Deciphering the Archaeological Record: Cosmological Imprints of Non-Minimal Dark Sectors

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Based on

[arXiv: 2001.02193] with *Keith Dienes, Jeff Kost, Shufang Su, Brooks Thomas*

[arXiv:2101.10337] with *Keith Dienes, Jeff Kost, Kevin Manogue, Brooks Thomas*

Kavli IPMU

04/14/2021

Basic idea…

early-universe dynamics

dark-matter phase-space distribution *f(p)*

cosmic structure

P(k), dn/dlogM, ……

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dark-matter phase-space distribution $f(p)$

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$P(k)$, $dn/d\log M$, ……

Can we invert this to learn about early-universe dynamics from observables of structure formation?

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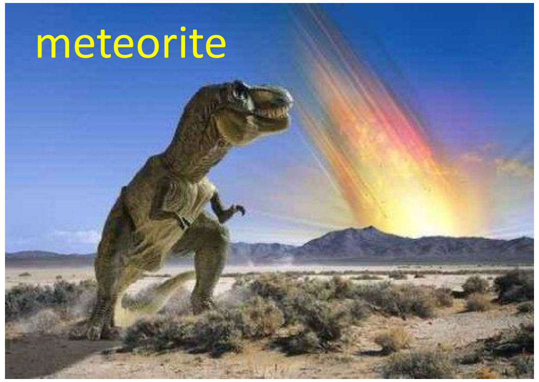

Multiple different early-universe dynamics could result in the same $f(p)$

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cosmic structure
$P(k)$, $dn/d\log M$, ……

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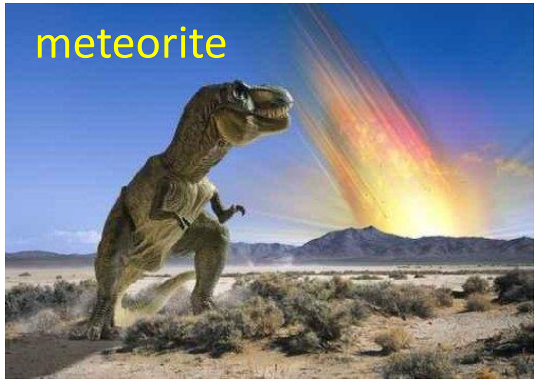


The focus of this talk:

- **What can we learn from structure formation?**
- **To what extent is an inversion possible?**

Multiple different early-universe dynamics could result in the same $f(p)$

Phase-Space Distribution

$$f(\vec{x},\vec{p},t) \approx f(p,t)$$

homogeneity and isotropy

For any particle species in the universe, its properties can be described through its phase space distribution $f(p,t)$

$$n(t) \equiv g\int \frac{d^3p}{(2\pi)^3} f(p,t)$$

$$\rho(t) \equiv g\int \frac{d^3p}{(2\pi)^3} E f(p,t) \qquad w(t) \equiv \frac{P(t)}{\rho(t)}$$

$$P(t) \equiv g\int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} f(p,t)$$

It also tells us whether a particle species is cold, warm or hot, thermal or nonthermal – especially important for DM. Therefore, $f(p,t)$ is the <u>***central***</u> quantity in understanding the properties of DM.

In general, $f(p,t)$ could take any reasonable functional form. Its evolution in time is governed by the Boltzmann equation:

$$\frac{\partial f}{\partial t} = Hp\frac{\partial f}{\partial p} + C[f] \qquad H(t) \equiv \frac{\dot{a}}{a}$$

basically, two effects: (1) cosmological redshift and (2) particle interactions

Cosmological redshift in FRW universe:

$$x(t) = x(t')\frac{a(t)}{a(t')} \Longrightarrow p(t) = p(t')\frac{a(t')}{a(t)} \quad \longleftarrow \quad \Delta p \sim a^{-1}$$

If plotting $f(p)$ vs p or $p^2 f(p)$ vs p, the redshift will make the width of the distribution more and more narrow. However, notice that

$$\frac{d\log p}{dt} = -H(t)$$

Time evolution amounts to additive shifts in $\log p$

This motivates us to switch from p to $\log p$

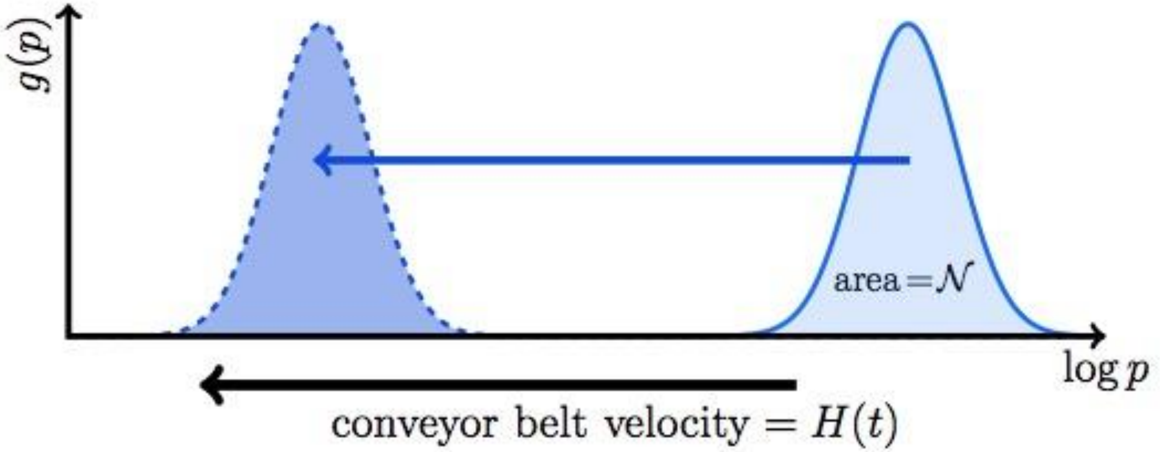
physical number density $\quad n(t) \sim \int d^3p\, f(p,t) \sim \int dp\, p^2 f(p,t) \sim \int d\log p\;\, p^3 f(p,t)$

comoving number density $\quad N(t) \sim n(t)a^3 \sim \int d\log p\; \boxed{(ap)^3 f(p,t)}$

We therefore define: $\quad g(p,t) \equiv a^3(t) p^3 f(p,t) \quad \longleftarrow \quad g(p(t),t) = g(p(t'),t')$

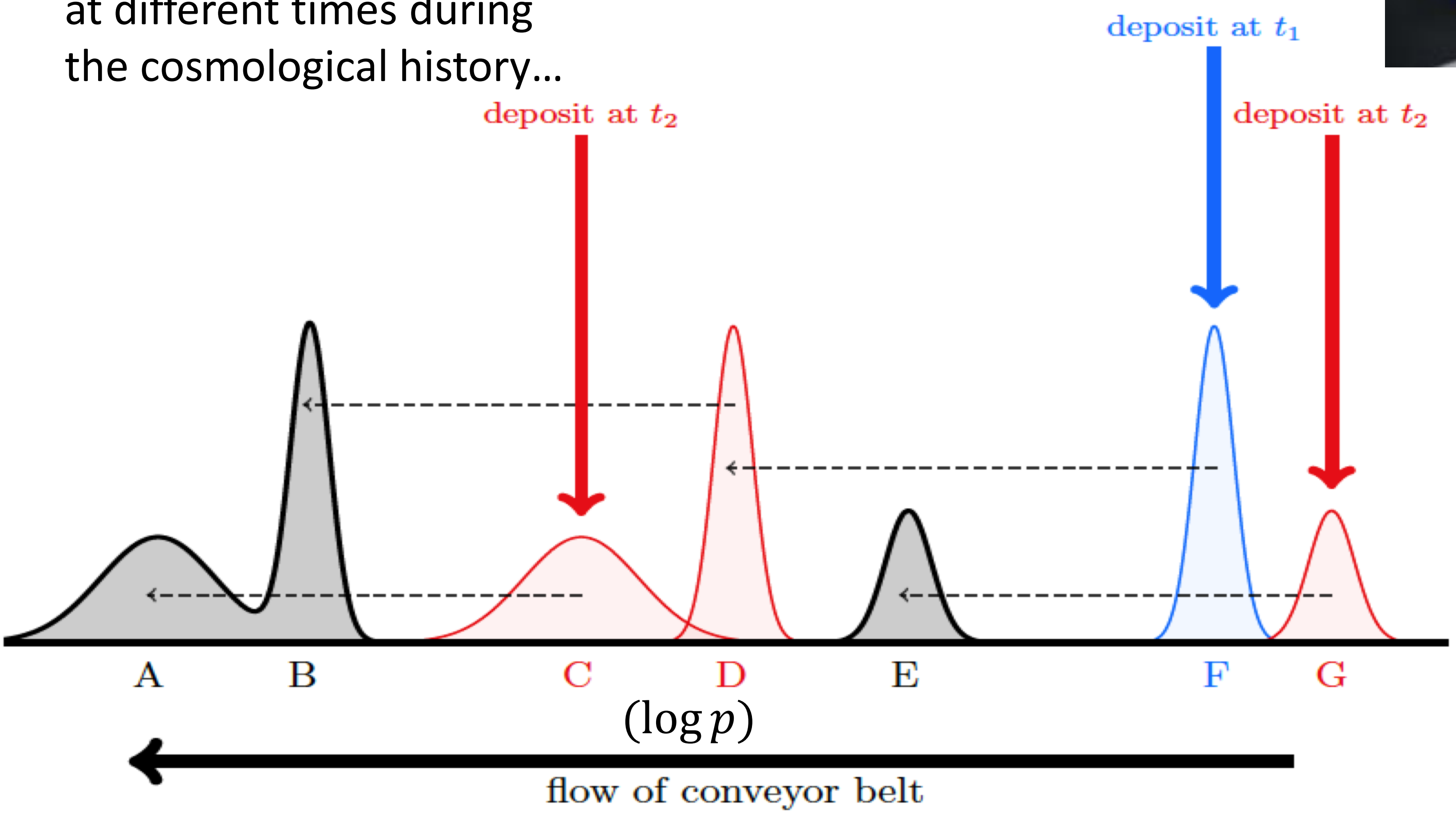
If we plot $g(p,t)$ against $\log p$:

- once DM is produced, area under the curve (comoving number density) is fixed under time evolution
- time evolution: $g(p,t)$ slides rigidly to smaller $\log p$, as if carried along a ***"cosmological conveyor belt"***



Similar to dropping groceries on a conveyor belt,
DM production → depositing particles on a conveyor belt in the momentum space

Example: if deposits occur at different times during the cosmological history…



Nontrivial, multi-modal distribution can result at present time!

We have seen that separate deposits occurring at different moments lead to multi-modal distribution.

Is such deposit pattern natural?

