Holographic Interfaces in Symmetric Product Orbifolds

AdS_2 branes in $AdS_3 \times S^3 \times \mathbb{T}^4$ at $k = 1$

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HELMHOLTZ

DESY.

Overview

Two Questions

1. Defects in holography
2. Strings on AdS_2 branes

Some Context

3. Tensionless holography
4. Branes in AdS_3

Our Answer

5. Holographic interfaces
6. One-loop partition functions.
7. General correlators

The talk is based on [2504.00078] by

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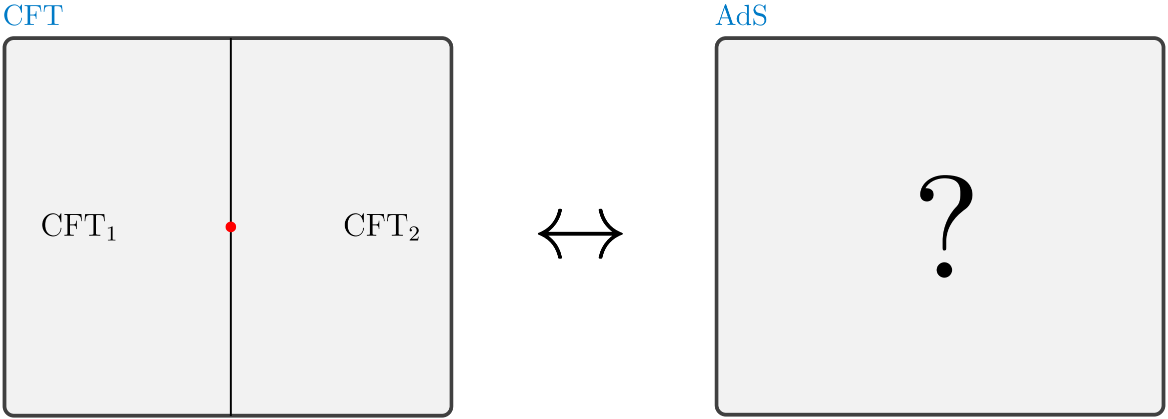
  - Deutsches Elektronen-Synchrotron, Hamburg.
- Yasuaki Hikida
  - Osaka Institute of Technology, Osaka.
- Volker Schomerus
  - Deutsches Elektronen-Synchrotron, Hamburg.
- Takashi Tsuda
  - Yukawa Institute for Theoretical Physics, Kyoto.

Two Questions.

1. Defects in holography



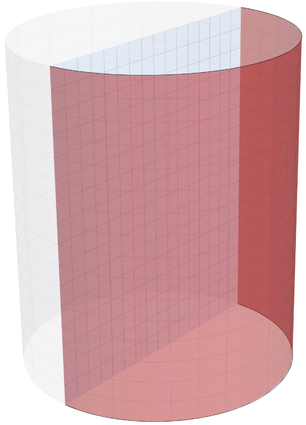
2. Strings on AdS_2 branes

CFT

?

$\leftrightarrow$

AdS



Some Context.

3. Tensionless holography – deriving AdS/CFT

> One of the many exciting features of AdS/CFT is that it is a strong weak duality.

$$\frac{R}{\ell_s} \sim g_{\rm YM}^2 N = \lambda \tag{1}$$

> However, this makes checks of the correspondence notoriously hard.
- small ℓ_s super gravity $\leftrightarrow$ perturbative gauge theory breaks down.
- small λ perturbative gauge theory $\leftrightarrow$ classical geometry breaks down.

> Remarkably, [Eberhardt, Gaberdiel, Gopakumar, '18,'19] established the example

Pure NS-NS strings on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ with $k = 1$	$\Longleftrightarrow$	$\mathrm{Sym}^N(\mathbb{T}^4)$

where both sides of the duality are under full comutational control.

3. Tensionless holography – string theory

- String theory on $\mathrm{AdS}_3 \times \mathrm{S}^3$ is special due to its relation to WZW models.

$$\text{RNS strings on } \mathrm{AdS}_3 \times \mathrm{S}^3 \cong \mathfrak{sl}(2,\mathbb{R})_{k+2} \oplus \mathfrak{su}(2)_{k-2} \oplus (\text{6 free fermions}) \tag{2}$$

- A non-negative level of $\mathfrak{su}(2)_{k-2}$ requires $k \geq 2$. Through a chain of field redefinitions,

$$\begin{gathered} \text{hybrid strings on } \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4 \\ \Longleftrightarrow \\ \mathfrak{psu}(1,1|2)_k \oplus [(\rho,\sigma) \text{ ghosts}] \oplus [\text{topologically twisted } \mathbb{T}^4] \end{gathered} \tag{3}$$

allows us to analyse minimal $k = 1$ case. But $\mathfrak{psu}(1,1|2)_1$ has free field realisation.

