IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

CAPES

The expanding Universe, PCA and supernova:

an attempt to avoid dark energy parameterizations

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IPMU - ACP Seminar
October 19th, 2010

Summary

Part 1:
Introduction to
Cosmology 0

accelerated

The expanding Universe

Part 3:
Is it possible?
How?

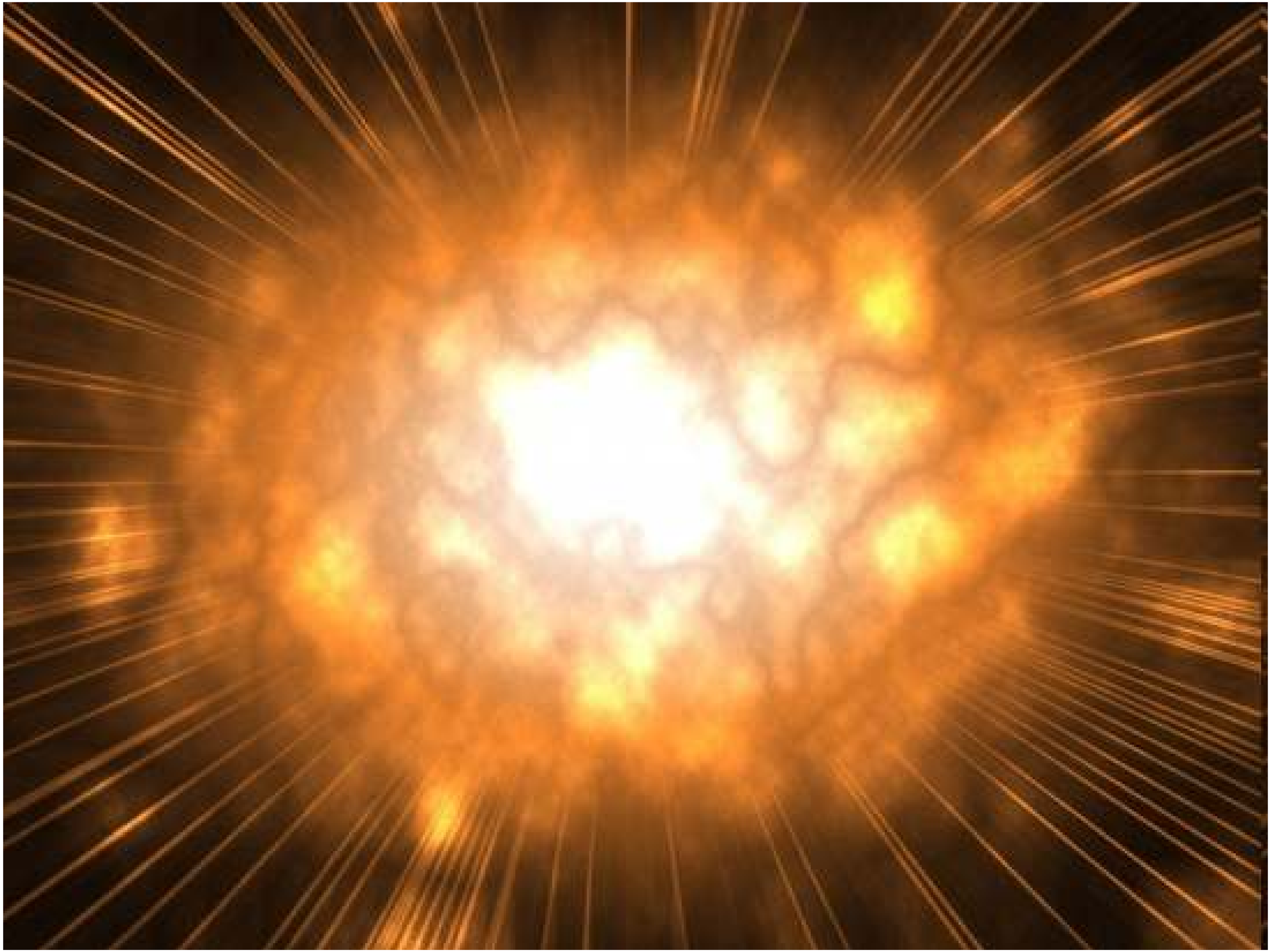
PCA and supernova

OBS: Didn't you say
"*theoretical cosmology*" in the
abstract?

Part 4:
Can you measure it?

an attempt to avoid dark energy
parameterizations

Part 2:
Why?