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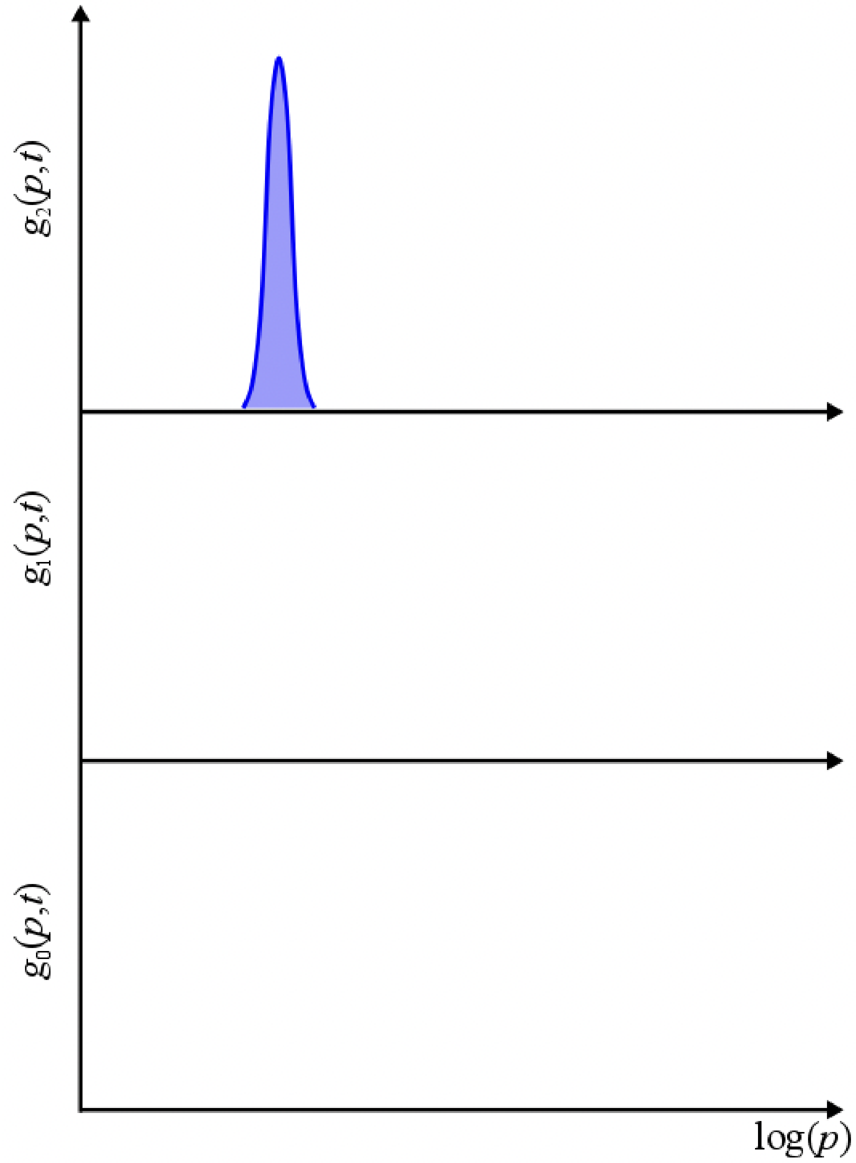
In fact, such a pattern of deposits can naturally arise from decays within a multi-state system…



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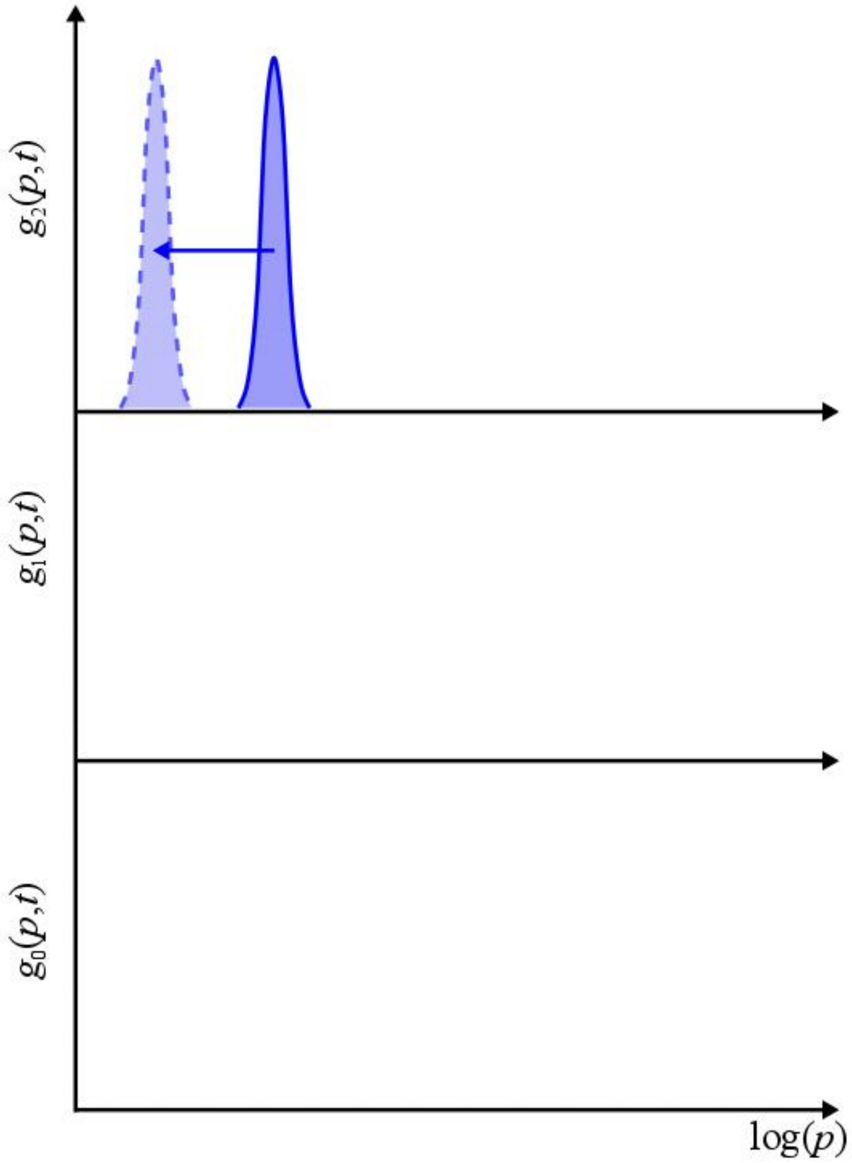

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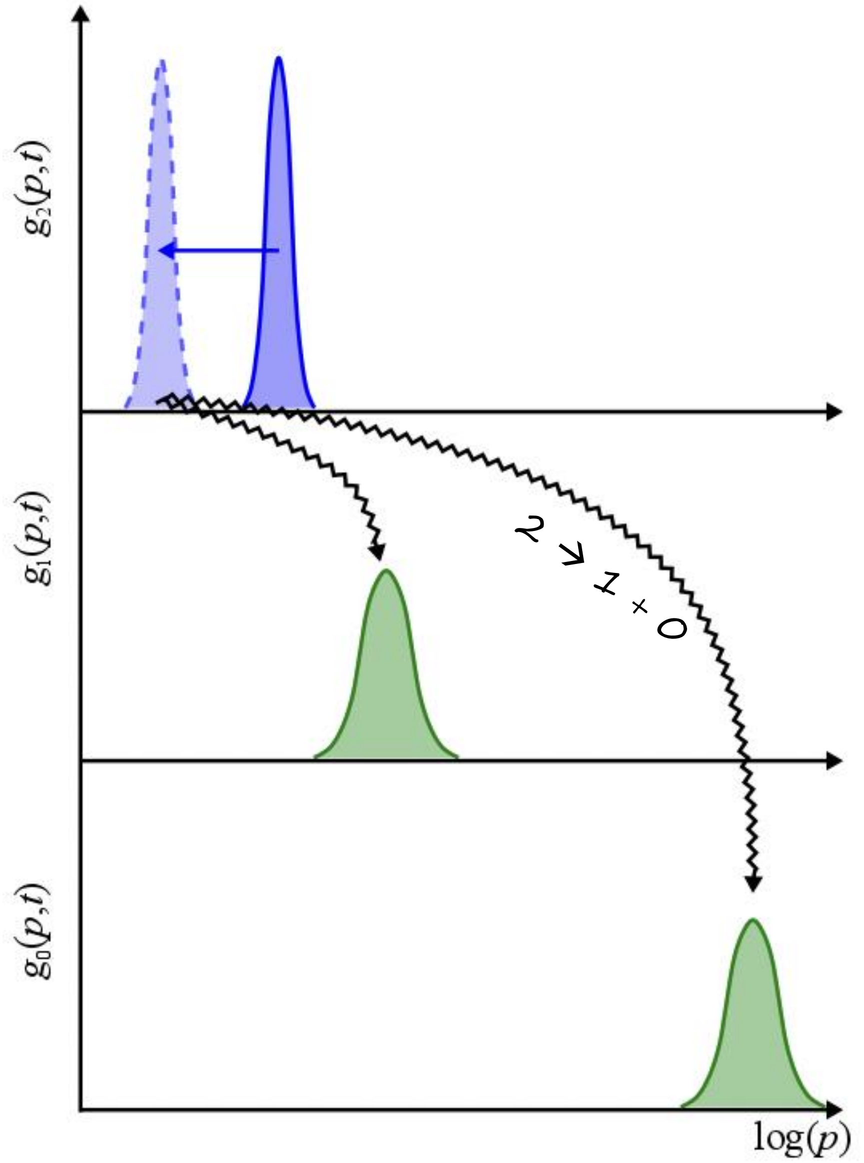

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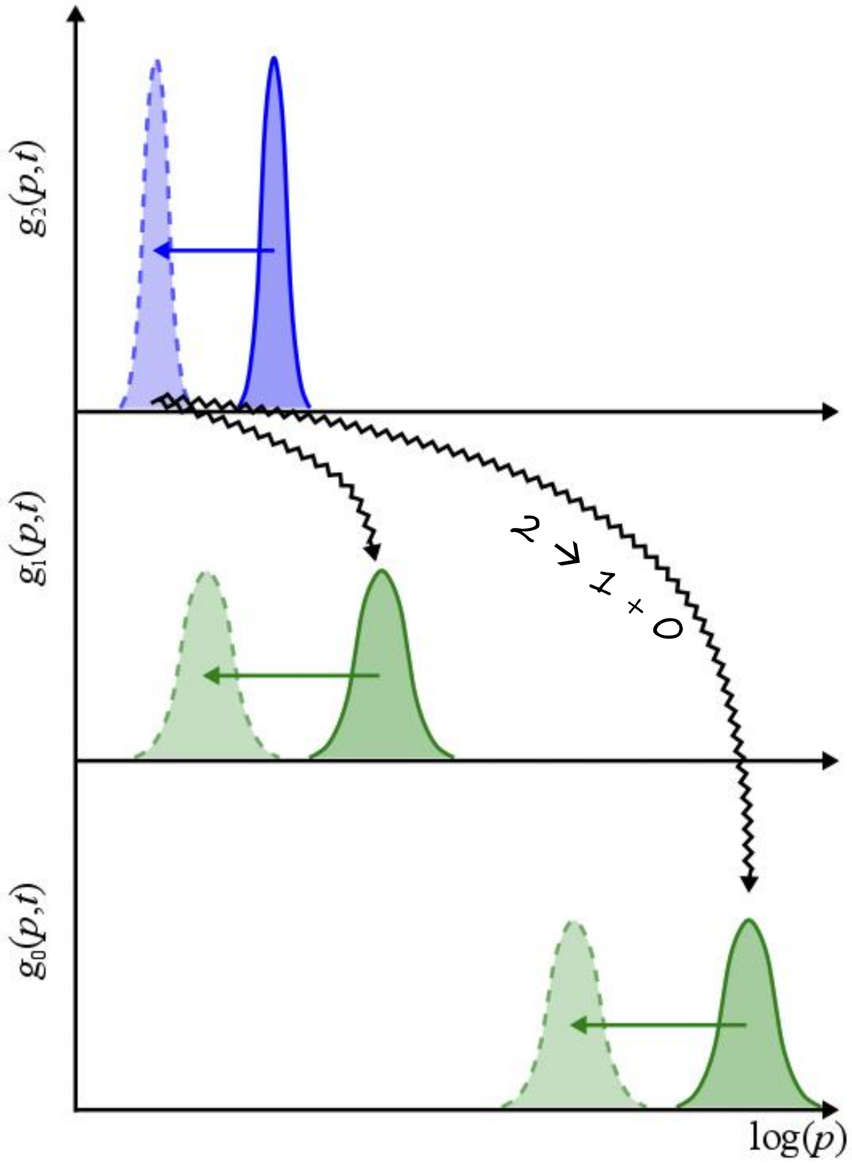

