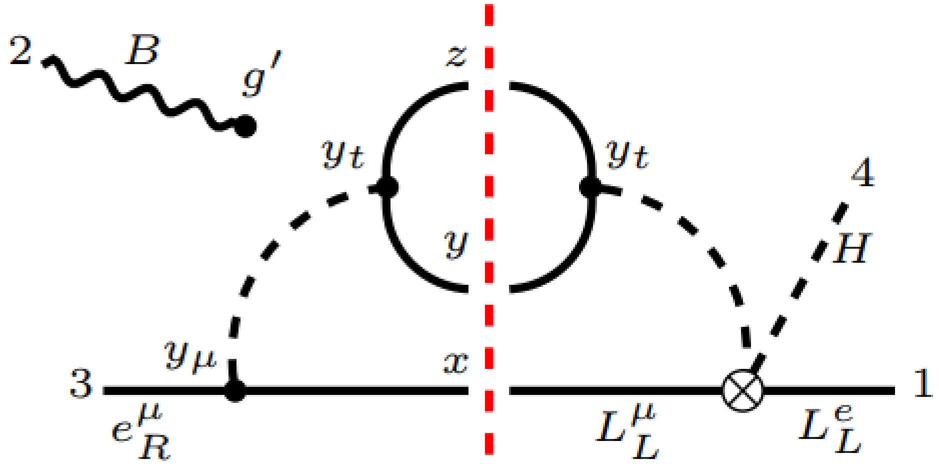


Gearing up for the next generation of LFV experiments, via on-shell methods

Mehmet Asım Gümüş, IPMU, 06.07.2022

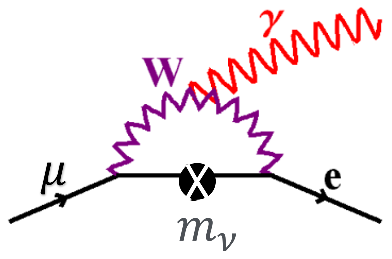
arXiv:2112.12131 (with J. Elias-Miró, Clara Fernandez, Alex Pomarol)

Why LFV observables?

- LF is an accidental symmetry in SM.

- They are “clean“ observables.

  ν oscillations + small m_ν $\Rightarrow$ (almost) no LFV through the SM.



$$\mathrm{Br}(\mu \to e\gamma) \approx 10^{-54}$$

Petcov 1977

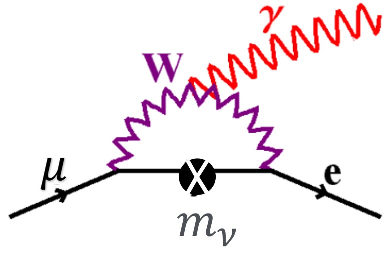
- Many BSMs suggests $\not{L}_i$ without necessarily $\not{L}$.

- Try to understand from IR physics: SM EFT
  - For a heavy new physics sector at $\Lambda \gg E$ breaking L_i but not L, we will focus on dim-6 operators and renormalization group running effects above electroweak breaking scale $E \gg m_W$.
  - The RG analysis between m_W and m_μ can be found in Crivellin et al. 2017 1702.03020

Why LFV observables?

- Standard Model as an Effective Field Theory of a bigger UV complete theory.

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \underbrace{\frac{c_5}{\Lambda^2}(\nu H)(\nu H)}_{\text{Weinberg op.}} + \frac{c_6^{(i)} O_6^{(i)}}{\Lambda^2} + \frac{c_7^{(i)} O_7^{(i)}}{\Lambda^3} + \dots$$



$\mathrm{Br}(\mu \to e\gamma) \approx 10^{-54}$

Petcov 1977

- An expansion of operators of increasing mass dimension $d \geq 4$.
- Heavy new physics → Small corrections to SM at low energies.

Which LFV observables?

We will focus on three processes.

- $\mu^- \to e^- \gamma$
- $\mu^- \to e^- e^+ e^+$
- $\mu^- N \to e^- N$

Reaction	Present limit	C.L.	Experiment	Year
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	90%	MEG at PSI	2016
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	90%	SINDRUM	1988
$\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}$ †	$< 6.1 \times 10^{-13}$	90%	SINDRUM II	1998
$\mu^-\mathrm{Pb} \to e^-\mathrm{Pb}$ †	$< 4.6 \times 10^{-11}$	90%	SINDRUM II	1996
$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$ †	$< 7.0 \times 10^{-13}$	90%	SINDRUM II	2006
$\mu^-\mathrm{Ti} \to e^+\mathrm{Ca}^*$ †	$< 3.6 \times 10^{-11}$	90%	SINDRUM II	1998
$\mu^+e^- \to \mu^-e^+$	$< 8.3 \times 10^{-11}$	90%	SINDRUM	1999
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	90%	BaBar	2010
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	90%	BaBar	2010
$\tau \to eee$	$< 2.7 \times 10^{-8}$	90%	Belle	2010
$\tau \to \mu\mu\mu$	$< 2.1 \times 10^{-8}$	90%	Belle	2010
$\tau \to \pi^0 e$	$< 8.0 \times 10^{-8}$	90%	Belle	2007
$\tau \to \pi^0 \mu$	$< 1.1 \times 10^{-7}$	90%	BaBar	2007
$\tau \to \rho^0 e$	$< 1.8 \times 10^{-8}$	90%	Belle	2011
$\tau \to \rho^0 \mu$	$< 1.2 \times 10^{-8}$	90%	Belle	2011
$\pi^0 \to \mu e$	$< 3.6 \times 10^{-10}$	90%	KTeV	2008
$K_L^0 \to \mu e$	$< 4.7 \times 10^{-12}$	90%	BNL E871	1998
$K_L^0 \to \pi^0\mu^+e^-$	$< 7.6 \times 10^{-11}$	90%	KTeV	2008
$K^+ \to \pi^+\mu^+e^-$	$< 1.3 \times 10^{-11}$	90%	BNL E865	2005
$J/\psi \to \mu e$	$< 1.5 \times 10^{-7}$	90%	BESIII	2013
$J/\psi \to \tau e$	$< 8.3 \times 10^{-6}$	90%	BESII	2004
$J/\psi \to \tau\mu$	$< 2.0 \times 10^{-6}$	90%	BESII	2004
$B^0 \to \mu e$	$< 2.8 \times 10^{-9}$	90%	LHCb	2013
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$B \to K^*\mu e$ ‡	$< 5.1 \times 10^{-7}$	90%	BaBar	2006
$B^+ \to K^+\tau\mu$	$< 4.8 \times 10^{-5}$	90%	BaBar	2012
$B^+ \to K^+\tau e$	$< 3.0 \times 10^{-5}$	90%	BaBar	2012
$B_s^0 \to \mu e$	$< 1.1 \times 10^{-8}$	90%	LHCb	2013
$\Upsilon(1s) \to \tau\mu$	$< 6.0 \times 10^{-6}$	95%	CLEO	2008
$Z \to \mu e$	$< 7.5 \times 10^{-7}$	95%	LHC ATLAS	2014
$Z \to \tau e$	$< 9.8 \times 10^{-6}$	95%	LEP OPAL	1995
$Z \to \tau\mu$	$< 1.2 \times 10^{-5}$	95%	LEP DELPHI	1997
$h \to e\mu$	$< 3.5 \times 10^{-4}$	95%	LHC CMS	2016
$h \to \tau\mu$	$< 2.5 \times 10^{-3}$	95%	LHC CMS	2017
$h \to \tau e$	$< 6.1 \times 10^{-3}$	95%	LHC CMS	2017

