Resonant Non-Gaussianity from Axion Monodromy Inflation

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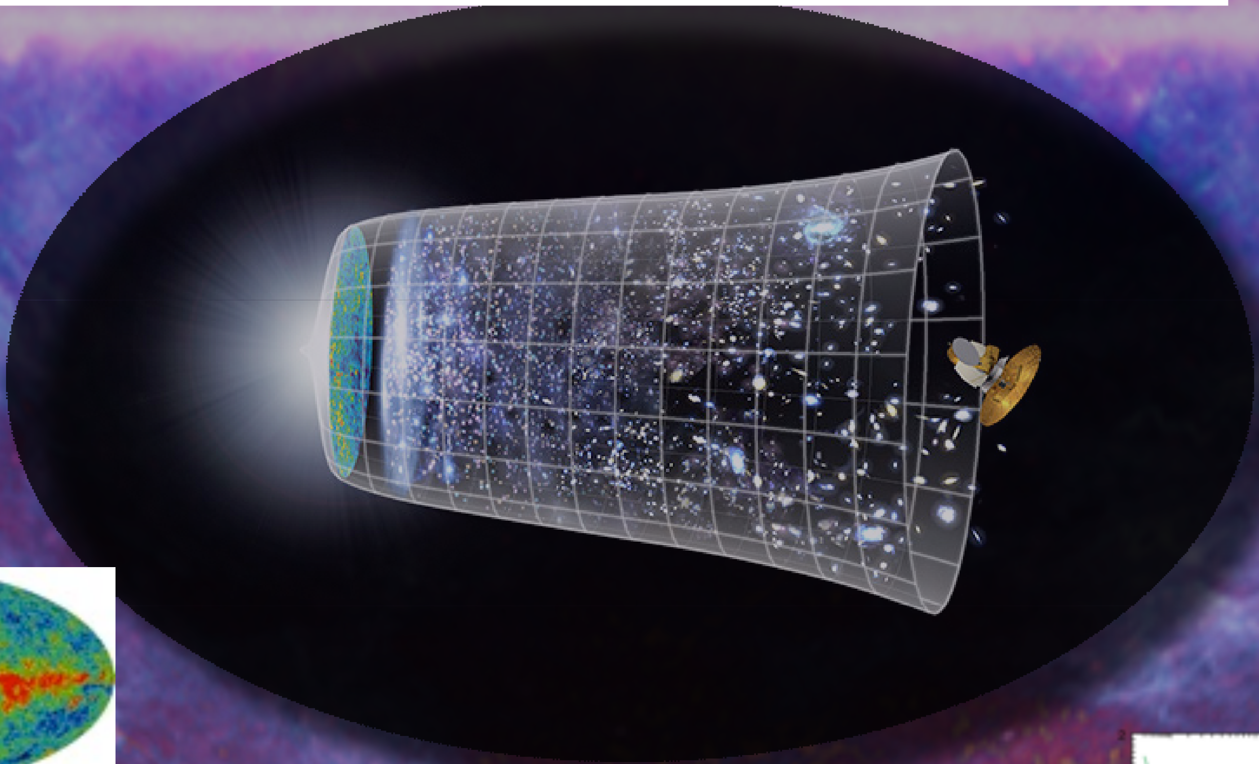
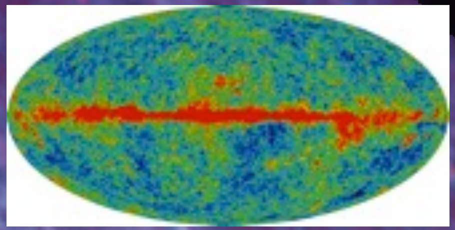
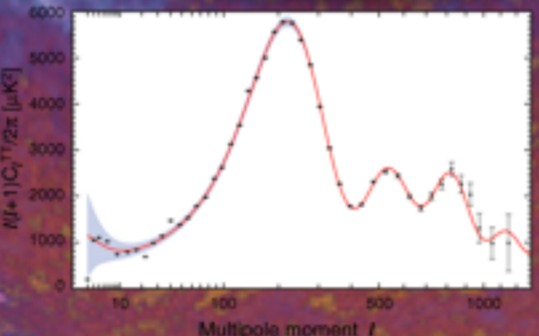
Outline

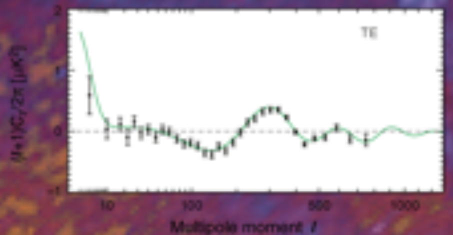
- Brief Introduction to CMB Observables
- From the CMB to inflation (and to strings?)
- Basic Ingredients of Axion Monodromy Inflation
- Signatures in the CMB
- Conclusions

Brief Introduction to CMB Observables



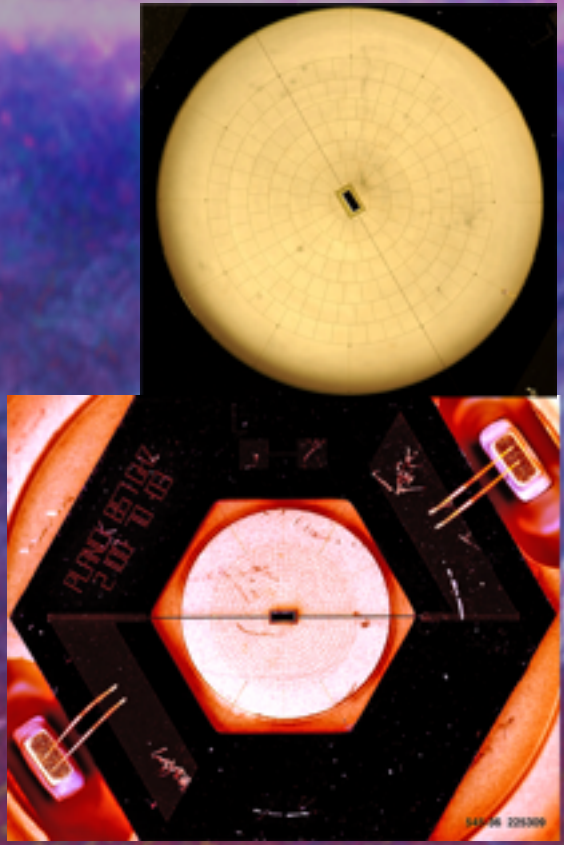






CMB Observables

(Holmes et al. 2008)



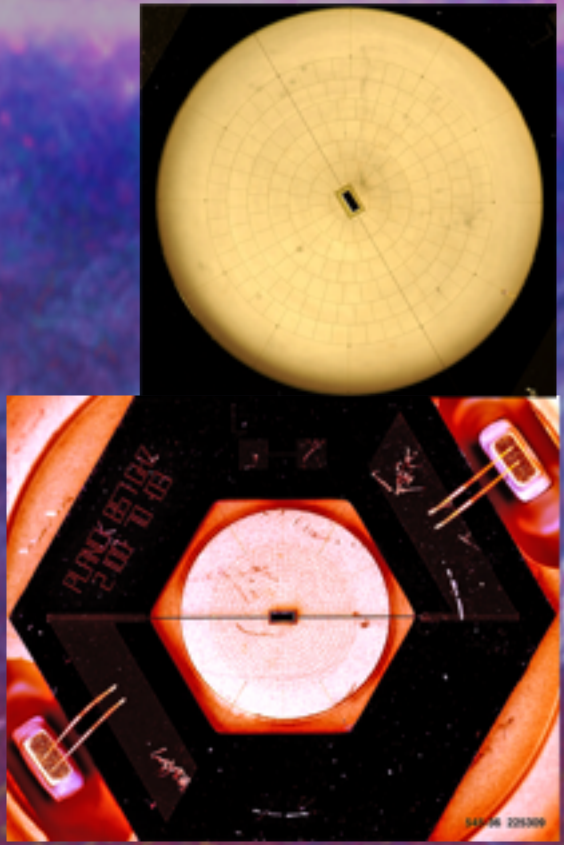
Think of the detector as a device that counts the number of photons hitting it per unit time per unit area with a given polarization.

It then measures the intensity

$$\mathcal{I}(\mathbf{x}, \hat{n}, t, \gamma)$$

CMB Observables

Think of the detector as a device that counts the number of photons hitting it per unit time per unit area with a given polarization.



(Holmes et al. 2008)

It then measures the intensity

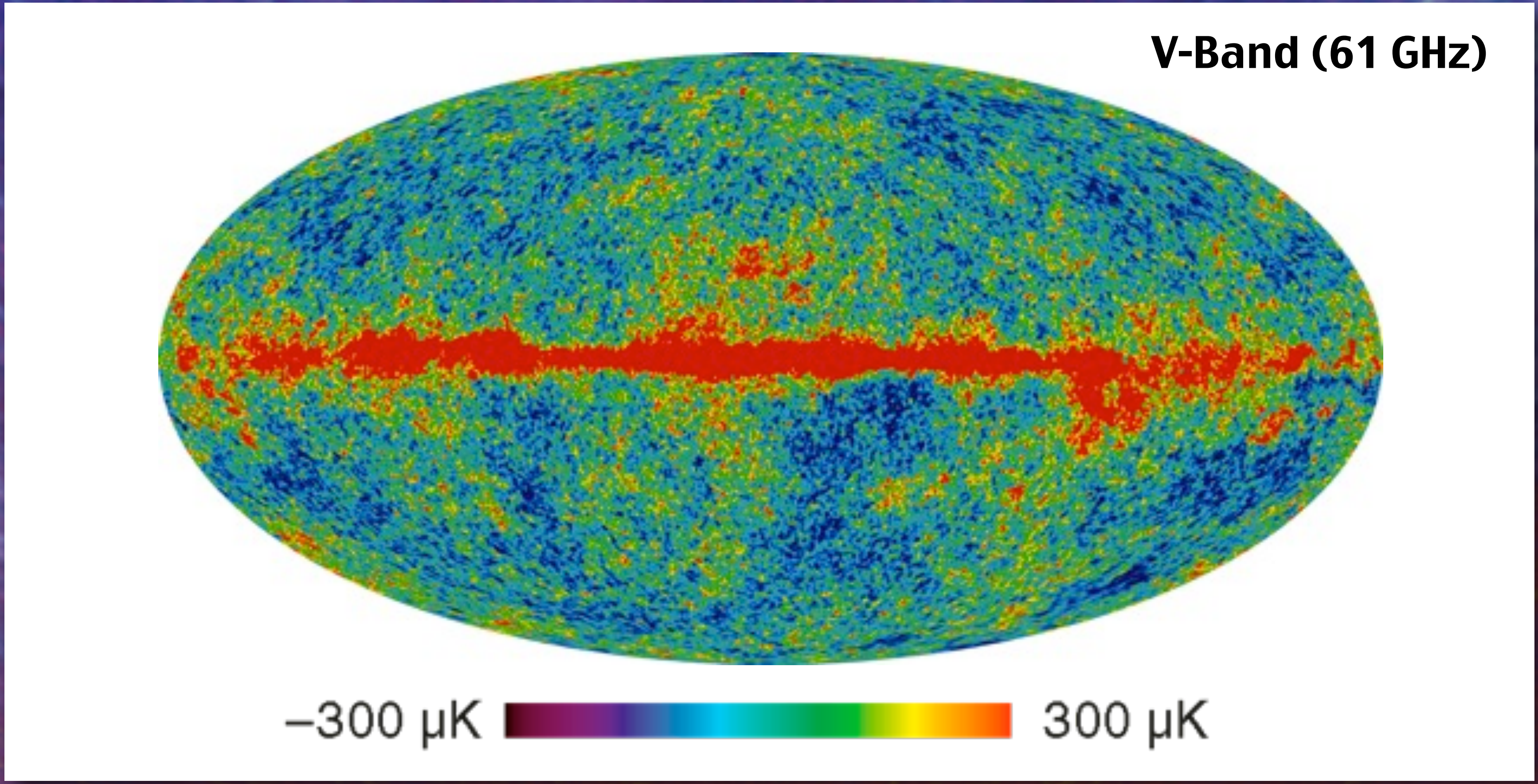
$$\mathcal{I}(0, \hat{n}, t_0, \gamma) = \mathcal{I}_0 \left(1 + \frac{4\Delta T(\hat{n})}{T_0} + \frac{4\sqrt{Q^2(\hat{n}) + U^2(\hat{n})}}{T_0} \cos\left(2\gamma(\hat{n}) - \arctan\frac{U(\hat{n})}{Q(\hat{n})}\right)\right)$$

