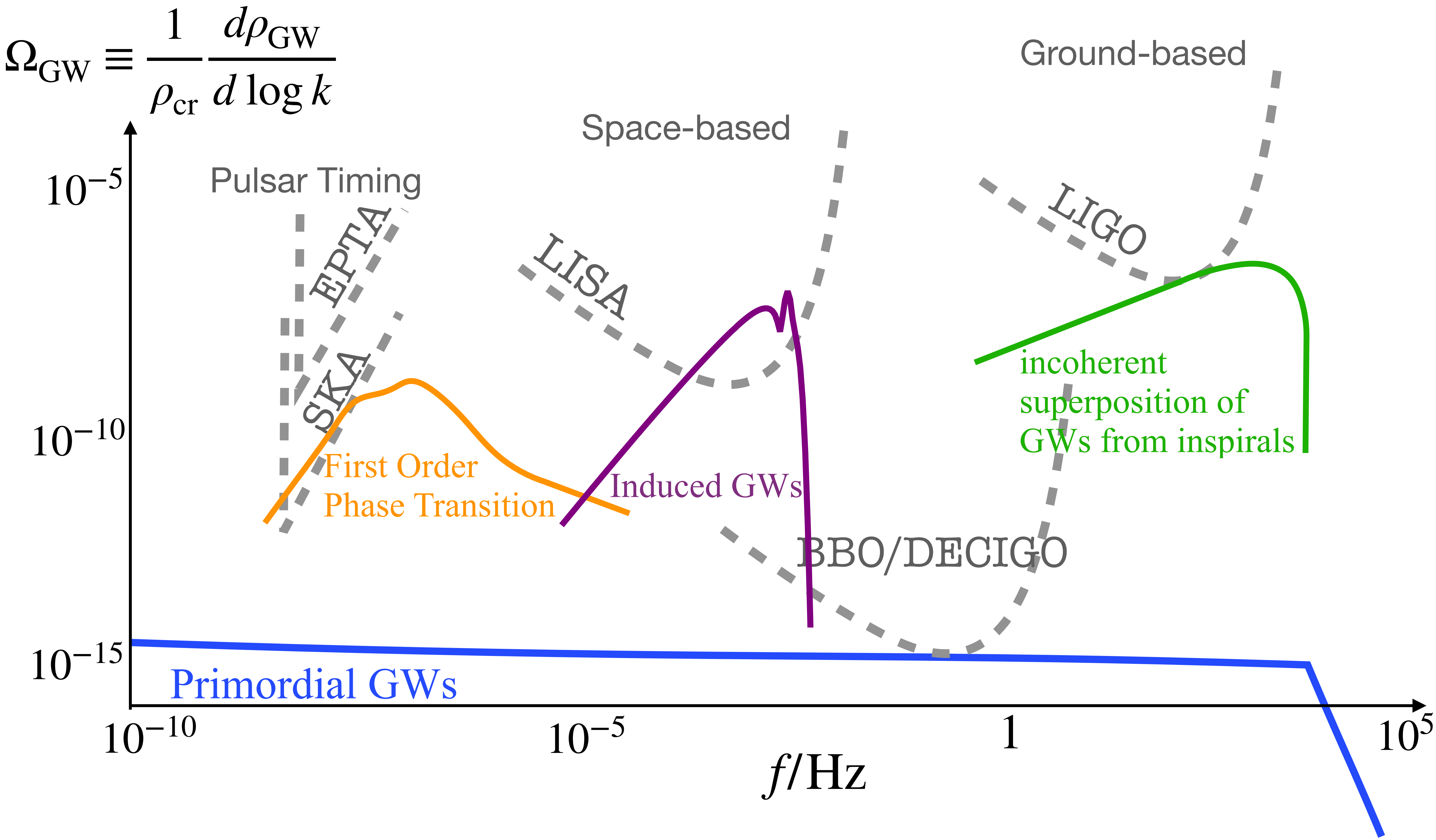
Future GW detectors

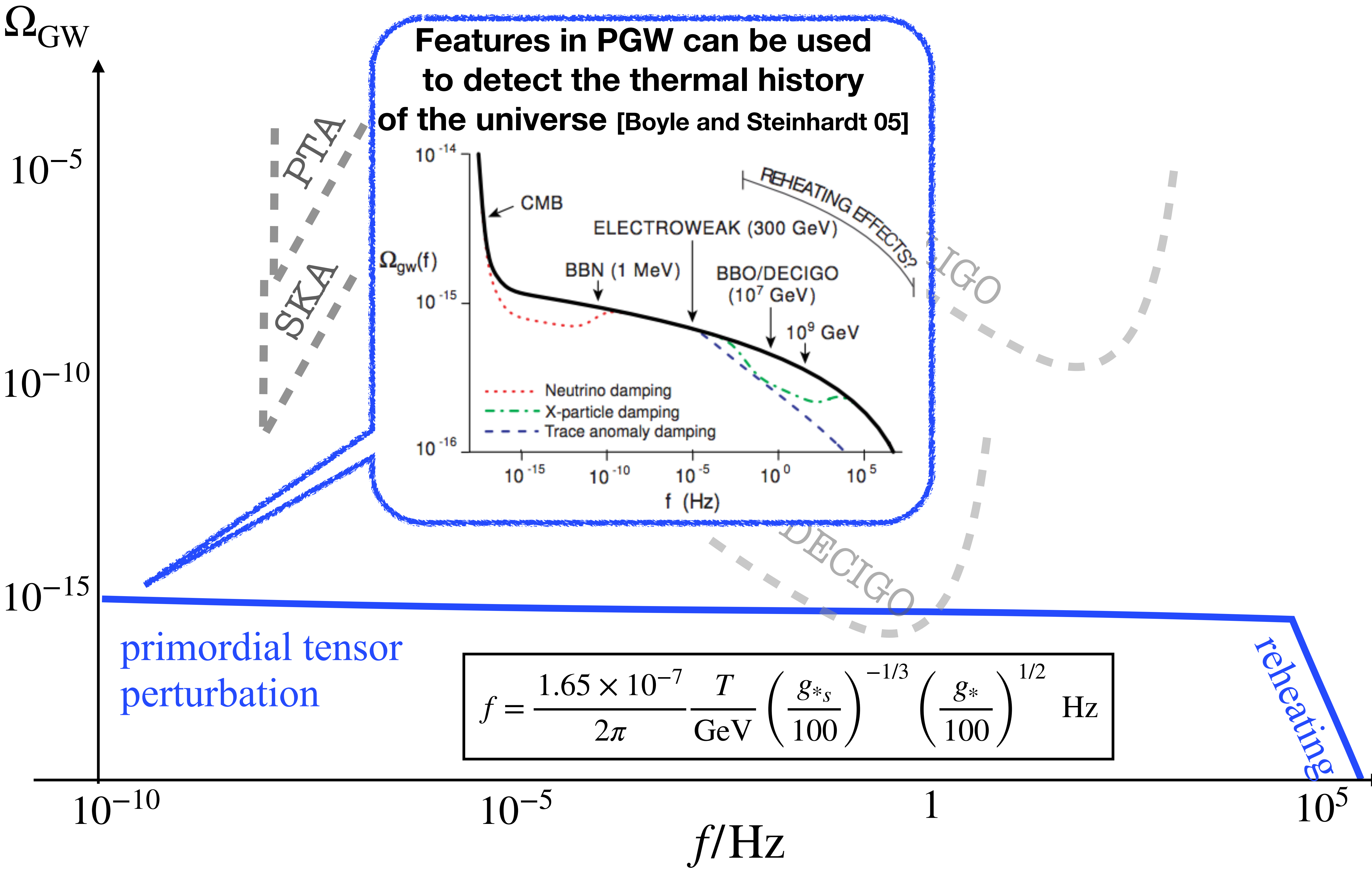
$$\Omega_{\rm GW} \equiv \frac{1}{\rho_{\rm cr}} \frac{d\rho_{\rm GW}}{d\log k}$$

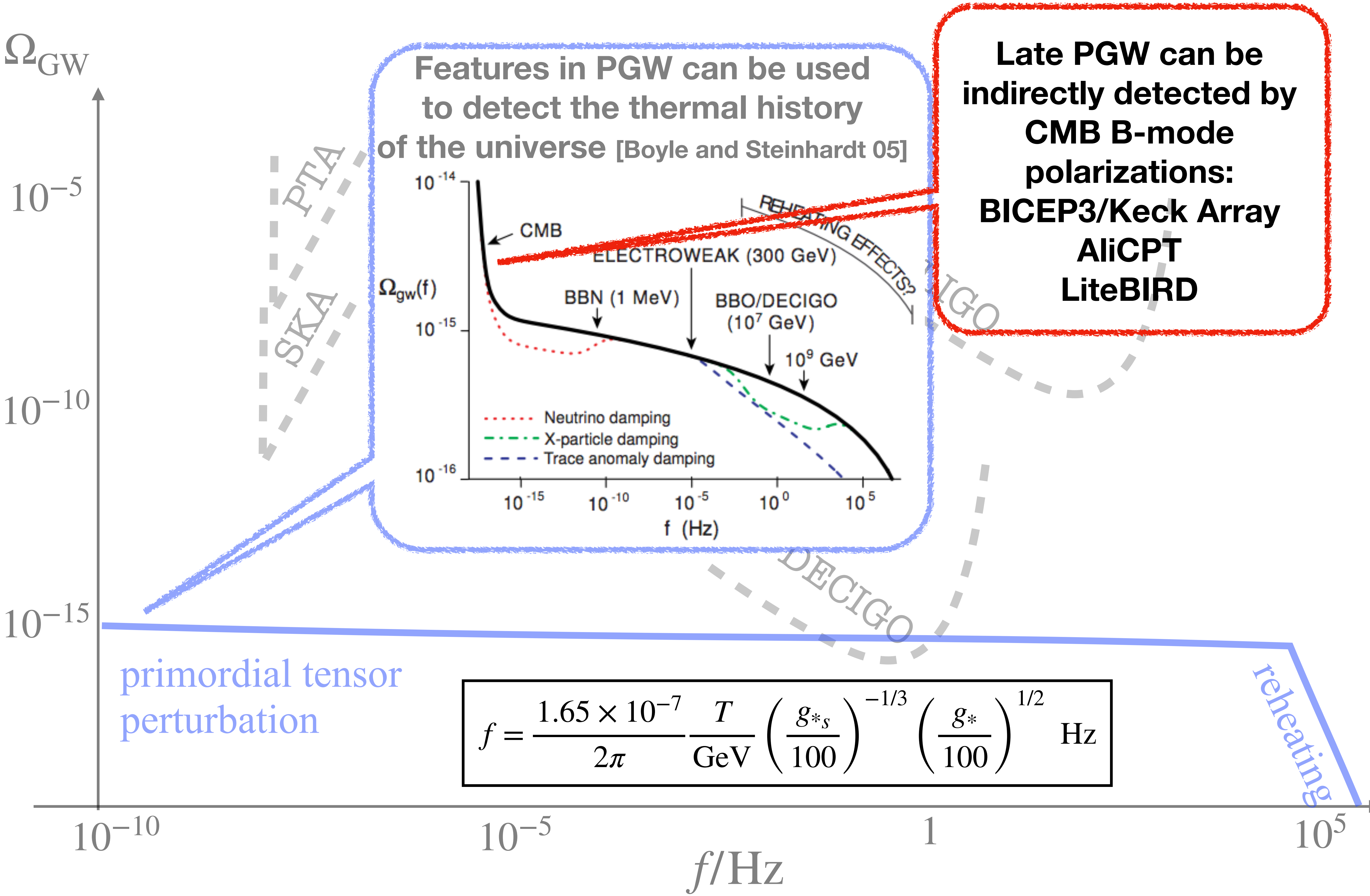


[Schimitz, 2002.04615]

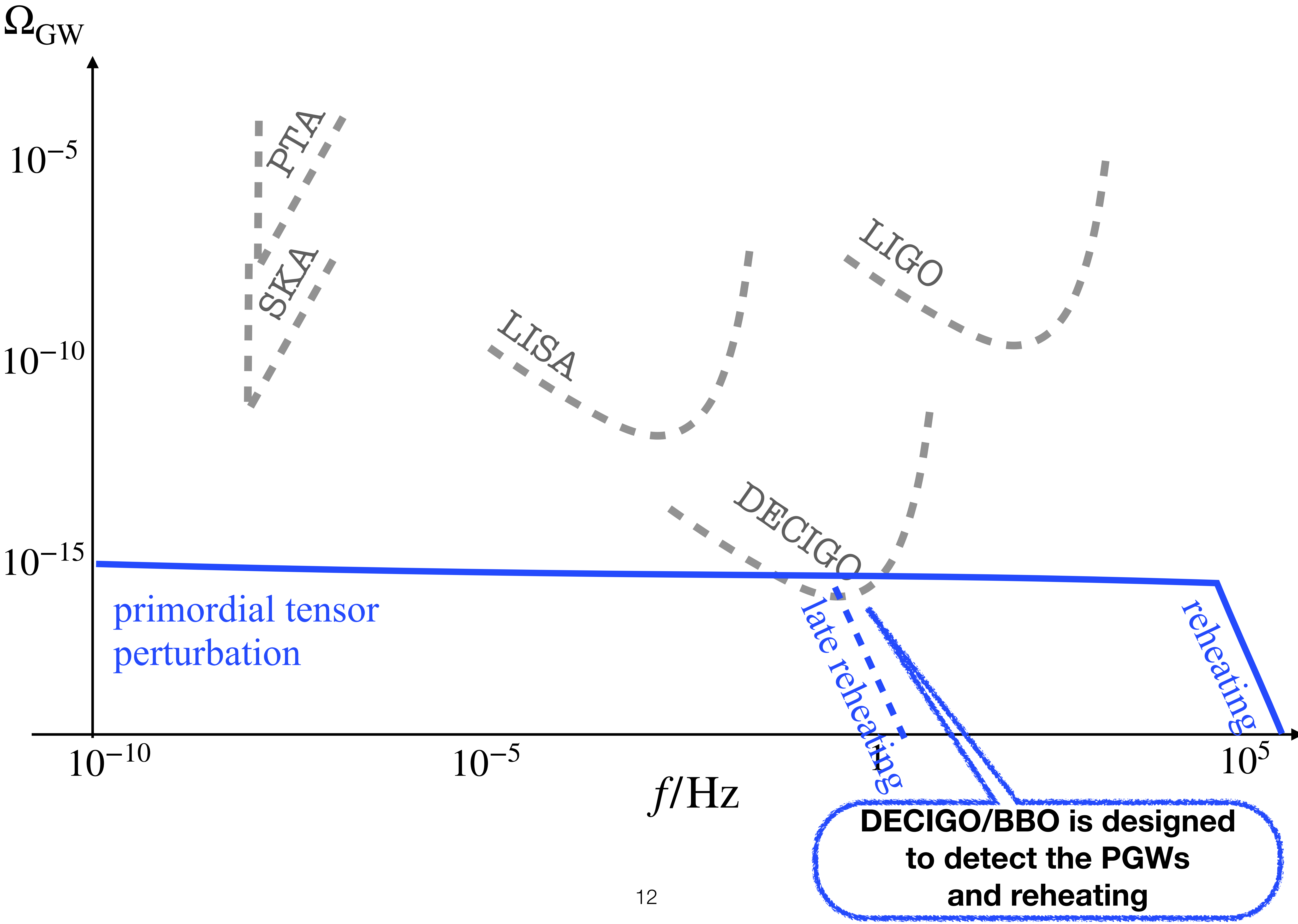
Possible SGWB Sources



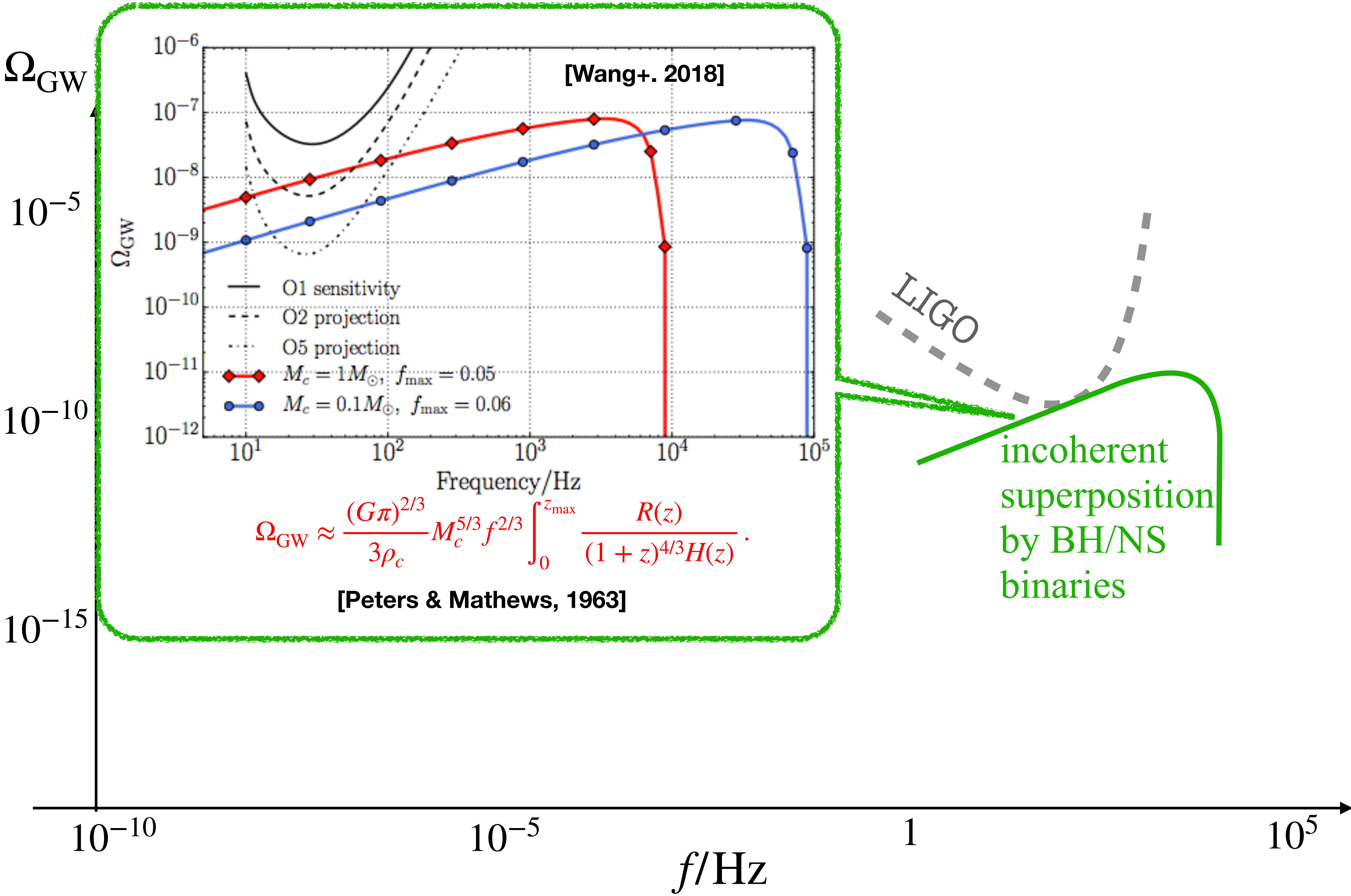

$\Omega_{\rm GW}$
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LIGO
DECIGO
Features in PGW can be used to detect the thermal history of the universe [Boyle and Steinhardt 05]
10^{-14}
$\Omega_{gw}(f)$
10^{-15}
10^{-16}
CMB
ELECTROWEAK (300 GeV)
BBN (1 MeV)
BBO/DECIGO (10^7 GeV)
10^9 GeV
REHEATING EFFECTS?
Neutrino damping
X-particle damping
Trace anomaly damping
10^{-15} 10^{-10} 10^{-5} 10^{0} 10^{5}
f (Hz)
primordial tensor perturbation
reheating
$f = \frac{1.65\times 10^{-7}}{2\pi}\frac{T}{\rm GeV}\left(\frac{g_{*s}}{100}\right)^{-1/3}\left(\frac{g_*}{100}\right)^{1/2}\ {\rm Hz}$
10^{-10}
10^{-5}
1
10^{5}
$f/{\rm Hz}$



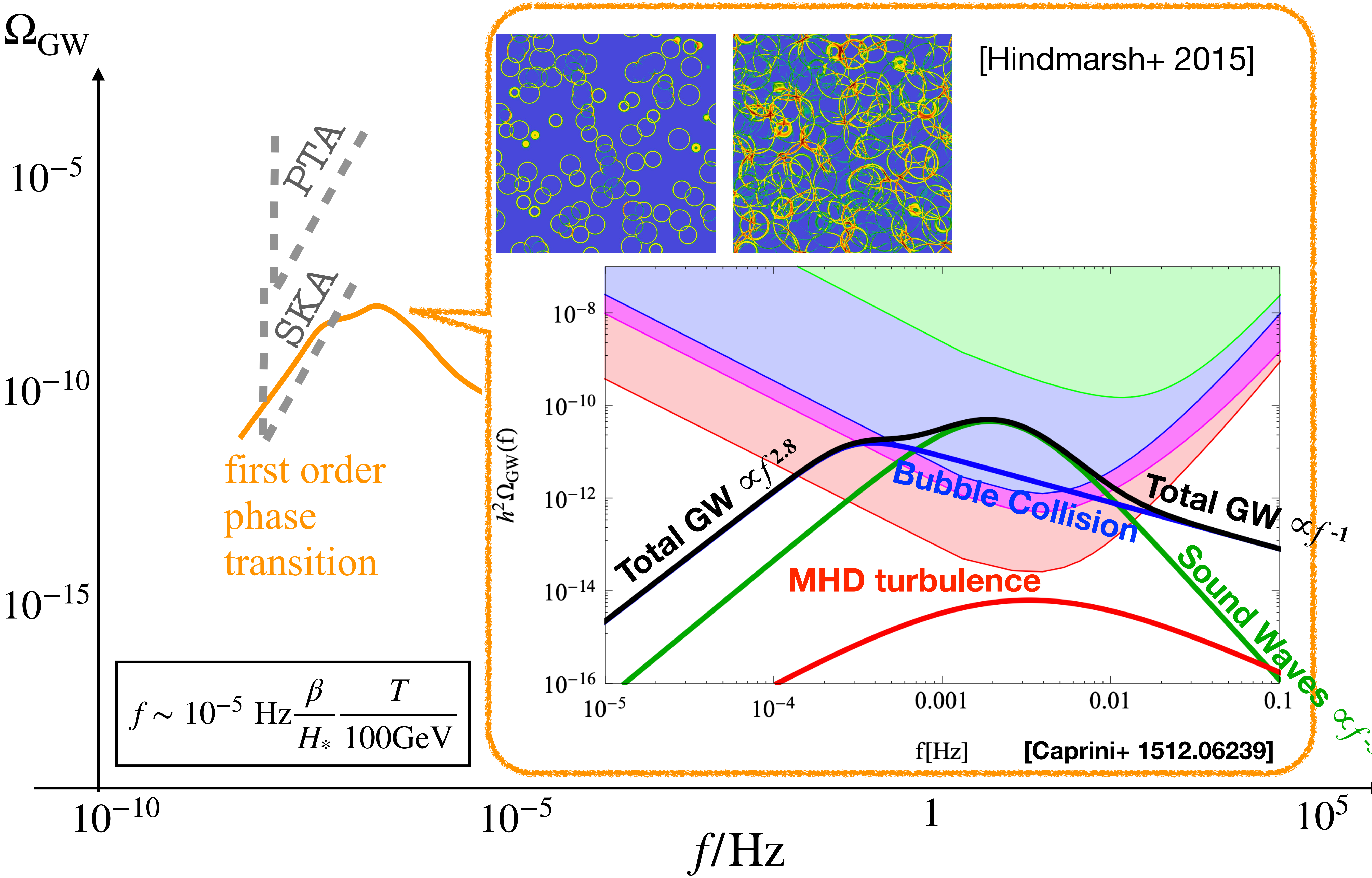
$$f = \frac{1.65 \times 10^{-7}}{2\pi} \frac{T}{\mathrm{GeV}} \left(\frac{g_{*s}}{100} \right)^{-1/3} \left(\frac{g_{*}}{100} \right)^{1/2} \mathrm{Hz}$$



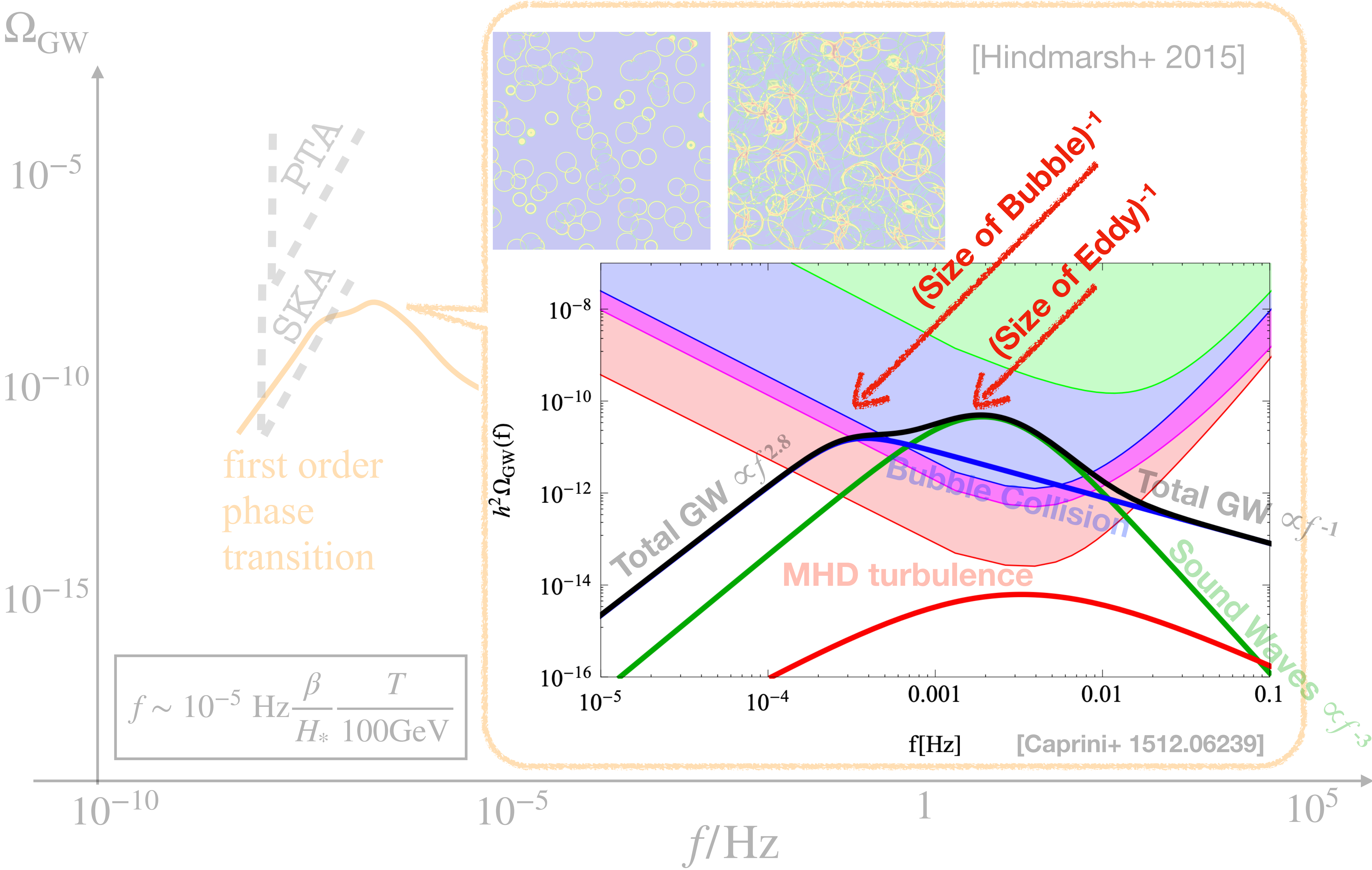
$\Omega_{\rm GW}$
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
LISA
LIGO
DECIGO
primordial tensor perturbation
late reheating
reheating
10^{-10}
10^{-5}
1
10^{5}
f/Hz
DECIGO/BBO is designed to detect the PGWs and reheating



