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FOR FUNDAMENTAL PHYSICS

The large charge expansion

Susanne Reffert
University of Bern

based on arXiv:1505.01537, 1610.04495, 1707.00710, 1809.06371, 1902.09542, 1905.00026, 1909.08642,1909.02571, 2008.03308, 2010.07942, 2102.12488, 2110.07617, 2110.07616, 2203.12624, 2211.15301 + work in progress

L. Alvarez-Gaume (SCGP), D. Banerjee (Kolkata),
Sh. Chandrasekharan (Duke), N. Dondi (Bern), S. Hellerman (IPMU),
I. Kalogerakis (Bern), R. Moser (Bern), O. Loukas,
D. Orlando (INFN Torino), V. Pellizzani (Bern), F. Sannino (Odense/Napoli),
T. Schmidt (Bern), M. Watanabe (Kyoto)

Introduction

Strongly coupled physics is notoriously difficult to access.
We do not have small parameters in which to do a perturbative expansion. Our most basic notions of field theory are of a perturbative nature.

Make use of symmetries, look at special limits/subsectors where things simplify.

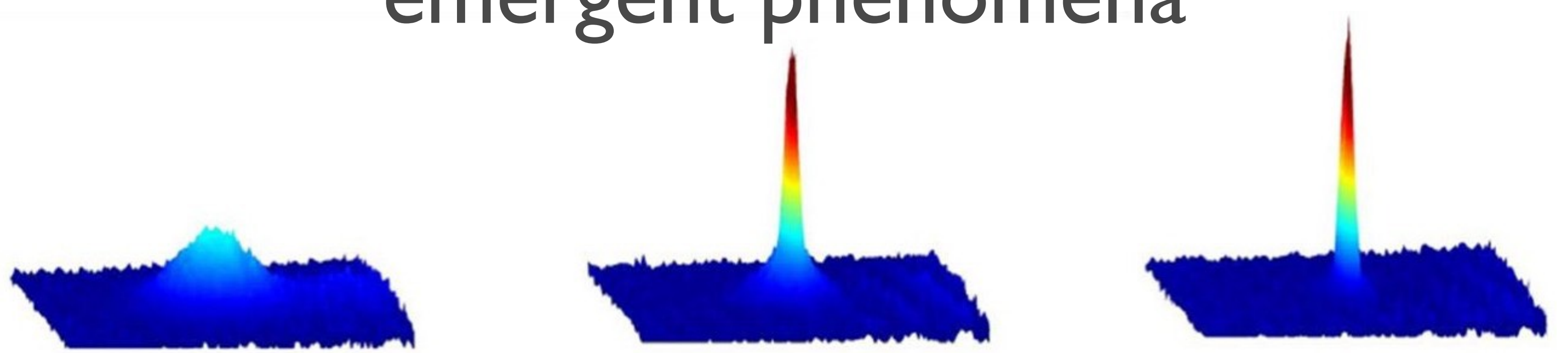
Here: study theories with a global symmetry group.
Hilbert space of the theory can be decomposed into sectors of fixed charge Q.
Study subsectors with large charge Q.

Large charge Q becomes controlling parameter in a perturbative expansion!

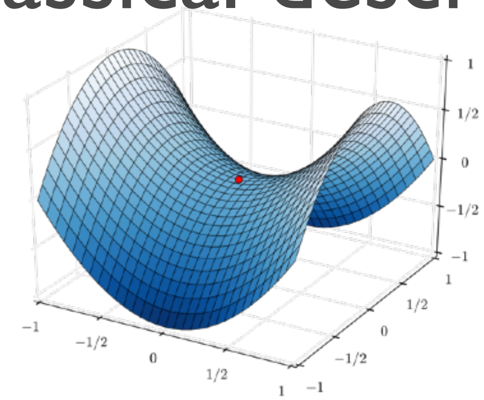
Introduction

Consider systems with large quantum number
many degrees of freedom

emergent phenomena



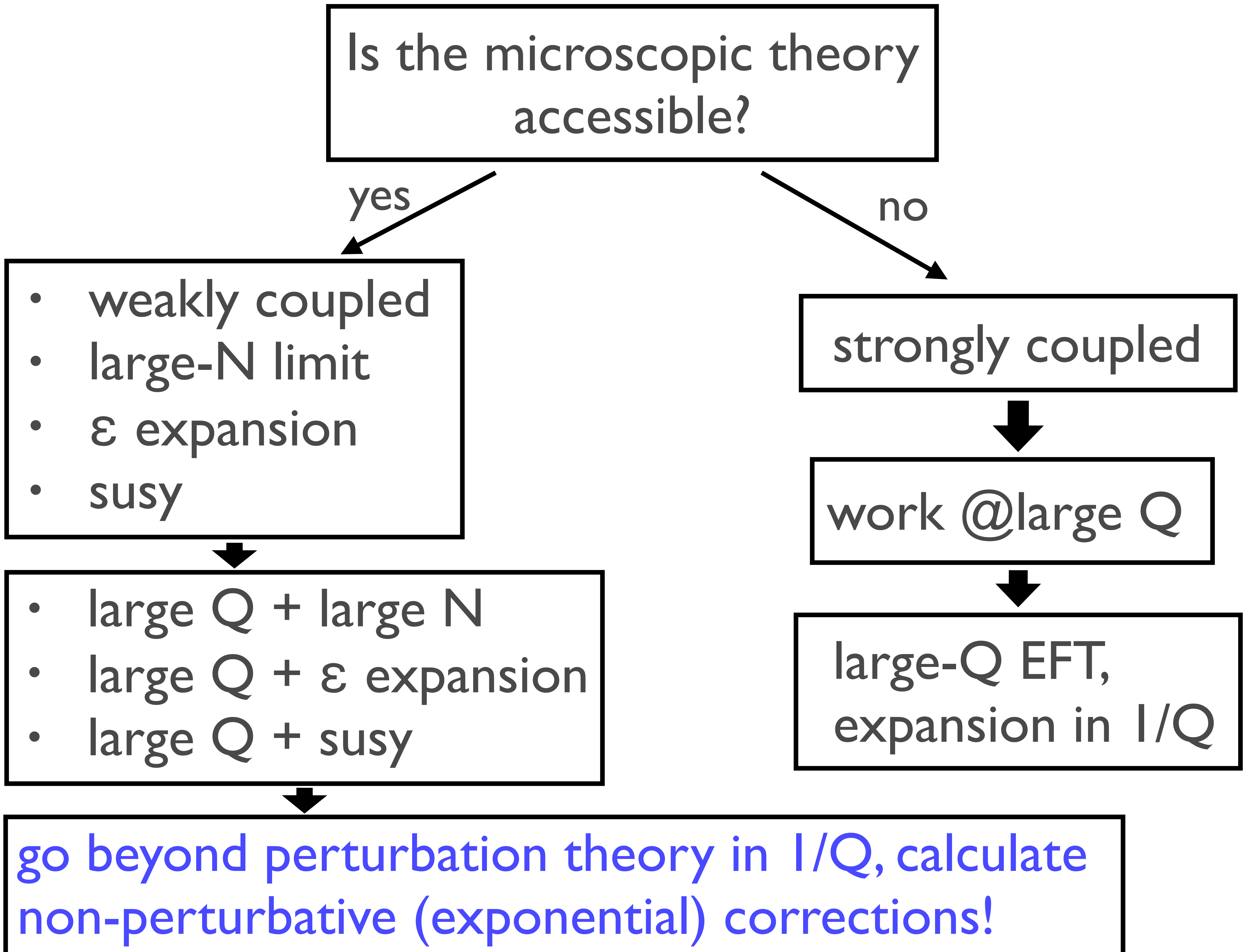
semiclassical description



e.g. superfluid

works also for strongly coupled systems!

Introduction



Introduction

The seem to be 2 main categories for systems at large quantum number:

<u>Superfluid</u>

isolated vacuum

- Wilson-Fisher CFT
- NRCFT (unitary Fermi gas)
- N=2 SCFT in 3d
- asymptotically safe model in 4d
- 3d chiral Gross-Neveu/NJL model

<u>EFT of the moduli space</u>

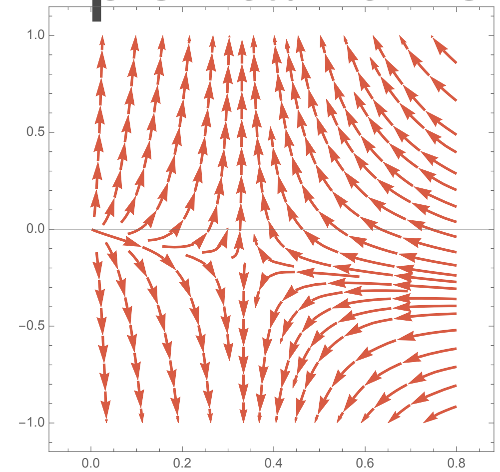
moduli space of vacua

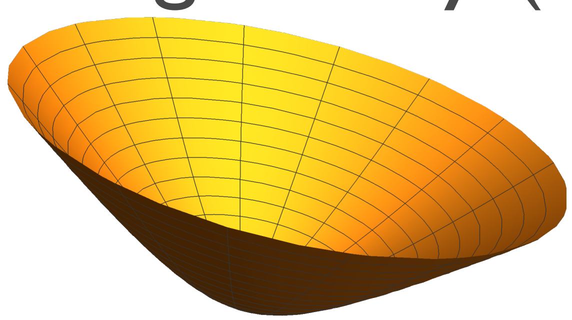
- free boson
- N=2 theories in 4d

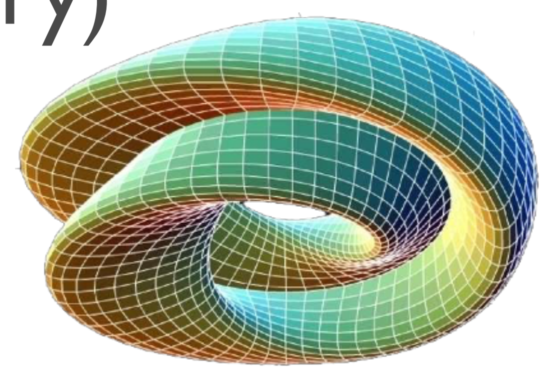
Introduction

Conformal field theories play an important role in theoretical physics:

- fixed points in RG flows
- critical phenomena
- quantum gravity (via AdS/CFT)
- string theory (WS theory)



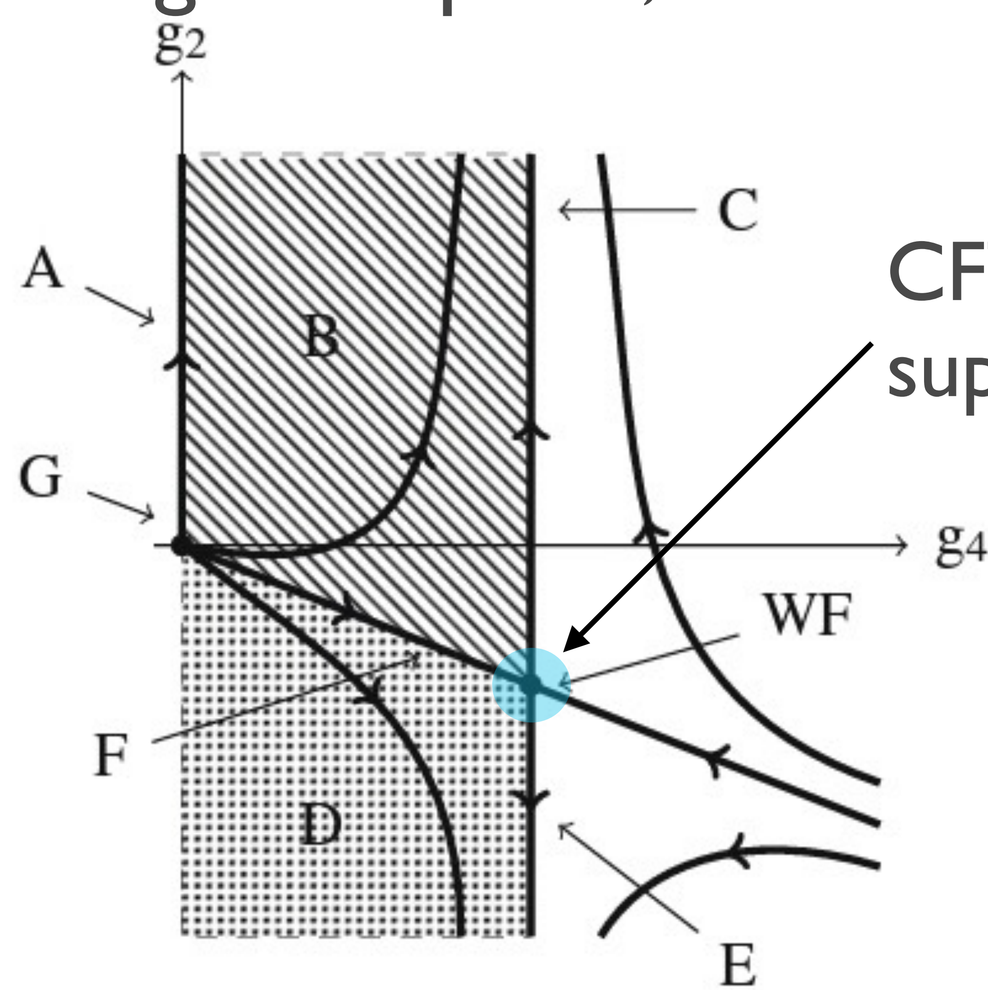




But: CFTs do not have any intrinsic scales, most have by naturalness couplings of O(1).
Possibilities: analytic (2d), conformal bootstrap (d>2), lattice calculations, non-perturbative methods…
Prime candidate for the large-charge approach.

Introduction

Example: Scalar field theories in 2<D<4 have a strongly-coupled interacting fixed point, the Wilson-Fisher FP.

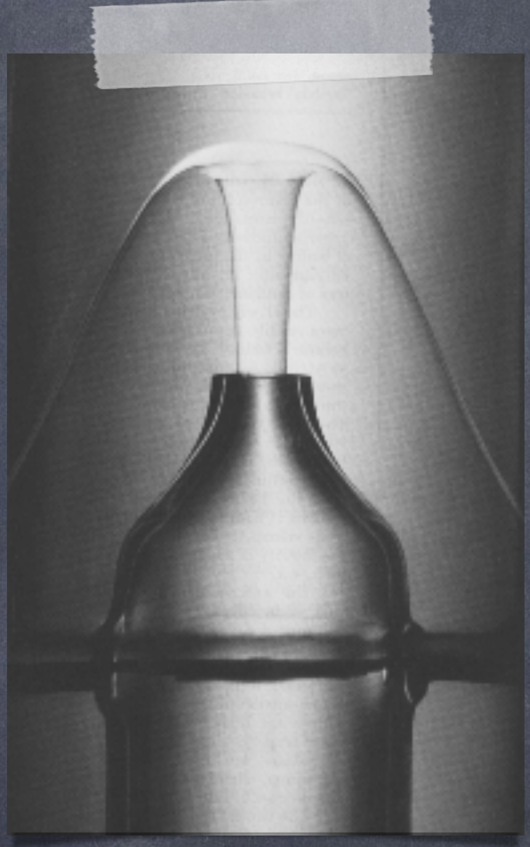


CFT, strongly coupled superfluid at large Q

O(2N) vector model in D=3:

$$S[\phi_i] = \sum_{i=1}^{N} \int \mathrm{d}t\mathrm{d}\Sigma \left[g^{\mu\nu}(\partial_\mu^i \phi_i)^\dagger (\partial_\nu^i \phi_i) + r(\phi_i^\dagger \phi_i) + \frac{u}{2N}(\phi_i^\dagger \phi_i)^2 \right]$$

For r=R/8, this flows to the WF fixed pt in the IR, $u \to \infty$



The O(2) model

The O(2) model

Simplest example: O(2) model in (2+1) dimensions

$$\mathcal{L}_{\mathrm{UV}} = \partial_\mu \phi^* \, \partial^\mu \phi - g^2 (\phi^* \phi)^2$$

Flows to Wilson-Fisher fixed point in IR.

Assume that also the IR DOF are encoded by cplx scalar

$\varphi_{\mathrm{IR}} = a\, e^{i\chi}$ Global U(1) symmetry: $\chi \to \chi + \mathrm{const.}$

Look at scales: put system in box (2-sphere) of scale R
Second scale given by U(1) charge Q:

$$\rho^{1/2} \sim Q^{1/2}/R$$

Study the CFT at the fixed point in a sector with

$$\frac{1}{R} \ll \Lambda \ll \frac{Q^{1/2}}{R} \ll g^2$$

UV scale

cut-off of effective theory

The O(2) model

LSM: Assume large vev for a: $\Lambda \ll a^2 \ll g^2$

scalar curvature (w. conf. coupling)

$$\mathcal{L}_{\mathrm{IR}} = \tfrac{1}{2}\,\partial_\mu a\,\partial^\mu a + \tfrac{1}{2} b^2 a^2\,\partial_\mu \chi\,\partial^\mu \chi - \frac{R}{16} a^2 - \frac{\lambda}{6} a^6 + \text{higher derivative terms}$$

dimensionless constants

ϕ has approximately mass dimension 1/2 and the action has a potential term $\propto |\varphi|^6$

Lagrangian is approximately scale-invariant.

Semi-classical analysis: solve classical e.o.m. at fixed Noether charge.

Solution must be homogeneous in space.

The O(2) model

Charge density: $\rho = b^2 a^2 \dot{\chi},$ $\quad Q = \rho \cdot \mathrm{Vol}(S^2)$

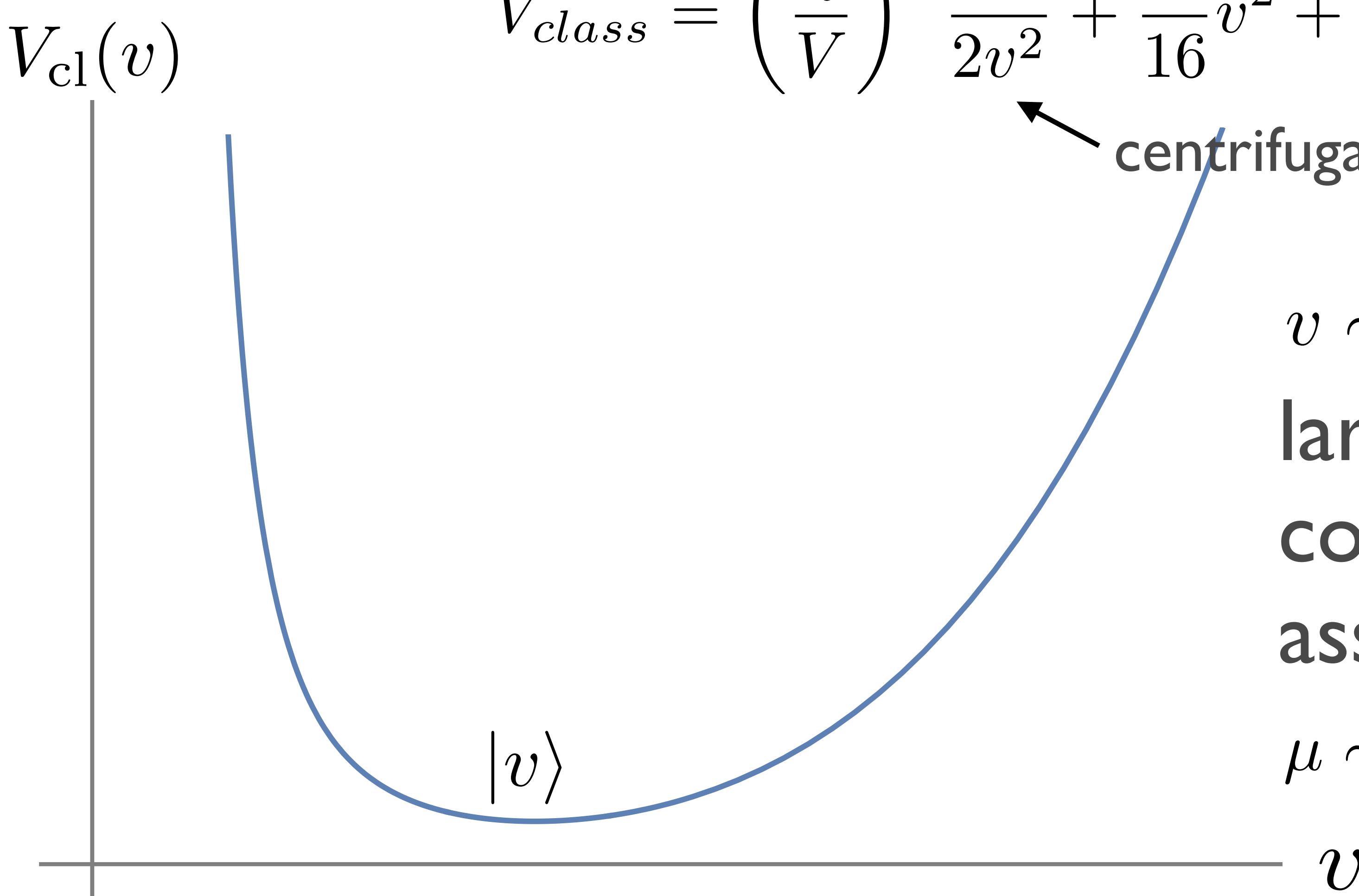
Lowest-energy solution: $a = v = \mathrm{const.}$