- Thus, for $k = 1$, a free WS theory describes strings on a curved background.

3. Tensionless holography – symmetric orbifold

> For every 2d CFT $\mathcal{M}$, one can construct the symmetric orbifolds $\mathrm{Sym}^N(\mathcal{M})$.

> Objects in the orbifolded theory are usually obtained through the following recipe.

1. Choose some initial data in the seed theory $\mathcal{M}$.
2. Lift to the product $\mathcal{M}^N$.
3. Add twisted sectors associated to S_N action on $\mathcal{M}^N$.
4. Project onto S_N invariant subspace.

> As an example, for the Hilbert space $\mathcal{H}$ of the theory, we have

$$\mathcal{H} \to \mathcal{H}^{\otimes N} \to \bigoplus_{g \in S_N} \mathcal{H}^{(g)} \to \Big[\bigoplus_{g \in S_N} \mathcal{H}^{(g)} \Big]^{S_N}. \quad (4)$$

3. Tensionless holography – symmetric orbifold

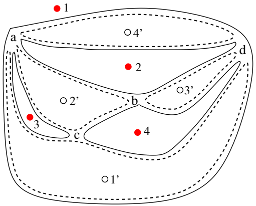
> holomorphic coverings play a crucial role in the theory of symmetric orbifolds.

[Lunin,Mathur,'00]: Symmetric orbifold correlators lift to seed theory correlation functions through holomorphic coverings of the space time Riemann surface X.

$$\langle \dots \rangle_X^{\mathrm{Sym}^N(\mathcal{M})} = \sum_{\Gamma:\Sigma\to X} W[\Gamma] \langle \dots \rangle_\Sigma^{\mathcal{M}} . \tag{5}$$

> Other uesful tools to study symmetric orbifolds include

- [Pakman,Rastelli,Razamat,'09]: Diagrammatic rules for the large N expansion of correlators.
- [Dei,Eberhardt,'19]: Computing symmetric orbifold correlation functions from Ward identities.



3. Tensionless holography – localisation principle

> A key mechanism underlying the duality is the localisation of the WS.

The integral over tensionless string WS localises to holomorphic coverings of ∂AdS_3 and is identified with the Lunin-Mathur sum over covering maps. In a cartoon of an equation, this could be expressed as

$$\text{“} \sum_{\Sigma} \int_{\Gamma:\Sigma\to\text{AdS}_3} \tilde{W}[\Gamma]\langle \dots \rangle_{\Sigma}^{\mathbb{T}^4} = \sum_{\Gamma:\Sigma\to\partial\text{AdS}_3} W[\Gamma]\langle \dots \rangle_{\Sigma}^{\mathbb{T}^4} = \langle \dots \rangle_{\partial\text{AdS}_3}^{\text{Sym}^N(\mathbb{T}^4)} \text{”} \tag{6}$$

> Evidence includes a matching of

- closed string spectrum. [Gaberdiel, Gopakumar,'18, Eberhardt,'20]
- closed string scattering amplitudes. [Eberhardt,Gaberdiel,Gopakumar,'18]
- annulus amplitudes of instantonic branes. [Gaberdiel, Knighton, Vosmera,'21]
- open string spectrum supported by AdS_2 branes. [SH, Hikida, Schomerus, Tsuda,'25]

4. Branes in AdS_3 – glueing conditions of $\mathrm{SL}(2,\mathbb{R})$ currents

> Strings on AdS_3 with pure NS-NS flux are described by the $\mathrm{SL}(2,\mathbb{R})_k$ WZW model.

$$(X^0)^2-(X^1)^2-(X^2)^2+(X^3)^2=R^2 \Leftrightarrow \frac{1}{R}\begin{pmatrix} X^0+X^1 & X^2+X^3 \\ X^2-X^3 & X^0-X^1 \end{pmatrix} \in \mathrm{SL}(2,\mathbb{R}).$$

> Symmetric D-branes can be classified by analysing glueing conditions

$$J^a(z)=\Omega^a_b\bar{J}^b(\bar{z}). \tag{7}$$

of the current algebra [Bachas, Petropoulos, '00].

