Universal constraints on primordial magnetogenesis from baryon isocurvature perturbations

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IPMU APEC seminar
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Courtesy: H. Oide

1. Introduction
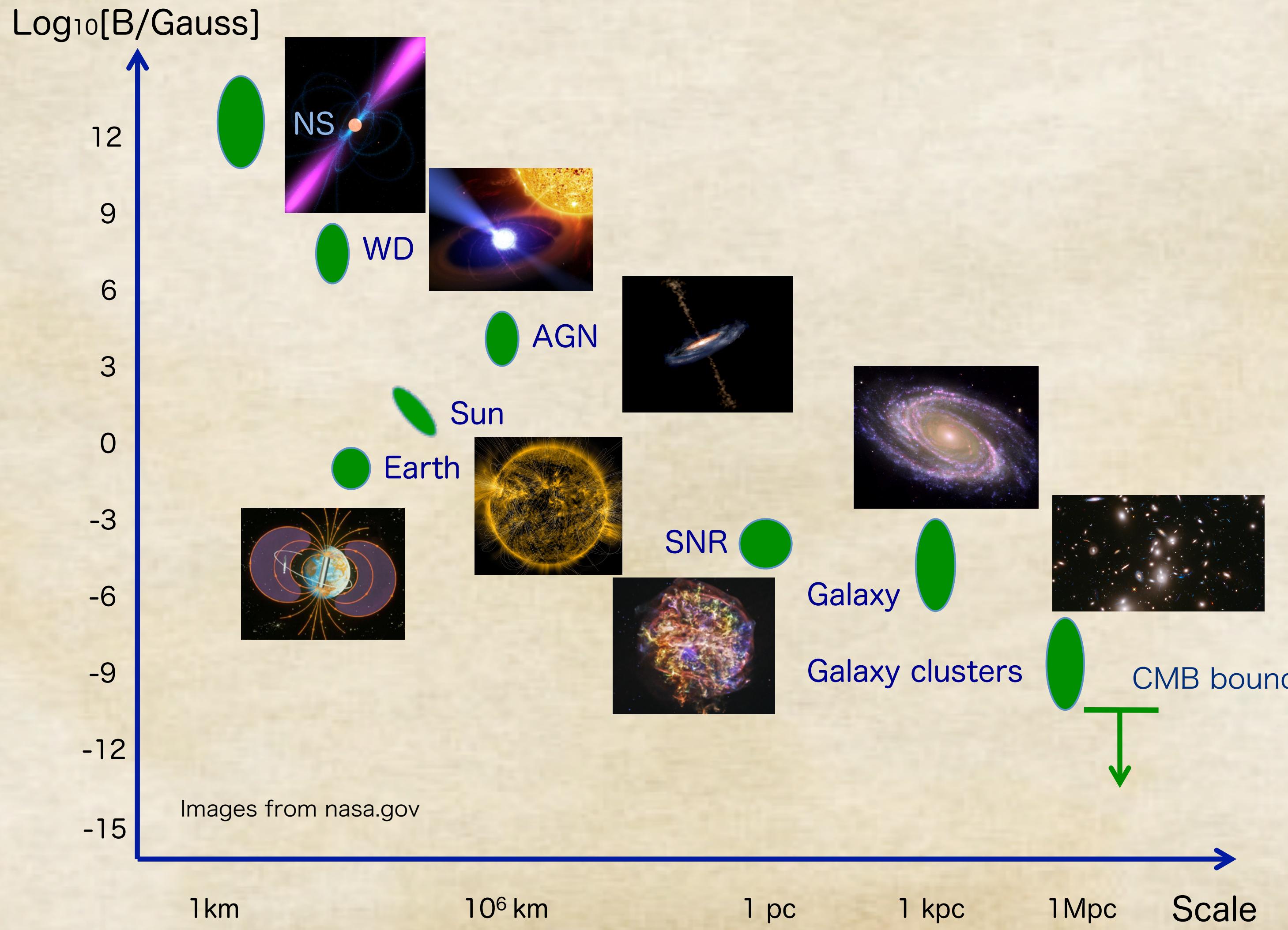   - Primordial magnetic fields (PMFs) -
2. Baryon asymmetry from PMFs
3. Baryon isocurvature constraints
4. Summary

Introduction

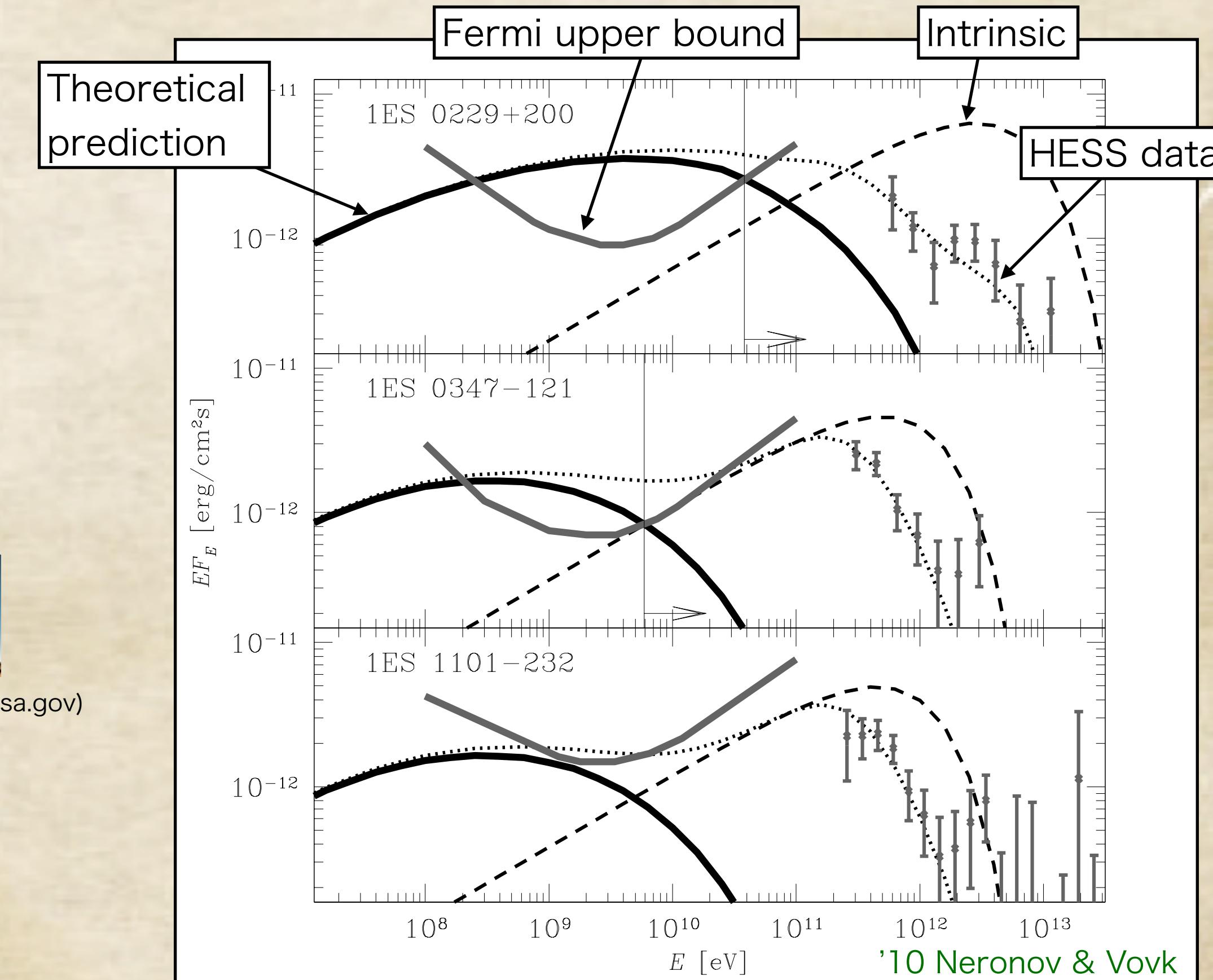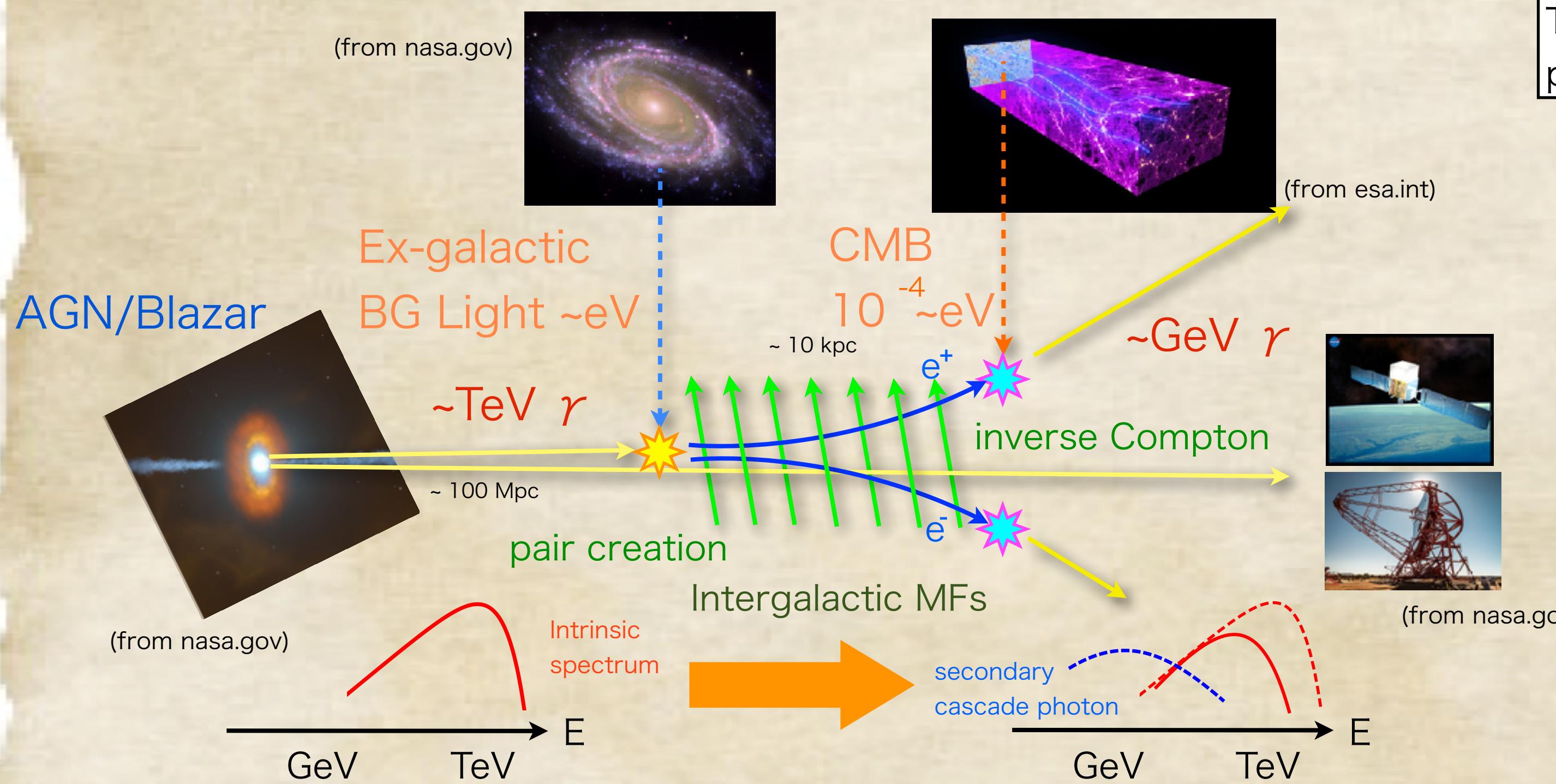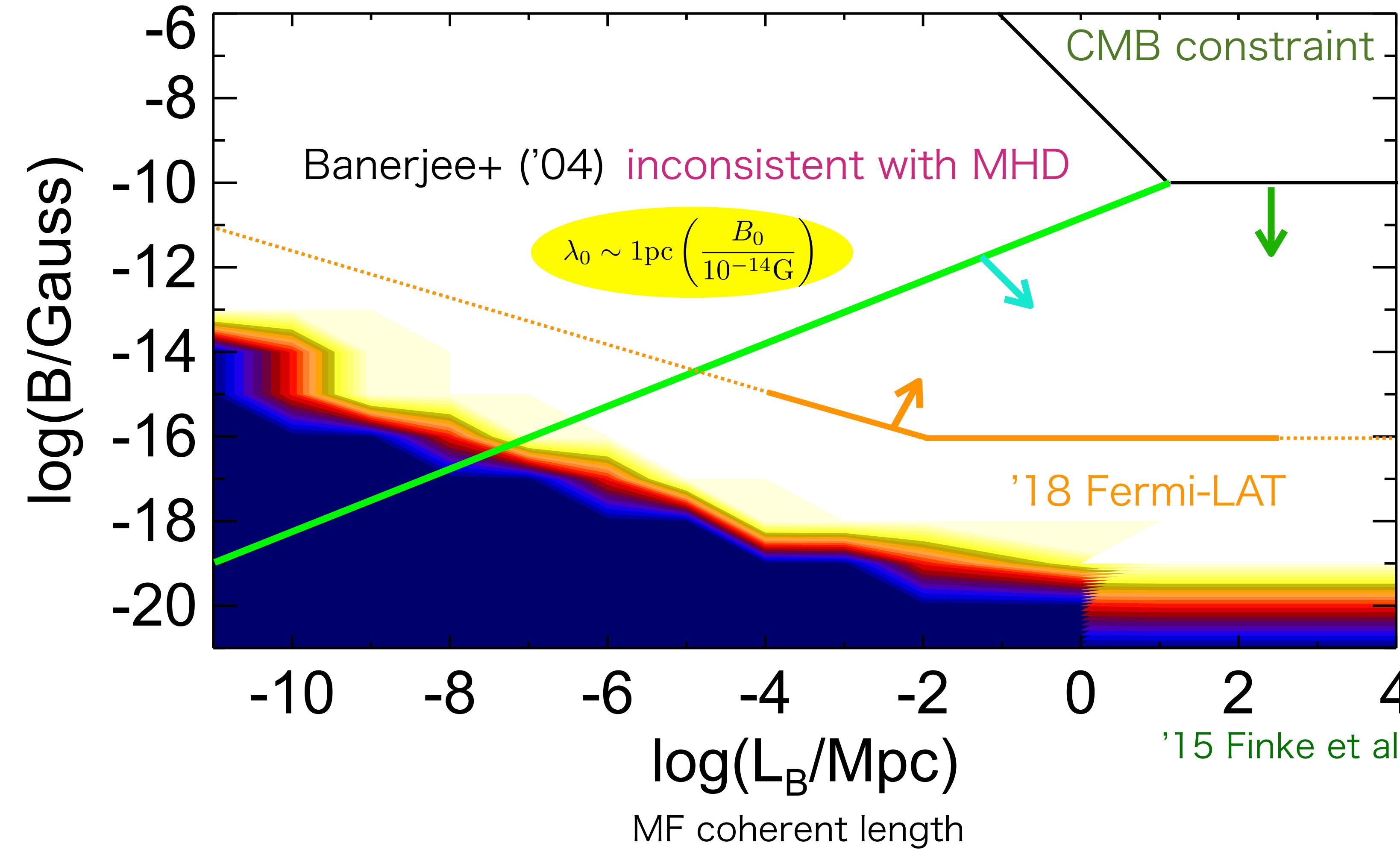# Magnetic fields are ubiquitous in the Universe.