$\chi = \mu t,$ non-const. vev $\quad \mu = \dfrac{\rho}{b^2 v^2}$

Determine radial vev v by minimizing the classical potential:

$$V_{class} = \left(\frac{Q}{V}\right)^2 \frac{1}{2v^2} + \frac{R}{16} v^2 + \frac{\lambda}{6} v^6$$

centrifugal term



$v \sim Q^{1/4}$

large condensate is compatible with our assumption $a \gg 1$

$\mu \sim \rho^{1/2}$

The O(2) model

Ground state at fixed charge breaks symmetries:

$$SO(3,2) \times O(2) \to SO(3) \times D \times O(2) \rightsquigarrow SO(3) \times D'$$

$D' = D - \mu O(2)$

Broken U(1) - superfluid!

Energy of classical ground state at fixed charge:

2 dimensionless parameters

cannot be calculated within EFT!

$$E_\Sigma(Q) = \frac{c_{3/2}}{\sqrt{V}} Q^{3/2} + \frac{c_{1/2}}{2} R\sqrt{V} Q^{1/2} + \mathcal{O}(Q^{-1/2})$$

dependence on manifold

Classical solution at lowest energy and fixed global charge becomes the vacuum of the quantum theory.

The O(2) model

Quantum story: study the low-energy spectrum

Parametrize fluctuations on top of the classical vacuum

$$a = v + \hat{a} \qquad \chi = \mu t + \frac{\hat{\chi}}{v}$$

Goldstone

massive mode, not relevant for low-energy spectrum $m \sim \mathcal{O}(\sqrt{Q})$

Go to NLSM: Integrate out a (saddle point for LO).

Dynamics is described by a single Goldstone field χ:

$$\mathcal{L}_{LO} = k_{3/2}(\partial_\mu \chi \, \partial^\mu \chi)^{3/2}$$

can get this purely by dimensional analysis

Use dimensional analysis, parity and scale invariance to determine (tree-level) operators in effective action beyond LO (Lorentz scalars of scaling dimension 3, including couplings to geometric invariants)

Use ρ-scaling to determine which terms are not suppressed: $\partial\chi \sim \rho^{1/2}, \quad \partial \ldots \partial\chi \sim \rho^{-1/4}$

The O(2) model

Result for NLSM action:

scale-inv. but NOT conformally inv.

LO Lagrangian

$$\mathcal{L} = k_{3/2}(\partial_\mu \chi \partial^\mu \chi)^{3/2} + k_{1/2} R (\partial_\mu \chi \partial^\mu \chi)^{1/2} + \mathcal{O}(Q^{-1/2})$$

dimensionless parameters

suppressed by inverse powers of Q

To be understood as an expansion around the classical ground state $\mu t + \hat{\chi}$

Expand action around GS to second order in fields:

$$\mathcal{L} = k_{3/2}\mu^3 + k_{1/2} R \mu + (\partial_t \hat{\chi})^2 - \frac{1}{2}(\nabla_{S^2}\hat{\chi})^2 + \dots$$

Compute zeros of inverse propagator and get dispersion relation:

$$\omega_{\vec{p}} = \frac{|\vec{p}|}{\sqrt{2}}$$

dictated by conf. invariance $1/\sqrt{d}$

$\Rightarrow \chi$ is indeed a "conformal" Goldstone (type I)

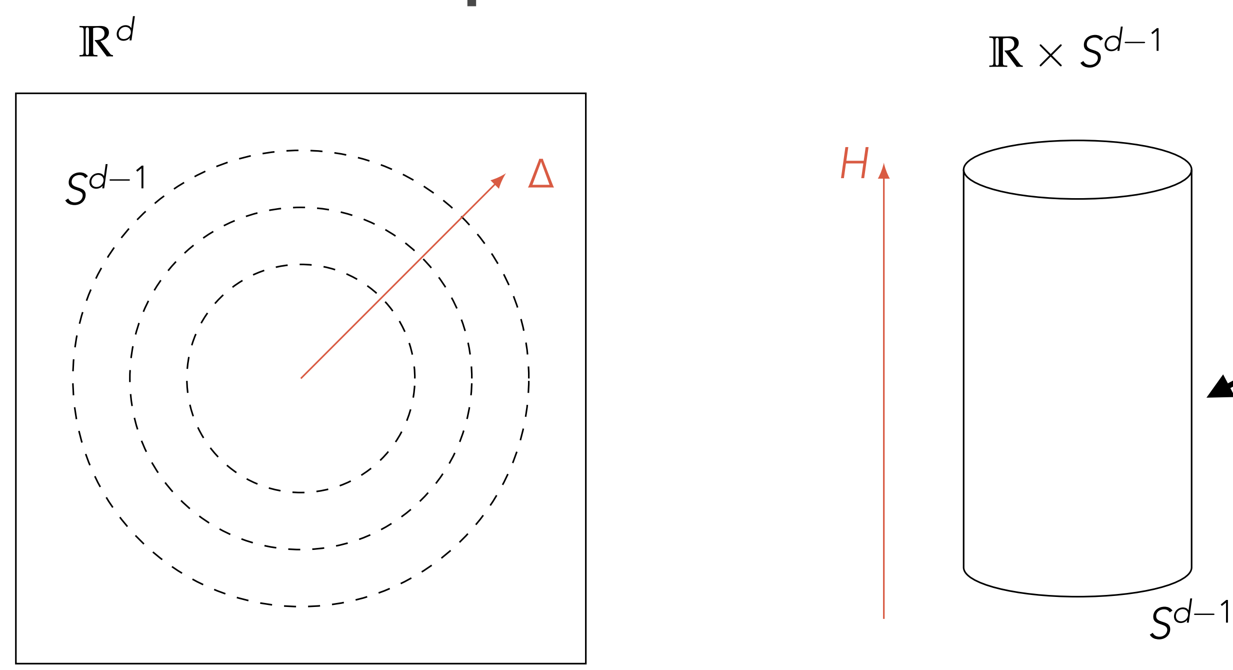
The O(2) model

Let's calculate observables: CFT data!

Scaling dimensions, fusion coefficients.

Use state-operator correspondence of CFT:

conformal dimension



energy

Scaling dimension of lowest operator of charge Q:

energy of class. ground state

$$D(Q) = R_0(E_0 + E_{Cas}) = c_{3/2}Q^{3/2} + c_{1/2}Q^{1/2} - 0.0937\cdots + \mathcal{O}(Q^{-1/2})$$