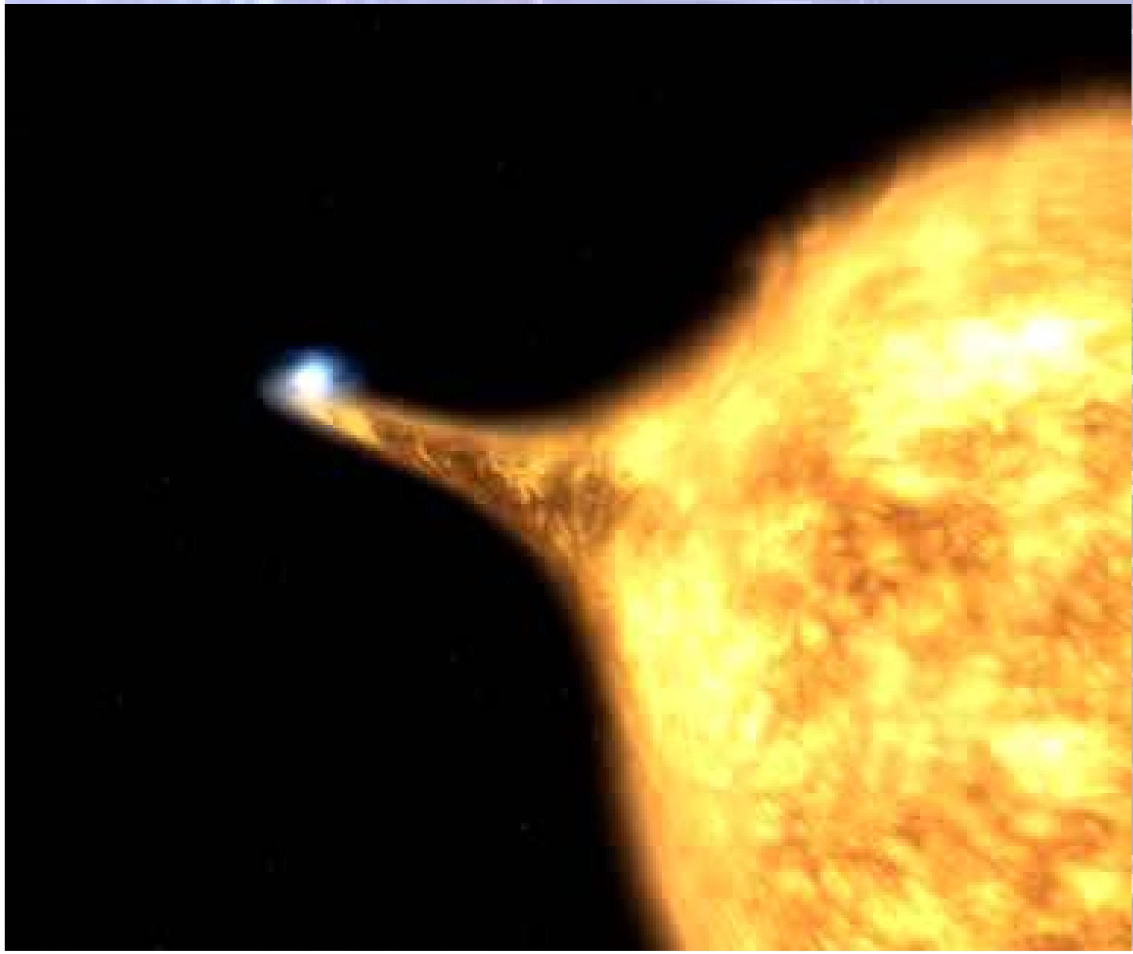
1. Introduction to Cosmology 0

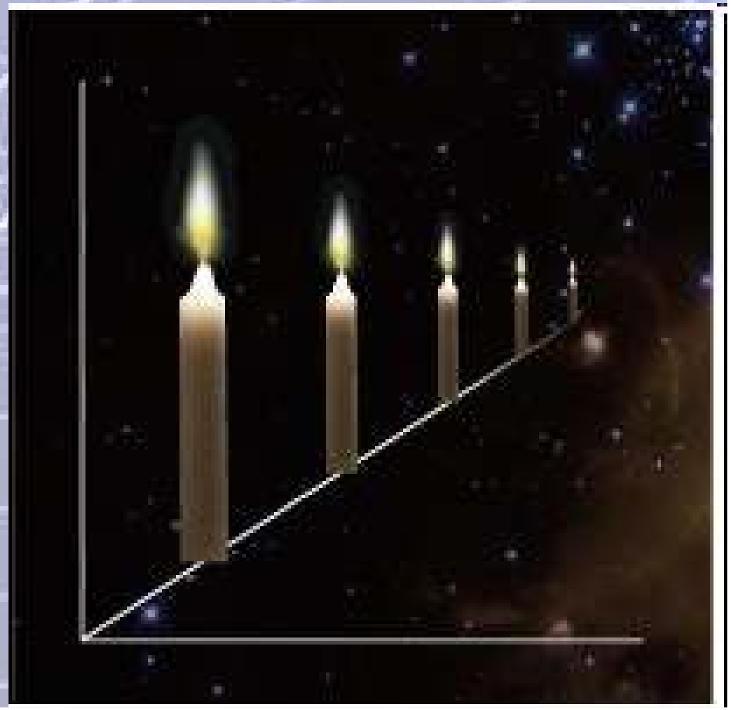
How to test it?