CMB Observables

This is usually shown in the form of color-coded maps

$$\Delta T(\hat{n})$$

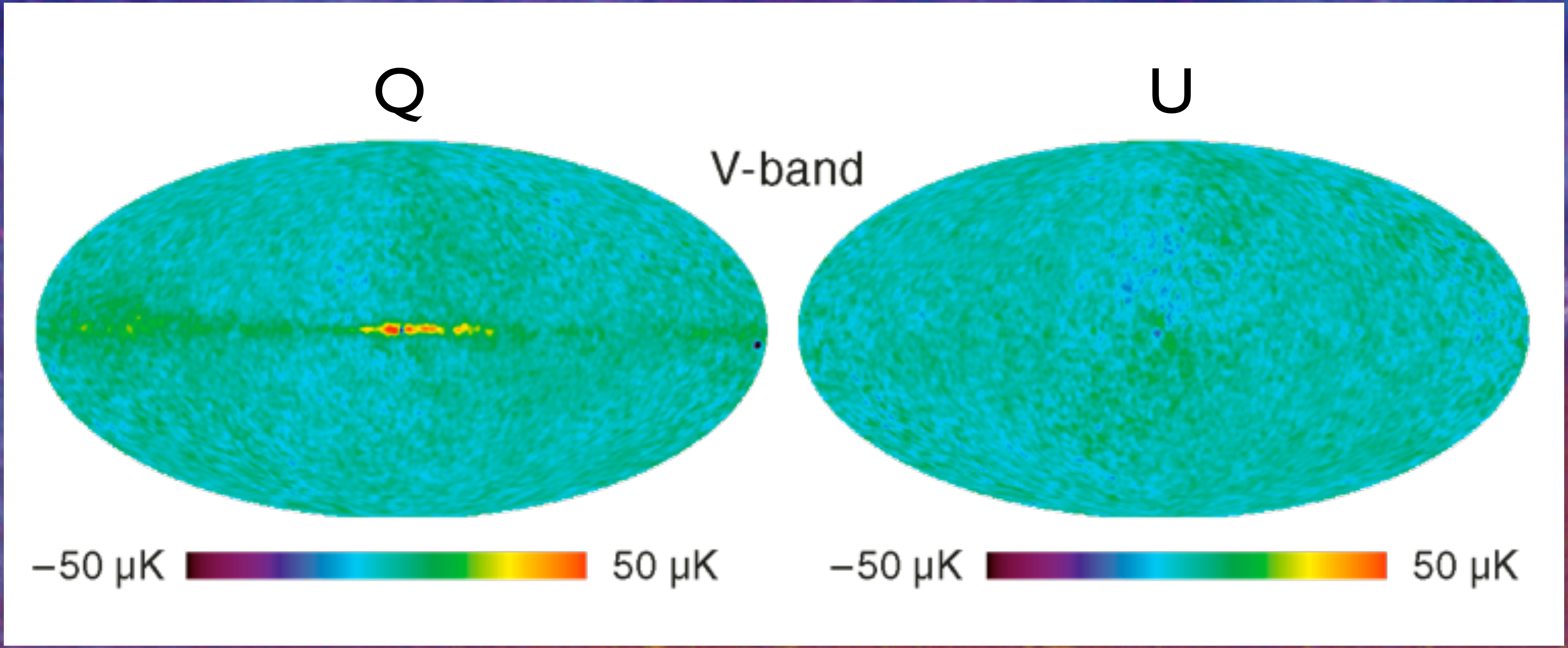
(Jarosik et al. 2010)



CMB Observables

The Stokes parameters Q and U can then also be shown as color-coded maps

(Jarosik et al. 2010)



CMB Observables

Comparison with theory

These quantities depend on initial conditions and cannot be predicted theoretically. What can be predicted are the correlations

$$\langle \Delta T(\hat{n}) \Delta T(\hat{n}') \rangle \quad ,$$

$$\langle \Delta T(\hat{n}) \left[Q(\hat{n}') + iU(\hat{n}') \right] \rangle \quad ,$$

$$\langle \left[Q(\hat{n}) + iU(\hat{n}) \right] \left[Q(\hat{n}') + iU(\hat{n}') \right] \rangle \quad ,$$

$$\langle \left[Q(\hat{n}) + iU(\hat{n}) \right] \left[Q(\hat{n}') - iU(\hat{n}') \right] \rangle \quad .$$

as well as higher n-point functions.

CMB Observables

Comparison with theory

It turns out to be more convenient to use coefficients

$$a_{T,\ell m} = \int d^2\hat{n}\, Y_\ell^{m*}(\hat{n})\Delta T(\hat{n})$$

$$a_{P,\ell m} = \int d^2\hat{n}\; {}_2Y_\ell^{m*}(\hat{n})\left(Q(\hat{n}) + iU(\hat{n})\right)$$

$$a_{E,\ell\, m} \equiv -(a_{P,\ell\, m} + a^*_{P,\ell\, -m})/2$$
$$a_{B,\ell\, m} \equiv i(a_{P,\ell\, m} - a^*_{P,\ell\, -m})/2$$

CMB Observables

Comparison with theory

The correlations between temperature fluctuations and polarization can then be given in terms of multipole coefficients $C_{XY,\ell}$ defined by

$$\left\langle a_{T,\ell\,m} a^*_{T,\ell'\,m'} \right\rangle = C_{TT,\ell} \delta_{\ell\ell'} \delta_{mm'} \,,$$

$$\left\langle a_{T,\ell\,m} a^*_{E,\ell'\,m'} \right\rangle = C_{TE,\ell} \delta_{\ell\ell'} \delta_{mm'} \,,$$

$$\left\langle a_{E,\ell\,m} a^*_{E,\ell'\,m'} \right\rangle = C_{EE,\ell} \delta_{\ell\ell'} \delta_{mm'} \,,$$

$$\left\langle a_{B,\ell\,m} a^*_{B,\ell'\,m'} \right\rangle = C_{BB,\ell} \delta_{\ell\ell'} \delta_{mm'} \,.$$

CMB Observables

Comparison with theory

These multipole coefficients can be calculated for a given model, and they can be estimated from the sky maps by

$$C^{\mathrm{obs}}_{TT,\ell} \equiv \frac{1}{2\ell+1}\sum_m \left|a^{\mathrm{obs}}_{T,\ell\,m}\right|^2$$

$$a^{\mathrm{obs}}_{T,\ell m} = \int d^2\hat{n}\, Y^{m*}_\ell(\hat{n})\Delta T(\hat{n})$$

and similarly for the others.

CMB Observables

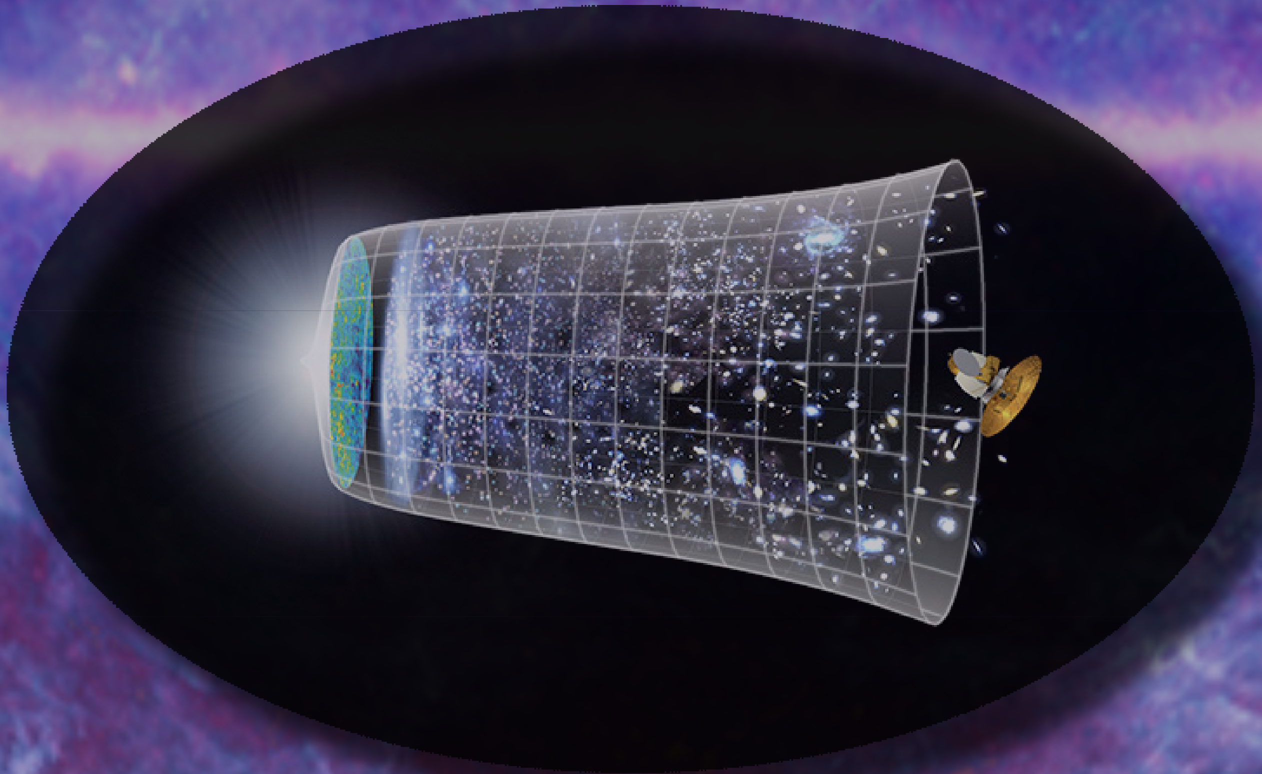
Comparison with theory

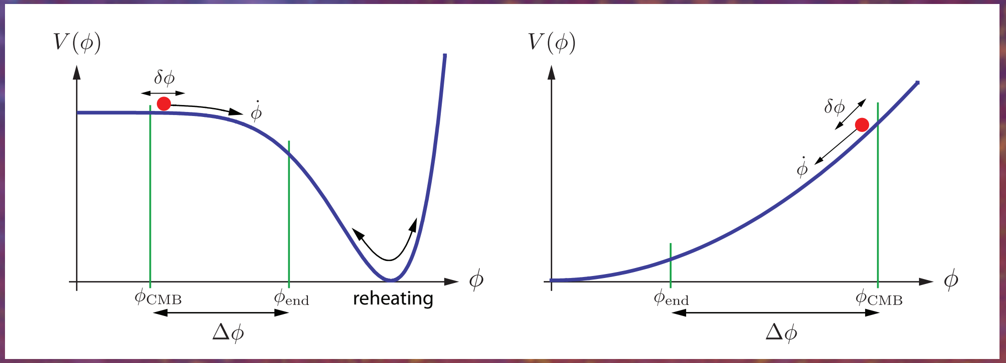
Similarly, the information about three-point correlations are contained in

$$\langle a_{X,\ell_1 m_1} a_{Y,\ell_2 m_2} a_{Z,\ell_3 m_3} \rangle = \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} b_{XYZ,\ell_1 \ell_2 \ell_3}$$

Currently neither data nor analysis tools are good enough to measure this bispectrum directly. Instead the magnitude of the temperature bispectrum is constrained for different shapes.

