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Exploring the vector modes in ΛCDM with general-relativistic simulations

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Based on 2010.08257 (**C. B-H**, B. Li, M. Bruni, J. He, 2020)

ICC

The vector modes of metric and matter fields

$$\boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}}$$

- Most general line element:

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + 2B_i \mathrm{d}x^i \mathrm{d}t + a^2[(1-2\Psi)\delta_{ij} + h_{ij}]\mathrm{d}x^i \mathrm{d}x^j$$

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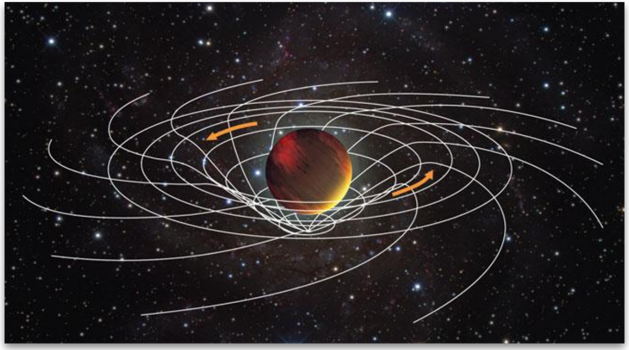
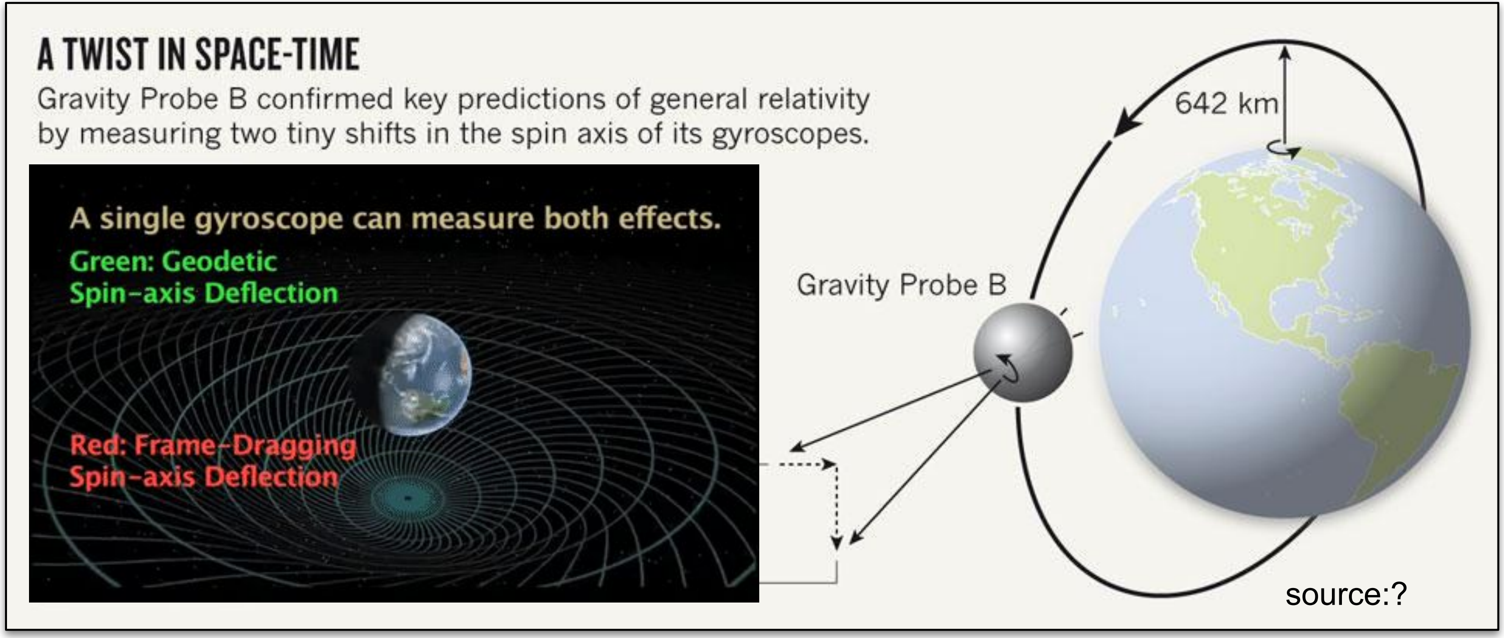
- Each mode encodes different physical effects.
    - Scalar = gravitational pull (Newtonian)
    - Vector = frame-dragging
    - Tensor = ~GWs
- From the matter sector, we are interested in **vorticity**.



Detecting gravitomagnetic effects

- Solar System: ✔️Gravity Probe B (Everitt+‘11)



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Detecting gravitomagnetic effects in cosmology

- Galactic scales: ❓Flat rotation curve of galaxies (Crosta+ ‘20).
- Cosmological scales: ❌ observables currently dominated by scalar modes.
  - Gravitomagnetic effects need to be successfully isolated.
  - E.g., dipole feature in lensing convergence due to dark matter halo rotation (Tang+ ‘20).
- Cosmological vector modes may become non-negligible in future observations:
  - Ratio of vector potential and Newtonian potential can almost reach the percent level (Lu+ ‘09).
  - Vorticity can contaminate future observations (Bonvin+ ‘18).