quantum correction from Casimir energy of Goldstone

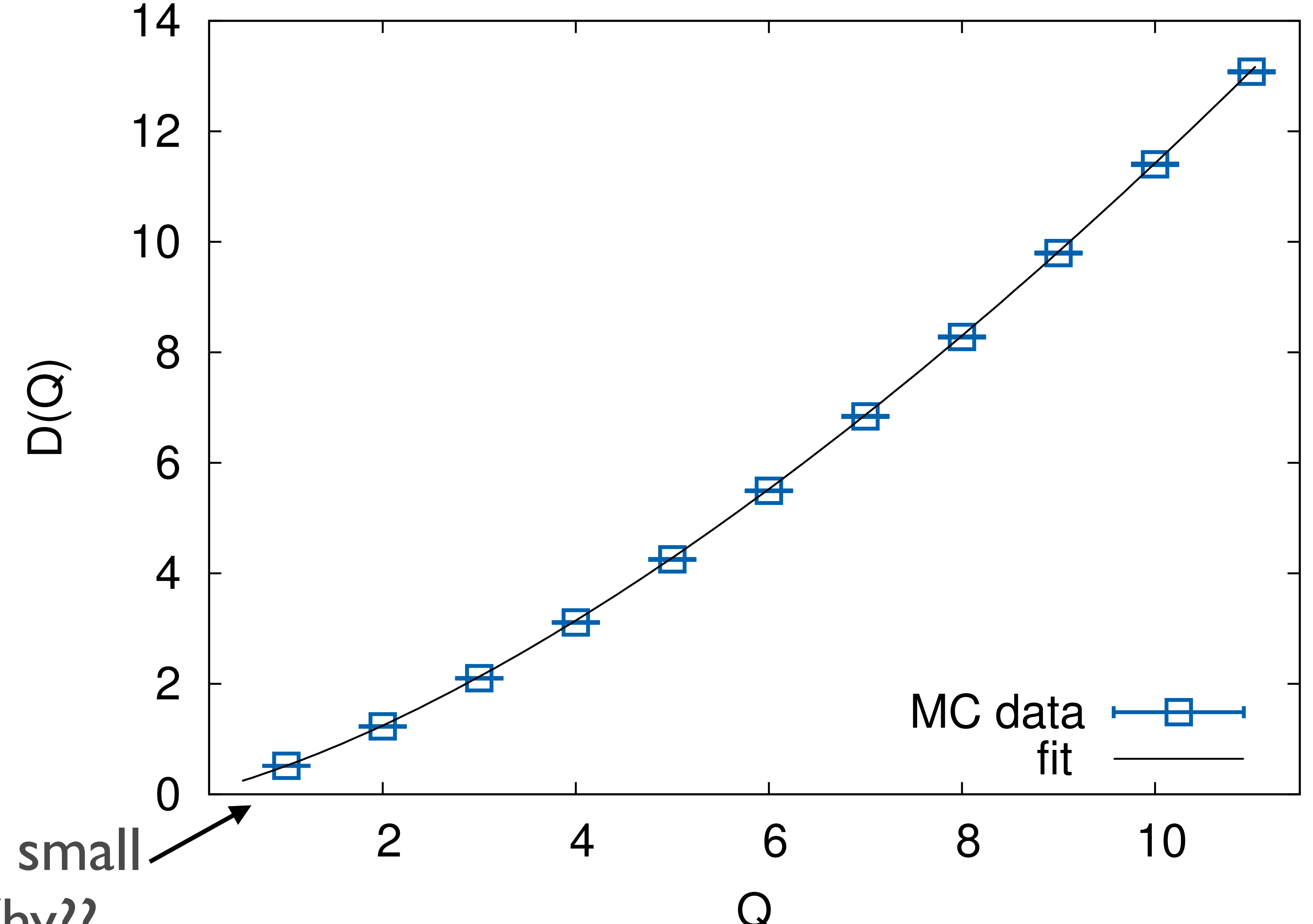
S. Hellerman, D. Orlando, S. R., M. Watanabe, arXiv:1505.01537 [hep-th]

The O(2) model

Testing our prediction:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

Independent calculation on the lattice:



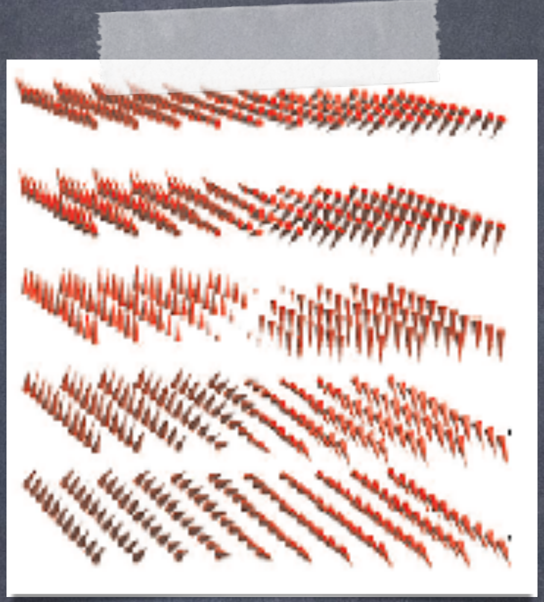
Excellent agreement!!

$$c_{3/2} = 1.195(10)$$
$$c_{1/2} = 0.075(10)$$

works for small charge. Why??

D. Banerjee, Sh. Chandrasekharan, D. Orlando [hep-th/1707.00711]

Large-charge expansion works extremely well for O(2).



Beyond O(2): 3d O(2N) vector model

Beyond O(2)

Where else can we apply the large-charge expansion?

Obvious generalization in 3d: O(2N) vector model

non-Abelian global symmetry group: new effects

Different symmetry breaking patterns possible, inhomogeneous ground states possible.

Homogeneous case: same form of ground state,

$$SO(3,2) \times O(2N) \to SO(3) \times D \times U(N) \to SO(3) \times D' \times U(N-1)$$

We expect dim[U(N)/U(N-1)] = 2N-1 Goldstone d.o.f.

On top of the conformal Goldstone of O(2), a new sector with N-1 non-relativistic type II Goldstones and N-1 massive modes with m=2μ appears.

The O(2N) vector model

Dispersion relation: $$\omega = \frac{p^2}{2\mu} + \mathcal{O}\mu^{-3}$$

The non-relativistic Goldstones count double.

Nielsen and Chadha; Murayama and Watanabe

Counting type I and type II modes, indeed,

$$1 + 2(N-1) = 2N - 1 = \dim(U(N)/U(N-1))$$

Non-relativistic Goldstones contribute to the conformal dimensions only at higher order.
The ground-state energy is again determined by a single relativistic Goldstone!

Same formula for scaling dimensions as for O(2):

N-dependent

universal for O(2N)

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

verified at large N for CP(N-1) model de la Fuente

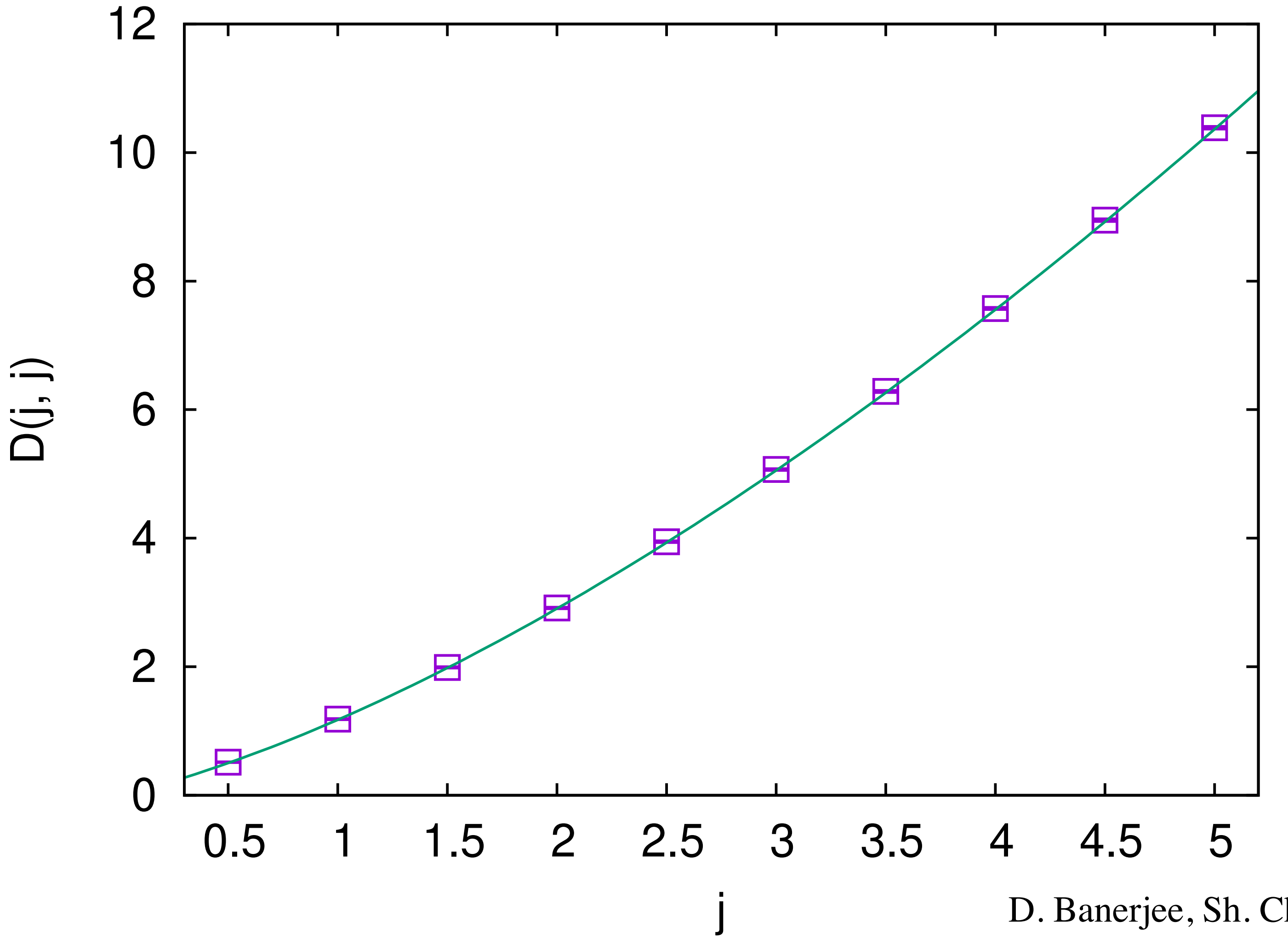
 L. Alvarez-Gaume, O. Loukas, D. Orlando and S. R., arXiv:1610.04495 [hep-th]

The O(2N) vector model

Testing our prediction:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

New lattice data for O(4) model:



$c_{3/2} = 1.068(4)$

$c_{1/2} = 0.083(3)$

D. Banerjee, Sh. Chandrasekharan, D. Orlando, S.R. 1902.09542

Again excellent agreement with large-Q prediction!

The large-N limit

Start in the UV with

$$S[\phi_i] = \sum_{i=1}^{N} \int \mathrm{d}t\mathrm{d}\Sigma \left[g^{\mu\nu} (\partial_\mu^i \phi_i)^\dagger (\partial_\nu^i \phi_i) + r(\phi_i^\dagger \phi_i) + \frac{u}{2N} (\phi_i^\dagger \phi_i)^2 \right]$$

For r=R/8, this flows to the WF fixed pt in the IR, $u \to \infty$

Instead: keep u finite - explore the RG flow.