From the CMB to inflation





From the CMB to inflation

$$ds^2 = -dt^2 + a(t)^2((1 + A)\delta_{ij} + \partial_i\partial_j B + h_{ij})dx^i dx^j$$

scalar modes $C_{TT,\ell}, C_{TE,\ell}, C_{EE,\ell}$

additional scalar modes include

$$\delta\rho_b, \delta\rho_c, \ldots$$

tensor modes $C_{TT,\ell}, C_{TE,\ell}, C_{EE,\ell}, C_{BB,\ell}$

From the CMB to inflation

The system of equations that governs the evolution of the scalar modes from around few keV to the present contains equations like

$$\frac{k^2}{a^2}A_k + H\left(3\dot{A}_k - k^2\dot{B}_k\right) =$$

$$8\pi G\left(\delta\rho_{b\,k} + \delta\rho_{c\,b} + \bar{\rho}_\gamma\Delta^{(S)}_{T,0} + \bar{\rho}_\nu\Delta^{(S)}_{\nu,0}\right)$$

$$\delta\dot{\rho}_{c\,k} + 3H\delta\rho_{c\,k} + \frac{1}{2}\bar{\rho}_{c\,k}\left(3\dot{A}_k - k^2\dot{B}_k\right) = 0$$

$$\vdots$$

From the CMB to inflation

$$C_{XX,\ell}^{(S)} = 4\pi T_0^2 \int \frac{dk}{k} \Delta_{\mathcal{R}}^2(k) \left| \int_0^{\tau_0} d\tau S_X^{(S)}(k,\tau) j_\ell(k(\tau_0 - \tau)) \right|^2$$

From the CMB to inflation

Initial Conditions

Late time evolution

$$C^{(S)}_{XX,\ell} = 4\pi T_0^2 \int \frac{dk}{k} \Delta^2_{\mathcal{R}}(k) \left| \int_0^{\tau_0} d\tau\, S_X^{(S)}(k,\tau)\, j_\ell(k(\tau_0 - \tau)) \right|^2$$

Physics of Recombination

Geometry

From the CMB to inflation

In single field inflation (at linear order in spatially flat gauge):

$$\mathcal{R}(\mathbf{k},t) = -H\frac{\delta\phi(\mathbf{k},t)}{\dot{\phi}}$$

Initial conditions are set by

$$\langle \mathcal{R}(\mathbf{k},t)\mathcal{R}(\mathbf{k}',t)\rangle = (2\pi)^6\delta(\mathbf{k}+\mathbf{k}')\frac{\Delta^2_{\mathcal{R}}(k)}{4\pi k^3}$$

$$\langle \mathcal{R}(\mathbf{k_1},t)\mathcal{R}(\mathbf{k_2},t)\mathcal{R}(\mathbf{k_3},t)\rangle =$$

$$(2\pi)^7\Delta^4_{\mathcal{R}}\frac{1}{k_1^2k_2^2k_3^2}\delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3}$$

From the CMB to inflation

For standard single field *slow-roll* inflation, the primordial spectrum of scalar perturbations is

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^2(t_k)}{8\pi^2\epsilon(t_k)} \approx \Delta^2_{\mathcal{R}} \left(\frac{k}{k_*}\right)^{n_s - 1}$$

with $n_s = 1 - 4\epsilon_* - 2\delta_*$

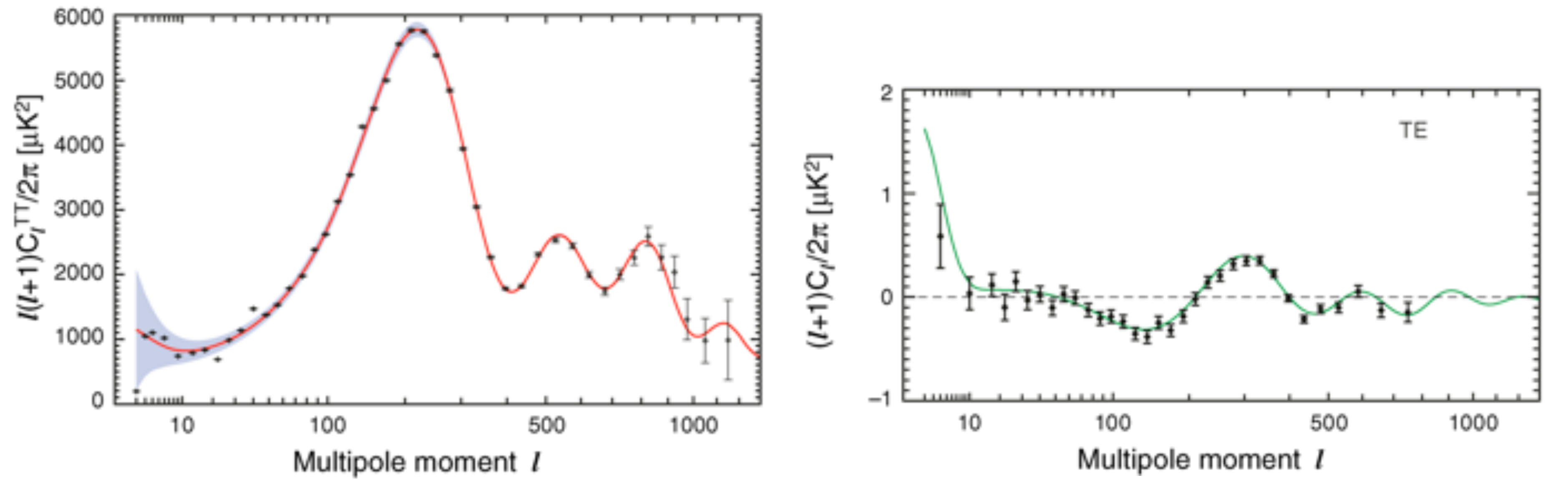
and $\epsilon = -\frac{\dot{H}}{H^2}$ $\delta = \frac{\ddot{H}}{2H\dot{H}}$

and the 3-pt function is too small to be observed.

From the CMB to inflation

Comparison with theory

(Larson et al. 2010)



From the CMB to inflation

In addition to the scalar modes, inflation also predicts a nearly scale invariant spectrum of tensor modes

$$\Delta_h^2(k) = \frac{2H^2(t_k)}{\pi^2}$$

A measurement of the tensor contribution would provide a direct measurement of the energy scale of inflation!

From the CMB to inflation

The tensor-to-scalar ratio r

Whether the tensor contribution is observable is often discussed in terms of the tensor-to-scalar ratio

$$r = \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2} = 16\epsilon$$

$$V_{\rm inf}^{1/4} = 1.06 \times 10^{16}\, GeV \left(\frac{r}{0.01}\right)^{1/4}$$

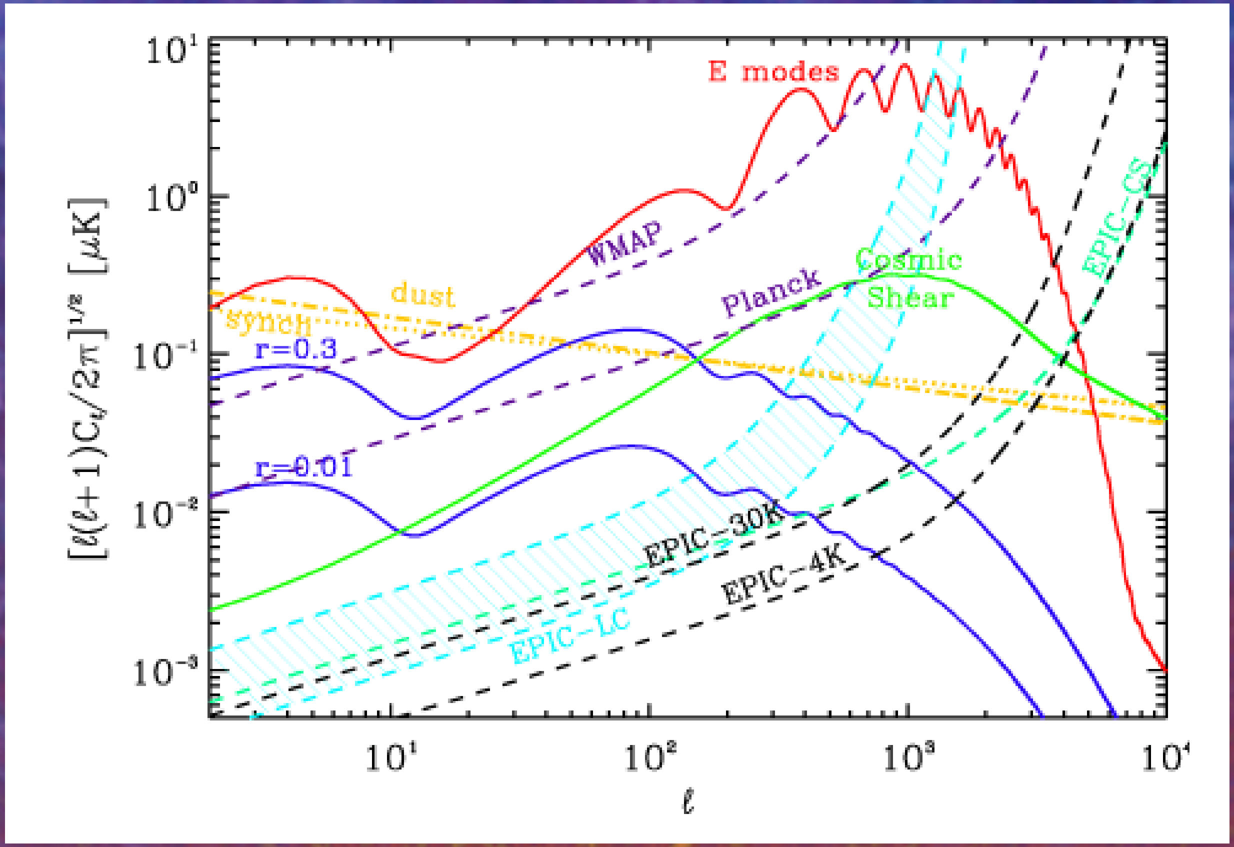
$$\Delta\phi \approx \Delta N \sqrt{\frac{r}{8}} \approx \sqrt{\frac{r}{0.01}}$$

From the CMB to inflation

WMAP+BAO+H_0 : $r < 0.24 \ (95\% \, C.L.)$ **(Komatsu et al. 2010)**

Future experiments: $r \sim 0.001$

(Bock et al. 2009)



... and to strings

- Inflation is UV sensitive and should be studied in a UV complete theory.
- For r>0.01 the inflaton must have moved over a distance in field space larger than the Planck mass.
- One must ensure the required flatness of the potential over distances large compared to M_p, e.g. by a shift symmetry.

... and to strings

- This makes axions natural candidates assuming one can break their shift symmetries in a controlled way.
- At the level of effective field theories, this can be done. Can it be done in a theory of quantum gravity? (string theory)
- Instanton effects may generate periodic contributions to the potential if the inflaton is an axion.

$$V(\phi) = V_0(\phi) + \Lambda^4 \cos\left(\frac{\phi}{f}\right)$$

Basic Ingredients for Axion Monodromy Inflation

Axion Monodromy Inflation

Consider string theory on $M \times X$

Axions arise from integrating gauge potentials over non-trivial cycles in the compactification manifold.

$$b_I(x) = \int_{\Sigma_I^{(2)}} B$$

$$c_\alpha(x) = \int_{\Sigma_\alpha^{(p)}} C^{(p)}$$

where $\Sigma_\alpha^{(p)}$ is an element of an integral basis of $H_p(X, \mathbb{Z})$

Axion Monodromy Inflation

These fields possess a shift symmetry to all orders in string perturbation theory.

The vertex operator for $b_I(x)$ in the limit of vanishing momentum is

$$V_{b_I}(0) = \int_{\mathcal{W}} d^2\xi \epsilon^{\alpha\beta} \partial_\alpha Y^i \partial_\beta Y^j \omega^I_{ij}(Y(\xi)) = \int_{\varphi(\mathcal{W})} \omega^I$$

vanishes if $\varphi(\mathcal{W}) = \partial\mathcal{C}$ so that coupling vanishes.