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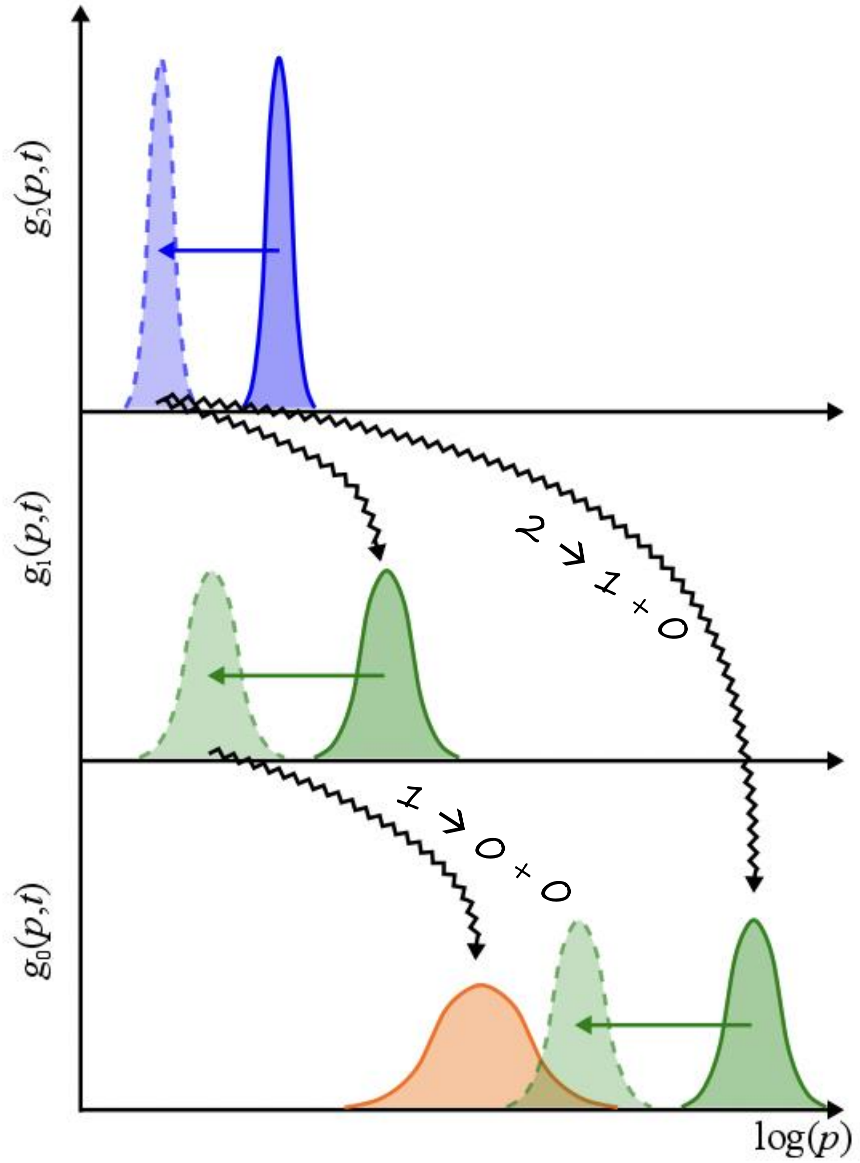

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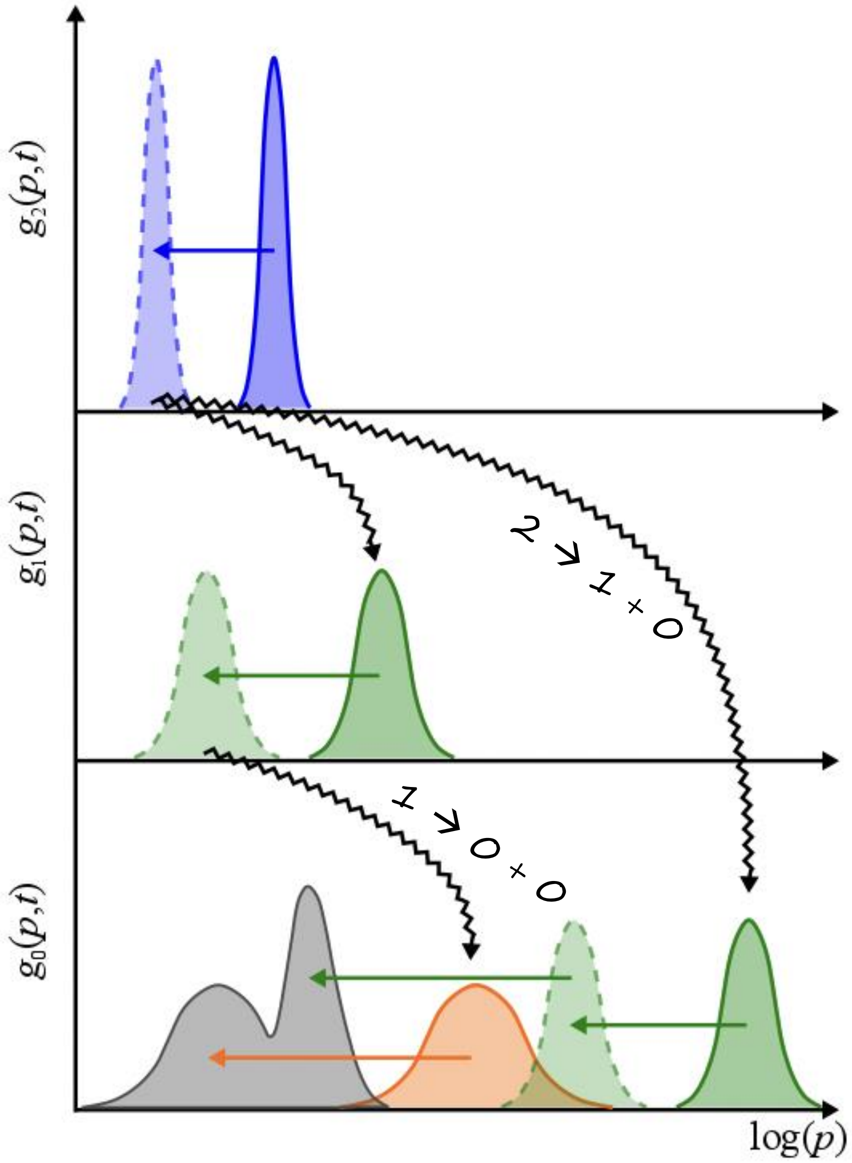

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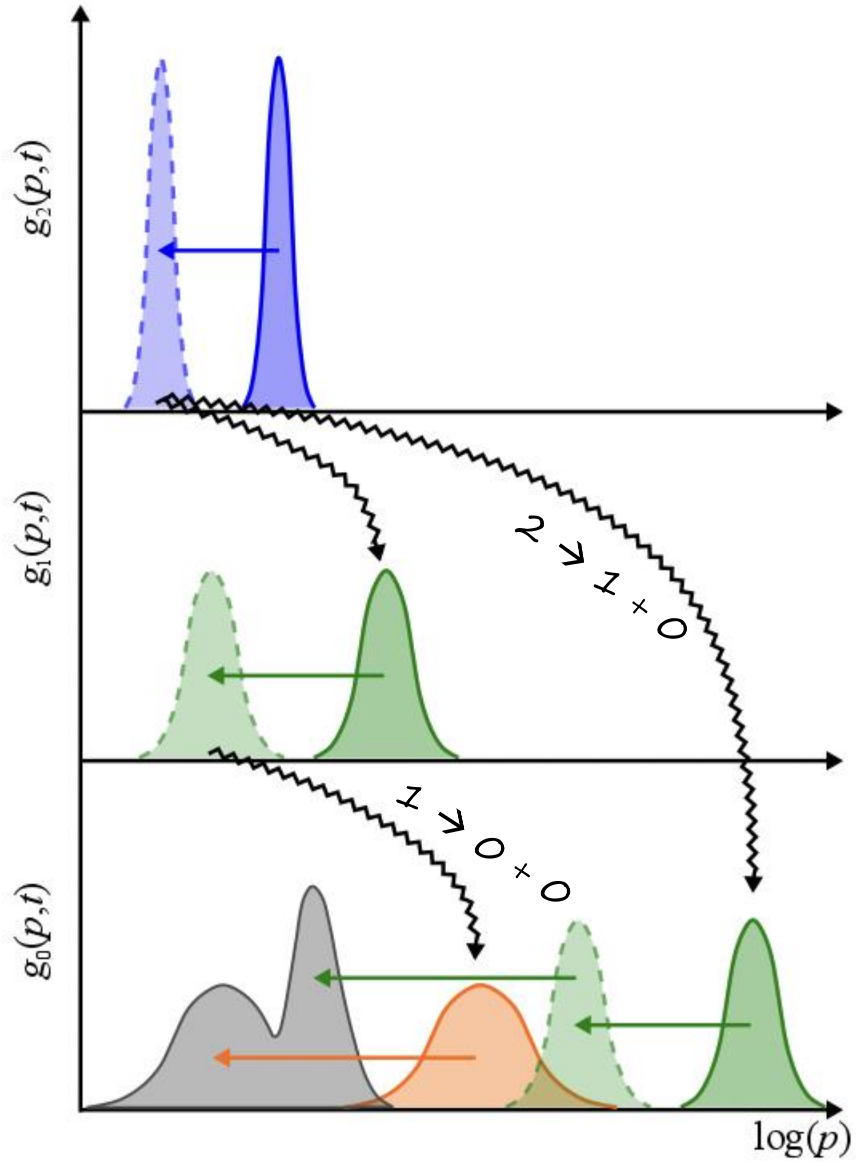
A non-trivial DM phase-space distribution at late times can represent the imprint of complex dynamics at earlier points in the cosmological history.

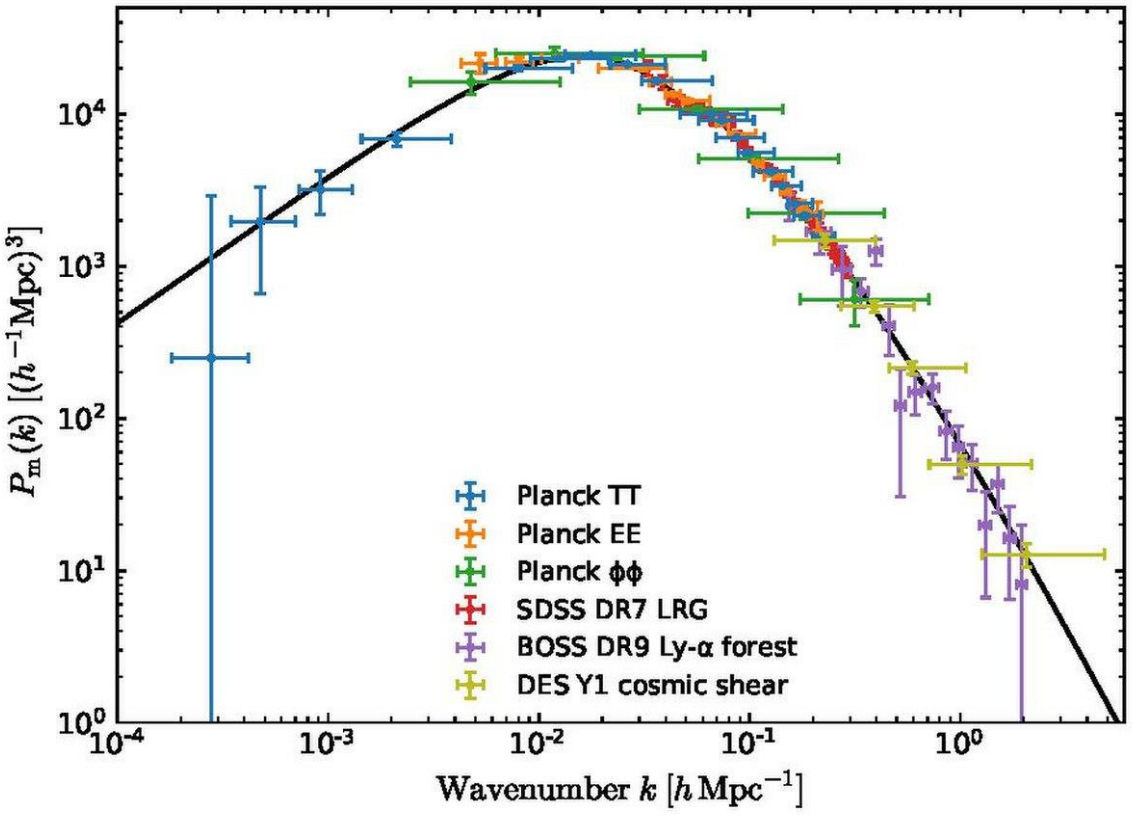


Why is the DM phase-space distribution important?

It turns out that the ***formation of structure in the early universe (clusters, galaxies, etc.) is sensitive to the velocity of DM!***

Structure formation is ***suppressed*** if DM has non-negligible velocity and therefore deviates from what is expected for CDM!

In fact, the cosmic structure carries an ***imprint*** of the DM velocity distribution.



e.g., in the linear regime, can be reflected in the shape of the matter power spectrum $P(k)$.

