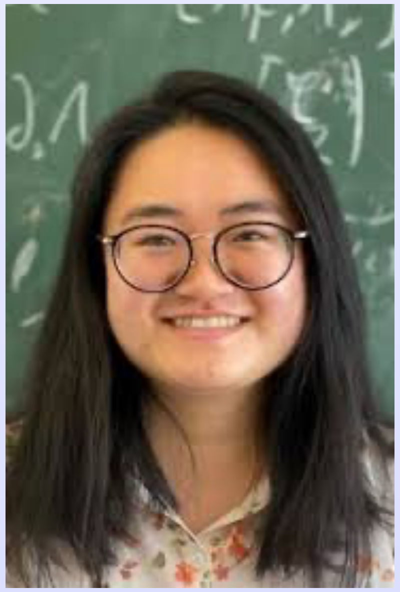
A PT symmetric non-hermitian holographic metal

Priv.-Doz. Dr. René Meyer
Lehrstuhl für Theoretische Physik III
Institut für Theoretische Physik & Astrophysik
Julius Maximilians Universität Würzburg

People behind the research

M.Sc. Zhaohui Chen

Dr. Zhuoyu Xian

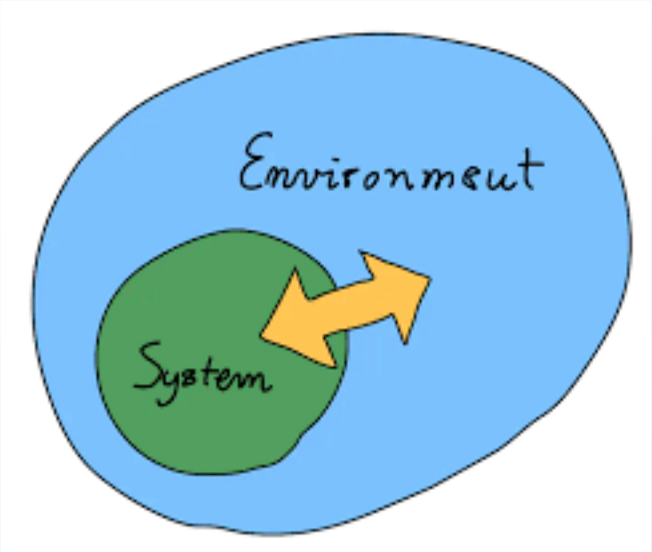


Dr. Yang Liu

Dr. David Rodriguez-Fernandez

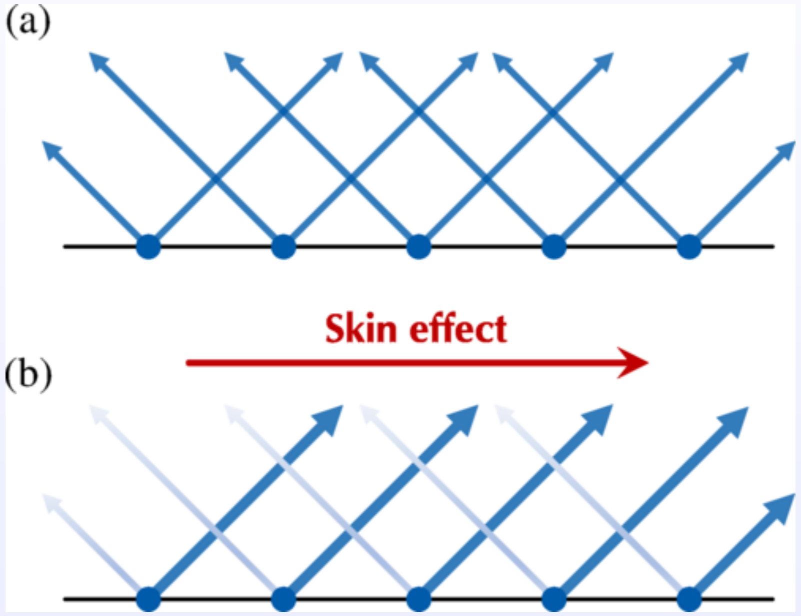
PD Dr. René Meyer

Motivations for Non-Hermiticity



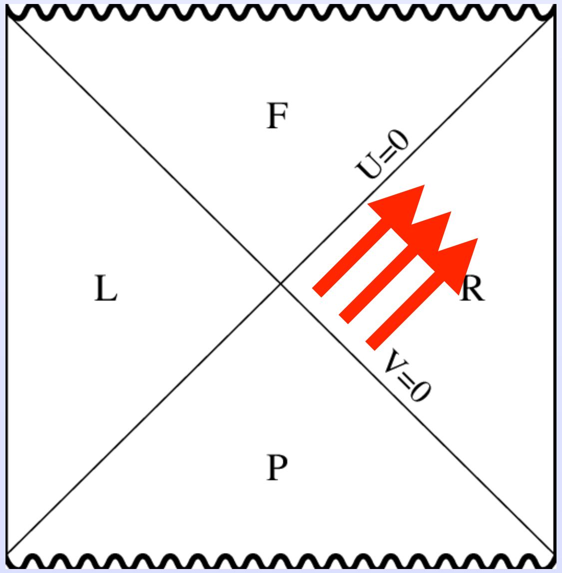
Open Quantum Systems

https://www.matteoacrossi.com/

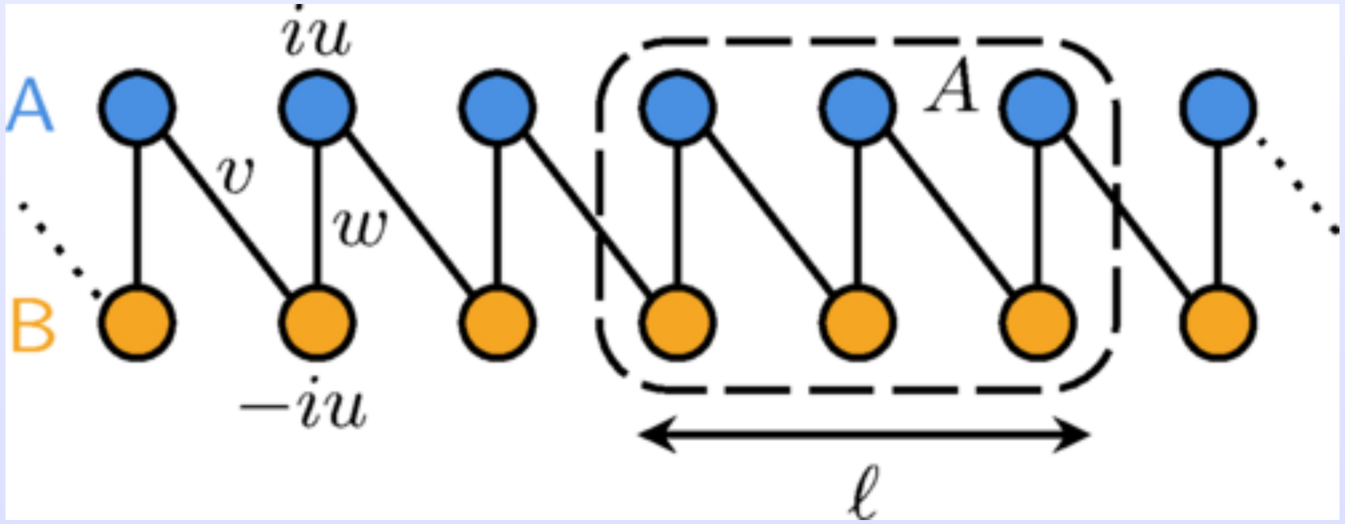


New Transport Phenomena

PRX 13, 021007



Bulk vs. Boundary Unitarity



Generalizations of QFTs

PRB 107, 205153

Outline

1. Motivation

2. PT Symmetric Non-Hermitian QM/QFT

3. Non-Hermitian AdS/CFT
   Landsteiner, Arean SciPost 9 (2020)

4. Conductivity and Sum Rule
   Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

5. Conclusions and Outlook

Unitarity in Quantum Mechanics

Quantum Mechanics:
Closed systems, unitary time evolution

$$i\partial_t \left|\Psi(t)\right\rangle = H \left|\Psi(t)\right\rangle \qquad \left|\Psi(t)\right\rangle = e^{-iHt} \left|\Psi(0)\right\rangle$$

Measurement outcomes: Probability distributions

$$p_i(t) = \left| \left\langle i|\Psi(t)\right\rangle \right|^2 \qquad \sum_i p_i(t) = 1$$

Probability conservation iff H is hermitian:

$$\sum_i p_i(t) = 1 = \sum_i p_i(0) \quad \Longleftrightarrow \quad H = H^\dagger$$

Non-unitary systems gain or loose probability with time through interactions with environment

Why PT symmetric Non-Hermiticity?

Quantum Mechanics: Real energy spectrum

$$H\,|\Psi(0)\rangle = E\,|\Psi(0)\rangle \qquad H = H^\dagger$$

$$\langle\Psi(0)|\,H = \langle\Psi(0)|\,E \qquad E = E^*$$

Generic non-unitarity: Complex spectrum or worse

$$[H, H^\dagger] = 0 \quad \Longrightarrow \quad \text{H is diagonalizable}$$

PT symmetry: Diagonalizability and real spectrum

$$H \neq H^\dagger, \quad H = \mathcal{PT}H\mathcal{PT}, \qquad [H, \mathcal{PT}] = 0, \qquad H\phi = E\phi, \quad \mathcal{PT}\phi = \lambda\phi,$$

Due to $[\mathcal{P}, \mathcal{T}] = 0$ and $\mathcal{P}^2 = \mathcal{T}^2 = 1$, set $\lambda = 1$,

$$[\mathcal{PT}]\,H\phi = [\mathcal{PT}]\,E\phi = E^\star\phi,$$
$$H\,[\mathcal{PT}\phi] = H\phi = E\phi.$$

Unbroken $\mathcal{PT}$ symmetry: $E = E^\star$; broken $\mathcal{PT}$ symmetry: $E \neq E^\star$.

