Holography for the IKKT matrix model

Franz Ciceri École normale supérieure (Lyon)

Based on works with H. Samtleben [2309.10073] [2503.xxxxx]

and G. Bossard, A. Kleinschmidt and G. Inverso [2209.02729]

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(Euclidean) IKKT matrix model

$$S_{\mathrm{IKKT}} = -\mathrm{Tr}\left(\frac{1}{4}[X_a\,,X_b]\,[X^a\,,X^b] + \frac{1}{2}\bar{\Psi}\,\Gamma^a\,[\Psi\,,X_a]\right)$$

[Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

- Theory of $N \times N$ matrices in **0 dimension**, with 16 supersymmetries and **SO(10) invariance**.
- Originally proposed as a **non-perturbative definition IIB superstring theory**.
- « Worldvolume theory » for the D(-1) brane.

⟶ Appears on the field theory side of the holographic dualities (for the extremal case p=-1):

strings/supergravity on the near-horizon geometry of Dp-branes ⟷ (p+1)-dimensional super Yang-Mills theories with 16 supercharges

$\sim AdS_{p+2} \times S^{8-p}$

[Boonstra, Skenderis, Townsend '98]

[Itzhaki, Maldacena, Sonnenschein, Yankielowicz '98]

Despite intriguing and highly non-trivial results in the IKKT model, this duality has so far remained largely unexplored…

(Euclidean) IKKT matrix model

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Goal of the talk:

Lowest KK fluctuations around S^9 described by a one-dimensional « gravity » theory ⟷ Lowest BPS multiplet of gauge invariant operators in the IKKT matrix model

Simplest example of holography?

KK truncation: textbook example

Scalar field Φ on a D-dimensional spacetime $\mathcal{M}_D = \mathcal{M}_d \times S_1$ with coordinates $X^M = (x^\mu, y)$

Expand on a Fourier basis: $\Phi(x,y) = \sum_{n\in\mathbb{Z}} \phi_n(x)\, e^{i\frac{n}{R}y}$ — circle radius

Dynamics: $\hat{\Box}\,\Phi(x,y) = 0 \longrightarrow \Box\phi_n(x) - \frac{n^2}{R^2}\phi_n(x) = 0$

$\hat{\Box} = \Box + \partial_y^2$

Free massless scalar

$n = 0$: massless mode

+

$n \neq 0$: Infinite tower of massive modes with $m_n = \frac{|n|}{R}$

Consistent Kaluza-Klein truncations: truncate to a finite subset of Kaluza-Klein modes (that do not source those that are the truncated).

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Consistent Kaluza-Klein truncations: truncate to a finite subset of Kaluza-Klein modes (that do not source those that are the truncated).

Always consistent on a circle (or torus): truncate to massless modes $\Phi(x,y) = \phi_0(x)$

Independent of the dynamics

$$\hat{\Box}\Phi = \Phi^2 \longrightarrow \Box\phi_n - \frac{n^2}{R^2}\phi_n = \phi_0^2$$

impossible for $n \neq 0$ (group theory)

Sphere truncations of gravity

D: Gravity theory

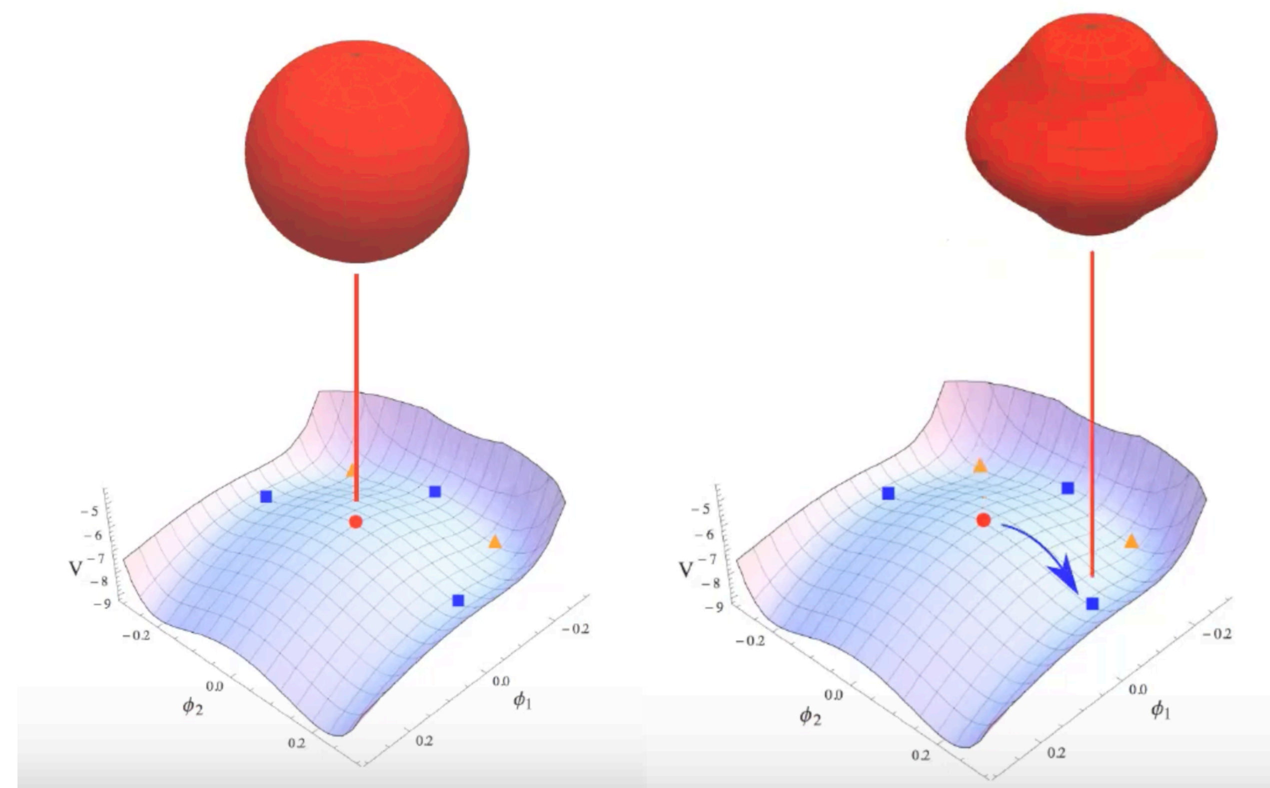
d=D-q: Gravity theory with $SO(q+1)$ YM and scalar potential