Modelling the vector modes in ΛCDM

Cosmological Perturbation Theory

Presureless perfect fluid (dust)

Vorticity ❌ (Lu+ ‘09).

$$\Phi, \Psi, \mathbf{B}, h_{ij}$$

2nd order

Modelling the vector modes in ΛCDM

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2nd order

N-body Simulations

Newtonian

Vorticity ✓

$\mathbf{\Phi}$

B-spectrum can be estimated (Bruni+ '14)

GR

Vorticity ✓

$\mathbf{\Phi}, \mathbf{\Psi}, \mathbf{B}, h_{ij}$

The **GR**AMSES N-body simulation

GR extension of the particle-mesh **RAMSES** code (Teyssier ‘02).

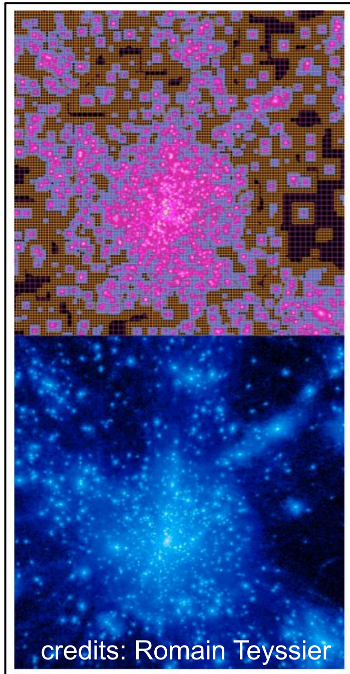
- Gravity solved in a constrained formulation of Einstein Equations including scalar and vector modes (Bonazzola+ ‘04).

3+1 metric: $$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\alpha^2 \mathrm{d}t^2 + \psi^4 \delta_{ij} \left(\mathrm{d}x^i + \beta^i \mathrm{d}t\right)\left(\mathrm{d}x^j + \beta^j \mathrm{d}t\right)$$

3+1 EOM: $$\frac{du_i}{dt} = -\alpha u^0 \partial_i \alpha + \boxed{u_j \partial_i \beta^j} - \frac{u_j u_k}{2u^0}\partial_i \gamma^{jk}$$
$$\frac{dx^i}{dt} = \frac{\gamma^{ij} u_j}{u^0} - \beta^i$$

~Gravitomagnetic force

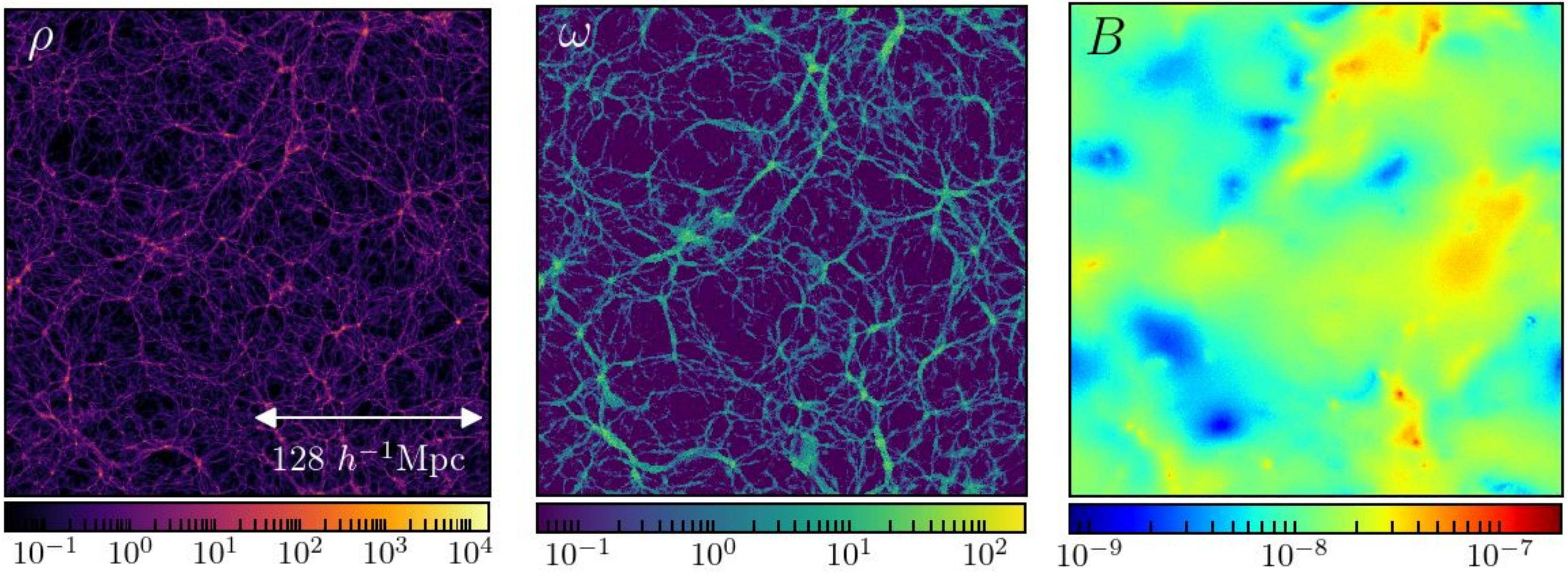
- L = 256 Mpc/h, 10^9 dark matter particles. Planck cosmology.
- Initial conditions set by linear PT (CAMB) at z=49.
- Maximum resolution (AMR) ~ 2 kpc/h.