1. Introduction to Cosmology 0

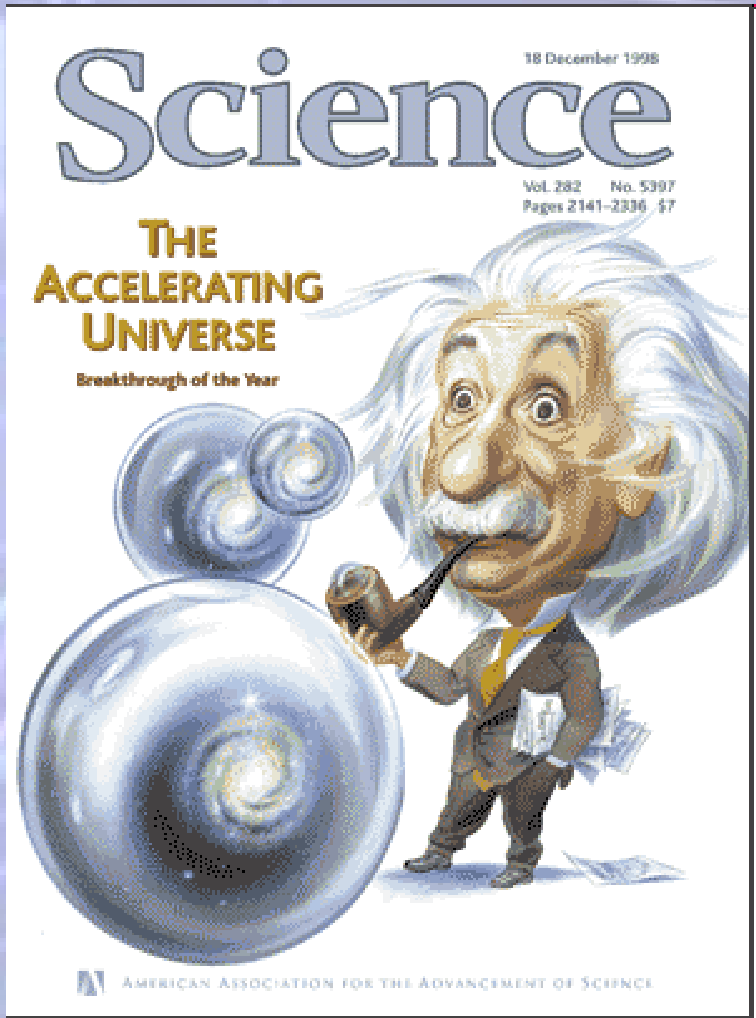
Find something that allows you to measure distances and velocities:

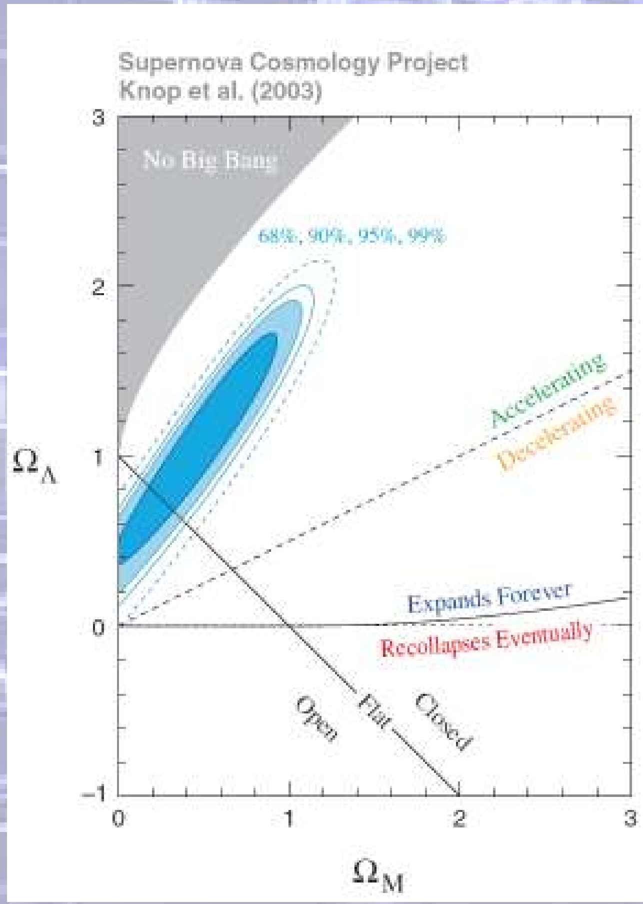
Standardizable Candle





1. Introduction to Cosmology 0





http://www.nersc.gov/news/annual_reports/annrep98/08expanding.html

Dark Energy

Cosmological Constant

Alternatives to GR

Sub-density regions, Bubbles

And a whole new set of possibilities emerged...

