M_{T2}-Assisted On-Shell (MAOS) Reconstruction of Missing Momenta and its Application to Spin Measurement at the LHC

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Outline:

1. Motivation

2. M_{T2}-Assisted On-Shell (= MAOS) Reconstruction of Missing Momenta in New Physics Event

   - Some features of MAOS momentum

3. Application to Spin Measurement at the LHC

   - Gluino Spin with $\tilde{g} \to q\bar{q}\chi$
   - Slepton Spin with $q\bar{q} \to Z/\gamma \to \tilde{\ell}\tilde{\ell}^* \to \ell\chi\bar{\ell}\chi$

4. Conclusion

Motivation:

Major motivations for new physics beyond the SM at the TeV scale:

- Hierarchy Problem

$$\delta m_H^2 \sim \frac{g^2}{8\pi^2}\Lambda_{\rm SM}^2 \sim M_Z^2 \quad \Longrightarrow \quad \Lambda_{\rm SM} \sim 1 \text{ TeV}$$

- Dark Matter

Thermal WIMP with $\Omega_{\rm DM} h^2 \sim \frac{0.1}{g^4}\left(\frac{m_{\rm DM}}{1\text{ TeV}}\right)^2 \sim 0.1$

$$\Longrightarrow \quad m_{\rm DM} \sim 1 \text{ TeV}$$

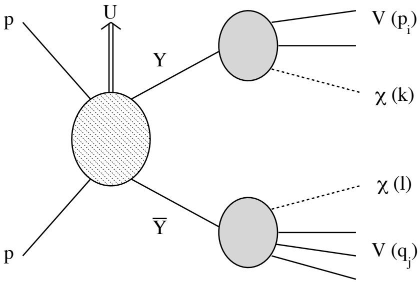
New physics models solving the hierarchy problem while giving a DM candidate typically predict new particles with a conserved discrete charge not carried by the SM particles:

* SUSY with R-parity
* UED with KK-parity
* Little Higgs with T-parity

LHC Signals:

Pair-produced new particle Y decaying into visible SM particles V plus an invisible WIMP χ:

$$pp \;\to\; Y + \bar{Y} \;\to\; \sum V(p_i) + \chi(k) + \sum V(q_j) + \chi(l)$$
$$\big(\text{jets} + \text{leptons} + \not{E}_T\big)$$



To identify the underlying theory, we need to determine the mass, spin and couplings of the produced new particles.

This is in fact a quite difficult job because
(i) each event involves two missing WIMPs,
(ii) the initial parton longitudinal momenta are not available.
$\Rightarrow$ A direct event reconstruction is not possible.

If one has a well defined scheme to assign a 4-momentum to each WIMP in new physics event under a minimal assumption, which can mimic well the true WIMP momentum, it would be greatly useful for experimental determination of new particle properties.

M_{T2}-Assisted On-Shell (MAOS) Reconstruction of Missing Momenta:

A scheme to assign a unique (up to two fold degeneracy) 4-vector (= MAOS momentum) to each WIMP in new physics event:

- MAOS momentum has a well-ordered correlation with the true WIMP momentum, so can be useful for extracting information on the properties of new particle.

- For given trial masses of mother particle Y and WIMP χ, reconstruction is possible for the largest event set.

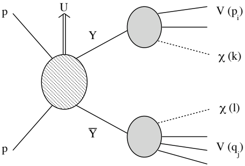
- Available without a good information on the mass and decay process of new particles.

MAOS Reconstruction:

Generic new physics event at the LHC:

$$pp \;\rightarrow\; Y + \bar{Y} \;\rightarrow\; \sum V(p_i) + \chi(k) + \sum V(q_j) + \chi(l)$$

$$\left(P = \sum p_i, \quad Q = \sum q_j \right)$$



Introduce trial masses m_Y and m_χ (true masses: $m_Y^{\rm true}$ and $m_\chi^{\rm true}$) of mother particle and WIMP, and impose the minimal constraints:

$$k^2 = l^2 = m_\chi^2, \quad (k+P)^2 = (l+Q)^2 = m_Y^2, \quad k_T + l_T = \not{E}_T$$

6 constraints for 8 unknowns (k^μ, l^μ), so two-parameter set of solutions which can be parameterized by k_T.