Results: Maps at z=0

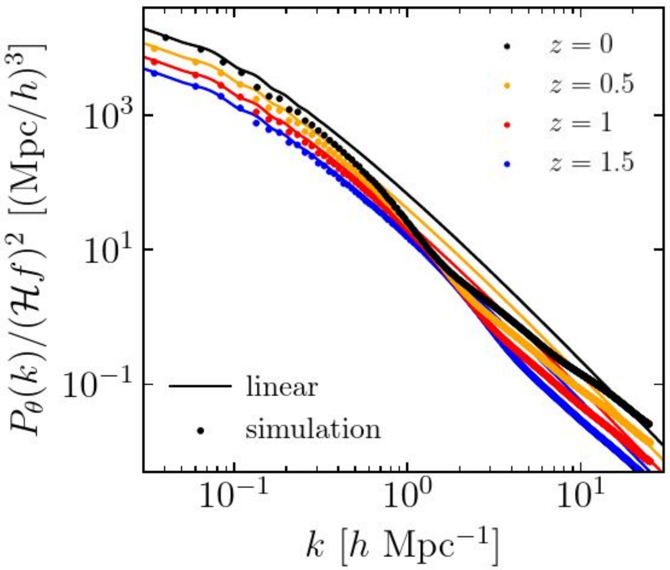
Vector mode of the shift

$$\boxed{B^i = \beta^i_V}$$

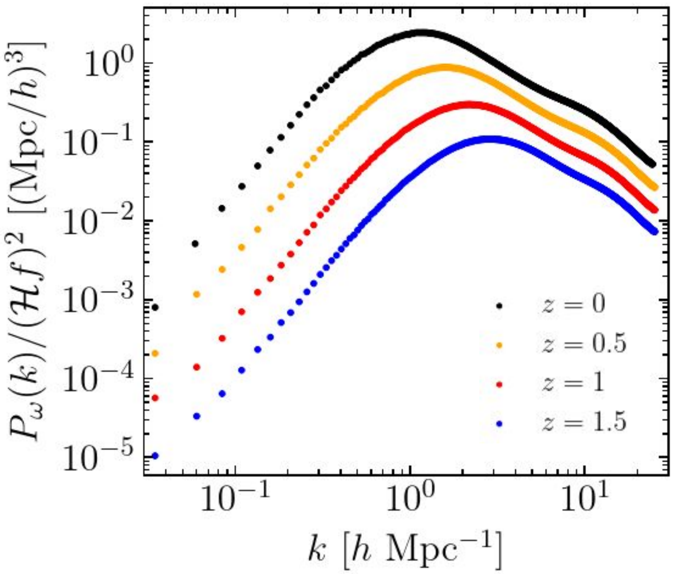


Velocity power spectra

Velocity divergence $\theta = \nabla \cdot \mathbf{u}$



vorticity $\omega = \nabla \times \mathbf{u}$



- On small scales $\theta \sim \omega$.

Vorticity power spectrum: shape

On large scales, the Effective Field Theory of the Large-scale Structure (EFTofLSS) predicts (Carrasco+ '14)

$$\boxed{P_\omega(k) = A_\omega k^{n_\omega}}$$

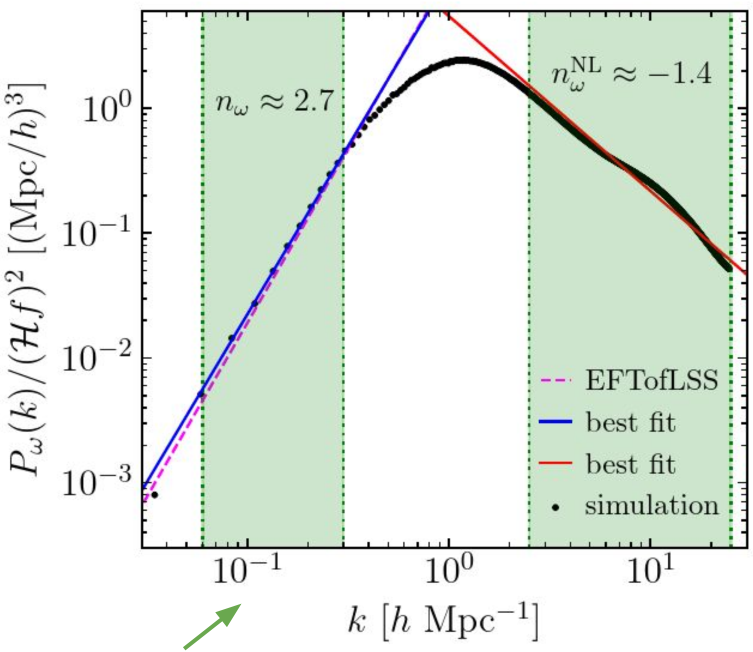
Literature:

Carrasco+ '14: **$n_\omega \approx 3.6, 2.8$ (theory)**

Jelic-Cizmek+ '15 : $n_\omega \approx 2.5$ (sim)