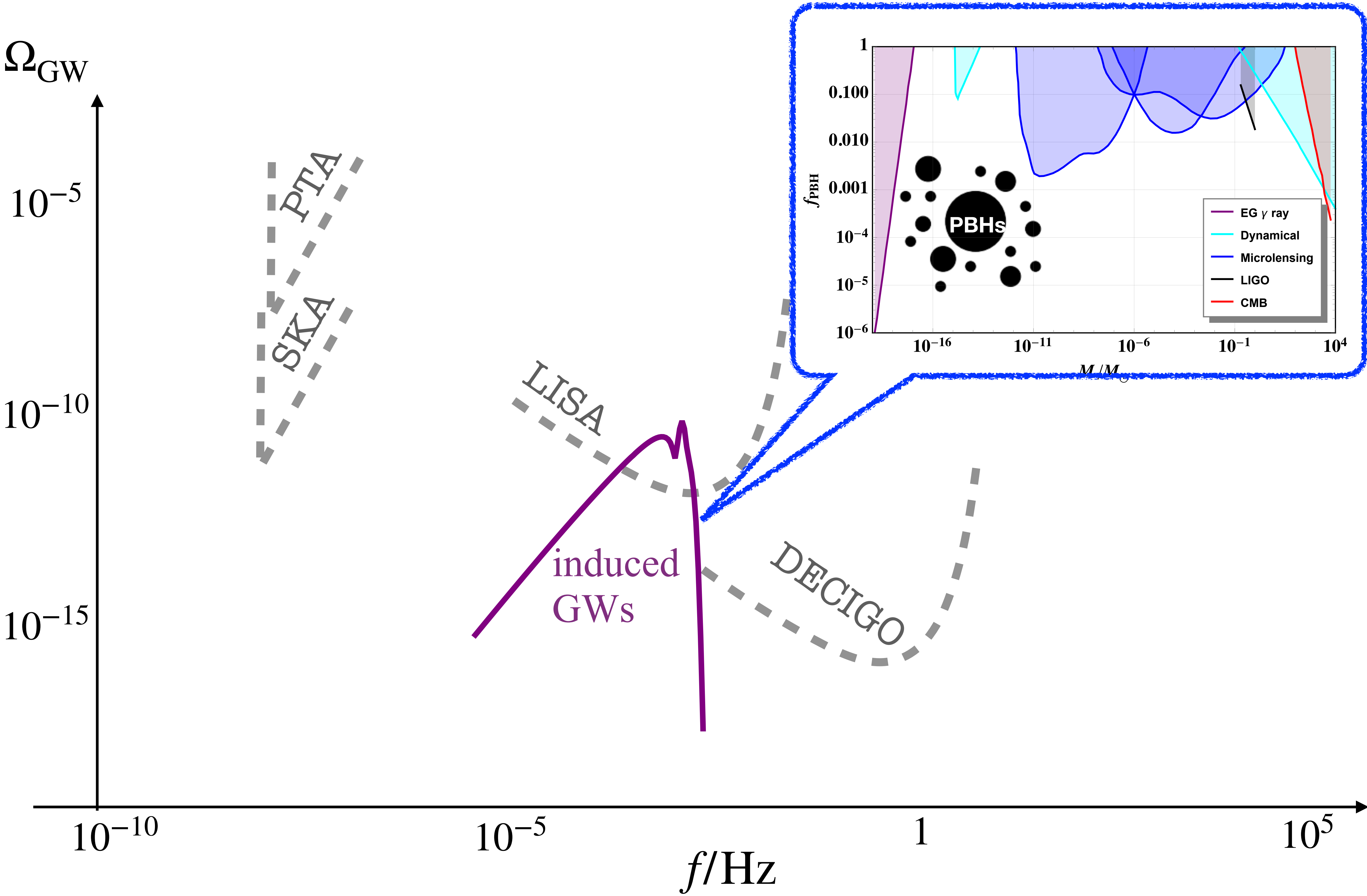
Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
10^{-10}
10^{-5}
1
10^{5}
f/Hz
LIGO
incoherent superposition by BH/NS binaries
[Wang+. 2018]
Ω_{GW}
Frequency/Hz
O1 sensitivity
O2 projection
O5 projection
$M_c = 1M_\odot, \; f_{\max} = 0.05$
$M_c = 0.1M_\odot, \; f_{\max} = 0.06$
$\Omega_{\mathrm{GW}} \approx \frac{(G\pi)^{2/3}}{3\rho_c} M_c^{5/3} f^{2/3} \int_0^{z_{\max}} \frac{R(z)}{(1+z)^{4/3} H(z)}.$
[Peters & Mathews, 1963]



Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
first order phase transition
$f \sim 10^{-5}\ \mathrm{Hz} \frac{\beta}{H_*} \frac{T}{100\mathrm{GeV}}$
[Hindmarsh+ 2015]
$h^2\Omega_{GW}(f)$
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
Total GW $\propto f^{2.8}$
Bubble Collision
Total GW $\propto f^{-1}$
MHD turbulence
Sound Waves $\propto f^{-3}$
10^{-5}
10^{-4}
0.001
0.01
0.1
f[Hz]
[Caprini+ 1512.06239]
10^{-10}
10^{-5}
1
10^{5}
f/Hz



$\Omega_{\rm GW}$
10^{-5}
10^{-10}
10^{-15}
PTA
SKA
first order phase transition
$f \sim 10^{-5}\ \mathrm{Hz} \frac{\beta}{H_*} \frac{T}{100\mathrm{GeV}}$
10^{-10}
10^{-5}
1
10^{5}
f/Hz
[Hindmarsh+ 2015]
(Size of Bubble)$^{-1}$
(Size of Eddy)$^{-1}$
$h^2\Omega_{\rm GW}(f)$
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-5}
10^{-4}
0.001
0.01
0.1
f[Hz]
Total GW $\propto f^{2.8}$
Bubble Collision
Total GW $\propto f^{-1}$
Sound Waves $\propto f^{-3}$
MHD turbulence
[Caprini+ 1512.06239]



Ω_{GW}
10^{-5}
10^{-10}
10^{-15}
10^{-10}
10^{-5}
1
10^{5}
f/Hz
PTA
SKA
LISA
DECIGO
induced GWs
1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
f_{PBH}
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_{\odot}$
PBHs
EG γ ray
Dynamical
Microlensing
LIGO
CMB

PTA discovered SGWB?

The NANOGrav 12.5-year Data Set:
Search For An Isotropic Stochastic Gravitational-Wave Background

ABSTRACT

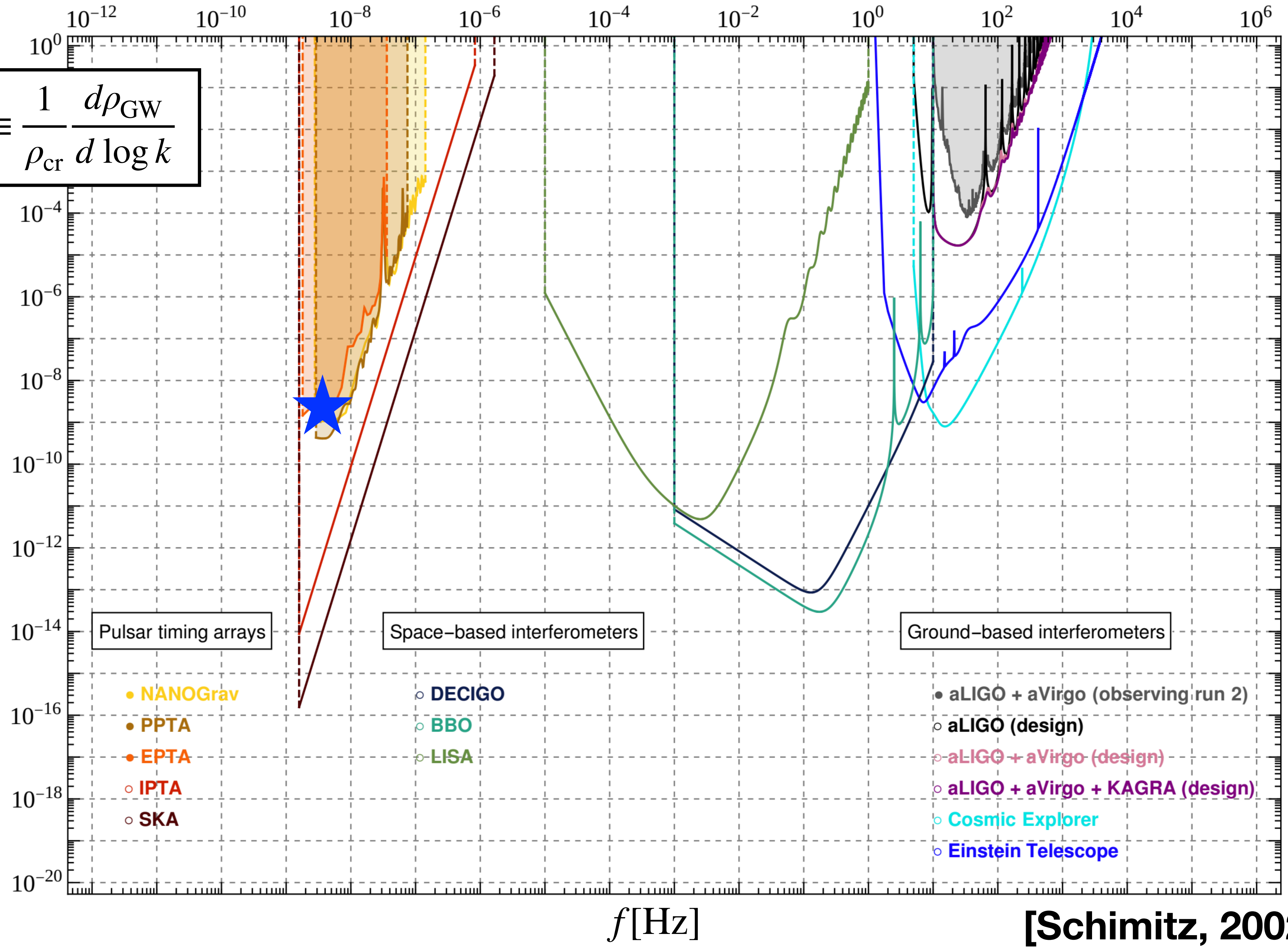
We search for an isotropic stochastic gravitational-wave background (GWB) in the 12.5-year pulsar timing data set collected by the North American Nanohertz Observatory for Gravitational Waves (NANOGrav). Our analysis finds strong evidence of a stochastic process, modeled as a power-law, with common amplitude and spectral slope across pulsars. The Bayesian posterior of the amplitude for a $f^{-2/3}$ power-law spectrum, expressed as characteristic GW strain, has median 1.92×10^{-15} and 5%–95% quantiles of 1.37–2.67×10^{-15} at a reference frequency of $f_{\rm yr} = 1\ \rm yr^{-1}$. The Bayes factor in favor of the common-spectrum process versus independent red-noise processes in each pulsar exceeds $10,000$. However, we find no statistically significant evidence that this process has quadrupolar spatial correlations, which we would consider necessary to claim a GWB detection consistent with General Relativity. We find that the process has neither monopolar nor dipolar correlations, which may arise from, for example, reference clock or solar-system ephemeris systematics, respectively. The amplitude posterior has significant support above previously reported upper limits; we explain this in terms of the Bayesian priors assumed for intrinsic pulsar red noise. We examine potential implications for the supermassive black hole binary population under the hypothesis that the signal is indeed astrophysical in nature.

[NANOGrav, 2009.04496]

$\Omega_{\rm GW} = 2.33 \times 10^{-9}$ $@f = 3.17 \times 10^{-8}/\rm hz$

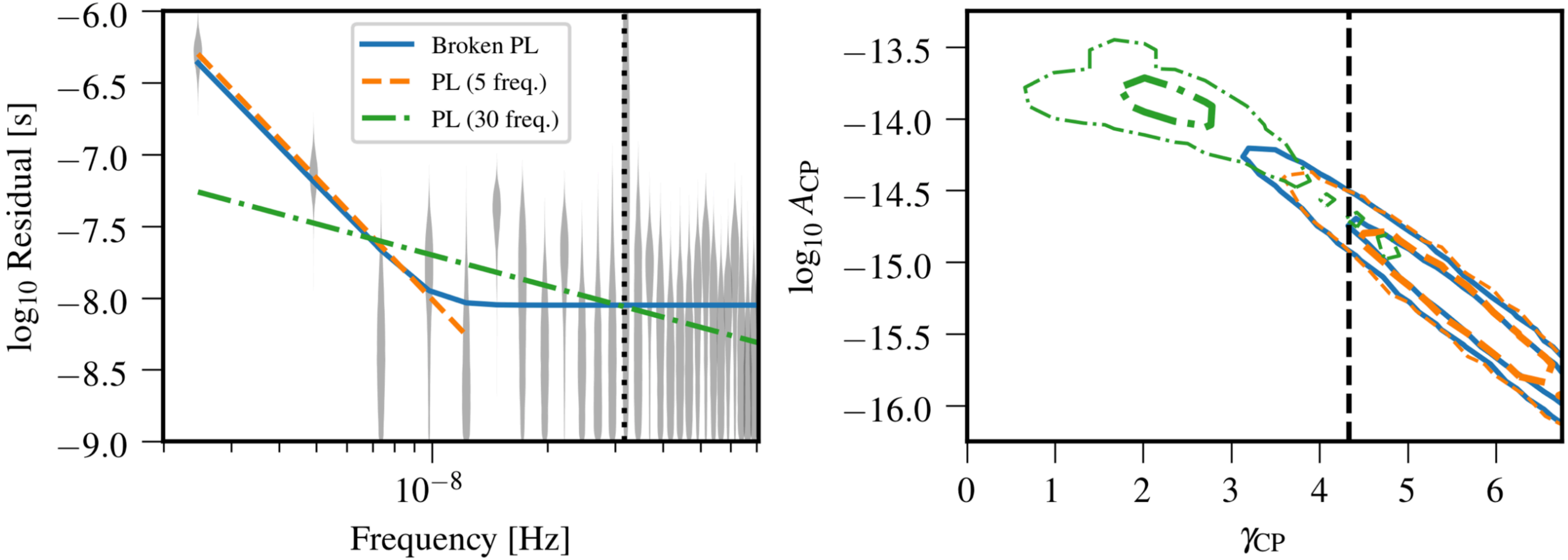
PTA discovered SGWB?

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_{\rm cr}} \frac{d\rho_{\rm GW}}{d\log k}$$



[Schimitz, 2002.04615]

PTA discovered SGWB?

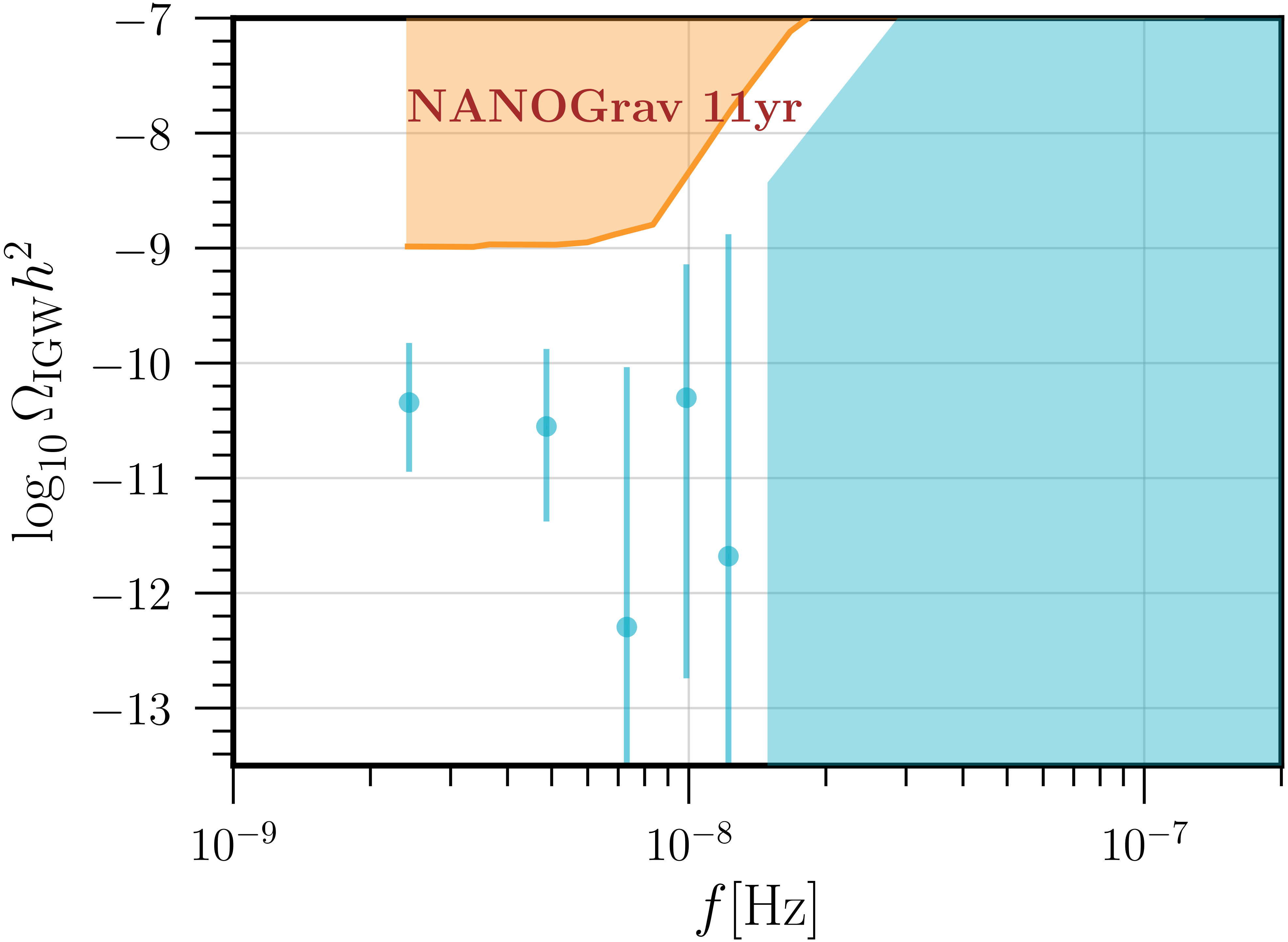


$$\Omega_{\rm GW} = \frac{2\pi^2 f_{\rm yr}^2}{3H_0^2} A_{\rm SGWB}^2 \left(\frac{f}{f_{\rm yr}}\right)^{5-\gamma}$$

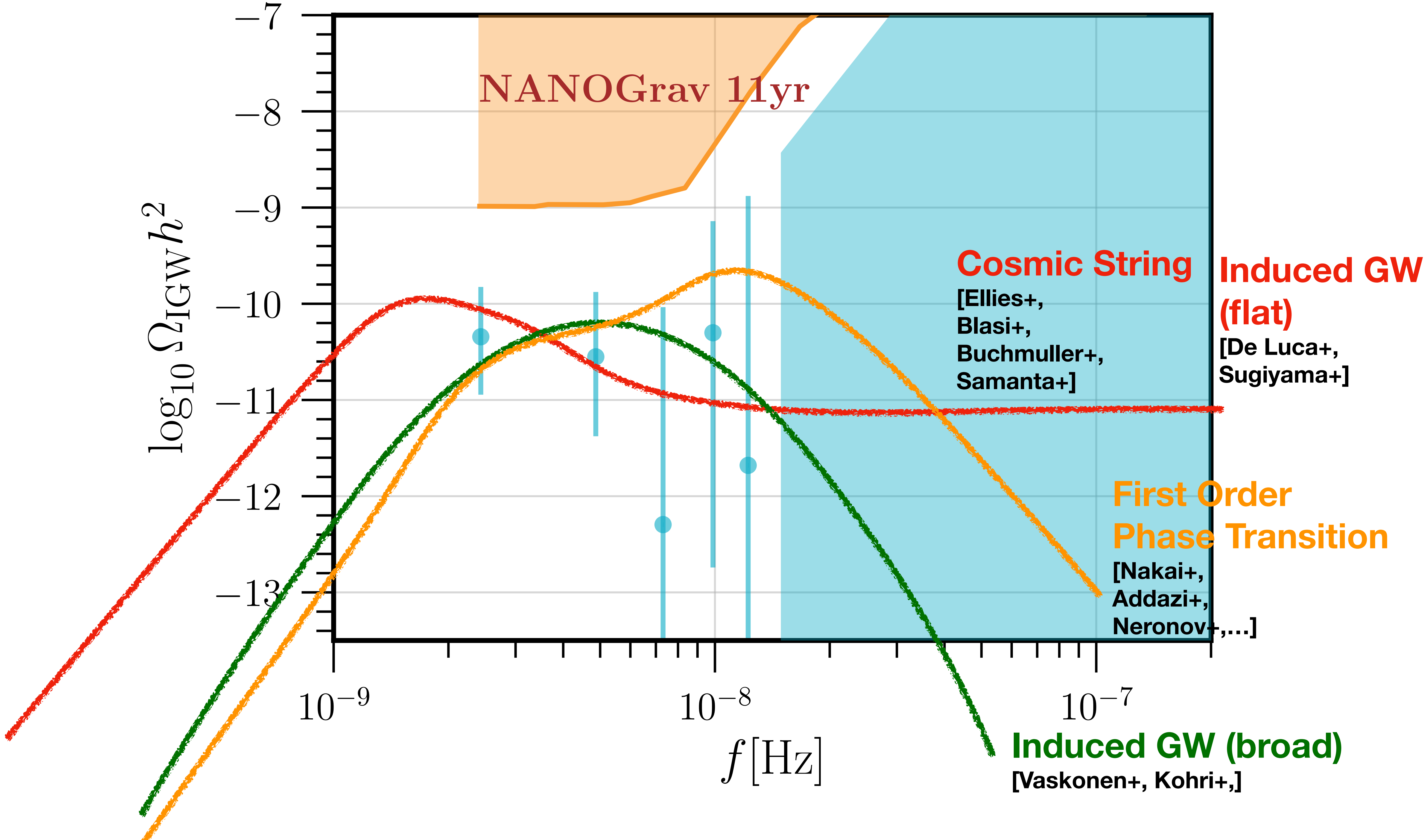
$$f_{\rm yr} = 3.17 \times 10^{-8}\ {\rm Hz}\,.$$

[NANOGrav, 2009.04496]

PTA discovered SGWB?



PTA discovered SGWB?



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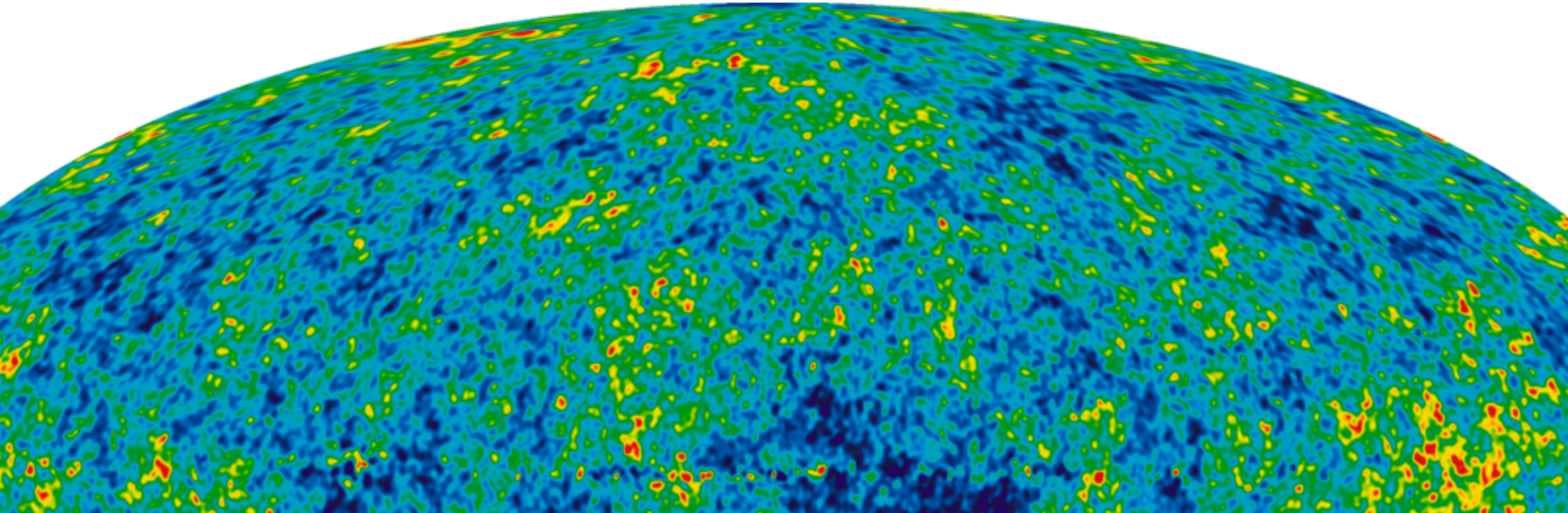
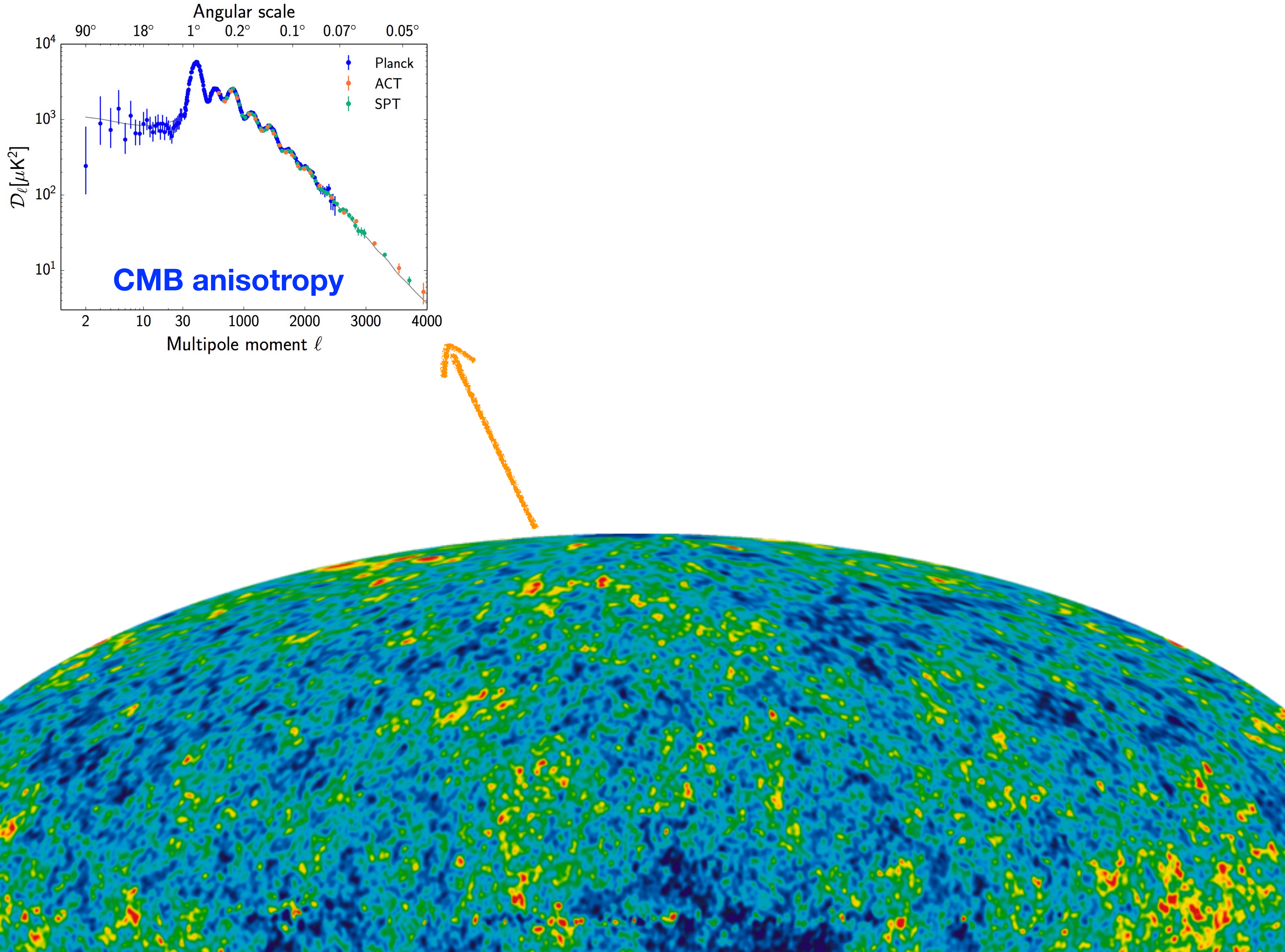
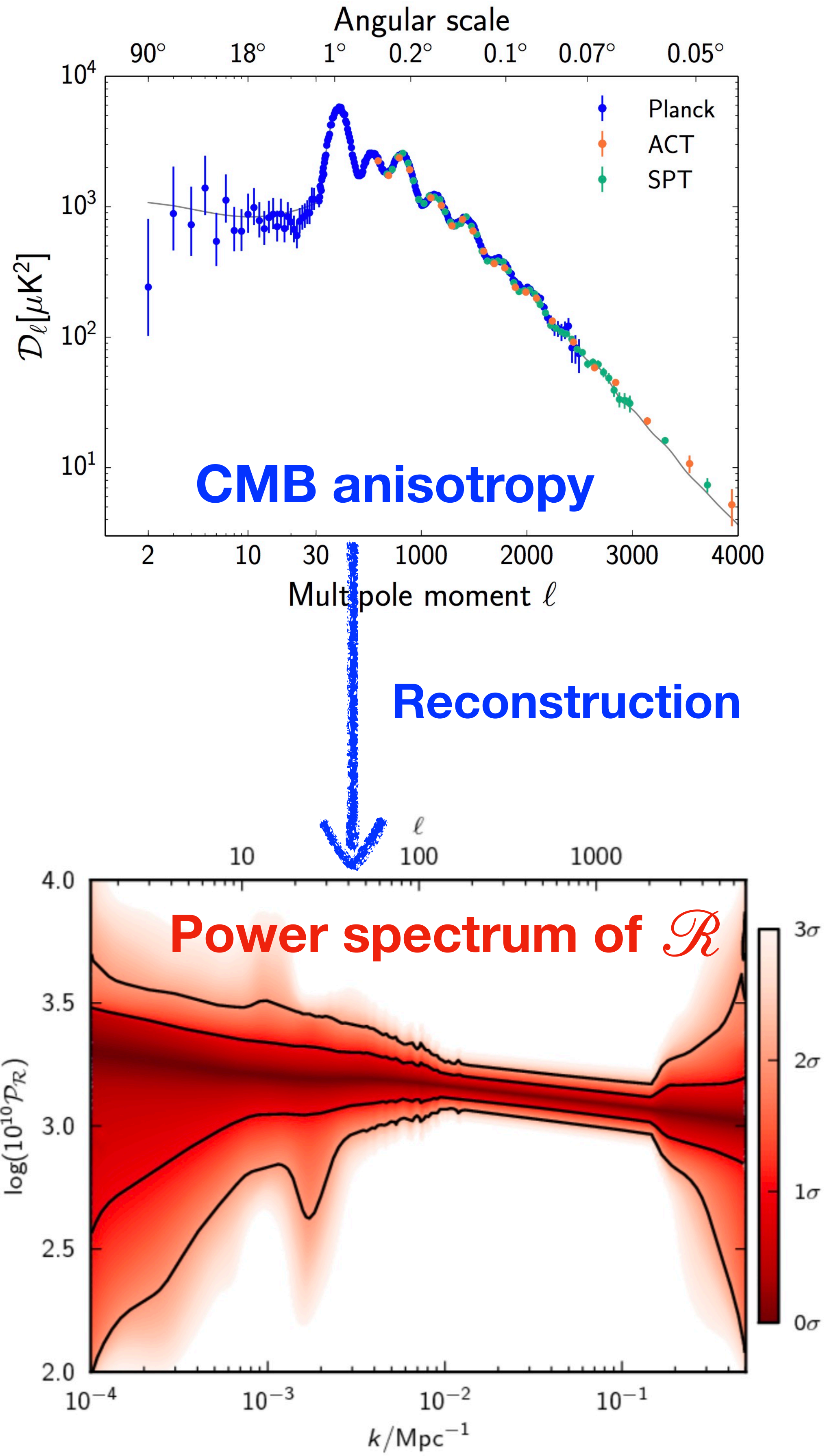
Introduction to Induced GW (and PBH)