Carl Bender, S. Boettcher, PRL 80 (24), 5243, 1998

PT symmetric anharmonic oscillator

One example of non-Hermitian $\mathcal{PT}$ symmetric theories:

$$H = p^2 + x^2(ix)^\epsilon \; (\epsilon \text{ is real}).$$

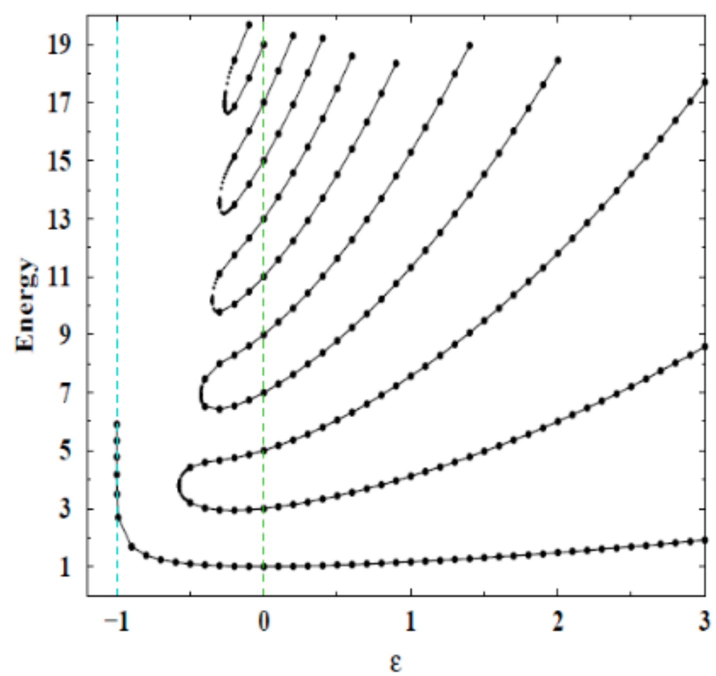


Figure 1: Energy levels of H as a function of ϵ

- $\epsilon \geq 0$: the spectrum is real and positive, $\epsilon = 0$: Harmonic oscillator;
- $-1 < \epsilon < 0$: finite number of real positive eigenvalues and an infinite number of complex conjugate pairs of eigenvalues;
- $\epsilon \leq -1$: no real eigenvalues

Carl Bender, Rep. Prog. Phys. 70 (6), 947

The PT symmetric Qubit

Two level system with gain loss balance

$$H = \begin{pmatrix} E - i\Gamma & g \\ g & E + i\Gamma \end{pmatrix}$$

$\mathcal{P}$: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\mathcal{T} : i \to -i$

$$[H, \mathcal{PT}] = 0$$

Energy spectrum: $\epsilon_+ = E + \sqrt{g^2 - \Gamma^2}$ $\epsilon_- = E - \sqrt{g^2 - \Gamma^2}$

Three phases: $g^2 > \Gamma^2$ PT symmetric

$g^2 = \Gamma^2$ Exceptional point

$g^2 < \Gamma^2$ PT broken

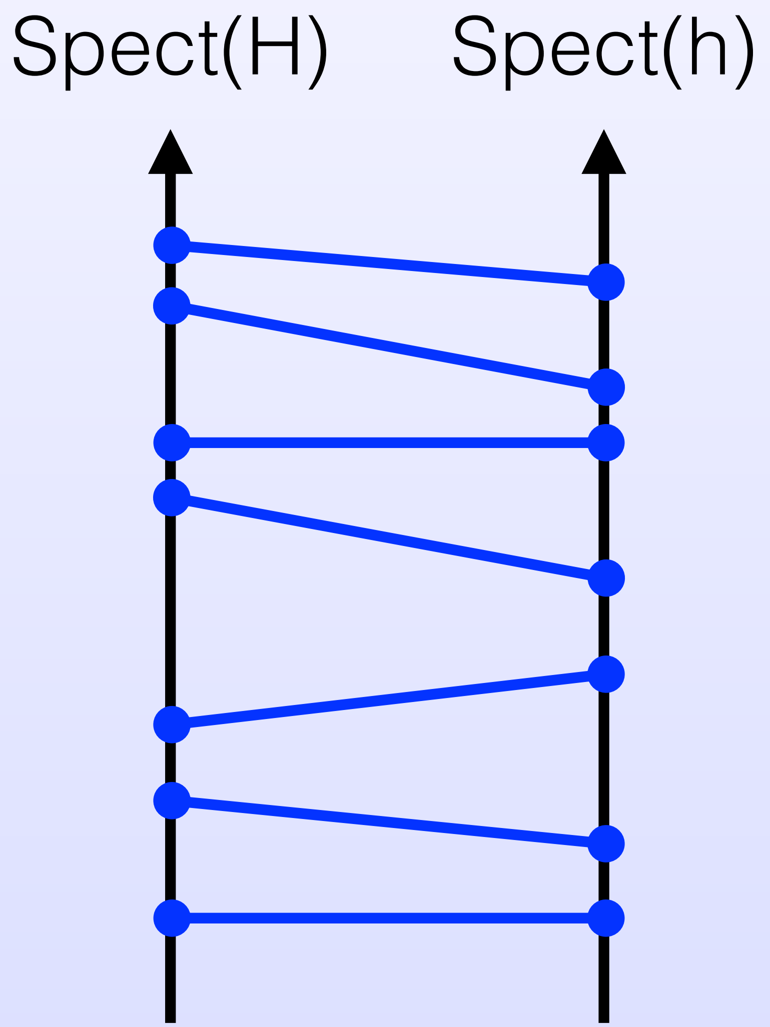
Dyson map: $H = \exp(\hat{\alpha}\sigma_1/2) \begin{pmatrix} E & g \\ g & E \end{pmatrix} \exp(-\hat{\alpha}\sigma_1/2)$ $\tanh \hat{\alpha} = \Gamma/g$

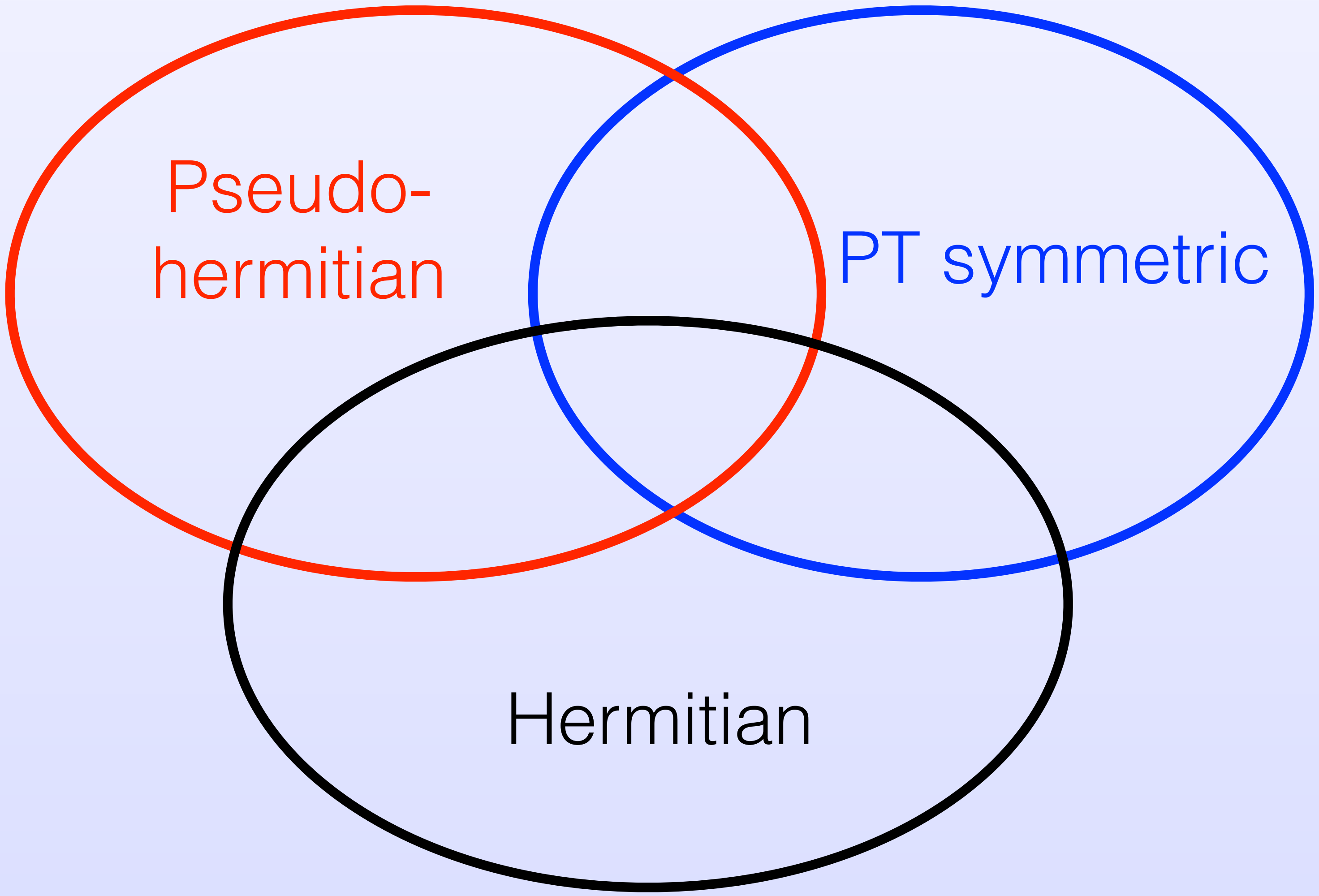
Complexified global SU(2): $\exp(i\frac{\alpha}{2}\sigma_1) \xrightarrow{\alpha \to i\hat{\alpha}} \exp(-\frac{\hat{\alpha}}{2}\sigma_1)$

Pseudohermiticity

PT symmetric phase always admits Dyson map:

$$H = \eta^{-1} h \eta \qquad h = h^\dagger$$





Dyson map can be complicated, generically SU(2)

PT symmetric 1+1D Fermions

Hamiltonian satisfying $H = H^\dagger = \mathcal{PT} H \mathcal{PT}$:

$$H = \int dx(-i\bar{\psi}\not{\nabla}\psi + NO_1) = \int dx(-i\bar{\psi}\not{\nabla}\psi + NO^\dagger + NO),$$
$$O_1 = \bar{\psi}\psi,\ O_5 = \bar{\psi}\gamma_5\psi,\ O^\dagger = (O_1 + O_5)/2,\ O = (O_1 - O_5)/2.$$