Log$_{10}$[B/Gauss]

12
9
6
3
0
-3
-6
-9
-12
-15

NS
WD
AGN
Sun
Earth
SNR
Galaxy
Galaxy clusters
CMB bound

Images from nasa.gov

1km   10^6 km   1 pc   1 kpc   1Mpc   Scale

Courtesy: H. Oide

Observations of the intergalactic magnetic fields

Fermi upper bound

Intrinsic

Theoretical prediction

HESS data

(from nasa.gov)

Ex-galactic
BG Light ~eV

CMB
10^{-4}~eV

(from esa.int)

AGN/Blazar

~GeV γ

~TeV γ

~ 10 kpc

e^+

inverse Compton

~ 100 Mpc

e^-

pair creation

Intergalactic MFs

(from nasa.gov)

(from nasa.gov)

Intrinsic spectrum

GeV   TeV   E

secondary cascade photon

GeV   TeV   E

1ES 0229+200

1ES 0347−121

1ES 1101−232

EF_E [erg/cm^{2}s]

10^8 10^9 10^{10} 10^{11} 10^{12} 10^{13}

E [eV]

'10 Neronov & Vovk

Non-observation of the secondary cascade GeV photon can give the lower bound of the intergalactic magnetic fields (indirect implication)

Latest constraints from Fermi

CMB constraint

Banerjee+ ('04) inconsistent with MHD

$$\lambda_0 \sim 1\mathrm{pc}\left(\frac{B_0}{10^{-14}\mathrm{G}}\right)$$

'18 Fermi-LAT

log(B/Gauss)

log(L$_B$/Mpc)

MF coherent length

'15 Finke et al.

Courtesy: H. Oide

We might expect that they are relics from the early Universe.

For the early Universe cosmologists, magnetogenesis mechanism originated
from a cosmological phenomena such as inflation or phase transition is of interest.

=> relation to CMB, gravitational wave background, ⋯ ?

Baryon asymmetry of the Universe can be also explained!

('98 Giovannini & Shaposhnikov,'16 Fujita & KK, KK & Long)

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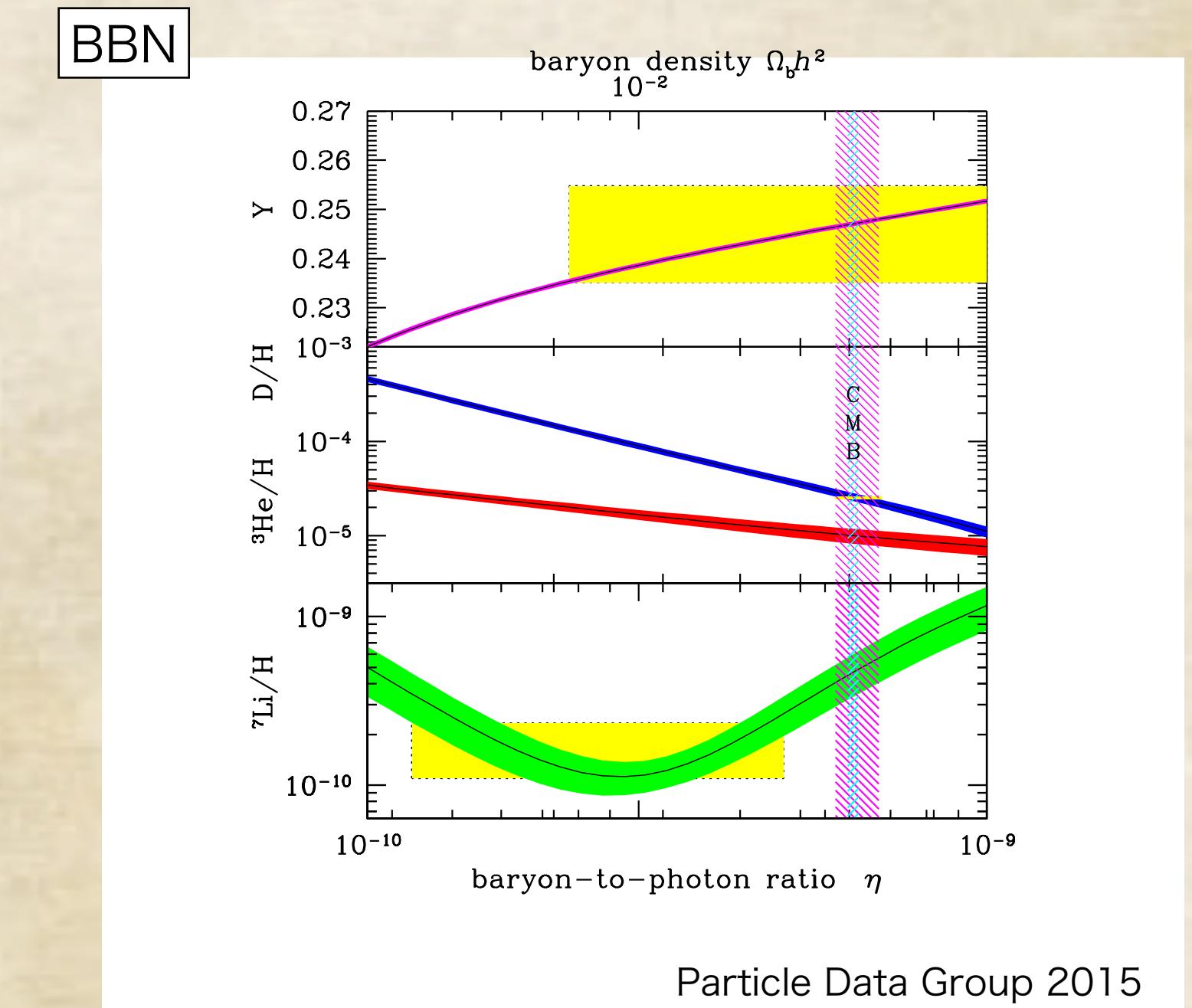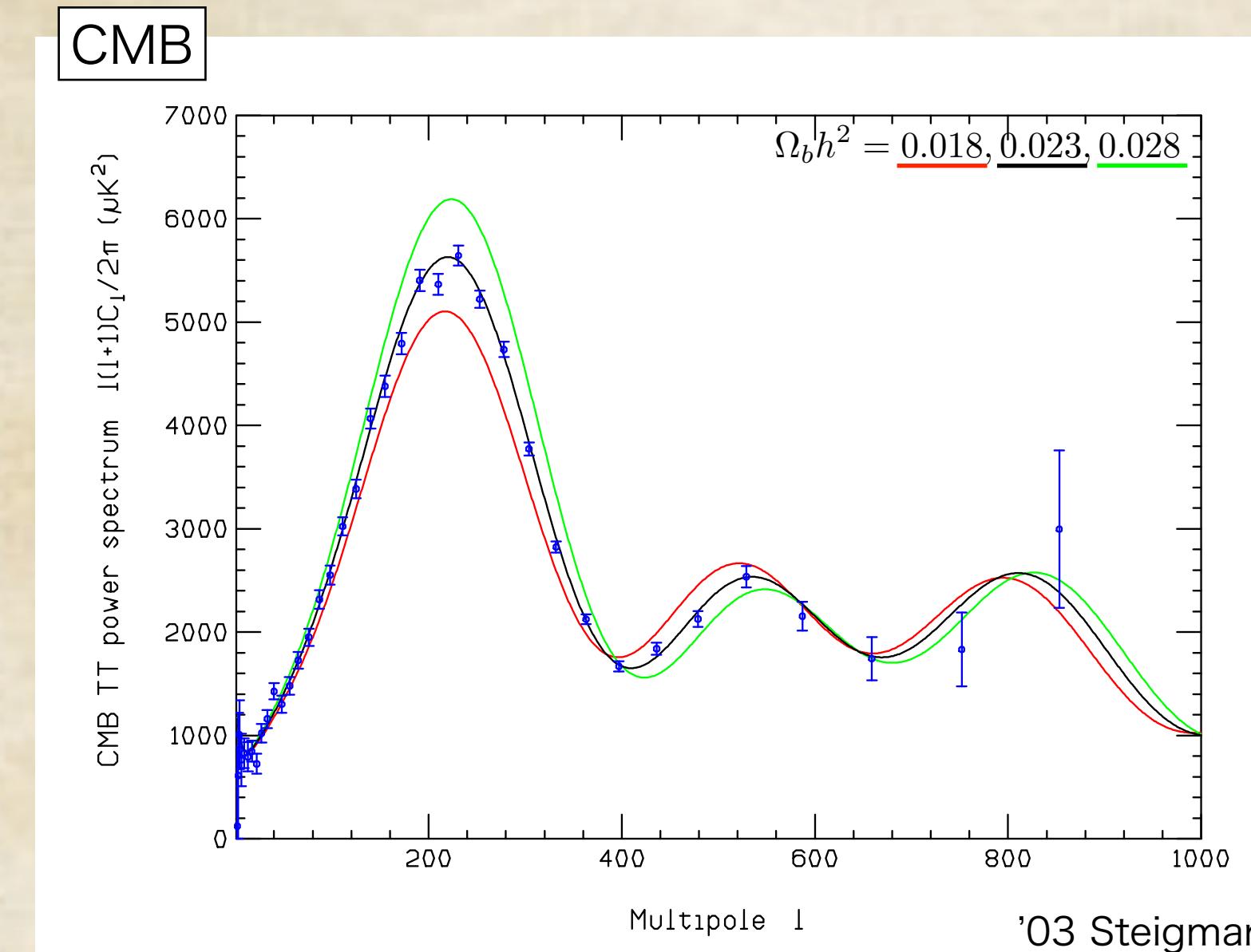Baryon asymmetry of the Universe can be also explained!

('98 Giovannini & Shaposhnikov,'16 Fujita & KK, KK & Long)

We however show that any magnetogenesis mechanisms before the EWSB
hardly explain the IGMFs suggested by blazar observations.

Baryogenesis from PMFs

Baryon Asymmetry of the Universe

- We live in a matter-antimatter asymmetric Universe.

- BBN and CMB can evaluate it quantitatively.

CMB
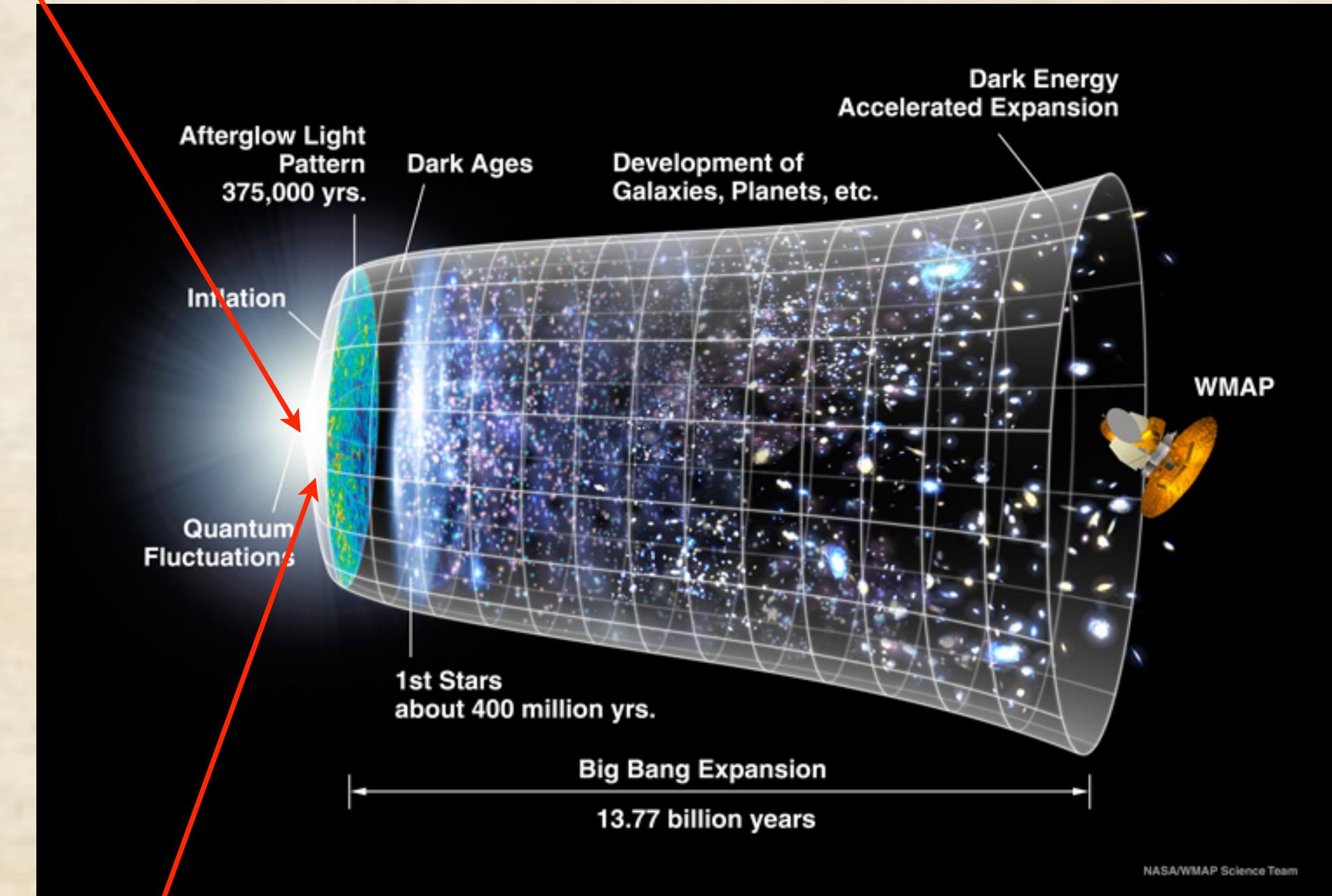

'03 Steigman

BBN

Particle Data Group 2015

$$\eta \equiv \frac{n_B}{n_\gamma} = (6.09 \pm 0.06) \times 10^{-10}$$

(Planck 2015)

$$\eta = (6.180 \pm 0.195) \times 10^{-10}$$

BBN+D ; '15 Cyburt+

Inflation dilutes the preexisting asymmetry.