1. The Standard Cosmological Model (in one slide)

$$R_{\mu\nu} - \frac{1}{2}(R + 2\Lambda)g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\nabla_\beta T^{\alpha\beta} = 0$$

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - p g^{\mu\nu}$$

+

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right]$$

+

$$w = \frac{p}{\rho}$$

$$H = \frac{\dot{a}}{a}$$

$$1 + z = \frac{a_0}{a}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\Omega_{i0} \equiv \frac{8\pi G \rho_{i0}}{3H_0^2}$$

$$\Omega_{K0} \equiv -\frac{K}{H_0^2}$$

$$\Omega_{\Lambda 0} \equiv \frac{\Lambda}{3H_0^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}.$$

$$\frac{H^2}{H_0^2} = (\Omega_{m0}(1+z)^3 + \Omega_{R0}(1+z)^4 + \Omega_{K0}(1+z)^2 + \Omega_{\Lambda 0})$$

2.Why? Standard Procedure to study DE properties

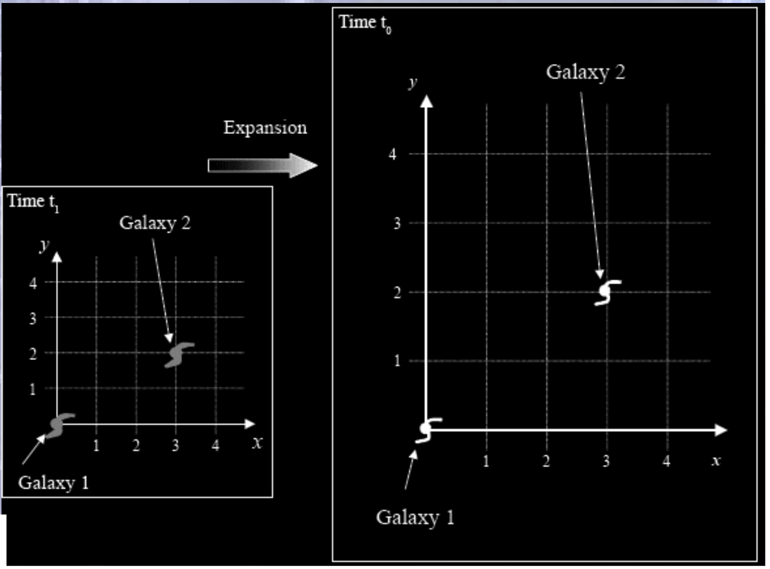
Pick a cosmological model:

Homogeneous and isotropic Universe: FRW metric

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]$$

$$1 + z = \frac{a_0}{a}$$

To make things easier: k=0



http://www.galacticfool.com/wp-content/uploads/2009/05/cosmic-scale-factor1.png

2. Why? Standard Procedure

General Relativity

Dark energy model :
Cosmological Constant

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}.$$

k=0

Cosmological
Constant

$$w = \frac{p}{\rho}$$

2. Why? Standard Procedure

Definitions:

$$\rho_{cr} = \frac{3H^2}{8\pi G}.$$

$$\Omega_m = \frac{\rho_m}{\rho_{cr}},$$

$$\Omega_\Lambda = \frac{\Lambda}{3H^2},$$

Hubble parameter (expansion velocity)

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$$

Used for distance calculations

$$D_C = \int_0^z \frac{dv}{H(v)}$$

2. Why? Standard Procedure

Data set:

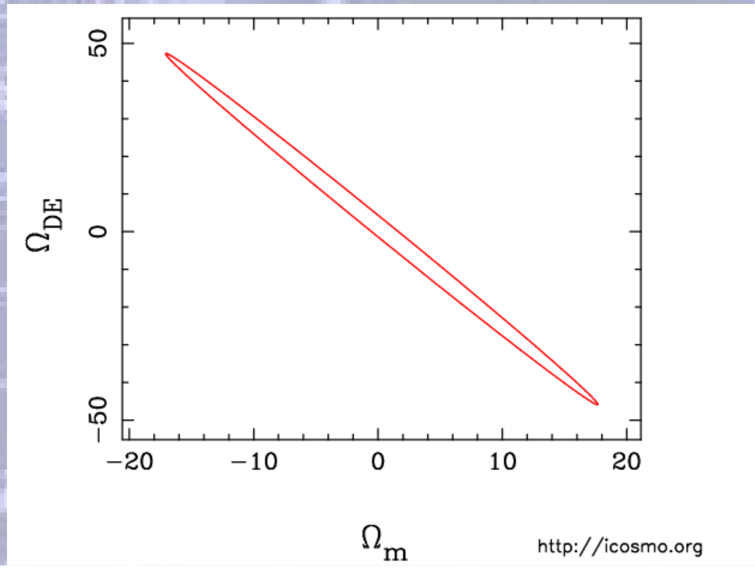
z	$D_c(z)$	σ
z_1	D_1	σ_1
z_2	D_2	σ_2
z_3	D_3	σ_3
…	…	…
z_n	D_n	σ_n