Calibbi, Signorelli 2018, 1709.00294

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$\mu^+e^- \to$ [illegible]	[illegible]	[illegible]	SINDRUM	1999
$\tau \to e\gamma$	[illegible]	[illegible]	BaBar	2010
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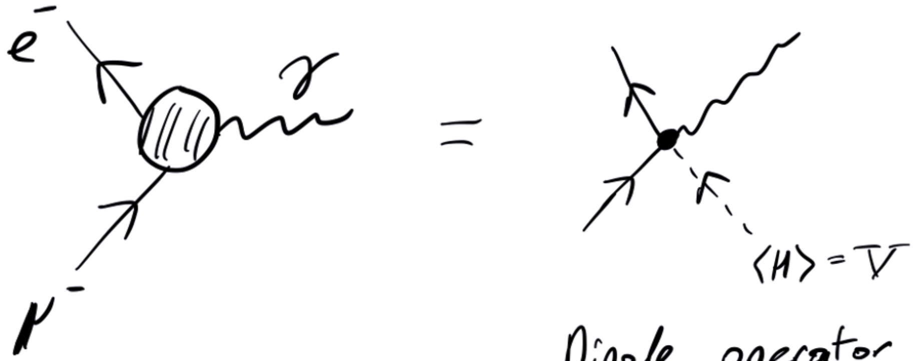
most sensitive ones

Which LFV observables?

- Tree-level analysis within SM EFT.

$$Br(\mu^- \to e^- \gamma) \propto \left| c^{(6)}_{\text{Dipole}} \right|^2$$



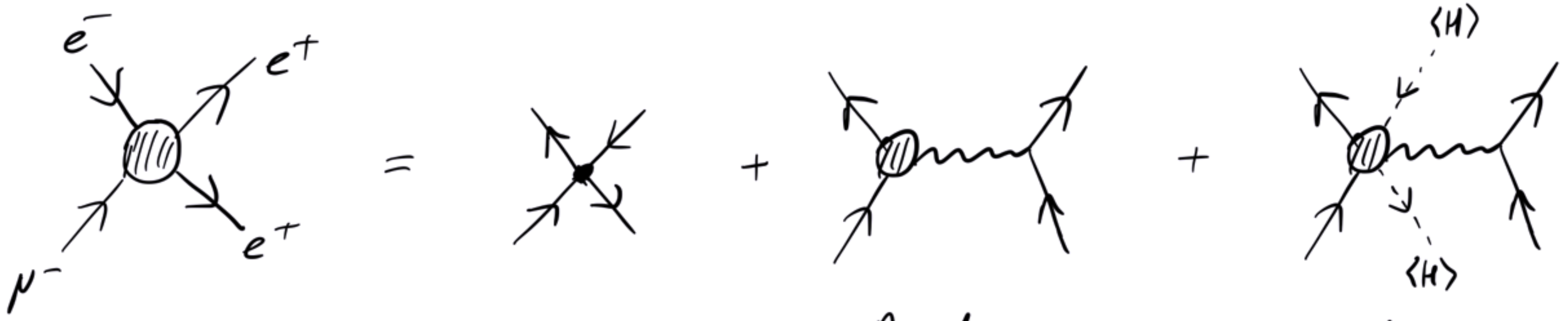
Dipole operator

$$\frac{c^{(6)}}{\Lambda^2} \cdot e y_\mu \cdot \bar{\Psi} \sigma^{\mu\nu} \Psi F_{\mu\nu} H$$

Which LFV observables?

- Tree-level analysis within SM EFT.

$$Br(\mu^- \to e^- e^+ e^+) \propto \left(C^{(6)}_{dipole} \,,\; C^{(6)}_{4\text{-}fermion} \,,\; C^{(6)}_{current^2} \right)^2$$

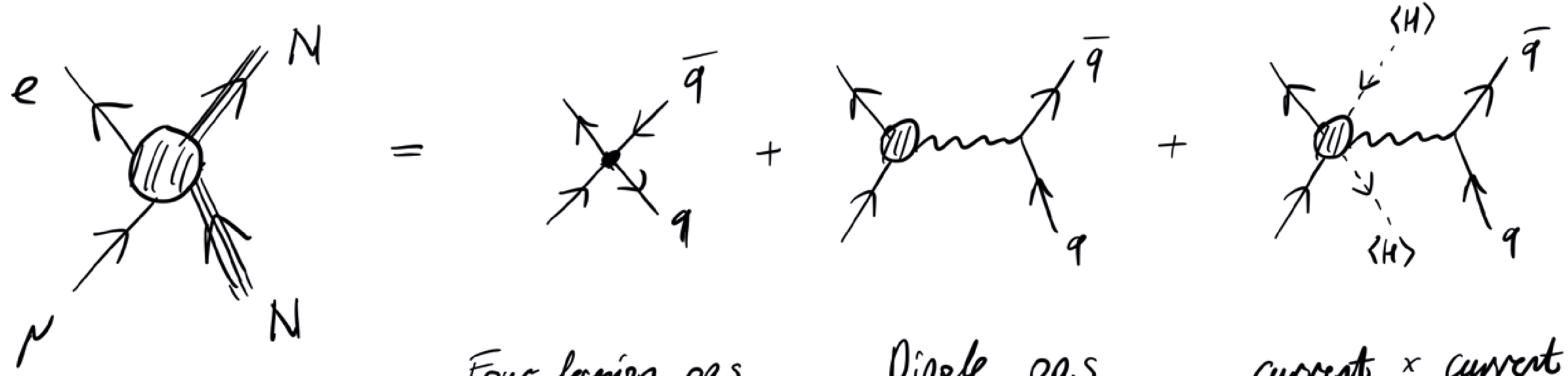


Four fermion op.s	Dipole op.s	current × current op.s
$(\bar{\mu}\gamma e)(\bar{e}\gamma e)$	$(\bar{\mu}\sigma^{\mu\nu}e)F_{\mu\nu}H$	$(\bar{\mu}\gamma e)(H^\dagger D H)$
$(\bar{\mu}e)(ee)$		

Which LFV observables?

- Tree-level analysis within SM EFT.

$$Br(\mu^- N \to e^- N) \propto \left(C^{(6)}_{\text{dipole}} \,,\; C^{(6)}_{\text{4-fermion}} \,,\; C^{(6)}_{\text{current}^2} \right)^2$$



Four fermion op.s | Dipole op.s | current × current op.s

$(\bar{\mu}\gamma e)(\bar{q}\gamma q)$ | $(\bar{\mu}\sigma^{\mu\nu} e) F_{\mu\nu} H$ | $(\bar{\mu}\gamma e)(H^\dagger D H)$

$(\bar{\mu} e)(\bar{q} q)$

Which LFV observables?

- A full list of operators we are interested in.