Solution of the Minimal Constraints for generic trial $k_T = \tilde{k}_T$:

$$\begin{aligned} \tilde{l}_T &= \not{E}_T - \tilde{k}_T \\ \tilde{k}_L^{\pm} &= \frac{1}{P^2 + P_T^2}\left(AP_L \pm P_0\sqrt{A^2 - (P^2 + P_T^2)(m_\chi^2 + \tilde{k}_T^2)}\right) \\ \tilde{l}_L^{\pm} &= \frac{1}{Q^2 + Q_T^2}\left(BQ_L \pm Q_0\sqrt{B^2 - (Q^2 + Q_T^2)(m_\chi^2 + \tilde{l}_T^2)}\right) \end{aligned}$$

$$\begin{aligned} P^\mu &= \sum p_i^\mu = (P_0, P_T, P_L) \\ Q^\mu &= \sum q_j^\mu = (Q_0, Q_T, Q_L) \\ A &= \frac{1}{2}(m_Y^2 - m_\chi^2 - P^2) + P_T \cdot \tilde{k}_T \\ B &= \frac{1}{2}(m_Y^2 - m_\chi^2 - Q^2) + Q_T \cdot \tilde{l}_T \end{aligned}$$

$$\left(P^2 \equiv P^\mu P_\mu = P_0^2 - P_T^2 - P_L^2\right)$$

Real solution of the Minimal Constraints exists iff

$$A^2 \geq (P^2 + P_T^2)(m_\chi^2 + \tilde{k}_T^2), \quad B^2 \geq (Q^2 + Q_T^2)(m_\chi^2 + \tilde{l}_T^2)$$

$$\Longleftrightarrow \quad m_Y \geq \max\Big(M_T(P, \tilde{k}_T, m_\chi), M_T(Q, \tilde{l}_T, m_\chi)\Big)$$

$$\begin{aligned} & M_T(P, \tilde{k}_T, m_\chi) \\ = & \left[P^2 + m_\chi^2 + 2\left(\sqrt{P^2 + P_T^2}\sqrt{m_\chi^2 + \tilde{k}_T^2} - P_T \cdot \tilde{k}_T\right)\right]^{1/2} \\ \equiv & \text{ Transverse mass of } Y \to V(P) + \chi(\tilde{k}) \end{aligned}$$

Best choice of $\tilde{k}_T$ giving real $\tilde{k}_L$, $\tilde{l}_L$ for the largest event set:

$$\begin{aligned} & \max\Big(M_T(P, \tilde{k}_T, m_\chi), M_T(Q, \tilde{l}_T, m_\chi)\Big) \\ = & \min_{k_T + l_T = \not{E}_T}\left[\max\Big(M_T(P, k_T, m_\chi), M_T(Q, l_T, m_\chi)\Big)\right] \\ \equiv & \; M_{T2}(P, Q, \not{E}_T; m_\chi) \quad \text{(Lester and Summers)} \end{aligned}$$

MAOS momentum of missing WIMP:

- Transverse component:

$$
\begin{aligned}
& \max\Big(M_T(P,\tilde{k}_T,m_\chi), M_T(Q,\tilde{l}_T,m_\chi)\Big) \\
= & \min_{k_T+l_T=\not{E}_T}\left[\max\Big(M_T(P,k_T,m_\chi), M_T(Q,l_T,m_\chi)\Big)\right] \\
\equiv & M_{T2}(P,Q,\not{E}_T;m_\chi)
\end{aligned}
$$

$$
\Longrightarrow \quad \tilde{k}_T = \tilde{k}_T(P,Q,\not{E}_T;m_\chi).
$$

For each event, there exists a unique transverse WIMP momentum $\tilde{k}_T(P,Q,\not{E}_T;m_\chi)$ giving the M_{T2} solution.