$$\chi^2(\Omega_m,\Omega_\Lambda)=\sum_{i=1}^{n}\frac{\left(D_i - D_C\left(z_i;\Omega_m,\Omega_\Lambda\right)\right)^2}{2\sigma_i^2}$$

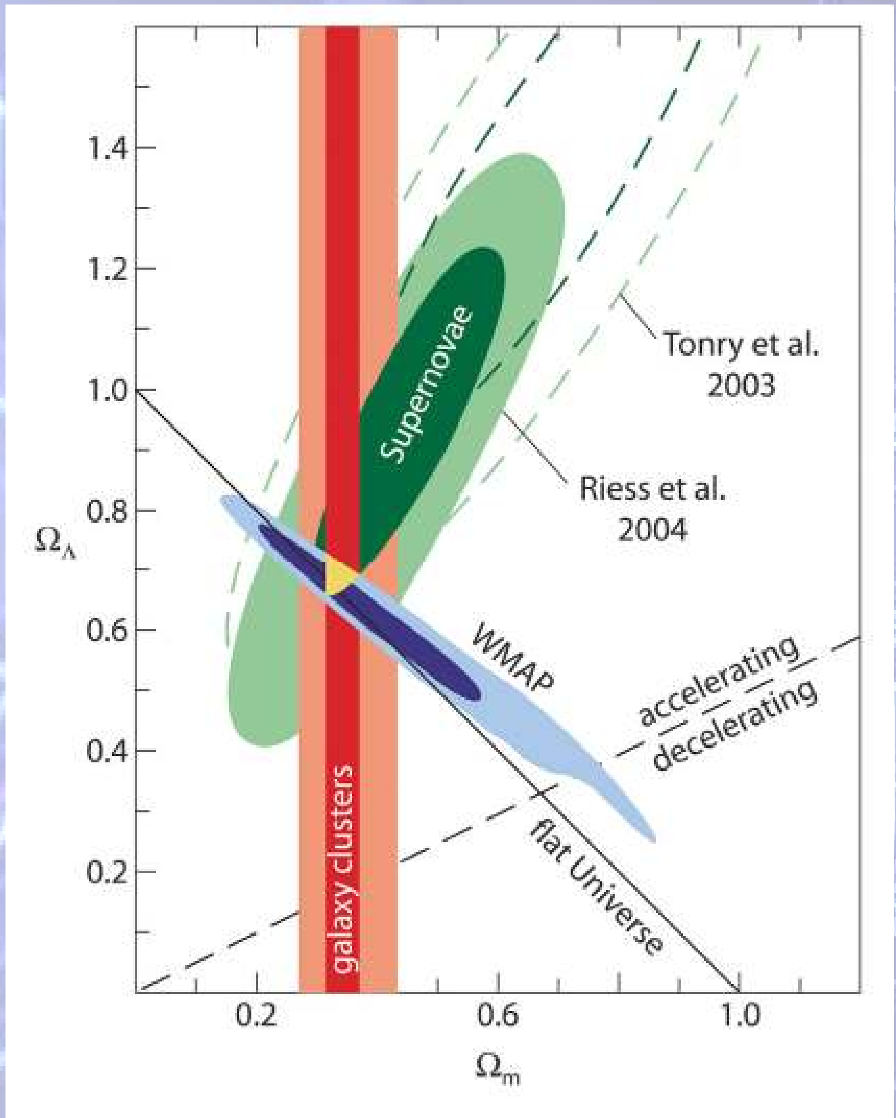
Find

$$(\Omega_m,\Omega_\Lambda) \Rightarrow \chi^2_{\min}$$

Questions about the velocity of the expansion almost all the time end up in allowed values for parameters Ω_m e Ω_Λ