We can use the standard large-N approach but we need to fix the charge:

$$\begin{aligned} Z(Q_1, \ldots, Q_N) &= \mathrm{Tr}\left[e^{-\beta H} \prod_{i=1}^{N} \delta(\hat{Q}_i - Q_i) \right] \\ &= \int_{-\pi}^{\pi} \prod_{i=1}^{N} \frac{d\theta_i}{2\pi} \prod_{i=1}^{N} e^{i\theta_i Q_i} \,\mathrm{Tr}\left[e^{-\beta H} \prod_{i=1}^{N} e^{-i\theta_i \hat{Q}_i} \right]. \end{aligned}$$

The charge fixing can be implemented via a boundary condition or via a covariant derivative:

The large-N limit

$$\mathrm{Tr}\left[e^{-\beta H - i\theta\cdot\hat{Q}}\right] = \int\limits_{\varphi_i(\beta,x)=\varphi_i(0,x)} \mathcal{D}\varphi_i\, e^{-S_\theta[\varphi_i]},$$

$$S_\theta[\varphi_i] = \sum_{i=1}^{N}\int dt\, d\Sigma\left[g^{\mu\nu}(D^i_\mu\varphi_i)^*(D^i_\nu\varphi_i) + r\varphi_i^*\varphi_i + \frac{u}{2}(\varphi_i^*\varphi_i)^2\right]$$

$$D^i_\mu\varphi_i = \begin{cases}\left(\partial_0 + i\frac{\theta_i}{\beta}\right)\varphi_i & \text{if } \mu = 0\\ \partial_i\varphi & \text{otherwise.}\end{cases}$$

Perform Stratonovich transform: introduce collective d.o.f. $(\varphi_i^*\varphi_i)$ and Lagrange multiplier λ, write EFT in terms of λ:

$$S_\theta[\varphi_i,\lambda] = \sum_{i=1}^{N}\int dt\, d\Sigma\left[(D^i_\mu\varphi_i)^*(D^i_\mu\varphi_i) + (r+\lambda)\varphi_i^*\varphi_i - \frac{\lambda^2}{2u}\right]$$

Can integrate out φ_i. From the saddle-point equations we find that due to the chemical potential, λ gets a vev m^2, where

$$\theta_i^2 = -m^2\beta^2$$

Adding the chemical potential gives us more structure to work with!

The large-N limit

From Z we find the grand potential (LO in N):

in flat space

$$\frac{\mathcal{L}}{2N} = -\frac{1}{2V}\zeta(-\frac{1}{2}|\Sigma, m) + \frac{(m^2-r)^2}{4u} = \left[\frac{m^3}{12\pi} + \frac{(m^2-r)^2}{4u}\right]$$

This is exactly the NLSM for $m^2 = \partial_\mu \chi \partial^\mu \chi$

This expression contains the full information about the model. More transparent, if we extract the effective potential. The LSM has the form

vev of radial mode

$$\mathcal{L}_{\mathrm{LSM}} = \Phi^2 m^2 - V(\Phi)$$

vev of angular mode

E.o.m. for radial mode:

$$m^2 - \frac{\mathrm{d}}{\mathrm{d}(\Phi^2)} V = 0$$

Plugging the solution back in, we must recover

$$\Phi^2 m^2 - V(\Phi)\Big|_{\Phi=\Phi(m^2)} = \mathcal{L}(m)$$

L is the Legendre transform of V in Φ^2

The large-N limit

Pay attention to convexity! Use more general definition of Legendre transform: $f^*(y) = \sup_x (xy - f(x))$

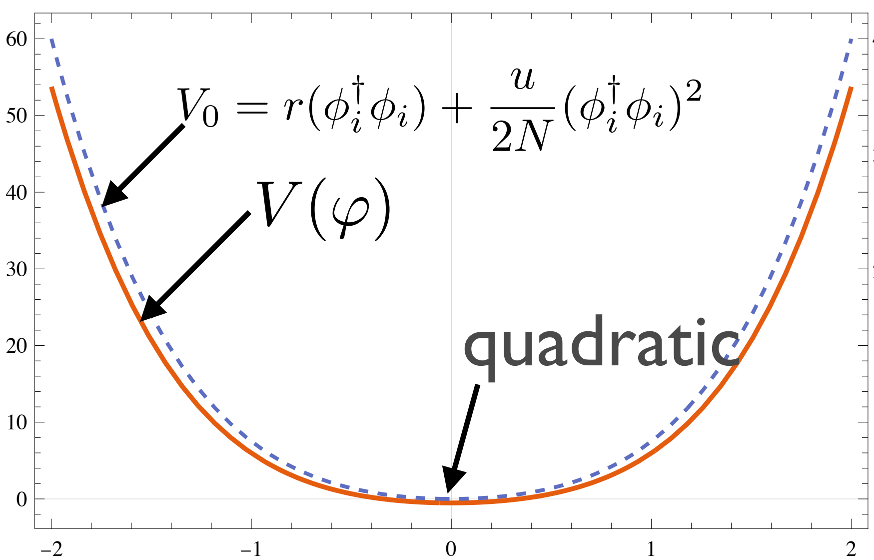
$$V(\Phi^2) = \sup_{m^2}(m^2\Phi^2 - \mathcal{L}(m))$$

$$V(\varphi) = \frac{u^3}{3 \cdot 2^{10}\pi^4}\left(1 + \frac{3}{2}\eta + \frac{3}{4}\eta^2 - (1+\eta)^{3/2}\right) \qquad \eta = 64\pi^2\left(\frac{|\varphi|^2}{Nu} + \frac{r}{u^2}\right)$$

r=0: Appelquist and Heinz

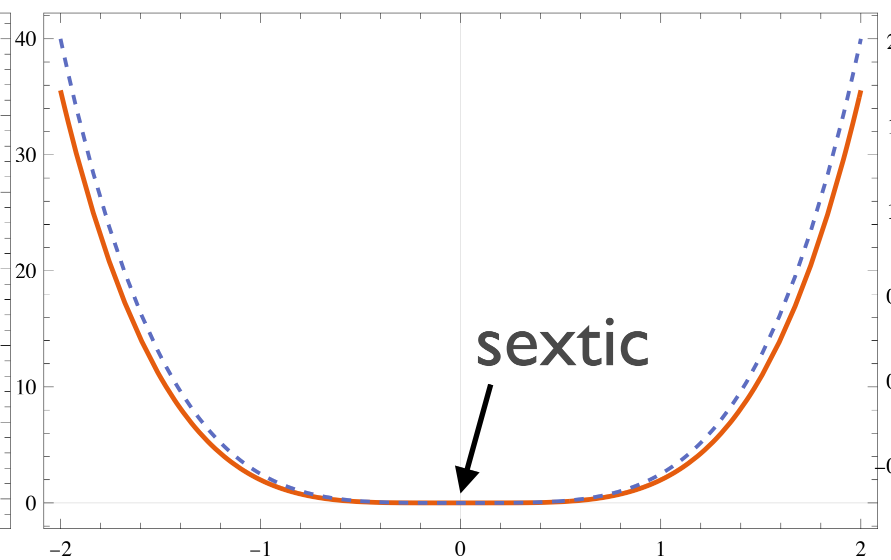
There are three cases, depending on the value of r:

r>0



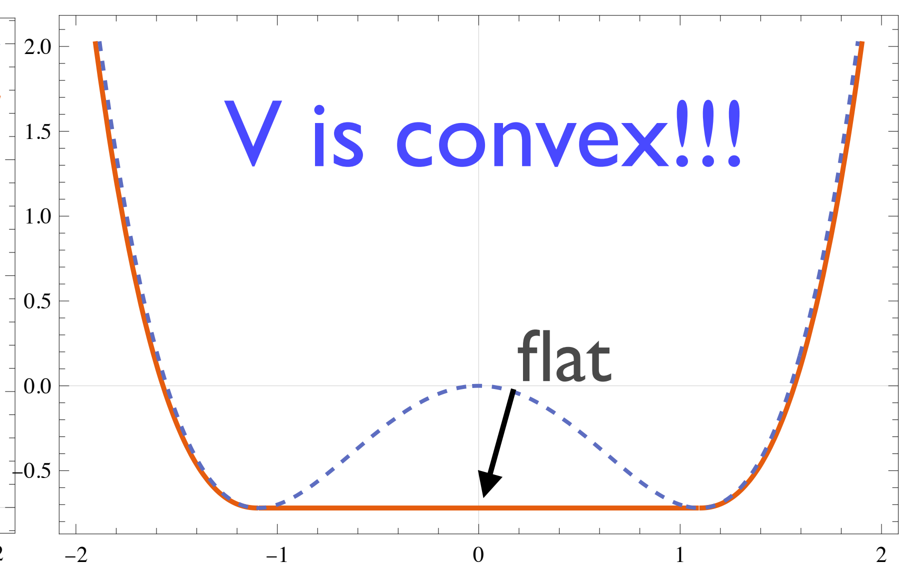
unbroken phase

r=0



critical point

r<0



broken phase

$$V(|\varphi|) = \begin{cases} -\frac{Nr^2}{2u} & \text{for } 0 < |\varphi| < \sqrt{-\frac{rN}{u}} \\ \hat{V}(|\varphi|) & \text{for } |\varphi| > \sqrt{-\frac{rN}{u}} \end{cases}$$

Orlando, Reffert, Schmidt 2110.07616

The large-N limit

Examine the critical case:

Scaling dimension of the lowest operator of a given charge is identified by the state-operator correspondence with the grand-canonical free energy:

$$\frac{\Delta(Q)}{2N} = \frac{2}{3}\left(\frac{Q}{2N}\right)^{3/2} + \frac{1}{6}\left(\frac{Q}{2N}\right)^{1/2} - \frac{7}{720}\left(\frac{Q}{2N}\right)^{-1/2} - \frac{71}{181440}\left(\frac{Q}{2N}\right)^{-3/2} + \dots$$

same Q-scaling as in EFT

L. Alvarez-Gaume, D. Orlando, S.R. 1909.02571

All these results are straightforwardly obtained thanks to the interplay between large Q and large N - no Feynman diagrams needed!

