Quantum Black Holes

Bridges between physics and mathematics

Kavli IPMU Postdoc Colloquium

(Seminar @ 12 00 pm)

Abhiram Kidambi

29 10 2021

Talk includes unnecessary stick figures

Partition functions

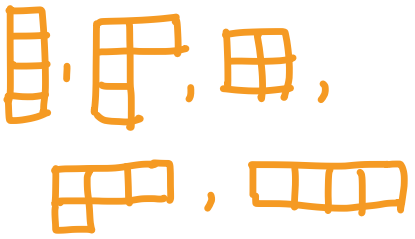
* Thermodynamic properties of the system
* How average phenomenon emerges from statistical distributions
* QFT. Euclidean path integral
* Entropy => Information / Microstates of the system

Computing the PF of your system / theory is crucial to your ability to fully understand it

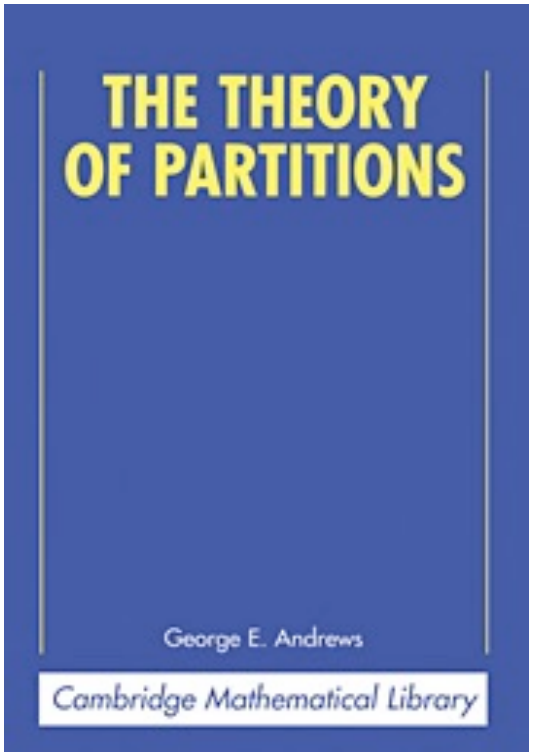
Also of mathematical interest → Studied intently by G H. Hardy, J Littlewood, S Ramanujan, H Rademacher

Ex. Simple integer partitions

$p(2) = \{\underbrace{1+1}_{2},\ 2+0\}$ → 

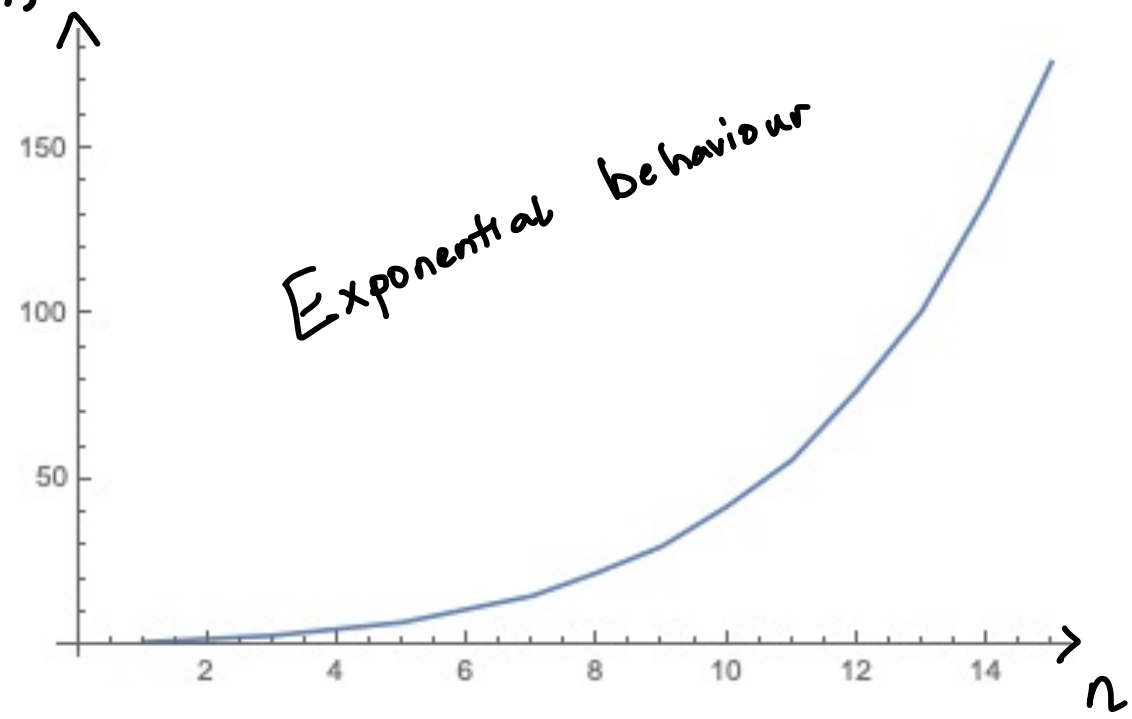
$p(4) = \{4+0,\ 3+1,\ 2+2,\ 2+1+1,\ \underbrace{1+1+1+1}_{5}\}$ → 