WMAP team

After inflation before BBN, asymmetry must be generated.

$$
\left\{
\begin{array}{ll}
\text{matter :} & \sim 10000000000+1 \\
\text{anti matter :} & \sim 10000000000 \\
\text{photon :} & \sim 10000000000
\end{array}
\right.
\quad
\underset{T \sim 100 \text{ MeV}}{\xrightarrow{\text{annihilation}}}
\quad
\left\{
\begin{array}{ll}
\text{matter :} & \sim 1 \\
\text{anti matter :} & \sim 0 \\
\text{photon :} & \sim 10000000000
\end{array}
\right.
$$

In order to generate baryon asymmetry...

Sakharov's condition is required. ('67 Sakharov)

1. B-violation

2. C & CP-violation

3. Deviation from thermal equilibrium

BSM is required!?

- Leptogenesis ('85 Fukugita&Yanagida)   : RH neutrinos

- Affleck-Dine   : SUSY with $\cancel{B}$ and $\cancel{CP}$ op.
   ('85 Affleck&Dine, '95 Dine,Randall&Thomas)

- EW baryogenesis   : 1st order EWPT + $\cancel{CP}$ op.
   ('85 Kuzmin, Rubakov&Shaposhnikov)

How helical hypermagnetic fields can satisfy the Sakharov's condition?

Giovannini&Shaposhnikov ('98), KK& Fujita, KK&Long ('16)

Imagine that the hypermagnetic fields exist before the EWSB as Gaussian stochastic fields,

$$\langle B_i(\boldsymbol{k}) \rangle = 0 \qquad \langle B_i(\boldsymbol{k}) B_j(\boldsymbol{k}') \rangle = (2\pi)^3 \left((\delta_{ij} - \hat{k}_i \hat{k}_j) S(k) + i\epsilon_{ijk} \hat{k}_k A(k) \right) \delta(\boldsymbol{k} - \boldsymbol{k}')$$

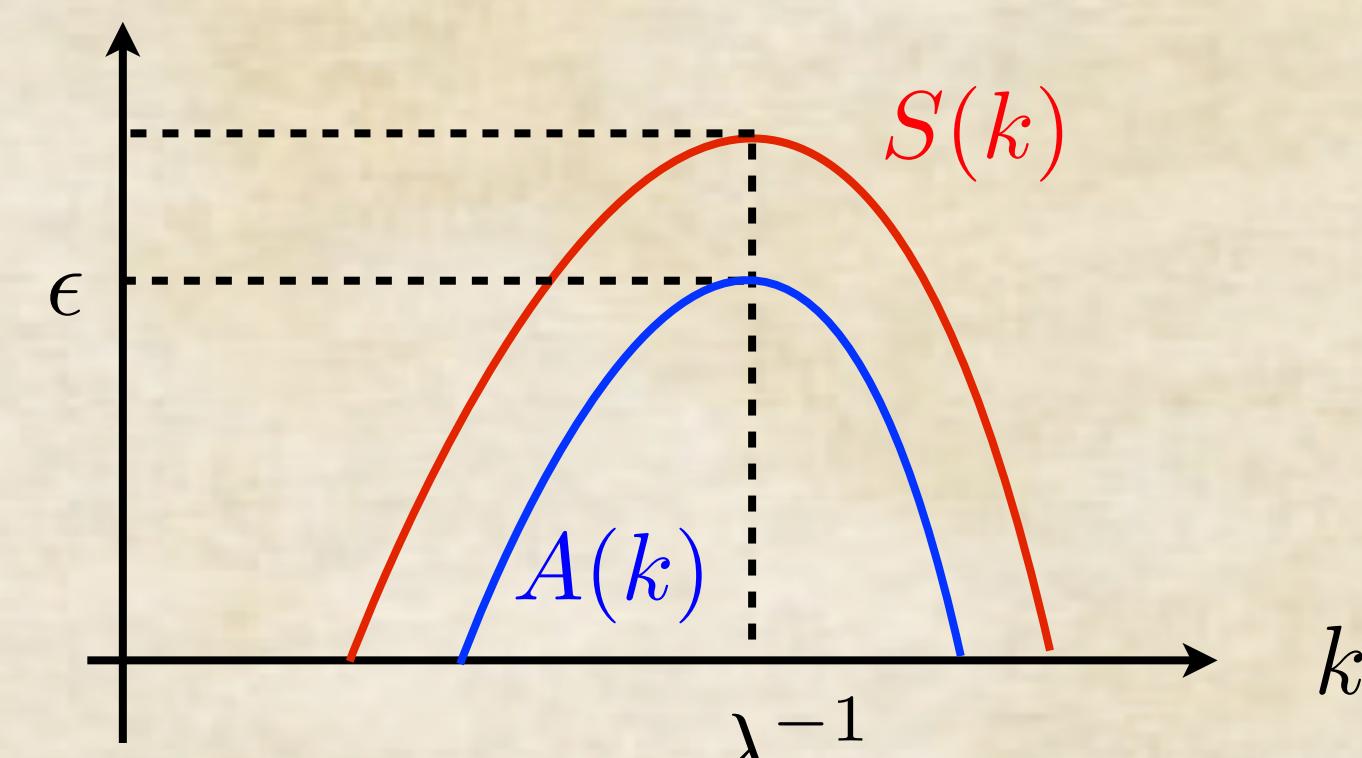$$(S(k) \geq A(k))$$

From these notations, characteristics of the magnetic fields are given by

$$\rho_B = \int \frac{d^3 k}{(2\pi)^3} k^2 S(k) \;\Rightarrow\; \overline{B} = \sqrt{2\rho_B} \qquad \mathcal{H} = 2 \int \frac{d^3 k}{(2\pi)^3} k A(k) \qquad \lambda = \frac{\int dk\, k^3 S(k)}{\int dk\, k^4 S(k)}$$

Suppose the spectrum is localized at a scale,

$$\mathcal{H} \simeq \epsilon \lambda \overline{B}^2 \qquad (\,\epsilon : \text{helicity fraction})$$

If the average of helicity density $\mathcal{H}$ decays,
B+L asymmetry is generated in the Universe?

$$\Delta Q_B = \Delta Q_L = N_g \left(\Delta N_{\mathrm{CS}} - \frac{g'^2}{16\pi^2} \Delta \mathcal{H}_Y \right)$$

Imagine that the hypermagnetic fields exist before the EWSB as Gaussian stochastic fields,

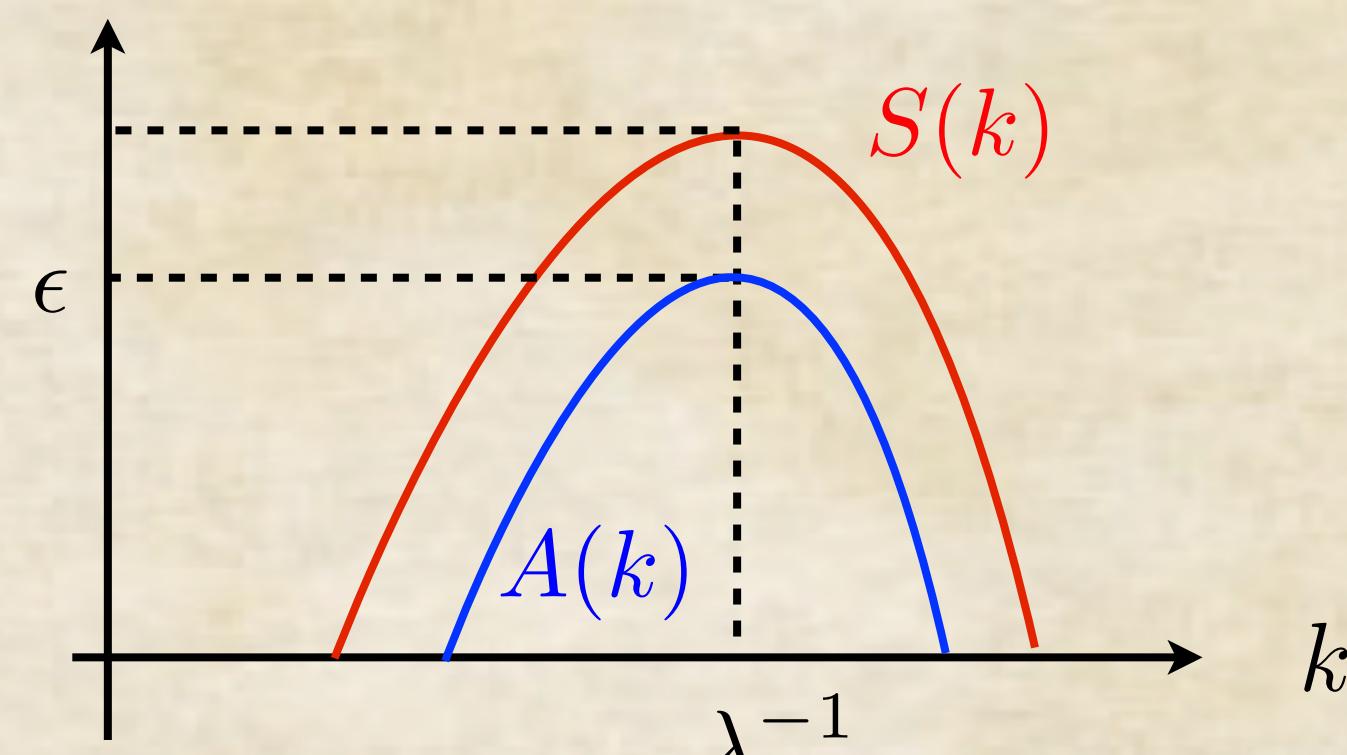$$\langle B_i(\boldsymbol{k})\rangle = 0 \qquad \langle B_i(\boldsymbol{k})B_j(\boldsymbol{k}')\rangle = (2\pi)^3 \left((\delta_{ij} - \hat{k}_i\hat{k}_j)S(k) + i\epsilon_{ijk}\hat{k}_k A(k) \right)\delta(\boldsymbol{k} - \boldsymbol{k}')$$

$$(S(k) \geq A(k))$$

From these no

$$\rho_B = \int \frac{d}{(2} \qquad \frac{{}^3 S(k)}{{}^4 S(k)}$$

If there were BG large-scale helical hyperMFs,
  B-violation: SM chiral anomaly
  C&CP-violation/non-equilibrium
          :existence of large-scale magnetic helicity
Sakharov's conditions are satisfied!

Suppose the spectrum is localized at a scale,

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How to realize helicity decay?

1. Decay due to MHD with finite conductivity ('98 Giovannini&Shaposhnikov)

The characteristic MF strength obeys the Maxwell eq. with the MHD approximation.

$$\boldsymbol{\nabla} \times \overline{\boldsymbol{B}} = \boldsymbol{J} = \sigma \overline{\boldsymbol{E}} \qquad \Rightarrow \overline{\boldsymbol{E}} = \frac{1}{\sigma} \boldsymbol{\nabla} \times \overline{\boldsymbol{B}}$$

$$\partial_t \mathcal{H} = -2\overline{\boldsymbol{E} \cdot \boldsymbol{B}} = -\frac{2}{\sigma}\overline{(\boldsymbol{\nabla} \times \boldsymbol{B}) \cdot \boldsymbol{B}} \simeq -\frac{2}{\sigma}\epsilon\frac{\overline{B}^2}{\lambda}$$

$$\sigma \simeq 100T$$

('97 Baym+, '00 Arnold+)

2. Electroweak symmetry breaking (16 KK&Long)

Gauge group

$$SU(2)_W \times U(1)_Y \to U(1)_{\text{em}}$$

Large-scale (massless) MFs

$$B_Y \to B_{\text{em}} = \cos\theta_W B_Y + \sin\theta_w B_{W^3}$$

Magnetic helicity

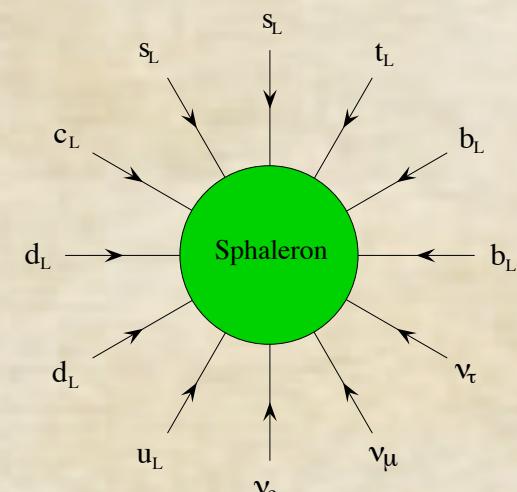$$H_Y^{\text{before}} \to H_{\text{em}}^{\text{after}} = H_Y^{\text{before}}$$

$$H_Y^{\text{after}} = \cos^2\theta_W H_{\text{em}}^{\text{after}} = \cos^2\theta_W H_Y^{\text{before}}$$

$$N_{\text{CS,W}^3}^{\text{after}} \sim \sin^2\theta_W H_{\text{em}}^{\text{after}} = \sin^2\theta_W H_Y^{\text{before}}$$

BAU:
$$\Delta H_Y = -\sin^2\theta_W H_Y^{\text{before}}$$
$$\Delta N_{\text{CS}} \sim \sin^2\theta_W H_Y^{\text{before}}$$

$$\Delta Q_B = \#\Delta N_{\text{CS}} - \#\Delta H_Y \sim \sin^2\theta_W H_Y^{\text{before}}$$

To evaluate the baryon asymmetry from the hypermagnetic helicity decay, we need to evaluate the washout effect.