- Longitudinal component:

$$
\tilde{k}^2 = m_\chi^2, \quad (P+\tilde{k})^2 = m_Y^2 \qquad (1)
$$

$$
\Longrightarrow \quad \tilde{k}_L = \tilde{k}_L^{\pm}(P,Q,\not{E}_T;m_\chi,m_Y)
$$

Some Features of MAOS momentum:

- Real MAOS momentum exists for all events if

$$m_Y \geq \max_{\{\text{events}\}} \left[M_{T2}(P, Q, \not{E}_T; m_\chi) \right] \equiv M_{T2}^{\max}(m_\chi)$$

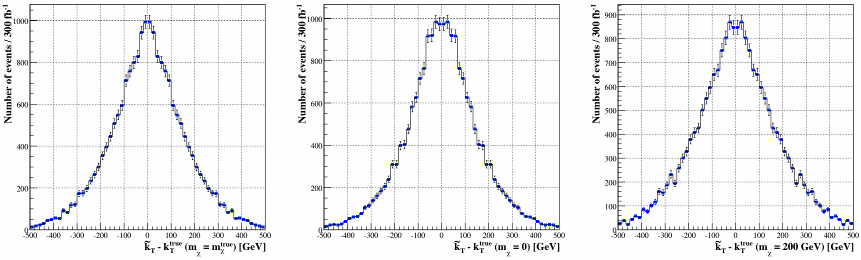
$$\left(m_Y^{\text{true}} = \max_{\{\text{events}\}} \left[M_{T2}(P, Q, \not{E}_T; m_\chi^{\text{true}}) \right] \right)$$

- $\tilde{k} = k^{\text{true}}$ for the M_{T2} endpoint events with $M_{T2} = M_{T2}^{\max}$, when $m_Y = m_Y^{\text{true}}$ and $m_\chi = m_\chi^{\text{true}}$.

- Even for generic events and generic trial masses (m_Y, m_χ), the MAOS momentum is well correlated to the true WIMP momentum.

- Regardless of m_χ, transverse MAOS momentum is distributed near the true momentum.

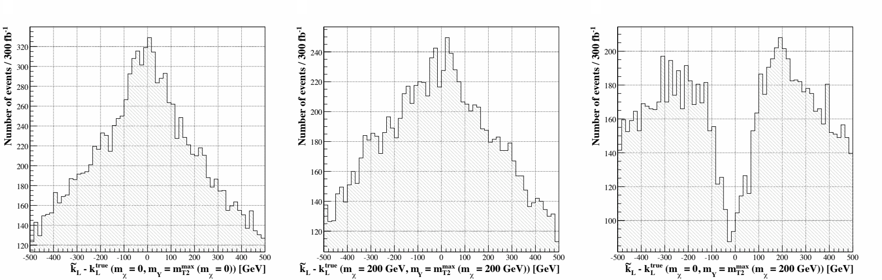
$\tilde{k}_T - k_T^{\rm true}$ distribution for $\tilde{g} \to q\bar{q}\chi$ with $m_\chi = 0,\ 122,\ 200$ GeV
(SPS2: $m_\chi^{\rm true} = 122$ GeV, $m_{\tilde{g}}^{\rm true} = 780$ GeV, $m_{\tilde{q}}^{\rm true} = 1500$ GeV)



- Longitudinal MAOS momentum is more chaotic, but still is distributed around the true momentum for a wide range of (m_Y, m_χ) (including the case with $m_Y = M_{T2}^{\max}(m_\chi)$), if one includes both $\tilde{k}_L^+$ and $\tilde{k}_L^-$ together.

  For certain range of (m_Y, m_χ), the distribution of $\tilde{k}_L - k_L^{\text{true}}$ is in bad shape, but this does not severely hurt the usefulness of the MAOS momentum.

Distribution of $\tilde{k}_L - k_L^{\text{true}}$ for (i) $m_\chi = 0,\ m_Y = M_{T2}^{\max}(0)$, (ii) $m_\chi = 200,\ m_Y = M_{T2}^{\max}(m_\chi)$, (iii) $m_\chi = 0,\ m_Y = 900$ GeV.