Axion Monodromy Inflation

Breaking by branes

For definiteness consider a D5-brane wrapping a two-cycle $\Sigma^{(2)}$ of size L$\sqrt{\alpha'}$.

$$S_{\mathrm{DBI}} = -\frac{1}{(2\pi)^5 {\alpha'}^3 g_s} \int d^6\xi \sqrt{det(-\varphi^*(G+B))}$$

$$\supset -\frac{\epsilon}{(2\pi)^5 {\alpha'}^2 g_s} \int d^4x \sqrt{{}^{(4)}g} \sqrt{L^4 + b^2}$$

Axion Monodromy Inflation

Breaking by branes

This implies the following potential

$$V(b) = \frac{\epsilon}{(2\pi)^5 \alpha'^2 g_s} \sqrt{L^4 + b^2}$$

similarly for the $C^{(2)}$ axion in the presence of NS5 branes

$$V(c) = \frac{\epsilon}{(2\pi)^5 \alpha'^2 {g_s}^2} \sqrt{L^4 + {g_s}^2 c^2}$$

Axion Monodromy Inflation

Breaking by branes

For large field values in terms of the canonically normalized fields the potential then becomes

$$V(\phi) \approx \mu^3 \phi$$

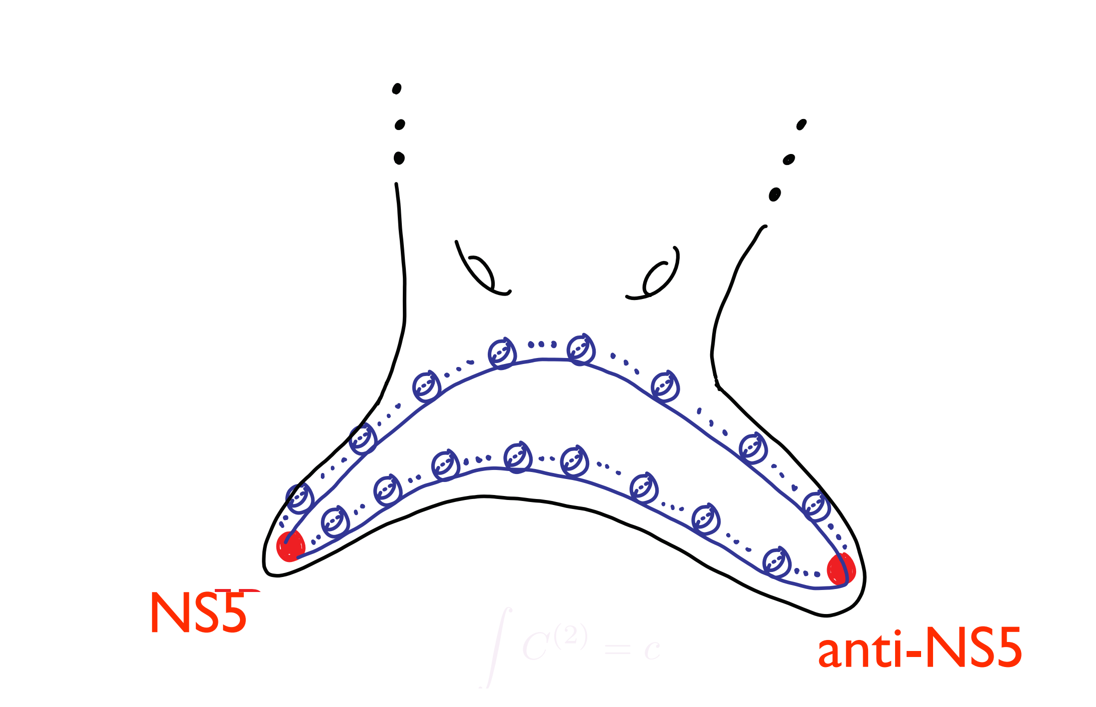
with $$\mu = \frac{\epsilon^{1/3}(2\pi)^3 g_s}{L^{10/3}} M_p$$ for b

$$\mu = \frac{\epsilon^{1/3}(2\pi)^3 g_s^{2/3}}{L^{10/3}} M_p$$ for c

Axion Monodromy Inflation

The basic setup

- Type IIB orientifolds with O3/O7
- Stabilize the moduli a la KKLT



Axion Monodromy Inflation

Consistency checks

The inflaton potential must be smaller than the potential barriers stabilizing the moduli.

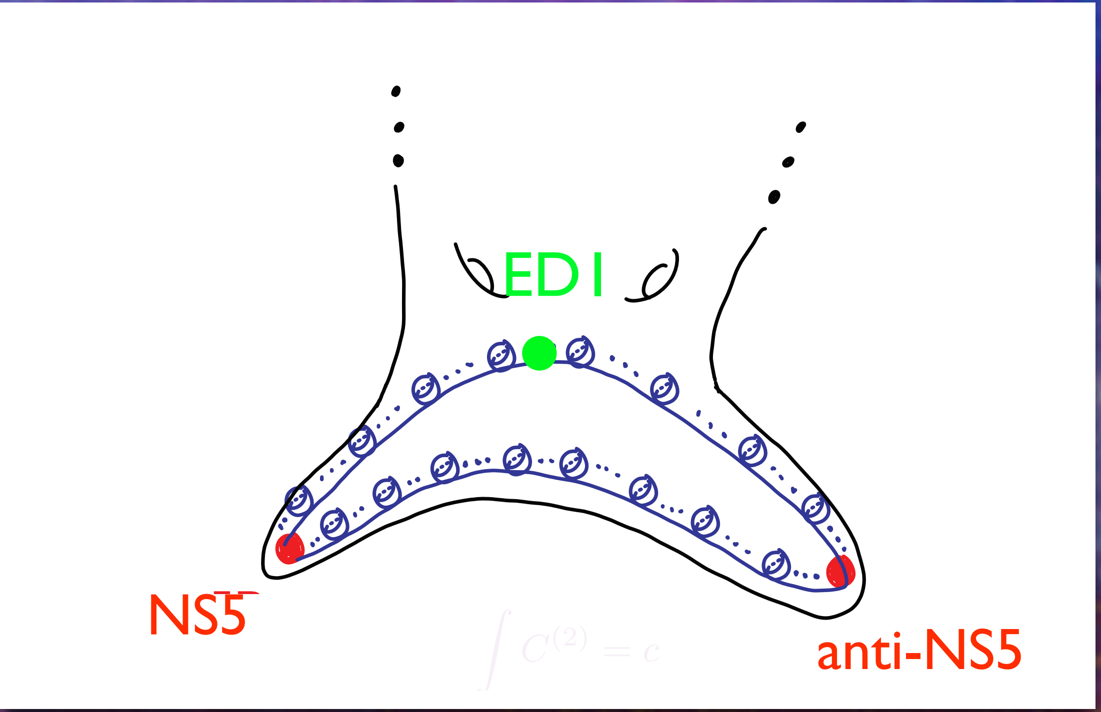
The backreaction on the geometry must be controlled.

Higher derivative corrections must be negligible.

Instanton corrections must be controlled.

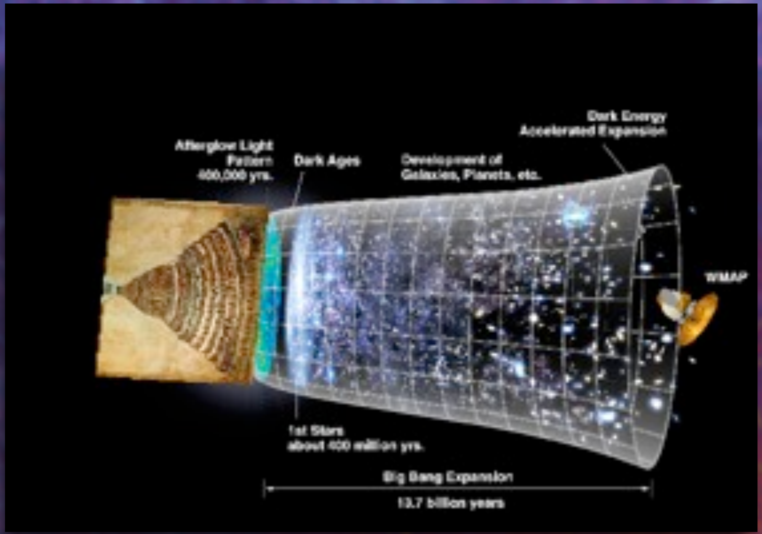
Axion Monodromy Inflation

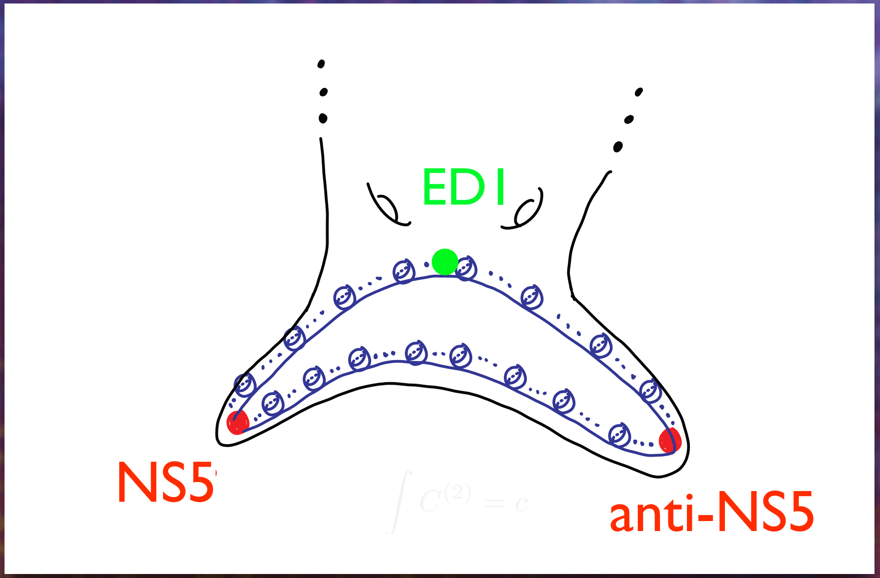
Instanton corrections may lead to interesting signatures.



$$K = -2\log(\mathcal{V}_E + e^{-S_{ED1}}\cos(c))$$

Signatures of Axion Monodromy Inflation in the CMB





Signatures of Axion Monodromy Inflation

The low energy effective field theory for Axion Monodromy Inflation is that of a single scalar field with canonical kinetic term, minimally coupled to gravity, with potential

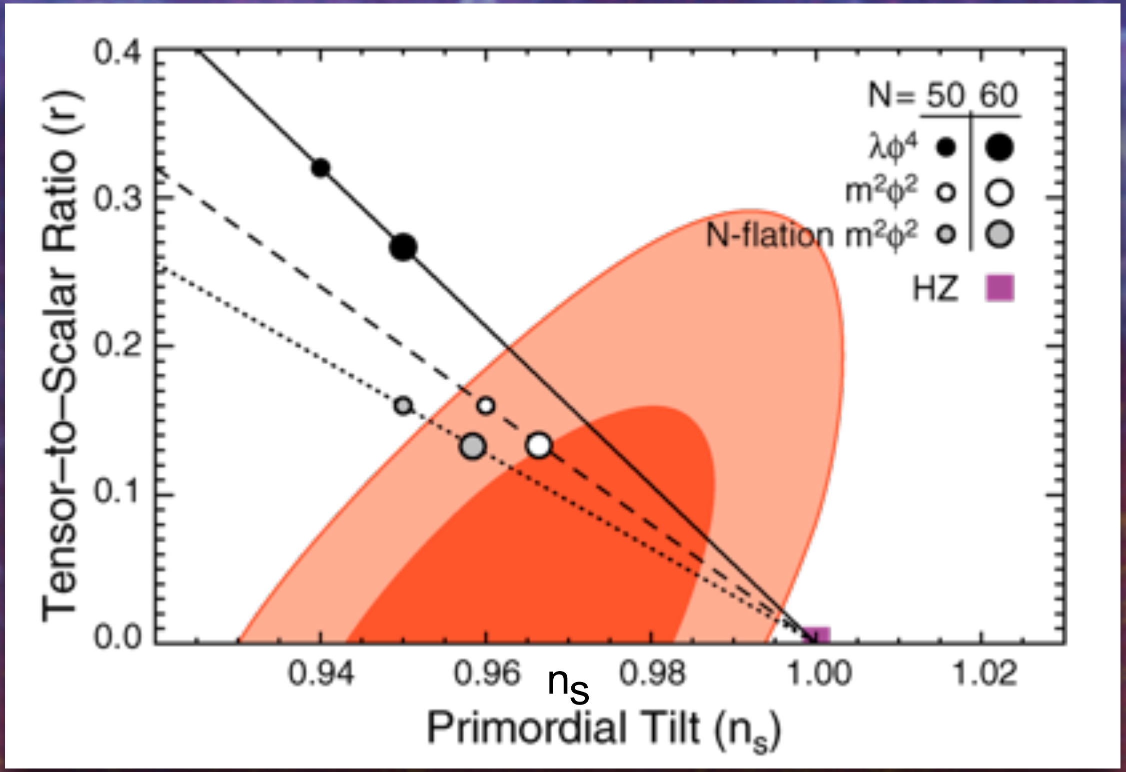
$$V(\phi) = \mu^3 \phi + b\mu^3 f \cos(\phi/f)$$