- **Studying the relation between DM phase-space distribution and large-scale structure enables us to learn about DM from its gravitational interaction only.**
- **This provides a way to learn about the dark sector even if the dark sector does not interact with the SM at all, except through gravity!**

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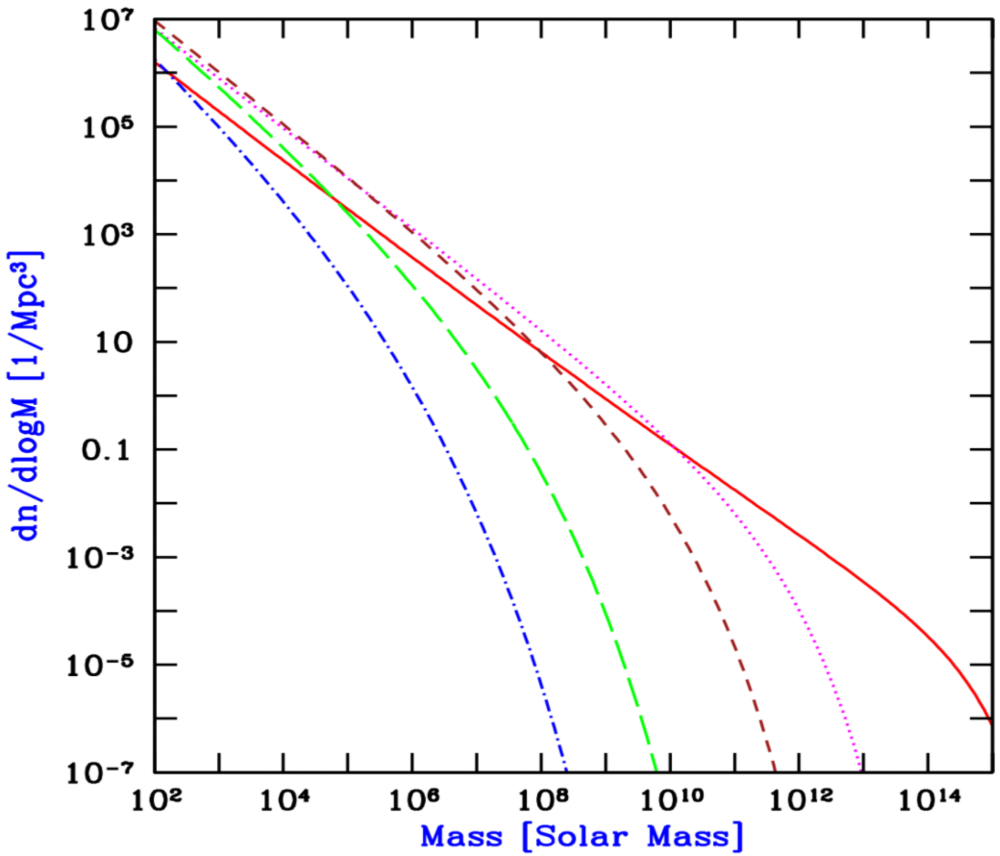


e.g., in the non-linear regime, can be reflected in the shape of the halo mass function $dn/d\log M$.

- **Studying the relation between DM phase-space distribution and large-scale structure enables us to learn about DM from its gravitational interaction only.**
- **This provides a way to learn about the dark sector even if the dark sector does not interact with the SM at all, except through gravity!**

Let us study the linear matter power spectrum *P(k)* first …

To study the impact of non-negligible velocities on *P(k)*, a standard approach is to define a single "free-streaming horizon" as a benchmark scale below which structure is suppressed

$$k_{\mathrm{FSH}} \equiv \left[\int_{t_{\mathrm{prod}}}^{t_{\mathrm{now}}} dt\, \frac{\langle v(t)\rangle}{a(t)}\right]^{-1}$$

Relies on averaging, not suitable for non-trivial or multi-modal distributions – average velocity might ***NOT*** be able to capture all the features in the distribution.
In some cases, the distribution might not even contain any DM particle with velocity $\langle v\rangle$!

Our approach

We begin by considering *momentum slices* through our dark-matter packet, relating each slice of momentum *p* to a corresponding value $k_{\rm hor}(p)$.

$$k_{\rm hor}(p) \;\equiv\; \xi\left[\int_{t_{\rm prod}}^{t_{\rm now}} dt\; \frac{v(t)}{a(t)}\right]^{-1}$$

Normally, $k_{\rm hor}$ would be interpreted as defining the ***minimum*** value of k which can be affected by dark matter in that momentum slice.

However, we shall instead take the defining relation for $k_{\rm hor}(p)$ as defining a ***mapping*** between the p-variable of $g(p)$ and the k-variable of $P(k)$.
In other words, we shall ***identify*** $k_{\rm hor}(p)$ with k and thereby consider $g(p)$ as having a corresponding profile in k-space:

$$\tilde{g}(k) \;\equiv g\left(k_{\rm hor}^{-1}(k)\right)\left|\frac{d\log p}{d\log k}\right|$$

Inverse of $k_{\rm hor}(p)$

It then follows

$$N(t) \sim \int d\log p \;\; g(p) = \int d\log k \;\; \tilde{g}(k)$$

Thus $\tilde{g}(k)$ describes a **dark-matter distribution in k-space**!

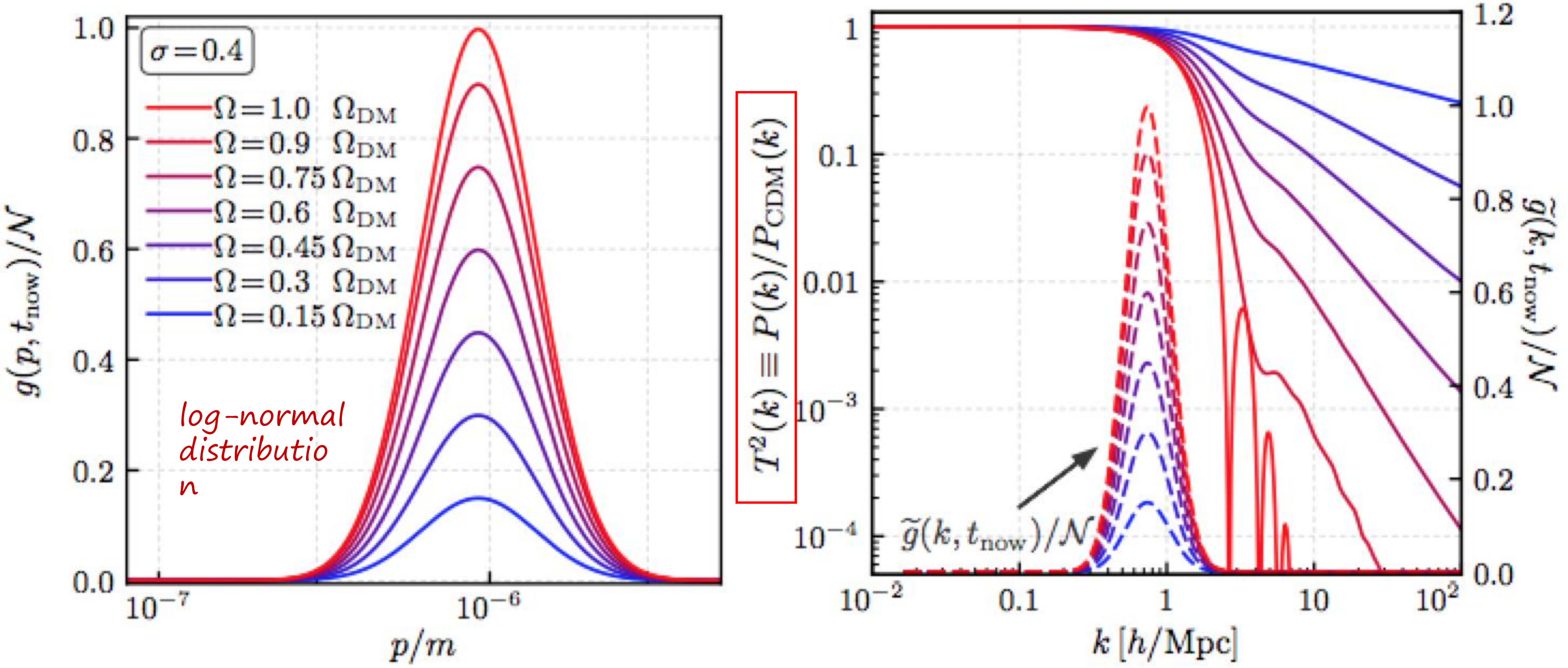
Moreover, because this $\tilde{g}(k)$ lives in the same space as $P(k)$, these two functions can even be plotted together along the same axis!

Now it makes sense to ask:
Can we discover/conjecture any relation between *these* two functions?

Examine the relations

<u>Vary height/area with width fixed</u>

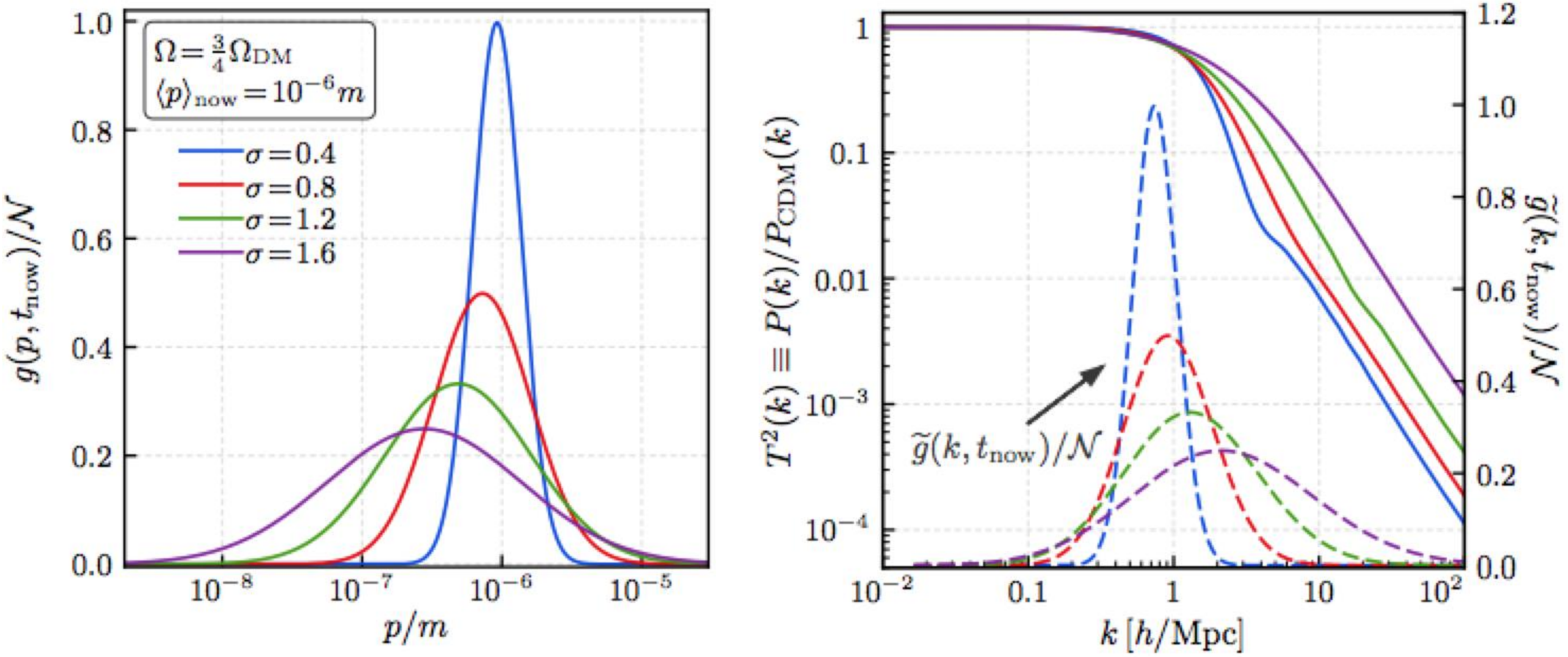
(a complementary CDM component is added to get the total DM abundance)



- No power suppression until we approach where $\tilde{g}(k)$ is concentrated
- Main observation: Larger packet → Stronger <u>suppression</u> and steeper <u>slope</u>

<u>Vary width with average/area fixed</u>

(a complementary CDM component is added to get the total DM abundance)