Consistency not ensured by group theory…

« Pauli truncation » [1953]

Consistent sphere truncations of gravity are rare and *difficult* to construct, but:

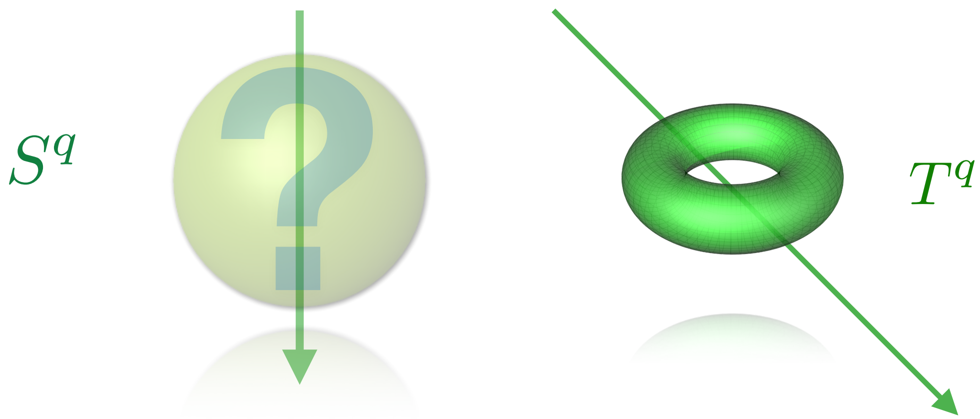
- Allow to uplift all solutions of the lower-dimensional theory.



- Powerful tools for holography

Sphere truncations of gravity

D: Gravity theory



d=D-q: Gravity theory with $SO(q+1)$ YM and scalar potential

Theory with rigid symmetry $\mathcal{G}$

Necessary condition:

$$SO(q+1) \subset \mathcal{G}$$

[Cvetic, Gibbons, Lü, Pope '03]

Sphere truncations of gravity

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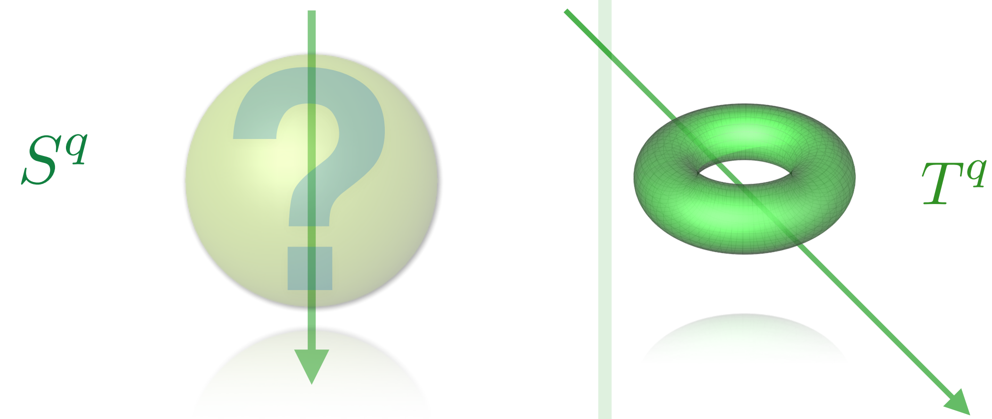

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Plan of the talk:

1. A few words on $\mathcal{G}$ ...

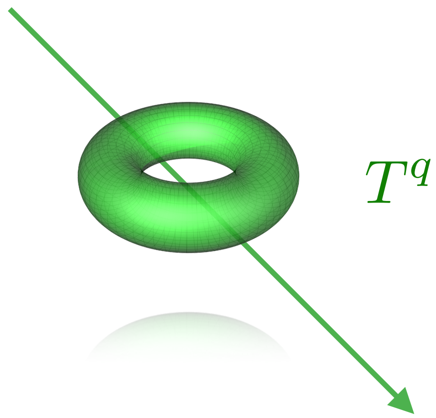
2. Back to spheres

3. Holography for the IKKT model

Pure gravity on the torus

D: Pure gravity

Einstein-Hilbert action



$$G_{MN}(x,\cancel{y}) = \begin{pmatrix} g_{\mu\nu} & \rho^{2/q} A_\mu{}^p\, T_{pn} \\ \rho^{2/q} A_\nu{}^p\, T_{mp} & \rho^{2/q} T_{mn} \end{pmatrix}$$

KK vectors — dilaton — Unimodular symmetric matrix $\in SL(q)$

d=D-q: Theory with rigid symmetry $\mathcal{G}$

$$S_d = \frac{1}{\kappa_d^2} \int d^d x\, e\, \rho \Big[R^{(d)} - \tfrac{1}{4} \rho^{2/q}\, T_{mn}\, F^m_{\mu\nu}\, F^{\mu\nu\, n} + \tfrac{q-1}{q} \rho^{-2}\, \partial_\mu \rho\, \partial^\mu \rho + \tfrac{1}{4} \partial_\mu T^{-1}_{mn}\, \partial^\mu T_{mn} \Big]$$

d-dimensional gravity

$U(1)^q$ Maxwell
$F^m_{\mu\nu} = 2\, \partial_{[\mu} A^m_{\nu]}$

dilaton

$SL(q)/SO(q)$ sigma model

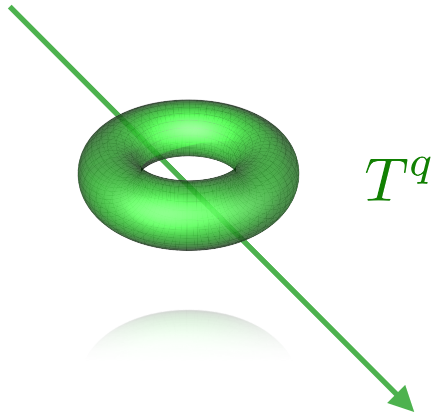
'Free' theory: *no non-abelian gauge interactions, no scalar potential.*

Deformations obtained from truncations on compact manifolds.

Pure gravity on the torus

D: Pure gravity

Einstein-Hilbert action



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d=D-q: Theory with rigid symmetry $\mathcal{G}$

$$S_d = \frac{1}{\kappa_d^2} \int d^d x\, e\, \rho \left[R^{(d)} - \tfrac{1}{4} \rho^{2/q}\, T_{mn}\, F^m_{\mu\nu}\, F^{\mu\nu\, n} + \tfrac{q-1}{q} \rho^{-2}\, \partial_\mu \rho\, \partial^\mu \rho + \tfrac{1}{4} \partial_\mu T^{-1}_{mn}\, \partial^\mu T_{mn} \right]$$

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dilaton

$SL(q)/SO(q)$ sigma model

Rigid symmetry $\mathcal{G} = R^+ \times SL(q)$

Inherited from higher-dimensional diffeomorphisms

$T \longrightarrow \Lambda\, T\, \Lambda^T$

$A_\mu \longrightarrow \lambda^{-1/q} \Lambda^{-1} A_\mu$

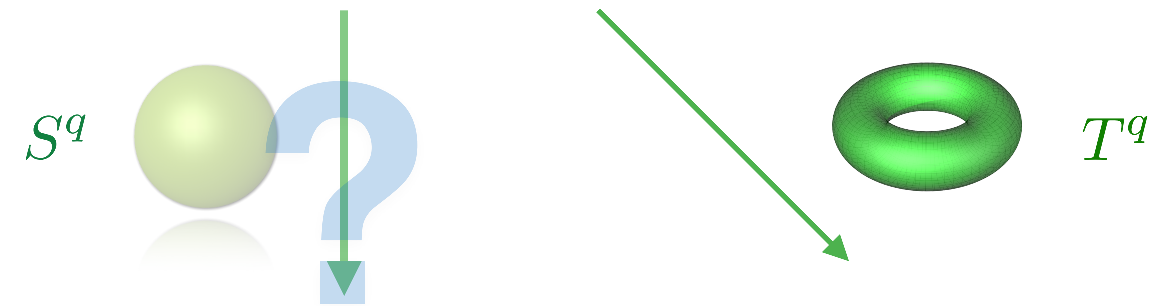
$\rho \longrightarrow \lambda^{1/q} \rho$

Back to sphere

Necessary condition:

$$SO(q+1) \subset \mathcal{G}$$

D: Gravity theory



d=D-q:

Theory with $SO(q+1)$ gauge symmetry

Theory with rigid symmetry $\mathcal{G}$

Doesn't work for pure gravity: $SO(q+1) \not\subset GL(q)$

Back to sphere

Necessary condition:

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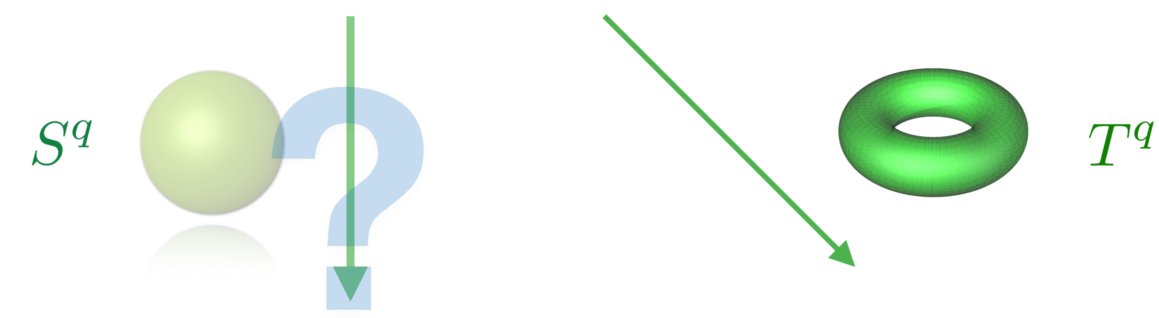


d=D-q:

Theory with $SO(q+1)$ gauge symmetry

Theory with rigid symmetry $\mathcal{G}$

Doesn't work for pure gravity: $SO(q+1) \not\subset GL(q)$

Requires symmetry enhancement of $\mathcal{G}$

→ Only few examples: must start from matter-coupled gravity

<u>Two families</u>

- Gravity + p-form
- Gravity + p-form + dilaton

Famous sphere truncations

- 1st family: Gravity + p-form

D=11 Gravity + 3-form

↓ S^7

d=4 Gravity + SO(8) YM + scalars

[De Wit, Nicolai '80's]

Maximal SUSY solution:

$AdS_4 \times S^7$ ⟷ M2-brane

Conformal branes

AdS/CFT correspondence

Famous sphere truncations

- 1st family: Gravity + p-form

D=11 Gravity + 3-form

↓ S^4

d=7 Gravity + SO(5) YM + scalars+…

[Nastase, Vaman, van Nieuwenhuizen '99]

Maximal SUSY solution:

$AdS_4 \times S^7$	⟷	M2-brane
$AdS_7 \times S^4$	⟷	M5-brane

Conformal branes

AdS/CFT correspondence

Famous sphere truncations

- 1st family: Gravity + p-form

D=10 Gravity + self-dual 4-form

$\downarrow$ S^5

d=5 Gravity + SO(6) YM + scalars

[Baguet, Hohm, Samtleben '15]

Maximal SUSY solution:

$AdS_4 \times S^7$	$\longleftrightarrow$	M2-brane
$AdS_7 \times S^4$	$\longleftrightarrow$	M5-brane
$AdS_5 \times S^5$	$\longleftrightarrow$	D3-brane

Conformal branes

AdS/CFT
correspondence

Less famous sphere truncations

- 2nd family: Gravity + p-form + dilaton

D: Gravity + Maxwell + dilaton

$D \neq 4$

$\downarrow S^2$

D-2: Gravity + $SO(3)$ YM + scalars

[Cvetic, Lü, Pope '00]

1/2-SUSY solution:

$e^{\Phi}(AdS_8 \times S^2)$	$\longleftrightarrow$	D6-brane
$e^{\Phi}(AdS_7 \times S^3)$	$\longleftrightarrow$	D5-brane
$e^{\Phi}(AdS_3 \times S^7)$	$\longleftrightarrow$	D1-brane

Non-conformal branes

Domain wall / QFT correspondence

+ 2 other examples

"Bosonic string" on S^3 and S^{D-3}

Sphere truncations to two dimensions

- 2nd family: Gravity + p-form + dilaton

D: Gravity + Maxwell + dilaton

$D \neq 4$

S^2

D-2: Gravity + $SO(3)$ YM + scalars

D: Gravity + Maxwell + dilaton

S^q

d=2: Gravity + $SO(q+1)$ YM + scalars

[F.C, Samtleben '23]

[Bossard, F.C, Inverso, Kleinschmidt '22 '23]

Why do these work…?