Dyson map with $Q = -\frac{1}{2}\int dx\psi^\dagger\gamma_5\psi$:

$$H_\theta = e^{\theta Q} H e^{-\theta Q} = \int dx(-i\bar{\psi}\not{\nabla}\psi + e^{-\theta} NO^\dagger + e^{\theta} NO).$$

Non-Hermiticity and $\mathcal{P}_\theta\mathcal{T}$ symmetry ($\mathcal{P}_\theta = e^{2\mathrm{Im}\theta Q}\mathcal{P}$) :

$$H_\theta^\dagger \neq H_\theta, \quad \mathcal{P}_\theta\mathcal{T} H_\theta \mathcal{P}_\theta\mathcal{T} = H_\theta.$$

Dyson map: Complexified axial U(1) rotation

Outline

1. Motivation

2. PT Symmetric Non-Hermitian QM/QFT

3. **Non-Hermitian AdS/CFT**
   Landsteiner, Arean SciPost 9 (2020)

4. Conductivity and Sum Rule
   Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

5. Conclusions and Outlook

Holographic S-Wave Superconductor

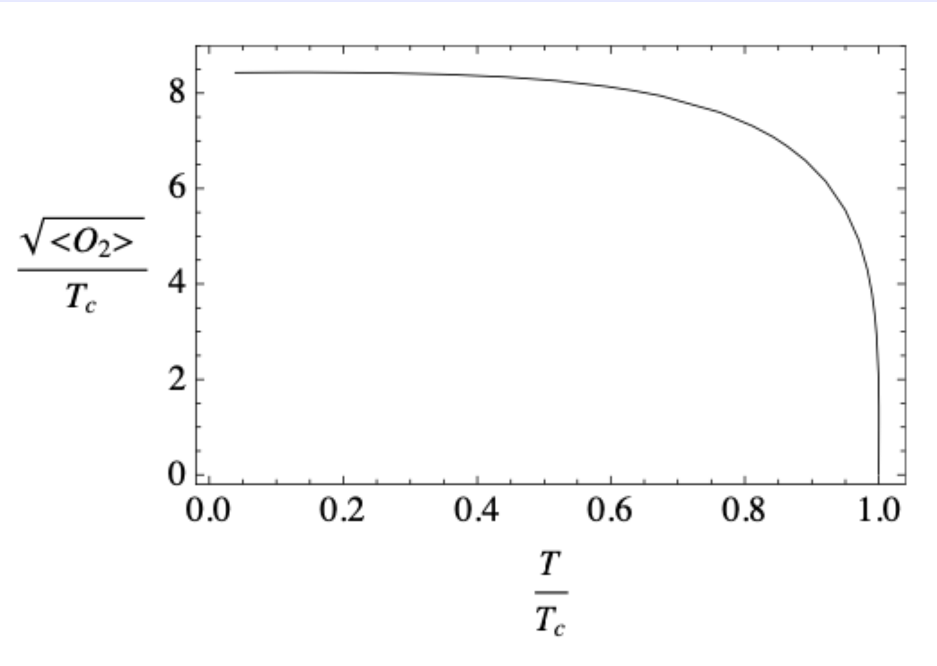
Bulk action with a local $U(1)$ symmetry:

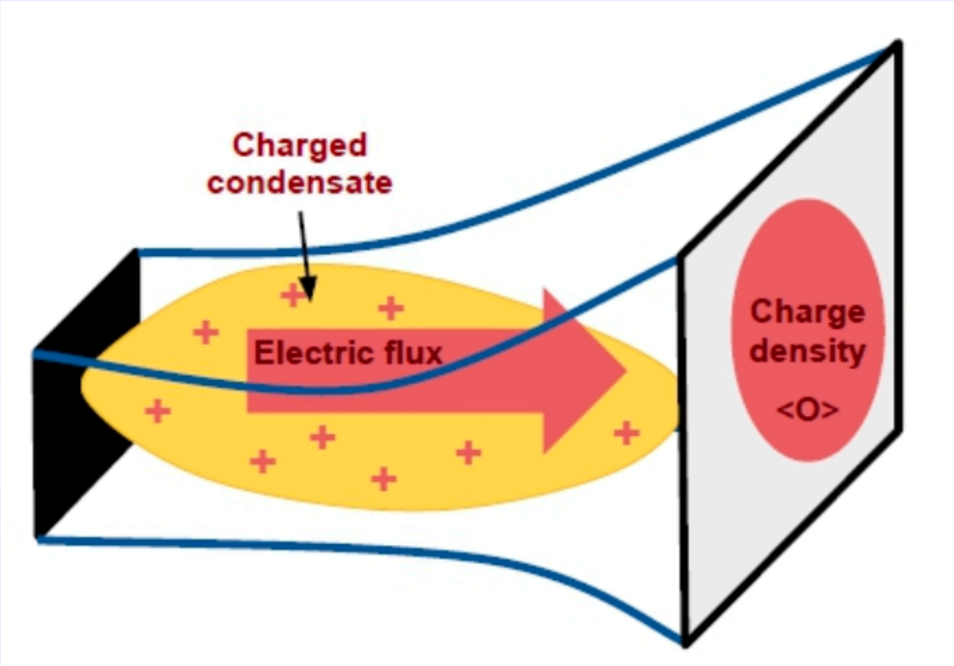
$$S = \int d^4_X \sqrt{-g}\left(R + \frac{6}{L^2} - D^{\dagger}_a \bar{\phi} D^a \phi - m^2 \bar{\phi}\phi - v\bar{\phi}^2\phi^2 - \frac{1}{4}F_{ab}F^{ab}\right)$$

Hartnoll, Herzog, Horowitz 2008

Probe Limit: Charged AdS-RN Black Brane

Scalar field asymptotics: $\phi \simeq J z^{d-\Delta} + \langle \mathcal{O}_\phi \rangle z^{\Delta}$ $\quad m^2 L^2 = \Delta(\Delta - d)$





Condensed solutions spontaneously break U(1) at low T

PT symmetric deformation

Bulk action with a local $U(1)$ symmetry:

$$S = \int d^4_X \sqrt{-g}\left(R + \frac{6}{L^2} - D^\dagger_a \bar{\phi} D^a \phi - m^2 \bar{\phi}\phi - v\bar{\phi}^2\phi^2 - \frac{1}{4}F_{ab}F^{ab}\right)$$

Arean, Landsteiner 2019

HHH with sources: Explicit breaking at high enough T

$$H = H_{\mathrm{CFT}} - \int d^2x (M\mathcal{O}^\dagger + \bar{M}\mathcal{O}) \qquad \bar{M} = M^\star$$

Complexified U(1) deformation:

$$\bar{M} \neq M^\star, \quad M \to Me^{-\theta}, \quad \bar{M} \to \bar{M}e^{\theta}$$

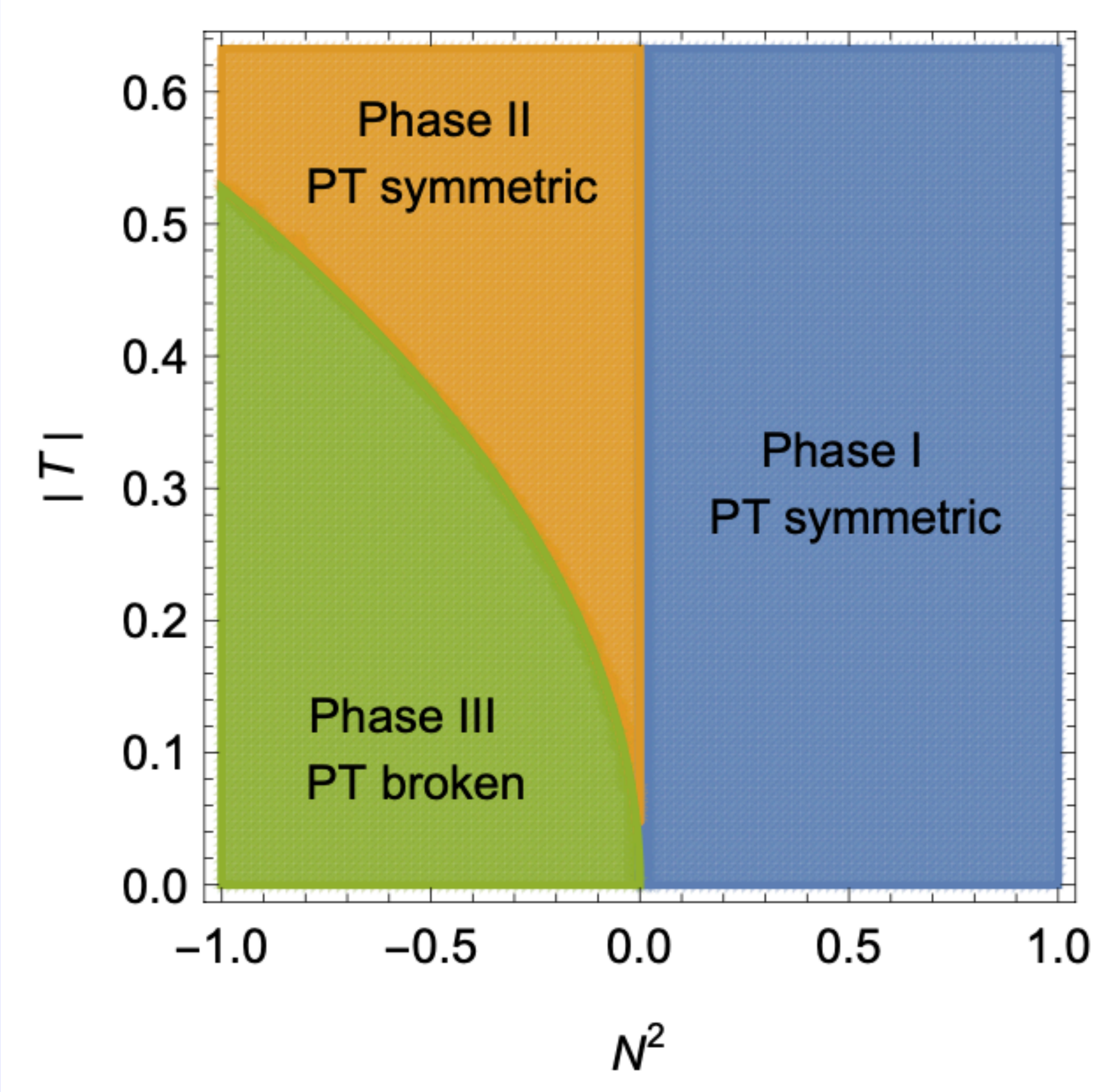
Dyson map:

$$H_\theta = e^{\theta Q} H e^{-\theta Q} = H_{\mathrm{CFT}} - \int d^2x (Me^{-\theta}\mathcal{O}^\dagger + \bar{M}e^{\theta}\mathcal{O})$$

Phases and their Properties

Partition function: $Z[M, \bar{M}] = Z[e^{-\theta} M, e^{\theta} M] = Z[N^2], \quad N^2 = M\bar{M},$

- $N^2 \geq 0$: $M\bar{M}$ is invariant under the Dyson map;
- $N^2 < 0$: non-Hermitian Hamiltonian cannot be mapped to Hermitian one via a Dyson map.