EW sphalerons+chirality flip by electron Yukawa

W. Buchmüller, 1212.3554

Thermal fluctuations of W-boson induces ΔN_{CS}

Together with Yukawa int., try to washout Q_B

Chiral Magnetic Effect ('80 Vilenkin, '08 Fukushima, Kharzeev, &Warringa)

Ampere's law

$$\boldsymbol{\nabla} \times \boldsymbol{B}_Y = \boldsymbol{J} = \sigma(\boldsymbol{E}_Y + \boldsymbol{v} \times \boldsymbol{B}_Y) + \frac{2\alpha_Y}{\pi}\mu_5 \boldsymbol{B}_Y$$

Ohm's current   Chiral magnetic current

$$\Rightarrow \quad \boldsymbol{E}_Y = -\boldsymbol{v} \times \boldsymbol{B}_Y + \frac{1}{\sigma}\left(\boldsymbol{\nabla} \times \boldsymbol{B}_Y - \frac{2\alpha_Y}{\pi}\mu_5 \boldsymbol{B}_Y\right)$$

$$\frac{d}{dt}n_f \ni \#\langle Y_{\mu\nu}\tilde{Y}^{\mu\nu}\rangle \left(= -4\langle \boldsymbol{E}_Y \cdot \boldsymbol{B}_Y\rangle\right)$$

$$= \#\frac{1}{\sigma}\left(\langle \boldsymbol{B}_Y \cdot (\boldsymbol{\nabla}\times\boldsymbol{B}_Y)\rangle - \frac{2\alpha}{\pi}\boxed{\mu_5}\langle|\boldsymbol{B}_Y|^2\rangle\right)$$

$$= \#\frac{1}{\sigma}\left(\frac{B_p^2}{\lambda_B} - \frac{2\alpha}{\pi}\boxed{\mu_5}B_p^2\right)$$

$$\boxed{\mu_5 = \sum_{f'}(-)^{q_{R/L}}6y_{f'}^2 n_{f'}/T^2}$$

Schematically evolution equation is given by

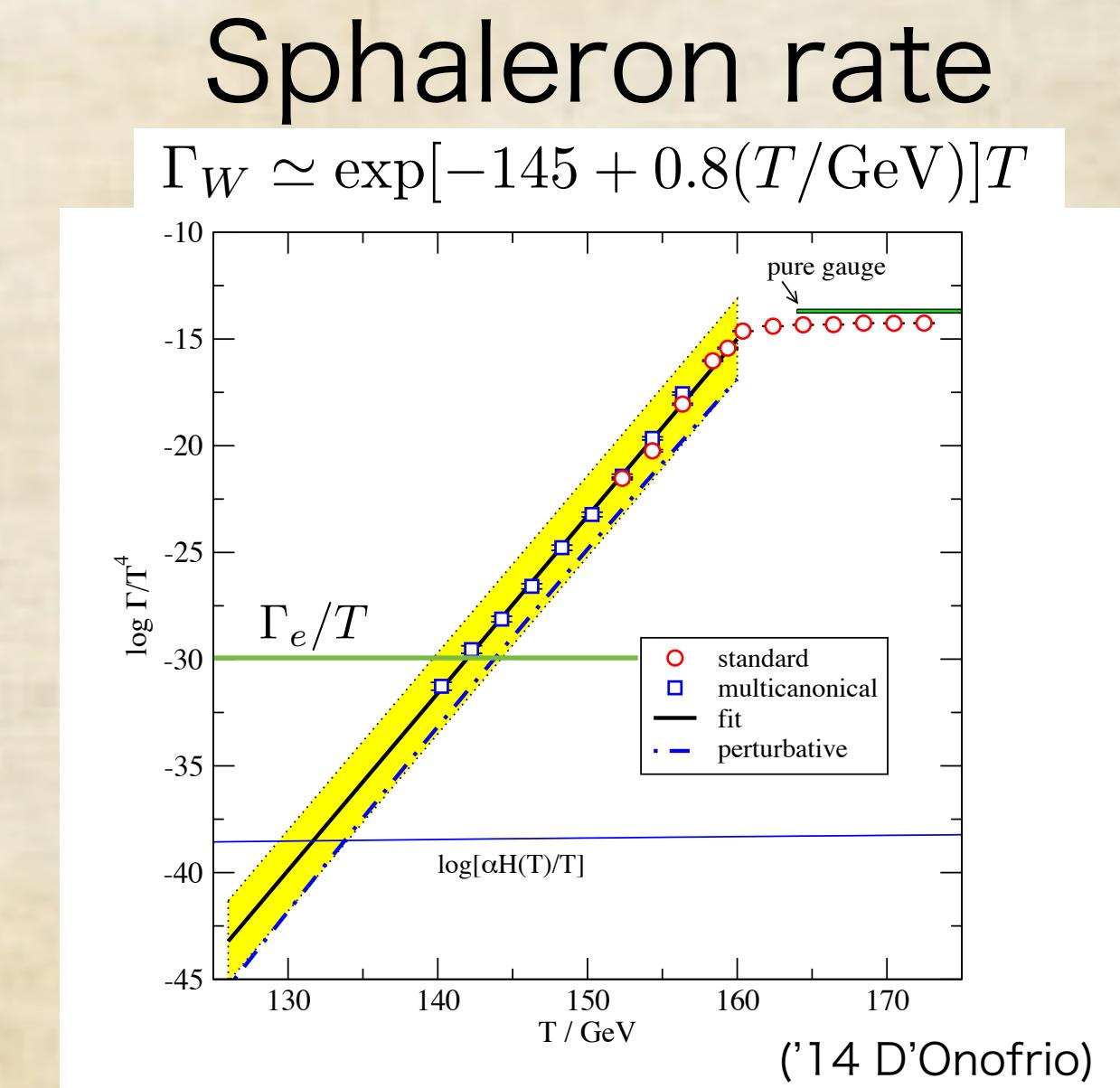$$\frac{dn_B}{dt} = \left(\# \frac{B^2}{\sigma \lambda} + \# \dot{\theta}_W \lambda B^2 \right) - \Gamma_{\text{w.o.}} n_B$$

MHD decay   EWSB   EW sphaleron   chirality-flip   CME

Reaches at "terminal" asymmetry⋯

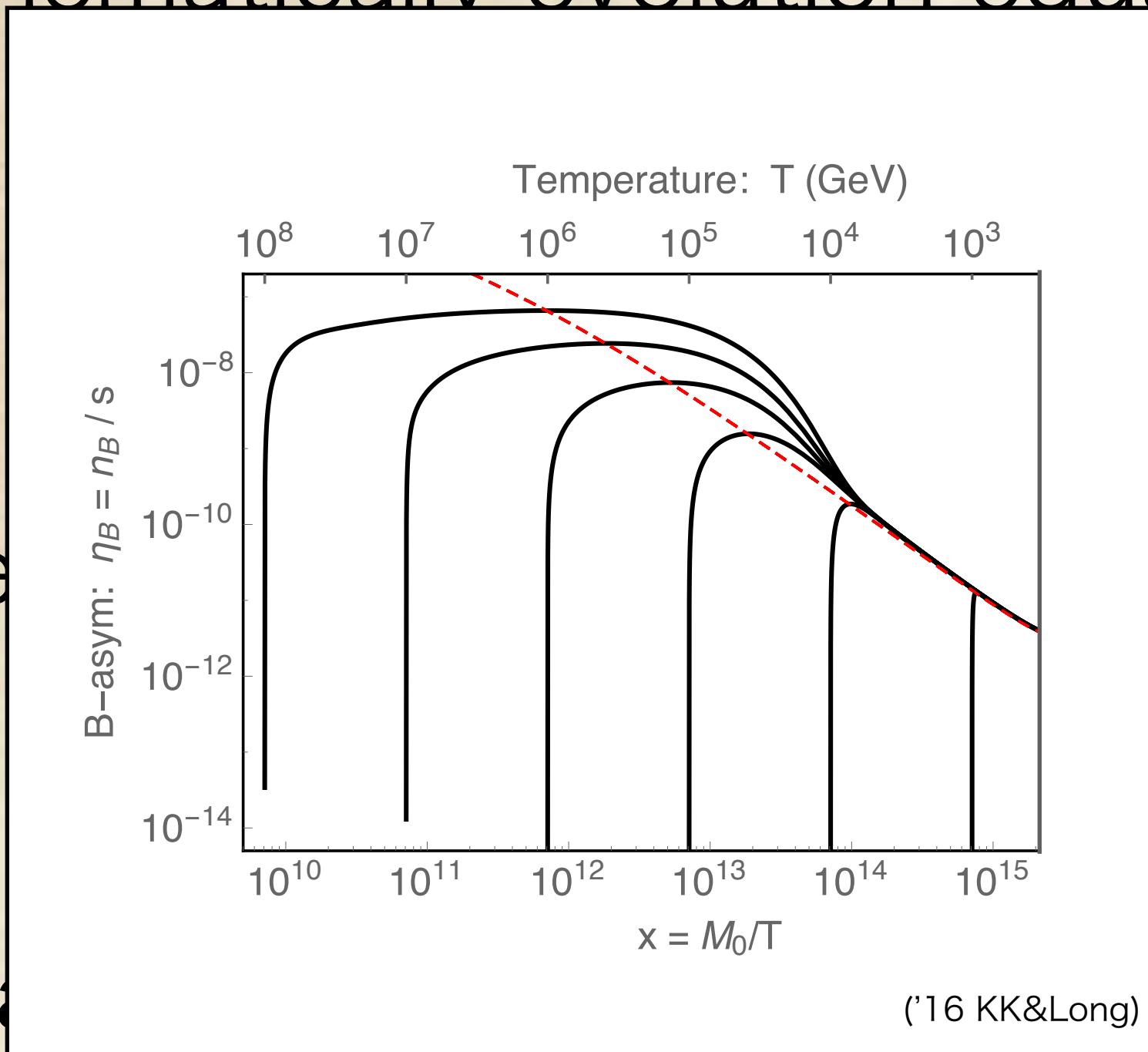$$n_B \simeq \frac{\# B^2/\sigma \lambda + \# \dot{\theta}_W \lambda B^2}{\Gamma_{\text{w.}}}$$

Sphaleron rate

$$\Gamma_W \simeq \exp[-145 + 0.8(T/\text{GeV})]T$$

Washout term $\Gamma_{\text{w.o}}$

High temperature (T>140 GeV): electron Yukawa or CME
Low temperature (T<140 GeV): EW sphaleron

('14 D'Onofrio)

Courtesy: H. Oide

Schematically evolution equation is given by

term   washout

$$\#\dot{\theta}_W \lambda B^2\Big) - \Gamma_{\text{w.o.}} n$$

EWSB   EW sphale

Re   netry⋯

$$B^2/\sigma\lambda + \#\dot{\theta}_w \lambda B$$

$$\Gamma_{\text{w}}$$

Wa

High temperature (T>140 GeV): electron Yukawa or CME

Low temperature (T<140 GeV): EW sphaleron

('16 KK&Long)
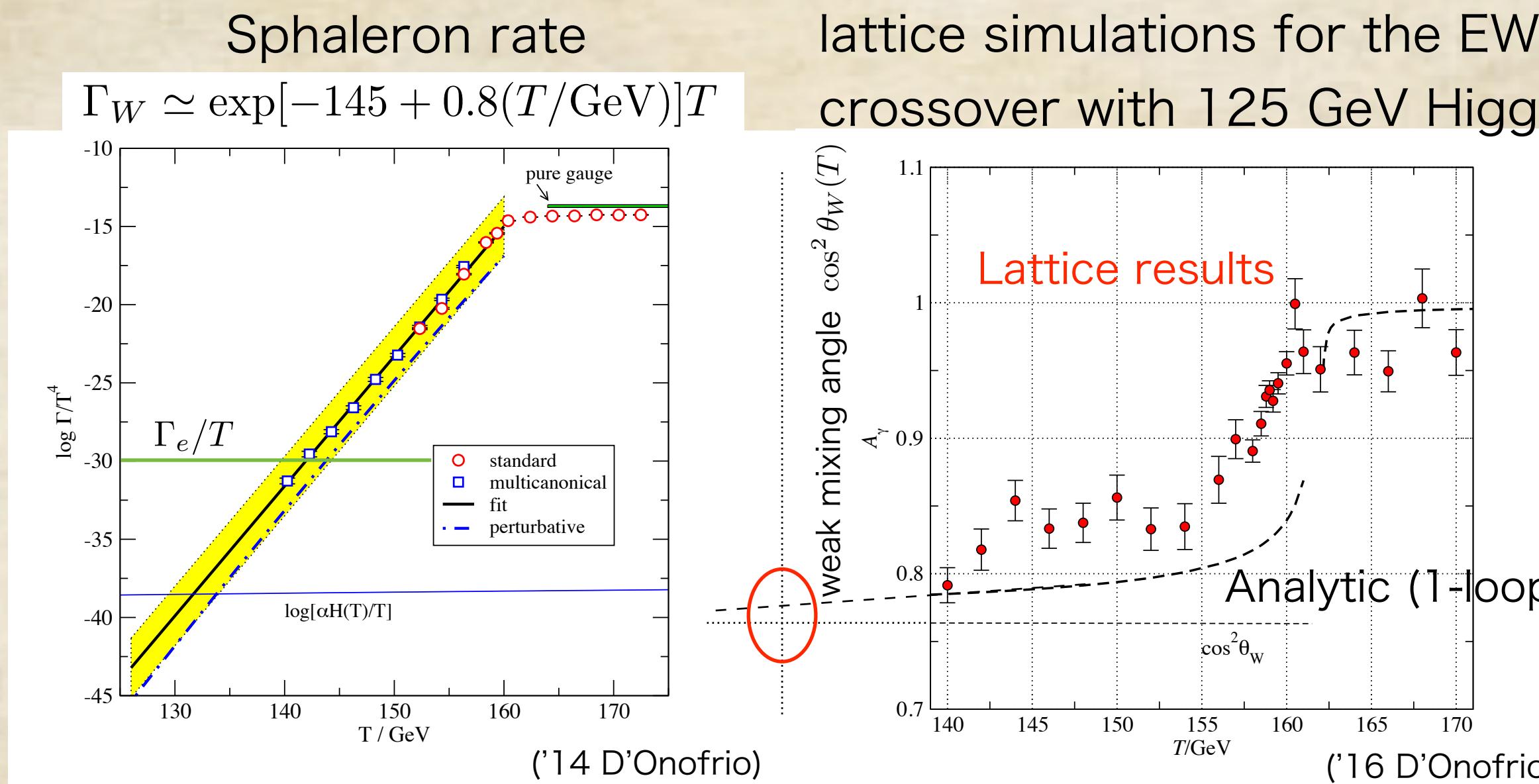
('16 KK&Long)

('14 D'Onofrio)

Subtle issue behind that⋯

Early EWSB (crossover) completion, late sphaleron freeze out

=> Net BAU is suppressed ('98 Giovannini&Shaposhnikov)

Early sphaleron freeze out, late EWSB (crossover) completion

=> Net BAU is efficiently remained

Sphaleron rate

$$\Gamma_W \simeq \exp[-145 + 0.8(T/\text{GeV})]T$$

lattice simulations for the EW crossover with 125 GeV Higgs

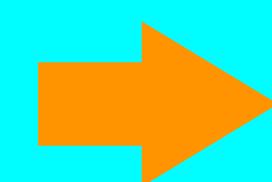('14 D'Onofrio)

('16 D'Onofrio)

BAU is very likely to remain!
Quantitative results are sensitive to $\theta_W(t)$

Finally analytic formula for the average baryon asymmetry is given.