Sphere truncations to two dimensions

- 2nd family: Gravity + p-form + dilaton

D+1: Gravity

↓ S^1

D: Gravity + Maxwell + dilaton

$D \neq 4$

↓ S^2

D-2: Gravity + $SO(3)$ YM + scalars

D+1: Gravity

↓ S^1

D: Gravity + Maxwell + dilaton

↓ S^q

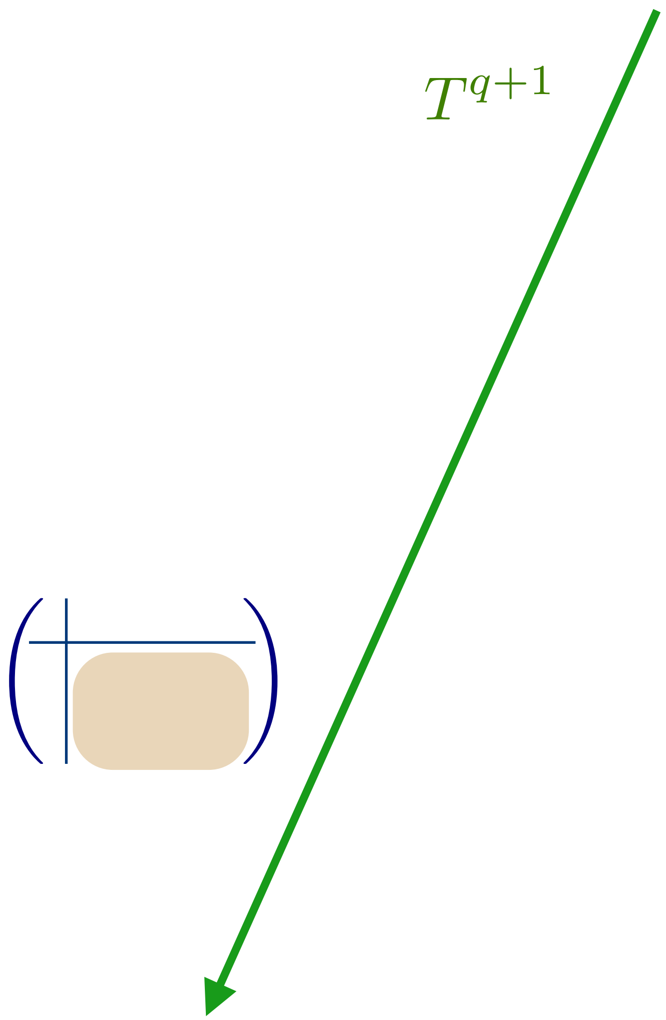
d=2: Gravity + $SO(q+1)$ YM + scalars

Something special
about this case…

Different paths to d=2

$D = 2 + q$

D+1: Gravity



d=2: $R^+ \times SL(q+1)$ symmetry

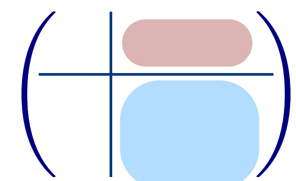
Different paths to d=2

$D = 2 + q$

D+1: Gravity

T^{q+1}

T^{q}

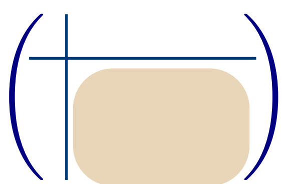


d=3: $R^{+} \times SL(q)$ symmetry

Dualisation of the KK vectors
into scalars $\Longrightarrow SL(q+1)_E$ symmetry

[Ehlers 56']

S^{1}



d=2: $R^{+} \times SL(q+1)$ symmetry

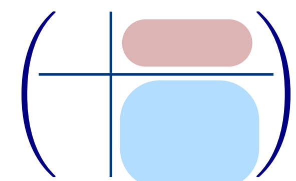
$R^{+} \times SL(q+1)_E$ symmetry

Different paths to d=2

D+1: Gravity

T^{q+1}

T^q



d=3: $R^+ \times SL(q)$ symmetry

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$\Longrightarrow SL(q+1)_E$ symmetry

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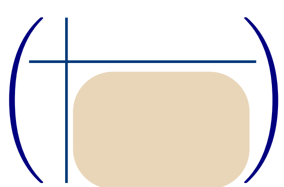
S^1



d=2: $R^+ \times SL(q+1)$ symmetry $\Longleftrightarrow$ $R^+ \times SL(q+1)_E$ symmetry

Realised on scalars dual to each other

Two (on-shell) equivalent versions of the D=2 theory

" $SL(q+1) \times SL(q+1)_E \sim \widehat{SL(q+1)}$ " [Geroch '71]

Affine extension

Holography for BFSS

- 2nd family: Gravity + p-form + dilaton

1/2-SUSY solution:

$e^{\Phi}(AdS_2 \times S^8)$ ⟷ D0-brane

D=11 Supergravity

↓ S^1

D=10: IIA supergravity

↓ S^9

d=2: SO(9) gauged supergravity

[Ortiz, Samtleben '12]

Used to holographically compute correlation functions in the BFSS matrix model

[Ortiz, Samtleben, Tsimpis '14]

Kaluza-Klein truncations around all Dp brane solutions have been constructed

except for the …

D(-1) brane ?

Half-supersymmetric solution of (Euclidean) IIB supergravity

$$ds_{10}^2 = dt^2 + t^2\,\Omega_9^2$$

$$e^{\Phi} = \frac{1}{g_s(1+\frac{Q}{t^8})} = \chi^{-1}$$

Flat space!

[Gibbons, Green, Perry '96]

Decoupling limit → *Holographic dual of the IKKT matrix model*

[Ooguri, Skenderis '98]

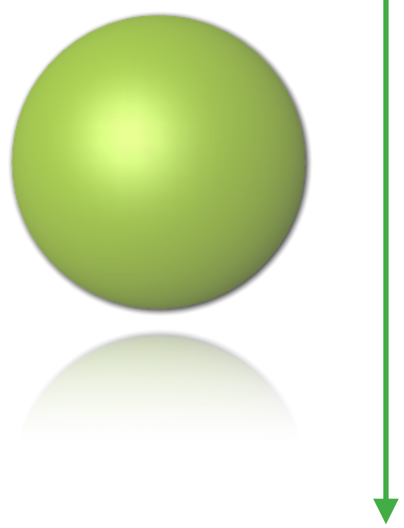
Holographic coordinate t : emergence of time?