	Phase I	Phase II	Phase III
$\mathcal{PT}$ symmetry	preserved	preserved	broken
(free) energy	real	real	complex
scalar stability	stable	unstable	unstable
vector stability	stable	stable	stable
superfluid density	positive	negative	complex
QC conductivity	suppressed	enhanced	complex
FGT sum rule	holds	holds	holds
NEC	holds	violated	ill-defined

Arean, Landsteiner 2019

Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

Backreacted Solution Structure

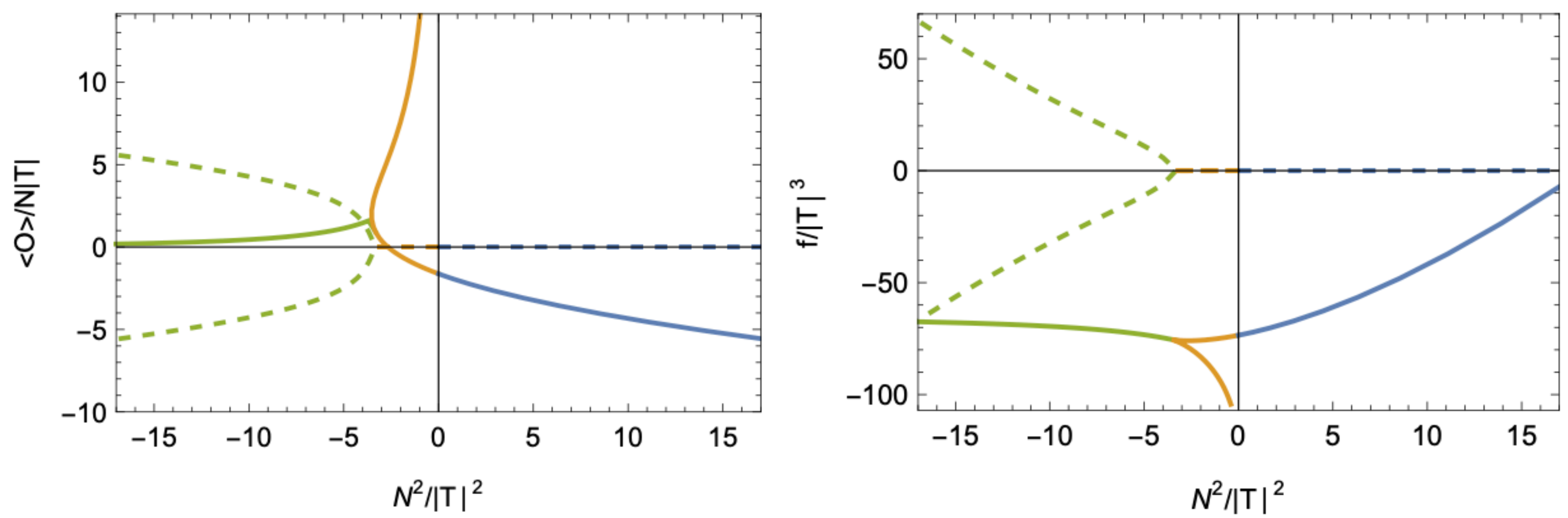
$\mathcal{PT}$ symmetry:

- M, $\bar{M} \in \mathbb{R}$, $\mathcal{PT}$ symmetric, $\mathcal{P}: \phi \to \bar{\phi}$, $\mathcal{T}: \phi \to \bar{\phi}$;

The solutions: $\phi\bar{\phi} \in \mathbb{R}$, $\mathcal{PT}$ symmetric; $\phi\bar{\phi} \notin \mathbb{R}$, $\mathcal{PT}$ broken.

Boundary condition: $\phi(z) = N\,z + \langle \mathcal{O} \rangle\, z^2 + \cdots$,

Free energy: $f = -\frac{1}{3}(Ts + 2N\langle \mathcal{O} \rangle)$.

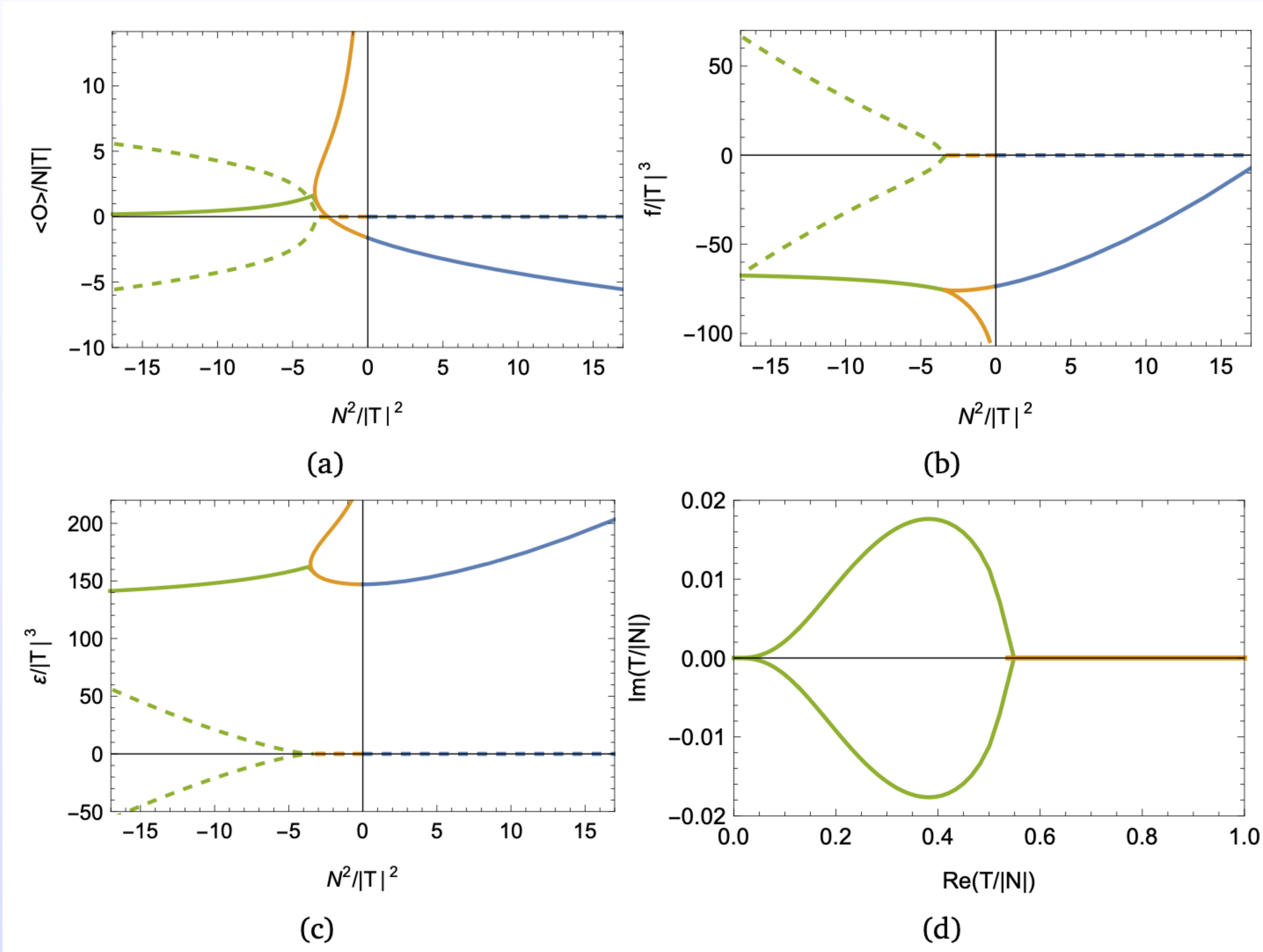


- $N^2 \geq 0$, $\phi\bar{\phi}(z) \geq 0$, $\mathcal{PT}$ symmetry is preserved;
- $(N/T)_c^2 \leq (N/T)^2 < 0$, $\phi(z) \notin \mathbb{R}$, $\phi\bar{\phi}(z) < 0$, $\mathcal{PT}$ symmetry is preserved;
- $(N/T)^2 < (N/T)_c^2$, $\phi\bar{\phi}(z) \in \mathbb{C}$, $\mathcal{PT}$ symmetry is broken.

Arean, Landsteiner 2019

Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

Thermodynamics



Arean, Landsteiner 2019

Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

Scalar channel stability

Stability: No spontaneously growing VEVs at finite $\omega, \vec{k}$

Quasinormal modes must obey $\mathrm{Im}(\omega) \leq 0$

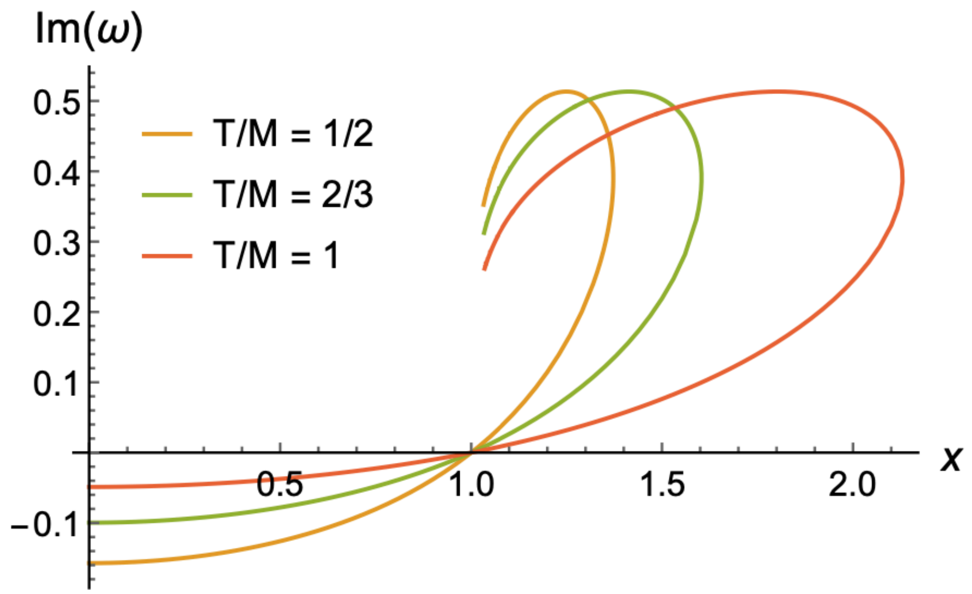


Figure 4: Pseudo-diffusive mode as a function of x for different values of T/M.