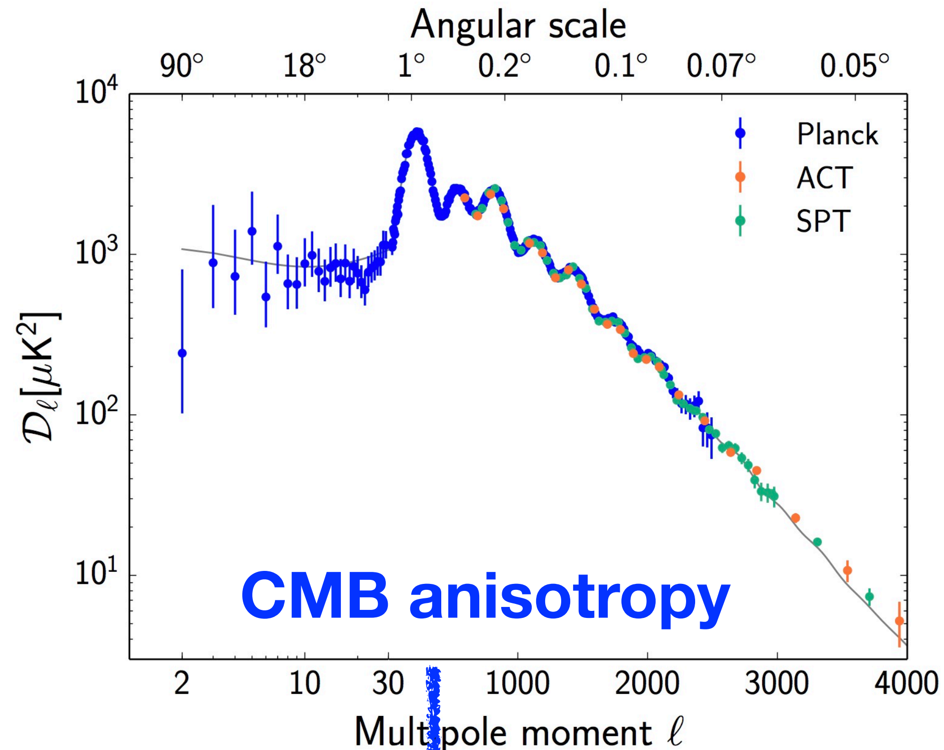




Angular scale
90°
18°
1°
0.2°
0.1°
0.07°
0.05°
10^4
10^3
10^2
10^1
$\mathcal{D}_\ell [\mu K^2]$
Planck
ACT
SPT
CMB anisotropy
2
10
30
1000
2000
3000
4000
Multipole moment ℓ

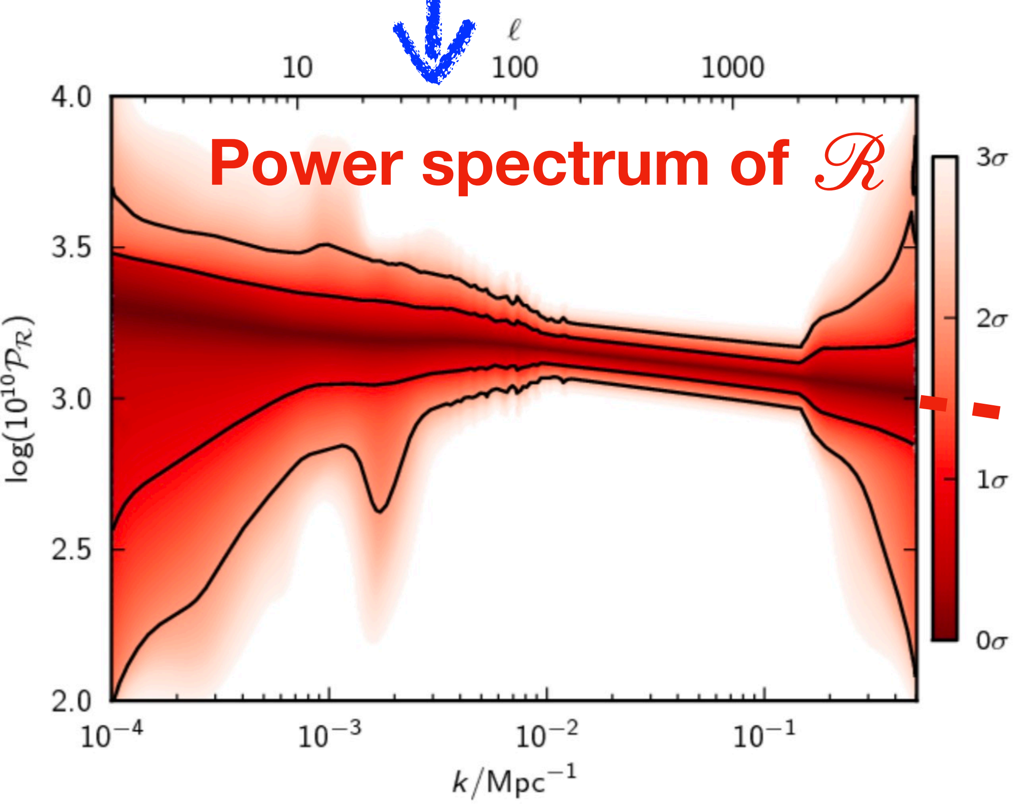


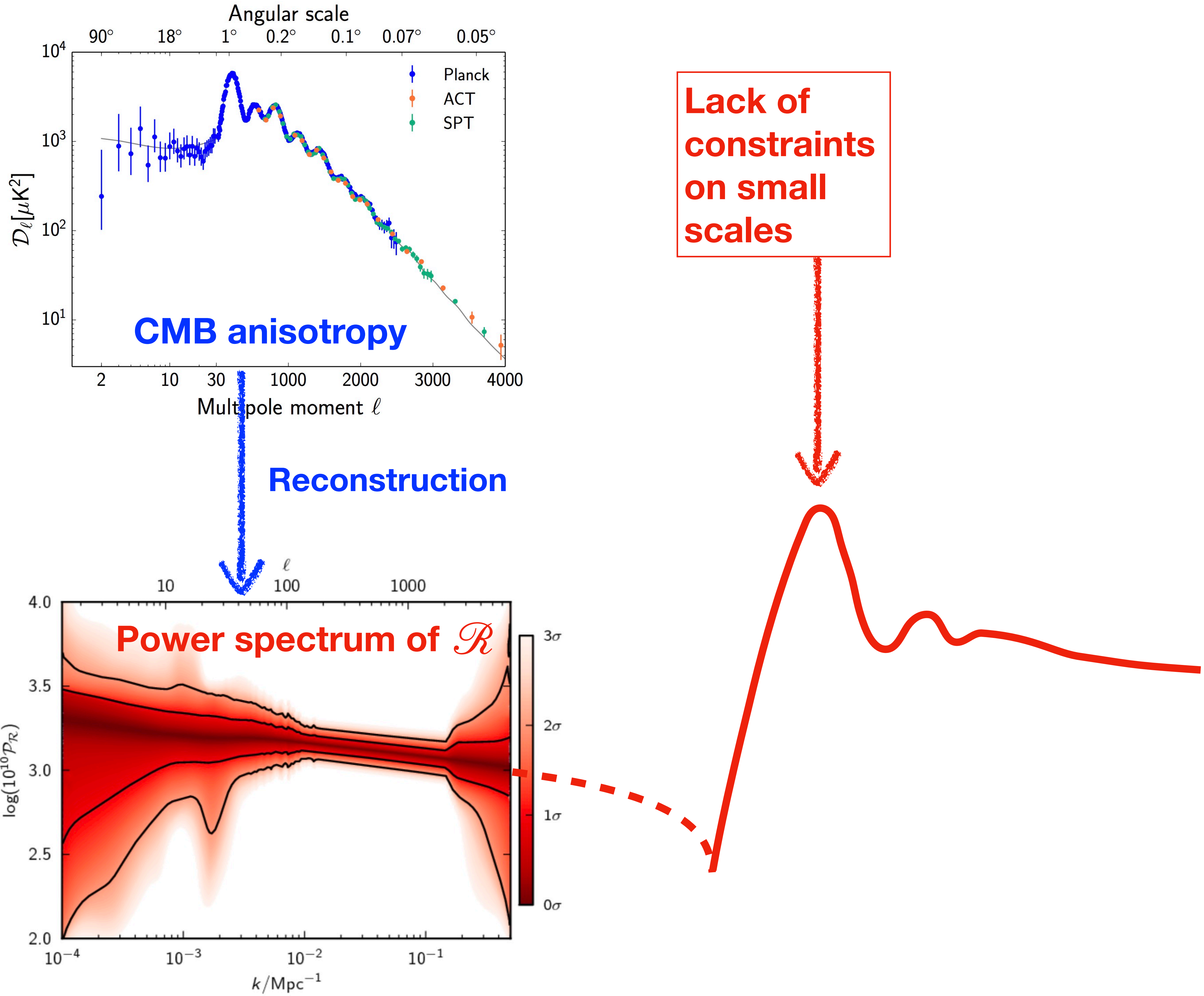
[Planck 2015, 4 knots]

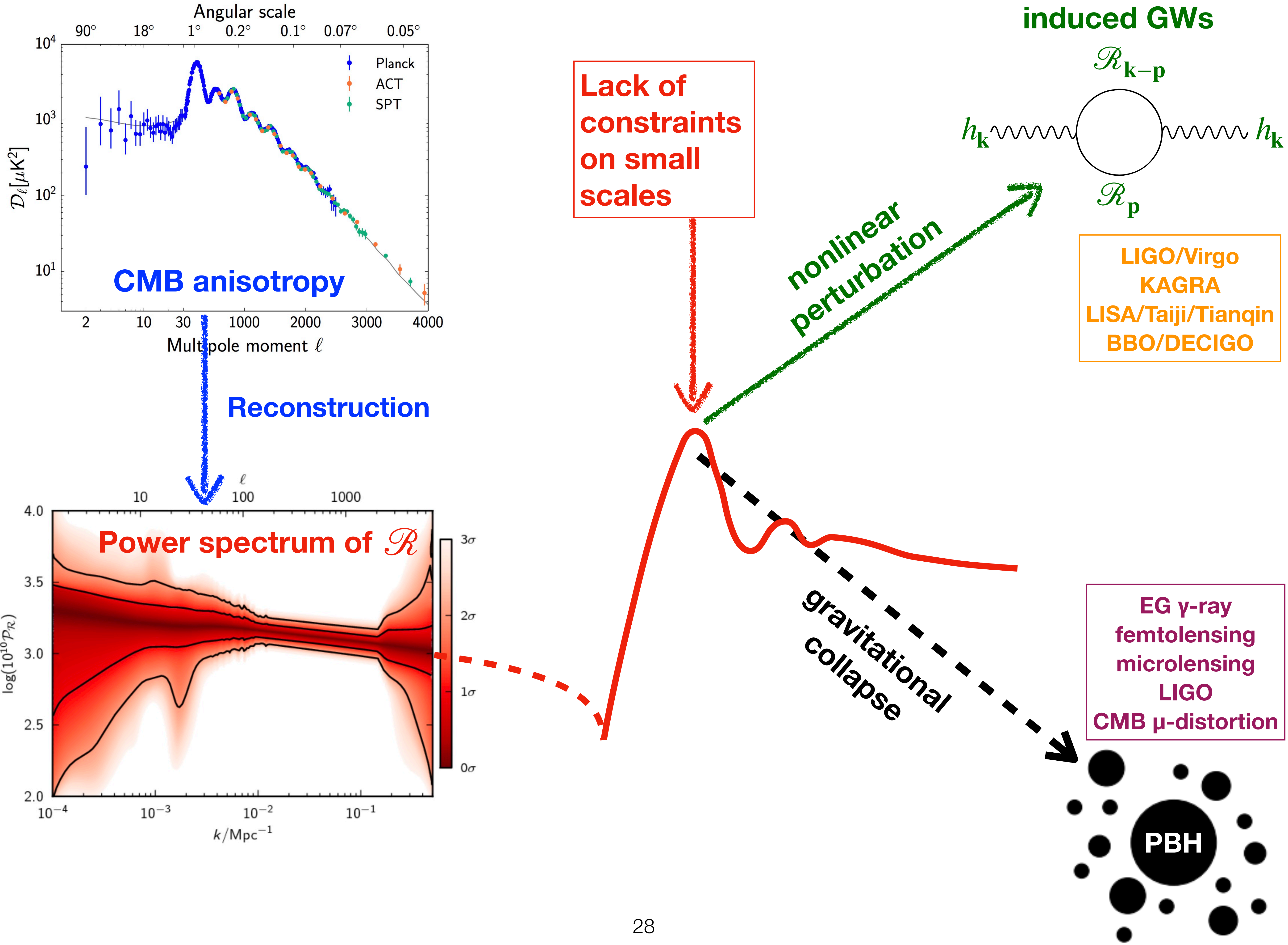


Reconstruction

Lack of constraints on small scales




Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell[\mu K^2]$
10^4 10^3 10^2 10^1
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
CMB anisotropy
Reconstruction
Lack of constraints on small scales
ℓ
10 100 1000
Power spectrum of $\mathscr{R}$
$\log(10^{10}\mathcal{P}_\mathcal{R})$
4.0 3.5 3.0 2.5 2.0
3σ 2σ 1σ 0σ
10^{-4} 10^{-3} 10^{-2} 10^{-1}
k/Mpc^{-1}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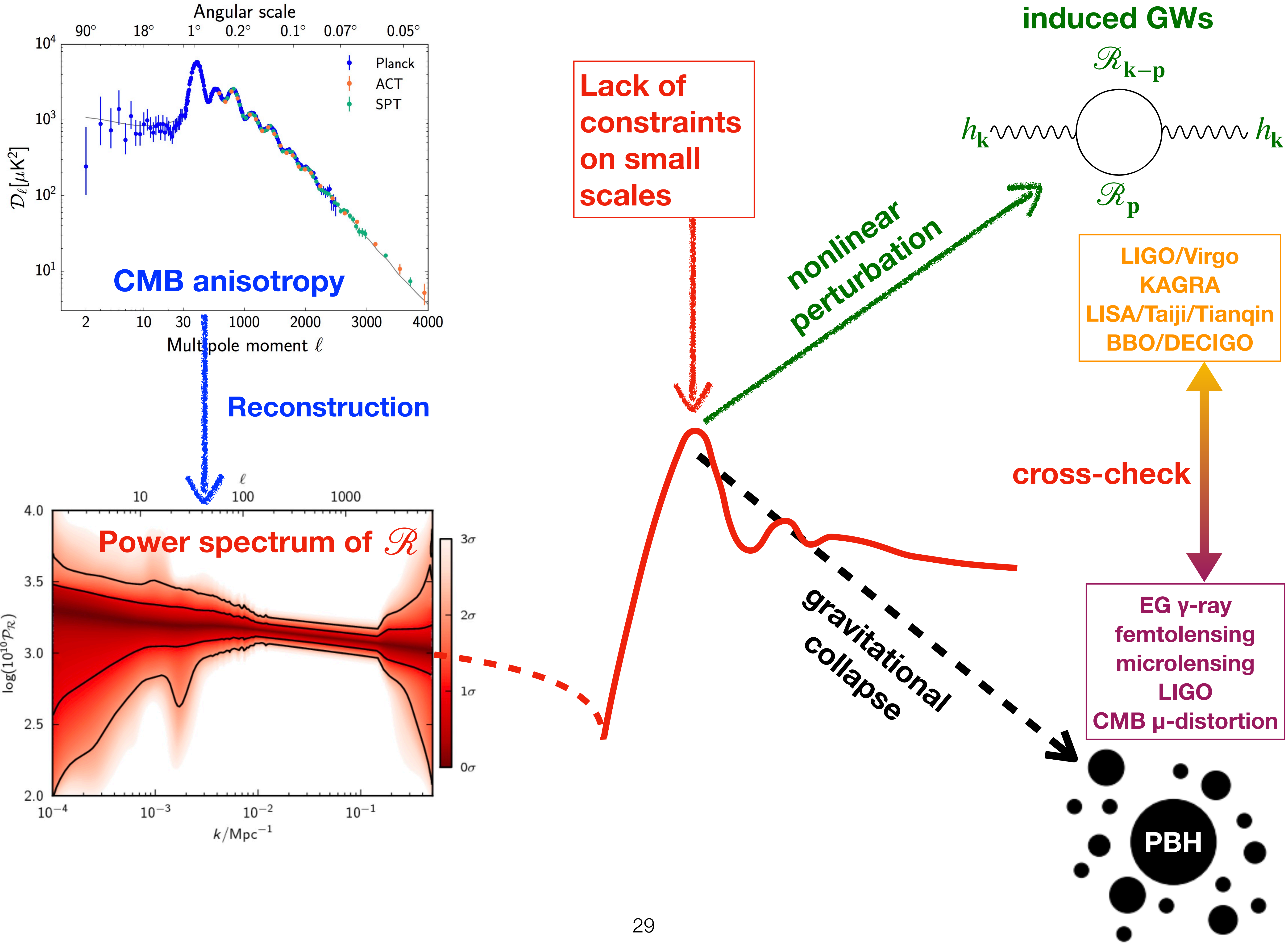
Multipole moment ℓ
CMB anisotropy
Reconstruction
ℓ
10 100 1000
Power spectrum of $\mathscr{R}$
$\log(10^{10}\mathcal{P}_\mathcal{R})$
k/Mpc^{-1}
3σ 2σ 1σ 0σ
Lack of constraints on small scales
nonlinear perturbation
induced GWs
$\mathscr{R}_{\mathbf{k-p}}$
$h_\mathbf{k}$
$h_\mathbf{k}$
$\mathscr{R}_\mathbf{p}$
LIGO/Virgo
KAGRA
LISA/Taiji/Tianqin
BBO/DECIGO
gravitational collapse
EG γ-ray
femtolensing
microlensing
LIGO
CMB μ-distortion
PBH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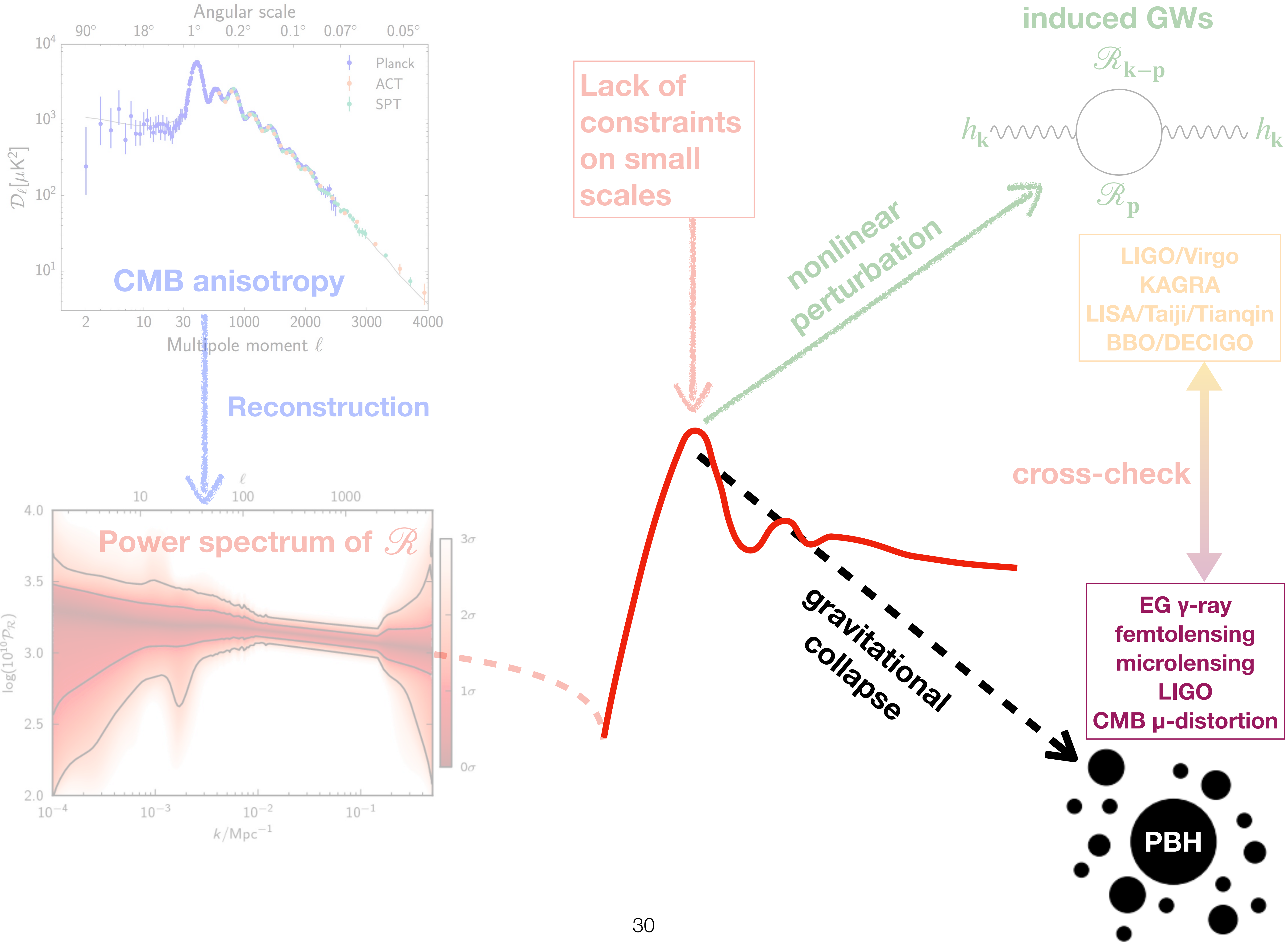
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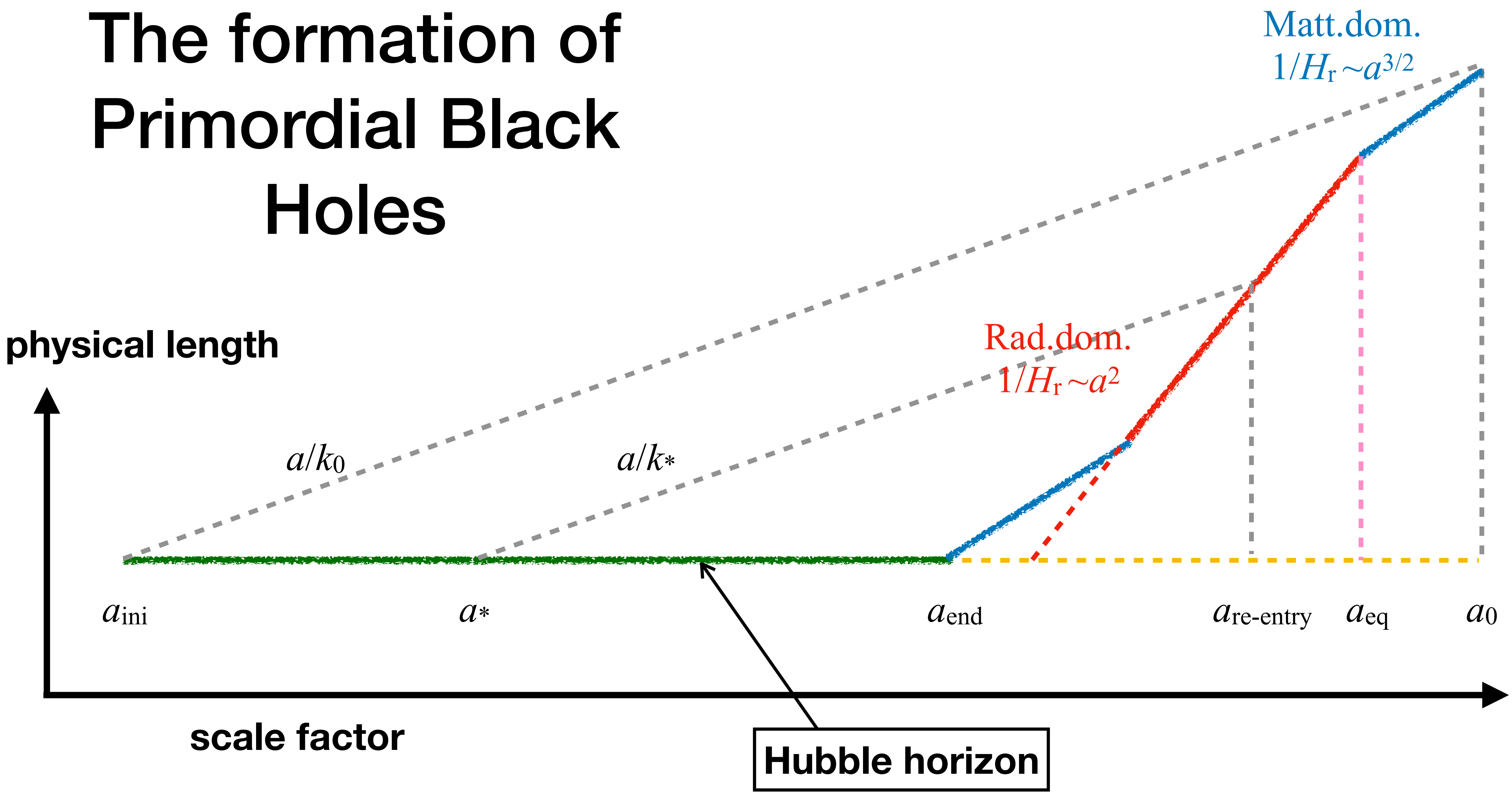
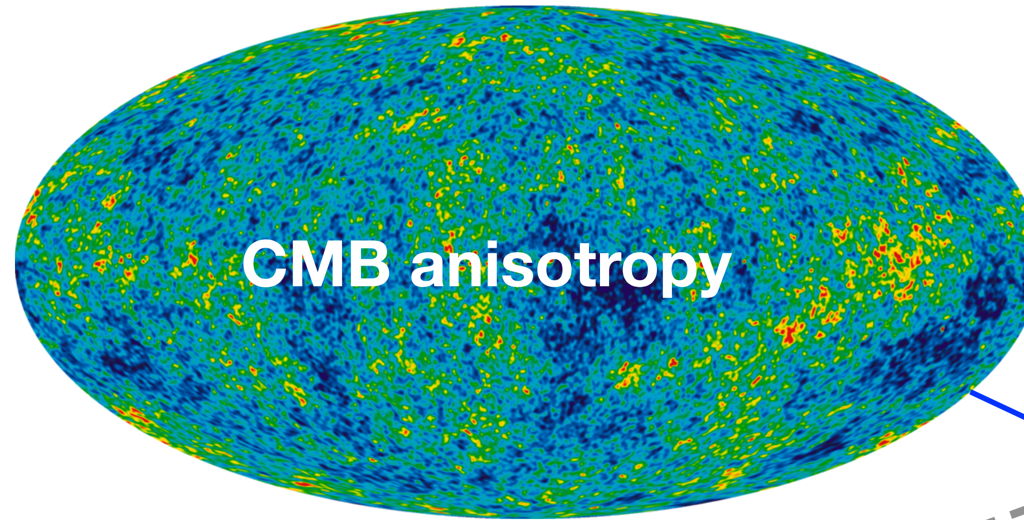
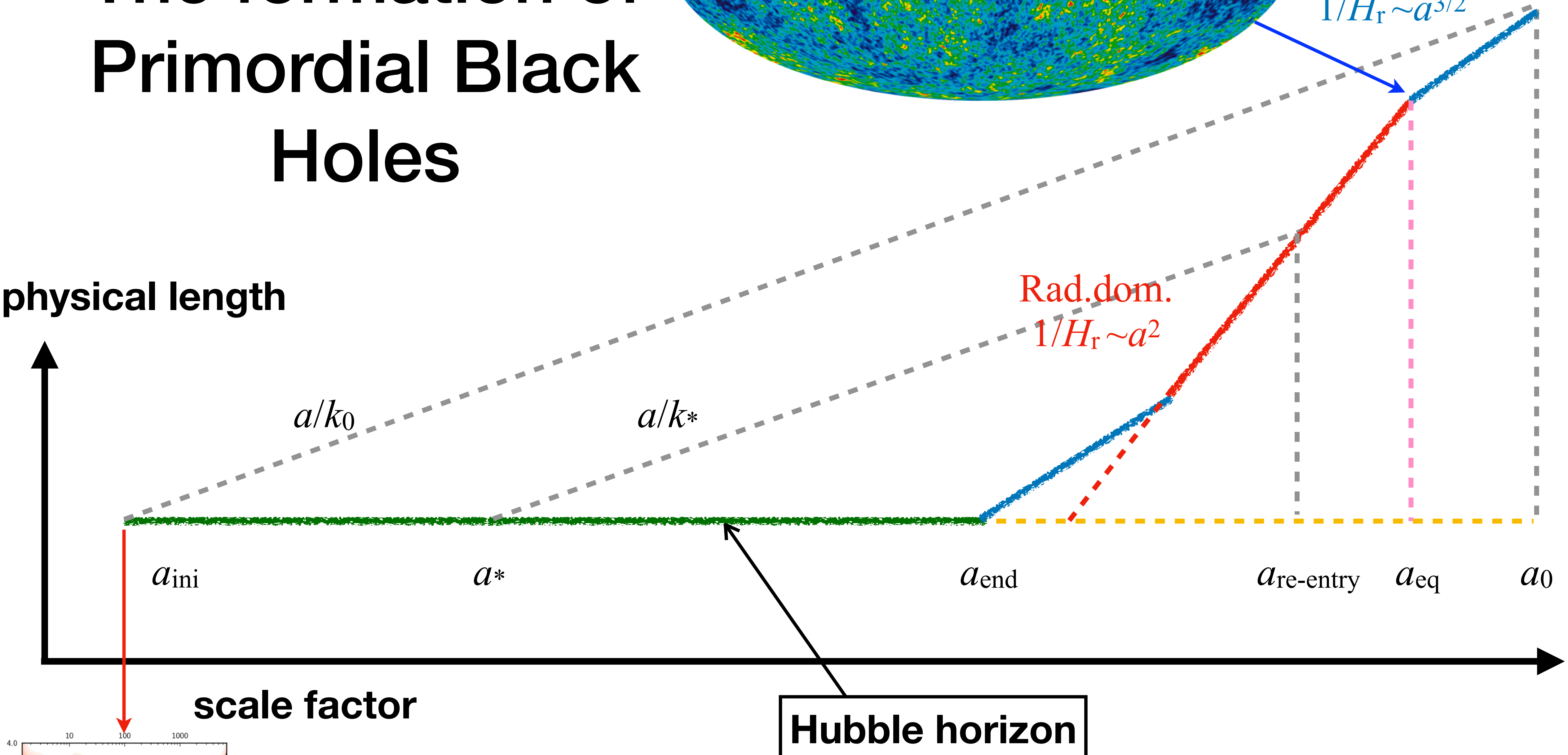
PBH