Signatures of Axion Monodromy Inflation

Observable 1: n_s and r

(no instanton corrections b=0)

(Komatsu et al. 2010)

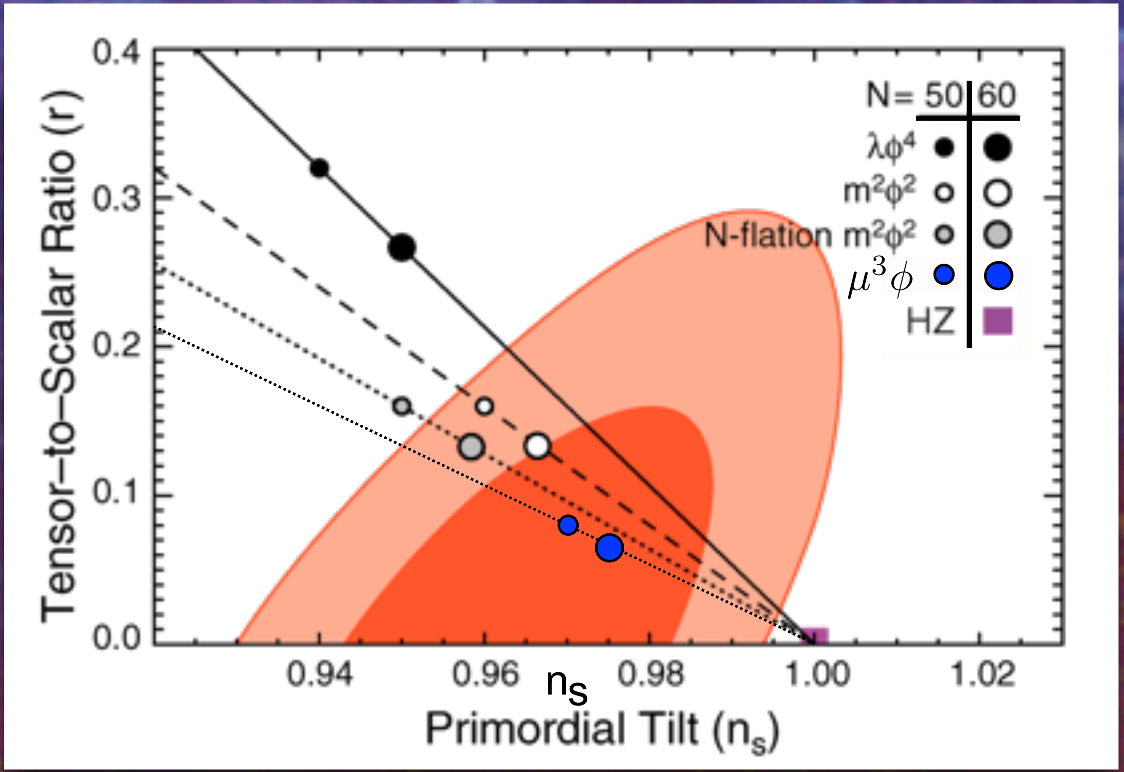


Signatures of Axion Monodromy Inflation

Observable 1: n_s and r

(no instanton corrections b=0)

(modification of Komatsu et al. 2010)



Signatures of Axion Monodromy Inflation

Observable II: the primordial power spectrum

In the presence of instanton corrections, the power spectrum gets modified.

This modification is not captured by the slow-roll approximation for the power spectrum because of parametric resonance, and the Mukhanov-Sasaki equation has to be solved carefully.

Signatures of Axion Monodromy Inflation

Observable II: the primordial power spectrum

$$\frac{d^2\mathcal{R}_k}{dx^2} - \frac{2(1+2\epsilon+\delta)}{x}\frac{d\mathcal{R}_k}{dx} + \mathcal{R}_k = 0$$

with

$$\epsilon = \epsilon_* - 3bf\sqrt{2\epsilon_*}\cos\left(\frac{\phi_k + \sqrt{2\epsilon_*}\ln x}{f}\right)$$

$$\delta = \delta_* - 3b\sin\left(\frac{\phi_k + \sqrt{2\epsilon_*}\ln x}{f}\right)$$

Signatures of Axion Monodromy Inflation

Observable II: the primordial power spectrum

$$\frac{d^2\mathcal{R}_k}{dx^2} - \frac{2(1+\delta_{\rm osc}(x))}{x}\frac{d\mathcal{R}_k}{dx} + \mathcal{R}_k = 0$$

Look for a solution

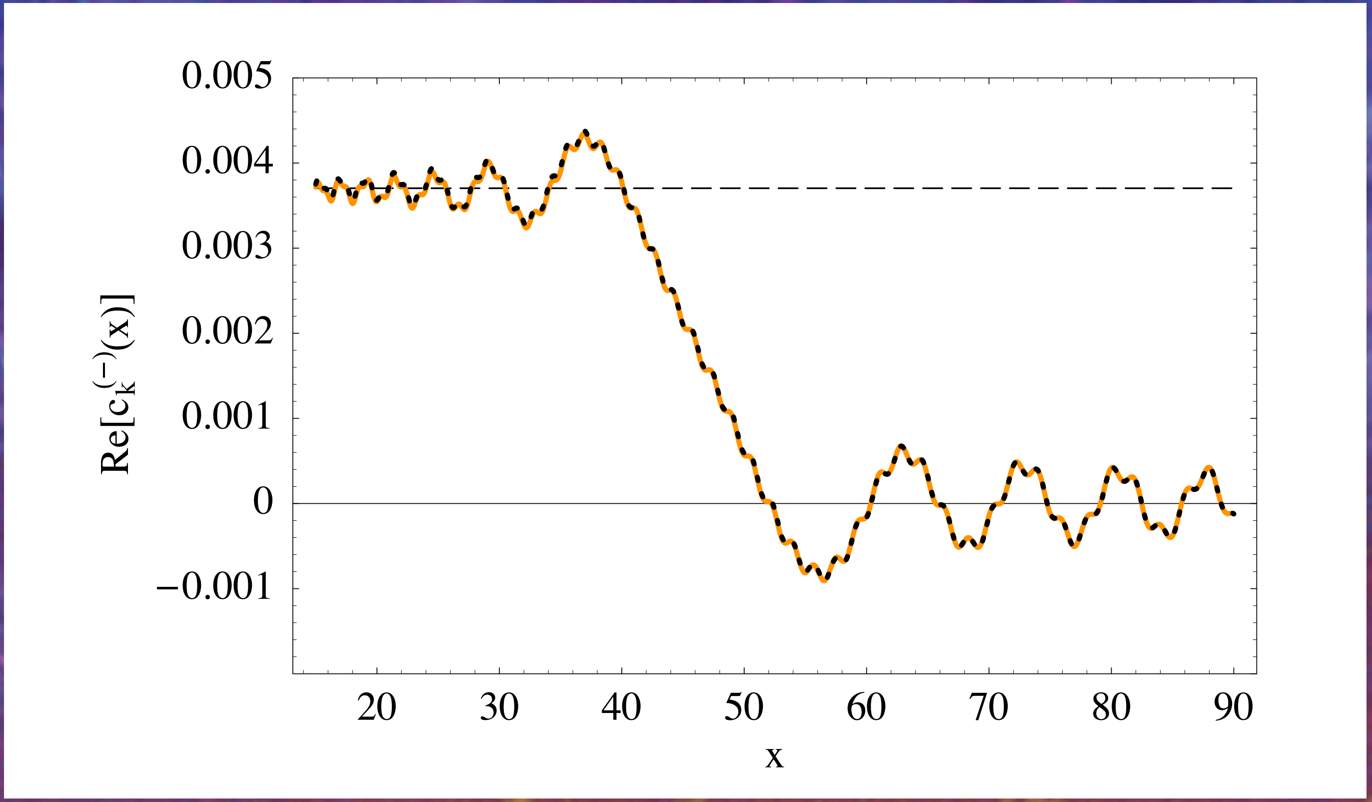
$$\mathcal{R}_k(x) = \mathcal{R}_{k,0}^{(o)}\left[i\sqrt{\frac{\pi}{2}}x^{3/2}H^{(1)}_{3/2}(x) - c_k^{(-)}(x)\,i\sqrt{\frac{\pi}{2}}x^{3/2}H^{(2)}_{3/2}(x)\right]$$

Then for large x

$$\frac{d}{dx}\left[e^{-2ix}\frac{d}{dx}c_k^{(-)}(x)\right] = -2i\frac{\delta_{\rm osc}(x)}{x}$$

Signatures of Axion Monodromy Inflation

Observable II: the primordial power spectrum



Signatures of Axion Monodromy Inflation

One finds

$$\Delta^2_{\mathcal{R}}(k) = \Delta^2_{\mathcal{R}}(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}\left[1+\delta n_s \cos\left(\frac{\phi_k}{f}\right)\right]$$

with

$$\delta n_s = 3b\left(\frac{2\pi f}{\sqrt{2\epsilon_*}}\right)^{1/2}$$

(This assumes $\frac{f}{\sqrt{2\epsilon_*}} \ll 1$. For the general case see our paper.)

Signatures of Axion Monodromy Inflation

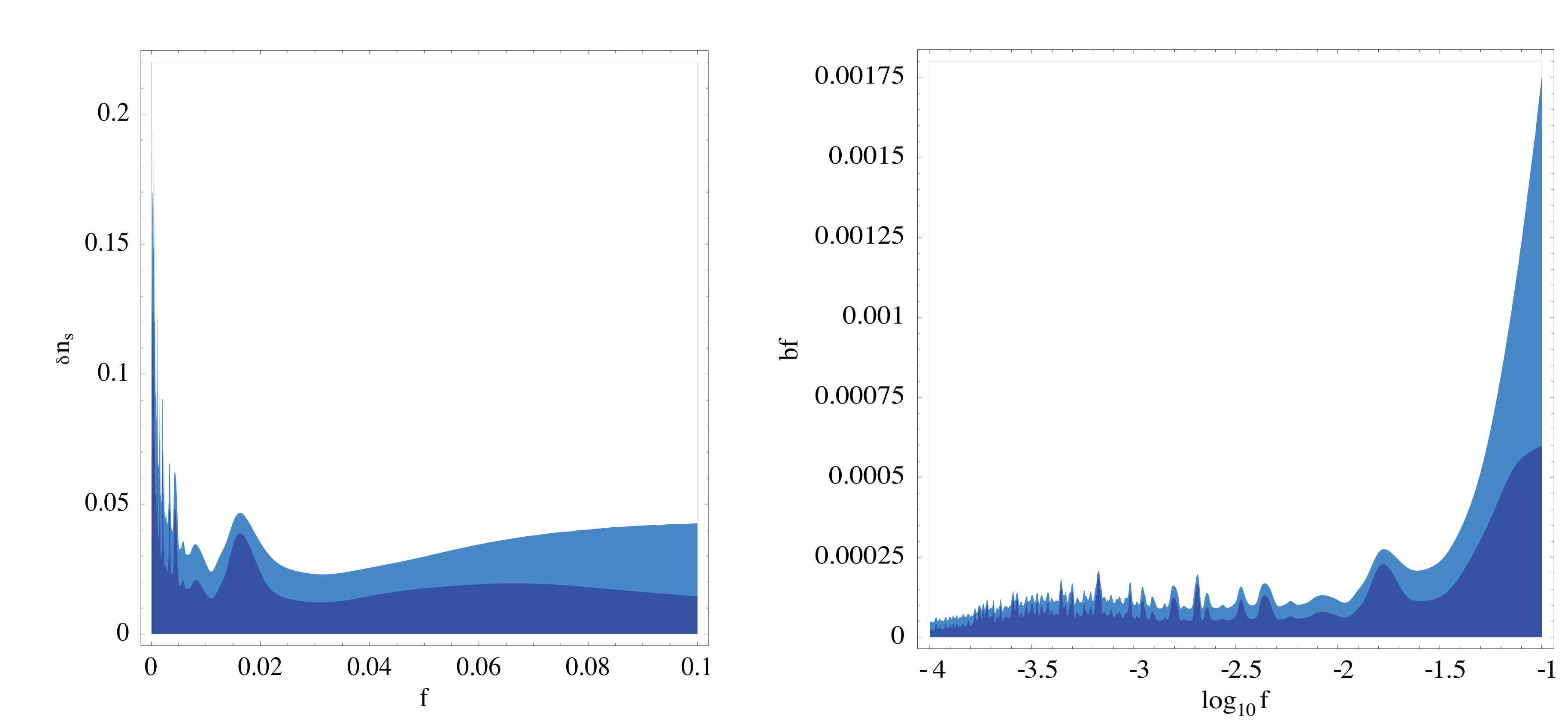
Constraints from WMAP5

	Min	Max	Points
$\Omega_b h^2$	0.0212	0.0266	16
f	0.00009	0.1	512
δn_s	0	0.44	128
$\Delta\varphi$	$-\pi$	π	32