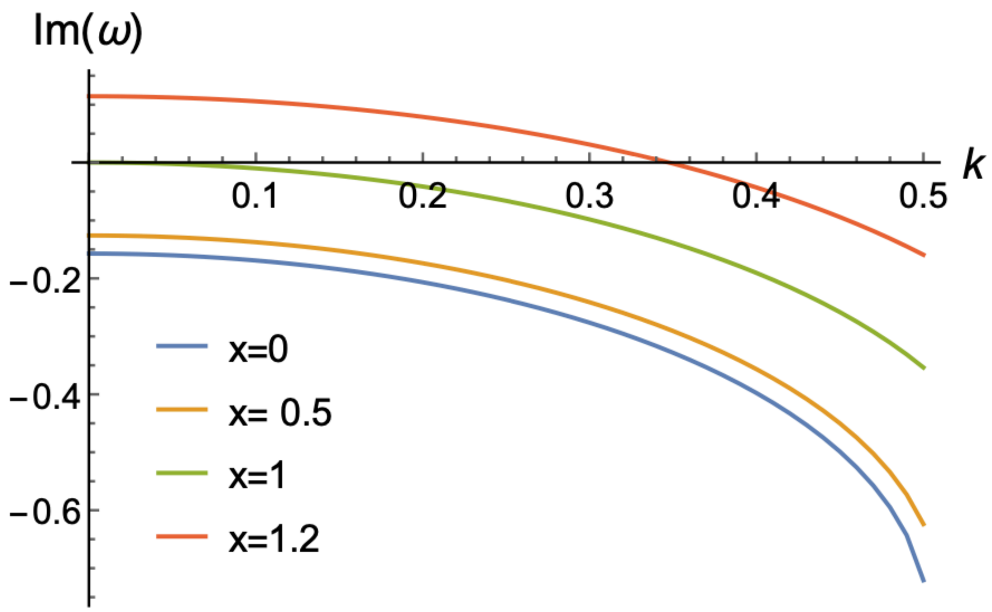


Figure 5: k dependence of the pseudo-diffusive mode for several values of x at $T/M = 1/2$.

Arean, Landsteiner 2019

Endpoint of instability?

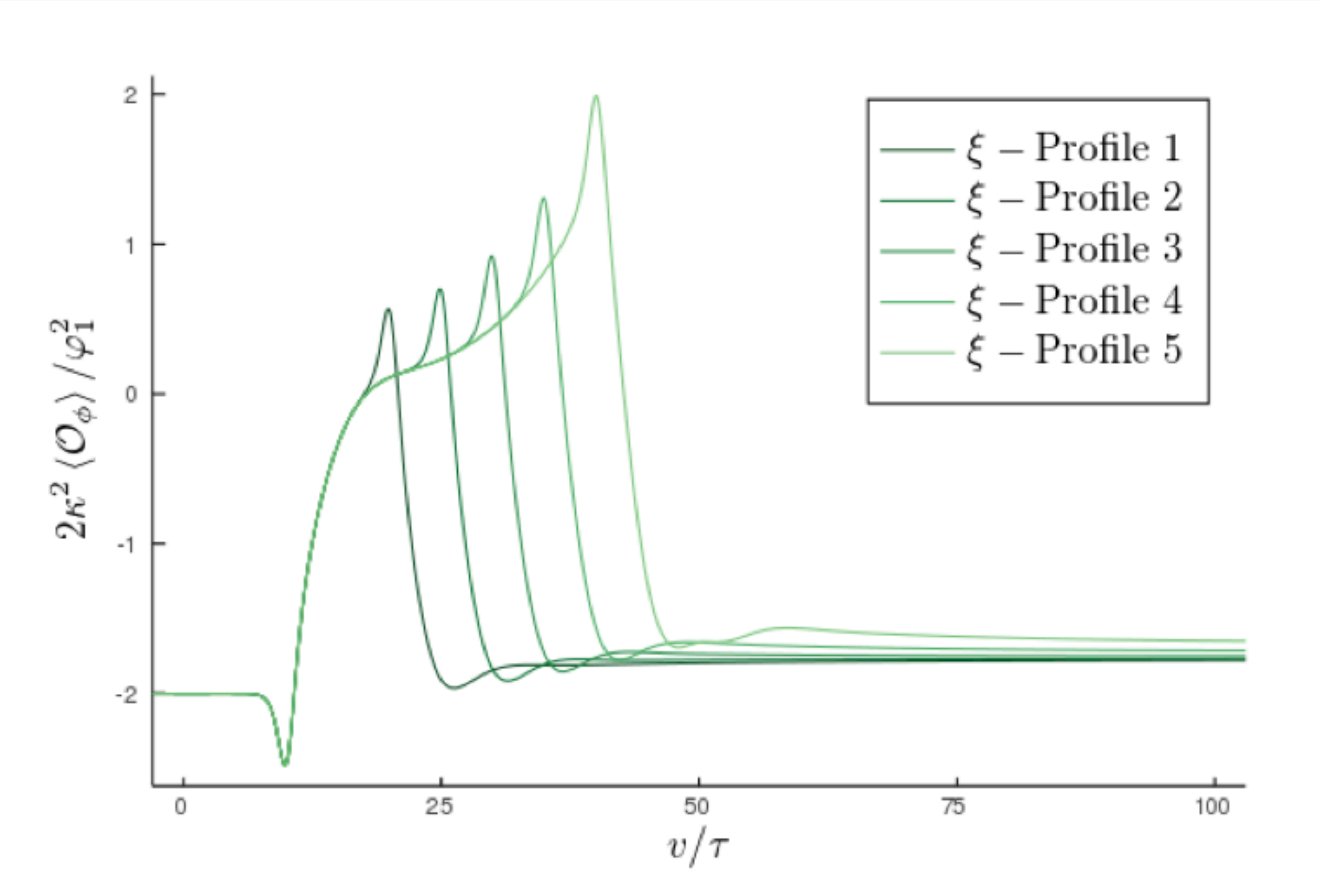


FIG. 5: Expectation values II.22 for a quench with profile III.2 for $\tilde{v}_m/\tilde{\tau} = \{20\,,25\,,30\,,35\,,40\}$. The initial and final states are located at $\xi = 0.8$, besides we set $\xi_m = 1.1$ and $\tau = \tilde{\tau} = 0.25$. We explore here the $\mathcal{PT}$-broken regime $|\xi| > 1$ remaining progresively more time into it. An instability starts to develop, see for instance the green and orange curves, but it eventually fades out as we re-enter the $\mathcal{PT}$-symmetric region.

Landsteiner, Morales Tejera 2203.02524

Outline

1. Motivation

2. PT Symmetric Non-Hermitian QM/QFT

3. Non-Hermitian AdS/CFT
   Landsteiner, Arean SciPost 9 (2020)

4. **Conductivity and Sum Rule**
   Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