2. Why? Standard Procedure



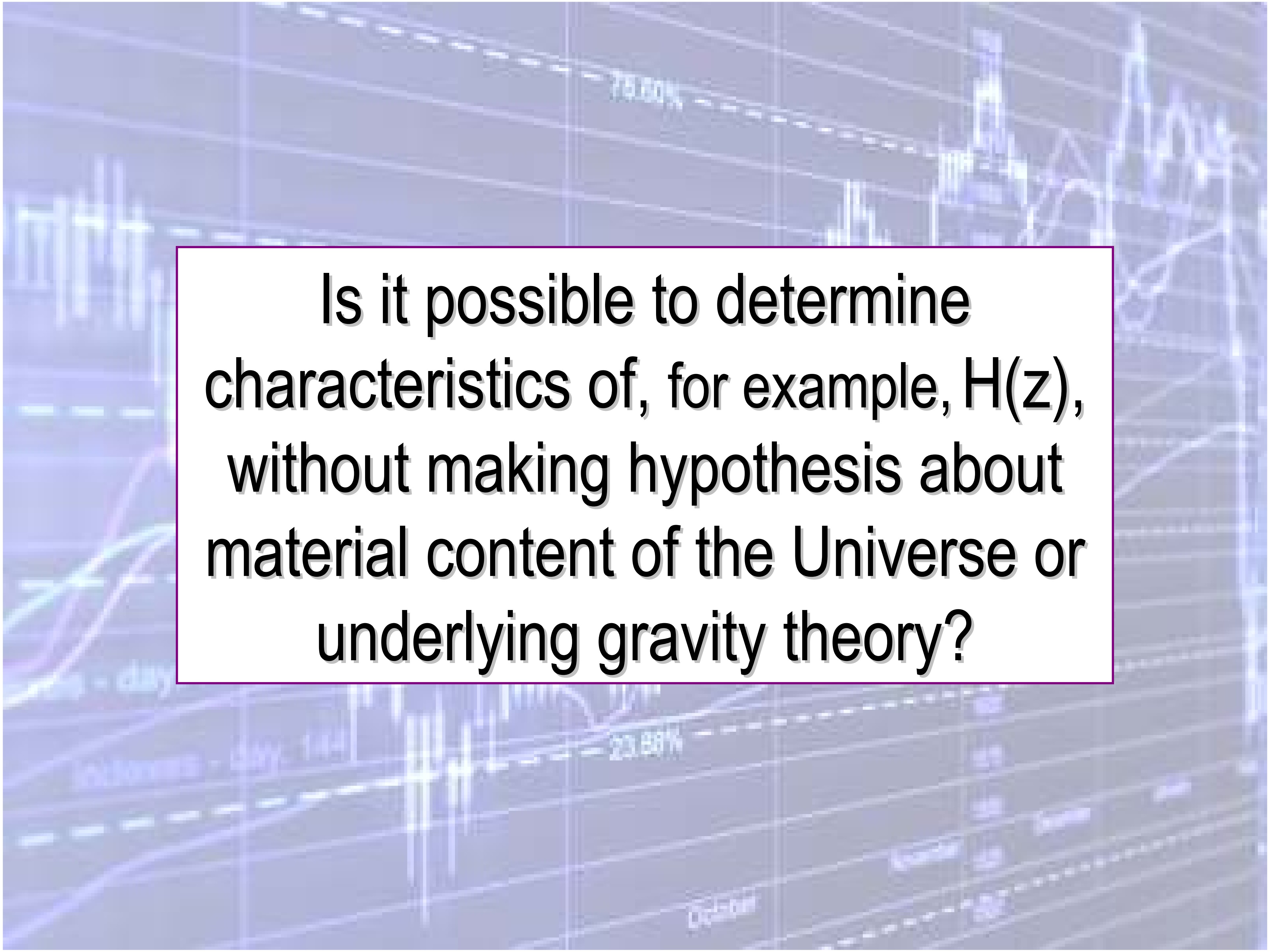
http://www.eso.org/public/archives/images/screen/eso0419d.jpg

Results restrict to hypothesis about:

Gravity theory

Energy content of the Universe

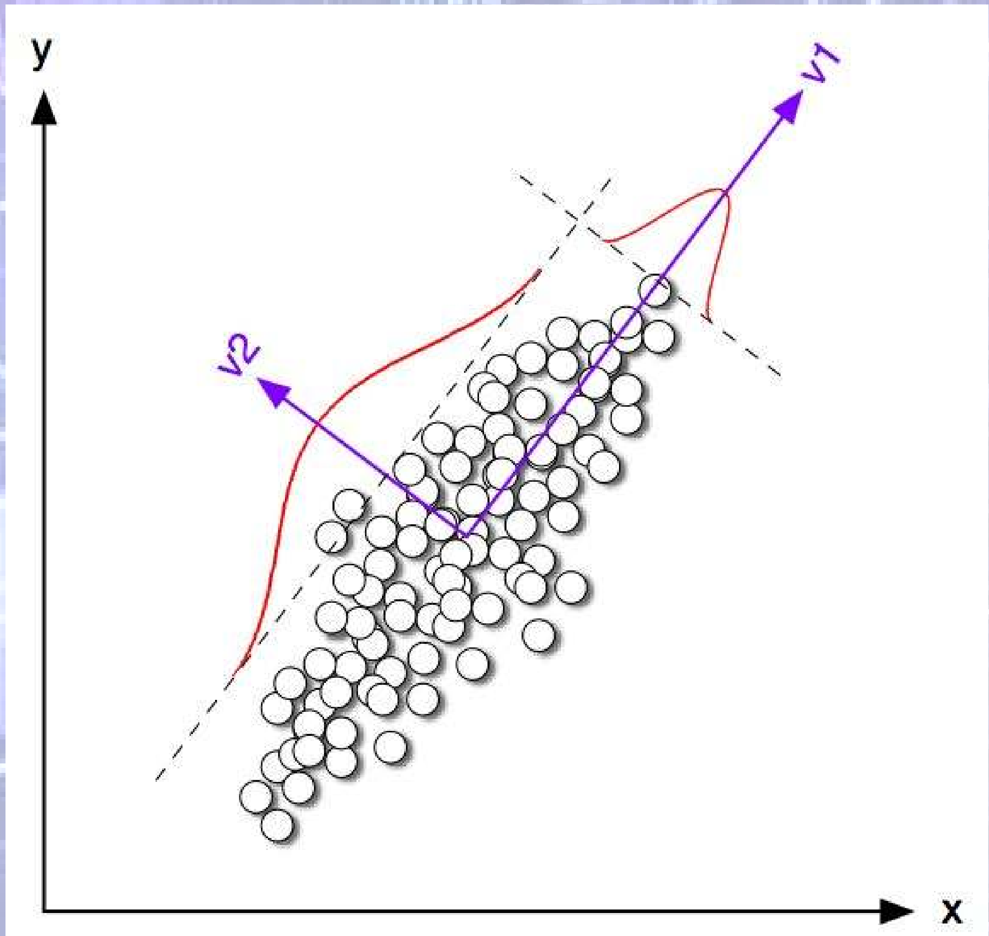
Specific Dark Energy model



Is it possible to determine characteristics of, for example, $H(z)$, without making hypothesis about material content of the Universe or underlying gravity theory?

3. How? Principal Component Analysis (PCA)

The main goal of PCA is to reduce the dimensionality of the initial parameter space



http://www.cs.cornell.edu/courses/cs322/2008sp/images/thumb_PCA.png

3. How? PCA: example

General idea:

N objects
p things we know about them

-height;
-n° publications;
- n°frequent flyer miles;
-gas consumption of their cars;

-height;
-n° publications;
- n°frequent flyer miles;
-gas consumption of their cars;

-height;
-n° publications;
- n°frequent flyer miles;
-gas consumption of their cars;

-height;
-n° publications;
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-height;
-n° publications;
- n°frequent flyer miles;
-gas consumption of their cars;

3. How? PCA: example

do astronomers who spend most of their lives in airports publish more?

Do people with inefficient cars fly more, or is it only the smart ones (with lots of publications) that do so?

Do these correlations represent real causal connection?

or..... or it is just that once you get tenure you buy a new car, stop publishing and give lots of invited talks in exotic foreign locations?

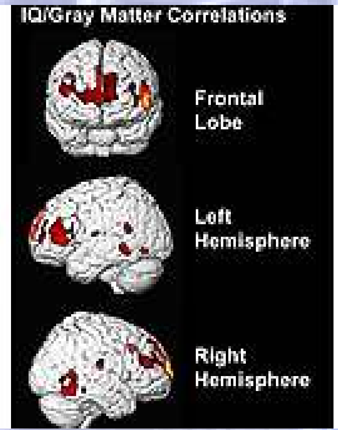
–astro-ph/9905079

3. How? PCA

First try:

Analyze all possible pair of parameters...

...which becomes impossible for not so large number of parameters!



PCA finds correlations between parameters
(in order to reduce the dimensionality of parameter space)

One of the first applications were in social sciences....

Ex: a set of *p* tests were applied to a group of *n* people, in order to test criativity, memory, mathematical skills... and tryied to find correlations

Result: almost all tests showed correlations with the others, indicating that one unic variable could be capable of predicting the result of one person in all the tests

IQ tests...