$$\overline{\eta_B} \simeq 10^{-10} \, \epsilon f(T, \theta_{\mathrm{w}}) \left(\frac{\lambda}{10^6 \, \mathrm{GeV}^{-1}} \right) \left(\frac{\overline{B}}{10^{-3} \, \mathrm{GeV}^2} \right)^2 \Bigg|_{T = 135 \mathrm{GeV}}$$

$$f(T, \theta_{\mathrm{w}}) \equiv -\sin 2\theta_{\mathrm{w}} T \frac{d\theta_{\mathrm{w}}}{dT} (\simeq 0.1) \quad \text{at} \quad T \simeq 135 \mathrm{GeV}$$
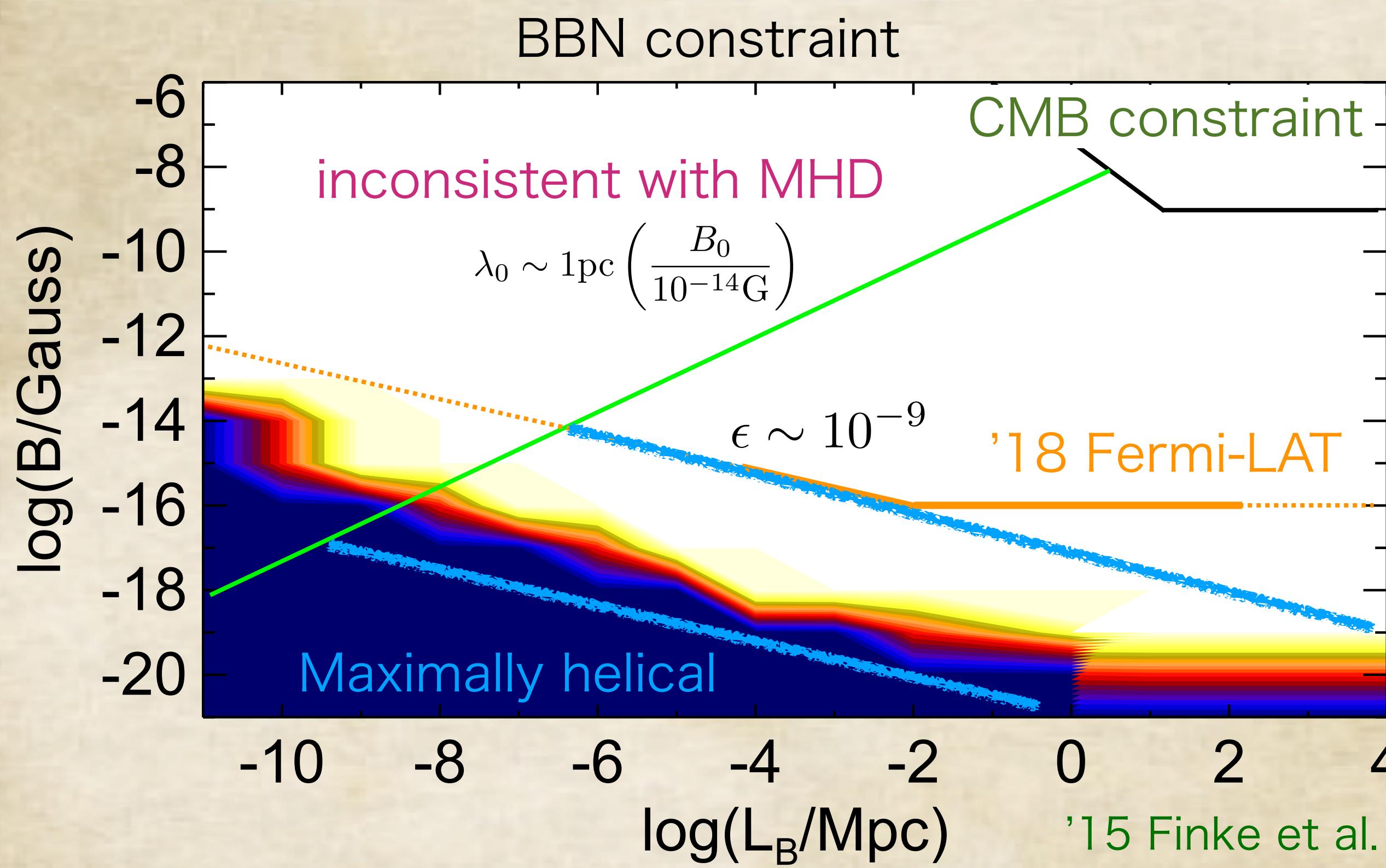
Magnetogenesis with positive helicity before EWSB.

➡ With appropriate properties of hyper MFs, present BAU can be explained.

※Since helicity is just the difference between the right and left helicity modes, the sign of helicity can be the same beyond the coherence length of MFs.

IGMFs that can explain the BAU '16 KK & Long

BBN constraint

CMB constraint

inconsistent with MHD

$\lambda_0 \sim 1\mathrm{pc}\left(\frac{B_0}{10^{-14}\mathrm{G}}\right)$

$\epsilon \sim 10^{-9}$

'18 Fermi-LAT

Maximally helical

log(B/Gauss)

log(L$_B$/Mpc)

'15 Finke et al.

IGMFs with tiny helicity ratio $\epsilon < 10^{-9}$ seems to explain the BAU.

IGMFs suggested by blazars with larger helicity should have generated after EWSB otherwise BAU is generated too much.
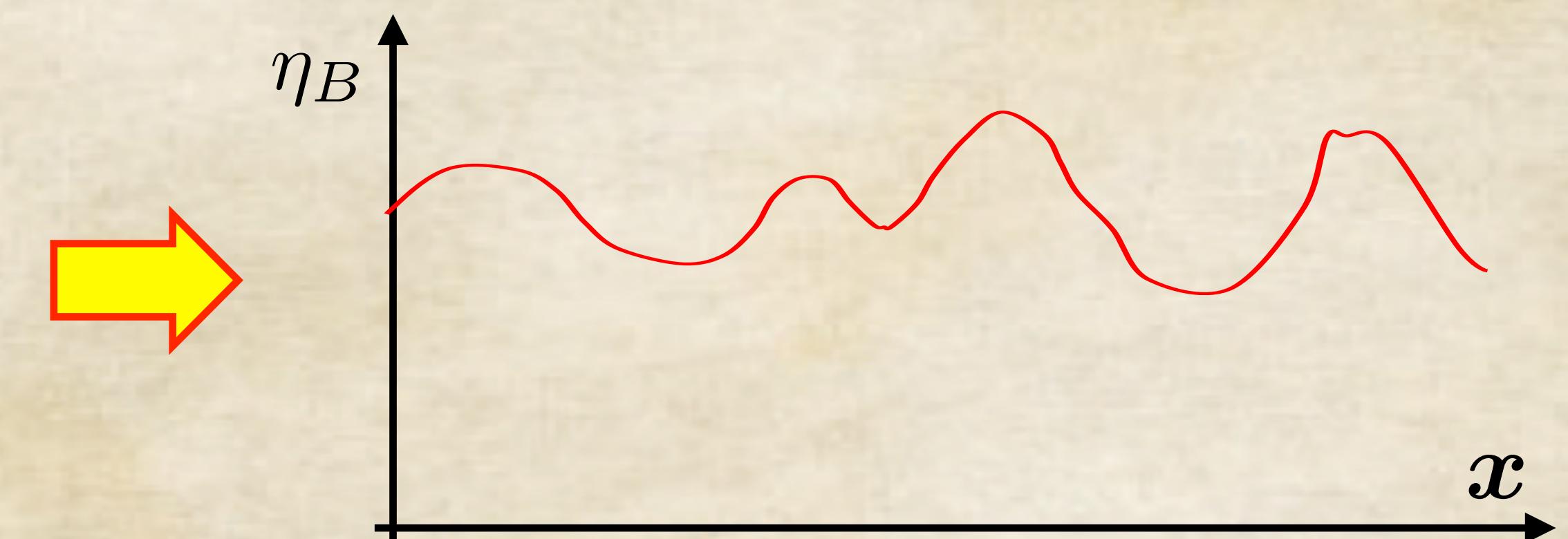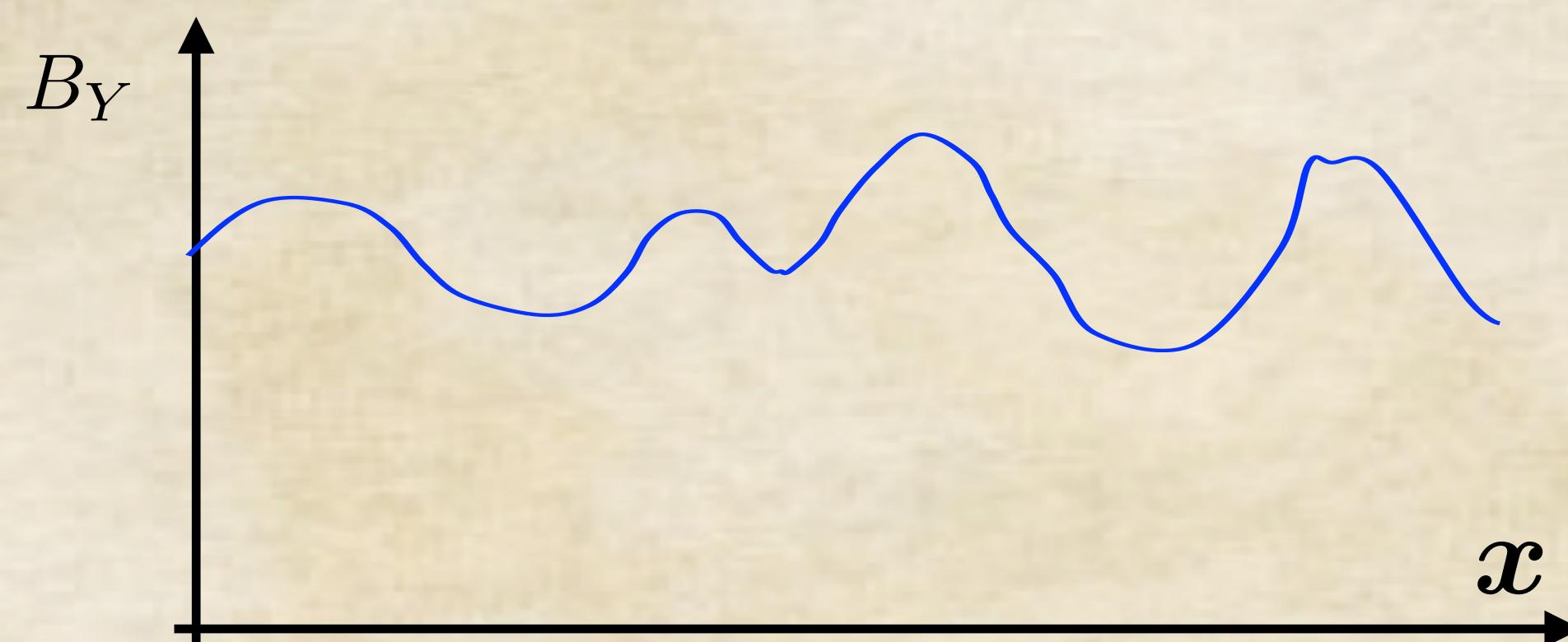
Courtesy: H. Oide

Baryon isocurvature constraints

KK, F. Uchida, J. Yokoyama (Tokyo), arXiv: 2012.14435 (astro-ph.CO)

Courtesy: H. Oide

Basic idea

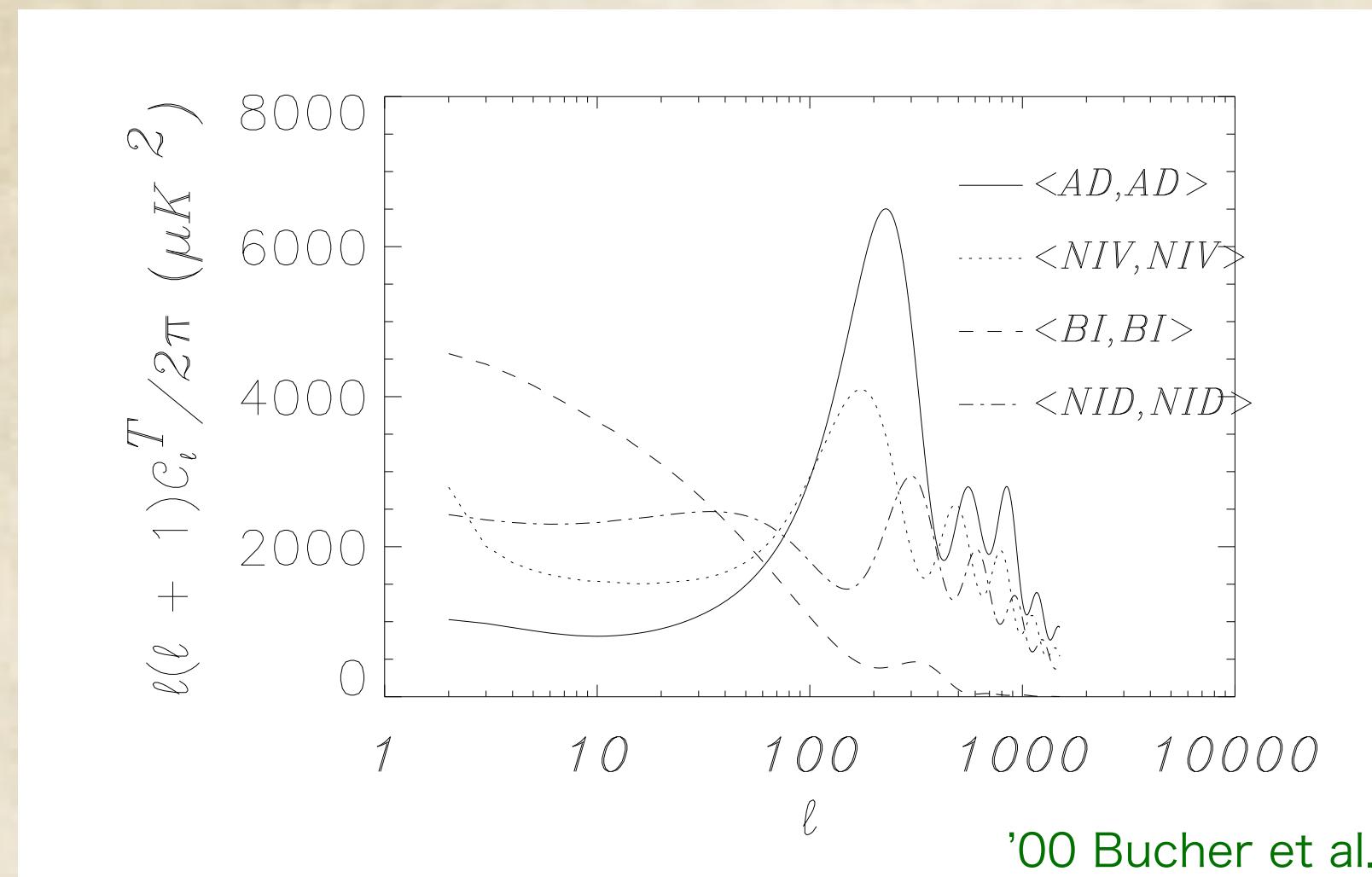Baryon asymmetry evaluated thus far is the spatially-averaged one

=> We expect that it has spatial dependence ("baryon isocurvature perturbation")
according to the spatial distributions of hypermagnetic fields.

constrained by observations?