$p(6) = 11$, $p(8) = 22$, .



Simple partitions

of n, $p(n)$



Exponential behaviour

Q: $\lim_{n \to \infty} p(n) = ?$

More interesting partitions?

$f(q) \stackrel{?}{=} \sum_n \#(\text{partitions of } n) * q^n$

Sometimes (*)

Highly symmetric functions

Partition functions

of certain Quantum

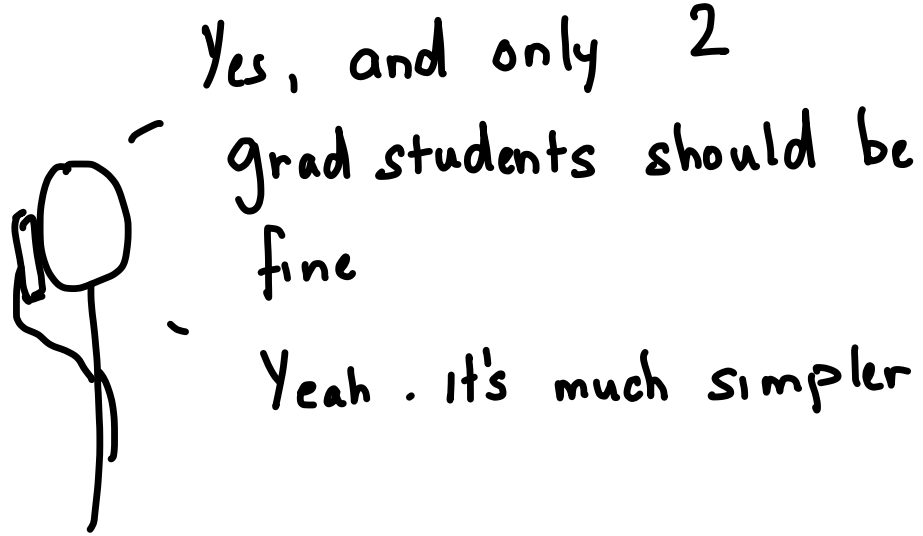
Black holes in string theory

Special partitions that appear in number theory and algebraic geometry

Physicists motivation

What is the smallest amount of information I need to give you in order to compute the full (indexed) black hole partition function?

How easy is it?



Mathematician's motivation

- What is the relation between the Ooguri-Strominger-Vafa conjecture, Gromov-Witten invariants and mock-modular forms?

How does mock modularity arise from the prepotential (of $\mathcal{N}=2$ supergravity)?

!