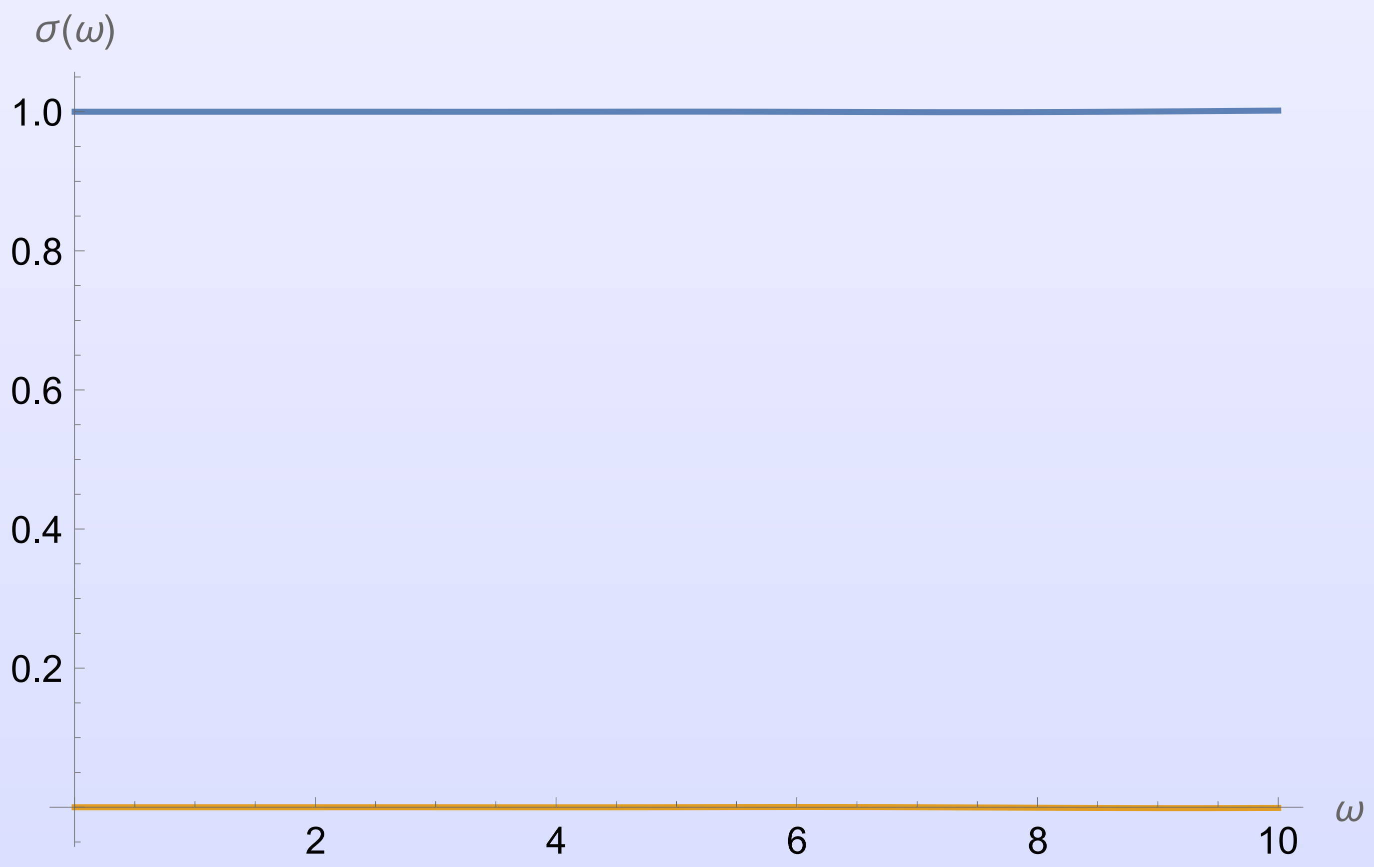
5. Conclusions and Outlook

AC conductivity in AdS-Schwarzschild

Linearized vector perturbations: $\delta A_x(z) = a(z)e^{-i\omega t}\, dx$

$$a(z) = a(0) + a'(0)z$$

Kubo formula: $\sigma_{xx}(\omega) = \frac{\langle J^x J^x \rangle_R(\omega)}{i\omega} = \frac{a'(0)}{i\omega a(0)}$



$$\sigma(\omega) = 1\frac{e^2}{h}$$

Particle-vortex duality

Witten 2003

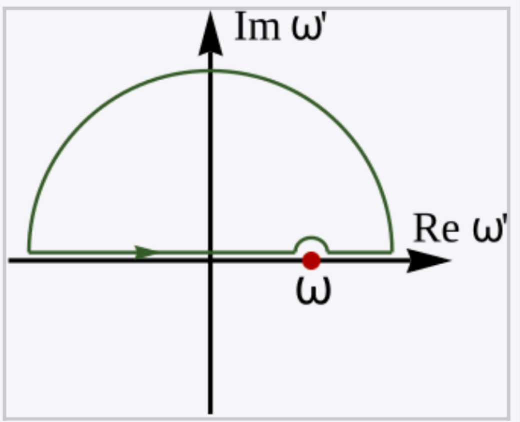
AC conductivity in AdS-RN

Finite density background: $A_t = \mu - \rho z$

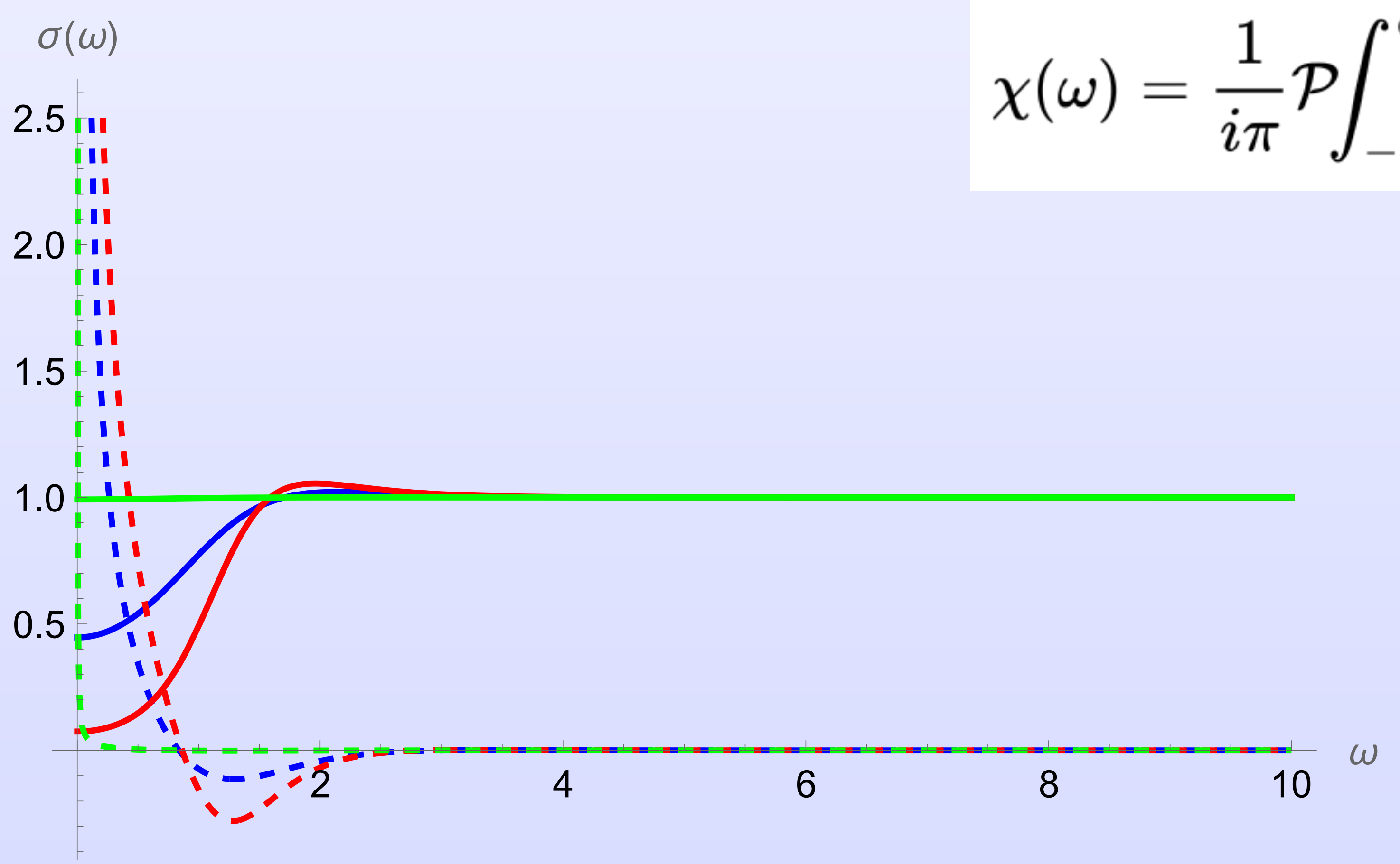
Kubo formula: $\sigma_{xx}(\omega) = \frac{\langle J^x J^x \rangle_R(\omega)}{i\omega} = \frac{a'(0)}{i\omega a(0)}$

Kramers-Kronig: $$0 = \oint \frac{\chi(\omega')}{\omega' - \omega} d\omega' = \mathcal{P}\int_{-\infty}^{\infty} \frac{\chi(\omega')}{\omega' - \omega} d\omega' - i\pi\chi(\omega).$$

$$\chi(\omega) = \frac{1}{i\pi}\mathcal{P}\int_{-\infty}^{\infty} \frac{\chi(\omega')}{\omega' - \omega} d\omega'$$



Integral contour for deriving Kramers–Kronig relations



At low frequencies:

$$\sigma^{xx}(\omega) = \rho_s q^2(\pi\delta(\omega) + \frac{i}{\omega}) + \cdots$$

Ferrel-Glover-Tinkham Sum Rule

Consequence of causality and electric charge conservation in the dissipative real part of the conductivity

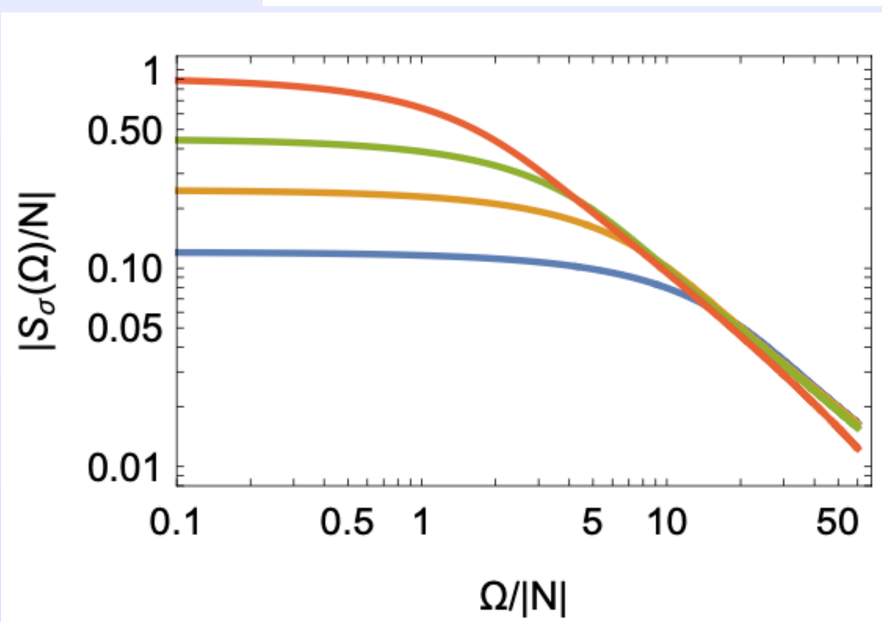
$$S_\sigma(\Omega) = \int_{-\Omega}^{\Omega} (\mathrm{Re}[\sigma(\omega)] - 1)\, d\omega\,.$$

$$\lim_{\Omega\to\infty} S_\sigma(\Omega) = 0$$

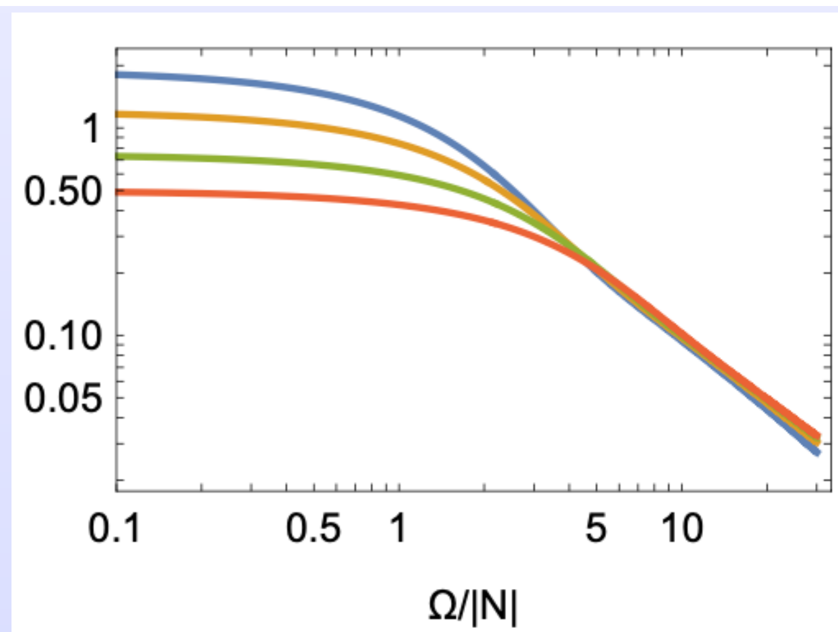
c.f. e.g. Erdmenger, RM, Schalm etal JHEP 2015