NLO in N: reproduce dispersion relations of Goldstones.

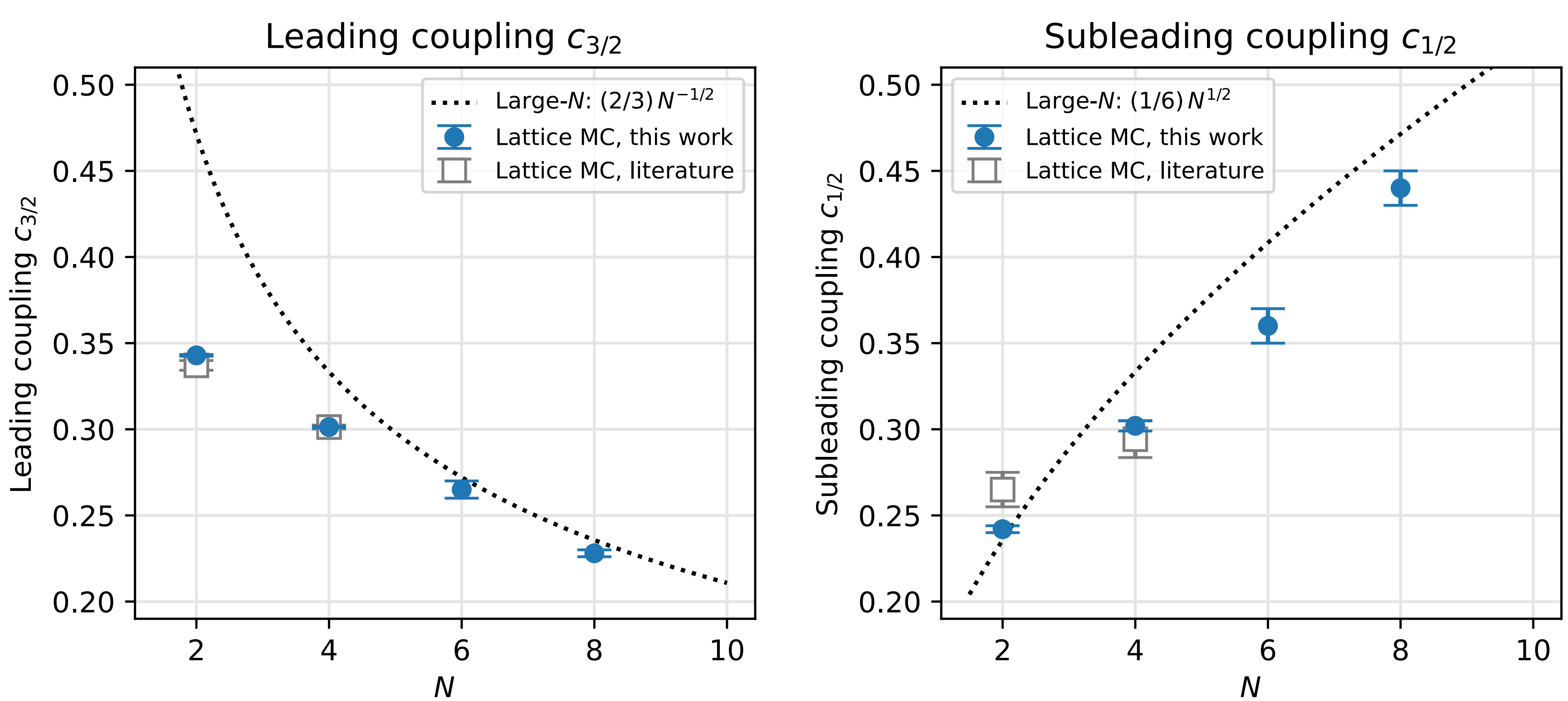
The large-N limit

Since we have an extra control parameter at large N, we can go beyond EFT results!

Find coefficients of the expansion (leading order in N):

$$c_{3/2} = \frac{1}{3}\sqrt{\frac{2}{N}} \qquad\qquad c_{1/2} = \frac{1}{3}\sqrt{\frac{N}{2}}$$

Comparison to lattice data:



Singh, arXiv:2203.00059 [hep-lat]

The large-N limit

Small charge limit: At large N, we now have more control and can also take the limit of $Q/N \ll 1$.

In this limit, the operator of charge Q whose dimension we are calculating is φ^Q.

$$\frac{\Delta(Q)}{Q} = \underbrace{\overbrace{\frac{1}{2}}^{\text{engineering dimension of } \phi}}_{\text{tree-level}} + \underbrace{\frac{4}{\pi^2}\frac{Q}{N}}_{\text{one-loop}} + \mathcal{O}\left(\frac{Q}{2N}\right)^2$$

Jack, Jones; Antipin et al.

Can be verified by a perturbative (loop) calculation around the zero-charge vacuum (Benvenuti, unpublished)!

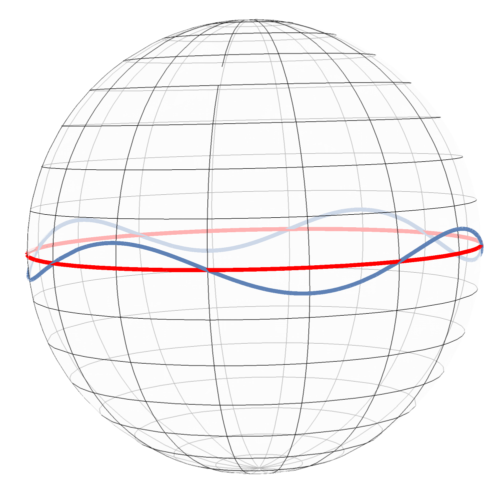
Resurgence analysis

Since we can compute all the coefficients of the large-Q expansion, we see that it is an asymptotic series which diverges as (2L)!

We can write the transseries and the non-perturbative corrections go like

$$e^{-2\pi k\sqrt{Q/(2N)}}$$

Geometric interpretation: particles of mass μ propagating on the equator of the 2-sphere.



CFT + resurgence: This picture must work for any N.

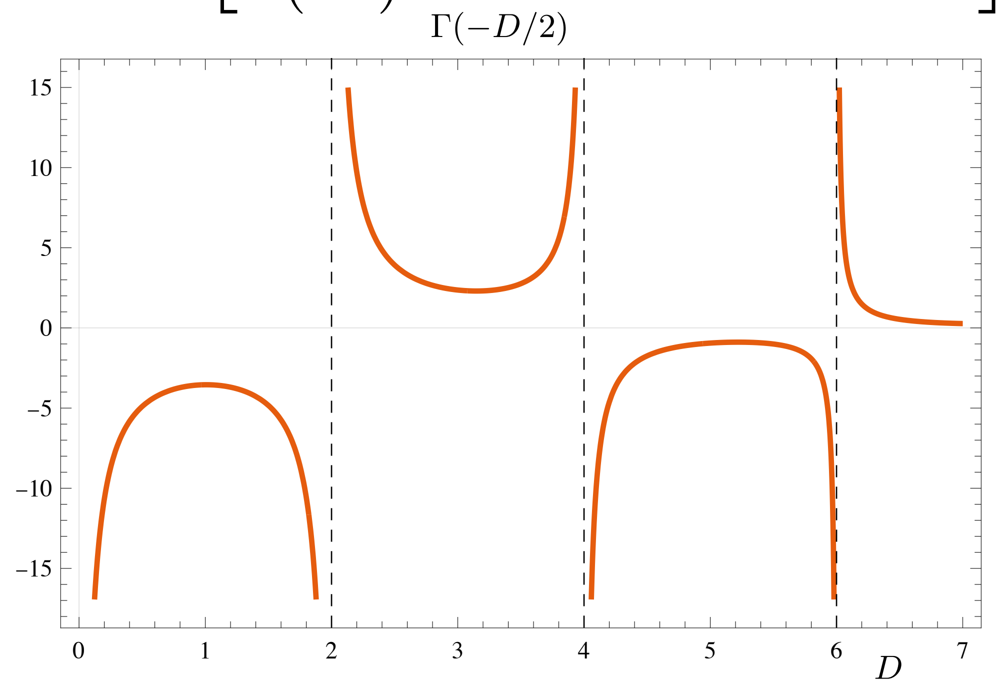
The optimal truncation is $\mathcal{O}(\sqrt{Q})$ terms. This explains why the comparison to the lattice calculation works so well.

A. Dondi, I. Kalogerakis, D.Orlando, S.R, arXiv: 2102.12488 [hep-th]

General dimensions

So far: D=3. Repeat the analysis for general dimension.

$$\mathcal{L} = (2N)\left[\frac{\Gamma(-D/2)}{2(4\pi)^{D/2}} m^D + \frac{(m^2 - r)^2}{4u}\right]$$



We see that for 4<D<6, L is unbounded from below. Instability!