- MAOS momentum mimics well the angular correlation of the true WIMP momentum.

Example: $Y + \bar{Y} \to V(p_1) + V(p_2) + \chi(k) + \sum_{j=1}^{j=n} V(q_j) + \chi(l)$

$$s = (p_1 + p_2)^2, \quad t = (p_1 + k^{\rm true})^2, \quad \tilde{t}_\pm = (p_1 + \tilde{k}^\pm)^2$$

$$\left(\tilde{k}^\pm = \tilde{k}^\pm(P, Q; m_\chi, m_Y), \; P = \sum p_i, \; Q = \sum q_j, \; \not{E}_T = -(P_T + Q_T) \right)$$

* Phase space distribution over (s,t) and $(s,\tilde{t})$:

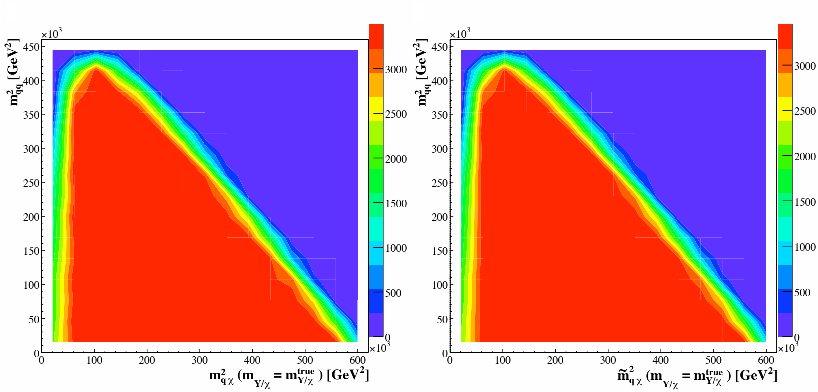
$$\int_{\Sigma(s,t)} \frac{d\Phi_{PS}^{(3)} d\Phi_{PS}^{(n+1)}}{ds\,dt} = \text{constant } C \qquad \left(d\Phi_{PS}^{(N)} = \delta^4(P_{IF}) \prod_a \frac{d^3 p_a}{E_a} \right)$$

$$\int_{\Sigma(s,\tilde{t})} \frac{d\Phi_{PS}^{(3)} d\Phi_{PS}^{(n+1)}}{ds\,d\tilde{t}_\pm} = C \frac{t_{\max}(s) - t_{\min}(s)}{\tilde{t}_{\max}(s) - \tilde{t}_{\min}(s)} = \tilde{t}_\pm\text{-independent}$$

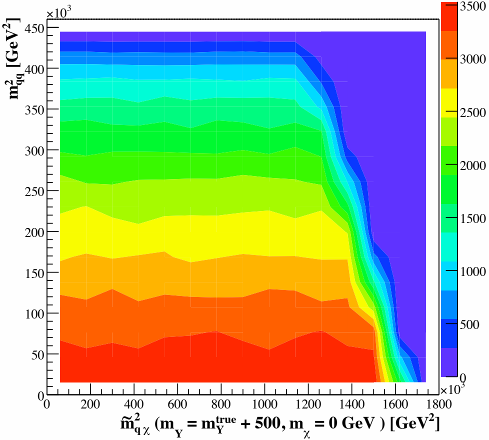
$\Sigma(x,y)$ = hypersurface with fixed x, y in the final state phase space,

$$t_{\max,\min}(s) = \mathcal{F}_{1,2}(m_Y^{\rm true}, m_\chi^{\rm true}; s), \quad \tilde{t}_{\max,\min} = \mathcal{F}_{1,2}(m_Y, m_\chi; s)$$

Phase space distribution over (s, t) and $(s, \tilde{t}_\pm)$ for $m_\chi = m_\chi^{\rm true}$, $m_Y = m_Y^{\rm true}$.