Observational constraints on the baryon isocurvature perturbations
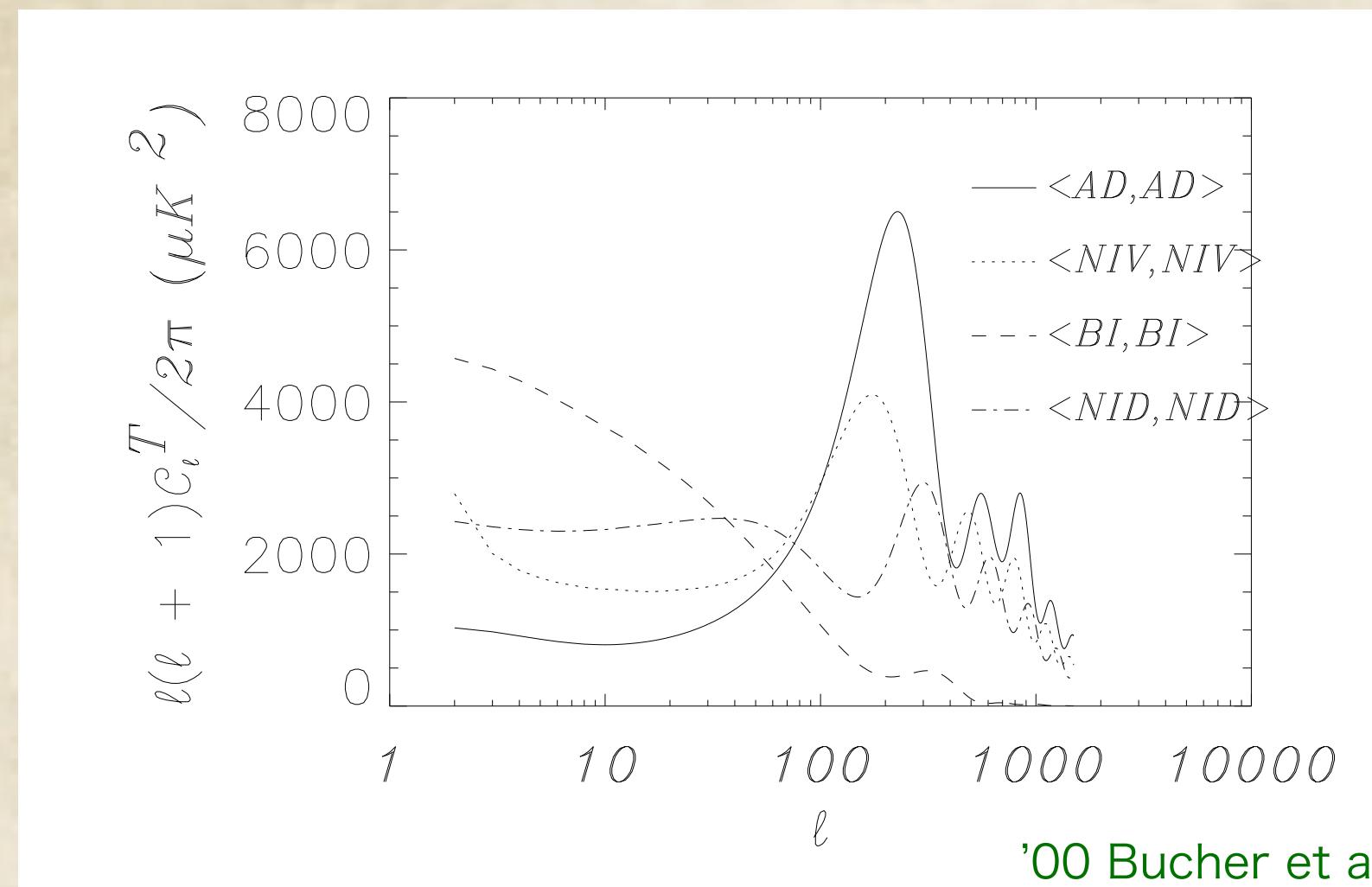
Mpc scales: CMB gives constraints.

'00 Bucher et al.

$$\beta_{\text{iso}} \equiv \frac{\mathcal{P}_{II}}{\mathcal{P}_{\mathcal{RR}} + \mathcal{P}_{II}} \lesssim 0.49 \quad @k = 0.1\text{Mpc}^{-1}$$

'18 Planck

Observational constraints on the baryon isocurvature perturbations

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'18 Planck

Much smaller scales: Inhomogeneous BBN

'87 Applegate+, Alock+

Baryon fluctuation with the scale larger than the neutron diffusion scale remains at BBN epoch and changes the prediction of light elements.
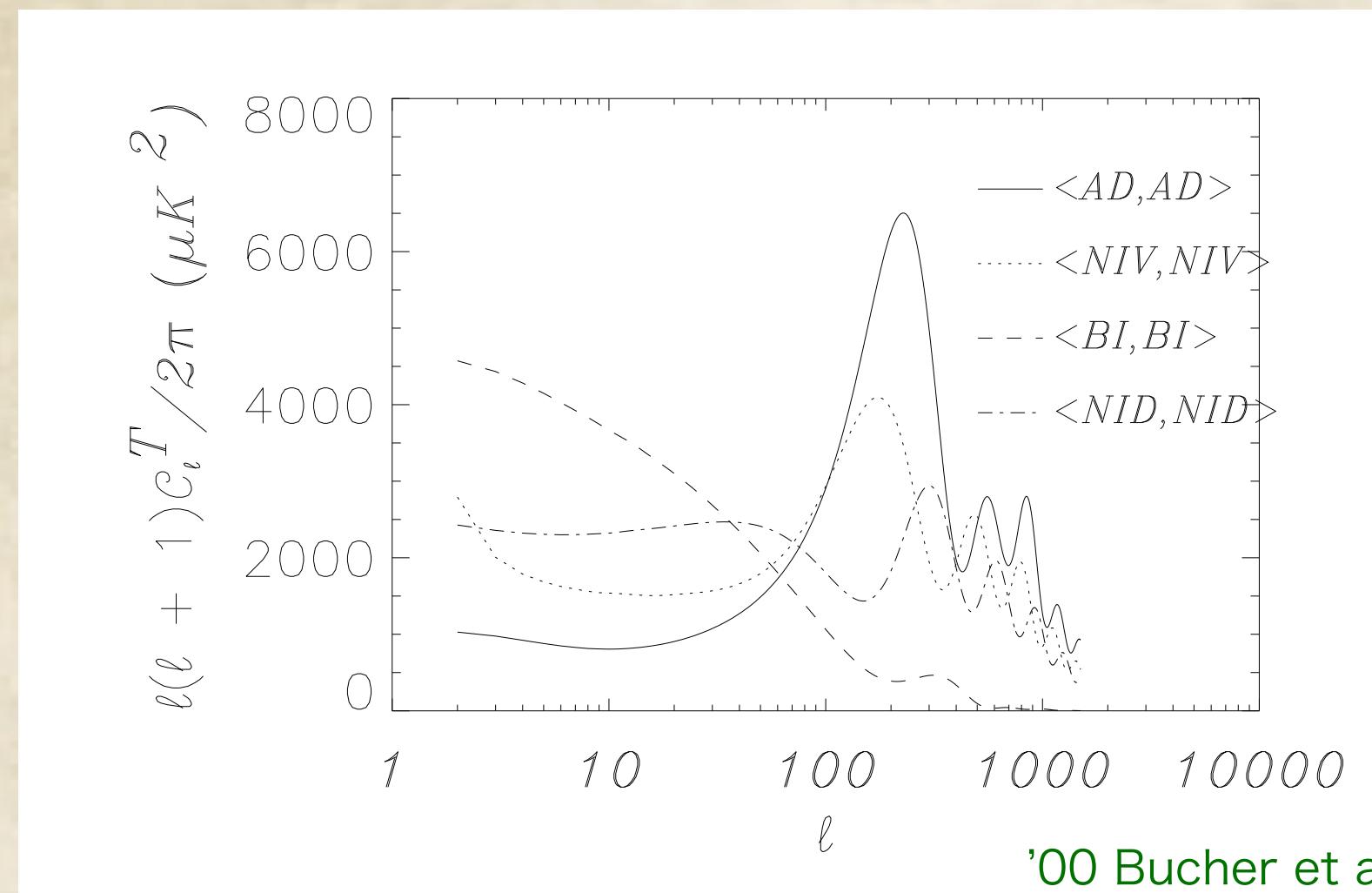
'08 Pisanti+, '15 Planck

$$10^5 (\mathrm{D/H})_p = 18.754 - 1534.4\,\omega_B + 48656\,\omega_B^2 - 552670\,\omega_B^3,$$

$$\omega_B = \Omega_B h^2$$

$$\overline{\omega}_B^{\ 2} + \langle \delta \omega_B^2 \rangle$$

Courtesy: H. Oide

Observational constraints on the baryon isocurvature perturbations

Mpc scales: CMB gives constraints.

'00 Bucher et al.

$$\beta_{\mathrm{iso}} \equiv \frac{\mathcal{P}_{II}}{\mathcal{P}_{\mathcal{R}\mathcal{R}} + \mathcal{P}_{II}} \lesssim 0.49 \quad @k = 0.1\mathrm{Mpc}^{-1}$$

'18 Planck

Much smaller scales: Inhomogeneous BBN

'87 Applegate+, Alock+

$$\langle S^2_{B,\mathrm{BBN}} \rangle = \frac{\langle \delta\eta^2_{B,\mathrm{BBN}}(\boldsymbol{x}) \rangle}{\overline{\eta_B}^2} = \frac{1}{V} \int \frac{d^3k}{(2\pi)^3} \mathcal{P}^{\mathrm{BBN}}_{\delta B} < 0.016$$

'18 Inomata +

Courtesy: H. Oide

Baryon isocurvature perturbations from hypermagnetic fields at EWSB

$$\eta_{B,\mathrm{EW}}(\boldsymbol{x}) = \mathcal{C}\boldsymbol{Y}(\boldsymbol{x}) \cdot \boldsymbol{B}_Y(\boldsymbol{x})(= \mathcal{C}\mathcal{H}_Y(\boldsymbol{x}))$$

$$\langle \delta\eta_{B,\mathrm{EW}}(\boldsymbol{x})\delta\eta_{B,\mathrm{EW}}(\boldsymbol{x}+\boldsymbol{r})\rangle = \mathcal{C}^2 \langle \boldsymbol{Y}(\boldsymbol{x}) \cdot \boldsymbol{B}_Y(\boldsymbol{x})\boldsymbol{Y}(\boldsymbol{x}+\boldsymbol{r}) \cdot \boldsymbol{B}_Y(\boldsymbol{x}+\boldsymbol{r})\rangle - \overline{\eta_{B,\mathrm{EW}}}^2$$

Fourier transform

$$\mathcal{G}(\boldsymbol{k}) = \frac{\mathcal{C}^2}{\overline{\eta}_B^2} \int \frac{d^3p}{(2\pi)^3} \left[p^2 S(|\boldsymbol{k}-\boldsymbol{p}|)S(p) + |\boldsymbol{k}-\boldsymbol{p}|p A(|\boldsymbol{k}-\boldsymbol{p}|)A(p) \right] \times \left[1 - \frac{2(\boldsymbol{k}-\boldsymbol{p})\cdot\boldsymbol{p}}{p^2} + \frac{((\boldsymbol{k}-\boldsymbol{p})\cdot\boldsymbol{p})^2}{|\boldsymbol{k}-\boldsymbol{p}|^2 p^2} \right].$$

Two-point function has nonzero value even for non-helical magnetic fields!
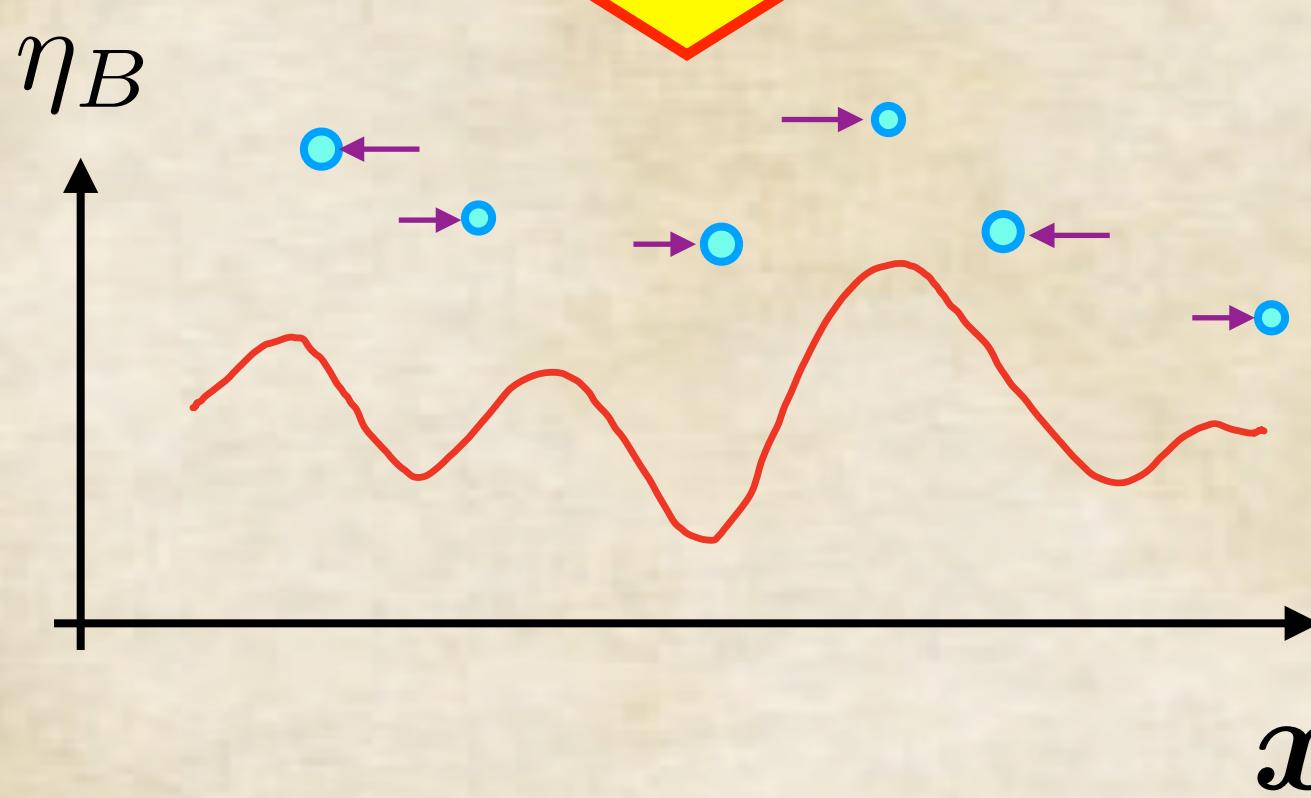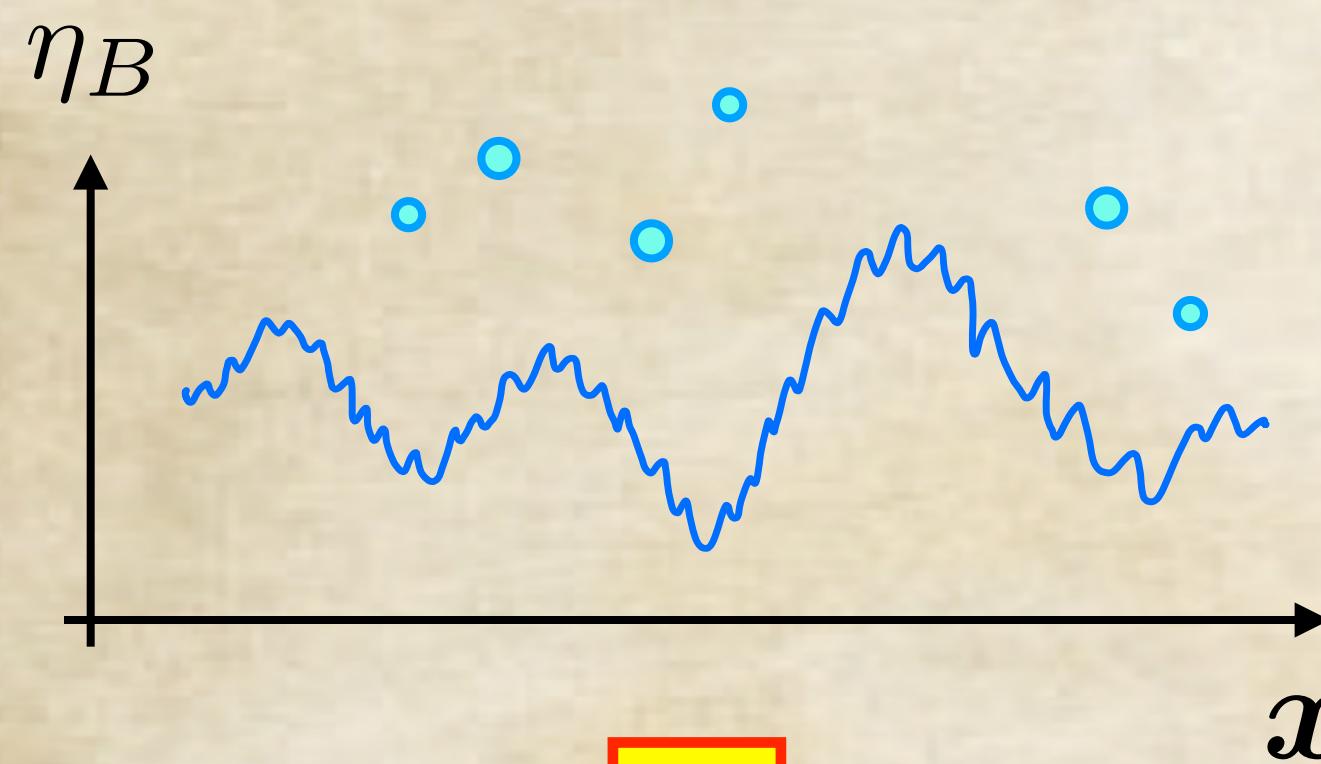
'98 Giovannini & Shaposhnikov