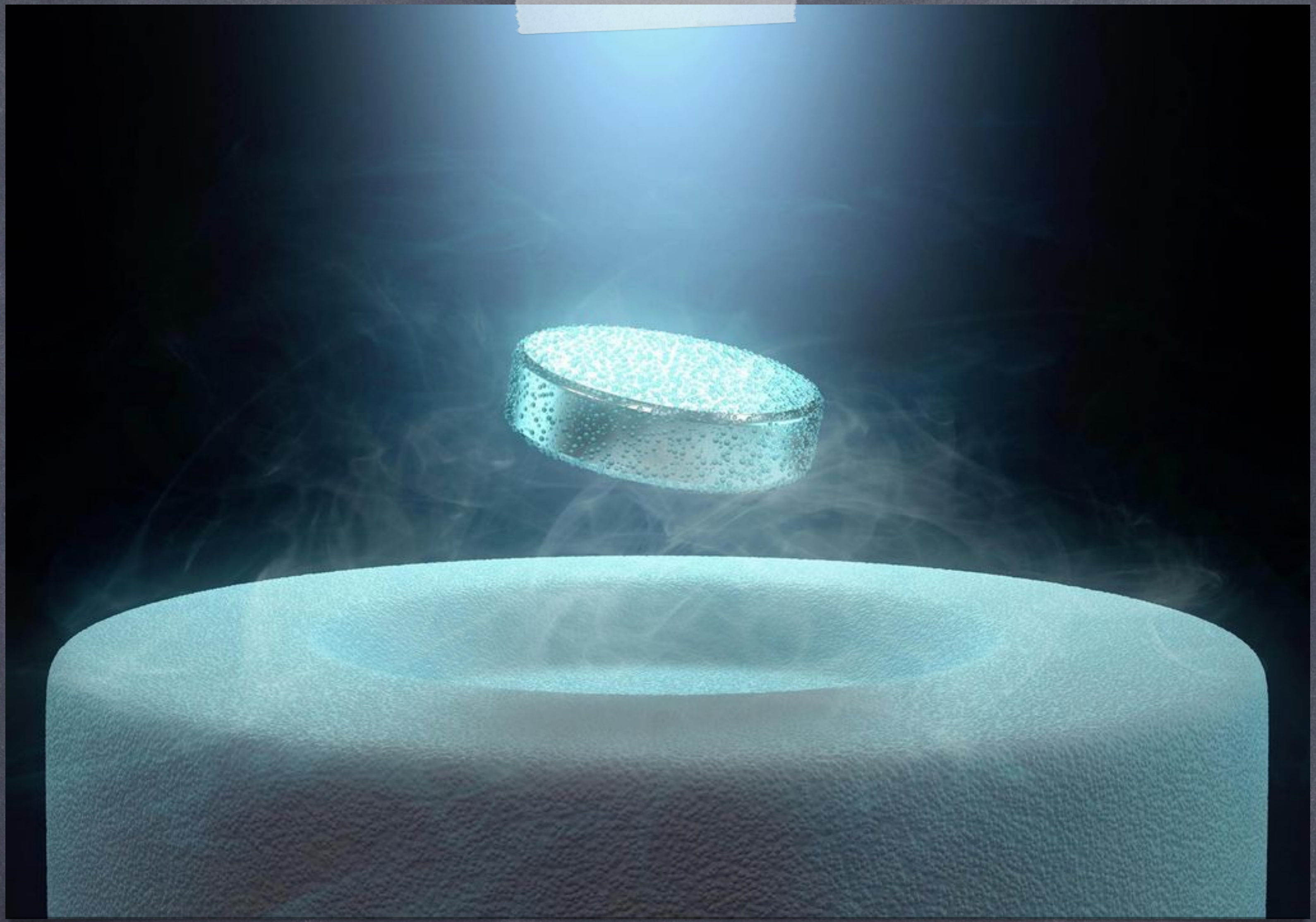
If we formally compute the conformal dimension for D=5:

$$\Delta(Q) = r_0 F_{S^4}(Q) = 2N\left[f_1 \frac{4\sqrt{3}}{5}\left(\frac{Q}{2N}\right)^{\frac{5}{4}} - \frac{f_2}{\sqrt{3}}\left(\frac{Q}{2N}\right)^{\frac{3}{4}}\right],$$

	branch 1	branch 2	branch 3	branch 4
f_1	$e^{i\pi/4}$	$e^{-i\pi/4}$	$e^{3\pi i/4}$	$e^{-3\pi i/4}$
f_2	$e^{3i\pi/4}$	$e^{-3i\pi/4}$	$e^{\pi i/4}$	$e^{-\pi i/4}$

Interpretation as non-unitary CFT.

Giombi, Hyman;
Moser, Orlando, Reffert 2110.07617



Fermions@large Q

Fermions@large Q

Will large Q work for fermionic models?

Antipin, Bersini, Panopoulos;

Let's start with the multicomponent Nambu-Jona-Lasinio (NJL) model, also known as the chiral Gross-Neveu (GN) model in 3D:

$$S_{\mathrm{cGN}} = -\int \mathrm{d}^3x \left[\bar{\psi}_a i\partial\!\!\!/ \psi_a + \frac{g}{2N}\left((\bar{\psi}_a \psi_a)^2 + (\bar{\psi}_a i\gamma_5 \psi_a)^2 \right) \right]$$

There are two conserved currents:

$$j^\mu = \bar{\psi}\gamma^\mu\psi, \qquad j^{5\mu} = \bar{\psi}\gamma^\mu\gamma^5\psi$$

We can study this model at large N.
After a Stratonovich transform, we have

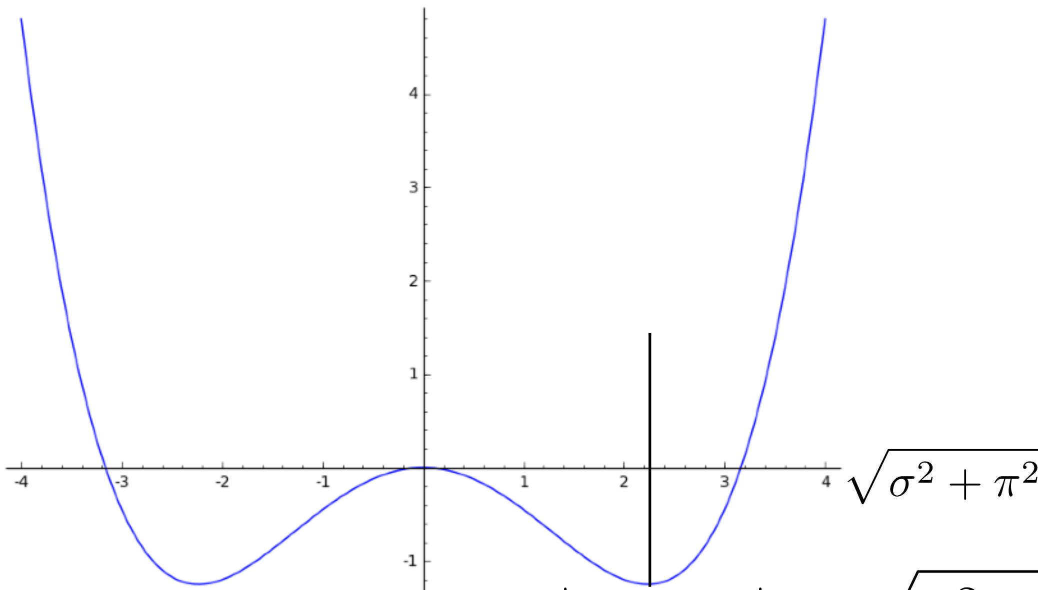
$$\mathcal{L} = \bar{\psi}_a i\partial\!\!\!/ \psi_a + \sigma\bar{\psi}_a\psi_a + i\pi\bar{\psi}_a\gamma^5\psi_a + \frac{N}{g}(\sigma^2+\pi^2) + \mu\,\bar{\psi}_a\gamma^0\gamma^5\psi_a$$

We can make the scalar fields dynamical and arrive at the Yukawa-NJL model which is the UV completion of the NJL model.

Fermions@large Q

Now we can integrate out the fermions and do the saddle-point analysis.

We find that only the axial charge gives rise to a condensate at criticality:



$$\log\det(i\partial\!\!\!/ + \sigma + i\pi\gamma^5 + \mu\gamma^0\gamma^5)$$

$$|\sigma + i\pi| = \sqrt{\kappa^2 - 1}\mu \qquad \kappa = 1.19968...$$

Scaling dimension:

large Q

$$\frac{\Delta}{N} = \frac{\sqrt{2}}{3}\left(\frac{Q}{\kappa N}\right)^{3/2} + \frac{1}{3\sqrt{2}}\left(\frac{Q}{\kappa N}\right)^{1/2} + \dots$$

$$= \frac{1}{2}\frac{Q}{N} + \frac{1}{\pi^2}\left(\frac{Q}{N}\right)^2 + \dots$$

small Q

Fermions@large Q

We find that like for the scalar case, we get a condensate at fixed charge, but not WF universality class.

The end result is similar in the sense that we have an EFT in terms of Goldstones fluctuating around a condensate.

Can go to a different frame using the Pauli-Gürsey transformation:

$$\psi_a \mapsto \frac{1}{2}(1-\gamma^5)\psi_a + \frac{1}{2}(1+\gamma^5)C\bar{\psi}_a^T$$

$$S_{\text{BCS}} = -\int \mathrm{d}^3x \left[\bar{\psi}_a i\partial\!\!\!/ \psi_a + \frac{g}{2N}\left(\bar{\psi}_a C\bar{\psi}_a^T\right)\left(\psi_b^T C\psi_b\right)\right]$$

This model gives rise to superconductivity from Cooper pair formation!

The condensate consists of Cooper pairs - superconductor!

Dondi, Hellerman, Kalogerakis, Moser, Orlando, Reffert, 2211.15318

Fermions@large Q

Different story in the standard Gross-Neveu model:

$$S_{GN} = -\int \mathrm{d}^3x \left[\bar{\psi}_a i\partial\!\!\!/ \psi_a + \frac{g}{2N}(\bar{\psi}_a\psi_a)^2\right]$$

Only one current, can fix its associated charge.

Result@leading order in N: the fixed-charge ground state is not a condensate, but a Fermi surface.