Black holes

Astrophysical: End point of gravitational collapse of a sufficiently massive star, where grav forces overcome neutron deg pressure

Quantum Info: A system that saturates the bound for info storage in a volume:

$$H_{\text{bekenstein}} \leq \frac{2\pi R M c}{\hbar \ln 2}$$

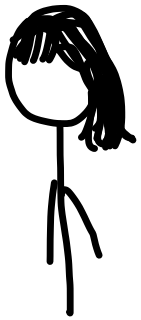
(R: Radius, M: Mass)

The human brain cannot store more than $\sim 10^{42}$ classical bits of information.

I will memorize Graham's number
Collapse my brain and the world into a black hole

OK you're immortal
Now what, Lex Luthor?

And we thought that the most insufferable billionaire was that Amazon guy



Discovery to dismissal to discovery

Schwarzschild's solution to Einstein Field eqs.

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{2M}{r}\right)} + r^2 d\Omega_2^2$$

$r = 0 \longrightarrow$ Singularity!

Einstein was dismissive of the solution

- 1940's - 50's work by Oppenheimer, Snyder and Wheeler re-instigated work on Black holes
- Today. Plenty of experimental evidence (*)

EHT photo, Stellar motion maps by Genzel + co & Ghez + co, GRB's....

Entropy of black holes

- Bekenstein and Hawking → Entropy of black holes

  Leading contribution comes from its area

S_{cl} of a black hole is zero.

  But.

$$S = \underbrace{\frac{A_{BH}}{4G_N^{(d)}}}_{\text{semi classical}} + \underbrace{\cdots\cdots\cdots}_{\text{Quantum | Subleading}}$$

  True for ANY black hole

- Computing the ellipsis terms, or subleading corrections to entropy is difficult Because it requires identifying all the saddle points to the gravitational path integral.

- Try working with a quantum theory of gravity.

A QG theory (in the BH context) should:

1. Tell us exactly what the entropy of a black hole is (Precision test of quantum gravity)
   ↳ What is the full partition function of a BH?

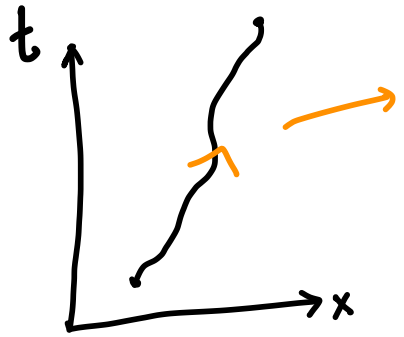
2 What exactly we are counting?
   What are physical microstates?

3 Dynamics of Black Holes + information....
   ↳ This is where the prize elephant sits

Black holes and string theory.

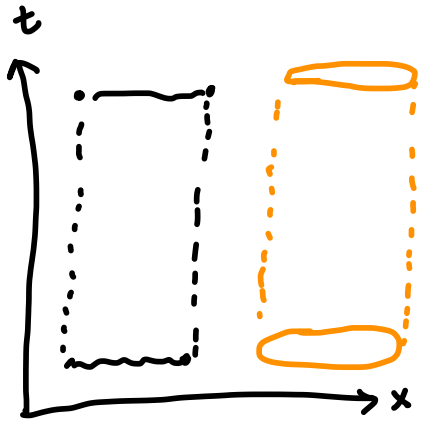
- String theory is an attempt to quantize gravity, where gravitational theories + QFT's arise in certain limits of a higher dimensional quantum theory (*).

- in classical/QM



particle trajectory w/ some world line parametrization

- in String theory.



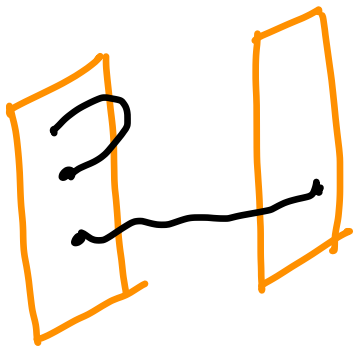
"world sheet"

'Conformal field theory'

String theory is a theory that turns a 2d CFT into a[(*)] gravitational / QFT in $d \leq 10$.