Baryon isocurvature perturbations at BBN

⋯ Neutron diffusion erases the small scale inhomogeneities.

=> Corresponds to the treatment that the baryon asymmetry
is convoluted with the Gaussian window function.

$$\langle S^2_{B,\mathrm{BBN}} \rangle = \int \frac{d^3 k}{(2\pi)^3} e^{-\frac{k^2}{3k_{\mathrm{d}}^2}} \mathcal{G}(\boldsymbol{k})$$

neutron diffusion scale: $k_{\mathrm{d}}^{-1} = 0.0025\mathrm{pc}$

$$= \frac{\mathcal{C}^2}{4\pi^4 \overline{\eta}_B^2} \int dk_1 dk_2 k_1^2 k_2^2 \sum_{\pm} \left(\pm \left\{ \frac{(k_1 \pm k_2)^2}{2} \left[S(k_1)S(k_2) \pm A(k_1)A(k_2) \right] \frac{3k_{\mathrm{d}}^2}{3k_1 k_2} \left(1 \mp \frac{3k_{\mathrm{d}}^2}{2k_1 k_2} \right) \right. \right.$$

$$\left. \left. + \left[\frac{k_1^2 + k_2^2}{2} S(k_1)S(k_2) + k_1 k_2 A(k_1)A(k_2) \right] \left(\frac{3k_{\mathrm{d}}^2}{2k_1 k_2} \right)^3 \right\} \exp\left[-\frac{2(k_1 \mp k_2)^2}{3k_{\mathrm{d}}^2} \right] \right).$$

For given the MF spectra ($S(k)$, $A(k)$), we can evaluate
the baryon isocurvature perturbation at BBN.

=> BBN constraint $\langle S^2_{B,\mathrm{BBN}} \rangle < 0.016$ can be given
with respect to any MF spectra :)

Some general features:

- BBN constrains the ensemble average of baryon isocurvature perturbations

$$\langle S_{B,\mathrm{BBN}}^2 \rangle < 0.016$$

=> perturbations at all the scales up to the present Hubble scale matters.

- Baryon isocurvature perturbation at small scale, $k > k_{\mathrm{d}}$, at the EWSB
becomes smaller by the neutron diffusion until BBN, but is not completely washed out.
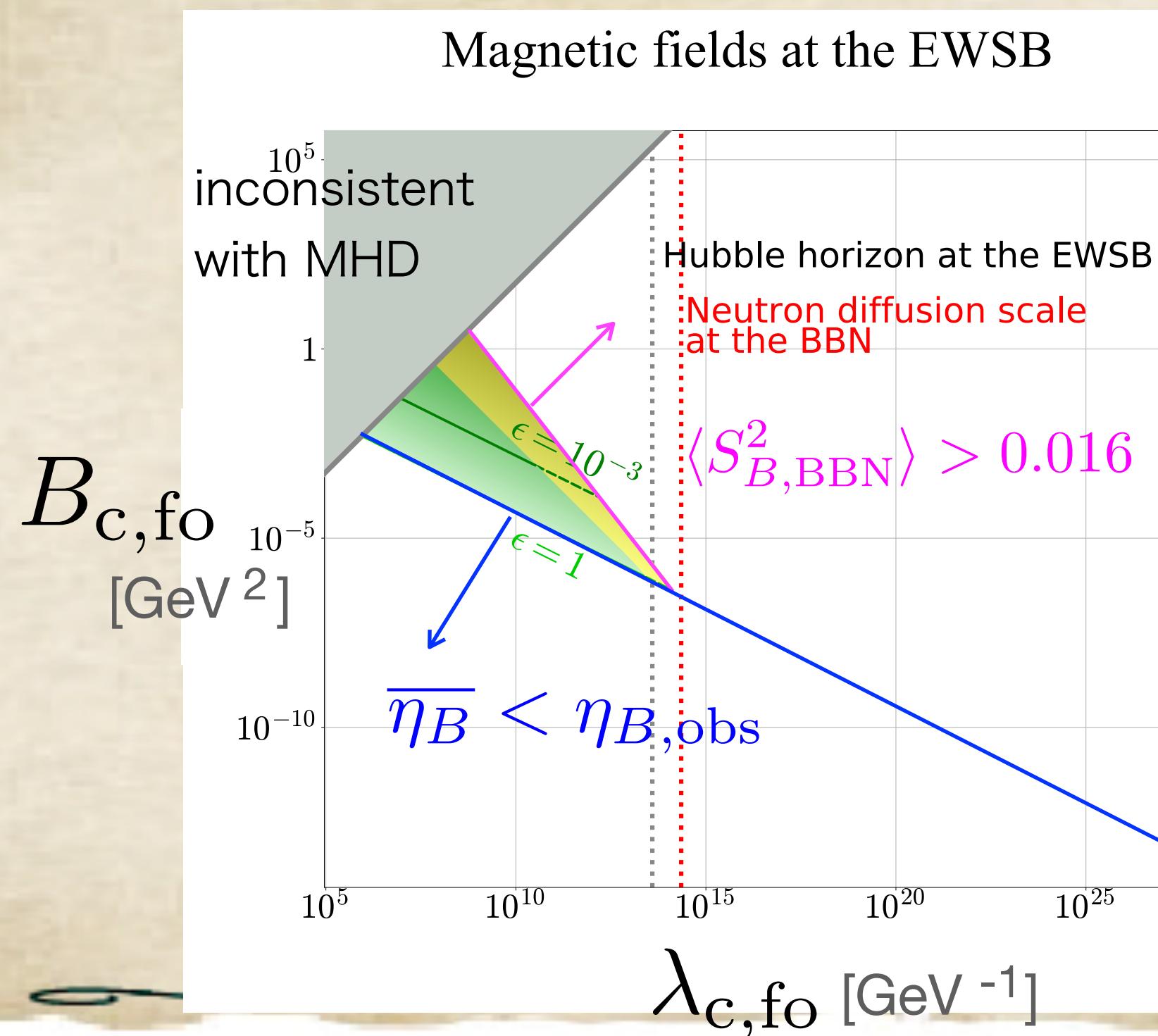
Constraints on peaky MF spectra

$$\rho_{B,c} \simeq \frac{1}{2} B_{c,fo}^2, \quad \lambda_{c,fo} \simeq k_\sigma^{-1}, \quad \mathcal{H}_Y = \epsilon_{fo} \lambda_{c,fo} B_{c,fo}$$
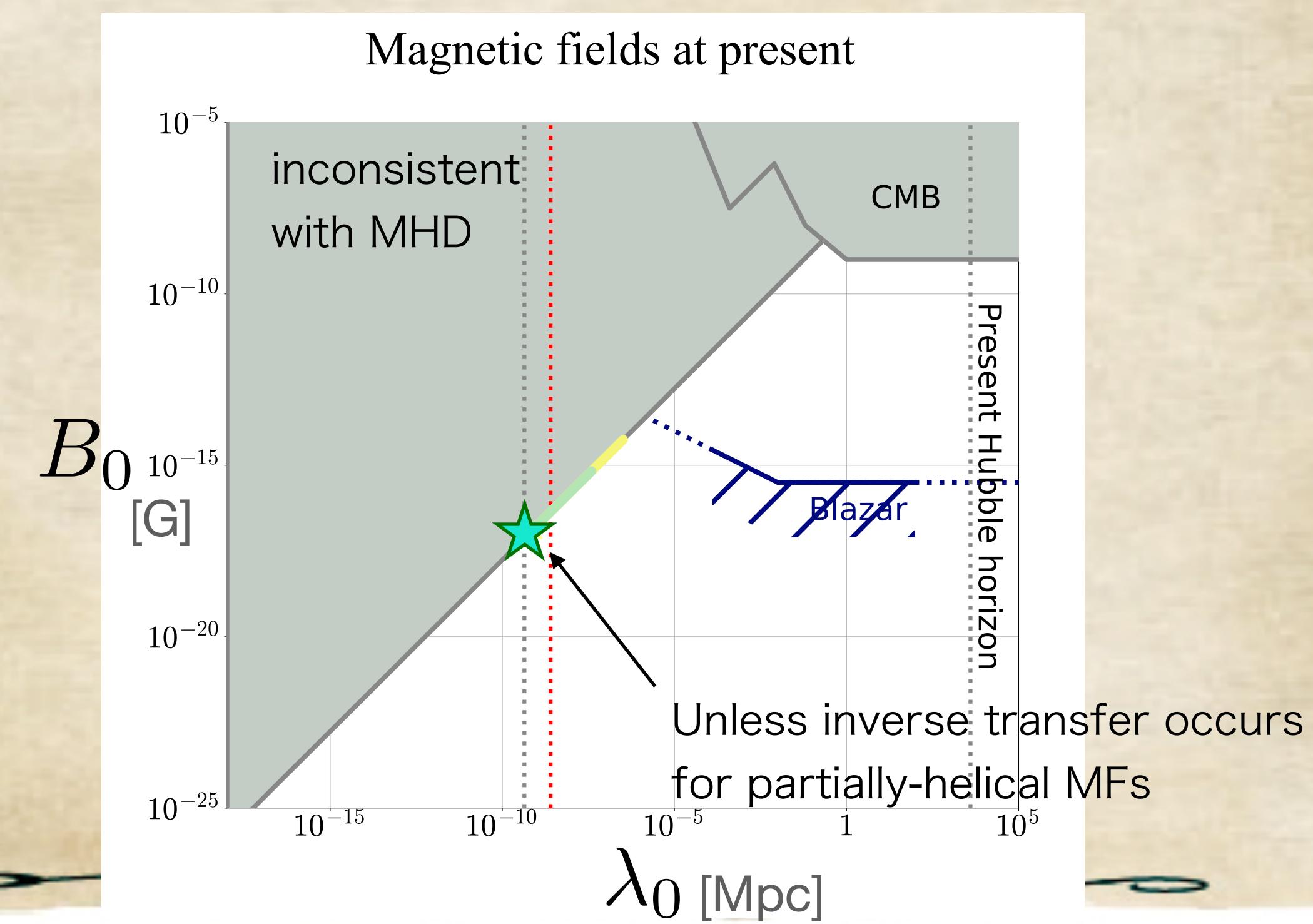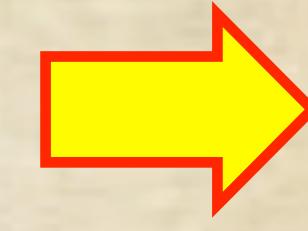
- delta-function model: $S(k) = \pi^2 \dfrac{B_{c,fo}^2}{k_\sigma^4} \delta(k - k_\sigma), \quad A(k) = \epsilon_{fo} S(k),$

- power-law with exponential cutoff: $S(k) = \dfrac{2\pi^2}{\Gamma(\frac{5+\alpha}{2})} \dfrac{B_{c,fo}^2}{k_\sigma^5} \left(\dfrac{k}{k_\sigma}\right)^\alpha \exp\left[-\left(\dfrac{k}{k_\sigma}\right)^2\right], \quad A(k) = \epsilon_{fo} S(k).$

$$(\alpha > -5/2)$$

If you would like to explain the BAU…

Magnetic fields at the EWSB

inconsistent with MHD

Hubble horizon at the EWSB

Neutron diffusion scale at the BBN

$\langle S_{B,BBN}^2 \rangle > 0.016$

$\epsilon = 10^{-3}$

$\epsilon = 1$

$B_{c,fo}$ [GeV^2]

$\overline{\eta_B} < \eta_{B,obs}$

$\lambda_{c,fo}$ [GeV^{-1}]

MF evolution with cascade being taken into account

Magnetic fields at present

inconsistent with MHD

CMB

Present Hubble horizon

Blazar

B_0 [G]