Different from all other cases studied before, as fixing the charge does not lead to SSB.

$$\frac{Q}{2N} = \lfloor \mu r_0 \rfloor (\lfloor \mu r_0 \rfloor + 1), \quad \frac{\Delta_{FS}}{2N} = \frac{1}{3}\lfloor \mu r_0 \rfloor (\lfloor \mu r_0 \rfloor + 1)(2\lfloor \mu r_0 \rfloor + 1) = \frac{Q}{6N}\sqrt{\frac{2Q}{N} + 1}.$$

$$\frac{\Delta_{FS}}{N} = \frac{2}{3}\left(\frac{Q}{2N}\right)^{3/2} + \frac{1}{12}\left(\frac{Q}{2N}\right)^{1/2} - \frac{1}{192}\left(\frac{Q}{2N}\right)^{-1/2} + \mathcal{O}\left(\left(\frac{Q}{2N}\right)^{-3/2}\right), \quad \frac{Q}{2N} \to \infty.$$

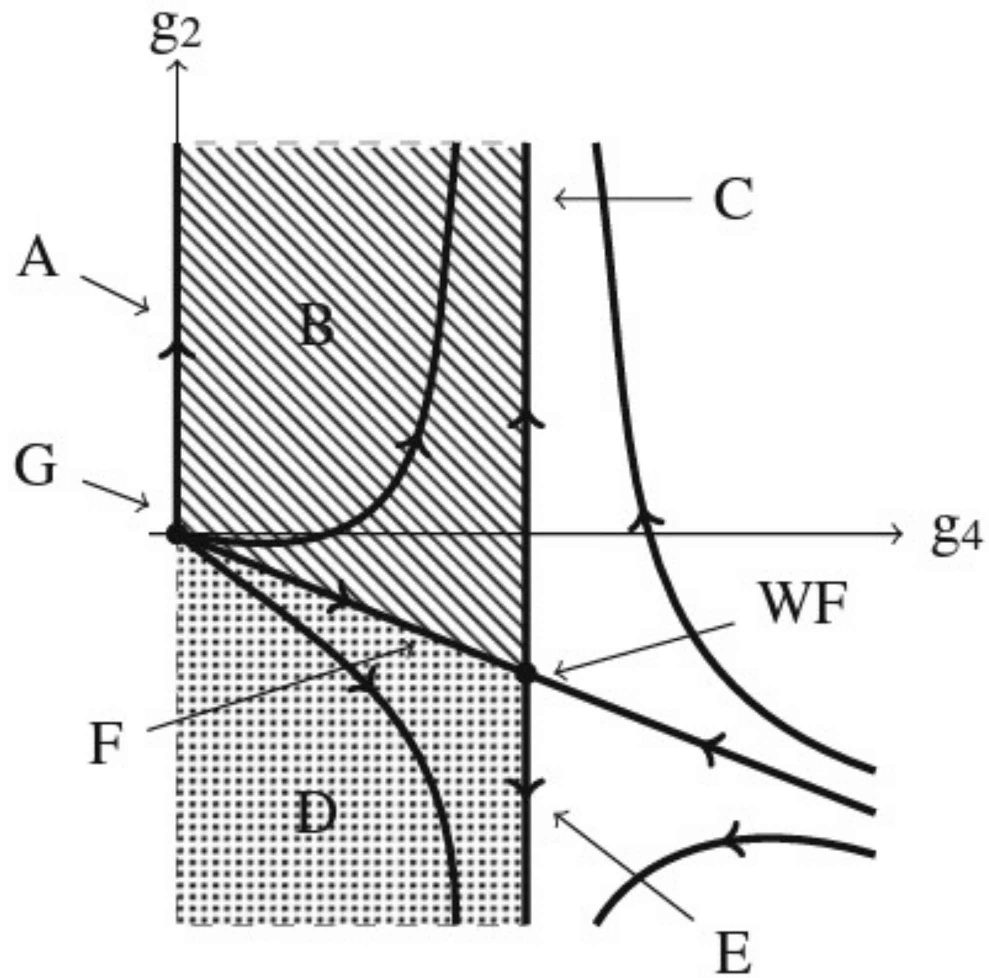
Interaction is exponentially suppressed in N, behaves like a free fermion.

Result consistent with free fermion results of Komargodski, Mezei, Pal, Raviv-Moshe.

Summary

Summary

Concrete examples where a strongly-coupled CFT simplifies at large charge.



O(2N) model in 3d: in the limit of large U(1) charge Q, we computed the conformal dimensions in a controlled perturbative expansion:

- Excellent agreement with lattice results for O(2), O(4)
- large Q and large N: path integral at saddle pt., more control than in EFT, can calculate coefficients
- can follow the flow away from conformal point
- find the full effective potential

New results for fermionic models.

Things I didn't say

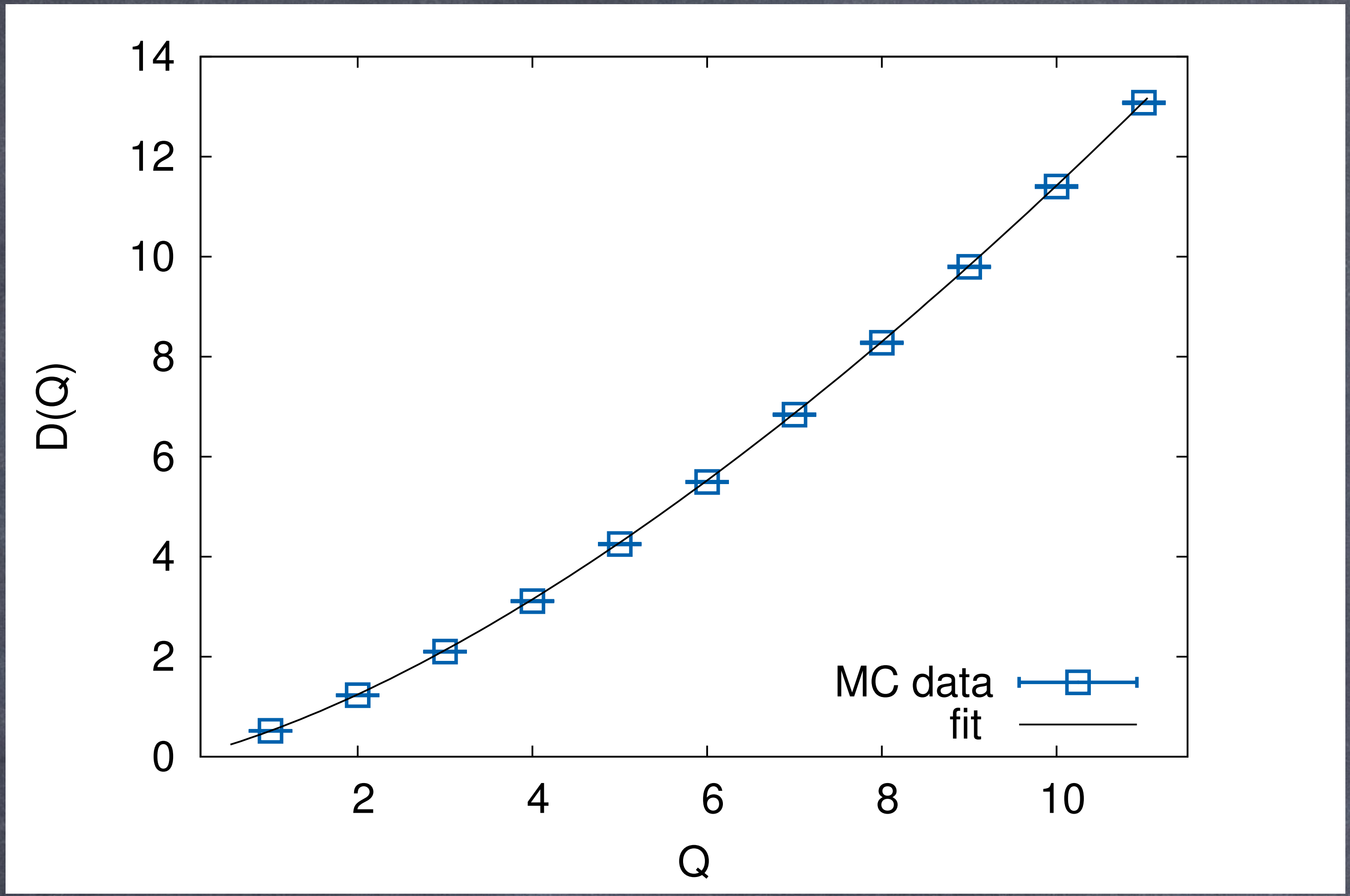
- Non-relativistic CFTs with global U(1).
  - Large-charge expansion exists, quantum corrections and higher-derivative terms are suppressed
  - results in 3+1D match eff. theory for unitary Fermi gas
  - qualitatively different behavior to relativistic case
- SCFT with and without moduli space behave very differently
  - w/o: same result as for O(2)
  - 1d: EFT is a free theory+WZ term
  - prediction matches localization results

Further directions

- Further study of supersymmetric models at large R-charge (higher-dim. moduli spaces) Hellerman, Maeda, Orlando, Reffert, Watanabe; Argyres et al.
- Connection to holography (gravity duals) Loukas, Orlando, Reffert, Sarkar; De la Fuente, Zosso; Giombi, Komatsu, Offertaler.
- Operators with spin; connection to large-spin results Cuomo, de la Fuente, Monin, Pirtskhalava, Rattazzi; Cuomo
- Understanding dualities semi-classically at large charge
- Use/check large-charge results in conformal bootstrap Jafferis and Zhiboedov
- Further lattice simulations: inhomogeneous sector, general O(N) Chandrasekharan et al.
- CFTs in other dimensions (2, 5, 6) Komargodski, Mezei, Pal, Raviv-Moshe; Araujo, Celikbas, Reffert, Orlando; Moser, Orlando, Reffert

Further directions

- Chern-Simons matter theories @large charge — Watanabe
- 4-ε expansion @large charge — Arias-Tamargo, Rodriguez-Gomez, Russo; Badel, Cuomo, Monin, Rattazzi; Watanabe; Antipin et al.
- going away from the conformal point — Orlando, Reffert, Sannino; Orlando, Pellizzani, Reffert
- non-relativistic CFTs — Favrod, Orlando, Reffert; Kravec, Pal; Orlando, Pellizzani, Reffert; Hellerman, Swanson; Pellizzani
- Boundary CFTs at large Q — Cuomo, Mezei, Raviv-Moshe
- Weak gravity conjecture — Aharony, Palti; Antipin et al.
- Study fermionic theories. Can large-charge approach be used for QCD (e.g. large baryon number)?
- Gauge theories @large charge — Komargodski, Mezei, Pal, Raviv-Moshe; Antipin, Bersini, Panopoulos; Dondi, Hellerman, Kalogerakis, Moser, Orlando, Reffert; Antipin et al.



Thank you for your attention!