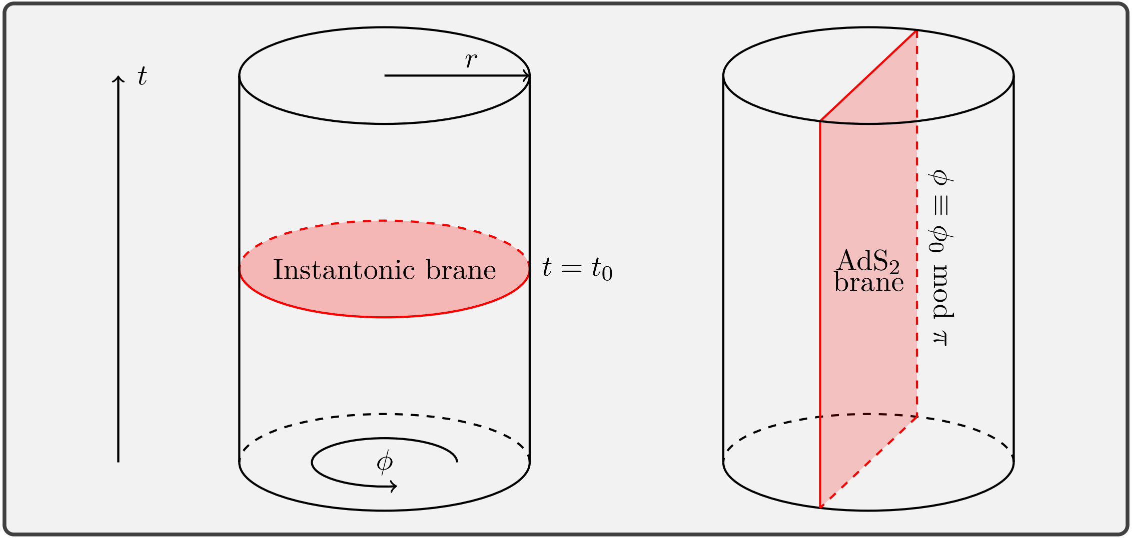
Instantonic branes

> Glueing "trivial".
> D-branes fixing time coordinate t.

AdS_2 branes

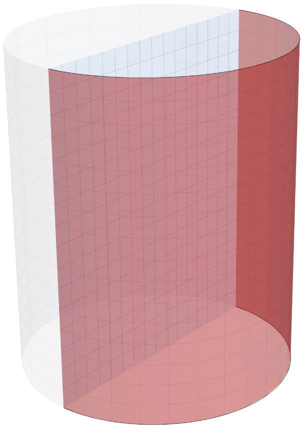
> Glueing twisted by outer automorphism.
> D-branes fixing angular coordinate ϕ.

4. Branes in AdS_3 – a picture



4. Branes in AdS_3 – WS perspective on AdS_2 branes

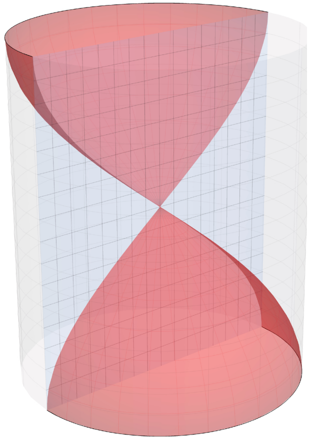
> [Gaberdiel, Knighton, Vosmera,'21] gave WS boundary states for AdS_2 branes.



$\leftarrow k = 1$

From the WS perspective, the AdS_2 brane splits into two half-branes at $k = 1$.

$k \gg 1 \rightarrow$



4. Branes in AdS_3 – History of AdS_2 branes

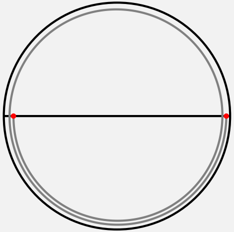
The 2000s, generic k string theory description.

- [Bachas, Petropoulos, '00]: Classification of branes in AdS_3.
- [Lee, Ooguri, Park, Tannenhauser,'01]: Open string spectrum.
- [Lee, Ooguri, Park, '01]&[Ponsot, Schomerus, Teschner, '01]: WS boundary states.
- [Ribault, '02]: Two brane system.
- [Ribault, '07]: WS boundary OPE.
- [Fateev, Ribault,'07]: WS boundary action.
- [Creutzig, Hikida, Rønne, '10]: Boundary FZZ duality.