Hahn+ '15: $n_\omega \approx 2.5$ (sim)

z=0



Best-fit value: $n_\omega \approx 2.7$

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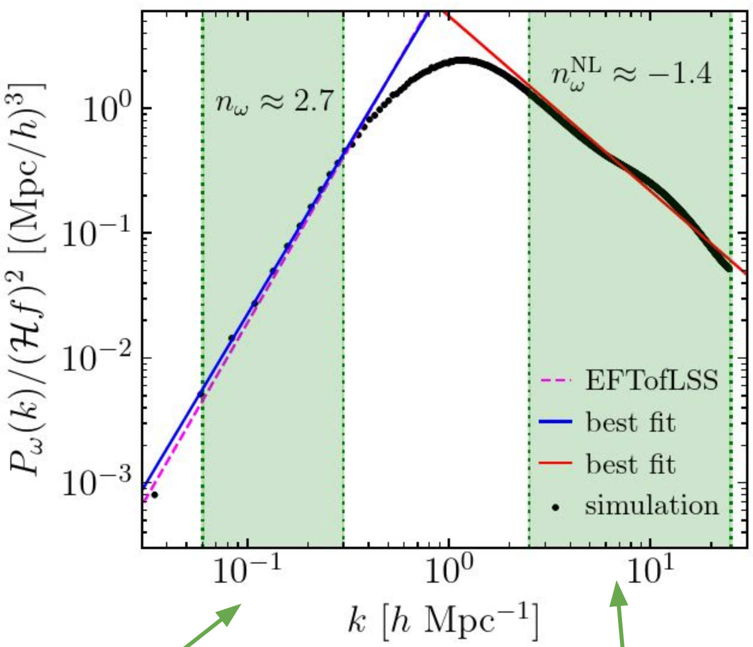


Best-fit value: n_ω≈2.7

Best-fit value: n_ω^{NL}≈-1.4

Hahn+ ‘15: n_ω^{NL}≈-1.5 (sim)

Vorticity power spectrum: time evolution

We can try to parametrise the evolution in terms of a spectral index γ_ω as (Pueblas & Scoccimarro '09)

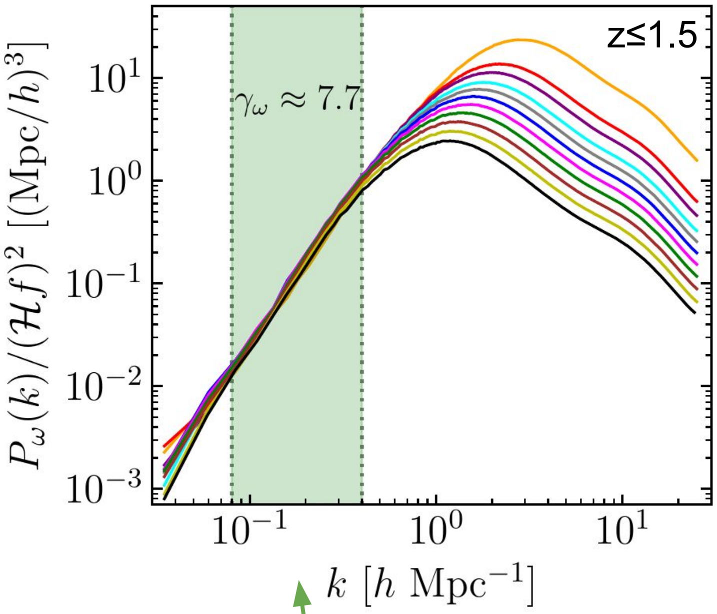
$$P_\omega(k;z) = \left(\frac{D_+(z)}{D_+(0)}\right)^{\gamma_\omega} P_\omega(k;z=0)$$

Literature:

Pueblas & Scoccimarro '09: $\gamma_\omega \approx 7.0 \pm 0.3$

Thomas+ 2015: $\gamma_\omega \gtrsim 7.0$

Jelic-Cizmek+ '15: $\gamma_\omega \gtrsim 7.0$



Best-fit value: $\gamma_\omega \approx 7.7$ ($z \leq 1.5$)

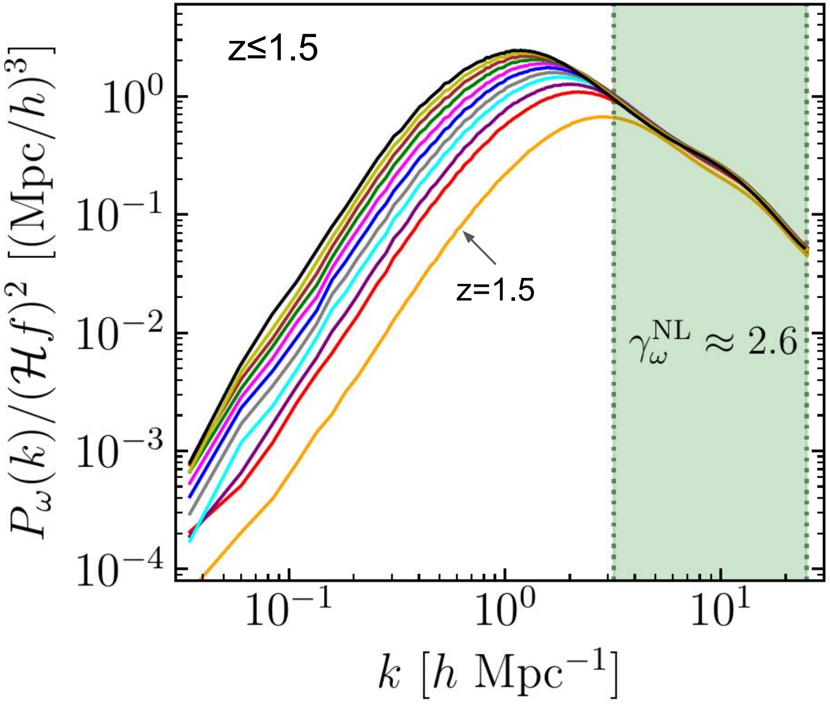
Vorticity power spectrum: time evolution (NL)