$\psi^2 FH$		$\overline{\psi}\psi H^2 D$	
$\mathcal{O}^{\mu e}_{DW}$ $\mathcal{O}^{\mu e}_{DB}$	$y_\mu g \overline{L}^{(2)}_L \tau^a \sigma^{\mu\nu} e^{(1)}_R HW^a_{\mu\nu}$ $y_\mu g' \overline{L}^{(2)}_L \sigma^{\mu\nu} e^{(1)}_R HB_{\mu\nu}$	$\mathcal{O}^{\mu e}_{L}$ $\mathcal{O}^{\mu e}_{L3}$ $\mathcal{O}^{\mu e}_{R}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\overline{L}^{(2)}_L \gamma^\mu L^{(1)}_L)$ $(H^\dagger i\overleftrightarrow{D}_\mu \tau^a H)(\overline{L}^{(2)}_L \gamma^\mu \tau^a L^{(1)}_L)$ $(H^\dagger i\overleftrightarrow{D}_\mu H)(\overline{e}^{(2)}_R \gamma^\mu e^{(1)}_R)$
$\overline{\psi}^2 \psi^2$		$\psi^4, \psi^2 H^3$	
$\mathcal{O}^{\mu eff}_{LL}$ $\mathcal{O}^{\mu eff}_{LL3}$ $\mathcal{O}^{\mu eff}_{RR}$ $\mathcal{O}^{\mu eff}_{LR}$	$(\overline{L}^{(2)}_L \gamma^\mu L^{(1)}_L)(\overline{F}_L \gamma_\mu F_L)$ $(\overline{L}^{(2)}_L \gamma^\mu \tau^a L^{(1)}_L)(\overline{F}_L \gamma_\mu \tau^a F_L)$ $(\overline{e}^{(2)}_R \gamma^\mu e^{(1)}_R)(\overline{f}_R \gamma_\mu f_R)$ $(\overline{L}^{(2)}_L \gamma^\mu L^{(1)}_L)(\overline{f}_R \gamma_\mu f_R)$ $\ldots$	$\mathcal{O}^{\mu eqq}_{LuQe}$ $\mathcal{O}^{\mu eqq}_{LeQu}$ $\mathcal{O}^{\mu e}_{y}$	$y_\mu(\overline{L}^{(2)}_L u_R)(\overline{Q}_L e^{(1)}_R)$ $y_\mu(\overline{L}^{(2)}_L e^{(1)}_R)(\overline{Q}_L u_R)$ $y_\mu(H^\dagger H)(\overline{L}^{(2)}_L e^{(1)}_R H)$

Remember the names in the first row. We will need them later.

Also notice:

$$C^{(\mu e)}_{Dipole} = C^{(\mu e)}_{DW} - C^{(\mu e)}_{DB} \quad \text{since}$$

$$F_{\mu\nu} = \sin\theta_W \cdot W^3_{\mu\nu} - \cos\theta_W \cdot B_{\mu\nu}$$

Which LFV observables?

- Great advances in precision limit in future on the experimental side.

	$\mathrm{BR}(\mu \to e\gamma)$	$\mathrm{BR}(\mu \to eee)$	$R(\mu N \to eN)$	$\mathrm{BR}(h \to \mu e)$
Current	$4.2 \cdot 10^{-13}$ [28]	$1 \cdot 10^{-12}$ [29]	$7 \cdot 10^{-13}$ [30]	$6.1 \cdot 10^{-5}$ [31]
Future	$6.0 \cdot 10^{-14}$ [32]	$1 \cdot 10^{-16}$ [33]	$8 \cdot 10^{-17}$ [34]	

$\times 10^1$ $\quad$ $\times 10^4$ improvement!

Which LFV observables?

- Great advances in precision limit in future on the experimental side.

- Dipole observable is extremely sensitive. Let us crack in some numbers:

$$Br(\mu^- \to e^- \gamma) \lesssim 10^{-13} \quad \text{translates to} \quad \frac{|c_{DW} - c_{DB}|^2}{\Lambda^4} \lesssim \frac{1}{10^{12}\,\text{TeV}^4}$$

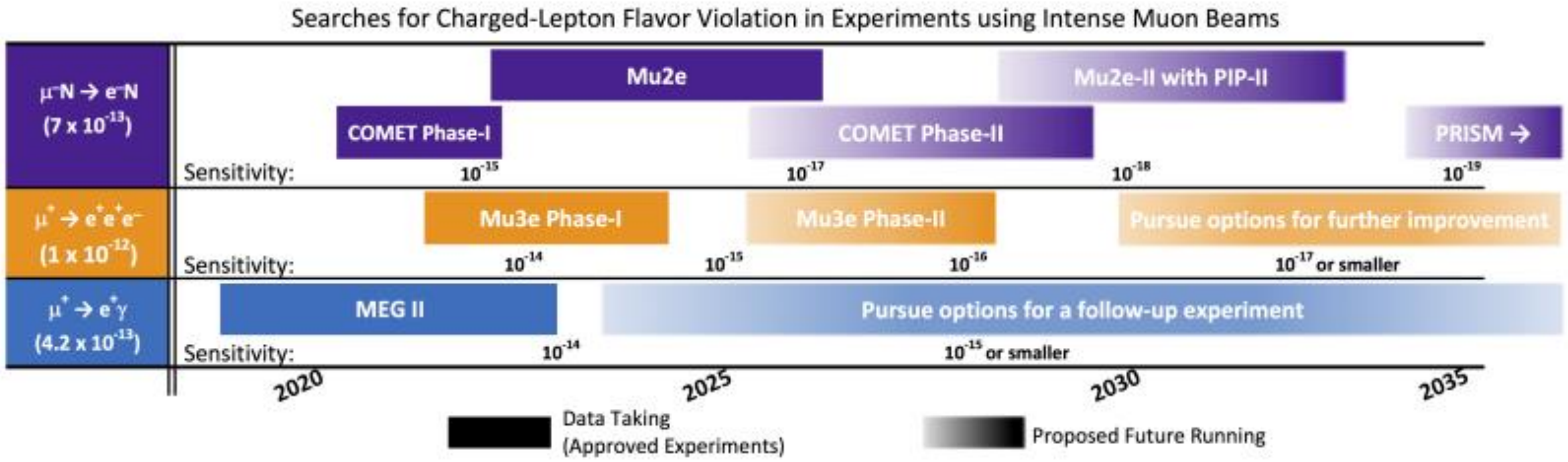
If there is an operator mixing with the dipole during RG flow, even with a two-loop suppression.

i.e.

$$\frac{c_i}{(16\pi^2)^2} \cdot \ln\left(\frac{\Lambda}{m_W}\right) \longrightarrow c_{DW} - c_{DB} \quad \text{would translate to} \quad \frac{|c_i|^2}{\Lambda^4} \lesssim \frac{1}{(6\,\text{TeV})^4}$$

Which LFV observables?

- Great advances in precision limit in future on the experimental side.
- Experimental road map:



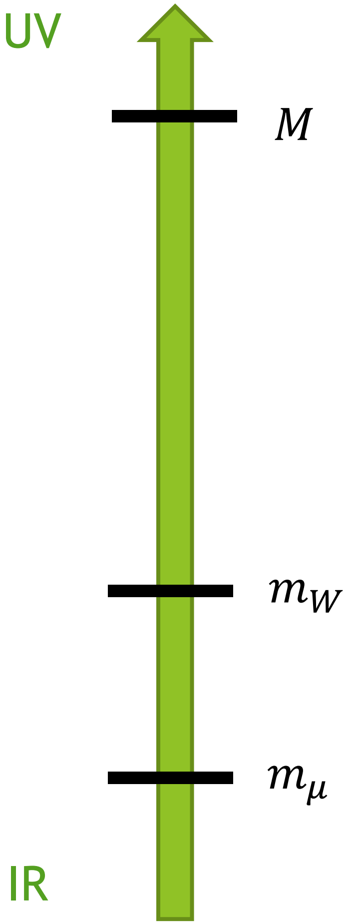
Baldini et al. 2018
From a report for the 2020 update of European Strategy for Particle Physics
1812.06540