Conditions for sum rule to hold:

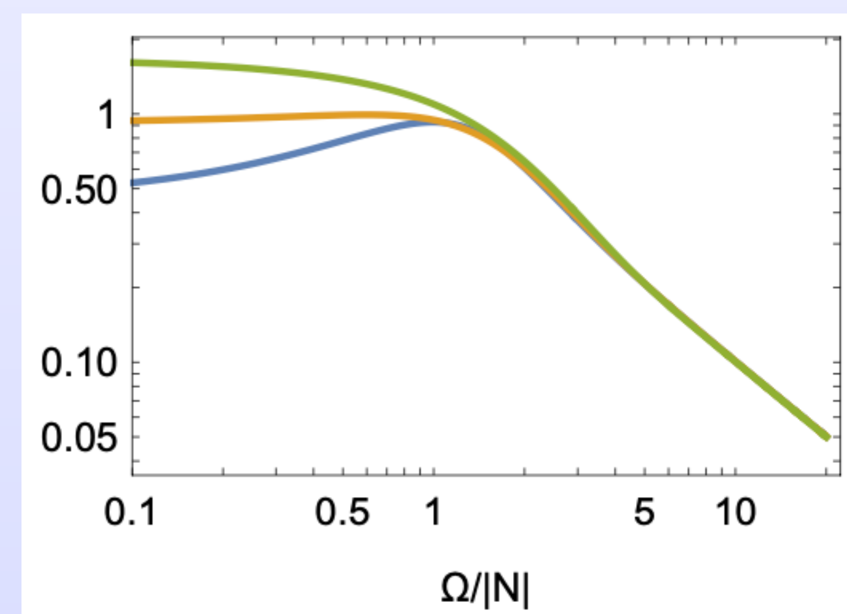
1. $G_R(\omega)$ is analytical on the upper half plane and on the real axis,
2. $\lim_{|\omega|\to\infty} G_R(\omega) = i\omega$.



Phase I



Phase II



Phase III

Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

Vector Channel Stability

Stability: No spontaneously growing VEVs at finite $\omega, \vec{k}$

Quasinormal modes must obey $\mathrm{Im}(\omega) \leq 0$

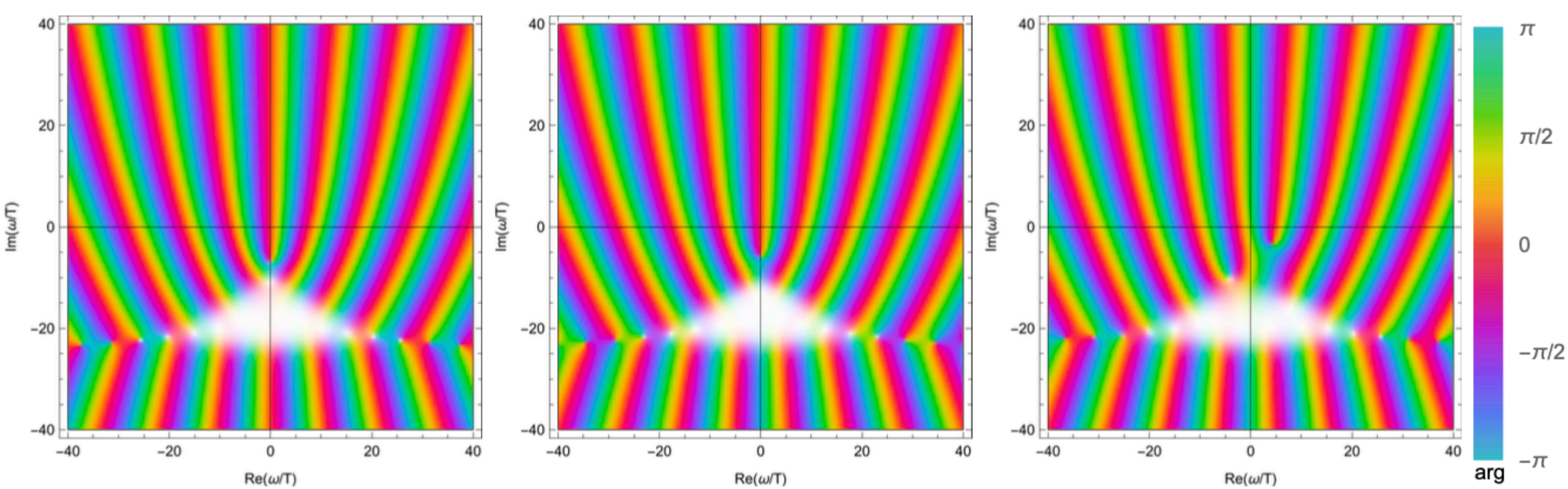


Figure 10: $1/\det[\mathcal{D}]$ in the complex ω/T plane in phase I, II, and III (from left to right), where the color denotes the argument and the shading denotes the absolute value of $1/\det[\mathcal{D}]$. The white points denote the poles. The three plots correspond to $(N^2/\mu^2, T/\mu) = (6.2, 2.4), (-6.2, 2.4), (-100, 2.4 + 0.1i)$, respectively.

Scalar channel was investigated before

Arean, Landsteiner 2019

Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

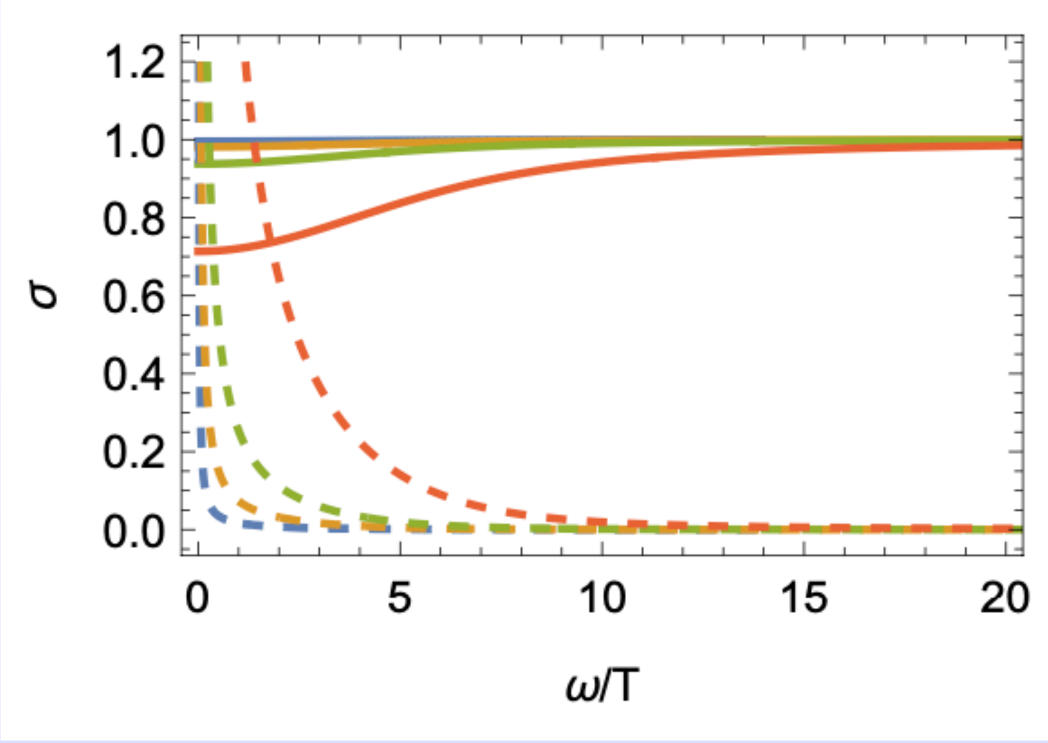
AC conductivity (I)

Linearized vector perturbations: $\delta A_x(z) = a(z)e^{-i\omega t}\,dx$

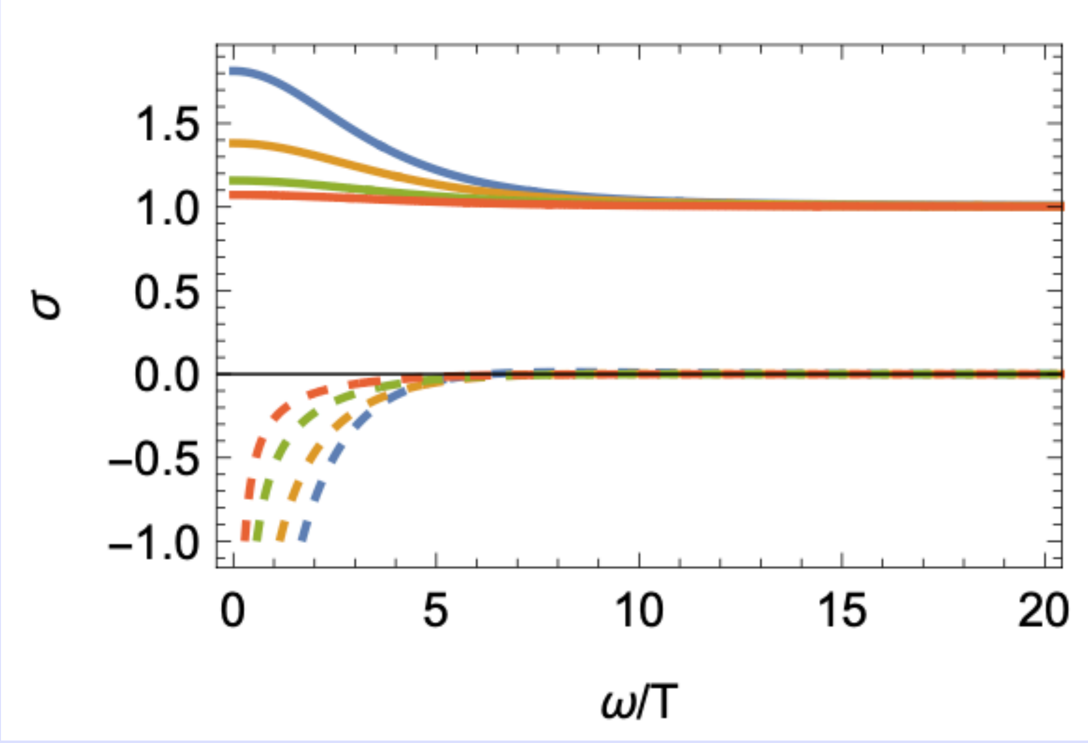
$$a(z) = a(0) + a'(0)z$$

Kubo formula: $\sigma_{xx}(\omega) = \frac{\langle J^x J^x \rangle_R(\omega)}{i\omega} = \frac{a'(0)}{i\omega a(0)}$