We can also parametrise the evolution on nonlinear scales with good approximation.

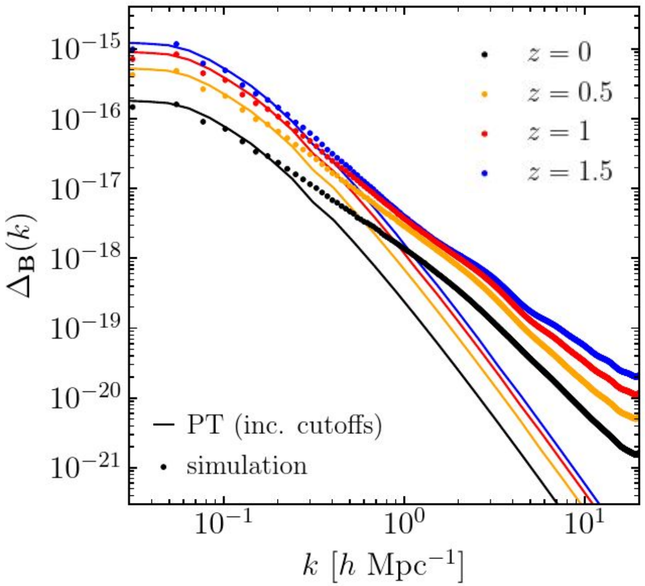
$$P_\omega(k;z) = \left(\frac{D_+(z)}{D_+(0)}\right)^{\gamma_\omega^{\mathrm{NL}}} P_\omega(k;z=0)$$

Best-fit value: $\gamma_\omega \approx 2.6$.

However, this seems to break down beyond z=1.

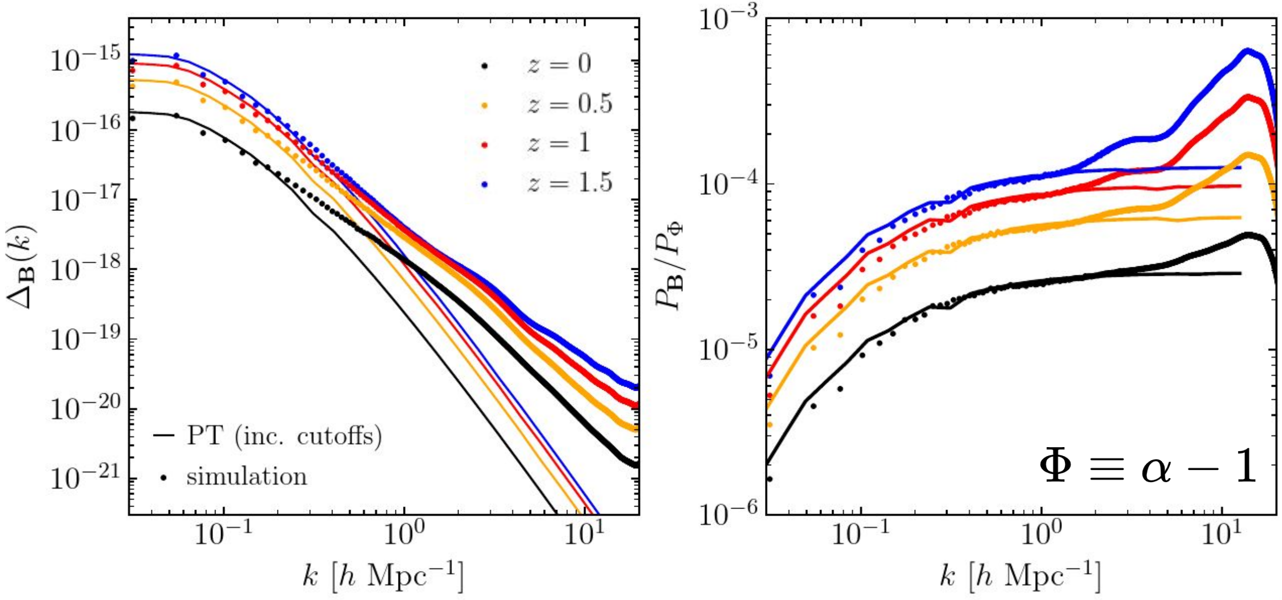


Gravitomagnetic potential: power spectrum



- **B**-spectrum consistent with Bruni+ '14, Thomas+ '15.