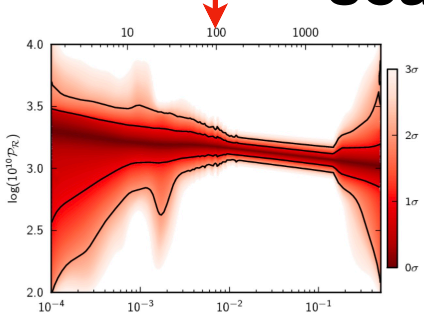
The formation of Primordial Black Holes



The formation of Primordial Black Holes

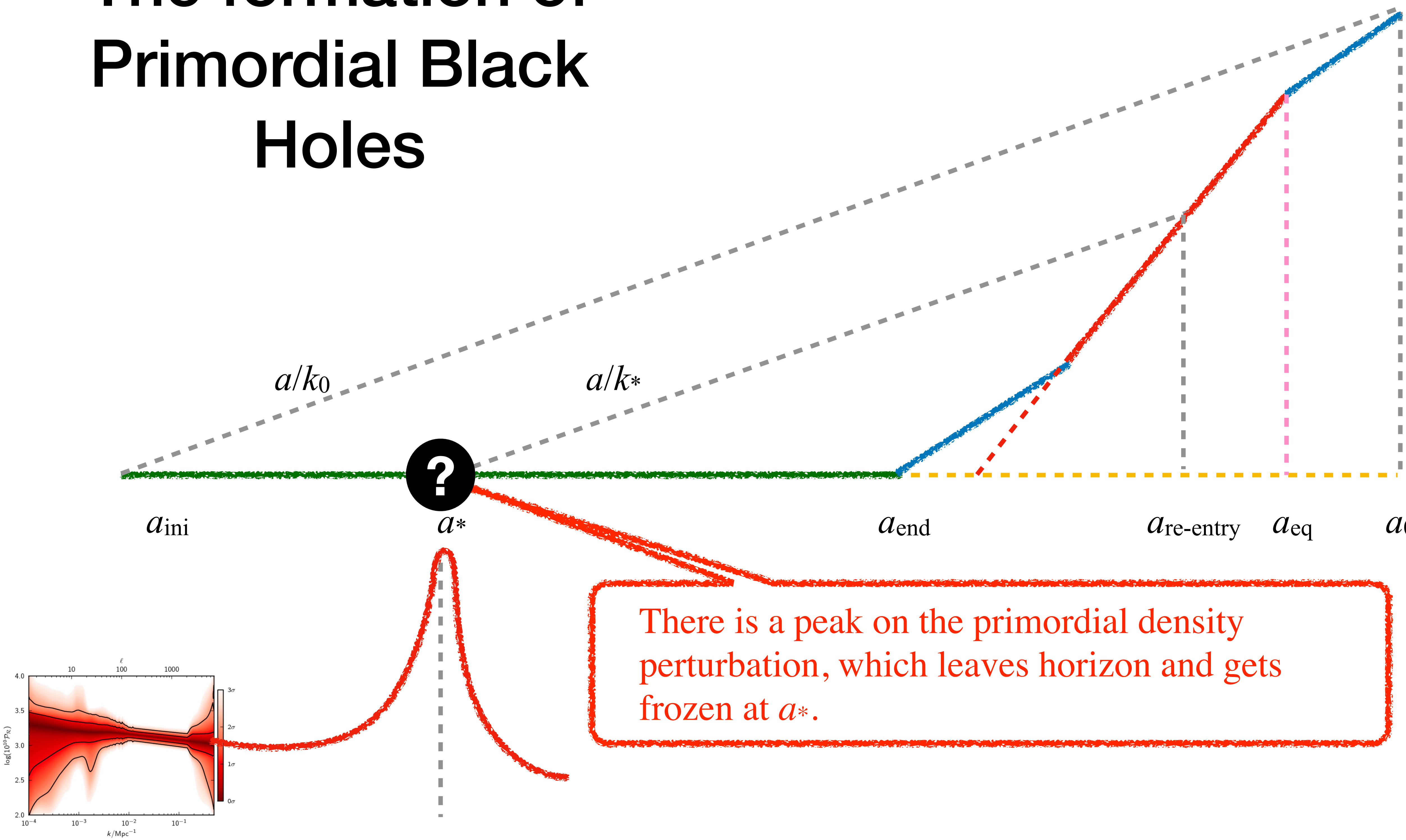






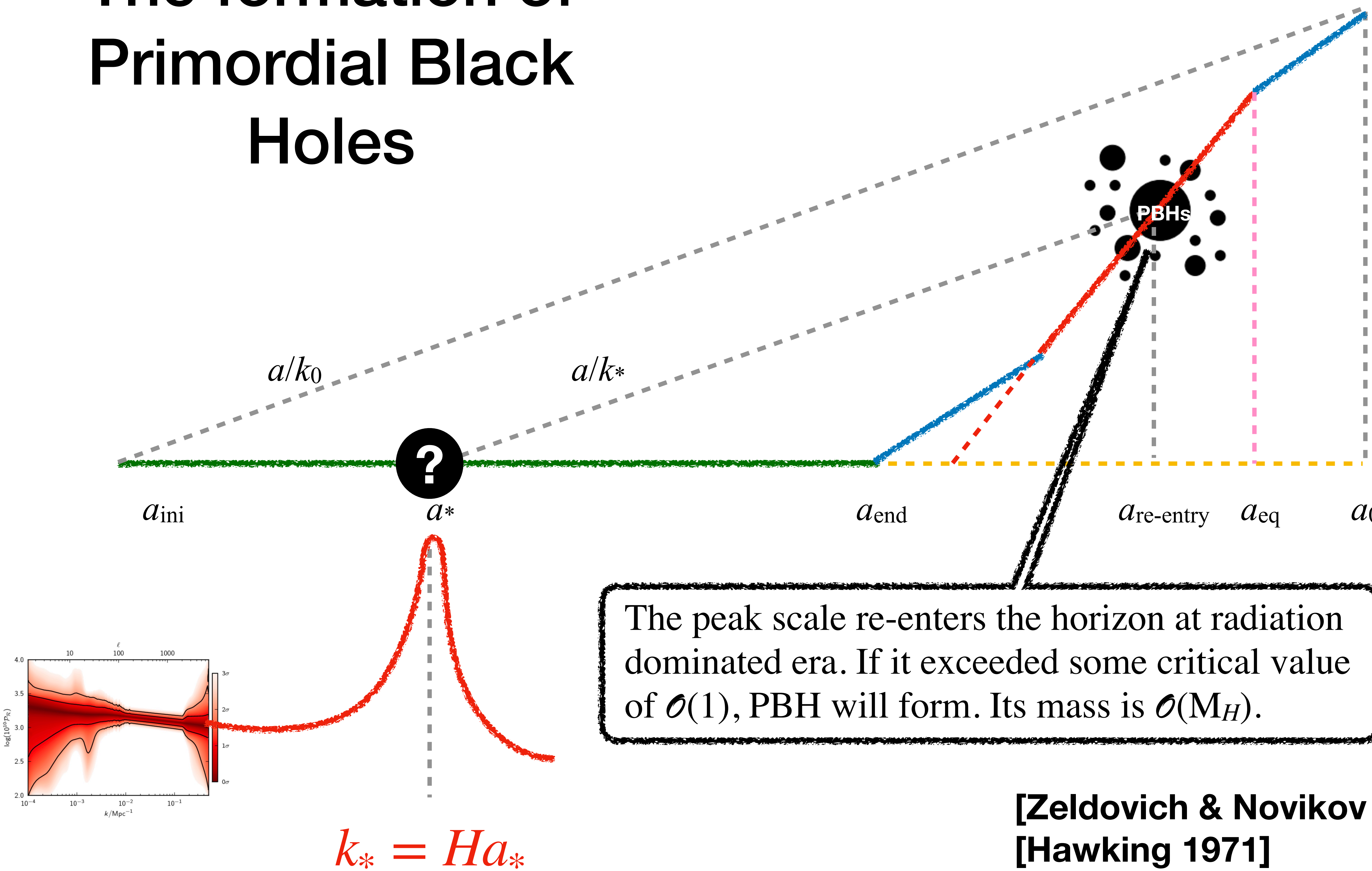
Power spectrum of $\mathscr{R}$

The formation of Primordial Black Holes

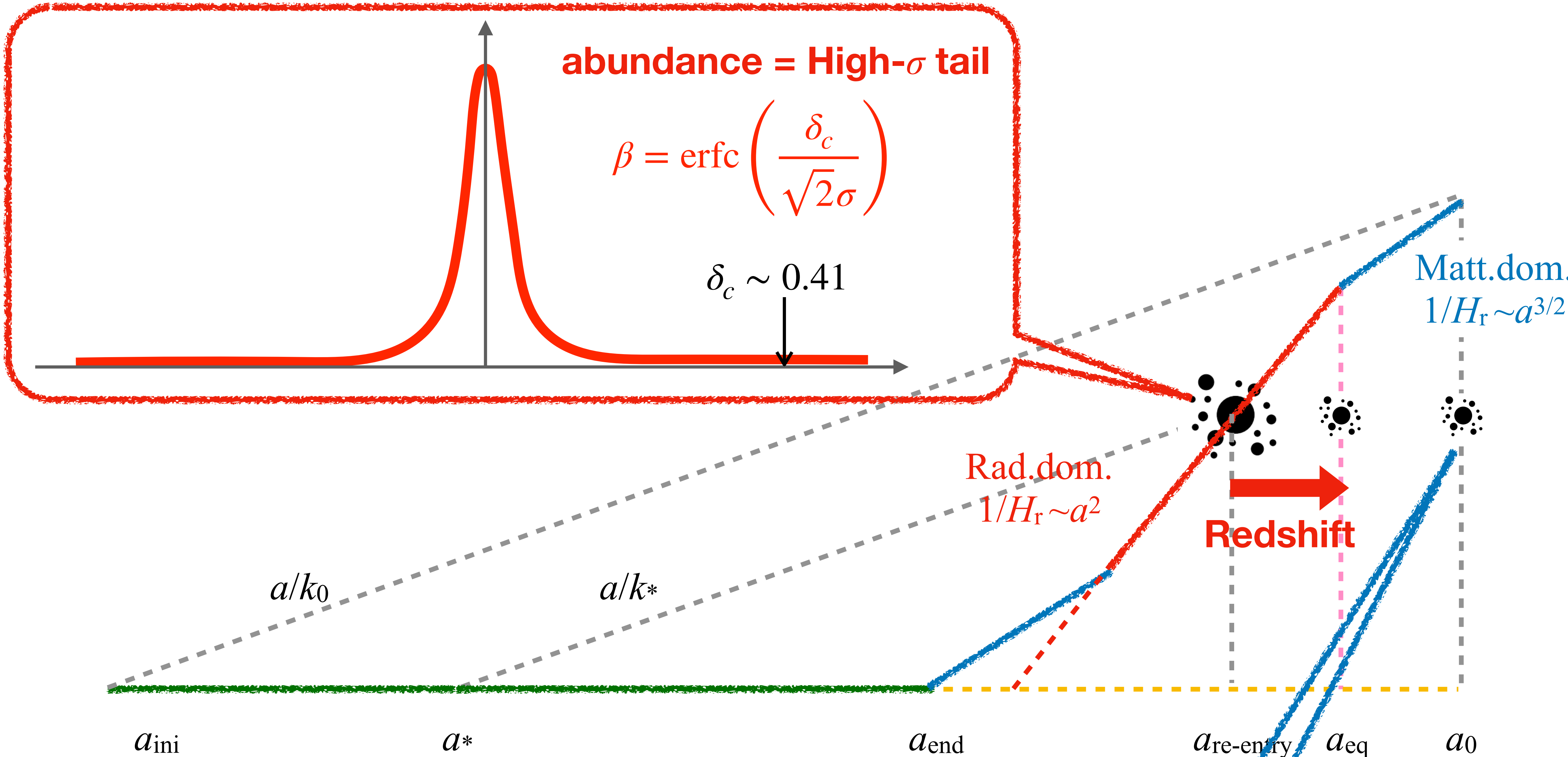


$$k_* = Ha_*$$

The formation of Primordial Black Holes



[Zeldovich & Novikov 1966]
[Hawking 1971]
[Carr & Hawking 1974]

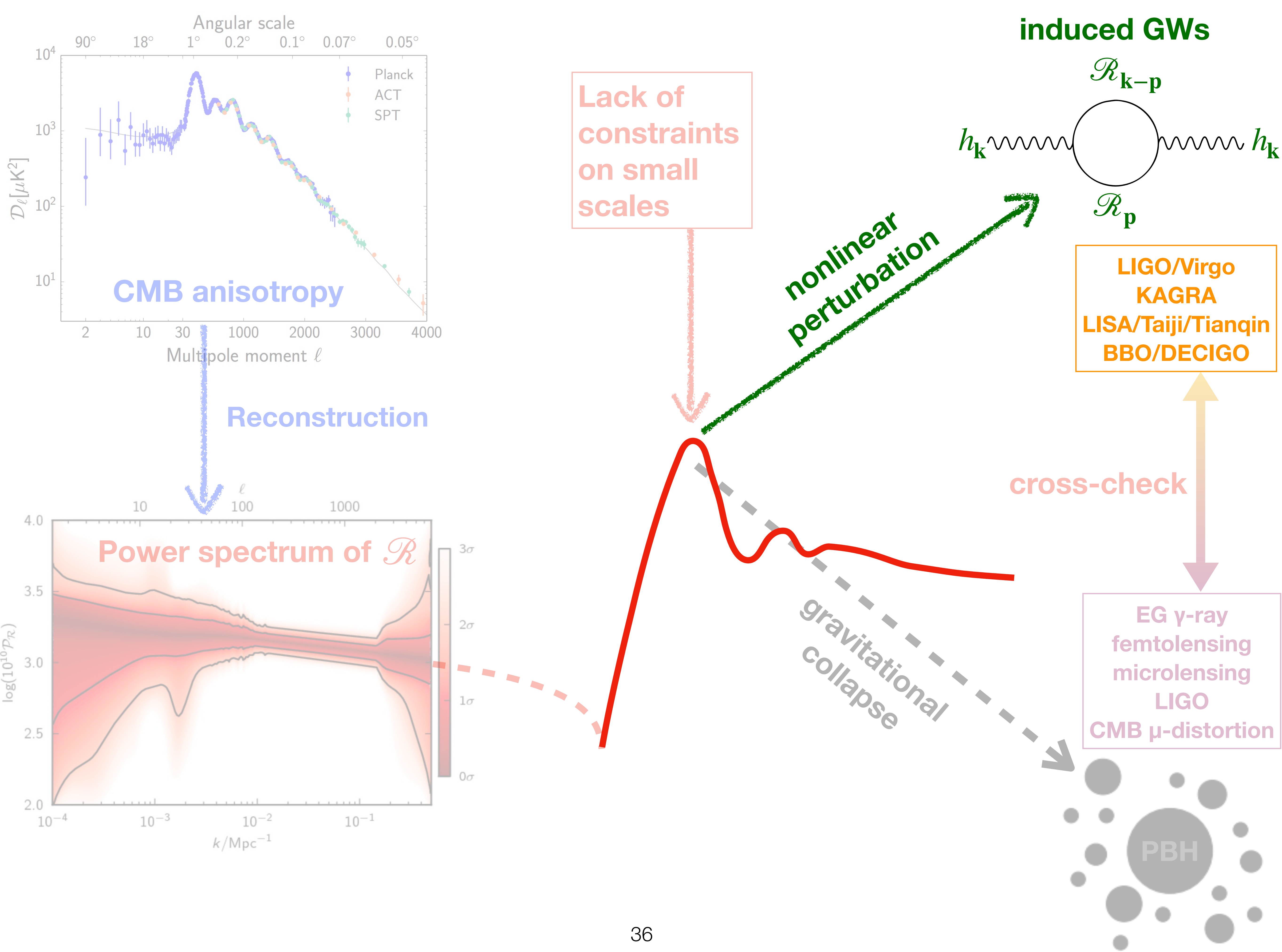


$$\Omega_\mathrm{PBH} = \beta \frac{a_\mathrm{eq}}{a_\mathrm{re}} = \beta \frac{a_\mathrm{eq}}{a_0} \frac{a_0}{a_\mathrm{re}} \simeq \beta \Omega_r (1 + z_\mathrm{re}(M))$$

$$M = \frac{c^3}{G H_\mathrm{re}} = \frac{c^3}{G \Omega_r^{1/2} (1+z)^2 H_0}$$

Redshifted after equality:

$$f_\mathrm{PBH} \equiv \frac{\Omega_\mathrm{PBH}}{\Omega_\mathrm{CDM}} = 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_\odot}\right)^{-1/2}$$



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$h_\mathbf{k}$ $\mathscr{R}_{\mathbf{k}-\mathbf{p}}$ $\mathscr{R}_\mathbf{p}$ $h_\mathbf{k}$
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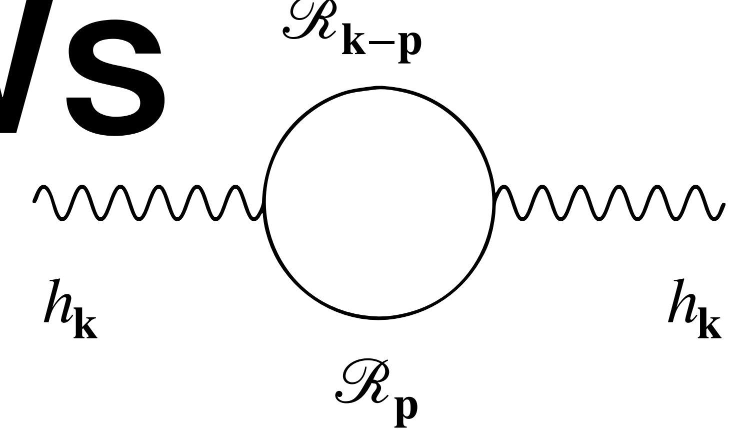
LIGO
CMB μ-distortion
PBH

Induced GWs



- EoM of the tensor perturbation:

$$h''_{\mathbf{k}} + 2\mathscr{H} h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} \sim \int d^3p \Phi_{\mathbf{p}} \Phi_{\mathbf{k}-\mathbf{p}} \times (\text{transfer functions})$$

- The induced (secondary) GWs are

$$h_{\mathbf{k}} \sim \int d\eta \times (\text{Green function}) \int d^3p \times (\text{Transfer function}) \times \Phi_{\mathbf{p}} \Phi_{\mathbf{k}-\mathbf{p}}$$

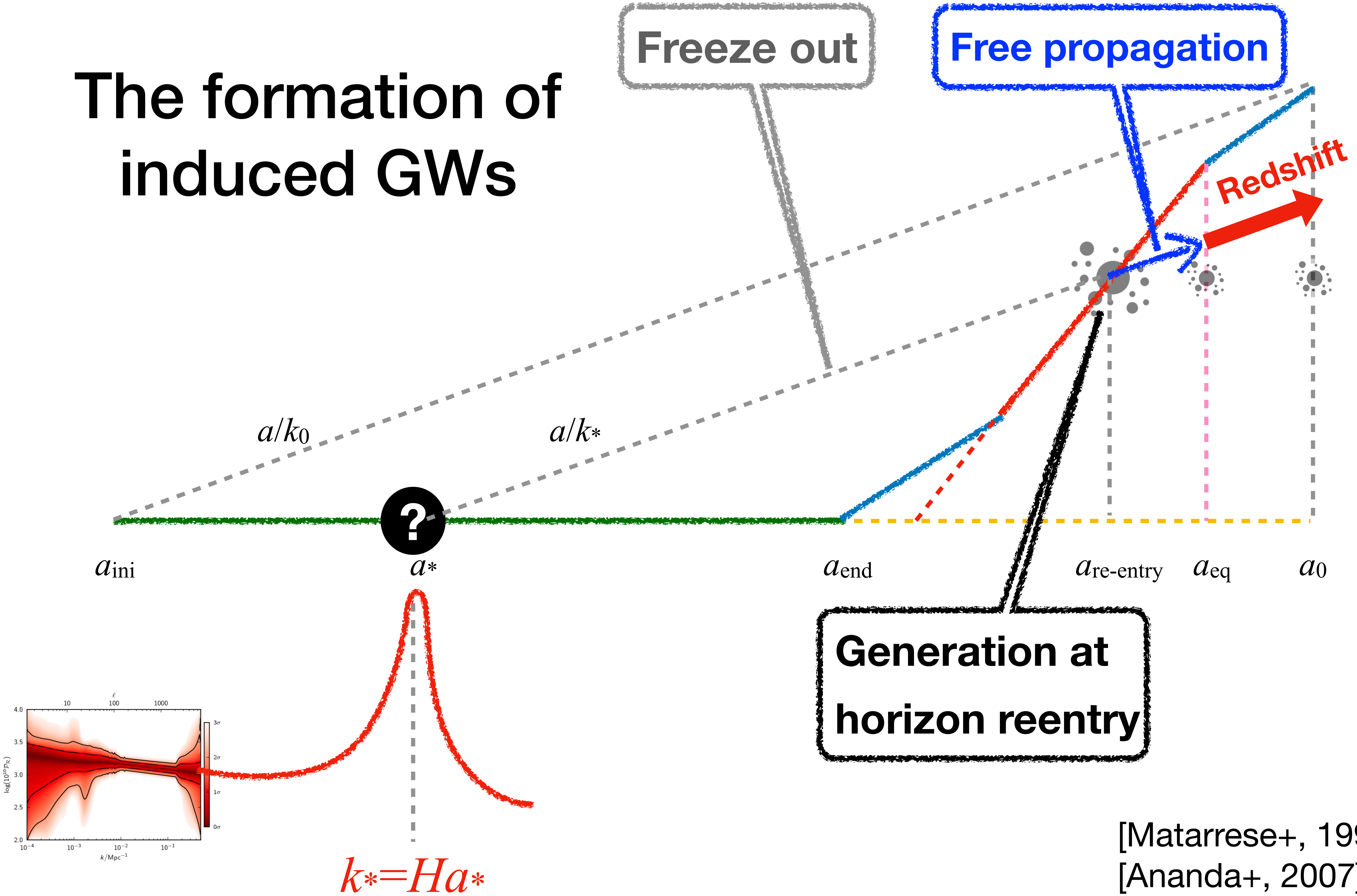
- The energy density parameter is then

$$\Omega_{\mathrm{GW}} \sim \langle hh \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathscr{P}_{\Phi}^2 \sim \mathscr{P}_{\mathscr{R}}^2$$

- In the radiation dominated universe we have $\mathscr{R} = \dfrac{2}{3}\Phi$

[Baumann+ 2007, Ananda+ 2007, Saito and Yokoyama 2008]

The formation of induced GWs



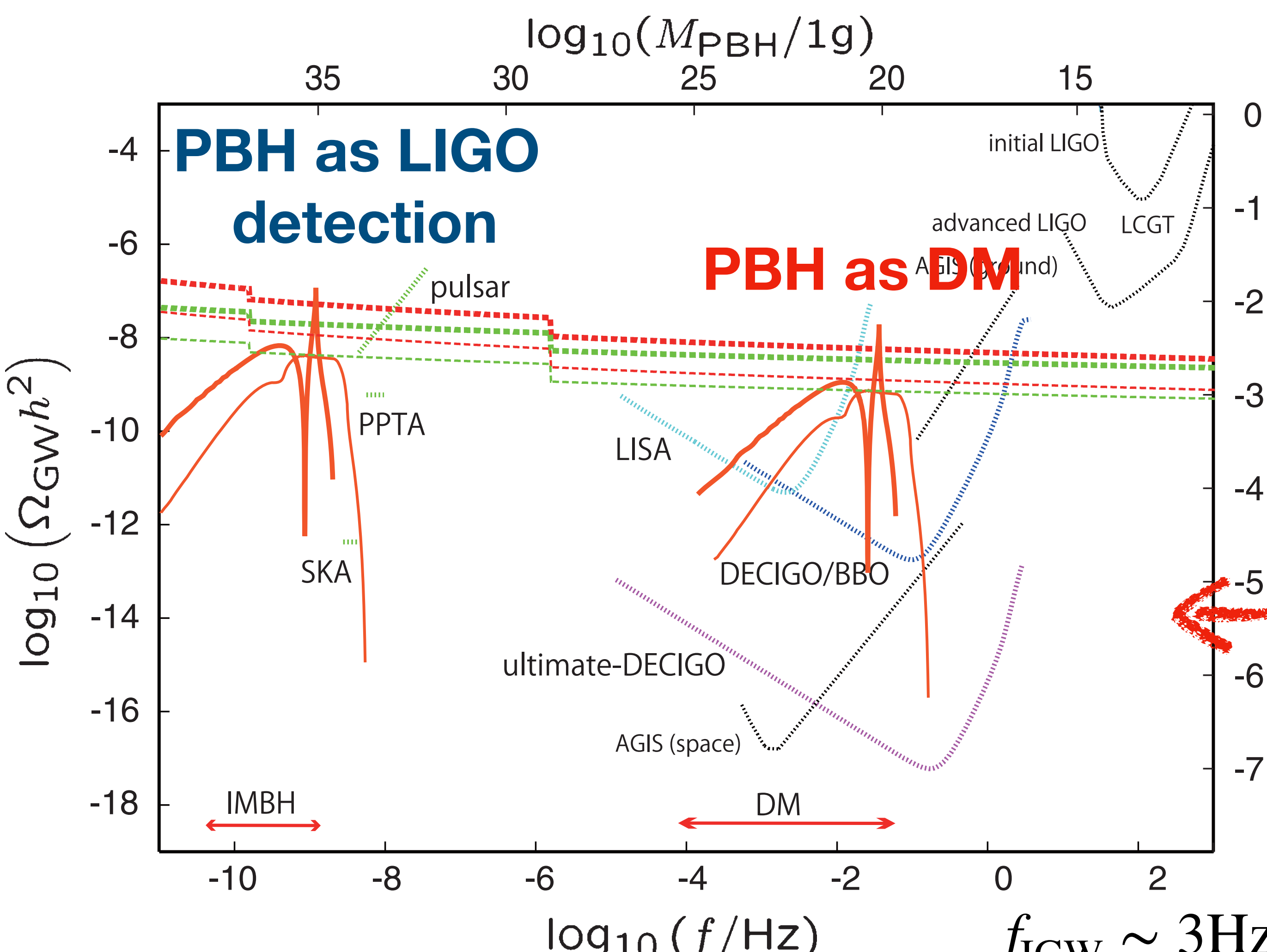
[Matarrese+, 1997]
[Ananda+, 2007]
[Baumann+, 2007]