Phase space distribution over $(s, \tilde{t}_{\pm})$ for $m_\chi = 0,\ m_Y = 1280$ GeV.
$(\ m_\chi^{\rm true} = 122$ GeV, $\ m_Y^{\rm true} = 780$ GeV $)$



$\tilde{t}_\pm = (p_1 + \tilde{k}^\pm)^2$ is a good event variable to mimic $t = (p_1 + k^{\text{true}})^2$ $=$ the invariant mass encoding the angular correlation between p_1 and k^{true}, thereby can provide information on the matrix element $|\mathcal{M}_{\text{TH}}|^2$ through the shape of its distribution:

$$\frac{dN_{\text{event}}}{ds\,d\tilde{t}_\pm} = \int_\Sigma |\mathcal{M}_{\text{TH}}|^2 \frac{d\Phi^{(3)}_{PS}\, d\Phi^{(n+1)}_{PS}}{ds\,d\tilde{t}\pm}$$

$$\Big(\Sigma = \text{hypersurface with fixed } (s, \tilde{t}_\pm) \Big)$$

Application to spin measurement:

MAOS reconstruction can provide various template distributions from which one can extract information on new particle properties, particularly the spin and possibly the mass also.

*** Spin measurement at the LHC:**

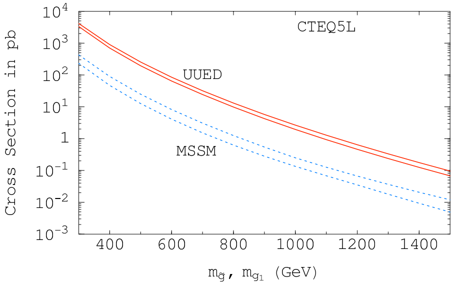
Barr; Datta, Kane, Toharia; Wang, Yavin; Csaki, Heinonen, Perelstein; Burns, Kong, Matchev, Park; Buckley, Murayama, Klemm, Rentala; ...

- Spin dependence of the production cross section
- Angular correlation encoded in the invariant mass distributions
- Pair-production (s-channel) angular distribution
- ...

- **Production cross section:**

Simple counting of the spin degrees of freedom in the production process: $pp \longrightarrow Y + \bar{Y}$

Example:
Gluino vs KK-gluon production rate (Datta, Kane, Toharia):



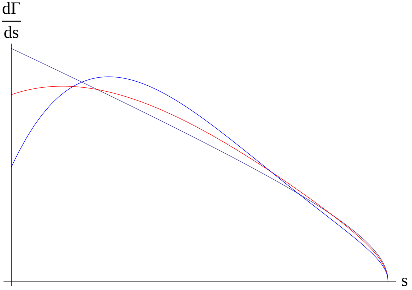
Can work if one has a good information on the mass and gauge charge of the produced new particle.

- **Angular correlation:**

Examine the shape of the invariant mass distribution dN_{event}/ds for the (cascade) decay process $Y \to \sum V(p_i) + \chi(k)$, where s is an appropriate invariant mass combination of visible particles.

Example:
3-body decay of gluino or KK-gluon for SUSY at SPS2a point and its UED equivalent (Csaki, Heinonen, Perelstein):

$$\frac{dN_{\text{event}}}{ds} \text{ of } Y \to q(p_1) + \bar{q}(p_2) + \chi(k) \quad \left(s = (p_1 + p_2)^2\right)$$

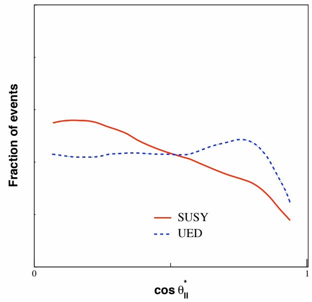


Production angular distribution:

Example:
Drell-Yan pair production of slepton or KK-lepton for SUSY at SPS1a point and its UED equivalent (Barr):

$$\frac{dN_{\text{event}}}{d\cos\theta_{\ell\bar{\ell}}} \quad \text{of} \quad q\bar{q} \to Z^0/\gamma \to Y\bar{Y} \to \ell\chi\bar{\ell}\chi$$

$$\left(\cos\theta_{\ell\bar{\ell}} = \hat{p}_\ell \cdot \hat{p}_{\text{beam}} \;\; \text{in the frame with} \;\; \eta_\ell = -\eta_{\bar{\ell}} \right)$$



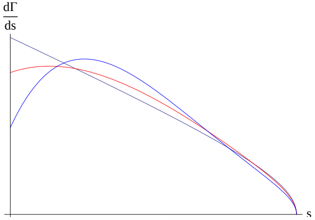
MAOS reconstruction is not relevant for the production rate,

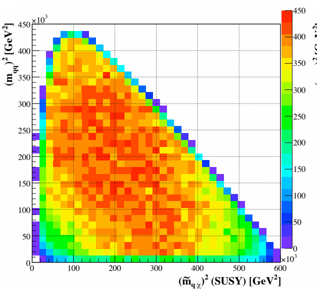
however can greatly enhance the discriminating power

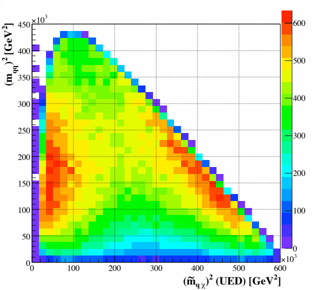
of the angular correlation and the production angular distribution.

- **Gluino and KK-gluon spin with** $Y \rightarrow q(p_1) + \bar{q}(p_2) + \chi(k)$

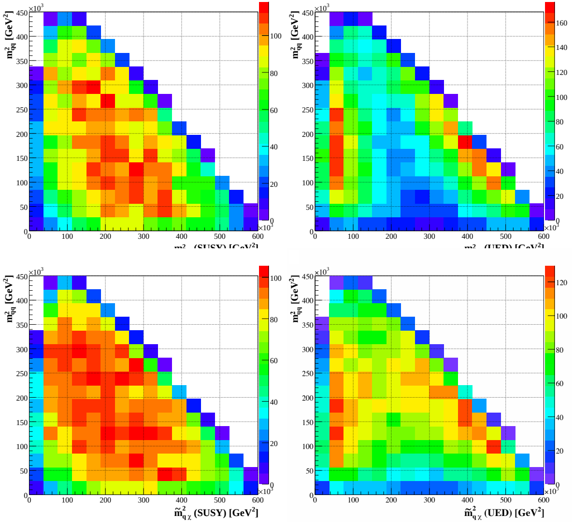
Without MAOS vs With MAOS $\Longrightarrow$ $\dfrac{dN_{\rm event}}{ds}$ vs $\dfrac{dN_{\rm event}}{ds d\tilde{t}}$