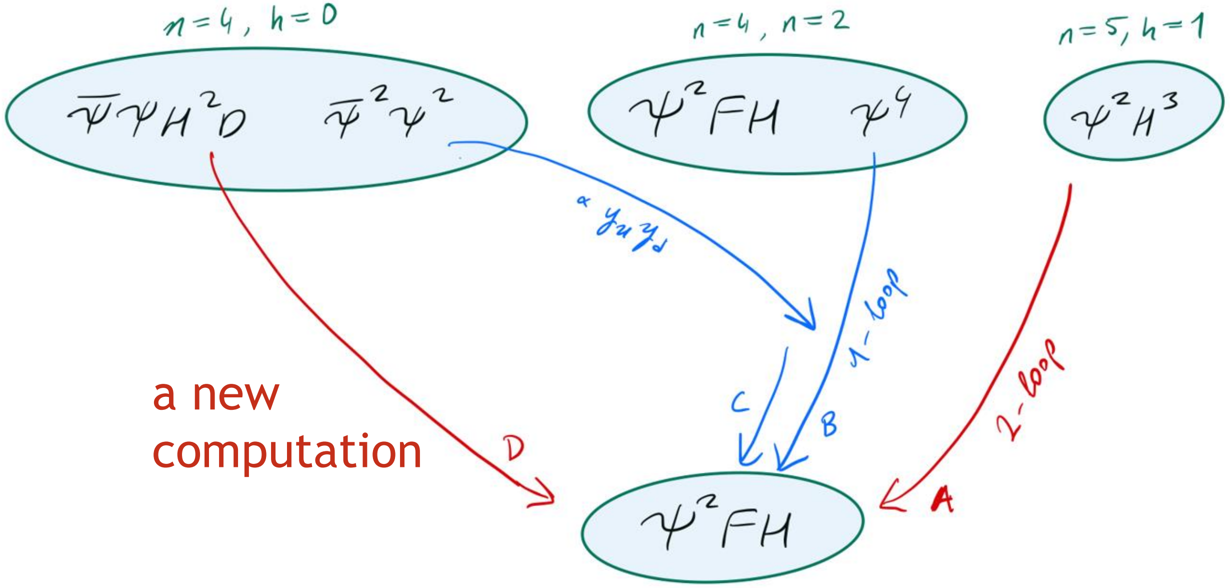
- MEG updates first, then Mu3e follows.

  ⇒ There is a time window to exploit high sensitivity on $\mu \to e\,\gamma$.

  To put direct bounds on c_{DW-DB} and indirect bounds on $c_{L,L3,R}$ through RG mixing.

Running of LFV Operators





power counting

A, D: $\dfrac{c_i}{(16\pi^2)^2} \ln\left[\dfrac{\Lambda}{m_W}\right]$

B: $\dfrac{c_i}{(16\pi^2)} \ln\left[\dfrac{\Lambda}{m_W}\right]$

C: $\dfrac{c_i c_j}{(16\pi^2)^2} \ln^2\left[\dfrac{\Lambda}{m_W}\right]$

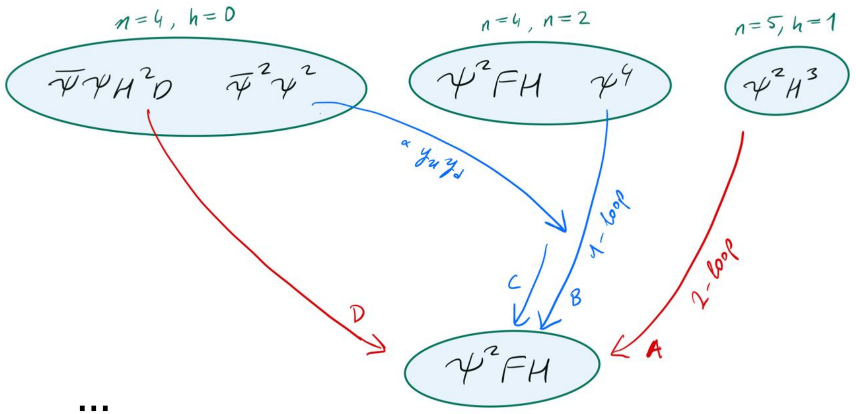
Cheung, Shen 2015, 1505.01844

- Helicity selection rules at work, protecting the dipole.

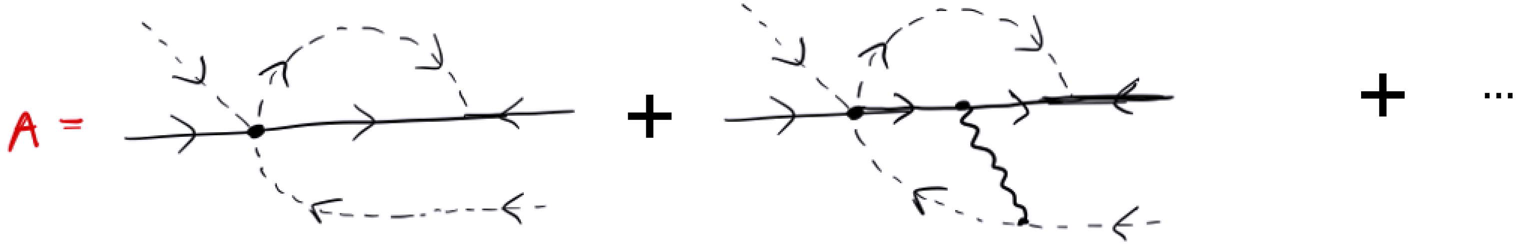
  n: number of fields   h: total helicity $\Delta n \geq \Delta h$

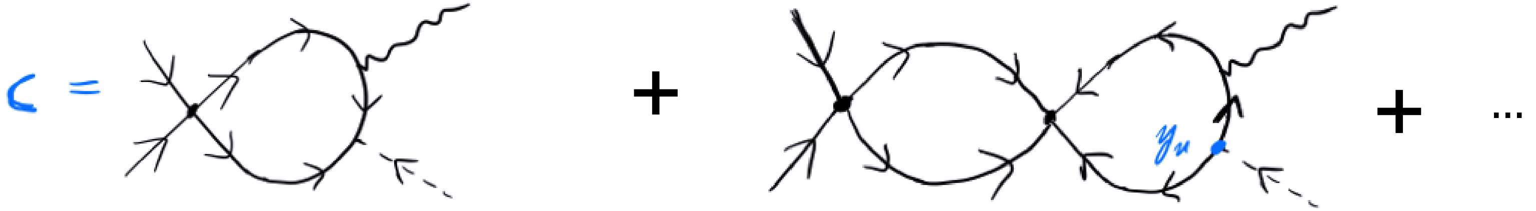
- Notice the leading log. two-loop arrows into dipole.
- One of them (D) wasn't computed previously. It turns out to be straightforward to calculate using an on-shell formula. We will see couple of slides later how.

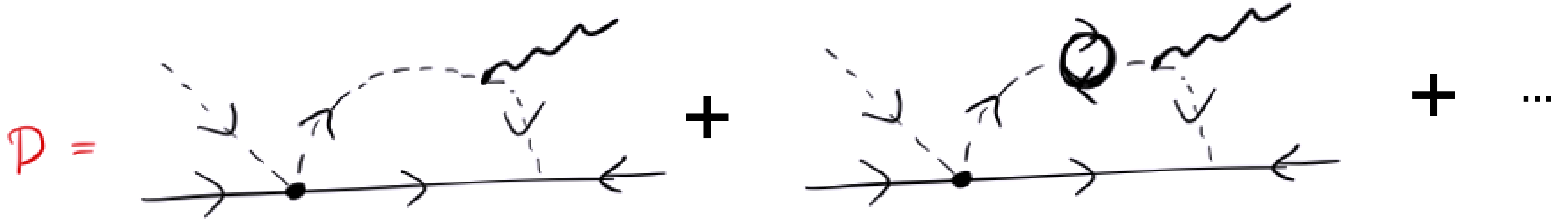
Running of LFV Operators



- Let us approach the same problem diagrammatically.