At low frequencies: $\sigma^{xx}(\omega) = \rho_s q^2(\pi\delta(\omega) + \frac{i}{\omega}) + \cdots$



Phase I



Phase II

Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

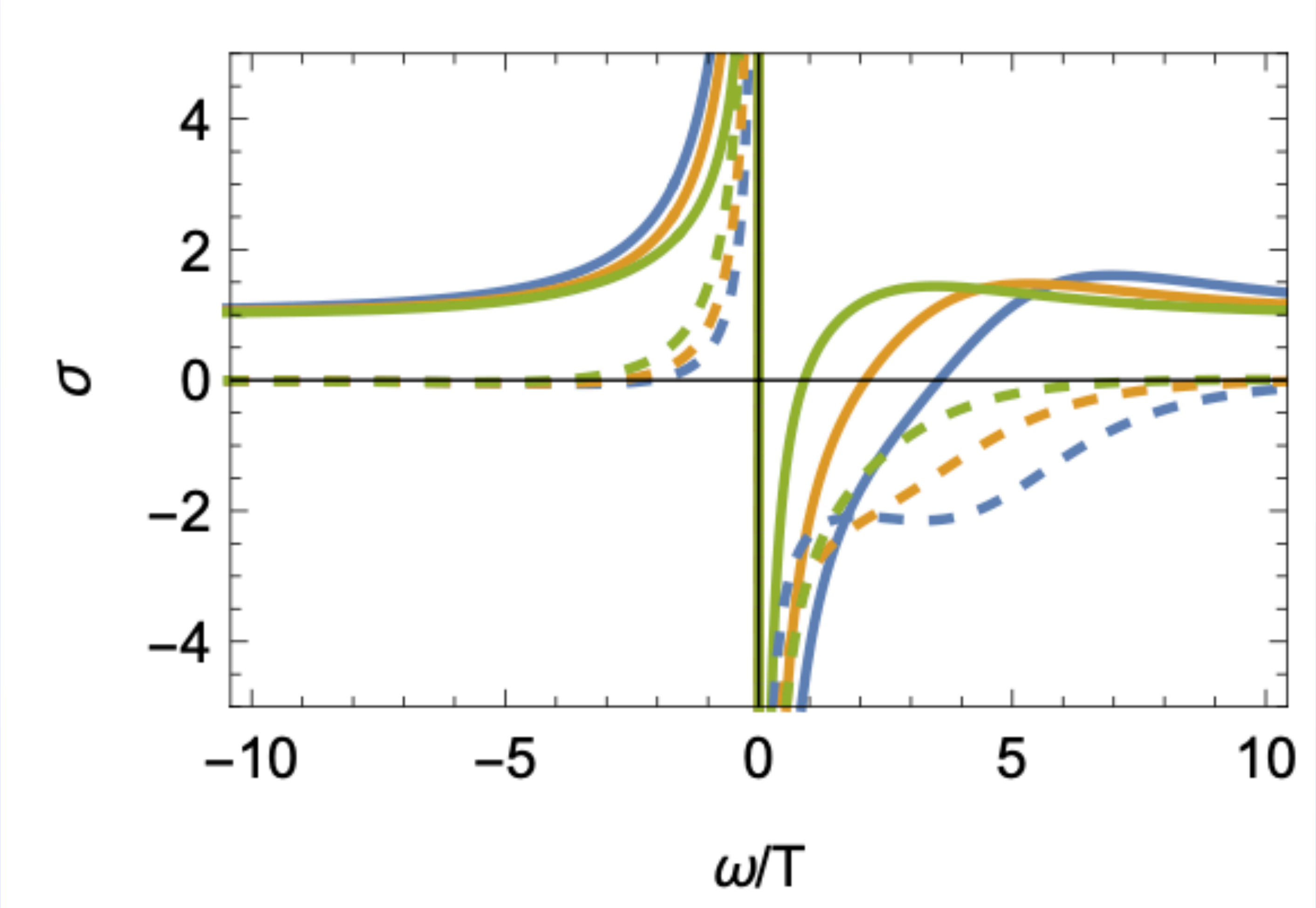
AC conductivity (II)

Linearized vector perturbations: $\delta A_x(z) = a(z)e^{-i\omega t}\,dx$

$$a(z) = a(0) + a'(0)z$$

Kubo formula: $\sigma_{xx}(\omega) = \dfrac{\langle J^x J^x \rangle_R(\omega)}{i\omega} = \dfrac{a'(0)}{i\omega a(0)}$

Phase III



Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

Landau-Ginzburg Theory

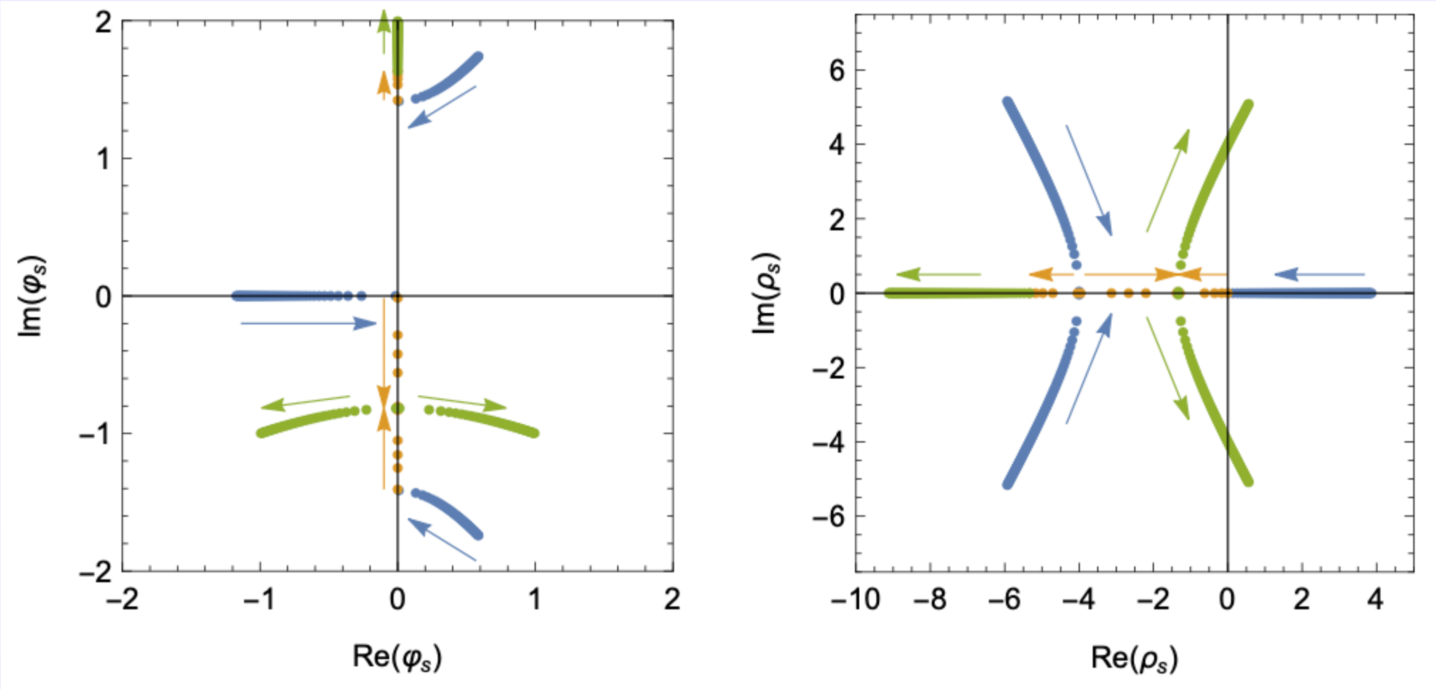
U(1) Rotor model

$$S_\phi = -\int d^d x \left(D_\mu \phi D^{\dagger\mu} \bar{\phi} + V(\phi, \bar{\phi}) + \bar{M}\phi + M\bar{\phi} \right), \quad V(\phi, \bar{\phi}) = r\phi\bar{\phi} + \frac{1}{2}u\phi^2\bar{\phi}^2,$$

Saddle point equations and superfluid density:

$$r_\mu \phi_s + u\phi_s^2 \bar{\phi}_s + M = 0, \quad r_\mu \bar{\phi}_s + u\bar{\phi}_s^2 \phi_s + \bar{M} = 0 \qquad \rho_s = 2\phi_s \bar{\phi}_s.$$

Superfluid density qualitatively follows holographic model



Full AC conductivity needs fluctuations and electrons

Xian ZY, Rodriguez, Chen ZH, Liu Y, RM, SciPost Physics 16 (2024)

Conclusions & Outlook (I)

Conclusions

- Interesting phase structure with QPT
- New behavior of AC conductivity
- Stability and sum rule in vector channel guaranteed
- Phase structure and zero frequency spectral weight reproduced by U(1) rotor model

Outlook

- Stability & nature of phases II/III unclear **Landsteiner et.al**
- Finite density, magnetic fields
- Superconductivity at low temperatures **Chen, Kornich, RM, Xian**
- Other transport coefficients?
- PT deformations of other holographic systems
- Relate to strongly correlated PT symmetric systems