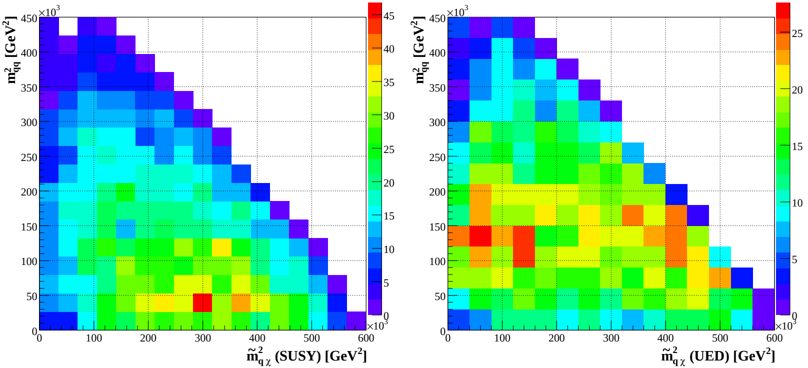




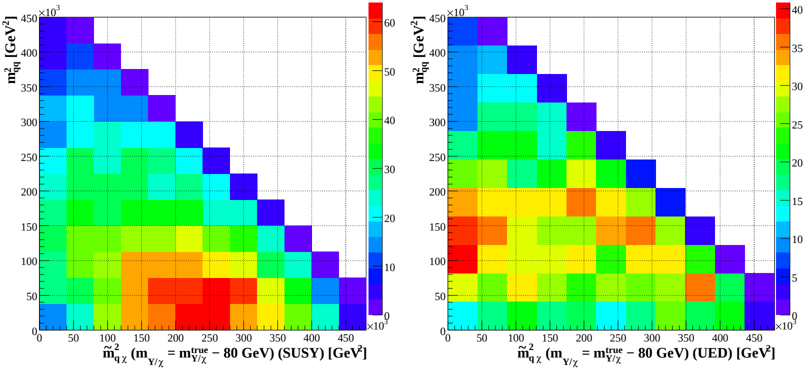
$$\frac{dN_{\rm event}}{ds\,dt} \quad \text{vs} \quad \frac{dN_{\rm event}}{ds\,d\tilde{t}} \quad \text{at parton level for } m_{\chi,Y} = m^{\rm true}_{\chi,Y},\ \mathcal{L} = 300{\rm fb}^{-1}$$



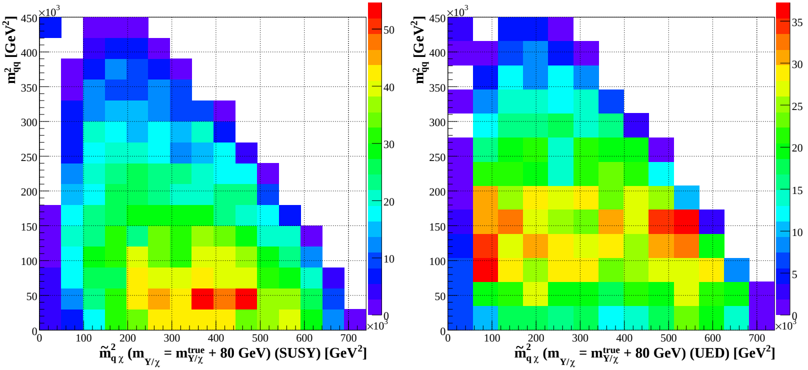
$\frac{dN_{\text{event}}}{ds d\tilde{t}}$ with appropriate event cut while including combinatoric errors (hemi-sphere method) and the detector smearing effects for $m_{\chi,Y} = m^{\text{true}}_{\chi,Y}$ and $\mathcal{L} = 300\text{fb}^{-1}$



$\frac{dN_{\text{event}}}{ds d\tilde{t}}$ with appropriate event cut while including combinatoric errors (hemi-sphere method) and the detector smearing effects for $m_{\chi,Y} = m^{\text{true}}_{\chi,Y} - 80$ GeV and $\mathcal{L} = 300\text{fb}^{-1}$



$\frac{dN_{\text{event}}}{ds\,d\tilde{t}}$ with appropriate event cut while including combinatoric errors (hemi-sphere method) and the detector smearing effects for $m_{\chi,Y} = m^{\text{true}}_{\chi,Y} + 80$ GeV and $\mathcal{L} = 300\text{fb}^{-1}$



- **Slepton** and **KK-lepton** spin with

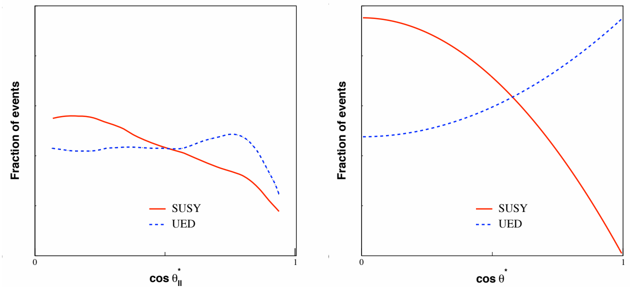
$$q\bar{q} \rightarrow Z^0/\gamma \rightarrow Y\bar{Y} \rightarrow \ell\chi\bar{\ell}\chi$$