33 million spectra

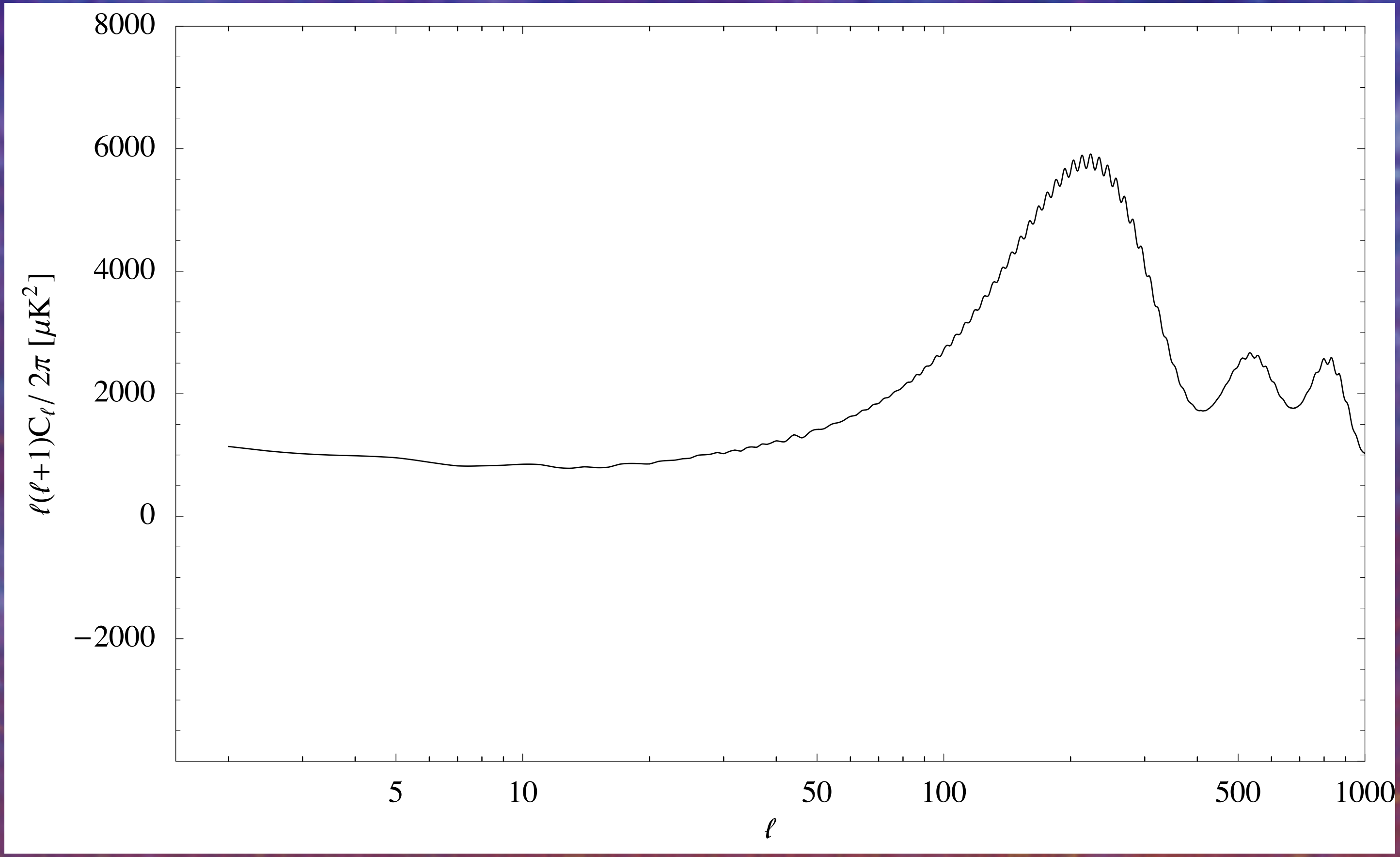
Signatures of Axion Monodromy Inflation

Constraints from WMAP5



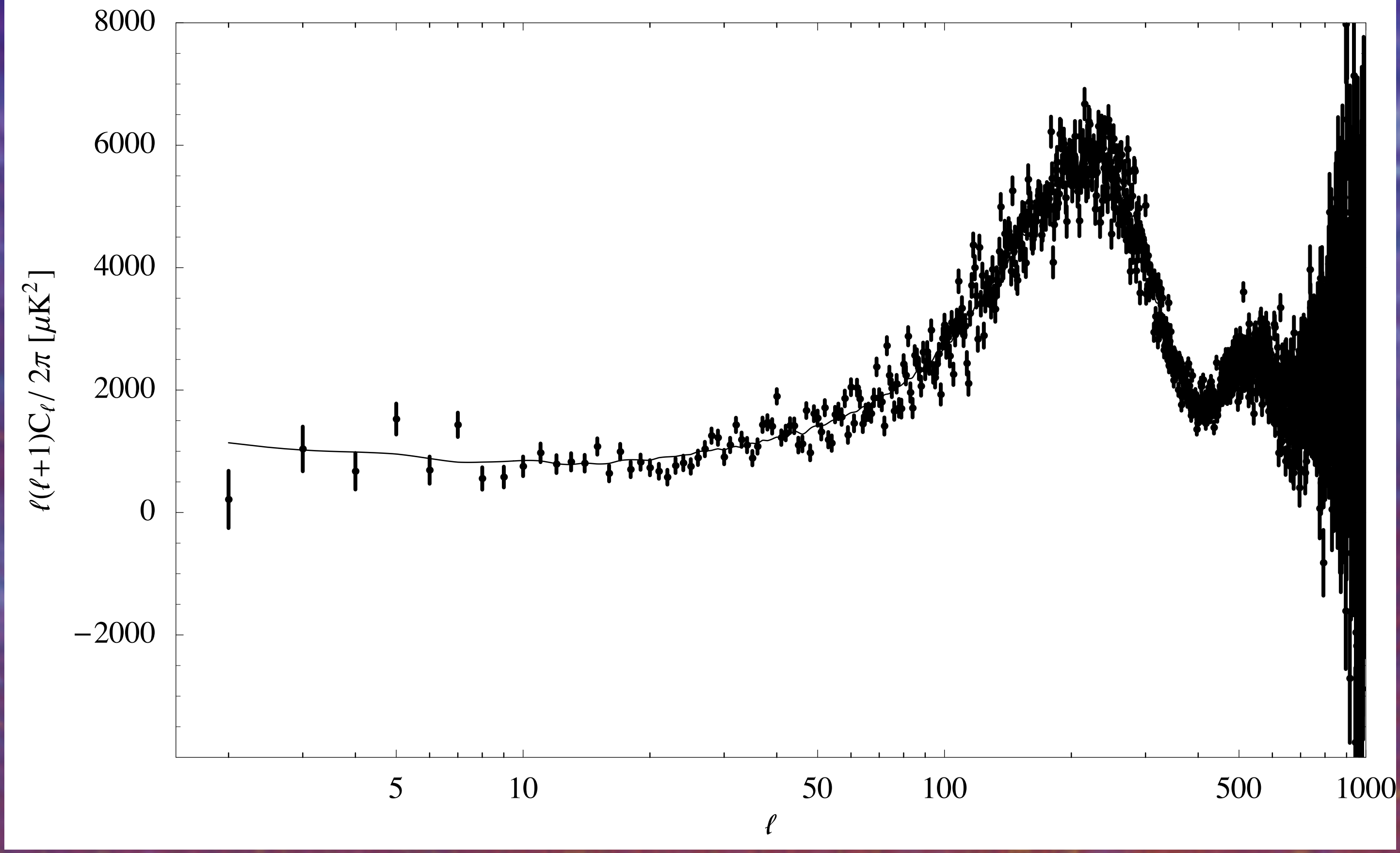
Signatures of Axion Monodromy Inflation

Constraints from WMAP5



Signatures of Axion Monodromy Inflation

Constraints from WMAP5



Signatures of Axion Monodromy Inflation

Observable III: Resonant Non-Gaussianity

Models with large $\dot{\delta}$ can lead to large non-Gaussianities

(Chen, Easther, Lim 2008)

$$\langle \mathcal{R}(\mathbf{k_1},t)\mathcal{R}(\mathbf{k_2},t)\mathcal{R}(\mathbf{k_3},t)\rangle =$$

$$-i\int_{-\infty}^{t} dt' \langle [\mathcal{R}(\mathbf{k_1},t)\mathcal{R}(\mathbf{k_2},t)\mathcal{R}(\mathbf{k_3},t), H_I(t')]\rangle$$

with

$$H_I(t) \supset -\int d^3x\, a^3(t)\epsilon(t)\dot{\delta}(t)\mathcal{R}^2(\mathbf{x},t)\dot{\mathcal{R}}(\mathbf{x},t)$$

Signatures of Axion Monodromy Inflation

Observable III: Resonant Non-Gaussianity

After some algebra

$$\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3} = \frac{1}{8}\int_0^\infty dX\,\frac{\dot{\delta}}{H}e^{-iX}$$

$$\left[-i - \frac{1}{X}\sum_{i\neq j}\frac{k_i}{k_j} + \frac{i}{X^2}\frac{K(k_1^2+k_2^2+k_3^2)}{k_1k_2k_3}\right] + c.c$$

Signatures of Axion Monodromy Inflation

Observable III: Resonant Non-Gaussianity

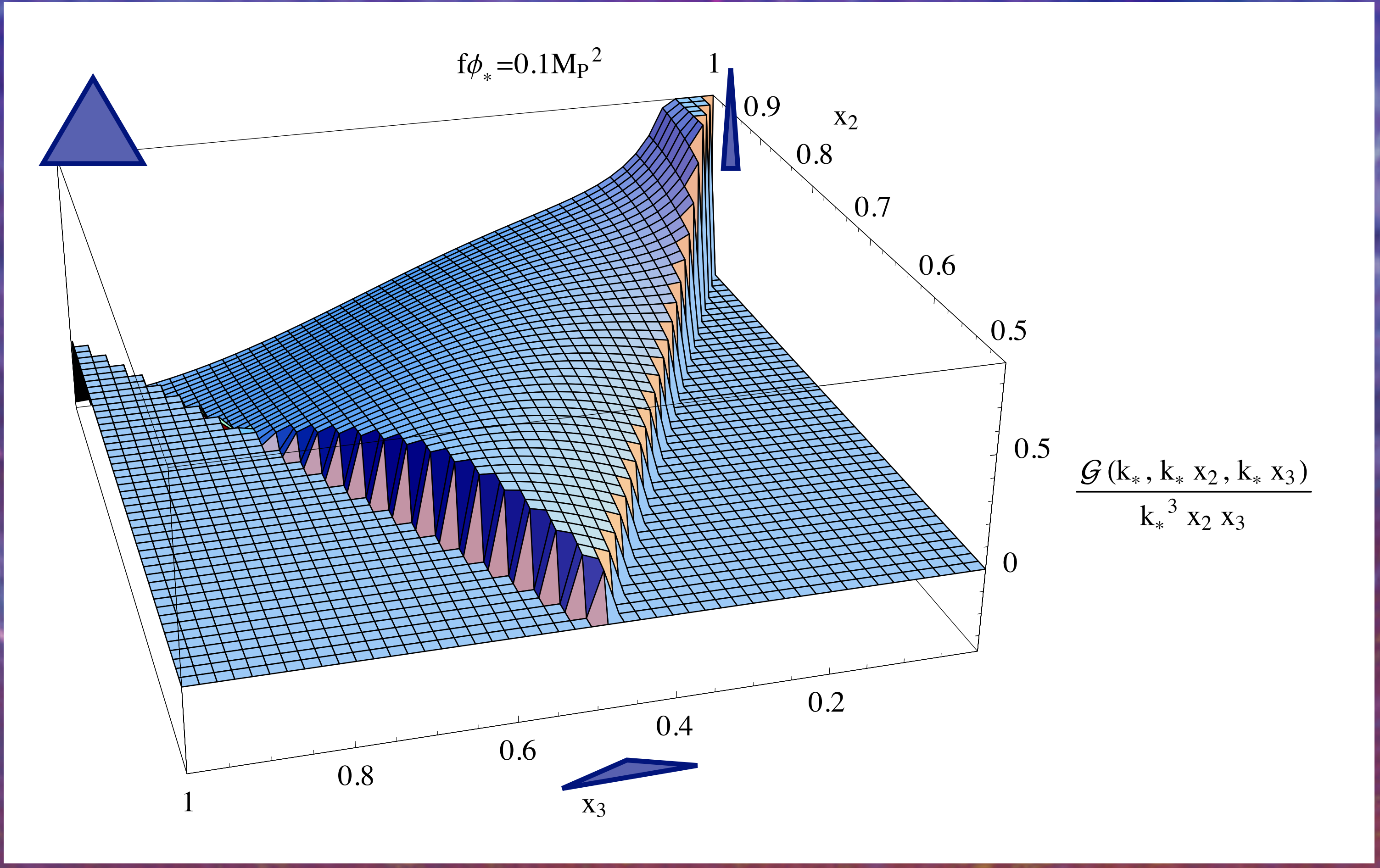
$$\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3} = f^{\rm res}\left[\sin\left(\frac{\ln K/k_*}{f\phi_*}\right) + f\phi_*\sum_{i\neq j}\frac{k_i}{k_j}\cos\left(\frac{\ln K/k_*}{f\phi_*}\right)\right]$$