- One-loop terms has no divergent contributions.
- Could we anticipate them? Is it possible to see such cases without doing a loop computation?

The on-shell way

- The answer is yes! We will need form factors (FF), so let us define it.

$$F_{\mathcal{O}}(1,2,\ldots,n\,|\,q) = \int d^d x \, \langle 1\,2\,3\cdots n|\,\mathcal{O}(x)\,|0\rangle \, e^{iqx}$$

- We need in particular their Fourier transform at zero momentum. In this form, it resembles a scattering amplitude with an operator insertion.

$$F_{\mathcal{O}}(1,2,\ldots,n\,|\,q) \xrightarrow[q\to 0]{} \langle 1\,2\,3\cdots n|\cdot\int d^d x\cdot\mathcal{O}(x)\,|0\rangle \equiv \langle \vec{n}|\,\mathcal{O}\,|0\rangle$$

- The beautiful formula connecting RG equations and on-shell amplitudes:

$$\underbrace{\langle\vec{n}|\,\mathcal{O}_j\,|0\rangle^{(0)}}_{\text{minimal FF}}\,\gamma_{ji}^{(1)} = -\frac{1}{\pi}\sum_{\vec{m}}\left(\overbrace{\langle\vec{n}|\mathcal{M}|\vec{m}\rangle}^{\text{amplitude}}\langle\vec{m}|\,\mathcal{O}_i\,|0\rangle\right)^{(1)}$$

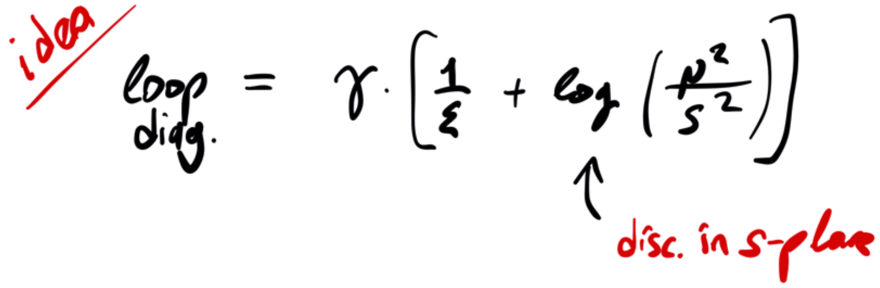
Caron-Huot, Wilhelm 2017
1607.06448

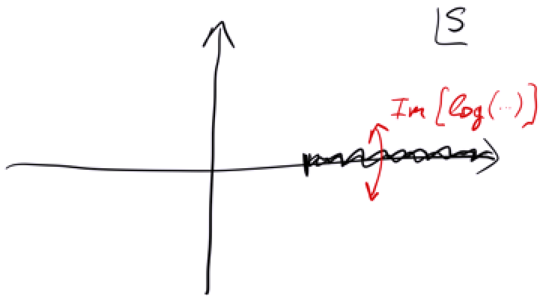
- Sum is over the phase space of particles $\vec{m} = \{1,2,\ldots,m\}$. Minimal FF means all couplings are turned off. Superscript (i) means i-th non-vanishing log. order.
- If time permits, we will go through an alternative derivation given in Elias Miro, Ingoldby, Riembau 2020 2005.06983

The on-shell way

$$\underbrace{\langle \vec{n}|\, \mathcal{O}_j\, |0\rangle^{(0)}}_{\text{minimal FF}}\, \gamma_{ji}^{(1)} = -\frac{1}{\pi}\sum_{\vec{m}}\left(\overbrace{\langle \vec{n}|\mathcal{M}|\vec{m}\rangle}^{\text{amplitude}}\langle \vec{m}|\, \mathcal{O}_i\, |0\rangle\right)^{(1)}$$

- But let us give ourselves quick intuition why it works.
- LHS is a local expression (=a polynomial in momenta) times γ_{ji}.
- At the leading log. order, one can read γ_{ji} from the coefficient in front of $1/\epsilon$ in dimensional regularization. It is also the same coefficient in front of the log.



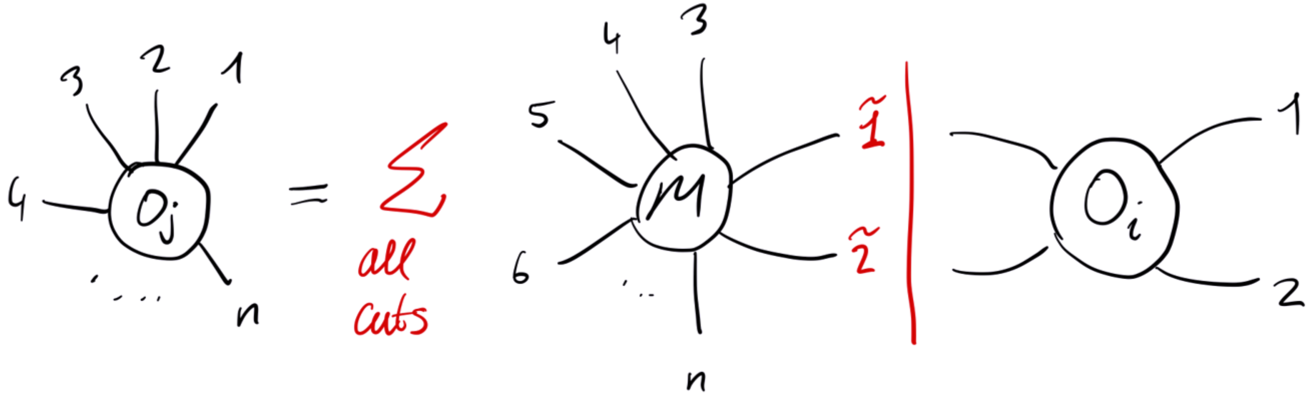


- One can also detect γ_{ji} by finding the discontinuity of FF in complex s-plane.
- It amounts to put some internal propagators on-shell. (Optical Theorem)
- Notice that a complicated RHS should turn out to give a local expression.

The on-shell way

- So we can have graphic way to represent the formula:

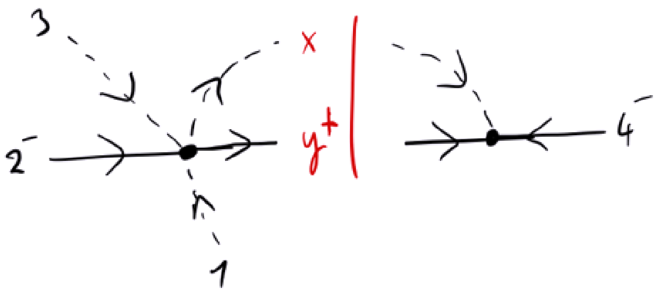
$$\underbrace{\langle \vec{n}|\, \mathcal{O}_j\, |0\rangle^{(0)}}_{\text{minimal FF}}\, \gamma_{ji}^{(1)} = -\frac{1}{\pi}\sum_{\vec{m}}\left(\overbrace{\langle \vec{n}|\mathcal{M}|\vec{m}\rangle}^{\text{amplitude}}\langle \vec{m}|\, \mathcal{O}_i\, |0\rangle\right)^{(1)}$$



$\{1, 2, \ldots, n, \tilde{1}, \tilde{2}\}$ are all on-shell momenta.