λ_0 [Mpc]

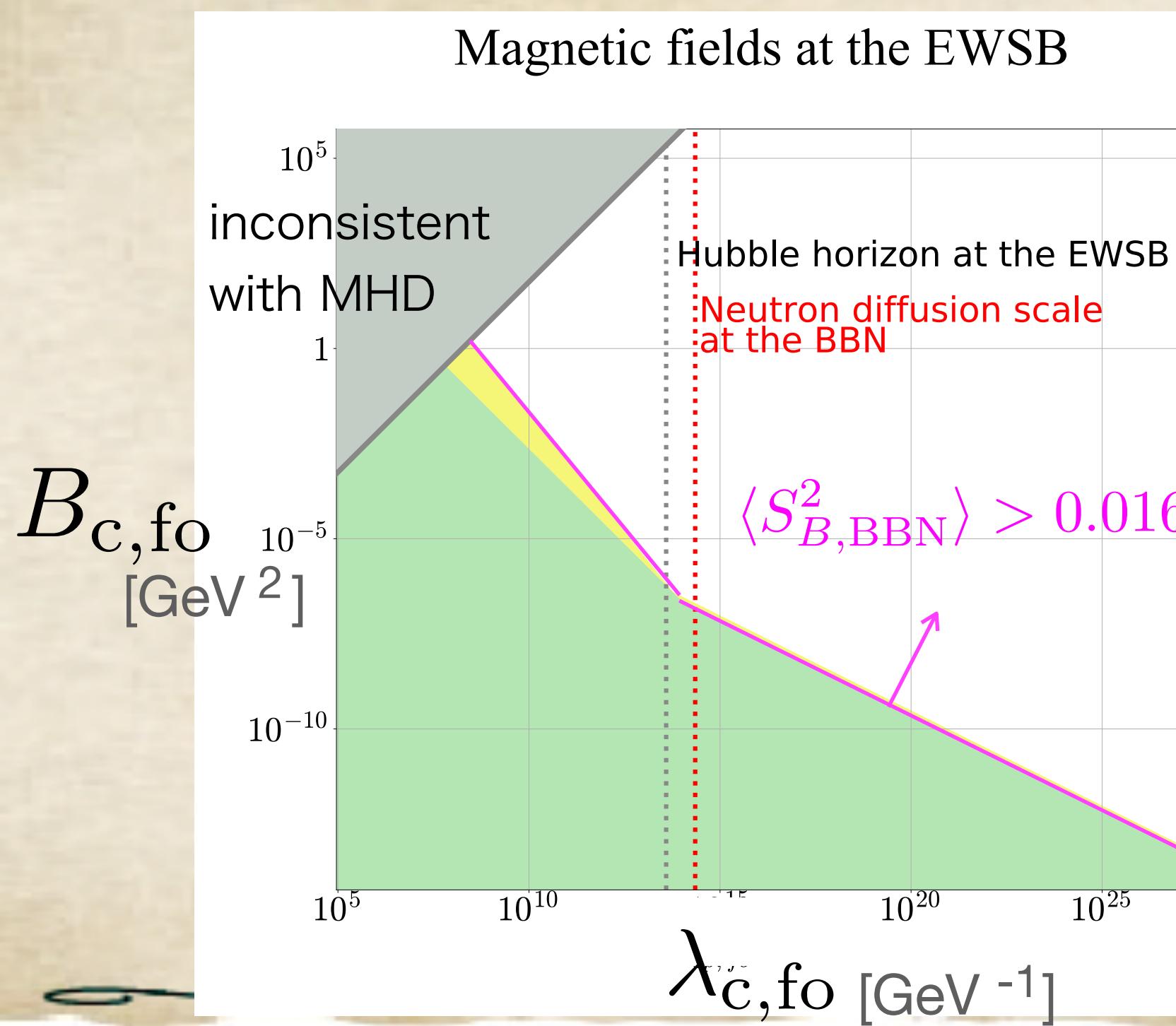Unless inverse transfer occurs for partially-helical MFs

Constraints on peaky MF spectra

$$\rho_{B,c} \simeq \frac{1}{2}B_{c,fo}^2, \quad \lambda_{c,fo} \simeq k_\sigma^{-1}, \quad \mathcal{H}_Y = \epsilon_{fo}\lambda_{c,fo}B_{c,fo}$$

- delta-function model: $S(k) = \pi^2 \dfrac{B_{c,fo}^2}{k_\sigma^4}\delta(k - k_\sigma), \quad A(k) = \epsilon_{fo}S(k),$
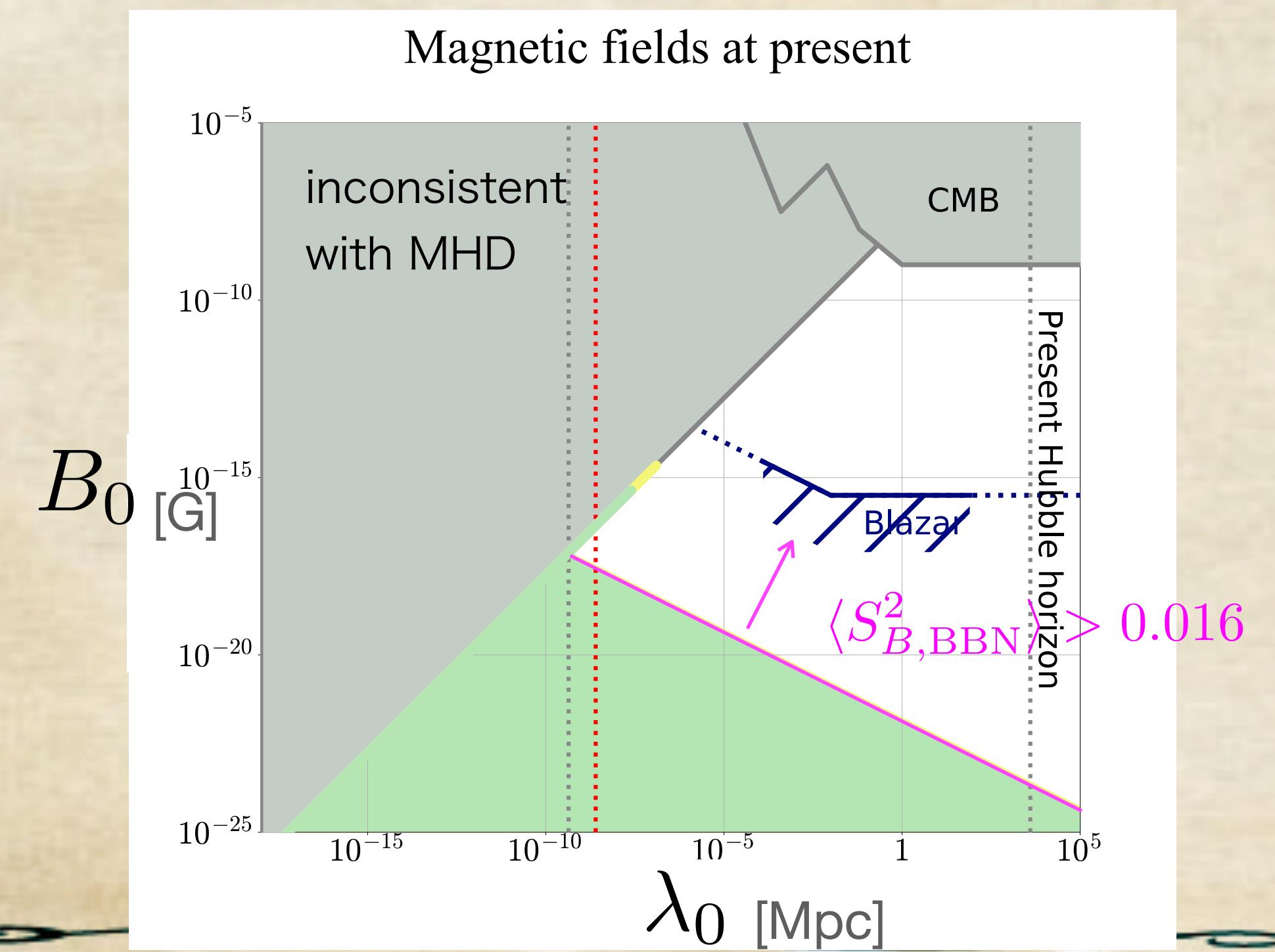
- power-law with exponential cutoff: $S(k) = \dfrac{2\pi^2}{\Gamma(\frac{5+\alpha}{2})}\dfrac{B_{c,fo}^2}{k_\sigma^5}\left(\dfrac{k}{k_\sigma}\right)^\alpha \exp\left[-\left(\dfrac{k}{k_\sigma}\right)^2\right], \quad A(k) = \epsilon_{fo}S(k).$

$$(\alpha > -5/2)$$

For non-helical MFs with forgetting about BAU

MF evolution with cascade being taken into account

Courtesy: H. Oide

For more flat spectrum such as those from inflationary magnetogenesis?

Just taking

$$S(k) = \frac{2\pi^2}{\Gamma(\frac{5+\alpha}{2})} \frac{B_{c,fo}^2}{k_\sigma^5} \left(\frac{k}{k_\sigma}\right)^\alpha \exp\left[-\left(\frac{k}{k_\sigma}\right)^2\right] \quad \text{with} \quad \alpha \simeq -5$$

➡ $\langle S_{B,\mathrm{BBN}}^2 \rangle$: IR divergent?

Reparameterize as

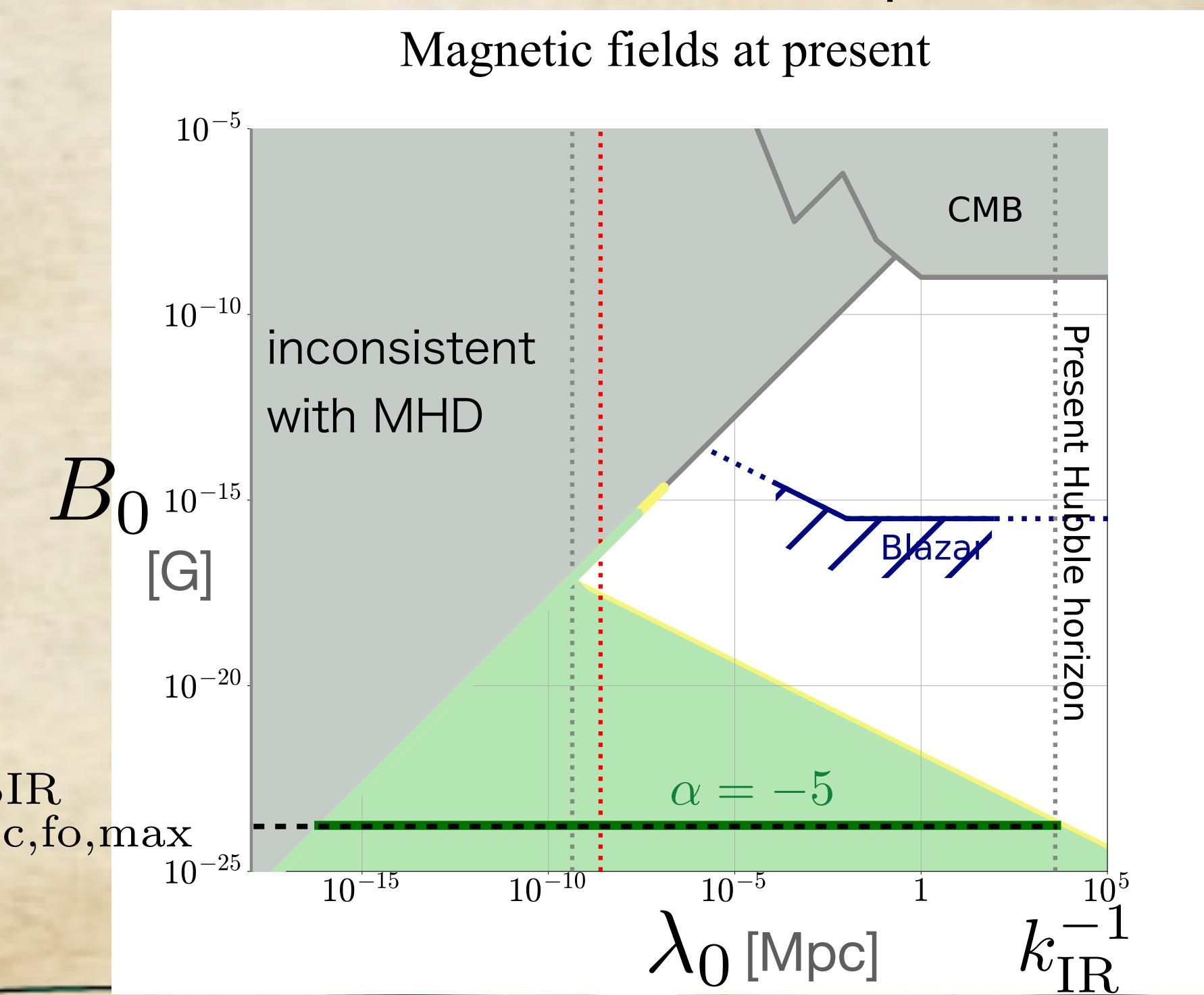$$S(k) = \frac{(B_{c,fo}^{\mathrm{IR}})^2}{k_{\mathrm{IR}}^5} \left(\frac{k}{k_{\mathrm{IR}}}\right)^\alpha \quad \text{with IR cutoff} \quad k_{\mathrm{IR}}$$

For long enough magnetogenesis during inflation, the IR cutoff k_{IR} should be taken as H_0

➡ $\langle S_{B,\mathrm{BBN}}^2 \rangle \sim \dfrac{\mathcal{C}^2 (B_{c,fo}^{\mathrm{IR}})^4}{\overline{\eta}_{B,\mathrm{obs}}^2 k_{\mathrm{IR}}^2} < 0.016$

$\longrightarrow B_{c,fo,\max}^{\mathrm{IR}}$

Constraint on flat MF spectrum

Magnetic fields at present

Summary

1. Absence of GeV cascade photon from blazars
   gave a motivation to study PMFs.

2. Hypermagnetic helicity decay is found
   to be able to generate BAU.

3. Baryon isocurvature constraint is so strong
   that magnetogenesis before the EWSB hardly explains
   the IGMFs suggested by blazar observations.

   1. 1st order (EW/QCD) PT would be more well-motivated.
   2. Inflationary magnetogenesis is possible
      only if its scale is smaller than the EW scale.
   3. EWSB is modified so that the EW sphaleron is more effective.

4. BAU can be still explained by the primordial hyper MFs,
   but we need to remove the "foreground" IGMFs
   to test the scenario.

B_0 [G]

λ_0 [Mpc]

Magnetic fields at present

inconsistent with MHD

CMB

Present Hubble horizon

Blazar

BAU?

$\langle S_{B,\mathrm{BBN}}^2 \rangle > 0.016$