→ Perturbatively : closed strings

→ On 'solitonic' backgrounds (Branes) : open strings



"gauge fields"

How do you create a black hole in string theory?

System of branes + strings ⟶ increase string coupling

Large g_s ⟶ effective black hole geometry

But! studying partition functions for generic black holes is still difficult.

$$\#\ \text{String states} \sim e^{M}$$

$$\#\ \text{of bh states} \sim e^{M^2}$$

Susskind suggested some renormalization should help alleviate this but we still don't know how/ what this procedure looks like.

BUT. Exist a class of black holes for which we don't need to worry about renormalization

When the theory has more supersymmetry ($N \geq 2$), there are so called "BPS" states

BPS states

- When # of supersymmetry ≥ 2, one can extend the supersymmetry algebra To this extension, you can associate a conserved charge.

"Central charge" $Z \in \mathbb{C}$ (*)

- $M = |Z| \longrightarrow$ BPS limit

BPS states are very robust Independent of physical coupling, or usually even values of moduli / parameters

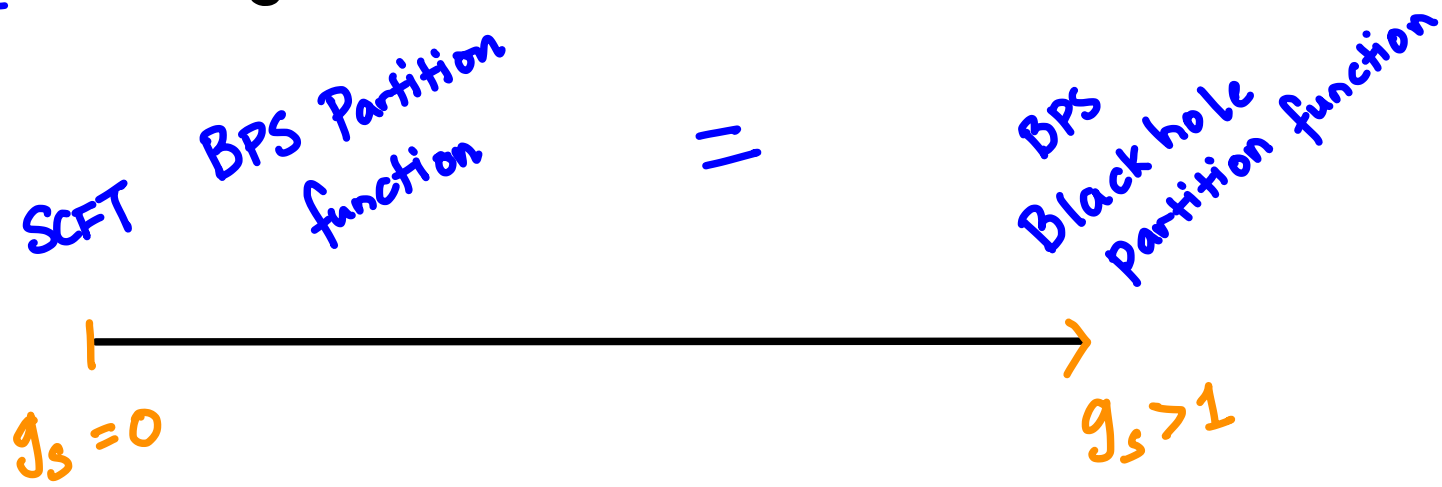
↓

enumerative geometry

and number theory.

BPS black holes.