No clear group theoretical argument for the existence of a sphere truncation to 1D…

Sphere truncations to one dimension

D: (**Euclidean**) gravity + dilaton + axion

$$\mathcal{L}_D = E\Big(R - \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}e^{2\Phi}(\partial\chi)^2\Big)$$



S^q

$q+1=D$

$$ds_D^2 = e^{\frac{2q\varphi}{q^2-1}}\,\Delta\, ds_1^2 + g^{-2}e^{\frac{2\varphi}{q^2-1}}T^{-1}_{ij}\mathcal{D}\mu^i\,\mathcal{D}\mu^j$$

$$e^{\Phi} = e^{-\frac{\varphi}{q+1}}\,\Delta^{-1}$$

$$\chi = -\frac{\mathrm{e}^{-1}}{2g}T^{-1}_{ij}\mathcal{D}_t T_{kj}\,\mu^i\mu^k + \frac{\mathrm{e}^{-1}}{(q^2-1)g}\,\dot{\varphi}$$

d=1: Gravity + $SO(q+1)$ YM + scalars

$$\mathcal{L}_1 = \frac{1}{2(q^2-1)}\mathrm{e}^{-1}\,\dot{\varphi}^2 + \mathrm{e}^{-1}\mathcal{D}_t T^{-1}_{ij}\mathcal{D}_t T_{ij} + \frac{1}{2}\mathrm{e}\,g^2\,e^{\frac{2\varphi}{q+1}}(2\,T_{ij}T_{ij} - T_{ii}^2)$$

SO(q+1) gauged quantum mechanics

- No curvature tensors in one dimension
  - → Variation w.r.t Einbein and gauge fields lead to first order constraints.
- For relation to IKKT: we set D=10.

Sphere truncations to one dimension

D=10: (**Euclidean**) gravity + dilaton + axion

$$\mathcal{L}_{10} = E\Big(R - \frac{1}{2}(\partial\Phi)^2 + e^{2\Phi}(\partial\chi)^2\Big)$$

S^9

$$ds^2_{10} = e^{\frac{9\varphi}{40}}\,\Delta\, ds^2_1 + g^{-2} e^{\frac{\varphi}{40}} T^{-1}_{ij} \mathcal{D}\mu^i \mathcal{D}\mu^j$$

$$e^{\Phi} = e^{-\frac{\varphi}{10}}\,\Delta^{-1}$$

$$\chi = -\frac{\mathrm{e}^{-1}}{2g} T^{-1}_{ij} \mathcal{D}_t T_{kj}\, \mu^i \mu^k + \frac{\mathrm{e}^{-1}}{80g}\,\dot{\varphi}$$

d=1: Gravity + SO(10) YM + scalars

$$\mathcal{L}_1 = \frac{1}{160}\mathrm{e}^{-1}\,\dot{\varphi}^2 + \mathrm{e}^{-1}\mathcal{D}_t T^{-1}_{ij}\mathcal{D}_t T_{ij} + \frac{1}{2}\mathrm{e}\, g^2\, e^{\frac{\varphi}{5}}\,(2\,T_{ij}T_{ij} - T^2_{ii})$$

$i\,,j = 1\,,\ldots 10$

SO(10) invariant solution: $T_{ij} = \delta_{ij}$ $\quad e^{\varphi} \sim t^{40}$ $\quad \mathrm{e} \sim t^{-9}$ — Uplifts to D=10 → D(-1) instanton solution (1/2-SUSY)

IKKT is supersymmetric, so let's discuss fermions…

Asking for 32 supercharges implies…

Bosons

e	Einbein		
φ	dilaton		
$A_t^{[ij]}$	$SO(10)_g$ gauge fields		
$T_{(ij)}$	$\frac{SL(10)}{SO(10)}$	scalars	**54**
$a_{[ijk]}$	scalars		**120**

SO(10) Fermions (32 comp. spinors)

ψ	gravitini	
λ	« spin-1/2 »	
χ_a	« spin-1/2 », Γ^a-traceless	**144**

$a\,,b = 1\,,\ldots 10$

$$\{\Gamma^a\,,\Gamma^b\} = 2\,\delta^{ab}\,\mathbf{1}_{32}$$

$$\mathcal{L}_1 = \frac{1}{160}\mathrm{e}^{-1}\dot{\varphi}^2 + \mathrm{e}^{-1}\mathcal{D}_t T^{-1}_{ij}\mathcal{D}_t T_{ij} - \frac{1}{12}\mathrm{e}^{-1}e^{-\frac{\varphi}{20}}\,T^{-1}_{ip}T^{-1}_{jq}T^{-1}_{kr}\,\mathcal{D}_t a_{ijk}\,\mathcal{D}_t a_{pqr} + \mathrm{e}\,\mathbb{V}[T\,,\varphi\,,a]$$
$$+\frac{1}{160}\bar{\lambda}\mathcal{D}_t\lambda - 2\,\bar{\chi}_a\Gamma^{ab}\,\mathcal{D}_t\chi_b + \mathcal{L}_{\text{Noether}} + \mathcal{L}_{\text{Yukawa}}$$

up to quadratic fermions

Full scalar potential

$$\mathbb{V}[T\,,\varphi\,,a] = g^2\,e^{\frac{\varphi}{5}}\left(2\,T_{ij}T_{ij} - T_{ii}^2\right) + \ldots + \mathcal{O}(a^8)$$