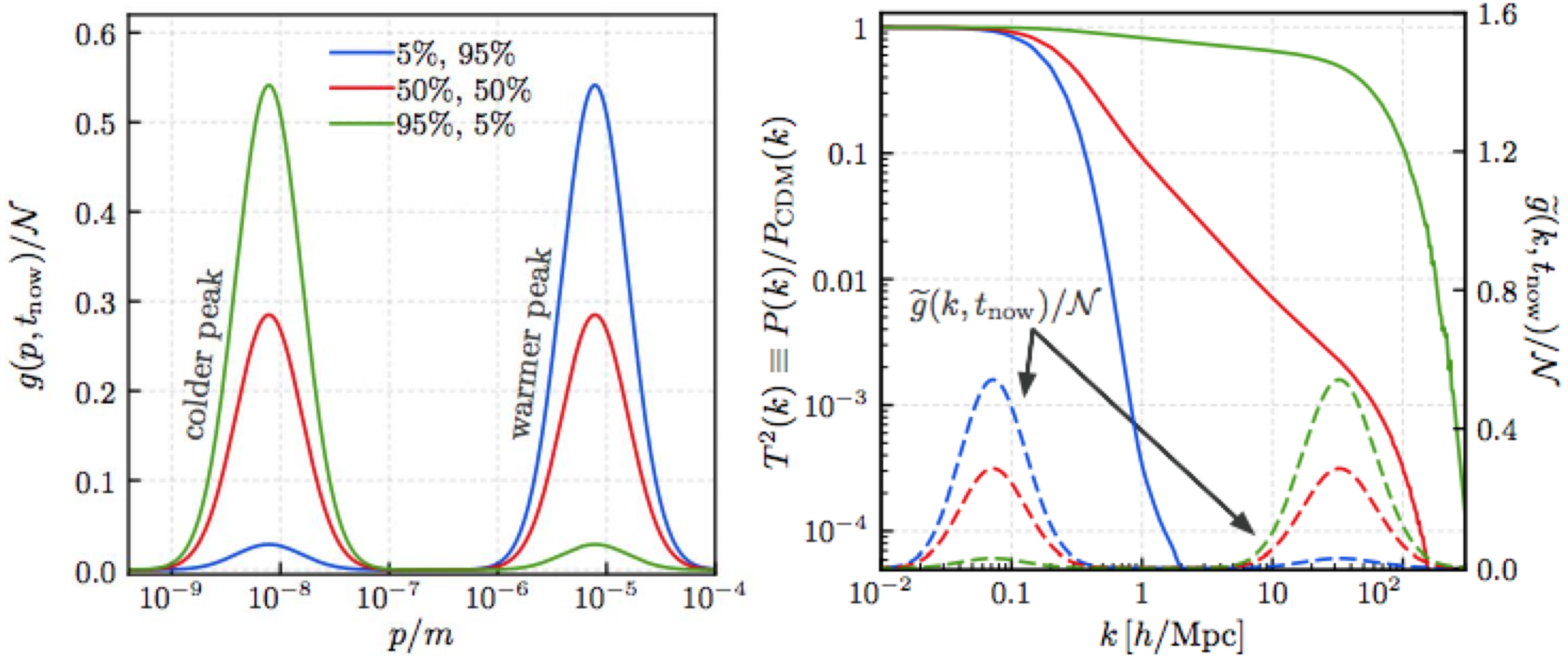


- The amount of <u>suppression</u> differs, but the ***<u>slope</u>*** at large k is essentially *<u>unaffected</u>* by widths! This suggests the ***<u>accumulative abundance is correlated with the slope</u>***, *NOT* with the net suppression.

Does this behavior survive for more complex $g(p)$?

Vary relative sizes of *two* packets

(two packets together carry the total DM abundance)



As we sweep from left to right in the k-space,

- *within* a peak: accumulative abundance increases → slope increases!
- *between* peaks: no change in accumulation of abundance → slope approximately constant!

Still find **accumulative abundance → slope!**

Based on our observation, our claim is that the ***slope*** of the transfer function at a particular scale k is related to the amount of DM particles that is able to freestream a distance larger than $\sim 1/k$,

Motivated by this observation, we define the ***hot-fraction function***,
i.e., the fraction of DM particles that is effectively "hot" at the scale k

$$\boxed{F(k) \equiv \frac{\int_{-\infty}^{\log k} \tilde{g}(k')\, d\log k'}{\int_{-\infty}^{+\infty} \tilde{g}(k')\, d\log k'}}$$

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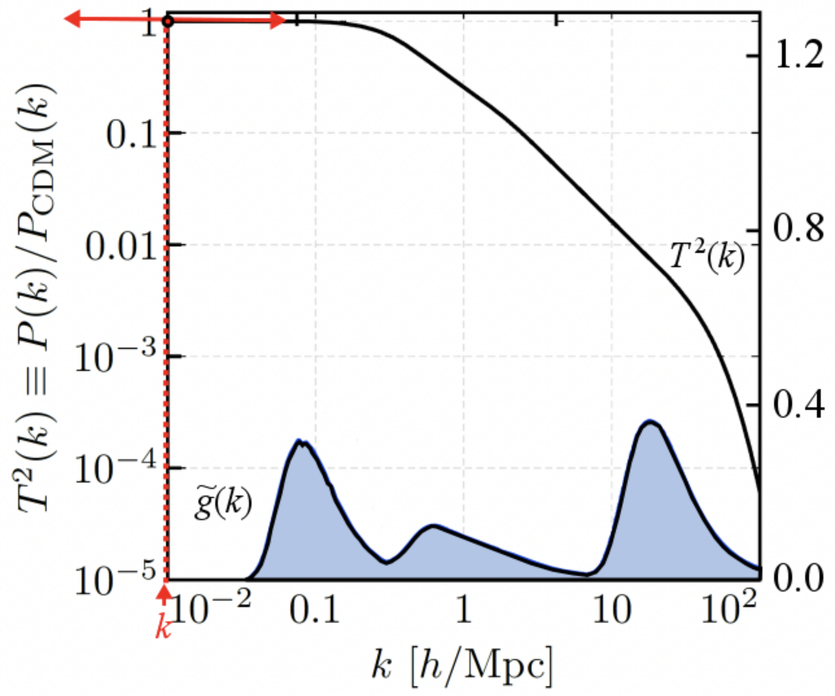

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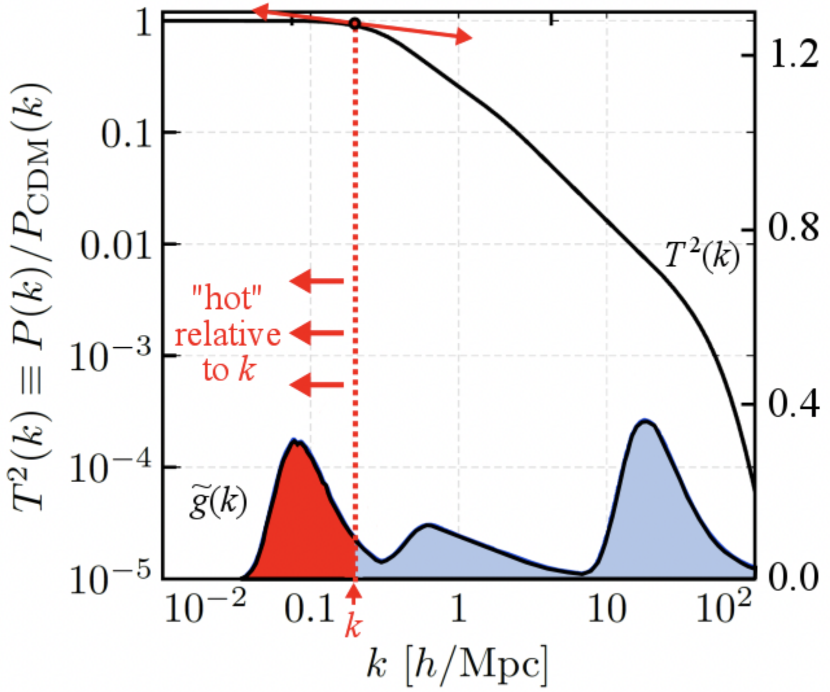

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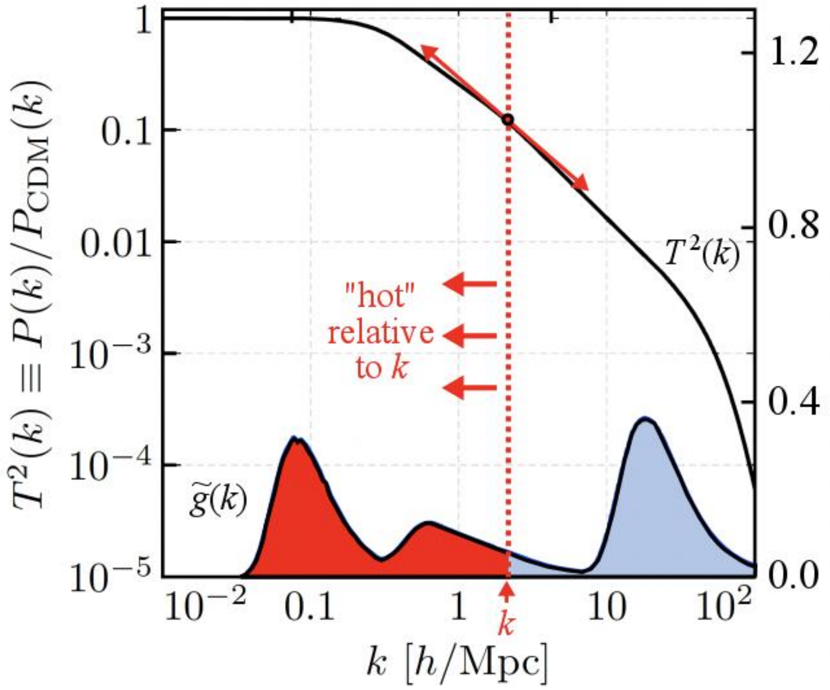

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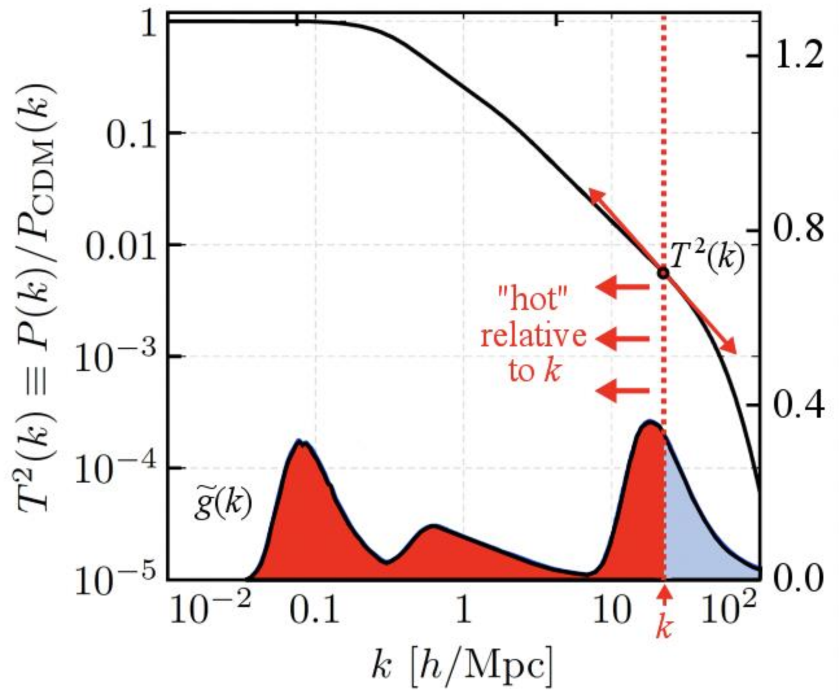


Our claim can then be formulated as

some as-yet unknown function

$$F(k) \approx \eta\left(\left|\frac{d\log T^2}{d\log k}\right|\right)$$

logarithmic slope of transfer function

Taking derivative of both sides

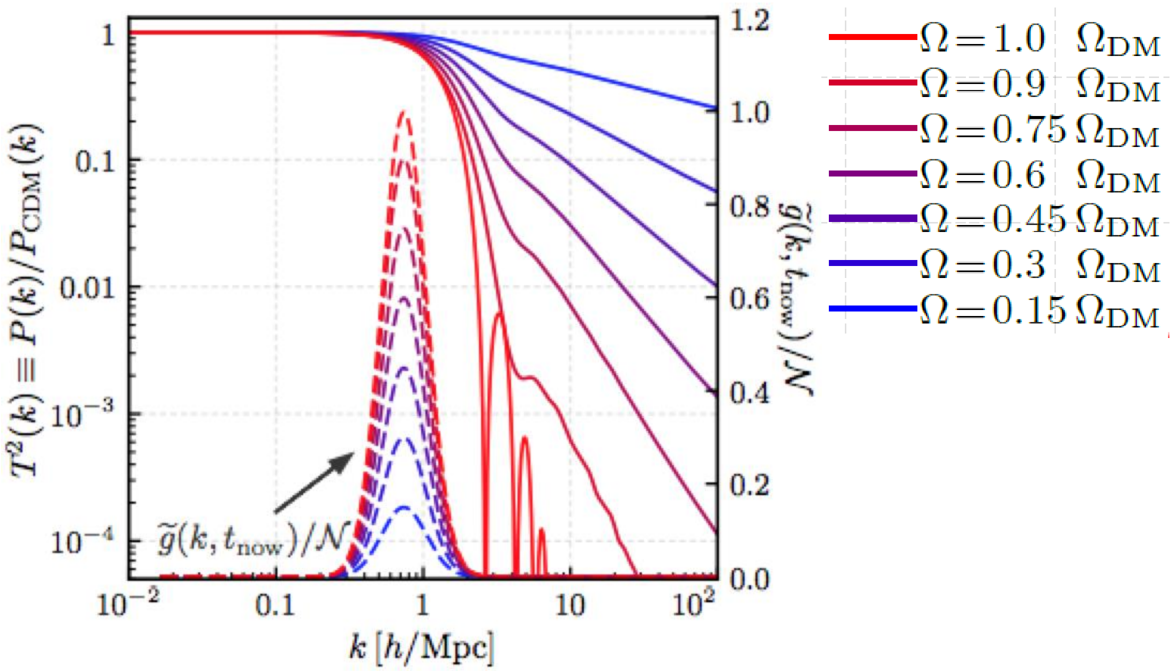
$$\frac{\widetilde{g}(k)}{\mathcal{N}} \approx \eta'\left(\left|\frac{d\log T^2}{d\log k}\right|\right)\left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

DM phase-space distribution

first derivative of T^2

second derivative of T^2

Once η is known, we will be able to resurrect the phase-space distribution!



$$\left| \frac{d \log T^2}{d \log k} \right| \approx [F(k)]^2 + \frac{3}{2} F(k)$$

relation holds to very high precision!