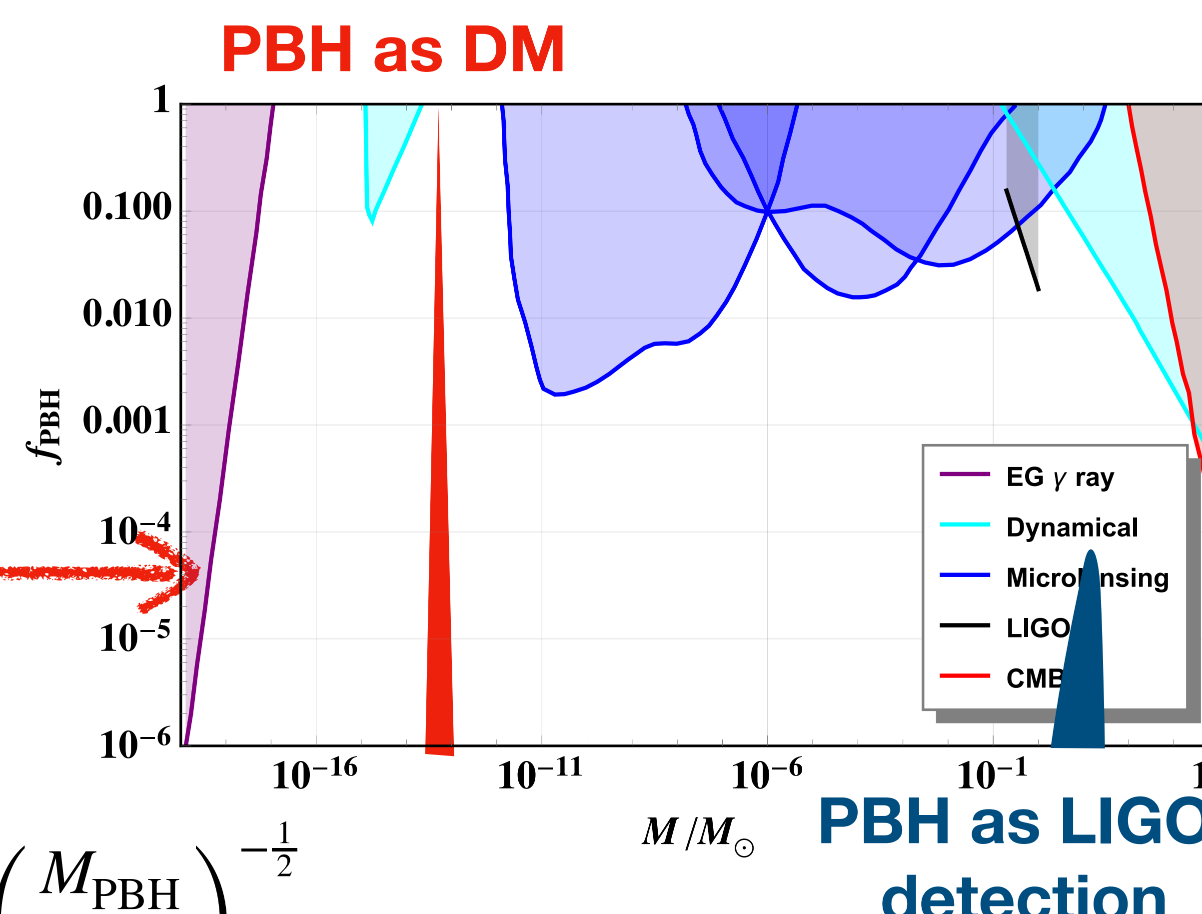
Induced GWs

$$\Omega_{\rm GW} \sim \langle hh \rangle \sim \langle \Phi\Phi\Phi\Phi \rangle \sim \mathscr{P}_\Phi^2 \sim \boxed{\mathscr{P}_\mathscr{R}^2}$$

PBH abundance

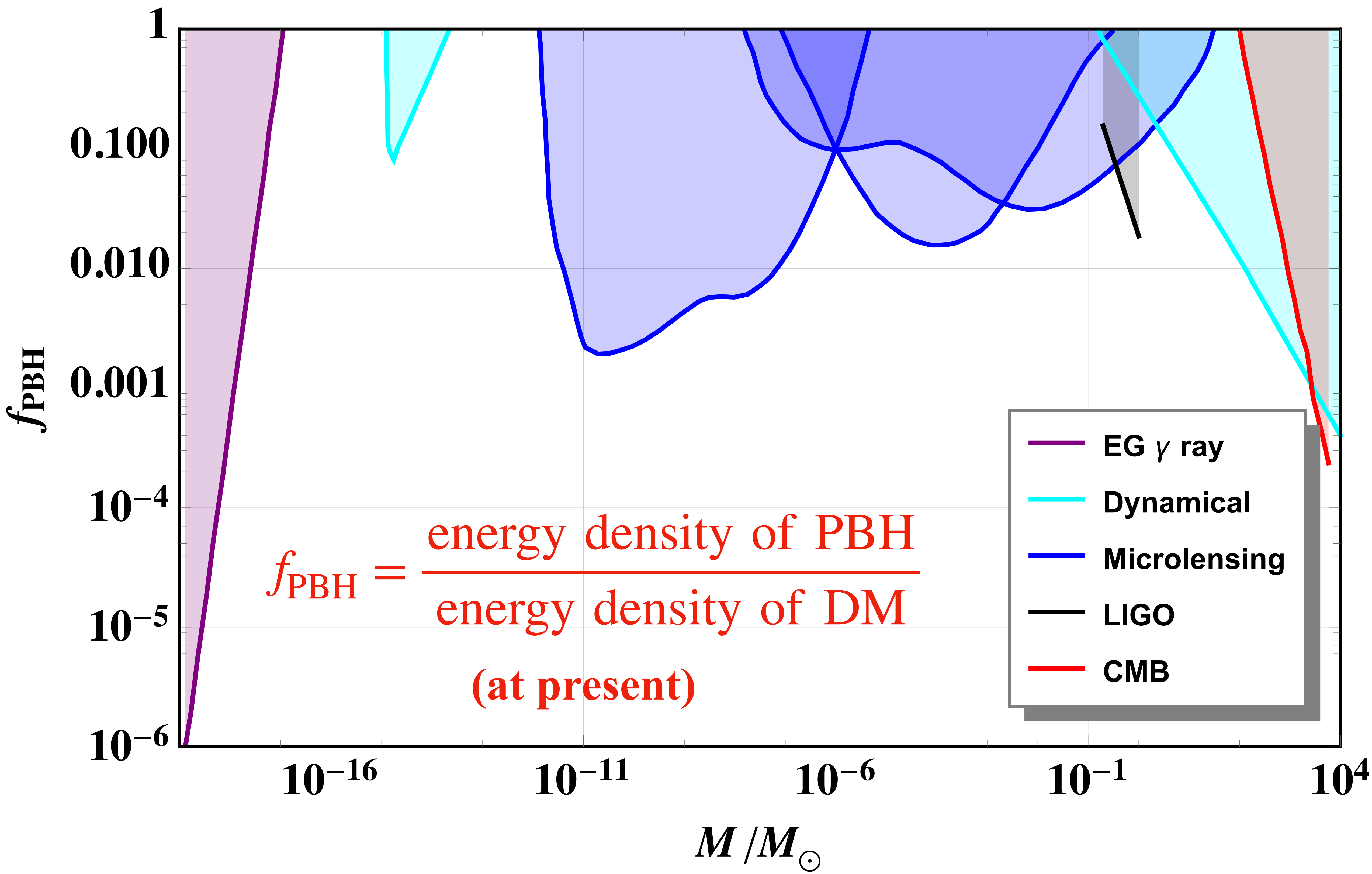
$$f_{\rm PBH} \sim 4.11 \times 10^8 \beta(M) \left(\frac{M}{M_\odot} \right)^{-1/2} \qquad \beta \sim \mathrm{erfc}\left(\frac{\mathscr{R}_c}{2\boxed{\mathscr{P}_\mathscr{R}^{1/2}}} \right)$$



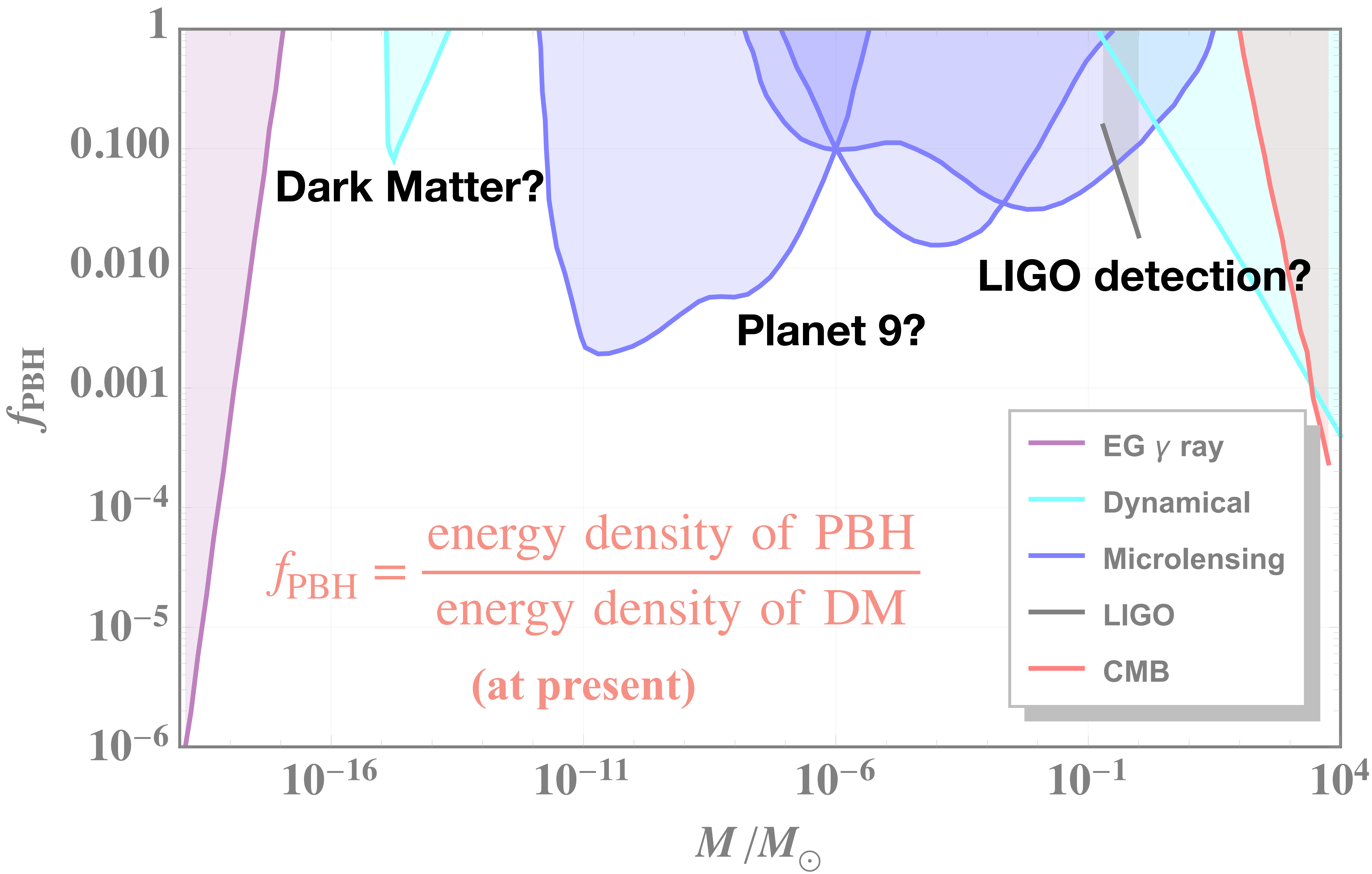


$$f_{\rm IGW} \sim 3{\rm Hz} \left(\frac{M_{\rm PBH}}{10^{16}{\rm g}} \right)^{-\frac{1}{2}}$$

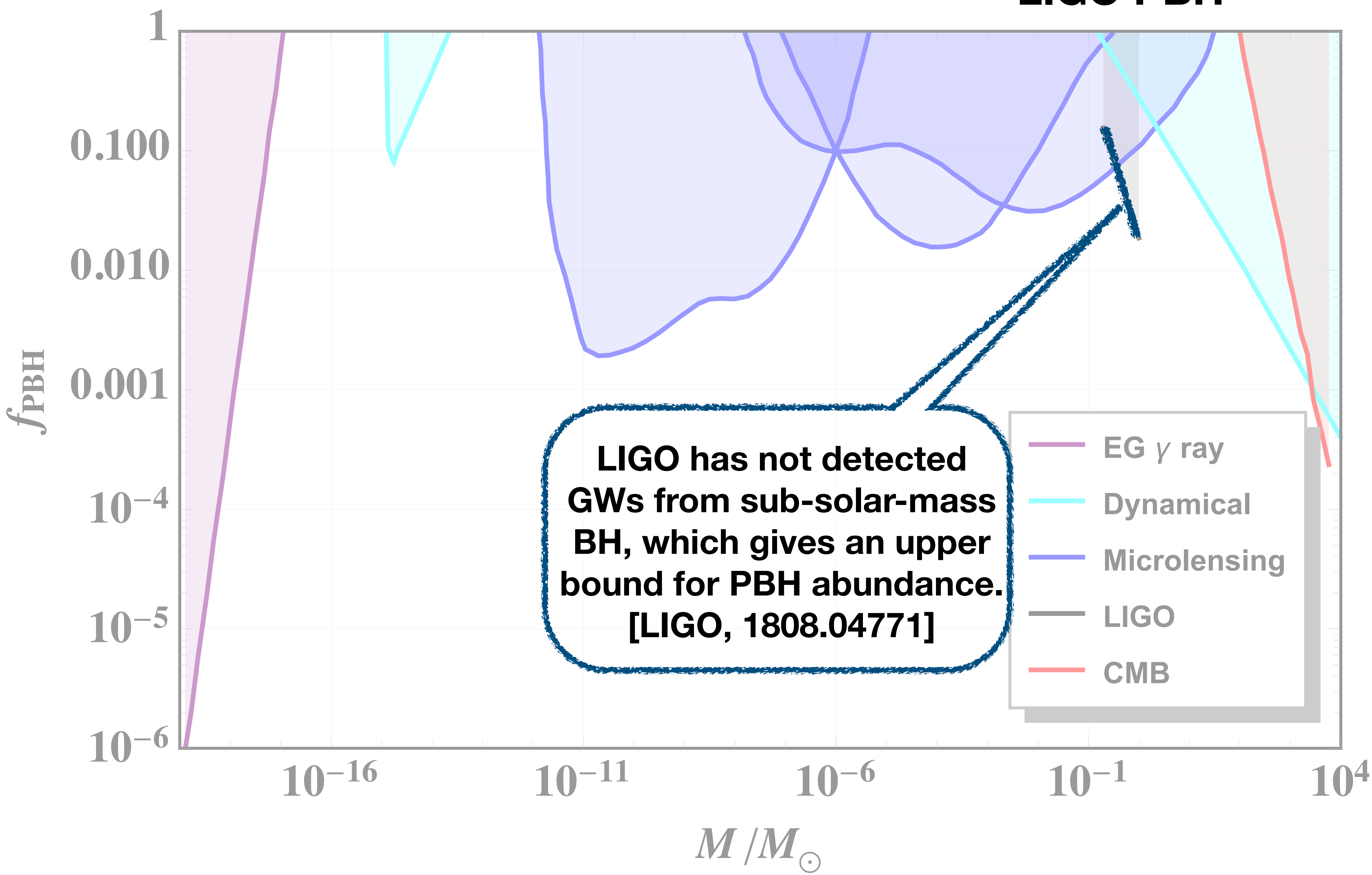
[Saito & Yokoyama 2008] [Bird+ 2016, Sasaki+ 2016]



1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
f_{PBH}
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_{\odot}$
EG γ ray
Dynamical
Microlensing
LIGO
CMB
$f_{\mathrm{PBH}} = \frac{\text{energy density of PBH}}{\text{energy density of DM}}$
(at present)

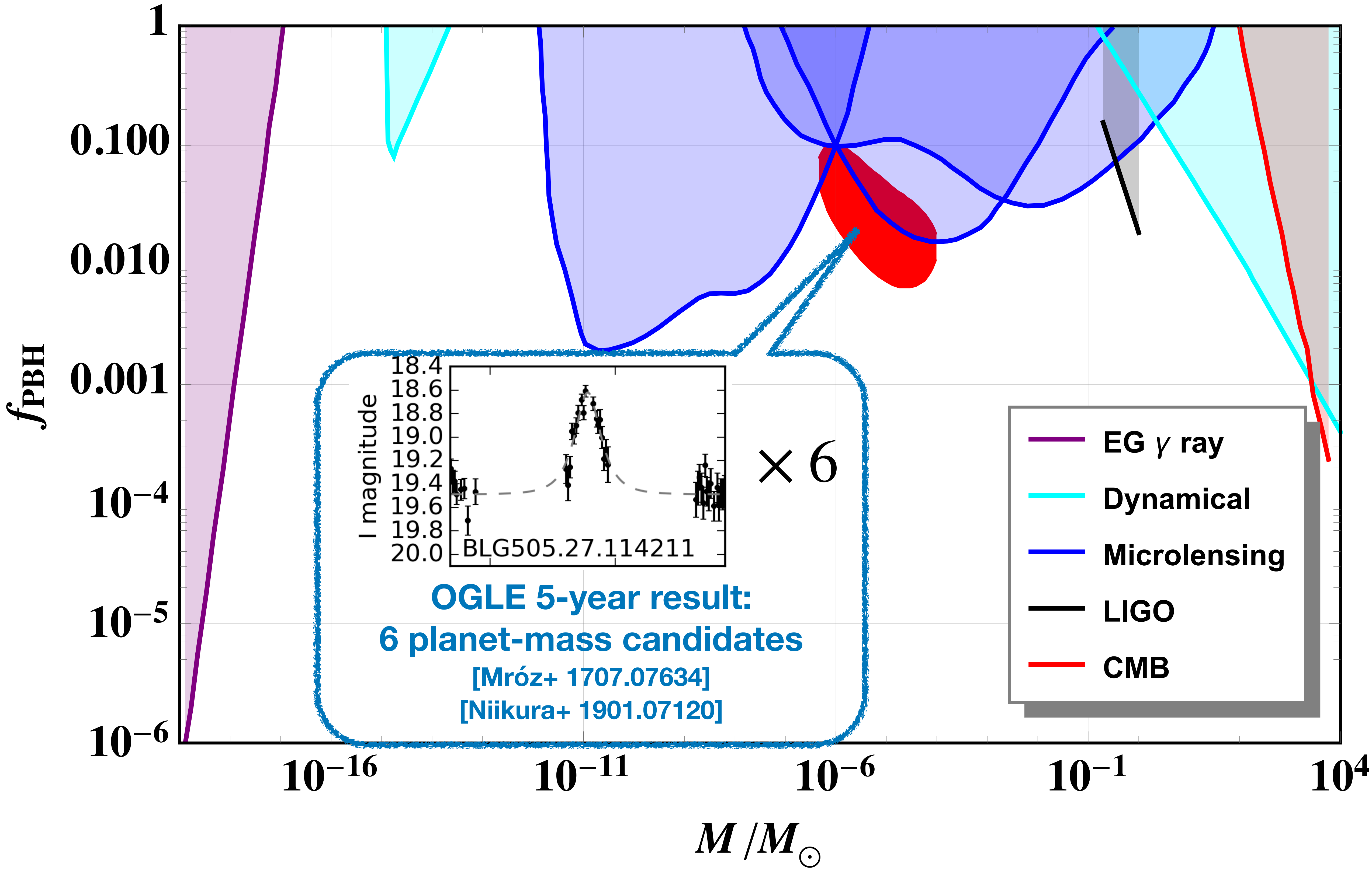

1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
$f_{\rm PBH}$
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_\odot$
Dark Matter?
Planet 9?
LIGO detection?
EG γ ray
Dynamical
Microlensing
LIGO
CMB
$f_{\rm PBH} = \dfrac{\text{energy density of PBH}}{\text{energy density of DM}}$
(at present)

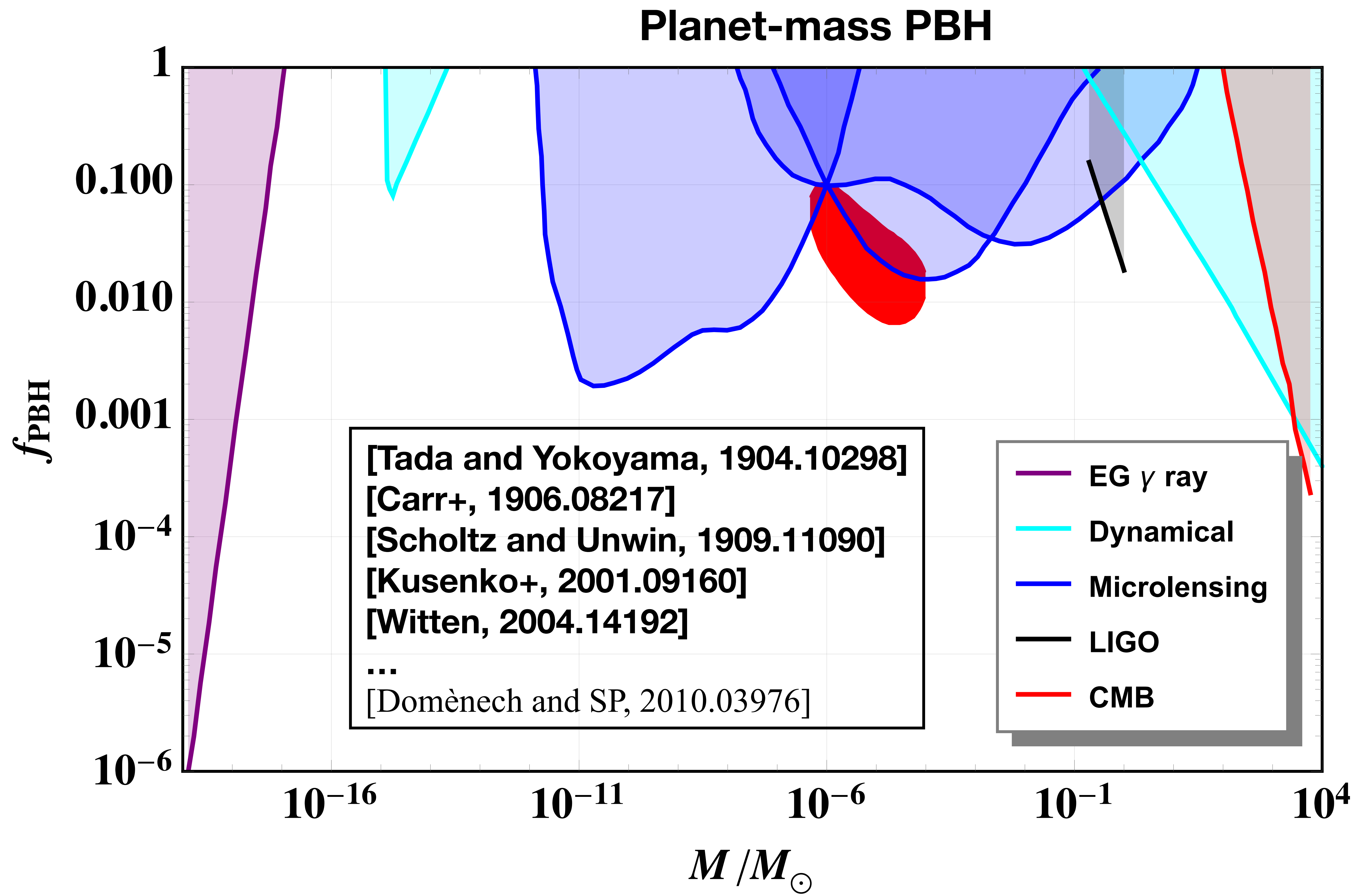
LIGO PBH



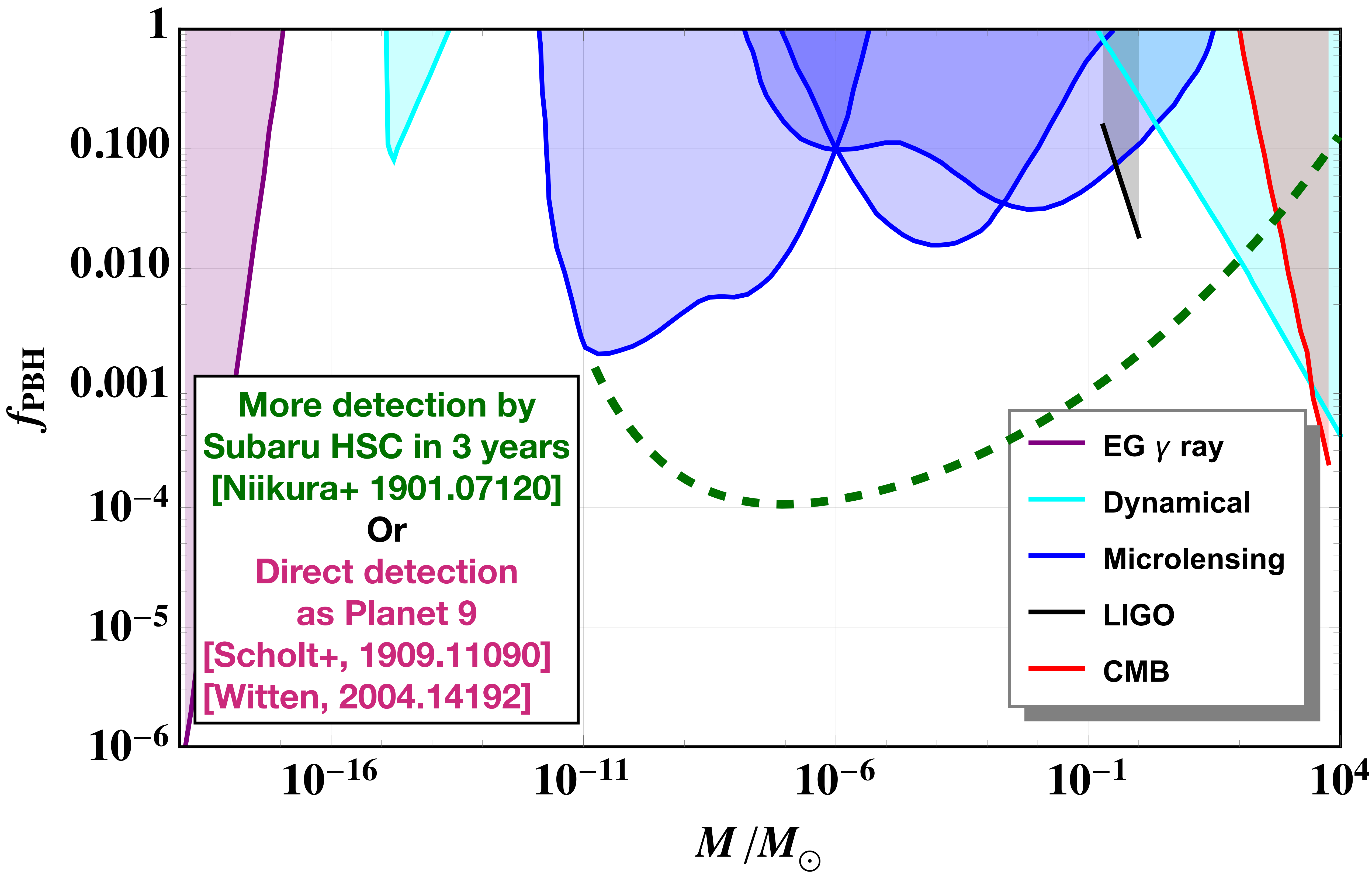
1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
f_{PBH}
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_{\odot}$
LIGO has not detected GWs from sub-solar-mass BH, which gives an upper bound for PBH abundance. [LIGO, 1808.04771]
EG γ ray
Dynamical
Microlensing
LIGO
CMB

Planet-mass PBH

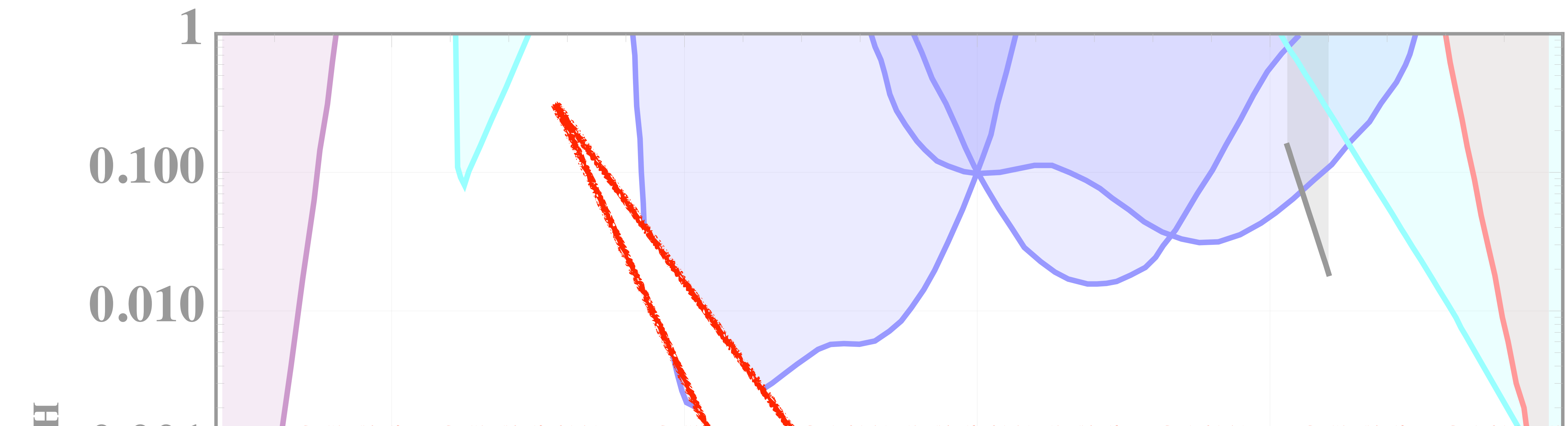




Planet-mass PBH
1
0.100
0.010
0.001
10−4
10−5
10−6
fPBH
10−16
10−11
10−6
10−1
104
M /M⊙
[Tada and Yokoyama, 1904.10298]
[Carr+, 1906.08217]
[Scholtz and Unwin, 1909.11090]
[Kusenko+, 2001.09160]
[Witten, 2004.14192]
...
[Domènech and SP, 2010.03976]
EG γ ray
Dynamical
Microlensing
LIGO
CMB



1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
$f_{\rm PBH}$
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_\odot$
More detection by
Subaru HSC in 3 years
[Niikura+ 1901.07120]
Or
Direct detection
as Planet 9
[Scholt+, 1909.11090]
[Witten, 2004.14192]
EG γ ray
Dynamical
Microlensing
LIGO
CMB