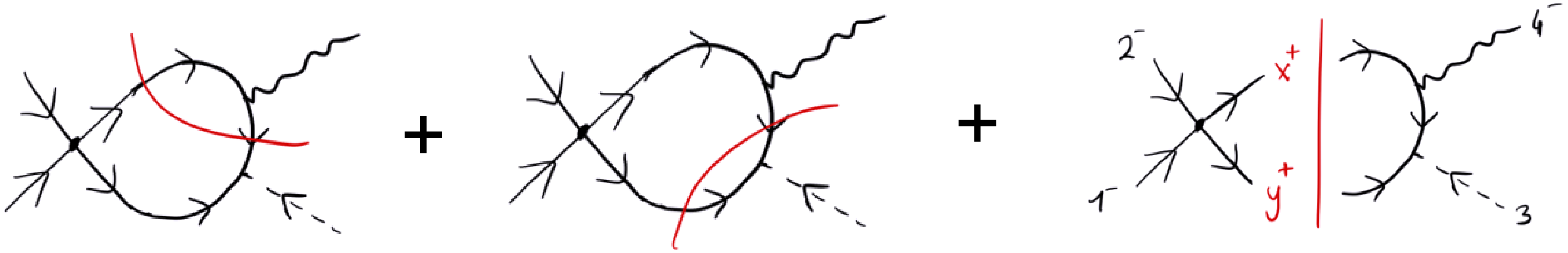
The on-shell way

- Let us see how we can use it, go back to the mixing diagram examples.
- For (A)



$$= -\frac{1}{\pi}\int dLIPS(x,y^+)\,\underbrace{\langle 4^-|x y^+\rangle}_{\text{vanishes kinematically!}}\langle x y^+ 1 2^- 3|\mathcal{O}_y|0\rangle$$

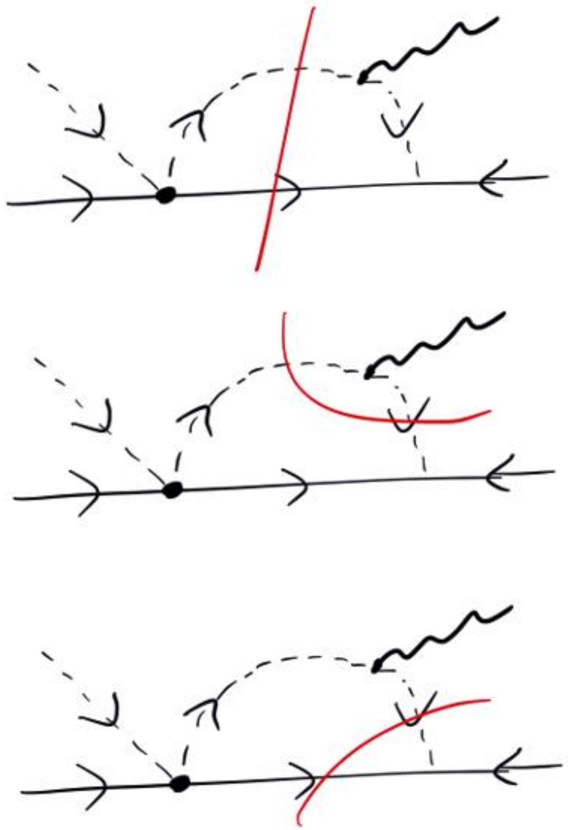
- For (C)



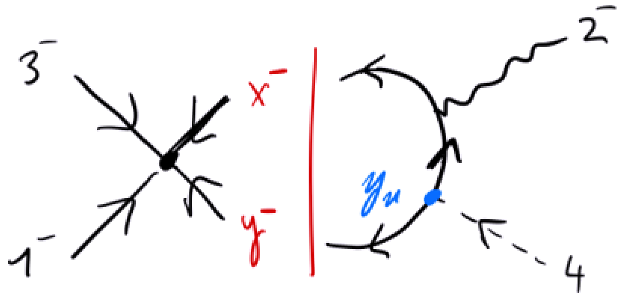
$$= \text{zero} + \text{zero} + \frac{1}{\pi}\int dLIPS(x^+,y^+)\,\underbrace{\langle 3 4^-|x^+ y^+\rangle}_{\text{vanishes in SM}}\langle x^+ y^+ 1^- 2^-|\mathcal{O}_{\varphi^2\bar{\varphi}^2}|0\rangle$$

The on-shell way

- For (D)



- A non-vanishing example finally, take (B).



$$= -\frac{1}{\pi}\int dLIPS(x^-,y^-)\, \mathcal{M}(2^- 4 \leftarrow x^- y^-)\, F_{\psi^4}(x^- y^- 1^- 3^-)$$

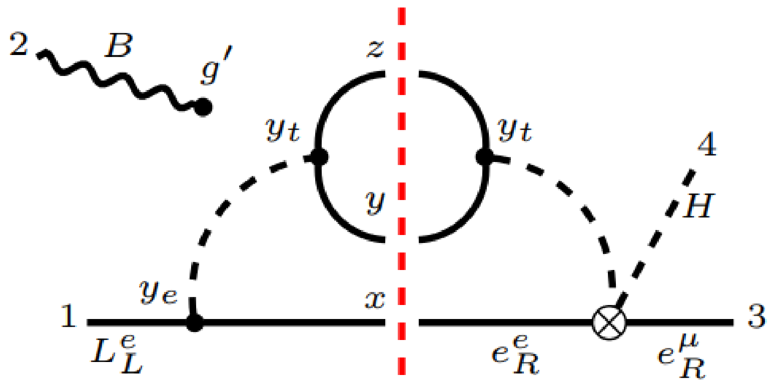
$$= \frac{1}{8\pi^2}\int d\Omega_2 \cdot \sqrt{2}\,(-y_u)\cdot\left(Y_{u_R}\frac{[xy]^2}{[x2][y2]} + Y_H \frac{[xy][4x]}{[42][x2]}\right) \times \langle 1y\rangle\langle x3\rangle$$

$$= 2\sqrt{2}\,\langle 12\rangle\langle 23\rangle \cdot y_u \frac{N_c/2}{(16\pi^2)}\,(2Y_{u_R} - Y_H)$$

$$= F_{F\varphi^2 H}(1^- 2^- 3^- 4)\cdot \gamma_{\psi^4 \to F\varphi^2 H}$$

The on-shell way

- Now we are warmed up to go bigger. Let us look at the new computation as promised.
- Next terms in (D) were two-loop diagrams, providing the leading log. since one-loop was 0 as we saw.
- Top Yukawa contribution:

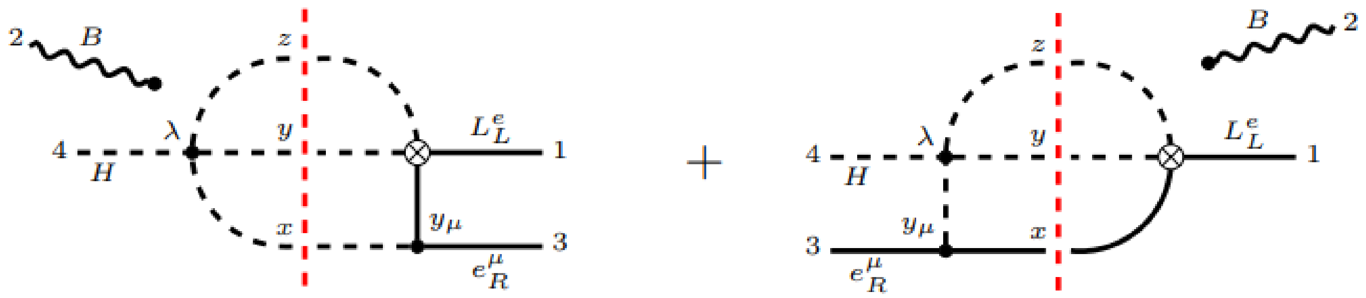


$$= \frac{\langle 12\rangle[12]}{(16\pi^2)^2}\int d\Omega_3\, M(12;xyz)\, F_{\mathcal{O}_i}(xyz34)$$

$$= \underbrace{2\sqrt{2}y_\mu g'\langle 12\rangle\langle 23\rangle\epsilon_{lk}}_{\text{dipole}}\,(Y_{e_R}-Y_H)y_t^2 y_e/y_\mu$$

The on-shell way

- Higgs quartic contribution at two-loop:



$$\text{cut }1 = \frac{12\sqrt{2}\lambda y_\mu g' Y_H}{(16\pi^2)^2}\frac{\langle 12\rangle}{[32]}s_{24}\left[1 + 2\frac{s_{34}}{s_{32}} + 2\frac{s_{34}}{s_{32}}\left(\frac{s_{34}+s_{32}}{s_{32}}\right)\ln\left(\frac{s_{34}}{s_{34}+s_{32}}\right)\right]\epsilon_{lk}$$

$$\text{cut }2 = \frac{12\sqrt{2}\lambda y_\mu g' Y_H}{(16\pi^2)^2}\frac{\langle 12\rangle}{[32]}s_{34}\left[1 - 2\frac{s_{13}}{s_{32}} + 2\frac{s_{13}^2}{s_{32}^2}\ln\left(\frac{s_{13}+s_{32}}{s_{13}}\right)\right]\epsilon_{lk}$$

notice non-local pieces

$$\text{cut }1 + \text{cut }2 = \underbrace{2\sqrt{2}y_\mu g'\langle 12\rangle\langle 23\rangle\epsilon_{lk}}_{\text{dipole}}\frac{6\lambda Y_H}{(16\pi^2)^2}$$

The on-shell way

- Results: The anomalous dimensions matrices, given by

$$(\gamma_{C^{e\mu}_{DB}}, \gamma_{C^{e\mu}_{DW}})^T = \gamma_{D1} \cdot (C^{e\mu}_L, C^{e\mu}_{L3}, C^{e\mu}_R)^T$$

$$\gamma_{D1} = \frac{N_c y_t^2}{(16\pi^2)^2} \begin{pmatrix} 0 & 0 & -3y_e/(2y_\mu) \\ 1 & -1 & y_e/(2y_\mu) \end{pmatrix} + \frac{\lambda}{(16\pi^2)^2} \begin{pmatrix} 3 & 3 & 3y_e/y_\mu \\ 1 & 3 & y_e/y_\mu \end{pmatrix}$$

and

$$(\gamma_{C^{\mu e}_{DB}}, \gamma_{C^{\mu e}_{DW}})^T = \gamma_{D2} \cdot (C^{\mu e}_L, C^{\mu e}_{L3}, C^{\mu e}_R)^T$$

$$\gamma_{D2} = \frac{N_c y_t^2}{(16\pi^2)^2} \begin{pmatrix} 0 & 0 & -3/2 \\ y_e/y_\mu & -y_e/y_\mu & 1/2 \end{pmatrix} + \frac{\lambda}{(16\pi^2)^2} \begin{pmatrix} 3y_e/y_\mu & 3y_e/y_\mu & 3 \\ y_e/y_\mu & 3y_e/y_\mu & 1 \end{pmatrix}$$

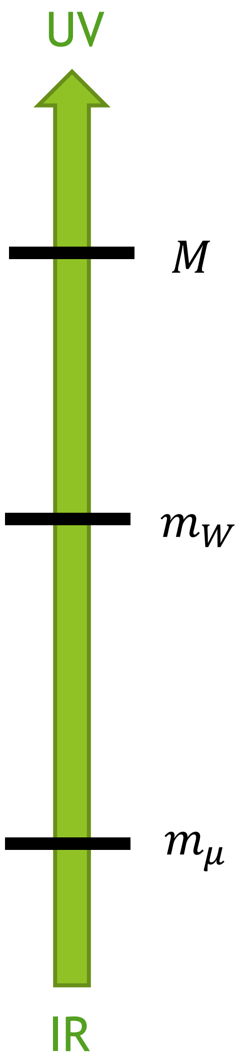
The on-shell way

- Putting in some numbers: We turn on one-by-one the operators that enter into LFV observables, taking into account RG mixing relations up to two-loops.

	$\mu \to e\gamma$	$\mu \to eee$	$\mu N \to eN$	$h \to \mu e$
$C_{DB}^{\mu e} - C_{DW}^{\mu e}$	951 TeV (1547 TeV)	218 TeV (2183 TeV)	208 TeV (1812 TeV)	
$C_{DB}^{\mu e} + C_{DW}^{\mu e}$	127 TeV (214 TeV)	26 TeV (309 TeV)	24 TeV (253 TeV)	
$C_{R}^{\mu e}$	35 TeV (59 TeV)	160 TeV (1602 TeV)	225 TeV (1535 TeV)	
$C_{L}^{\mu e} + C_{L3}^{\mu e}$	4 TeV (7 TeV)	164 TeV (1642 TeV)	225 TeV (1535 TeV)	
$C_{L}^{\mu e} - C_{L3}^{\mu e}$	24 TeV (41 TeV)	35 TeV (421 TeV)	50 TeV (395 TeV)	
$C_{y}^{\mu e}$	4 TeV (6 TeV)	1 TeV (9 TeV)	1 TeV (7 TeV)	0.3 TeV

Present (Future) bounds
Black: tree-level
Blue: one-loop
Red: two-loop

UV Example: Heavy Vector-Like Fermions



- Consider a vector-like fermion with mass $M \gg m_w$.
- Yukawa-like coupled to 1st and 2nd generation leptons.

$$\begin{aligned}
\Delta\mathcal{L}_S &= (y_S^{(1)}\bar{L}_L^{(1)} + y_S^{(2)}\bar{L}_L^{(2)})S_R i\tau_2 H^* + \text{h.c.}\,, \\
\Delta\mathcal{L}_E &= (y_E^{(1)}\bar{L}_L^{(1)} + y_E^{(2)}\bar{L}_L^{(2)})E_R H + \text{h.c.}\,, \\
\Delta\mathcal{L}_D &= (y_D^{*(1)}\bar{e}_R^{(1)} + y_D^{*(2)}\bar{e}_R^{(2)})D_L H^\dagger + \text{h.c.}\,.
\end{aligned}$$

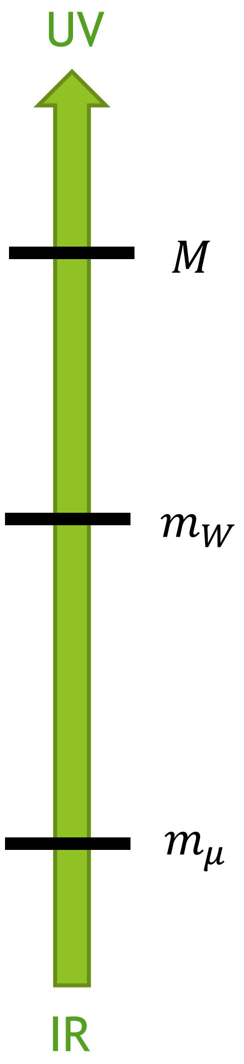
- We considered 3 different representations:

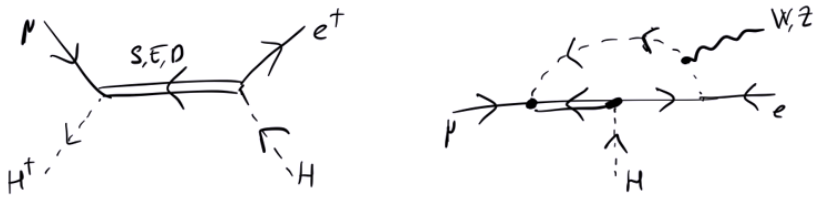
S_R : SM singlet

E_R : has hypercharge = -1

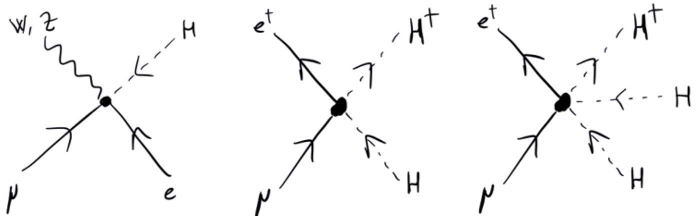
D_L : SU(2) doublet

UV Example: Heavy Vector-Like Fermions

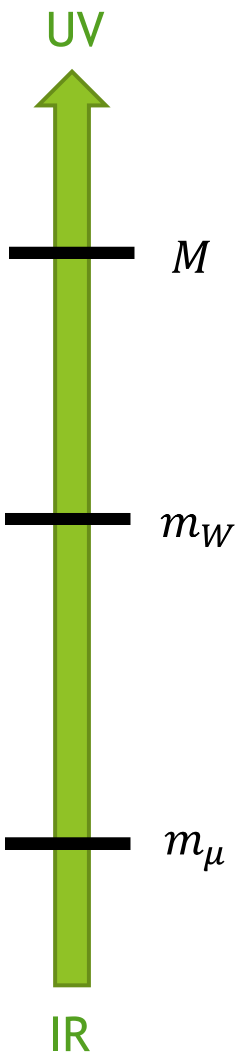


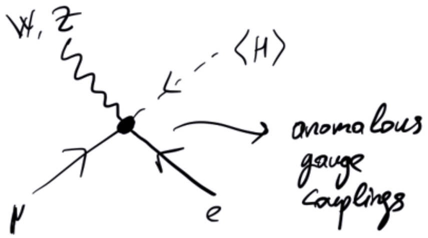


- Integrate it out and match it onto SM EFT.

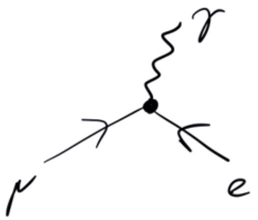


UV Example: Heavy Vector-Like Fermions

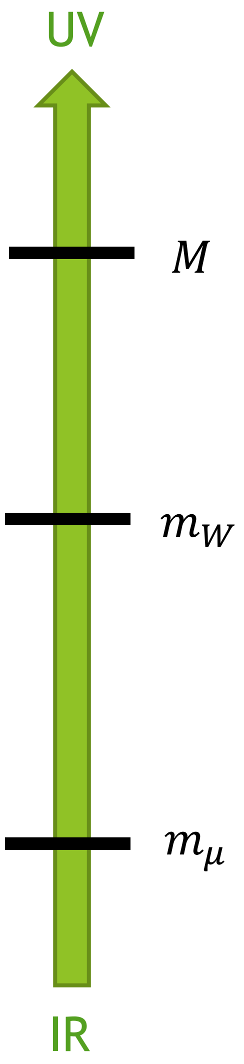




- Integrate W, Z, h, top and match again.

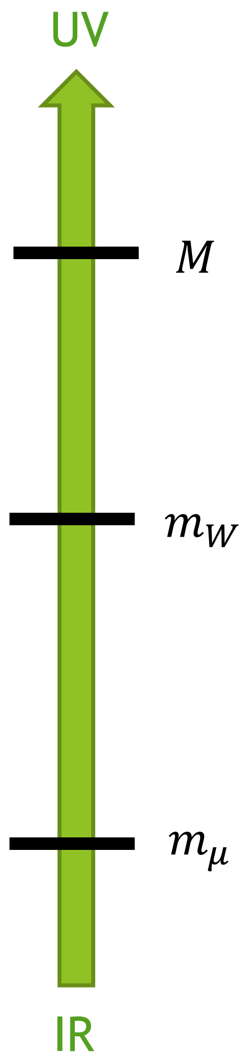


UV Example: Heavy Vector-Like Fermions



- Finally, run down to the mass scale of muon.

UV Example: Heavy Vector-Like Fermions



- In total, the RG equation for dipole looks like:

$$
\begin{aligned}
d_{e\mu}(m_W) &\simeq \frac{e}{2}\frac{v^2}{M^2}\Bigg[\Delta d_{e\mu}(m_W) + \left(C^{e\mu}_{DW}(M) - C^{e\mu}_{DB}(M)\right)\left(1 - N_c y_t^2 \frac{\ln(M/m_W)}{16\pi^2}\right) \\
&+ \left(\left(-N_c y_t^2 + 2\lambda\right) C^{e\mu}_{L}(M) + N_c y_t^2 C^{e\mu}_{L3}(M) - \frac{5}{8} g'^2 C^{e\mu}_{y}(M)\right)\frac{\ln(M/m_W)}{(16\pi^2)^2}\Bigg], \\
d_{\mu e}(m_W) &\simeq \frac{e}{2}\frac{v^2}{M^2}\Bigg[\Delta d_{\mu e}(m_W) + \left(C^{\mu e}_{DW}(M) - C^{\mu e}_{DB}(M)\right)\left(1 - N_c y_t^2 \frac{\ln(M/m_W)}{16\pi^2}\right) \\
&+ \left(\left(-2N_c y_t^2 + 2\lambda\right) C^{\mu e}_{R}(M) - \frac{5}{8} g'^2 C^{\mu e}_{y}(M)\right)\frac{\ln(M/m_W)}{(16\pi^2)^2}\Bigg].
\end{aligned}
$$

- Taking $y^{(i)}_{S,E,D} \sim O(1)$ bounds on the dipole implies that
  - For singlet case: $M \geq 40\, TeV$ and RG contribution is 20%
  - For doublet case: $M \geq 50\, TeV$ and RG contribution is 25%

Conclusions

- LFV observables at low-energies provides us a clean handle on new physics parametrized by high-dimensional operators in an EFT setting.
- Upcoming experiments are touching two-loop sensitivity. We are getting ready for the results from theory side.
  - As an EFT, our framework can capture signs of heavy new physics e.g. heavy vector-likes.
  - $(C_L - C_{L3})^{\mu e}$ and $C_y^{\mu e}$ can be well-bounded via dipole in general.
  - $C_y^{\mu e}$ bound is competitive with accelerator searches of $h \to \mu\, e$.
- Power of on-shell methods:
  - Complicated looking leading log. loop calculations rendered very simple.
  - Makes transparent the selection rules, such as helicity and angular momentum, provides a better understanding for vanishing anomalous dimensions.

Thank you!