Gravitomagnetic potential: power spectrum



- **B**-spectrum consistent with Bruni+ '14, Thomas+ '15.
- **B/|Φ|** remains close to PT (Lu+ '09), but the potentials can deviate ~1 order of magnitude.

Dark matter haloes

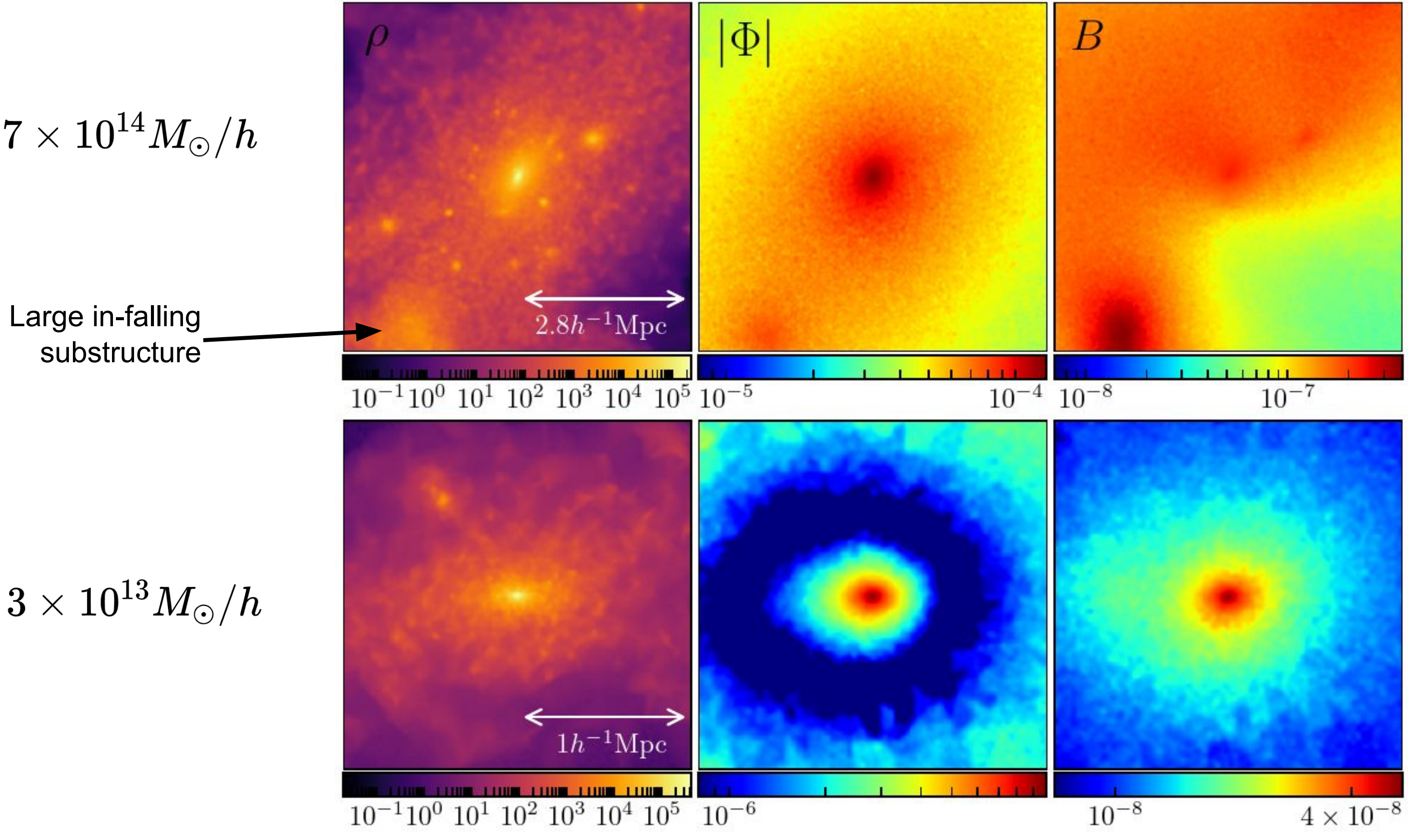
- Halo catalogue from the Friends-of-Friends halo finder *Rockstar*.
- From the catalogue, we select a sample of dark matter haloes split into different mass bins:

Higher mass range	$M_h \geq 10^{14.5}\ h^{-1} M_{\odot}$
Intermediate mass range	$\bar{M}_h = 10^{13.5}\ h^{-1} M_{\odot}$
Lower mass range	$\bar{M}_h = 10^{12.5}\ h^{-1} M_{\odot}$