with $$K = k_1+k_2+k_3$$

$$f^{\rm res} = \frac{3\sqrt{2\pi}b}{8(f\phi_*)^{3/2}}$$

This satisfies the consistency condition.

Signatures of Axion Monodromy Inflation



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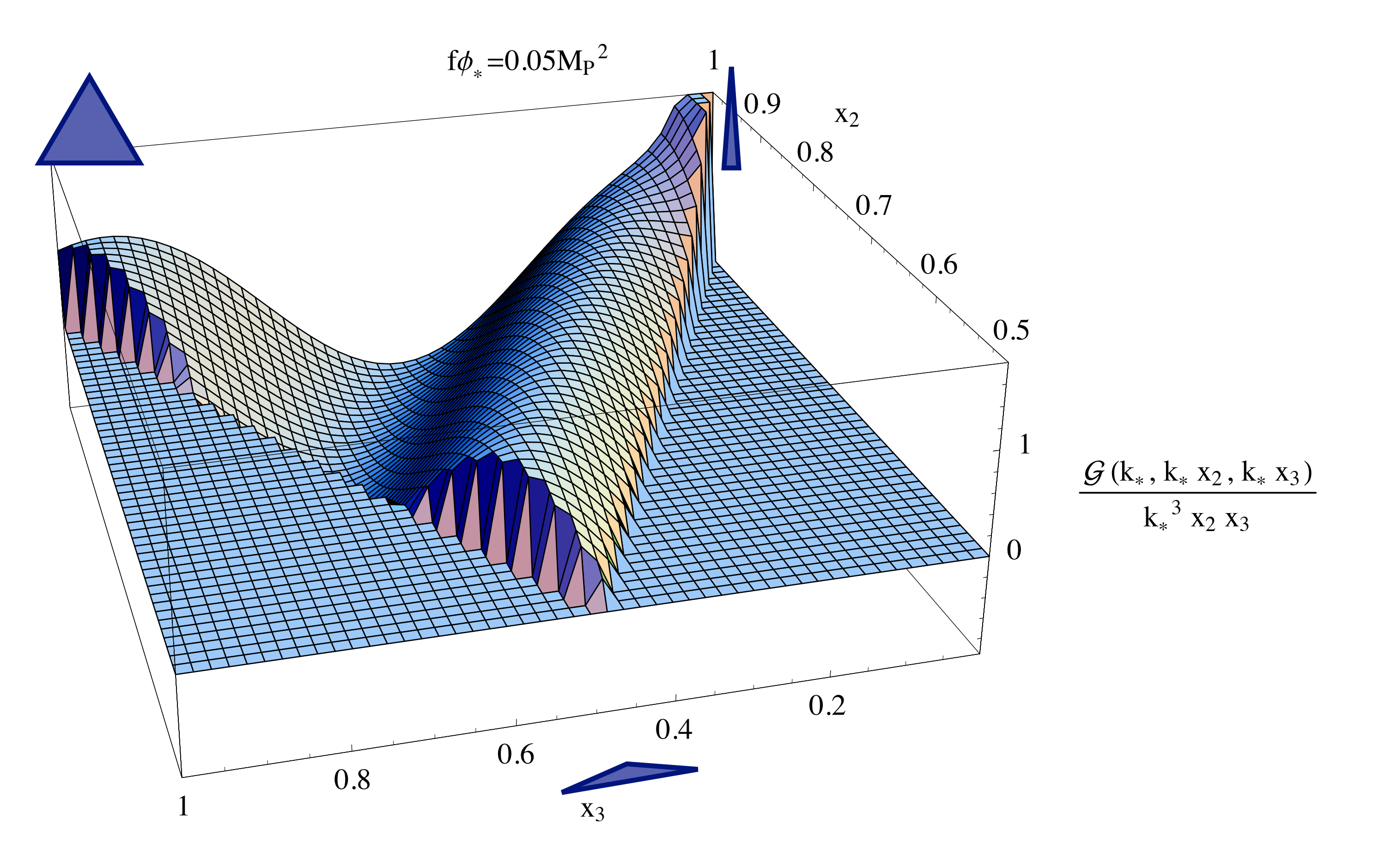

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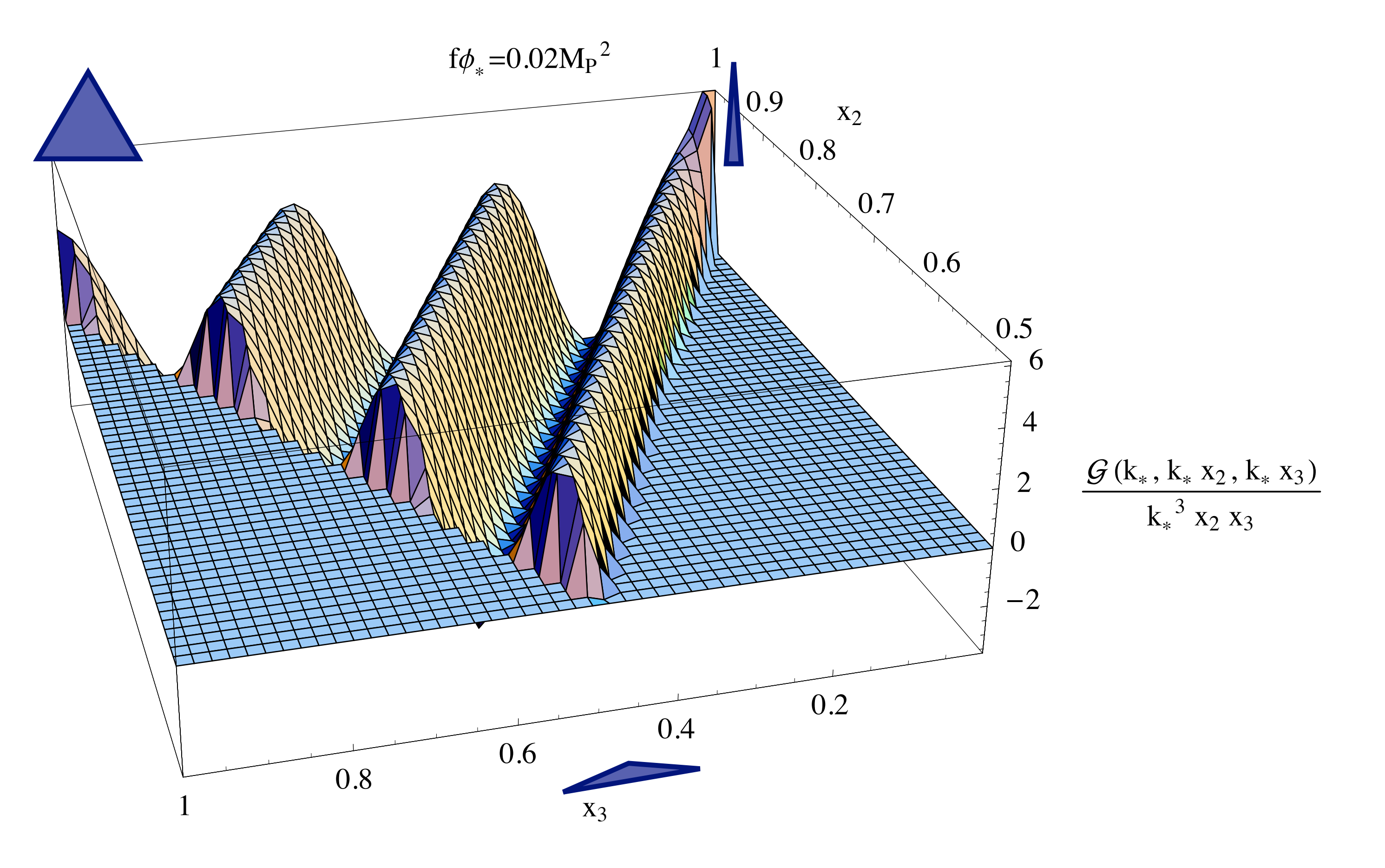

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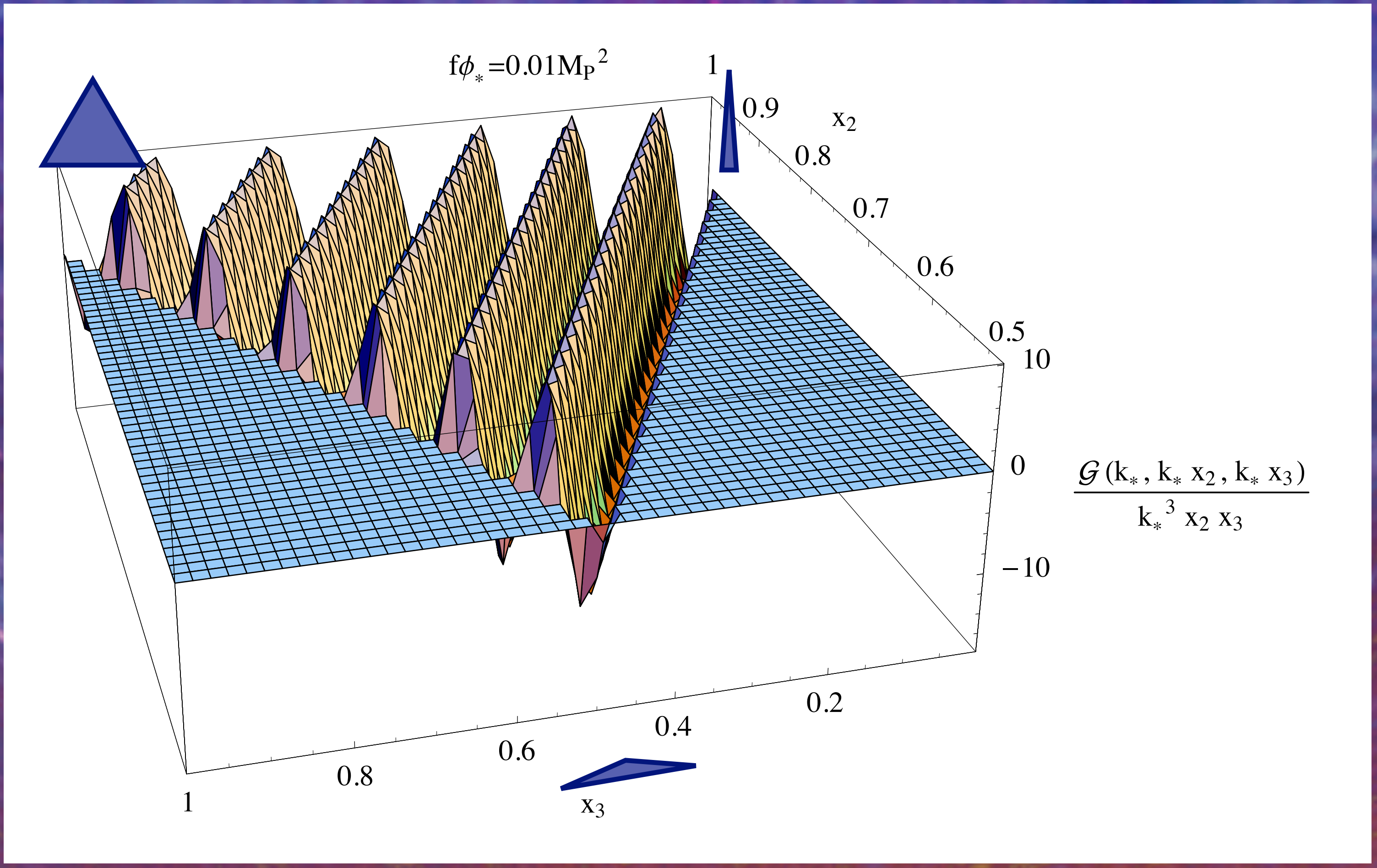