Our final conjecture:

$$\frac{\tilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2} \left(\frac{9}{16} + \left| \frac{d \log T^2}{d \log k} \right| \right)^{-1/2} \left| \frac{d^2 \log T^2}{(d \log k)^2} \right|$$

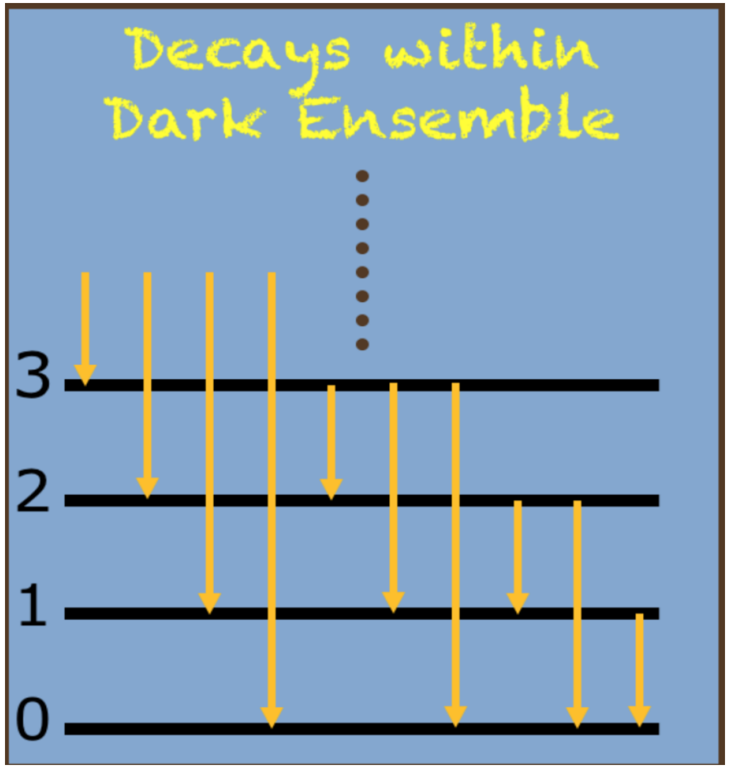
This allows us to "resurrect" $\tilde{g}(k)$ from the transfer function $T^2(k)$!

Let's now see how these ideas play out in practice!

As an example, we study a model in which the dark sector contains *many* components with many different masses and thus many possible decay chains.

Moreover, we assume decays within the dark sector are the dominant processes

How robust are our observations?



Toy Model: Parametrization

Dark ensemble consists of *N+1* real scalars ϕ_j with $j = 0,1, \dots N$, and a mass spectrum:

$$m_j = m_0 + j^\delta \Delta m$$

Lagrangian:

$$\mathcal{L} = \sum_{\ell=0}^{N} \left(\frac{1}{2} \partial_\mu \phi_\ell \partial^\mu \phi_\ell - \frac{1}{2} m_\ell^2 \phi_\ell^2 - \sum_{i=0}^{\ell} \sum_{j=0}^{i} c_{\ell ij}\, \phi_\ell \phi_i \phi_j \right) + \cdots$$

The trilinear coupling:

mass difference between parent and products

mass difference between products

$$c_{\ell ij} = \mu R_{\ell ij} \left(\frac{m_\ell - m_i - m_j}{\Delta m} \right)^{r} \left(1 + \frac{|m_i - m_j|}{\Delta m} \right)^{-s} \Theta(m_\ell - m_i - m_j)$$

Positive r → Decays with more kinetic energy
Negative r → Decays more marginal (less phase space)

Positive s → Decay products tend to have similar masses
Negative s → Decay products tend to have different masses

In our analysis we considered ***N=9*** ***(10 states)***

Toy Model: Parametrization

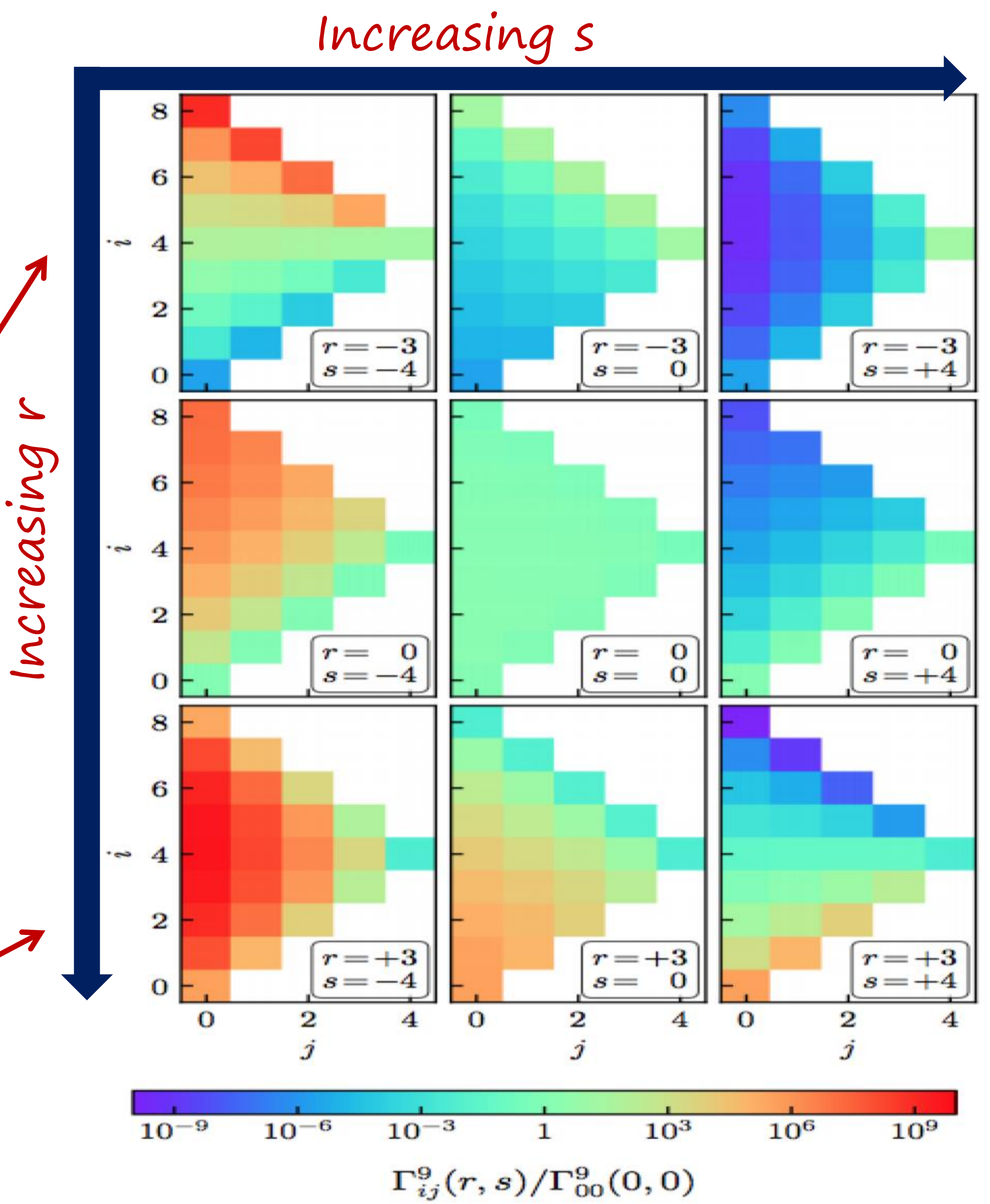
Given the explicit Lagrangian we can calculate the decay widths from a given parent state to a given pair of daughter states:

$$\phi_\ell \rightarrow \phi_i + \phi_j$$

For $\ell = 9$, we have…

Tend to minimize kinetic energy and symmetry
e.g., $\phi_9 \rightarrow \phi_8 + \phi_0$

Tend to produce light states but favor asymmetry
e.g., $\phi_9 \rightarrow \phi_4 + \phi_0$



Tend to minimize kinetic energy but favor symmetry
e.g. $\phi_9 \rightarrow \phi_4 + \phi_4$

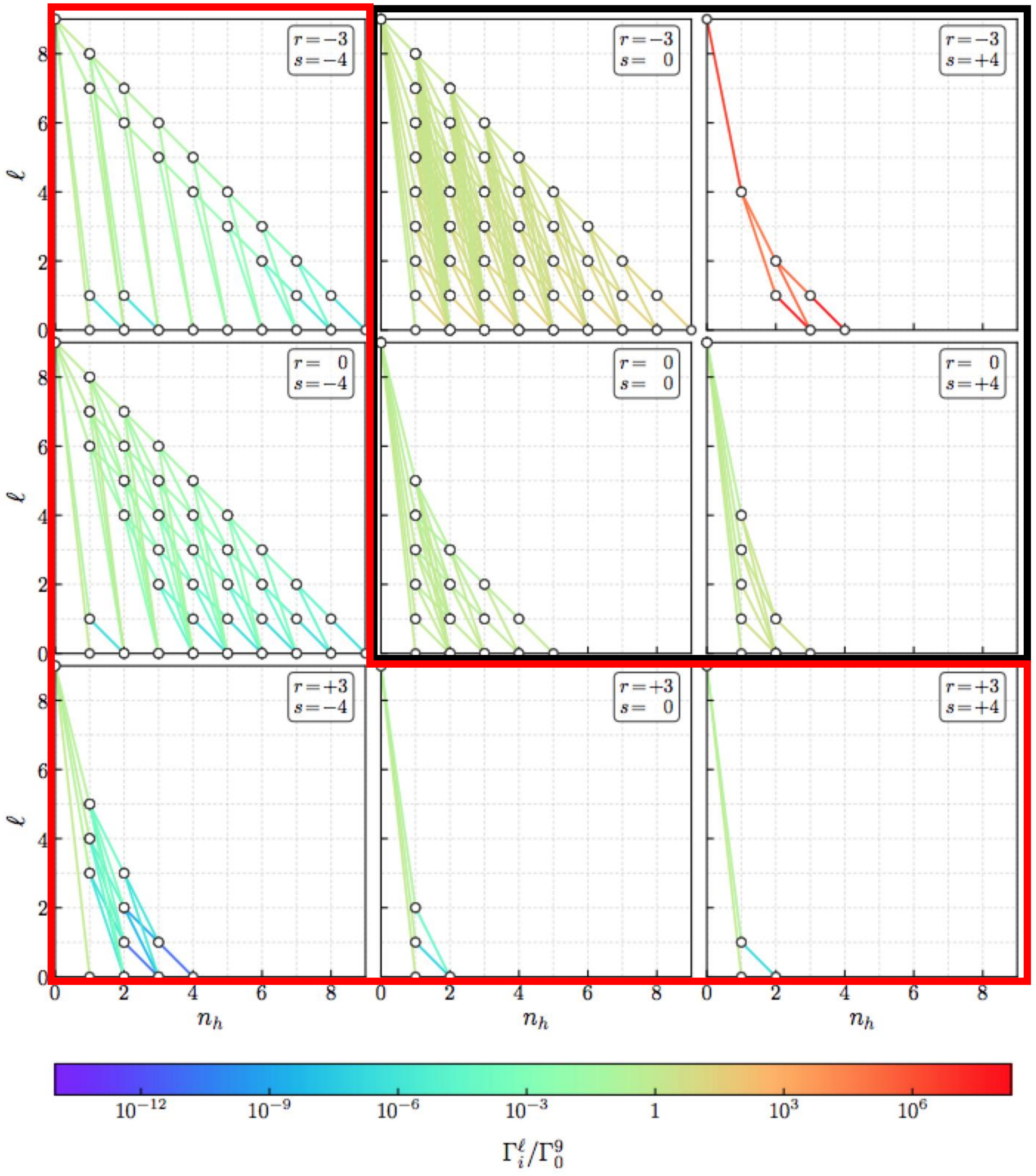
Tend to produce the lightest states
e.g., $\phi_9 \rightarrow \phi_0 + \phi_0$

Toy Model: Decay Chains

The figure shows how decays proceed step by step from a heavy state to the ground state. Only major decay chains are shown.