Dark matter haloes: potentials

$M_h = 7 \times 10^{14} M_\odot/h$

$M_h = 3 \times 10^{13} M_\odot/h$



Halo profiles: potentials

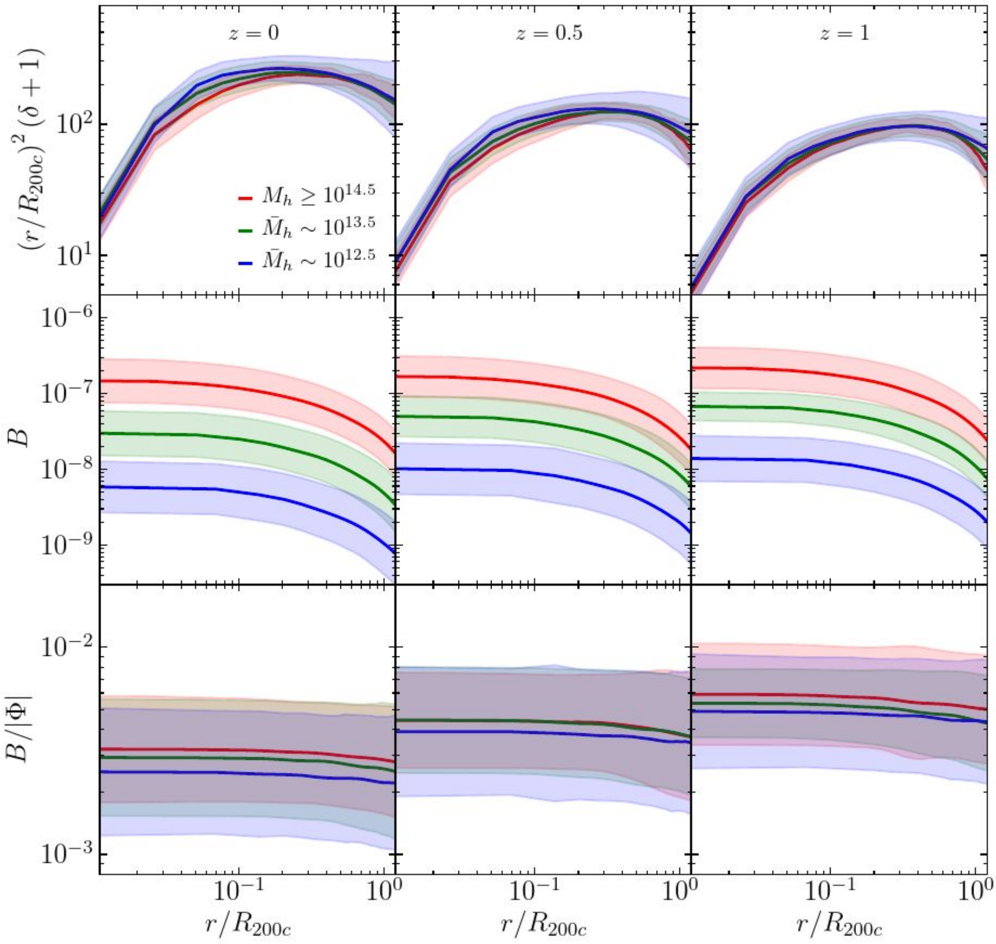
We calculate the **spherical average** of each field around halo centers. These are split into:

$$\Phi(r) = \boxed{\Phi_{\rm halo}} + \Phi_{\rm env}$$
$$B(r) = \boxed{B_{\rm halo}} + B_{\rm env}$$

Then, we subtract their values at r=2R$_{200c}$.

We calculate the **ensemble average** and 1σ deviation in a given mass bin.

B/|Φ| ratio has weak dependence on halo mass.



Dark matter haloes: forces

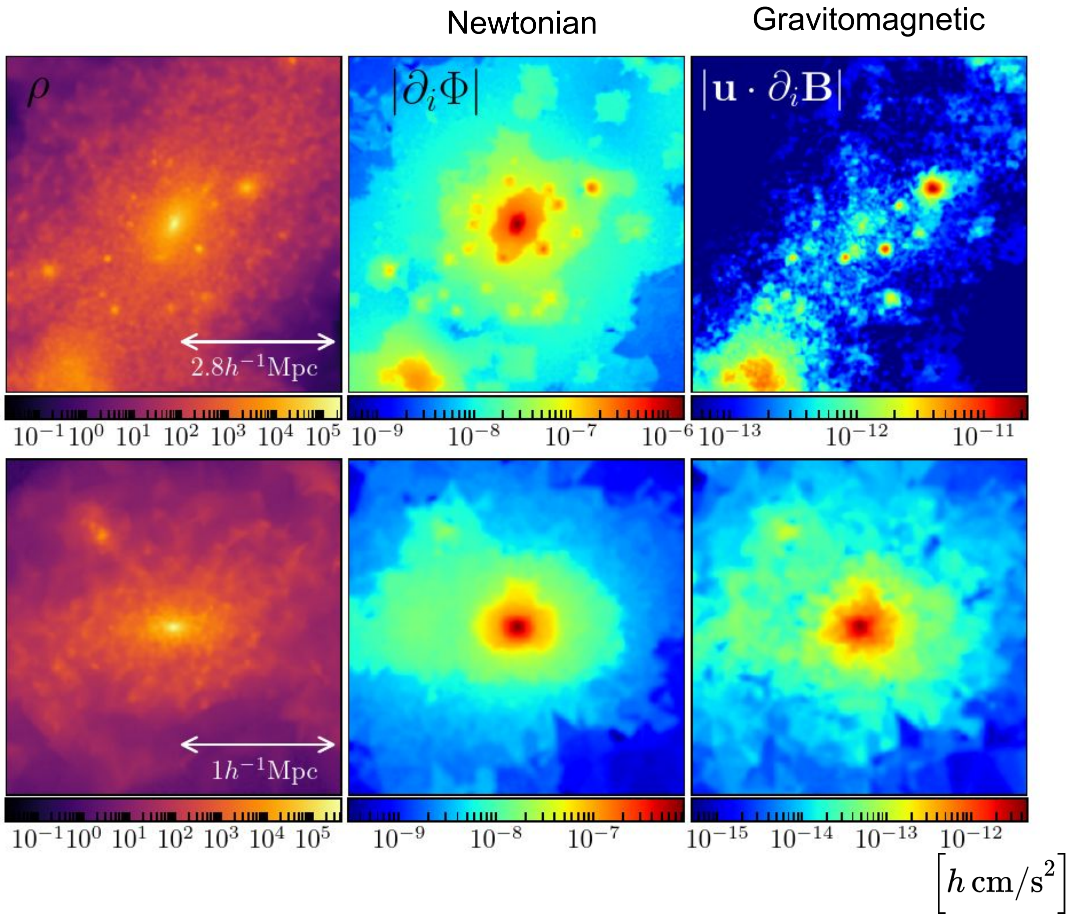
3+1 'acceleration' equation:

$$\frac{du_i}{dt} = -\boxed{\alpha u^0 \partial_i \alpha} + \boxed{u_j \partial_i \beta^j} - \underbrace{\frac{u_j u_k}{2u^0} \partial_i \gamma^{jk}}_{\text{Higher-order corrections}}$$

$\partial_i \Phi$

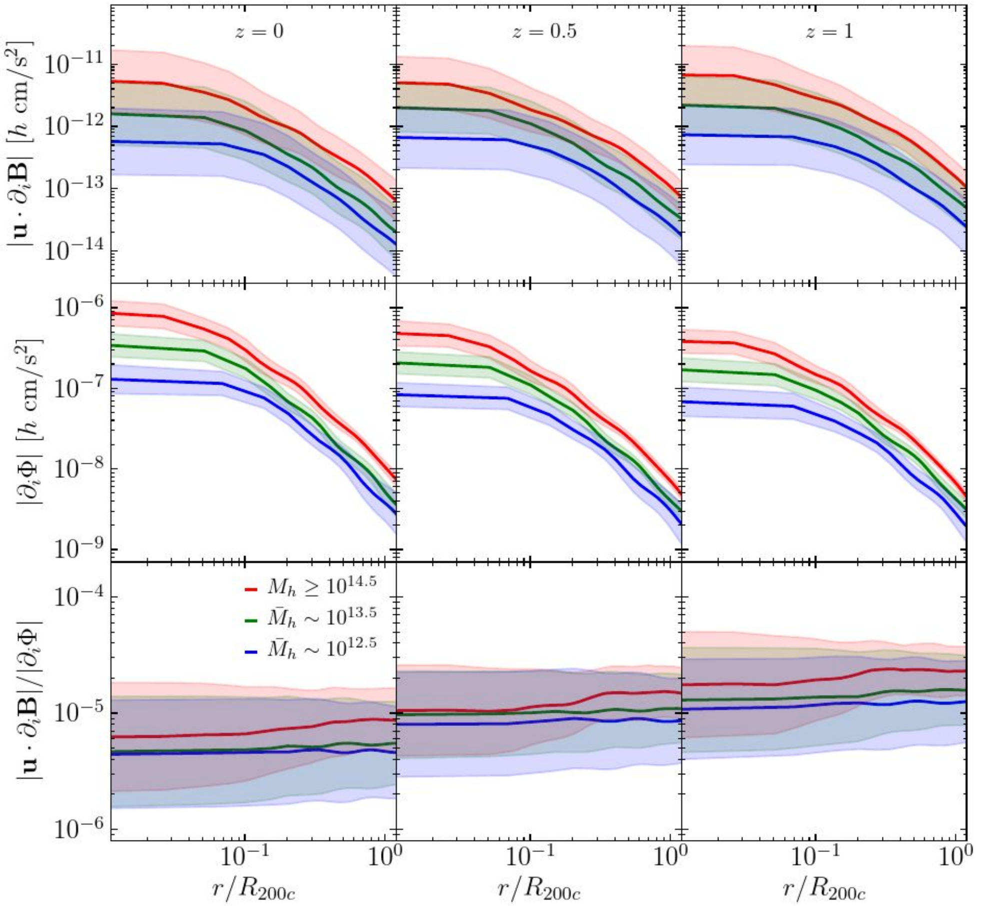
Newtonian limit

Gravitomagnetic force



Halo profiles: forces

- Results for upper halo mass range are consistent with Adamek+ ‘16 at the 1σ level.
- Nonetheless, the gravitomagnetic acceleration can be 1 order of magnitude larger than in Adamek+ ‘16.
- Force ratio **~10^{-5}**, also has weak dependence on halo mass.



Conclusions

I. We have studied the cosmological vector modes with a high-resolution GRAMSES simulation.

II. Power-law fit for the vorticity power spectrum shape and evolution on both large and small scales.

III. The gravitomagnetic potential and force peak towards the halo centers, correlating with halo mass.

IV. Ratio **B/|Φ|** remains flat around **~10^{-3}** inside haloes, regardless of halo mass.

V. Ratio between the gravitomagnetic and Newtonian force remains **~10^{-5}**, regardless of halo mass.

VI. Future work: study the impact of **B** on photons (e.g. lensing convergence) and cosmological observables.