The 2020s, $k = 1$ holographic description.

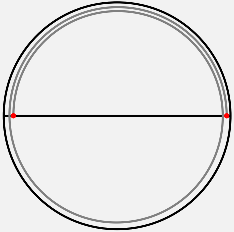
- [Gaberdiel, Knighton, Vosmera,'21]: $k = 1$ WS boundary states.
- [Martinec,'22]: $k < 1$ qualitative description of holographic dual.
- [SH, Hikida, Schomerus, Tsuda,'25]: $k = 1$ construction of holographic dual.

4. Branes in AdS_3 – open strings ending on AdS_2 branes



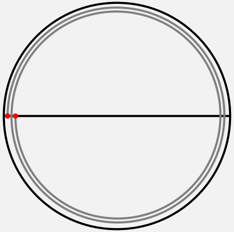
Odd winding LR

$$Z_{LR}^{(w)}$$



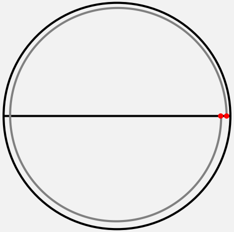
Odd winding RL

$$Z_{RL}^{(w)}$$



Even winding LL

$$Z_{LL}^{(w)}$$



Even winding RR

$$Z_{RR}^{(w)}$$

4. AdS_2 branes – holography of AdS_2 branes

> The [Gaberdiel, Knighton, Vosmera,'21] proposal cannot be extended to AdS_2 branes.

Only $w = 0$ states of the WS theory couple to the AdS_2 boundary states. But there are no $w = 0$ strings at $k = 1$. No closed strings couple to the branes!

> [Martinec, '22] provides an insightful qualitative discussion of AdS_2 branes in $\mathrm{AdS}_3/\mathrm{CFT}_2$ holography (focussing on the noncricitcal $k < 1$ case).

> Martinec proposed that the ST CFT description of the branes should be in terms of some ensemble of interfaces with varying degree of transmissivity.

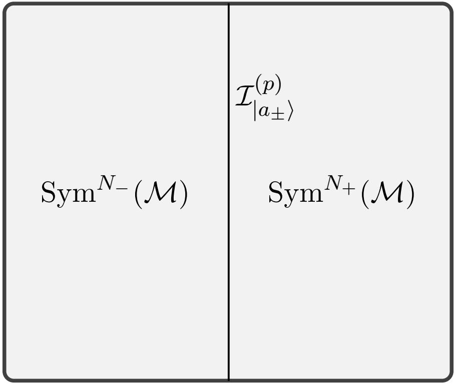
> However, no construction of the interfaces was given in [Martinec, '22].

What is the holographic dual of AdS_2 branes in $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ at $k = 1$?

Our Answer.

5. Holographic interfaces – general overview

> In our work, we constructed interfaces $\mathcal{I}^{(p)}_{|a_\pm\rangle}$ between $\mathrm{Sym}^{N_\pm}(\mathcal{M})$.



> The labels of $\mathcal{I}^{(p)}_{|a_\pm\rangle}$ are

- $|a_\pm\rangle$, two seed theory boundary states.
- $p \in \mathbb{Z}$ with $0 \leq p \leq \min(N_-, N_+)$ measuring transmissivity.

> In particular

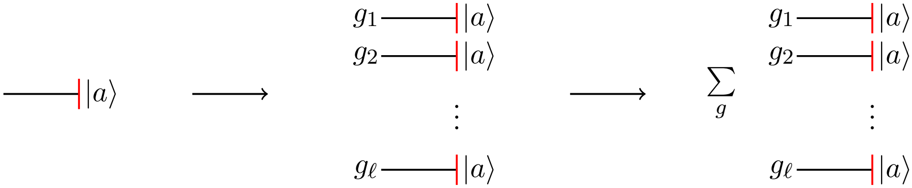
- $N_- = N_+ = p$ corresponds to the trivial defect.
- $p = 0$ corresponds to decoupled BCFTs.