Without MAOS vs With MAOS $\Longrightarrow \quad \dfrac{dN_{\rm event}}{d\cos\theta_{\ell\bar{\ell}}}$ vs $\dfrac{dN_{\rm event}}{d\cos\theta^*}$

$\cos\theta_{\ell\bar{\ell}} = \hat{p}_\ell \cdot \hat{p}_{\rm beam}$ in the frame with $\eta_\ell = -\eta_{\bar{\ell}}$

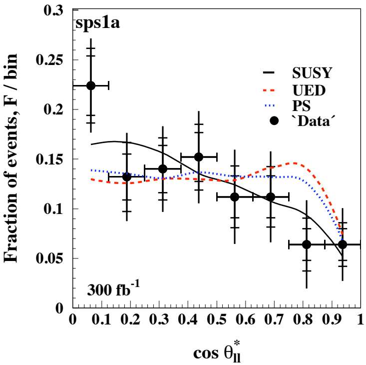
$\cos\theta^*_{\pm\pm} = \hat{p}_{\tilde{\ell}} \cdot \hat{p}_{\rm beam}$ in the CM frame of $q\bar{q}$ for $\tilde{k}^\pm, \tilde{\ell}^\pm$

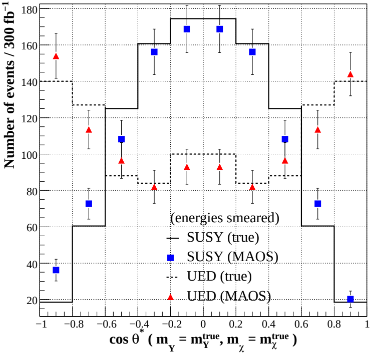
$$\frac{dN_{\rm event}}{d\cos\theta^*} \equiv \sum_\alpha \sum_\beta \frac{dN_{\rm event}}{d\cos\theta^*_{\alpha\beta}}$$



$$\frac{dN_{\text{event}}}{d\cos\theta_{\ell\bar{\ell}}}(\text{Barr}) \quad \text{vs} \quad \frac{dN_{\text{event}}}{d\cos\theta^*}(\text{Cho,KC,Kim,Park})$$

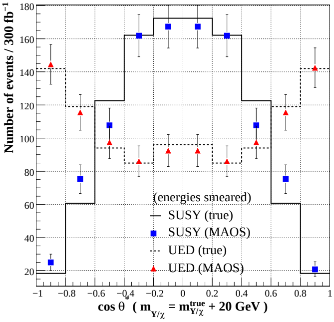
with appropriate event cut (involving the use of M_{T2}) while including the detector smearing effect: $m_{\chi,Y} = m_{\chi,Y}^{\text{true}}$ for SUSY SPS1a and its UED equivalent

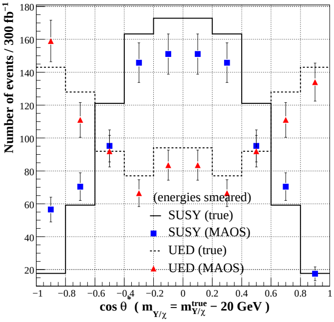




$$\frac{dN_{\text{event}}}{d\cos\theta^*}$$

with appropriate event cut (involving the use of M_{T2}) while including the detector smearing effect: $m_{\chi,Y} = m_{\chi,Y}^{\text{true}} + 20$ GeV and $m_{\chi,Y} = m_{\chi,Y}^{\text{true}} - 20$ for SUSY SPS1a and its UED equivalent





Conclusion

- MAOS reconstruction is a scheme to assign a 4-momentum (MAOS momentum) to each WIMP produced in new physics event under a minimal assumption.
- MAOS momentum mimics well the true WIMP momentum, and thus can provide various template distributions from which one can extract information on the properties of new particle, particularly the spin and possibly the mass also.
- MAOS reconstruction is available without a good information on the mass and decay process of new particles.