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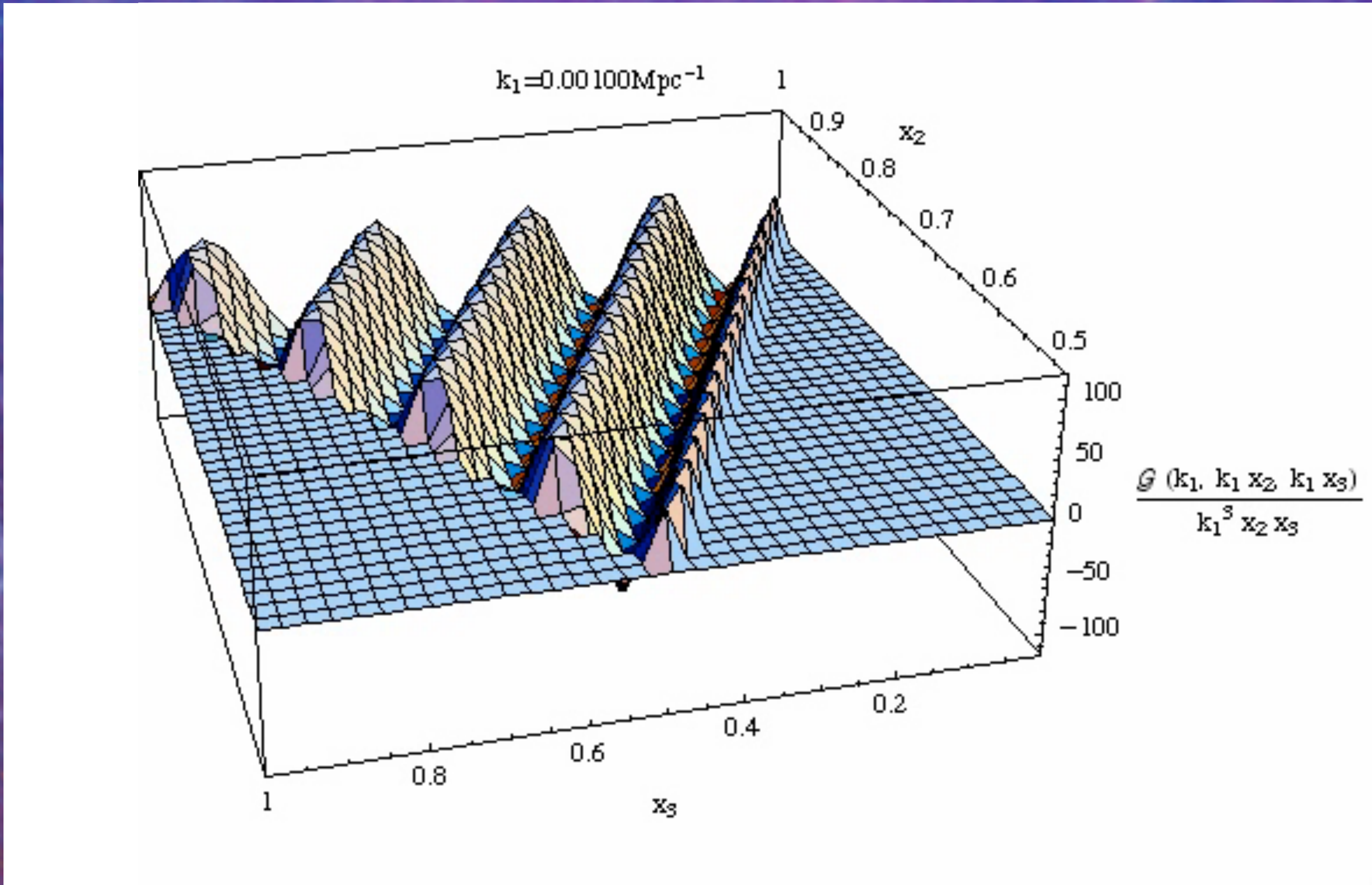
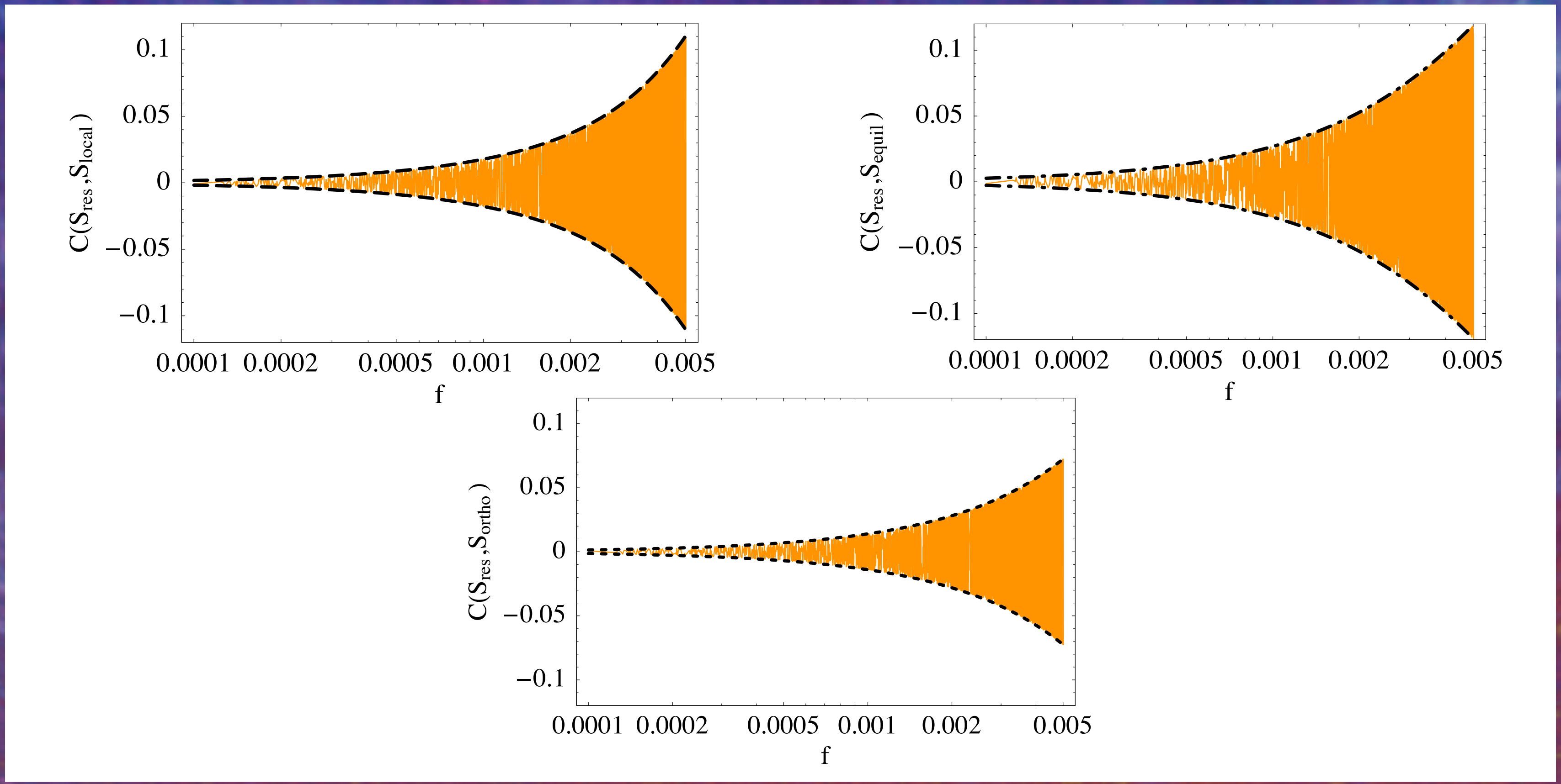


Signatures of Axion Monodromy Inflation

Can we convert existing constraints on local, equilateral, and orthogonal shapes into constraints on this shape?



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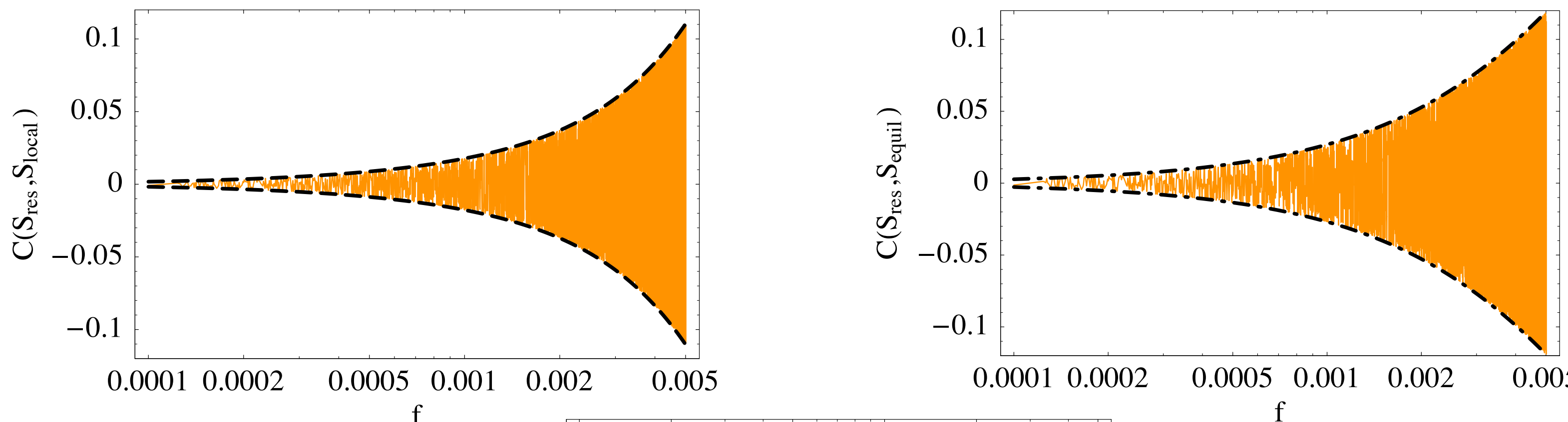

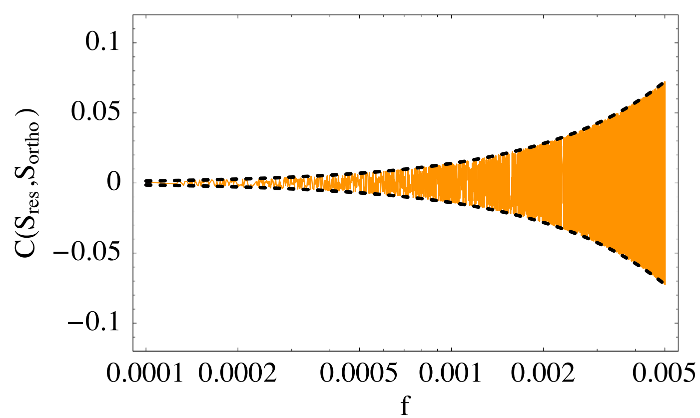

No

Conclusions

- The model has interesting signatures. These are a large tensor to scalar ratio, and potentially a modulated temperature anisotropy spectrum as well as resonant non-Gaussianities.
- This kind of non-Gaussianities is currently poorly constrained and deserves further study independent of the stringy scenario.


Thank you