5. Holographic interfaces – the special case $N_- = 0$

- $N_- = 0$ enforces $p = 0$. The interface reduces to a boundary state for $\mathrm{Sym}^{N_+}(\mathcal{M})$.
- A seed theory boundary state $|a\rangle$ can be lifted to the symmetric orbifold according to

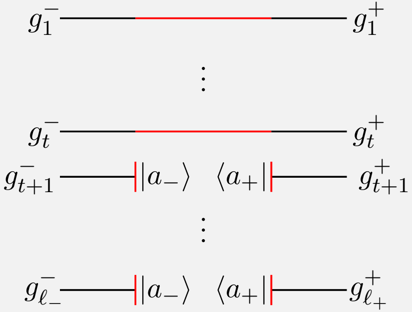
$$|a\rangle = \sum_j a_j |j\rangle\rangle \quad \rightarrow \quad |a\rangle_g = \frac{1}{\sqrt{|\mathcal{C}_g|}} \prod_{\nu=1}^{\ell} \sum_{j_\nu} a_{j_\nu} |j_\nu\rangle\rangle_{g_\nu} \quad \rightarrow \quad |a\rangle_\chi = \sum_{g\in S_N} \frac{\chi(g)}{|[g]|} |a\rangle_g \tag{8}$$

where χ is an S_N character and $g = g_1 \dots g_\ell$ with g_i cyclic. Pictorially,



5. Holographic interfaces – the general case

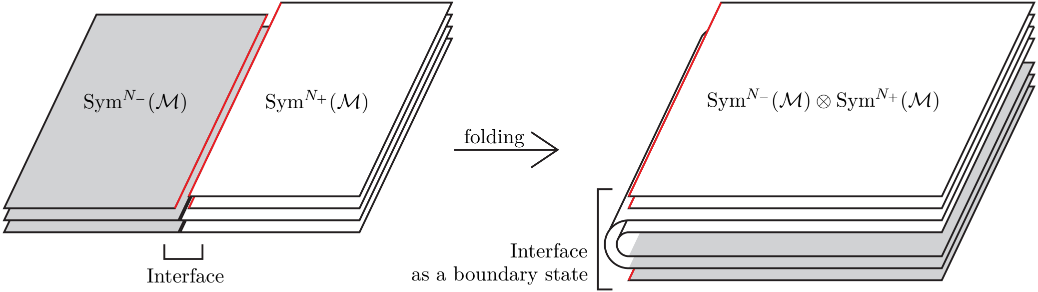
$$\mathcal{I}^{(p)}_{|a_\pm\rangle} = \sum_{g^\pm}$$



$$\text{where} \quad \sum_{i=1}^{t} |g_i^\pm| = p \,. \tag{9}$$

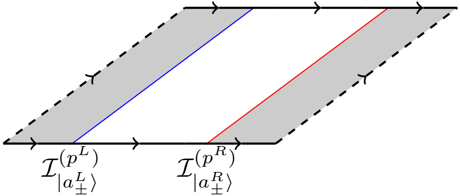
6. One-loop partition functions – folding trick

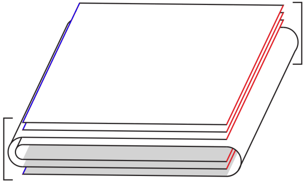
> Using the folding trick, we rewrite the interface $\mathcal{I}^{p}_{|a_\pm\rangle}$ as a boundary state $|p, a_\pm\rangle_{N_\pm}$.



6. One-loop partition functions – folding trick

> Folding maps the torus partition fn. with two interfaces to a cylinder amplitude.





> Hence, the folded quantity to compute is

$$\mathcal{Z}^{N_\pm}_{(p^L,a^L_\pm),(p^R,a^R_\pm)}(t) = \langle p^L, a^L_\pm | q^{L_0 - \frac{c(N_- + N_+)}{24}} | p^R, a^R_\pm \rangle. \tag{10}$$

6. One-loop partition functions – result

> Consider the grand canonical partition function

$$\mathcal{Z}_{a_\pm^{L/R}}[\mu_\pm, \rho_{L/R}; t] := \sum_{N_\pm=0}^{\infty} \sum_{p^{L/R}=0}^{\min(N_\pm)} \mu_+^{N_+} \mu_-^{N_-} \rho_L^{N_+ + N_- - 2p^L} \rho_R^{N_+ + N_- - 2p^R} \mathcal{Z}^{N_\pm}_{(p^L, a_\pm^L),(p^R, a_\pm^R)}(t).$$

> Like the bulk torus partition function it exponentiates

$$\mathcal{Z}_{a_\pm^{L/R}}[\mu_\pm, \rho_{L/R}; t] = \exp(\mathcal{Z}_C[\mu_\pm; t] + \mathcal{Z}_O[\mu_\pm, \rho_{L/R}; t]). \tag{11}$$