- Color of each segment measures how fast a state is being produced, ***<u>warmer color → faster production</u>***
- Timescales of a decay chain can be inferred by inverting the ***<u>"slowest color"</u>***

Many different patterns of decay chains could emerge!



Deposits to the ground state tend to occur around the same time

Deposits to the ground state tend to occur at different times

Toy Model: Final Phase-Space Distribution

$g(p)$ is phase-space distribution w.r.t. $\log p$

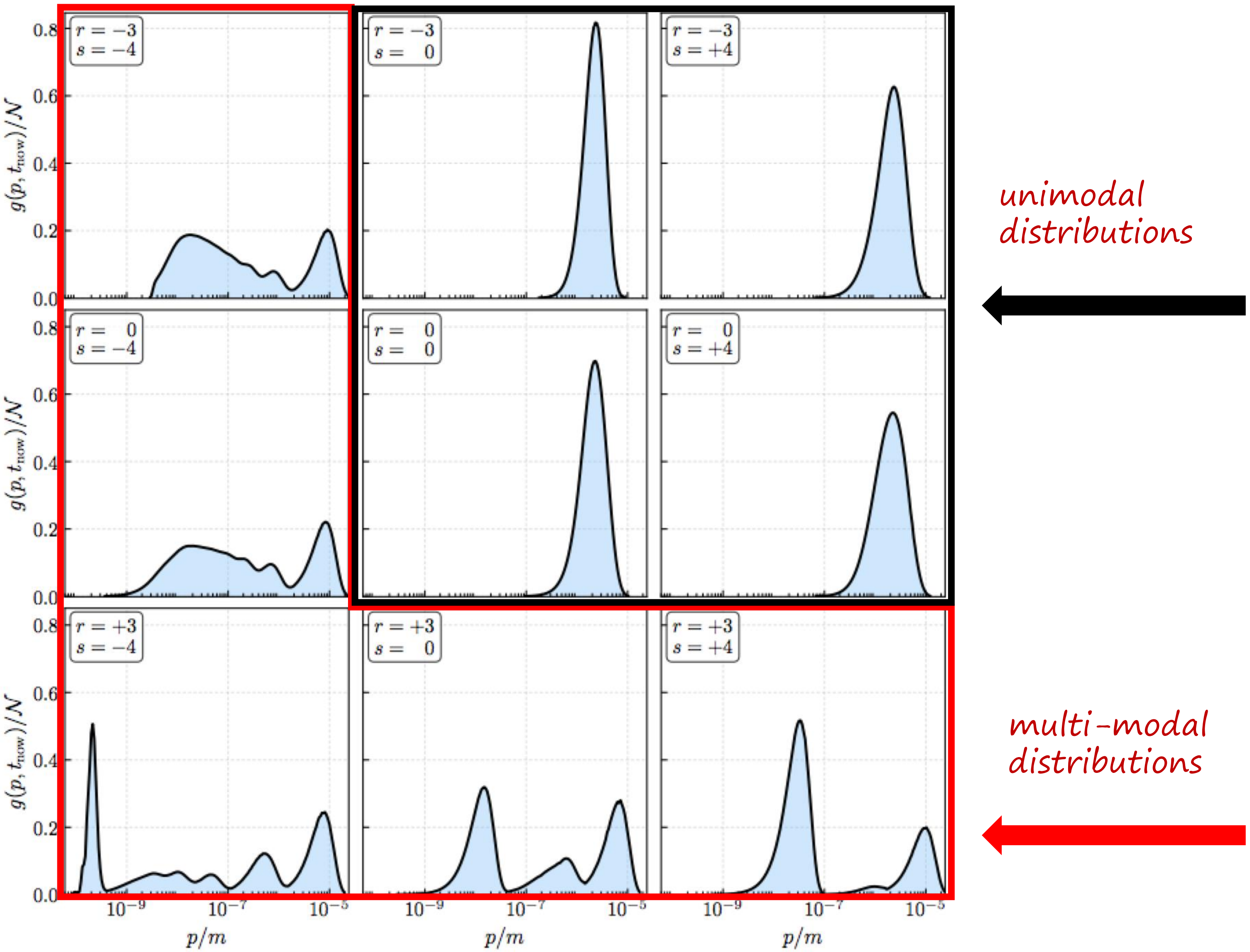
$$N(t) \;=\; \frac{g_{\rm int}}{2\pi^2} \int_{-\infty}^{\infty} d\log p \; g(p,t) \;\equiv\; \frac{g_{\rm int}}{2\pi^2}\mathcal{N}(t)$$

Numerically solve the Boltzmann equation assuming only the heaviest state is populated initially.

A rich variety of distributions emerges!

As expected!

- Cases in which decay chains land on the ground state at ***similar timescales*** tend to produce ***unimodal distributions***
- ***Multi-modal*** distributions could result if ***timescales*** of different decay chains ***differ significantly***



Linear regime: $g(p) \to P(k)$

$$k_{\rm hor}(p) \equiv \xi \left[\int_{t_{\rm prod}}^{t_{\rm now}} \frac{p/a(t)}{\sqrt{p^2/a(t)^2 + m^2}} \frac{dt}{a(t)} \right]^{-1}$$

Matter power spectrum $P(k)$ obtained by feeding $g(p)$ to CLASS code

Plot the squared transfer function

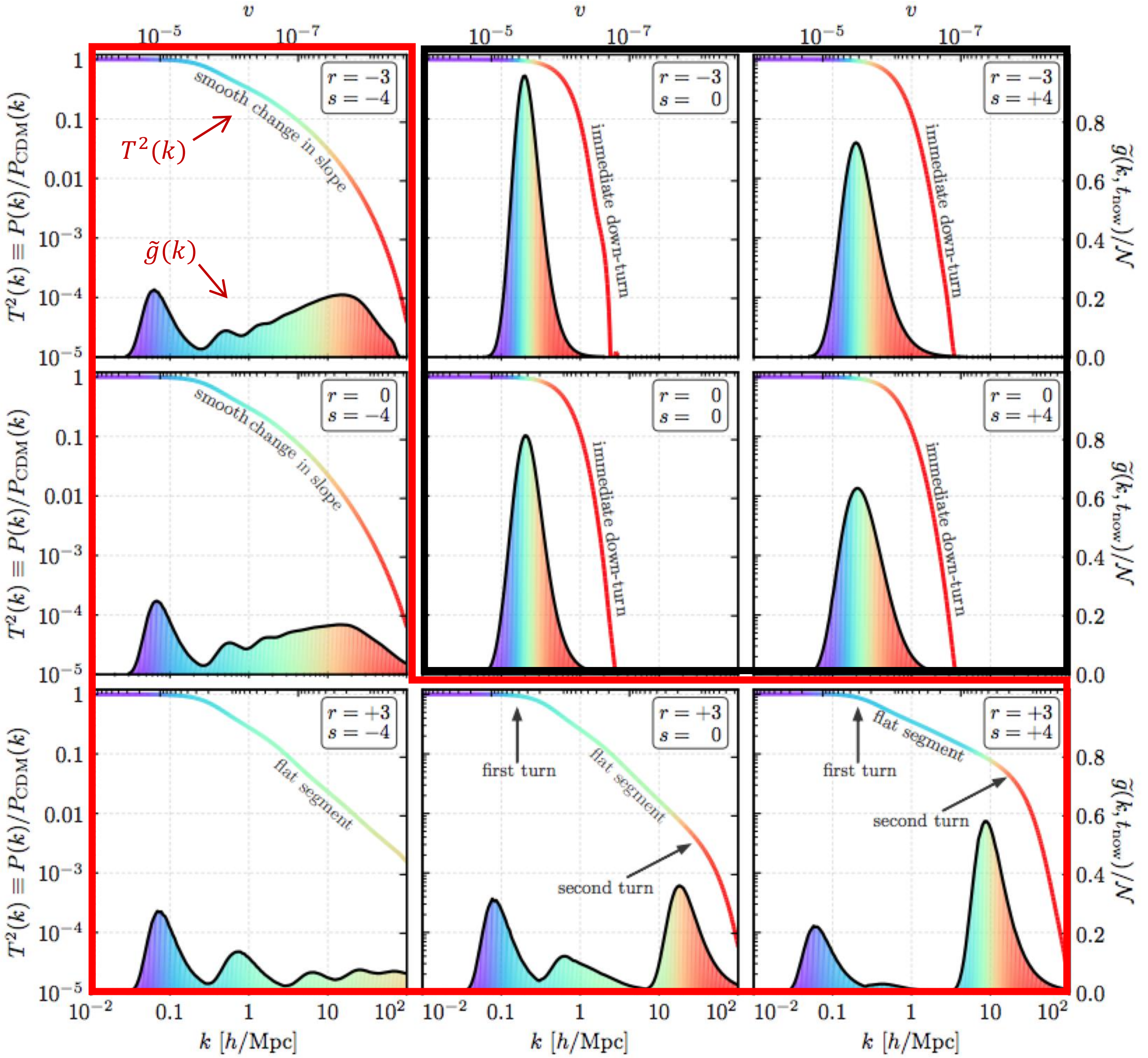
$$T^2(k) \equiv P(k)/P_{\rm CDM}(k)$$

to show relative suppression

Map $g(p)$ to $\tilde{g}(k)$ by mapping p to $k_{\rm hor}$

Rainbow colors correspond to ***hot-fraction function*** $F(k)$: fraction of DM particles with

$$k_{\rm hor} < k$$



$$F(k) \equiv \frac{\int_{-\infty}^{\log k} \tilde{g}(k')\, d\log k'}{\int_{-\infty}^{+\infty} \tilde{g}(k')\, d\log k'}$$

The ***Slope*** of $T^2(k)$ indeed appears to ***correlate*** with $F(k)$

Linear regime: Reconstruction Conjecture

To what extent can we "***resurrect***" the DM phase-space distribution from the transfer function?