No sensitivity on small scales

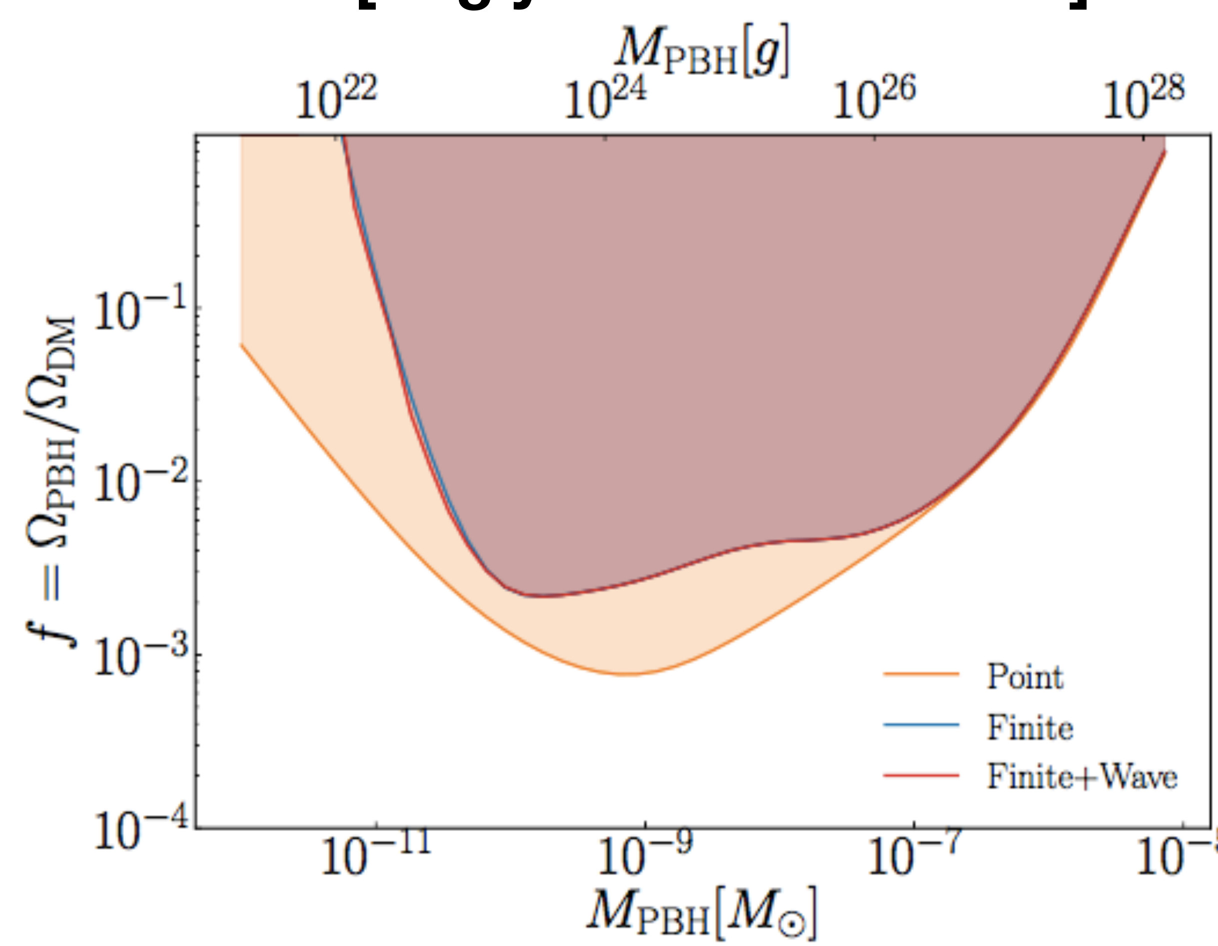
Finite Size Effect:

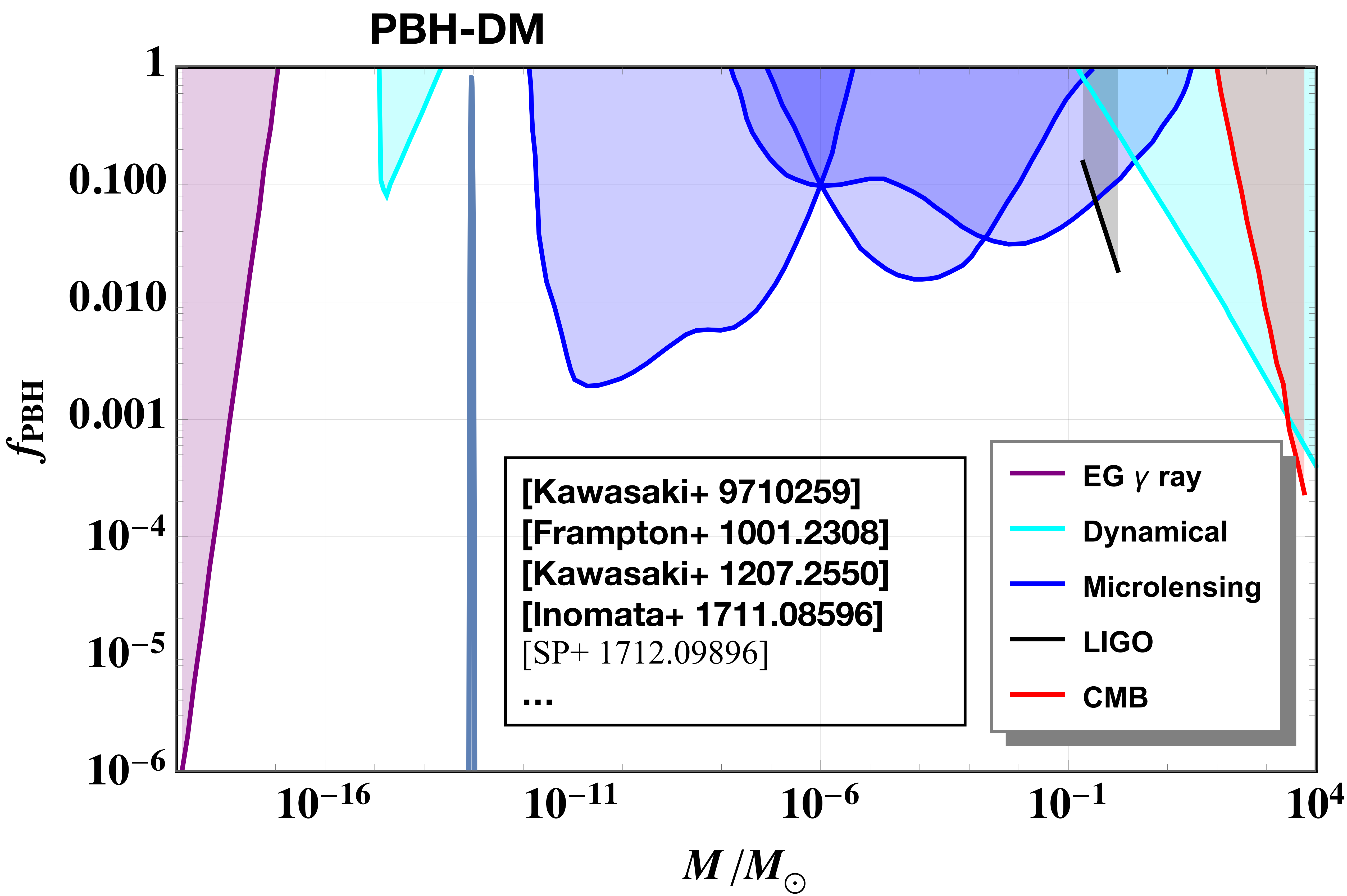
$$M < 5 \times 10^{-10} M_\odot \left(\frac{R_s}{R \odot} \right)^2$$

Wave effect:

$$M < 4 \times 10^{-10} M_\odot \left(\frac{\lambda_s}{\mu \mathrm{m}} \right)$$

[Sugiyama+ 1905.06066]

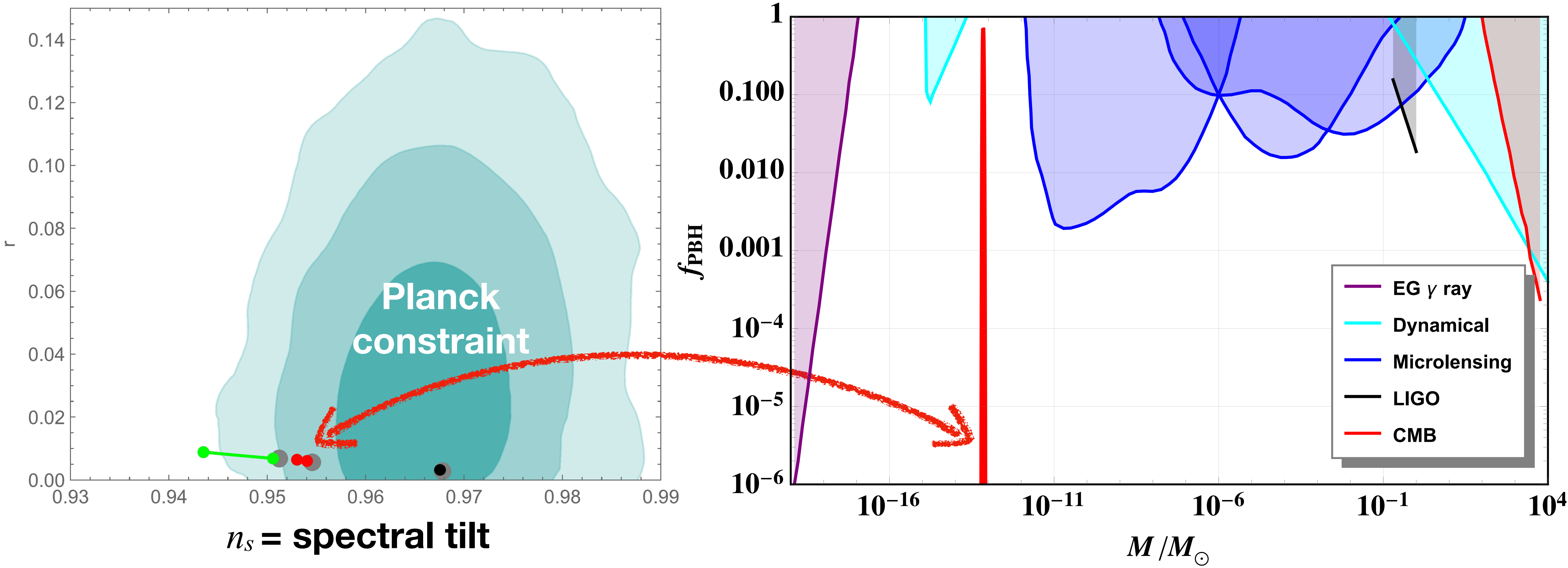




PBH-DM
1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
$f_{\rm PBH}$
10^{-16}
10^{-11}
10^{-6}
10^{-1}
10^{4}
$M/M_\odot$
[Kawasaki+ 9710259]
[Frampton+ 1001.2308]
[Kawasaki+ 1207.2550]
[Inomata+ 1711.08596]
[SP+ 1712.09896]
...
EG γ ray
Dynamical
Microlensing
LIGO
CMB

PBH from R^2-inflation

$r = \frac{\textbf{tensor spectrum}}{\textbf{scalar spectrum}}$ $\qquad$ $f_{\rm PBH} = \frac{\textbf{PBH energy density}}{\textbf{CDM energy density}}$



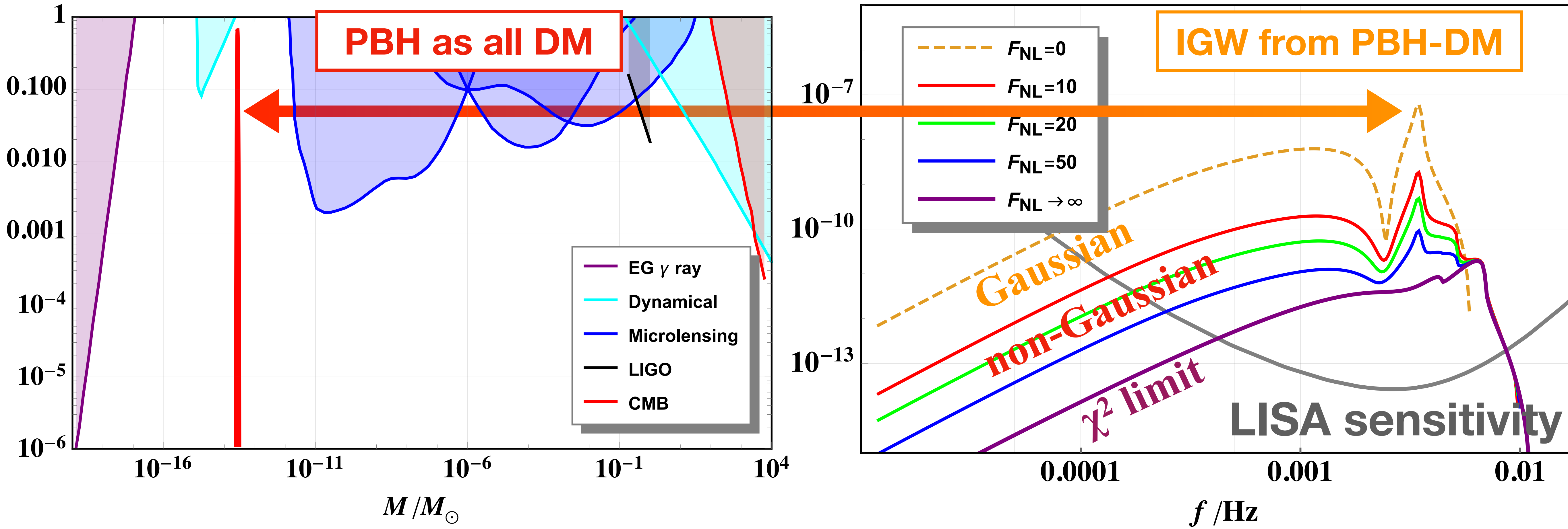
[SP, Zhang, Huang & Sasaki, JCAP1805, 042]

If PBH serves as all DM, the induced GWs must be detectable by LISA, independent of $\mathcal{A}_{\mathcal{R}}$ or $F_{\rm NL}$.

$$\mathcal{R} = \mathcal{R}_G + F_{NL}\mathcal{R}_G^2$$

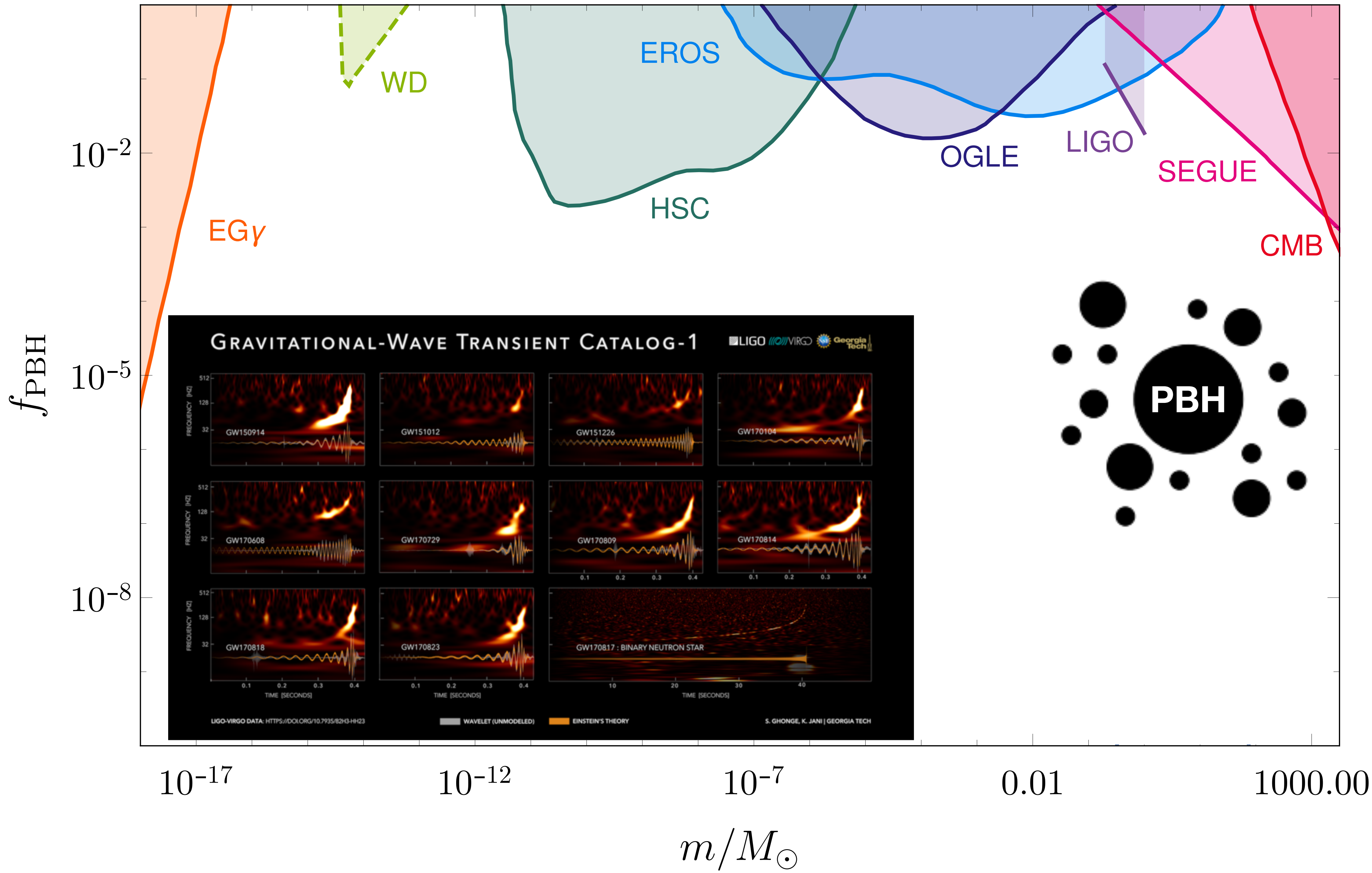
$$\Omega_{\rm GW} \sim \mathcal{P}^2_{\mathcal{R},\rm G} + F^2_{\rm NL}\mathcal{P}^3_{\mathcal{R},\rm G} + F^4_{\rm NL}\mathcal{P}^4_{\mathcal{R},\rm G}$$

$$f_{\rm PBH} = \frac{\text{PBH energy density}}{\text{CDM energy density}} \qquad \Omega_{\rm GW} = \frac{\text{GW energy density}}{\text{critical energy density}}$$



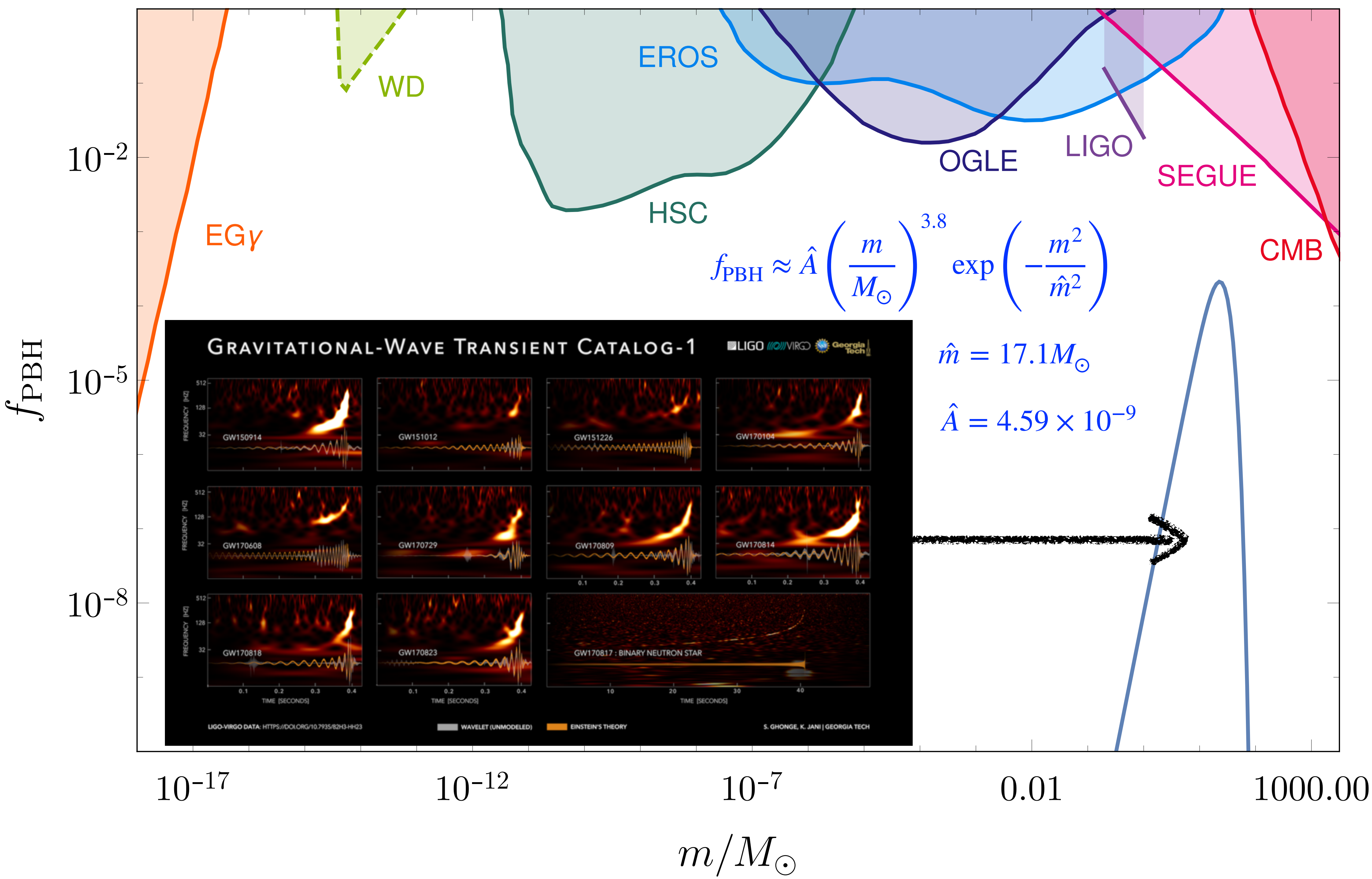
[Cai, SP, and Sasaki, PRL122, 201101]

PBHs as LIGO events



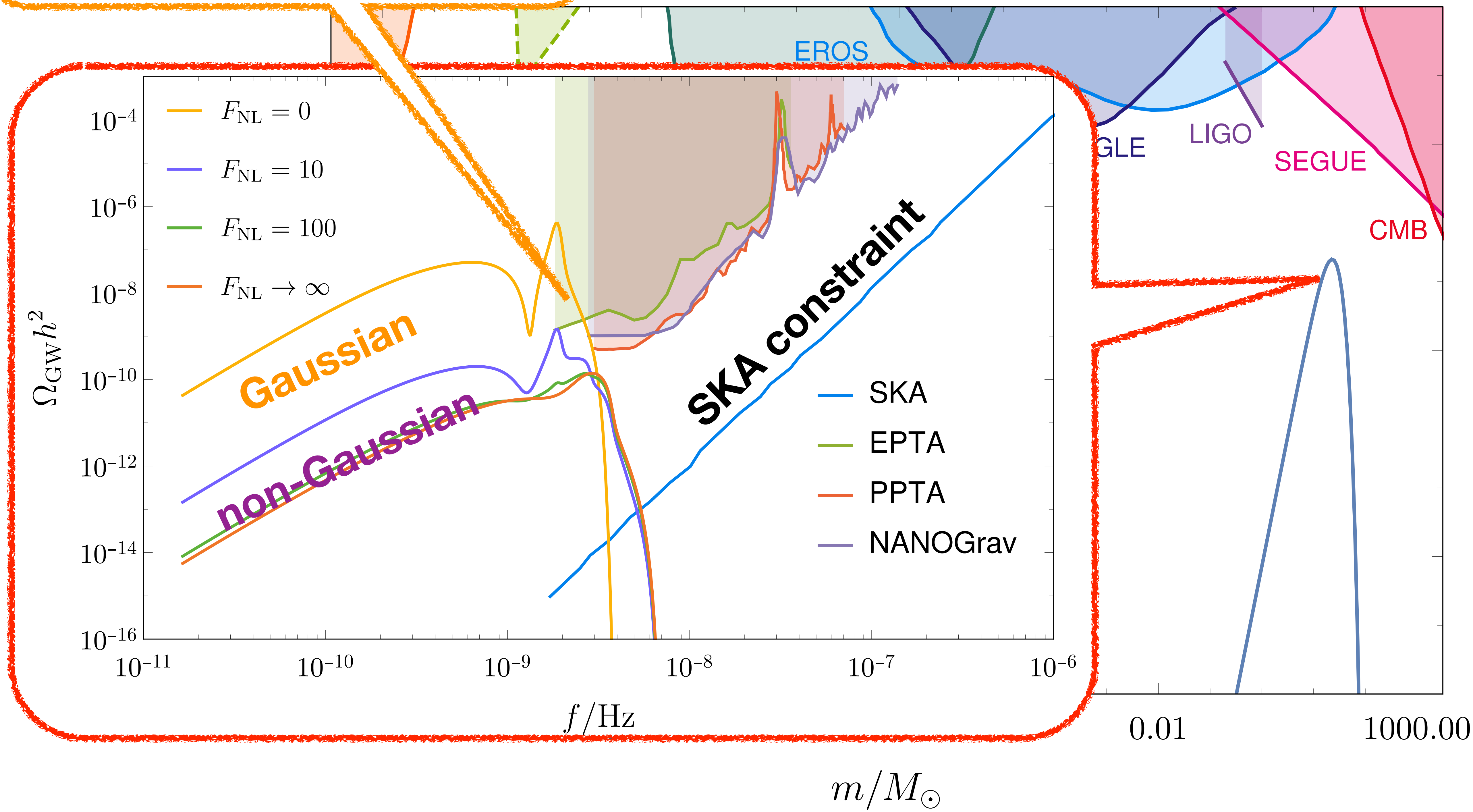
[Cai, SP, Wang, and Yang, JCAP1910, 059]

PBHs as LIGO events

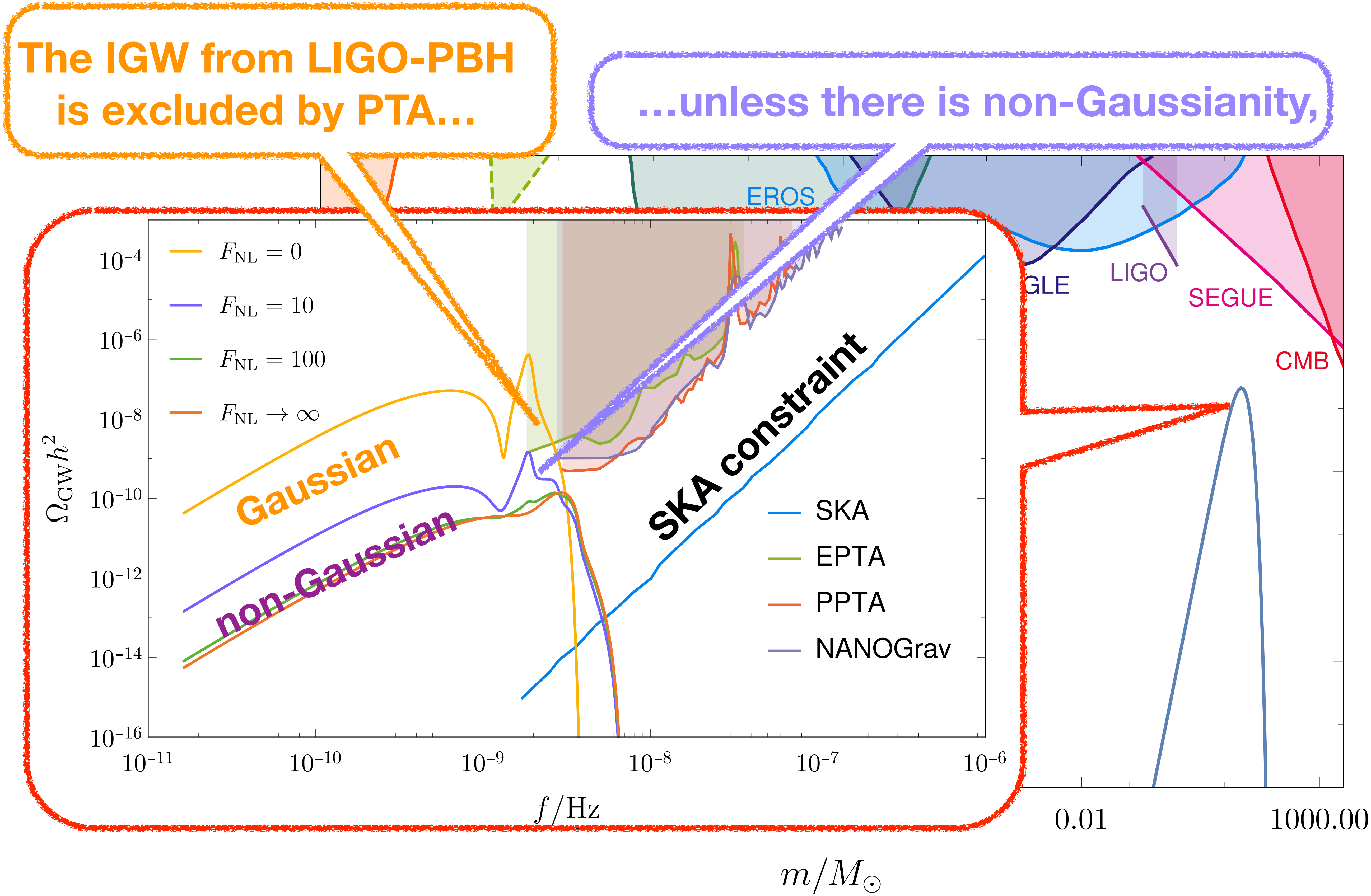


[Cai, SP, Wang, and Yang, JCAP1910, 059]