> In terms of $Z_c(t) = Z_{\mathcal{M}}(t, t)$, the "single closed string" contribution $\mathcal{Z}_C$

$$\mathcal{Z}_C[\mu_\pm; t] = \sum_{k=1}^{\infty} \mu_-^k \mu_+^k T_k Z_c(t) \quad \text{with} \quad T_k Z_c(t) = \frac{1}{k} \sum_{w|k} \sum_{j=0}^{w-1} Z_c\left(\frac{kt}{w^2} + \frac{j}{w}\right). \tag{12}$$

6. One-loop partition functions – result

> The "single open string" contribution $\mathcal{Z}_O$ is

$$\mathcal{Z}_O[\mu_\pm, \rho_{L/R}; t] = \sum_{A,B\in\{L,R\}} \sum_{k=1}^{\infty} \sum_{\ell|k} \rho_A^\ell \rho_B^\ell \mu_-^{k-\ell\delta_A^R\delta_B^L} \mu_+^{k-\ell\delta_A^L\delta_B^R} \frac{1}{\ell} Z_o^{AB}\left((2\tfrac{k}{\ell} - 1 + \delta_A^B)\tfrac{t}{\ell}\right).$$

> Here,

$$Z_o^{LL} = Z_{a_-^L,(a_+^L)^*}, \quad Z_o^{LR} = Z_{a_-^L,a_-^R}, \quad Z_o^{RL} = Z_{a_+^L,a_+^R} \quad \text{and} \quad Z_o^{RR} = Z_{(a_+^R)^*,a_-^R}. \tag{13}$$

> Specifying $\mathcal{M} = \mathbb{T}^4$, we showed that $\mathcal{Z}_O$ precisely agrees with the annulus partition function of open strings ending on an AdS_2 brane in thermal AdS_3.

7. General correlators

> The AdS_2 brane does not alter holography of pure closed string scattering.

> As first step to an open string extension, we matched the large N expansion of the orbifold with the g_s expansion of string pertrubation theory.

> Concretely, a genus g WS whose boundary has b connected components contributes to string scattering amplitudes of n_o open strings and n_c closed strings weighted by

$$g_s^{-2+2g+b+n_c+\frac{n_o}{2}}. \tag{14}$$

> We showed that terms contributing to orbifold correlators are weighted by

$$N^{1-g-\frac{b}{2}-\frac{n_c}{2}-\frac{n_o}{4}}, \tag{15}$$

where g and b are topological data of the covering space, consistent with $N \sim 1/g_s^2$.

Outlook and summary

In [SH, Hikida, Schomerus, Tsuda, 2504.00078], we provide

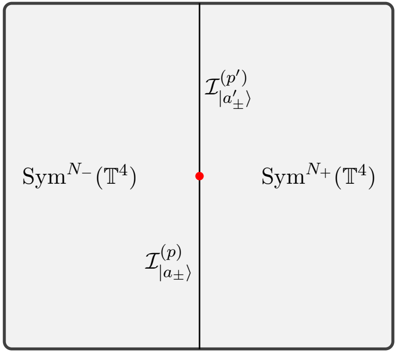
- an explicit construction for a novel class of interfaces in symmetric product orbifolds.
- a match between the spectrum of interface changing operators and the open string spectrum supported by an AdS_2 brane in tensionless holography.
- first evidence that correlation functions and string scattering amplitudes also match.

Next, we would like to

- extend our analysis of correlation functions and scattering amplitudes
  - in particular, study boundary correlation functions in generic symmetric orbifolds.
- deform away from $k = 1$ along the lines of [Eberhardt,'21].

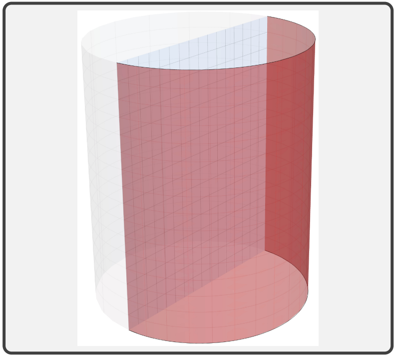
Thank you!

$$\bigoplus_{N_-=0}^{\infty} \mathrm{Sym}^{N_-}(\mathbb{T}^4) \times \bigoplus_{N_+=0}^{\infty} \mathrm{Sym}^{N_+}(\mathbb{T}^4)$$