[F.C, Samtleben to appear]

A simple set of Killing spinors

SUSY variations at

$a_{ijk} = 0$

$$\delta_\epsilon \psi = D_t \epsilon - \frac{\mathrm{e}}{4} e^{\frac{\varphi}{10}} T_{ii}\, \Gamma_* \epsilon$$

$$\delta_\epsilon \lambda = \mathrm{e}^{-1} \dot{\varphi}\, \Gamma_* \epsilon + 4\, e^{\frac{\varphi}{10}} T_{ii}\, \epsilon$$

$$\delta_\epsilon \chi_a = (\mathcal{D} T \ldots)_{ab}\, \Gamma^b \epsilon + (T \ldots)_{ab}\, \Gamma^b \Gamma_* \epsilon$$

Chirality matrix $\Gamma_* = \begin{pmatrix} \mathbf{1}_{16} & 0 \\ 0 & -\mathbf{1}_{16} \end{pmatrix}$

Killing spinor equations:

$$\delta_\epsilon(\text{fermions}) \overset{!}{=} 0$$

Only two possibilities: non-supersymmetric configurations or…

1/2-supersymmetric configurations:

$$\delta_\epsilon \lambda = \ldots \left(\mathbf{1}_{32} \pm \Gamma_*\right) \epsilon \overset{!}{=} 0$$

$$\delta_\epsilon \chi_a = (\ldots)_{ab}\, \Gamma^b \left(\mathbf{1}_{32} \pm \Gamma_*\right) \epsilon \overset{!}{=} 0$$

$\longrightarrow$ $\epsilon_- = \begin{pmatrix} \varepsilon(t) \\ 0 \end{pmatrix}$

A simple set of Killing spinors

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→ $\epsilon_- = \begin{pmatrix} \varepsilon(t) \\ 0 \end{pmatrix}$

This factorization requires the bosonic fields to satisfy 1st order equations that:

- imply all the field equations.
- can be solved exactly

General 1/2-SUSY solution:

$$T_{ij} = \begin{pmatrix} e^{\phi_1} & & \\ & \ddots & \\ & & e^{\phi_{10}} \end{pmatrix} \quad \text{with} \quad e^{\phi_i} = \frac{\prod_{j=1}^{10} (\tilde{t} + c_j)^{1/10}}{\tilde{t} + c_i}$$

→ uplifts to 10D flat space!

dual to v.e.v deformations of IKKT?

On the field theory side

$$S_{\rm IKKT} = -{\rm Tr}\Big(\frac{1}{4}[X_a\,,X_b]\,[X^a\,,X^b] + \frac{1}{2}\bar\Psi\,\Gamma^a\,[\Psi\,,X_a]\Big)$$

16 supercharges $\Gamma_*\epsilon = \epsilon$

algebra: $[\delta_{\epsilon_1}\,,\delta_{\epsilon_2}] = \delta_\theta^{SU(N)}$

- Toroidal reduction of 10D super YM to 0 dimension: no time…
- X_a and Ψ are $SU(N)$ matrices, transforming in the $\mathbf{10}$ and $\mathbf{16_s}$ of $SO(10)$.

- $SU(N)$ invariant single trace operators: $\mathcal{O} = {\rm Tr}\Big[XX\ldots\Psi\ldots X\ldots\Psi\ldots\Big]$ $\longrightarrow$ arrange in supermultiplets

Tower of BPS multiplets: $\sum_{n=2}^{\infty}\mathcal{B}_n$ which combine SO(10) rep.

$$\mathcal{B}_n = [n,0000]_n \oplus [n-1,0001]_{n+\frac{1}{2}} \oplus [n-2,0100]_{n+1} \oplus [n-3,1010]_{n+\frac{3}{2}} \oplus [n-3,0020]_{n+2} \oplus [n-4,2000]_{n+2} \oplus [n-4,1010]_{n+\frac{5}{2}} \oplus [n-4,0100]_{n+3} \oplus [n-4,0001]_{n+\frac{7}{2}} \oplus [n-4,0000]_{n+4}$$

$$\mathcal{O}_{a_1\ldots a_n} = {\rm Tr}\big[X_{((a_1}X_{a_2}\ldots X_{a_n))}\big]$$

[Morales, Samtleben '05]

- 1D supergravity fields $\longleftrightarrow$ lowest BPS multiplet of operators

$$\mathcal{B}_2 = \mathbf{54}_{+2} \oplus \mathbf{144}_{+\frac{5}{2}} \oplus \mathbf{120}_{+3} \ominus \mathbf{45}_{+4} \ominus \mathbf{16}_{+\frac{9}{2}}$$