Recall our conjecture…

$$\frac{\widetilde{g}(k)}{\mathcal{N}} \approx \frac{1}{2}\left(\frac{9}{16}+\left|\frac{d\log T^2}{d\log k}\right|\right)^{-1/2}\left|\frac{d^2\log T^2}{(d\log k)^2}\right|$$

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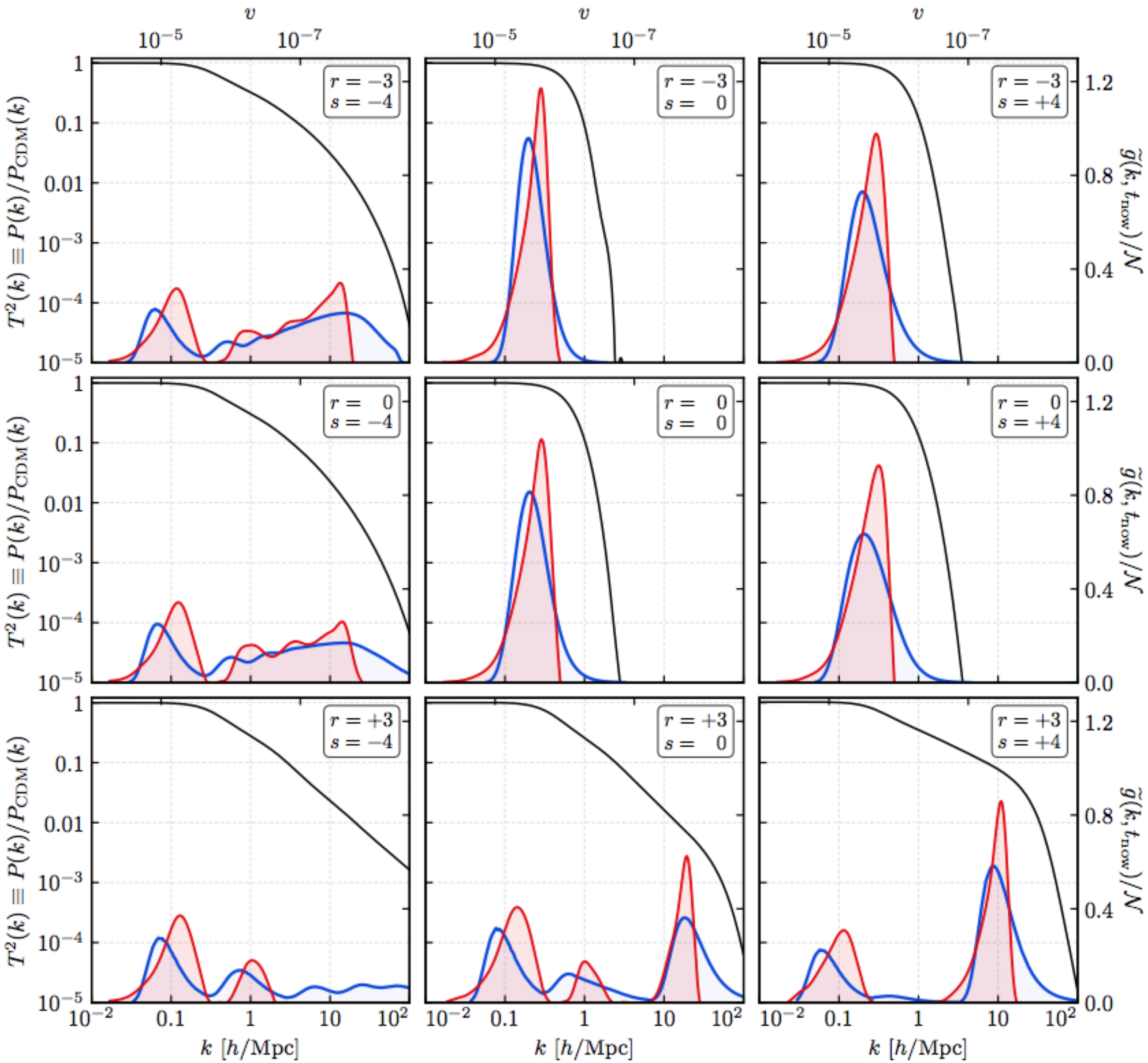
Blue: original DM distribution in k-space
Red: reconstruction directly from $T^2(k)$

Archaeological reconstruction is surprisingly accurate for a **variety** of possible DM distributions.
Able to resurrect the **salient features** of the original distribution!



Non-Linear regime: $g(p) \to dn/d\log M$

We study the non-linear regime using the ***halo mass function*** $dn/d\log M$

i.e., number density of haloes in $[\log M, \log M + d\log M]$

The halo mass function can be calculated using the ***Press-Schechter formalism***:

- Spherical collapse model: linearly extrapolate density perturbation $\delta(\vec{x}, t)$, regions have collapsed and formed DM haloes by time t if
$$\delta(\vec{x}, t) > \delta_c \approx 1.686$$
- Probability that a ***random point $\vec{x}$ is inside a matter clump whose $m > M$*** is proportional to probability that ***the density fluctuation averaged on a mass scale M is larger than δ_c***:
$$\mathcal{P}(m > M, t) = 2\mathcal{P}(\delta_M > \delta_c, t, M)$$
- Gaussian:
$$\mathcal{P}(\delta_M > \delta_c, t, M) = \int_{\delta_c}^{\infty} d\delta_M \frac{1}{\sqrt{2\pi\sigma^2(t, M)}} \exp\left[-\frac{\delta_M^2}{2\sigma^2(t, M)}\right]$$

Halo mass function can be related to the linear matter power spectrum
$$\frac{dn}{d\log M} = \frac{\bar{\rho}}{12\pi^2 M} \nu(M)\eta(M) \frac{P\big(1/R(M)\big)}{\delta_c^2 R^3(M)}$$
where $M = \bar{\rho} \times \frac{4\pi}{3}(c_W R)^3$ is the mass of the spherical region that collapses to form a halo, and $k = 1/R$.

Details:
relation between M and R depends on window function with which the density field is smoothed.
$c_W \approx 2.5$, for top-hat in k-space.
$\nu(M) = \delta_c^2 \sigma^2(t_{now}, M)$,
$\eta(M) = \frac{\sqrt{2\nu(M)}}{\pi} A[1 - \nu^{-\alpha}(M)]e^{-\nu(M)/2}$
for a more realistic ***ellipsoidal*** collapse

Non-Linear regime: $g(p) \to g_M(M)$

Following the same logic, we would like to plot the phase space distribution in the M-space. Recall that

$$k_{\rm hor}(p) \;\equiv\; \xi \left[\int_{t_{\rm prod}}^{t_{\rm now}} dt\, \frac{v(t)}{a(t)}\right]^{-1}$$

provides a functional map between p and k.

Moreover, $M = \bar{\rho} \times \frac{4\pi}{3}(c_W R)^3$ and $k = 1/R$ defines a functional map between k and M.

Together, we obtain a functional map between p and M:

$$M(p) \;\equiv\; \frac{4\pi\bar{\rho}c_W^3}{3\xi^3} \left[\int_{t_{\rm prod}}^{t_{\rm now}} dt\, \frac{v(t)}{a(t)}\right]^{3}$$

Non-Linear regime: $g(p) \rightarrow g_M(M)$

With this functional map, we have

$$g_M(M) = g(p) \left| \frac{d \log p}{d \log M} \right|$$

and we can also similarly define the ***hot-fraction function in the M-space***:

$$F(M) \equiv \frac{\int_{\log M}^{\infty} d \log M' \, g_M(M')}{\int_{-\infty}^{\infty} d \log M' \, g_M(M')}$$

the fraction of DM particles that is capable of free-streaming out of a region that would collapse into a halo of mass M,

in other words, the fraction of DM particles that is effectively "hot" at the mass scale M

Non-linear regime: Reconstruction Conjecture

Following a similar procedure, we define the
structure-suppression function

$$S(M) \equiv \sqrt{\frac{dn/d\log M}{(dn/d\log M)_{CDM}}}$$

and investigate the relation between $g_M(M)$ and $S(M)$.

We find that

$$\frac{g_M(M)}{\mathcal{N}} \approx \sqrt{\frac{5}{14}\left(\frac{d\log S^2(M)}{d\log M}\right)^{-\frac{1}{2}}\left|\frac{d^2\log S^2(M)}{(d\log M)^2}\right|}$$

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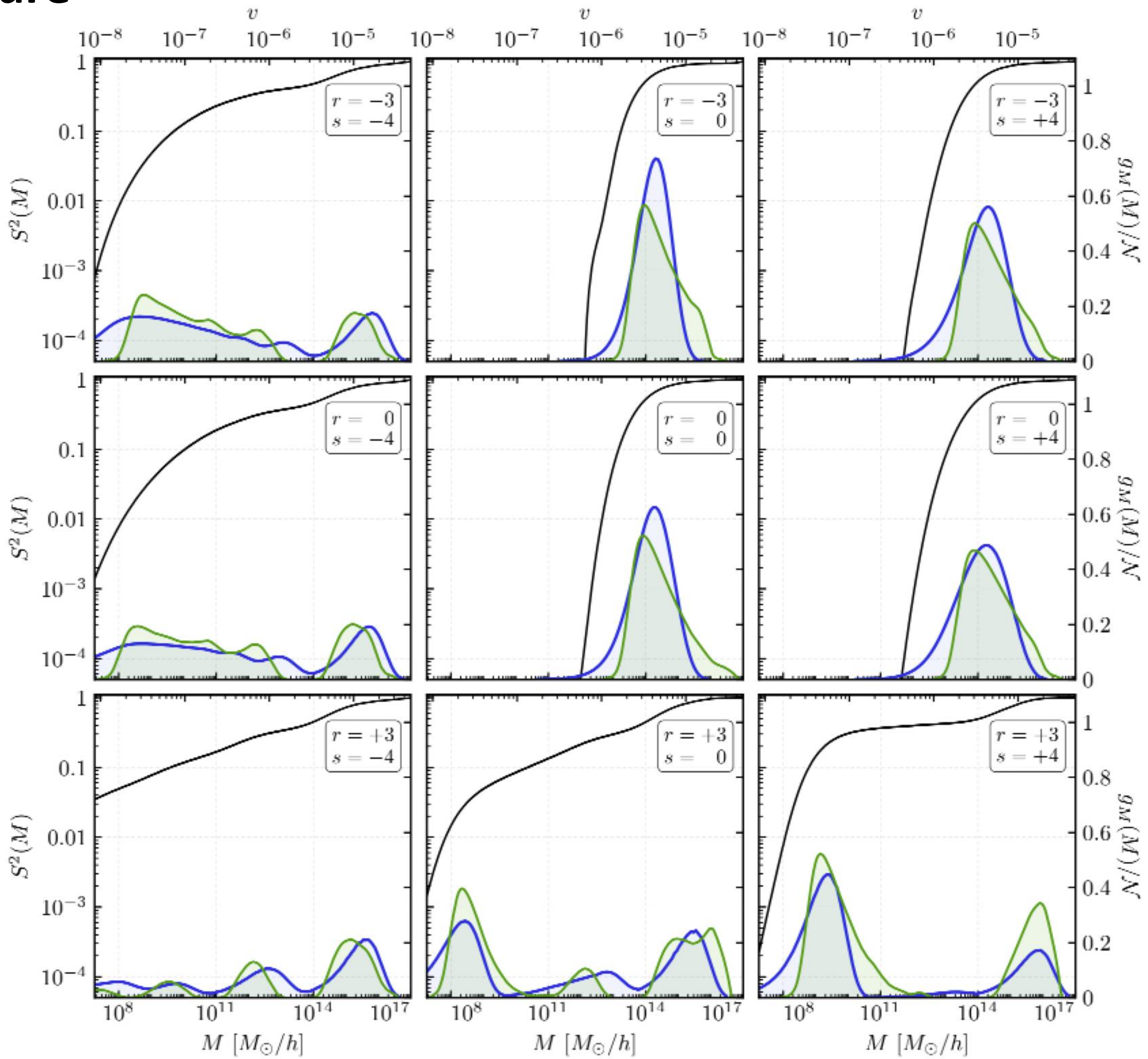
Blue: original DM distribution in M-space

Green: reconstruction directly from $S^2(M)$

Once again, able to resurrect the **salient features** of the original distribution!



Conclusions

- Early-universe processes leave ***identifiable patterns*** in the phase-space distribution $g(p)$ of dark matter.
- In particular, ***non-minimal dark sectors*** naturally give rise to non-trivial DM phase-space distributions which are then ***imprinted*** on the cosmic structure.
- The DM phase-space distribution $g(p)$ is ***correlated*** with the observables in both the linear and non-linear regimes such as the matter power spectrum $P(k)$ and the halo mass function $dn/d\log M$ through the ***hot-fraction function***.
- We proposed two ***reconstruction conjectures*** (one for each regime) which allow us to reproduce $g(p)$. These reconstruction conjectures are simple and allow us to ***resurrect the salient features*** of the phase-space distribution directly from $P(k)$ and $dn/d\log M$.
- Such approaches allow us to learn about dark-sector dynamics even ***if the dark sector has only gravitational couplings to the SM***.
- The dark sectors of string theory generically include unstable Kaluza-Klein towers, thus could potentially lead to multi-modal distributions and non-trivial $P(k)$ and $dn/d\log M$. This provides motivation to measure/bound those observables with increased precision.

Future Directions

- A systematic study of different production mechanisms which can give rise to nonthermal, multi-modal distributions.
- Incorporate effects that might come from couplings to SM. These effects could potentially affect evolution of phase-space distributions in additional subtle ways.
- Incorporation of observational bounds (Lyman α, etc.), in progress.
- Refine reconstruction conjecture for greater accuracy.
- Conjectures for other observables.