$\leftrightarrow$

Pure NS-NS strings on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$ with minimal $k = 1$



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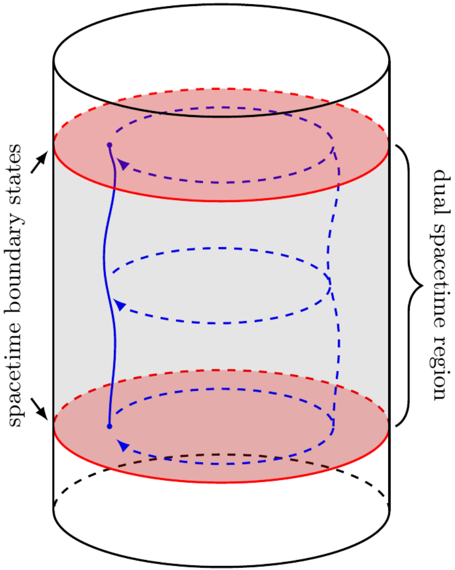
Additional material

8. Instantonic branes – string theory

- From the explicit knowledge of the WS boundary states, closed string cylinder amplitudes can be computed [Gaberdiel, Knighton, Vosmera,'21].
- In agreement with the localisation principle, the string WS wraps the boundary cylinder.
- Thus, the cylinder amplitude is a sum

$$Z_{u|v} = \sum_{w=1}^{\infty} Z_{u|v}^{(w)}$$

over sectors of closed strings that wind around the boundary w times.



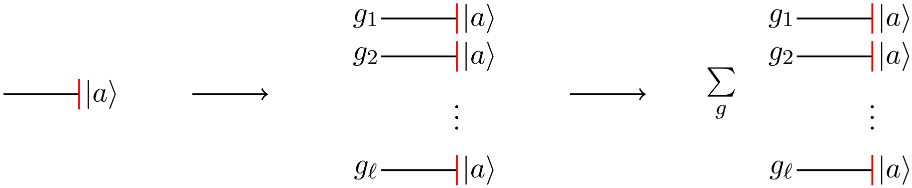
8. Instantonic branes – symmetric orbifold

> A seed theory boundary state $|a\rangle$ can be lifted to the symmetric orbifold according to

$$|a\rangle = \sum_j a_j |j\rangle\rangle \quad \rightarrow \quad |a\rangle_g = \frac{1}{\sqrt{|\mathcal{C}_g|}} \prod_{\nu=1}^{\ell} \sum_{j_\nu} a_{j_\nu} |j_\nu\rangle\rangle_{g_\nu} \quad \rightarrow \quad |a\rangle_\chi = \sum_{g \in S_N} \frac{\chi(g)}{|[g]|} |a\rangle_g \tag{16}$$

where χ is an S_N character and g is the product $g_1 \dots g_\ell$ of cyclic permutations.

> Pictorially,



8. Instantonic branes – symmetric orbifold

> To match the cylinder amplitude computed in string theory, consider

$$ {}_N\langle u | q^{L_0 - N\frac{c}{24}} | v\rangle_N = \sum_{g\in S_N} \begin{matrix} \langle u| \vdash\!\!\!-\!\!\!-\!\!\!-\, g_1 \,-\!\!\!-\!\!\!-\!\!\dashv |v\rangle \\ \vdots \\ \langle u| \vdash\!\!\!-\!\!\!-\!\!\!-\, g_\ell \,-\!\!\!-\!\!\!-\!\!\dashv |v\rangle \end{matrix} \quad . \tag{17} $$

> The grand canonical partition fn. is an exponential of a "single paricle" partition fn.

$$ \sum_{N=0}^{\infty} {}_N\langle u | q^{L_0 - N\frac{c}{24}} | v\rangle_N = \exp\left(\sum_{g \text{ cyclic}} \langle u| \vdash\!\!\!-\!\!\!-\!\!\!-\, g \,-\!\!\!-\!\!\!-\!\!\dashv |v\rangle \right) \tag{18} $$

> The single particle partition function agrees with that computed in string theory

$$ \sum_{g \text{ cyclic}} \langle u| \vdash\!\!\!-\!\!\!-\!\!\!-\, g \,-\!\!\!-\!\!\!-\!\!\dashv |v\rangle = \sum_{w=1}^{\infty} Z^{(w)}_{u|v} \qquad \text{with} \qquad w = |g| . \tag{19} $$