BPS Strings and branes — Spectrum Indendent of g_s



Q. What are these BPS partition functions?

Idea: Compute individually in SCFT and BH description and see if they agree

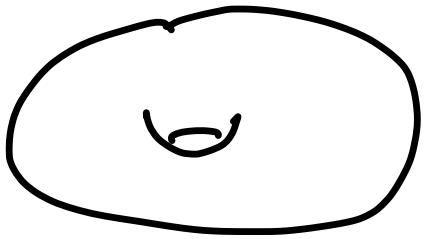
Example. $\frac{1}{4}$ BPS states in $N=4$ string theory in 5d.

Area law of BH = leading growth in SCFT.

[Strominger, Vafa]

SCFT's

→ Partition functions are 1-loop calculations (*)



reparametrization of torus

should not affect the PF

Such functions that are in/co-variant with reparametrization of torus => Modular forms, Jacobi forms

(* Siegel modular forms)

MF's | JF's | SMF's are highly symmetric functions that are periodic. → Fourier expansion

$$F(\bar{q}) = \sum_{\Delta} c(\Delta)\, \bar{q}^{\Delta}$$

Black hole degeneracies !

To compute these pfs EXACTLY.

→ "Modular" symmetry of PF

→ $C(\Delta)$ $\forall$ $\Delta < 0$ [Finitely many]

→ Some other functions known as multiplier systems and Kloosterman sums

Hardy - Ramanujan - Rademacher circle method.

$$C(n>0) = Rad\left[kl, M, c(\Delta<0)\right]$$

complexity of BPS

Smallest input black holes

# of charges	PF	Rad
• Only electric / mag Zero ang mom	$F(\tau) = \sum_{n>-\|N\|} c(n) q^n$ (MF)	✓
• Only elec / mag + ang mom	$F(\tau, z) = \sum_{n,L} c(n,L) q^n y^L$ (JF)	✓
• Dyonic + ang mom	$F(\tau, \sigma, z) = \sum_{n,m,L} c(n,m,L)\, q^n p^m y^L$ (SMF)	~ Subtle

$N = 4$, $N = 8$ ~ Okay $N = 2$ – ?

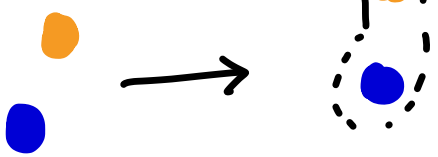
Analogy for subtlety Wall Crossing

→ You think you're computing an invariant But no Sudden jumps.

Basically..

→ If you were a bouncer at a club and you saw this guy, what would you do? This guy is obviously 2 kids in a trenchcoat

Dyonic black holes:



Sneaky guys!

→ Not genuine black holes But the SMF Sort of has counted them already

So you have to go take the effort of removing these guys from the SMF

In doing so, you lose the beautiful symmetry of the SMF

But if you remove these singly charged bound states, you get something beautiful (and fascinating)

$$SMF(q,p,y) = \sum_m JF(q,y)\, p^m$$

$$JF(q,y) = Z^{\text{Dyonic}}(q,y) + Z^{\text{fraudulent}}(q,y)$$

↘ Hol. Mock modular

↘ meromorphic

Types of functions predicted/written down by Ramanujan shortly before he died. Study gained more prominence only in 2003.

- Circle method of MJF's is much more complicated

+ Need to know what argument to feed it?

↓

Only need to know singly charged BH

PF. (Coloured partition of 24 colours)

Dedekind eta. $\eta(\tau)^{-24} = q^{-1} \prod_{n \geq 1} (1-q^n)^{-24}$

- $\frac{1}{\eta(\tau)^{24}} \longrightarrow$ 1-loop correction to K3 prepotential

[Yau-Zaslow]

From the gravity side, there are geometric techniques due to Atiyah, Bott, Duistermaat, Heckman :

(somewhat analogous to Anderson Localization)

generate full PF !

→ Agrees with SCFT for $N = 8$ (modular)

Q : How does gravity see mock-modularity?

Many more mathematical structures to explore.

reach out to me to have a chat about these things!

Thank

you.