? $\#_{\rm bos.} = \#_{\rm ferm.} + 1$

Lowest BPS multiplet

$$S_{\rm IKKT} = -{\rm Tr}\Big(\frac{1}{4}[X_a\,,X_b]\,[X^a\,,X^b] + \frac{1}{2}\bar\Psi\,\Gamma^a\,[\Psi\,,X_a]\Big)$$

16 supercharges $\Gamma_*\epsilon = \epsilon$

algebra: $[\delta_{\epsilon_1}\,,\delta_{\epsilon_2}] = \delta_\theta^{SU(N)}$

- Toroidal reduction of 10D super YM to 0 dimension: no time…
- X_a and Ψ are $SU(N)$ matrices, transforming in the $\mathbf{10}$ and $\mathbf{16_s}$ of $SO(10)$.

$\mathcal{B}_2$ multiplet of operators

SUSY variations

54	$\mathcal{O}_{ab} = {\rm Tr}[X_aX_b] - \frac{1}{10}\,\delta_{ab}\,{\rm Tr}[X^cX_c]$	$\delta_\epsilon\mathcal{O}_{ab} = \dfrac{9}{5}\bar\epsilon\,\Gamma^{[a}\,\mathcal{O}^{b]}$
144	$\mathcal{O}^a = {\rm Tr}[X^a\,\Psi] - \frac{1}{9}\,{\rm Tr}[X_b\,\Gamma^{ab}\Psi]$	$\delta_\epsilon\mathcal{O}_a = \dfrac{1}{18}\left(7\,\Gamma^{bc}\,\epsilon\,\mathcal{O}_{abc} - \Gamma_{abcd}\,\epsilon\,\mathcal{O}^{bcd}\right)$
120	$\mathcal{O}_{abc} = {\rm Tr}\,[X_a[X_b,X_c]] - \frac{1}{8}\,{\rm Tr}\,\left[\bar\Psi\Gamma_{abc}\Psi\right]$	$\delta_\epsilon\mathcal{O}_{abc} = 0$

[F.C, Samtleben to appear]

Nilpotent structure of SUSY variations → Deformations by $\mathcal{O}_{ab}$ preserve 16 supercharges

A dictionary

$$S_{\rm IKKT} = -{\rm Tr}\Big(\frac{1}{4}[X_a\,,X_b]\,[X^a\,,X^b] + \frac{1}{2}\bar{\Psi}\,\Gamma^a\,[\Psi\,,X_a]\Big)$$

16 supercharges $\Gamma_* \epsilon = \epsilon$

algebra: $[\delta_{\epsilon_1}\,,\delta_{\epsilon_2}] = \delta_\theta^{SU(N)}$

- Toroidal reduction of 10D super YM to 0 dimension: no time…
- X_a and Ψ are $SU(N)$ matrices, transforming in the $\mathbf{10}$ and $\mathbf{16_s}$ of $SO(10)$.

	$\mathcal{B}_2$ multiplet of operators	Holographic dictionary	1D supergravity fluctuations
54	$\mathcal{O}_{ab} = {\rm Tr}[X_a X_b] - \frac{1}{10}\,\delta_{ab}\,{\rm Tr}[X^c X_c]$	⟷	T_{ab}
144	$\mathcal{O}^a = {\rm Tr}[X^a\,\Psi] - \frac{1}{9}\,{\rm Tr}[X_b\,\Gamma^{ab}\Psi]$	⟷	χ^a
120	$\mathcal{O}_{abc} = {\rm Tr}\,[X_a[X_b, X_c]] - \frac{1}{8}\,{\rm Tr}\,\left[\bar{\Psi}\Gamma_{abc}\Psi\right]$	⟷	a_{abc}

Holographic dictionary only based on kinematical properties.

Conclusion

- We presented sphere truncations of gravity theories to one (Euclidean) time dimension that capture the lowest KK fluctuations around the D(-1) brane solution.

- We constructed the maximally supersymmetric completion (32 supercharges) of the resulting one-dimensional model ⟶ Powerful tool to study holography for IKKT.

- We discussed the holographic dictionary between the lowest BPS multiplet of gauge invariant operators in the IKTT model and the one-dimensional supergravity multiplet.

Open directions

- Compute mass spectrum and correlation functions in 1d SUGRA for quantitative comparisons with IKKT results.

- Compute the on-shell action for our IIB solutions ⟷ IKKT free energy.

  ⟶ Spontaneous symmetry breaking of SO(10) in IKKT: emergence of large dimensions?

  [Nishimura, Okubo, Sugino '11] [Kim, Nishimura, Tsuchiya '12] and many more…

- Can we describe the mass-deformed IKKT model (« polarized ») within our truncation?

  [Hartnoll, Liu '24] [Komatsu, Penedones, Vuignier, Zhao '24]

- Relation with Lorentzian version of IKKT model?

  For recent results see [Asano, Nishimura, Piensuk, Yamamori '24]

- Meaning of the large N limit (`erratic' behaviour of partition the IKKT partition function)..?

  [Krauth, Nicolai, Staudacher '98] [Moore, Nekrasov, Shatashvili '98]

ご清聴ありがとうございました

Thank you for your attention