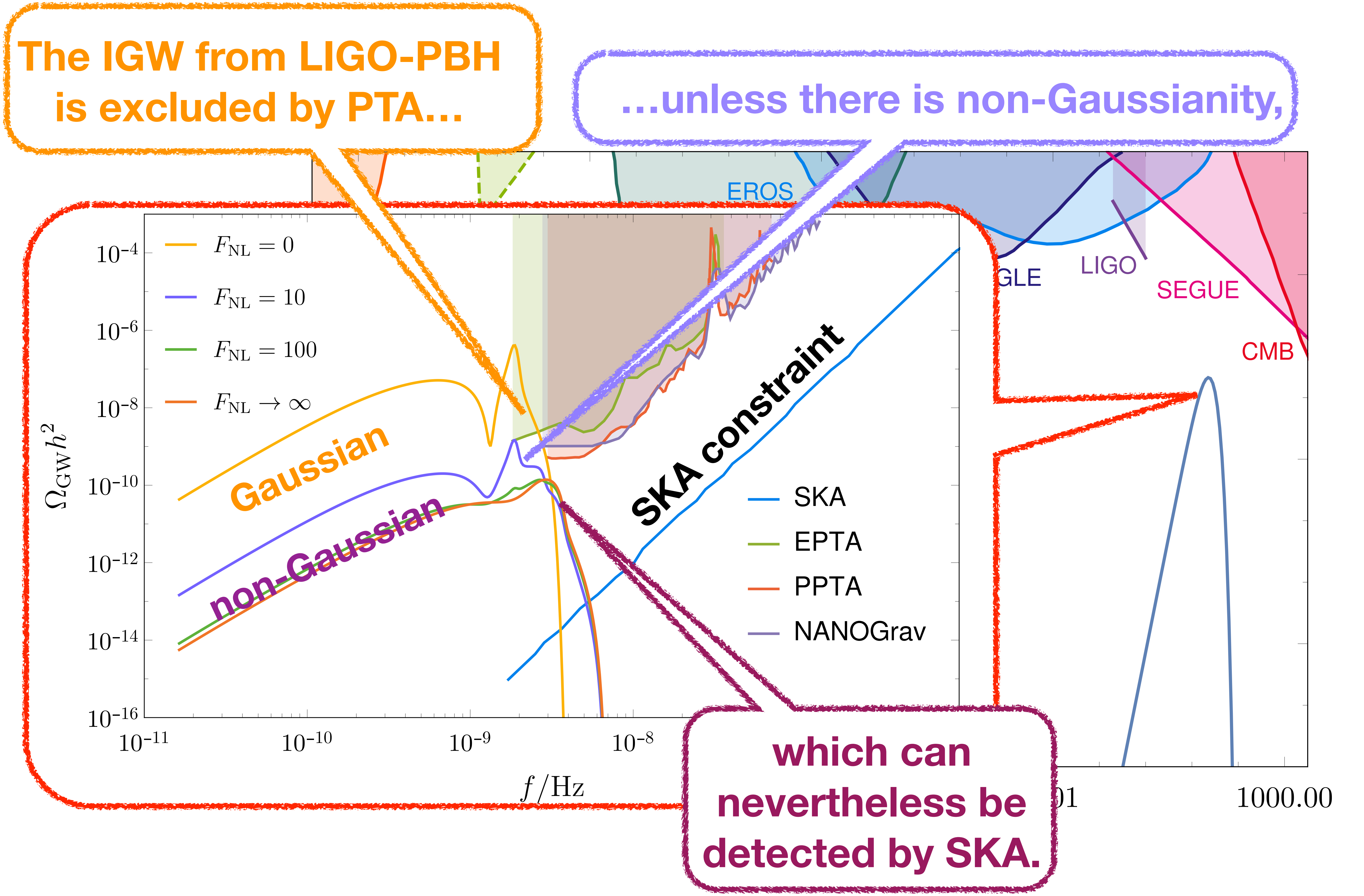
The IGW from LIGO-PBH is excluded by PTA…



[Cai, SP, Wang, and Yang, JCAP1910, 059]



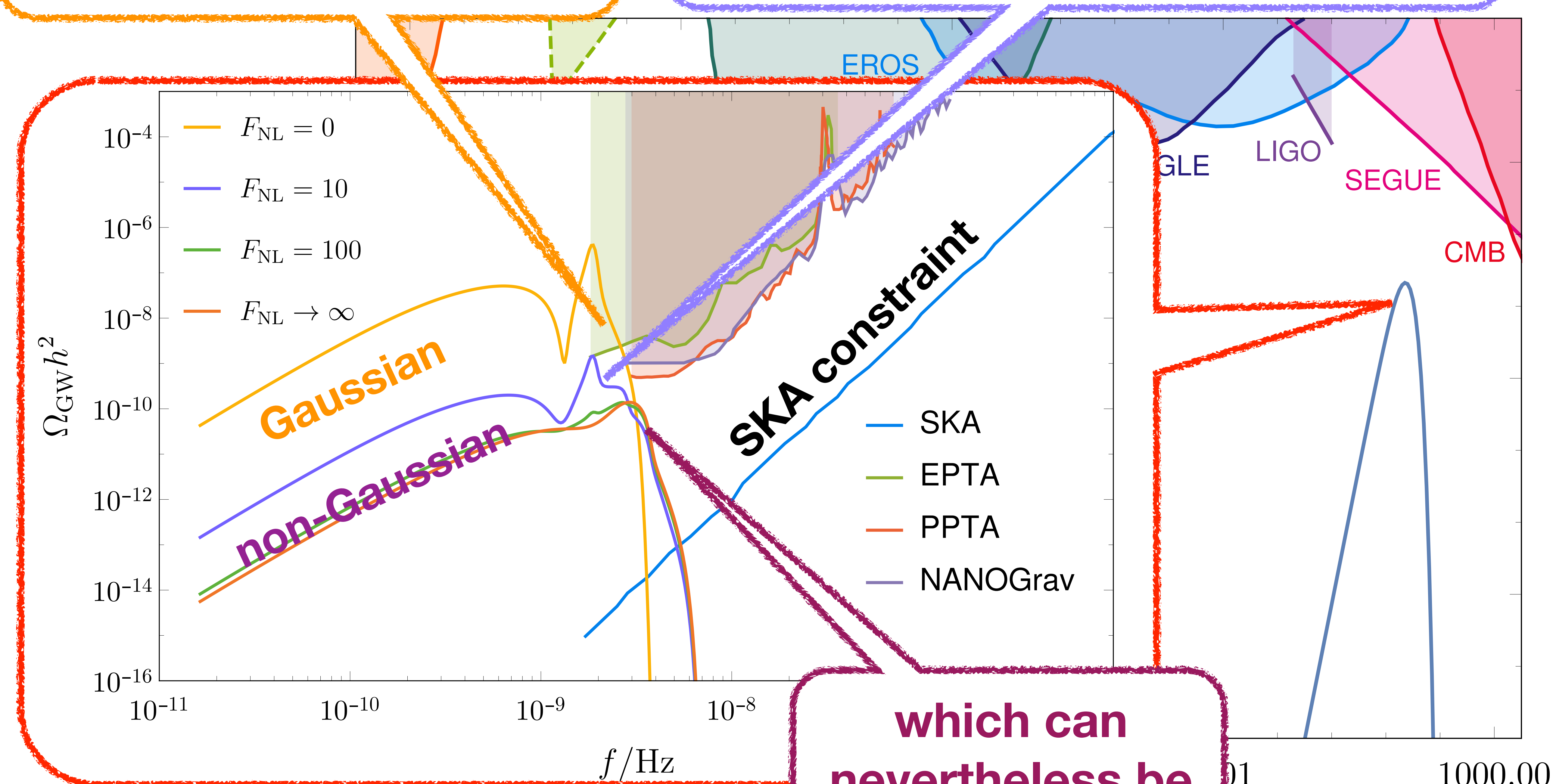
[Cai, SP, Wang, and Yang, JCAP1910, 059]



[Cai, SP, Wang, and Yang, JCAP1910, 059]

The IGW from LIGO-PBH is excluded by PTA…

…unless there is non-Gaussianity,



which can nevertheless be detected by SKA.

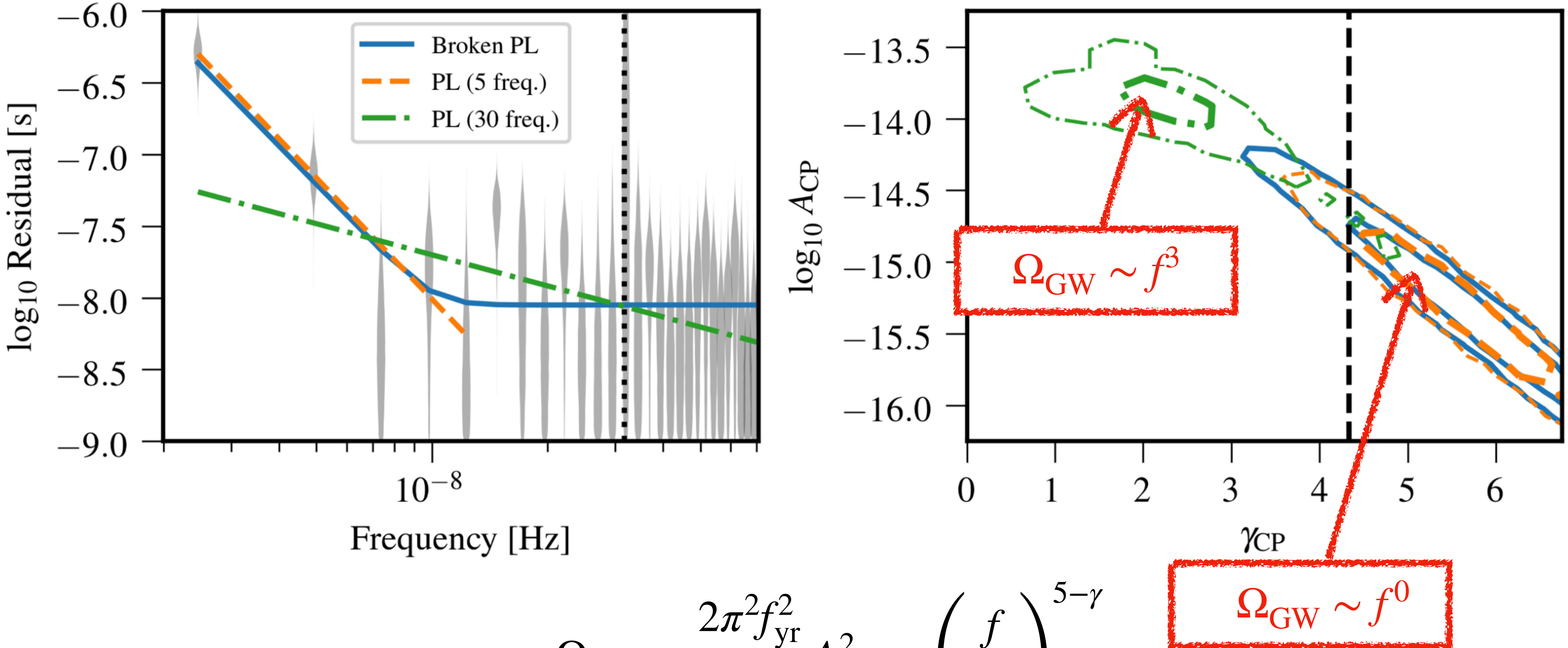
But wait…

[Cai, SP, Wang, and Yang, JCAP1910, 059]

Content

- Introduction
- Primordial perturbation, PBHs, and induced GWs
- Some properties for IGW: scaling and peak
- NANOGrav hints on planet-mass PBHs
- Summary

PTA discovered SGWB?

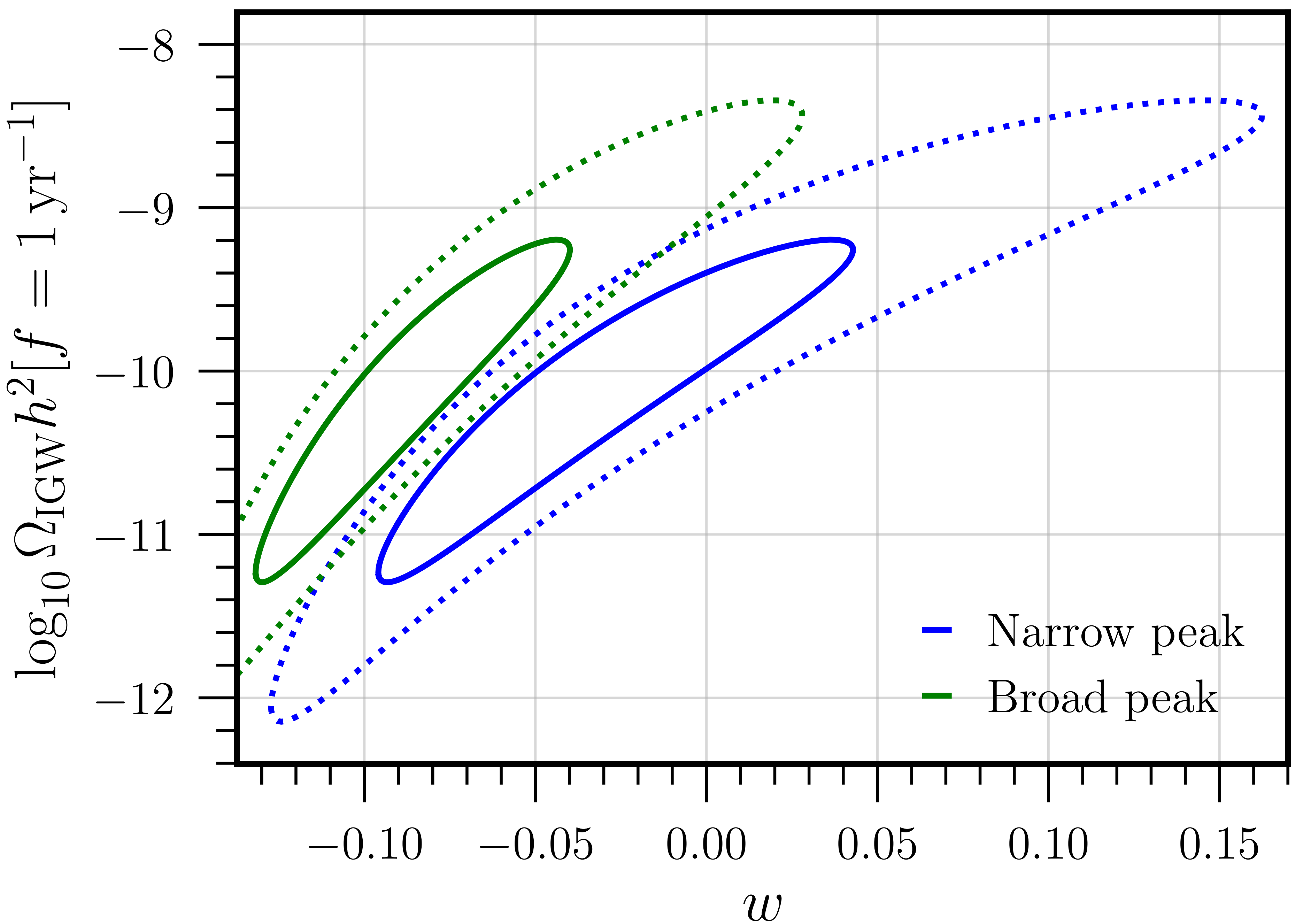


$$\Omega_{\rm GW} = \frac{2\pi^2 f_{\rm yr}^2}{3H_0^2} A_{\rm SGWB}^2 \left(\frac{f}{f_{\rm yr}}\right)^{5-\gamma}$$

$$f_{\rm yr} = 3.17 \times 10^{-8}\ \text{Hz}\,.$$

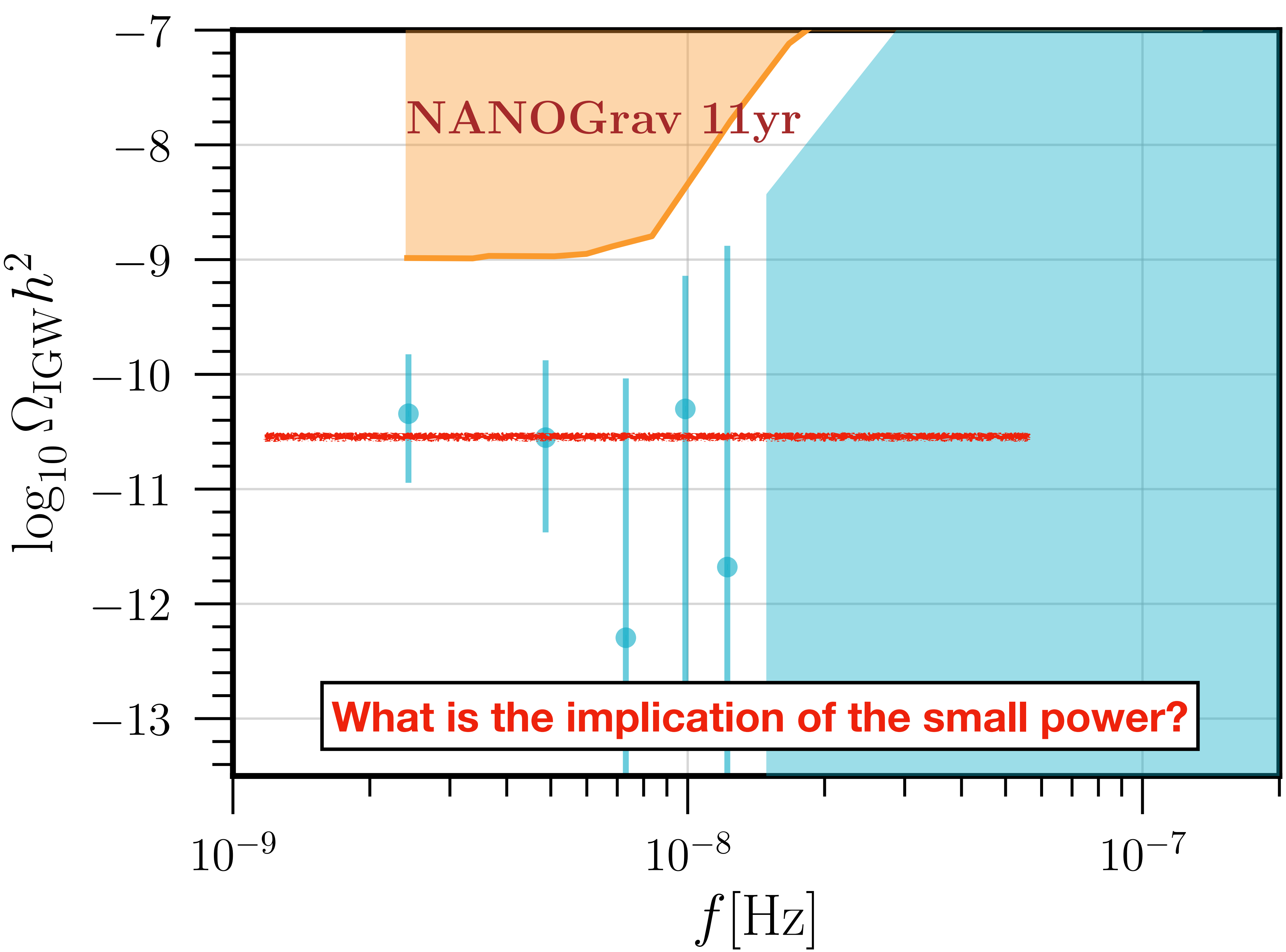
[NANOGrav, 2009.04496]

PTA discovered SGWB?



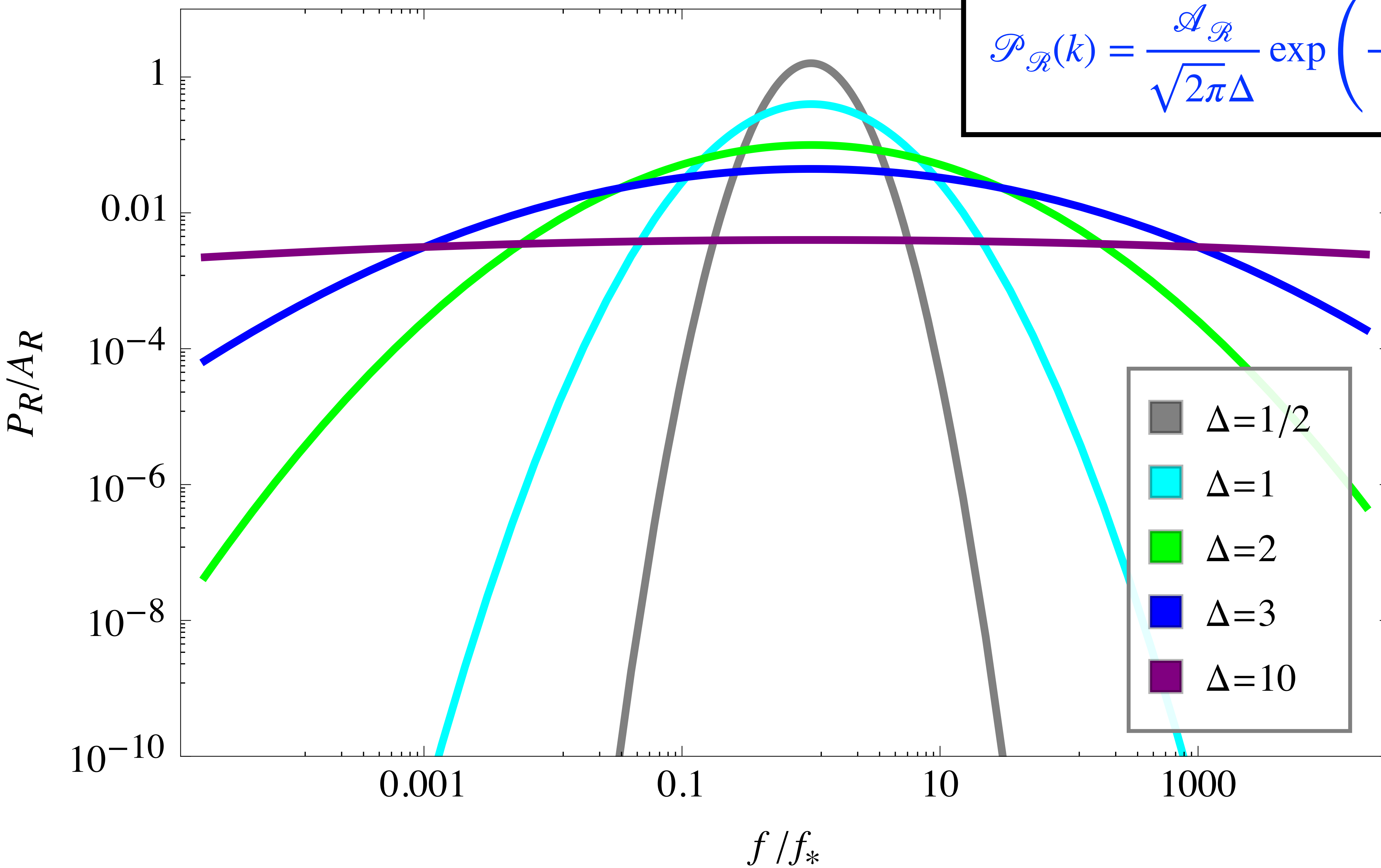
[Domènech and SP, 2010.03976]

PTA discovered SGWB?



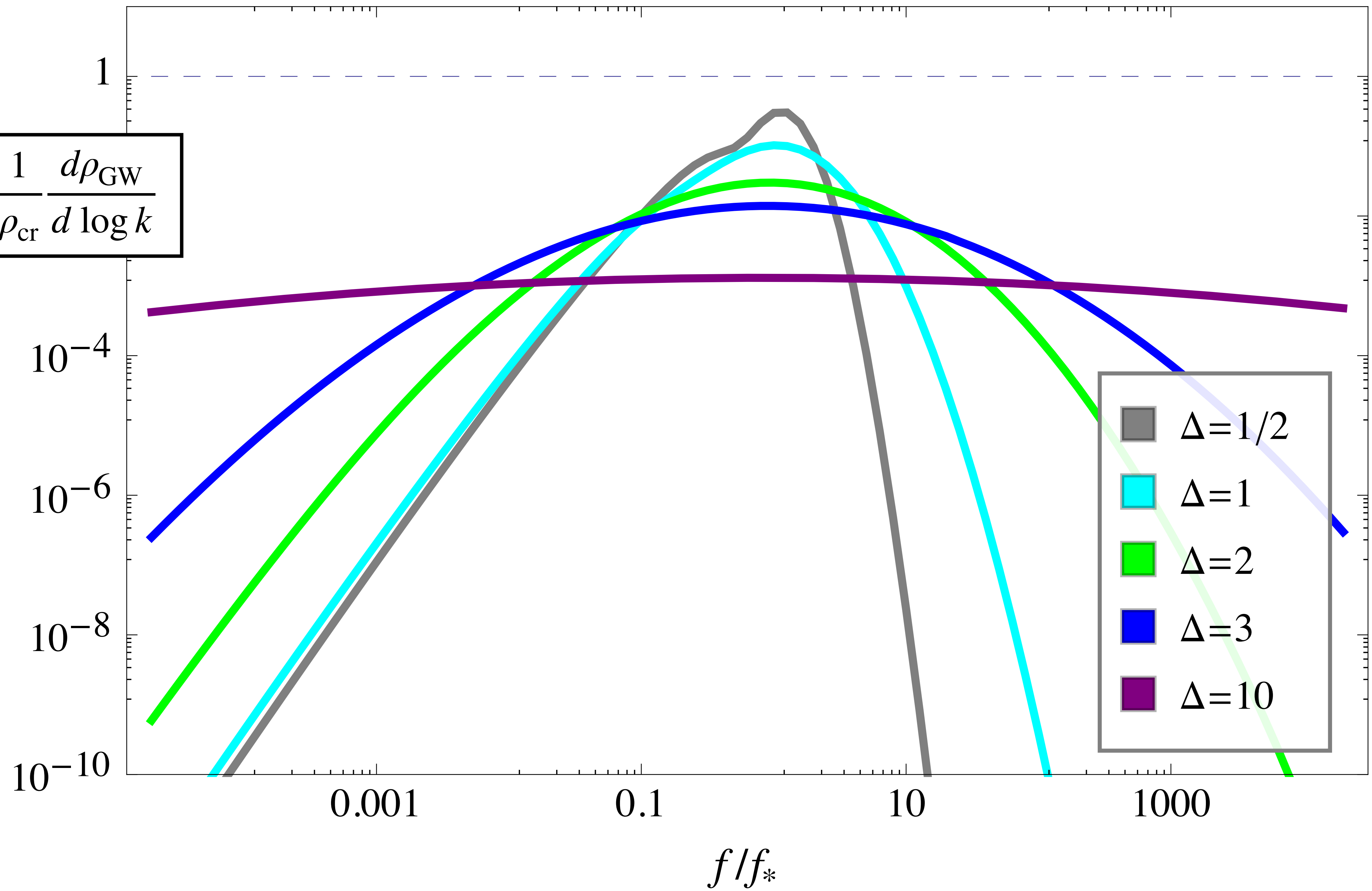
Broad Peak of $\mathscr{P}_{\mathscr{R}}$

$$\mathscr{P}_{\mathscr{R}}(k) = \frac{\mathscr{A}_{\mathscr{R}}}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\ln^2(f/f_*)}{2\Delta^2} \right)$$



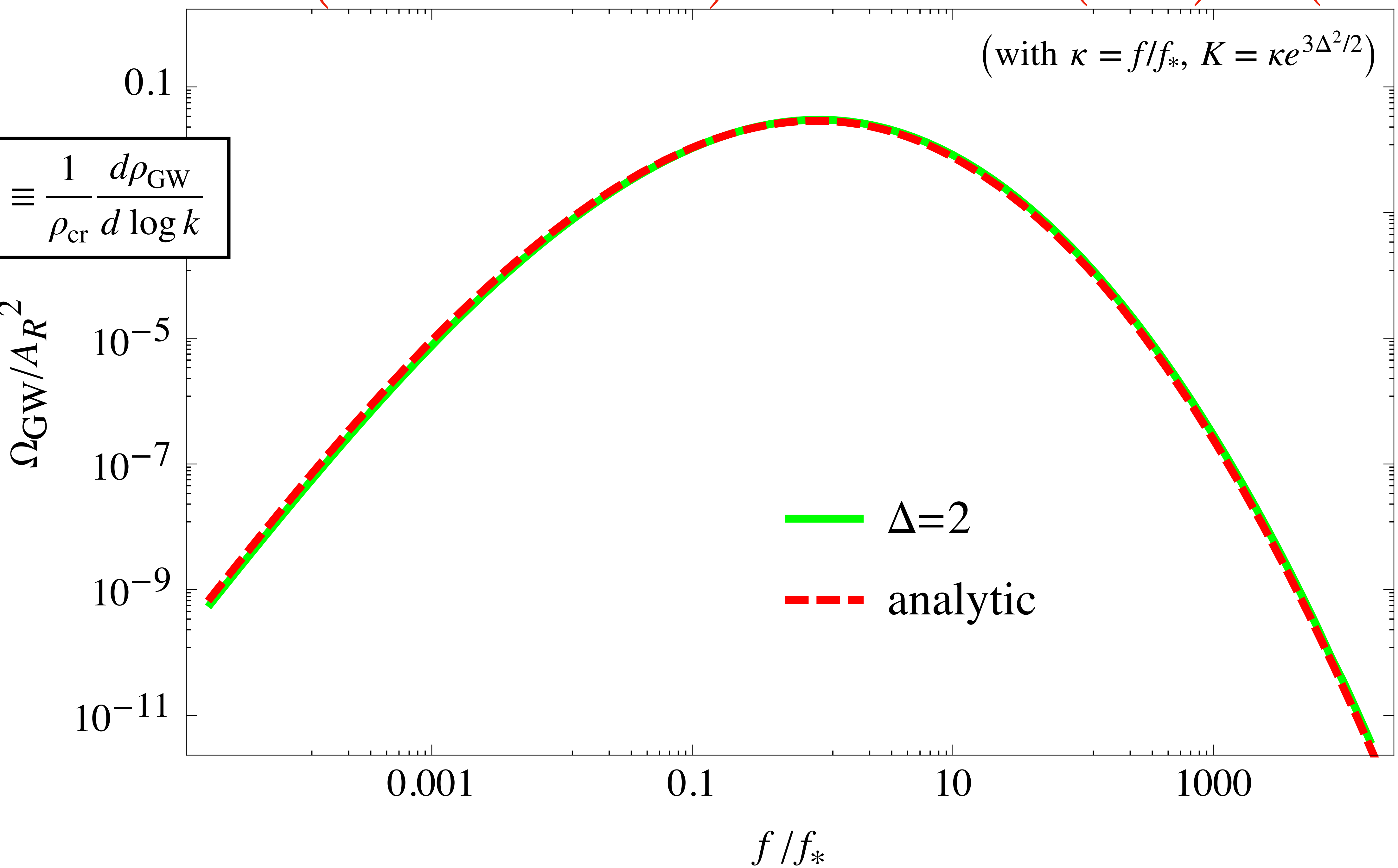
IGW from Broad Peak

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_{\rm cr}} \frac{d\rho_{\rm GW}}{d\log k}$$



$$
\frac{\Omega_{\rm GW}}{\mathcal{A}_{\mathcal{R}}^2} \approx \frac{4}{5\sqrt{\pi}}\kappa^3 \frac{e^{\frac{9\Delta^2}{4}}}{\Delta}\left[\left(\ln^2 K + \frac{\Delta^2}{2}\right)\mathrm{erfc}\left(\frac{\ln K + \frac{1}{2}\ln\frac{3}{2}}{\Delta}\right) - \frac{\Delta}{\sqrt{\pi}}\exp\left(-\frac{\left(\ln K + \frac{1}{2}\ln\frac{3}{2}\right)^2}{\Delta^2}\right)\left(\ln K - \frac{1}{2}\ln\frac{3}{2}\right)\right]
$$

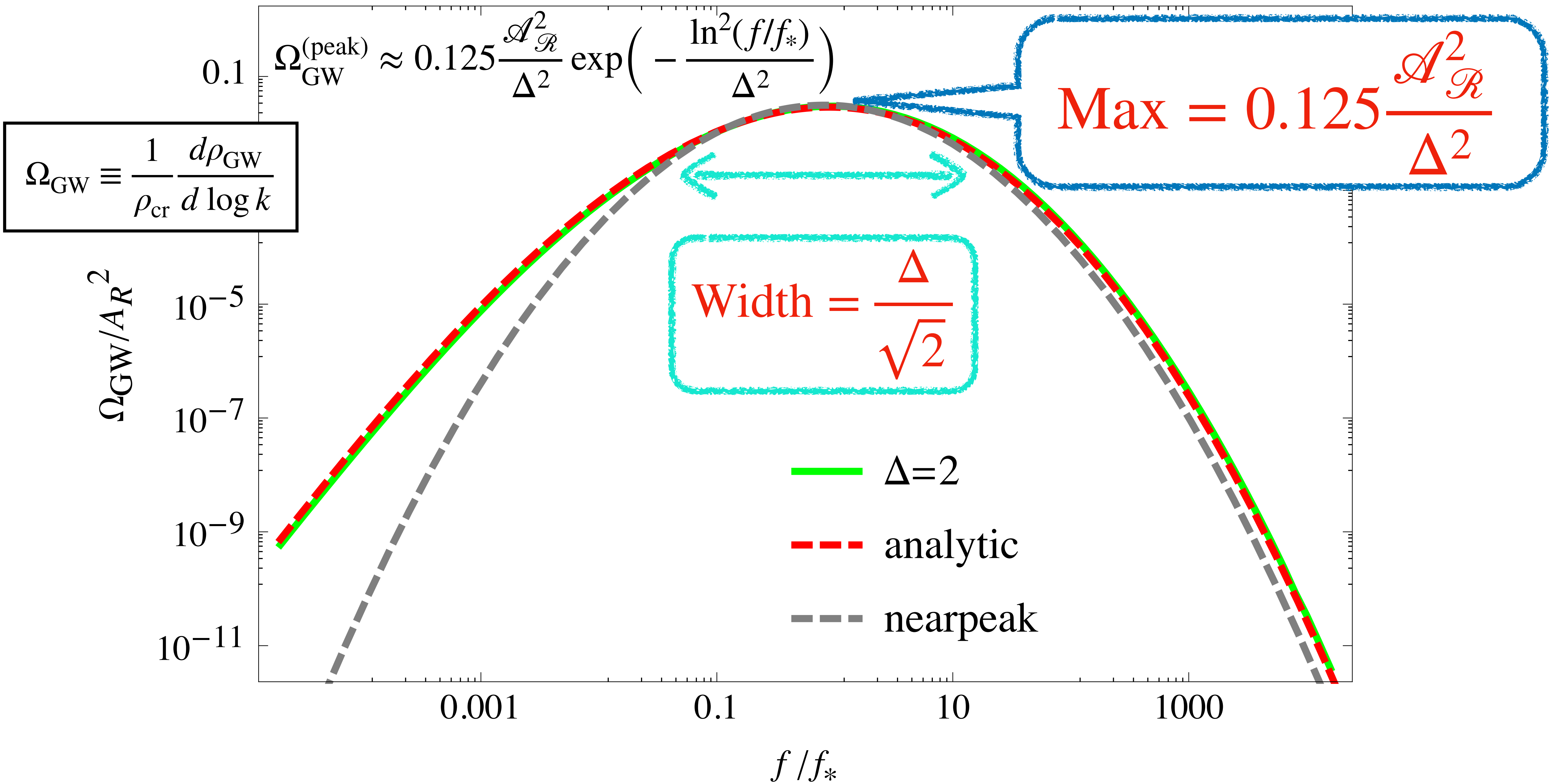
$$
+\frac{0.0659}{\Delta^2}\kappa^2 e^{\Delta^2}\exp\left(-\frac{\left(\ln\kappa + \Delta^2 - \frac{1}{2}\ln\frac{4}{3}\right)^2}{\Delta^2}\right) + \frac{1}{3}\sqrt{\frac{2}{\pi}}\kappa^{-4}\frac{e^{8\Delta^2}}{\Delta}\exp\left(-\frac{\ln^2\kappa}{2\Delta^2}\right)\mathrm{erfc}\left(\frac{4\Delta^2 - \ln(\kappa/4)}{\sqrt{2}\Delta}\right)
$$



$$
\boxed{\Omega_{\rm GW} \equiv \frac{1}{\rho_{\rm cr}}\frac{d\rho_{\rm GW}}{d\log k}}
$$

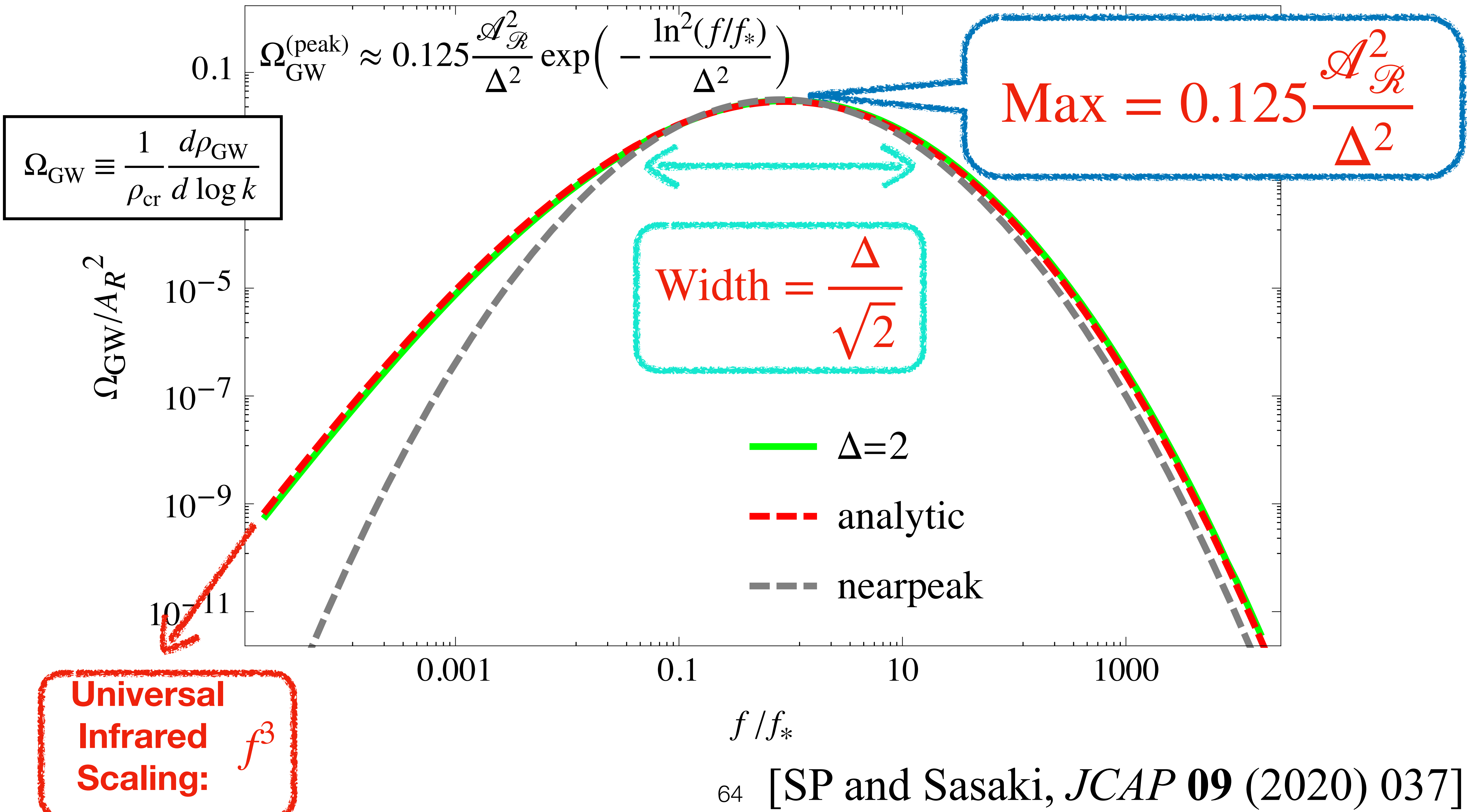
 [SP and Sasaki, *JCAP* **09** (2020) 037]

IGW from Broad Peak



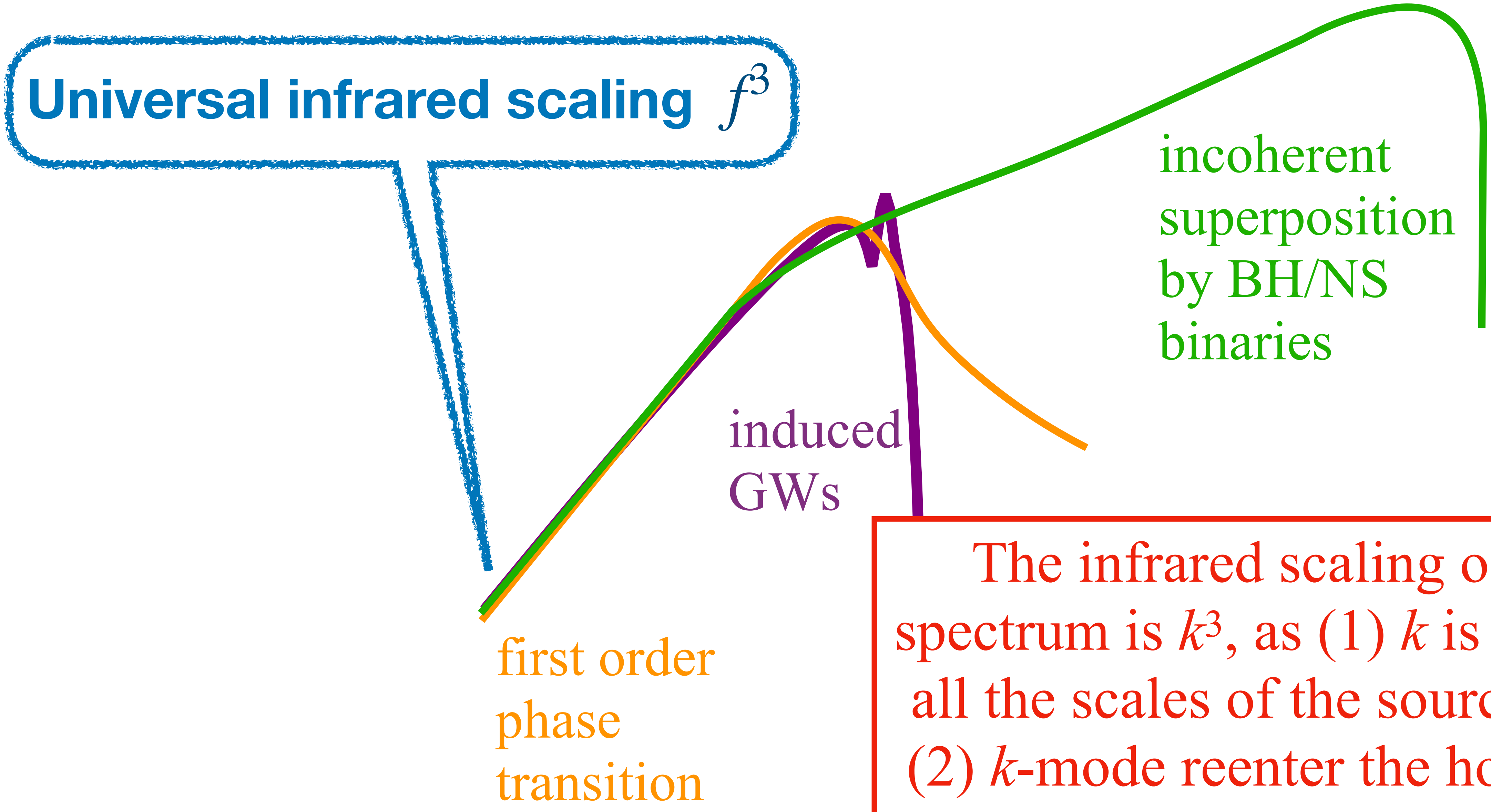
 [SP and Sasaki, *JCAP* **09** (2020) 037]

Infrared Scaling



[SP and Sasaki, *JCAP* **09** (2020) 037]

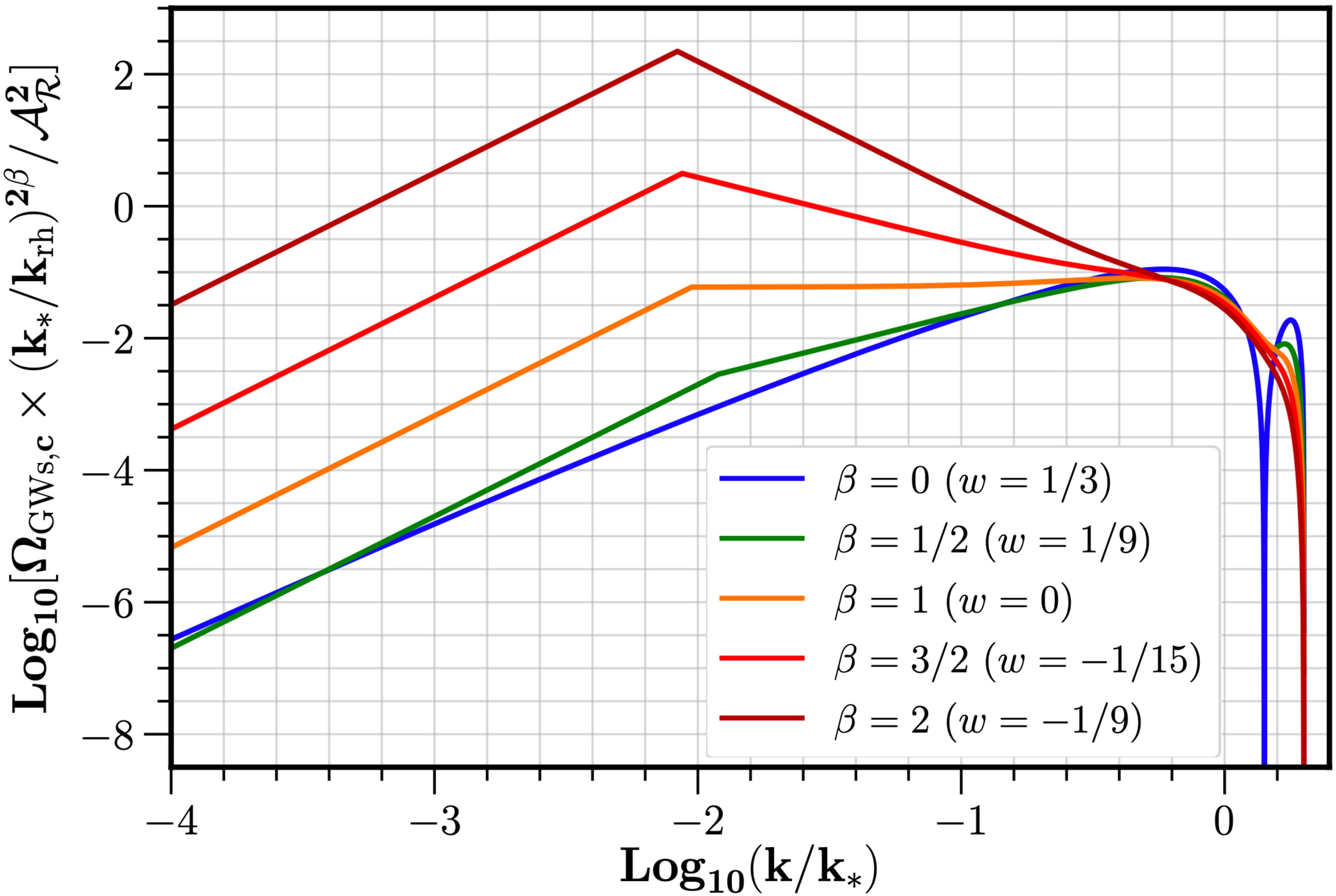
Infrared Scaling of SGWB



The infrared scaling of the GW spectrum is k^3, as (1) k is smaller than all the scales of the source term; and (2) k-mode reenter the horizon in the radiation dominated universe.

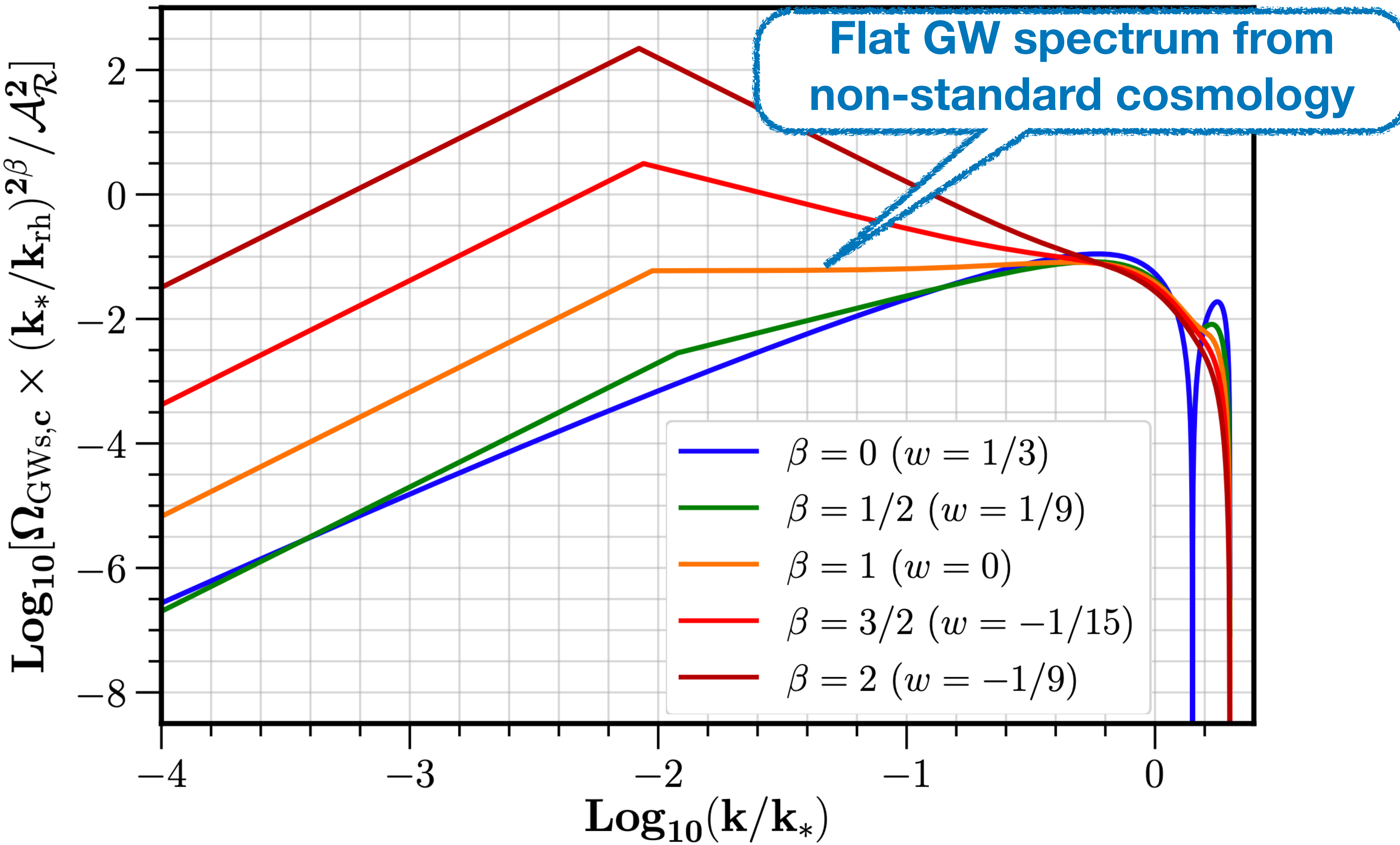
[Cai, SP, and Sasaki, arXiv:1909.13728]

Infrared Scaling of SGWB



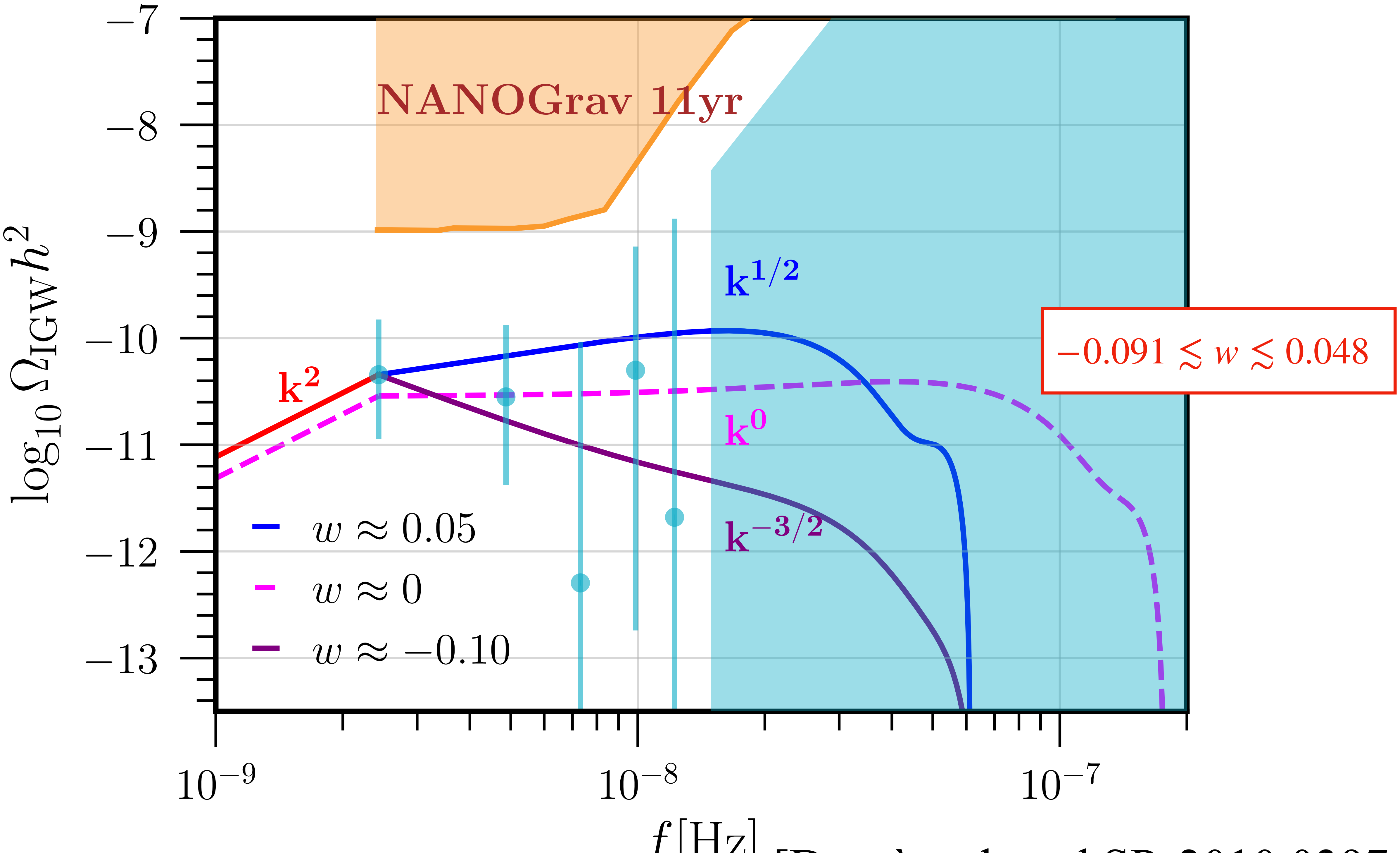
[Domènech, SP, and Sasaki, *JCAP* **08** (2020) 017]

Infrared Scaling of SGWB



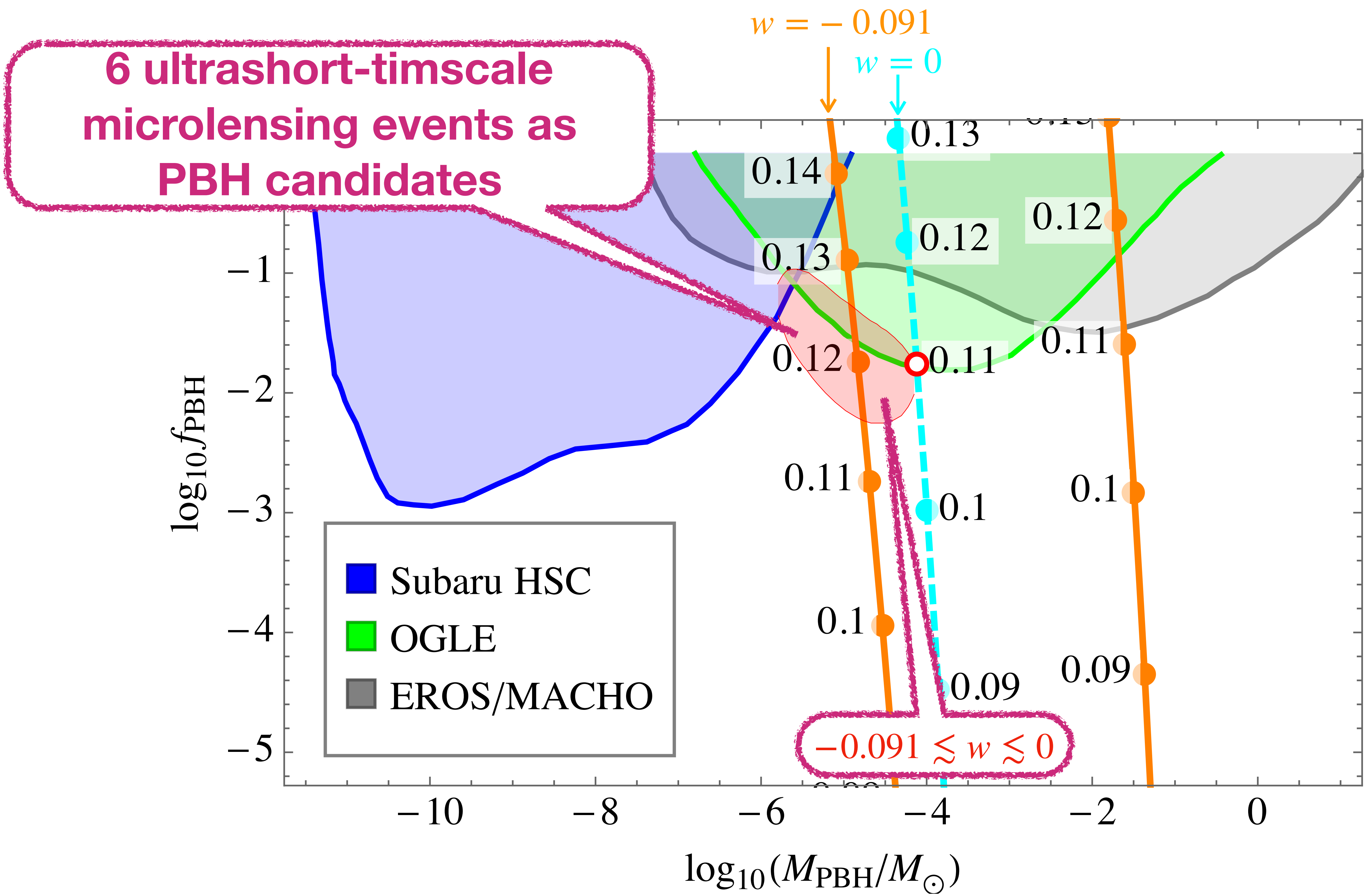
[Domènech, SP, and Sasaki, *JCAP* **08** (2020) 017]

NANOGrav SGWB



[Domènech and SP, 2010.03976]

NANOGrav PBHs



[Domènech and SP, 2010.03976]

Summary

- Induced GWs is one of the most important scientific goal of the next generation GW detectors, which has fruitful phenomena together with PBH physics.

- IGW and PBH-DM: If PBHs can serve as all the DM, induced GWs must be detectable by space-based GW detectors.

- Shape of IGW: It depends crucially on the width of the scalar peak. We found analytical formulae for both cases in the radiation-dominated universe, which is useful for signal searching in the future.

- NANOGrav has detected common-spectrum PTA time residuals which might be the first detection of SGWB. We show that this can be connected to the recently reported planet-mass PBHs.

Thank you
for your attendance