–astro-ph/9905079

3. How? PCA

Notation

sample $\rightarrow$ $\mathbf{x} = (x_1, \ldots, x_n)$

Covariance: $$\mathrm{cov}(x,y) = \sum_{i=1}^{n} \frac{(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

Covariance matrix (2D): $$C = \begin{pmatrix} \mathrm{cov}(x,x) & \mathrm{cov}(x,y) \\ \mathrm{cov}(y,x) & \mathrm{cov}(y,y) \end{pmatrix}$$

Eigenvectors $\Longrightarrow$ Principal Components – PCs -(new *uncorrelated* variables)

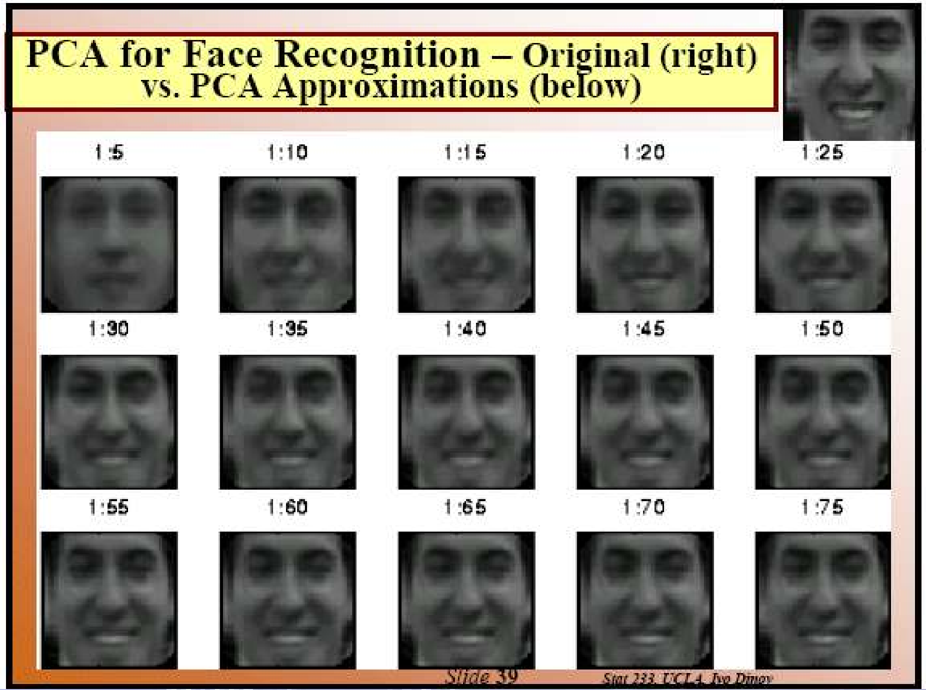
Eigenvalues $\Longrightarrow$ variances associated to the PCs

Largest eigenvalue $\Longrightarrow$ 1° Principal Component

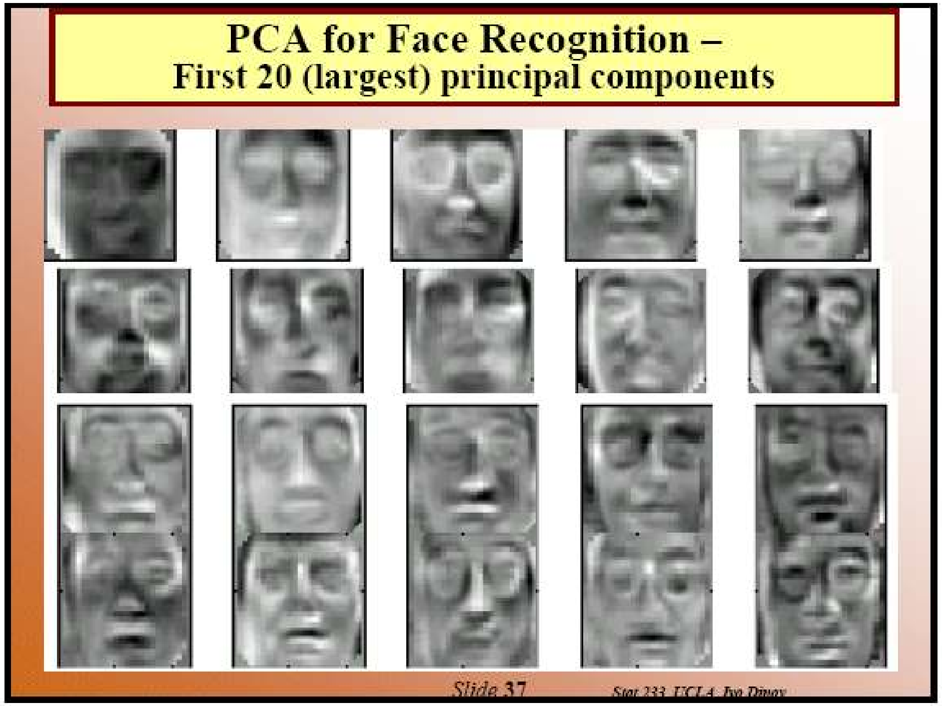
Re-write the data using the new set of parameters (PC)

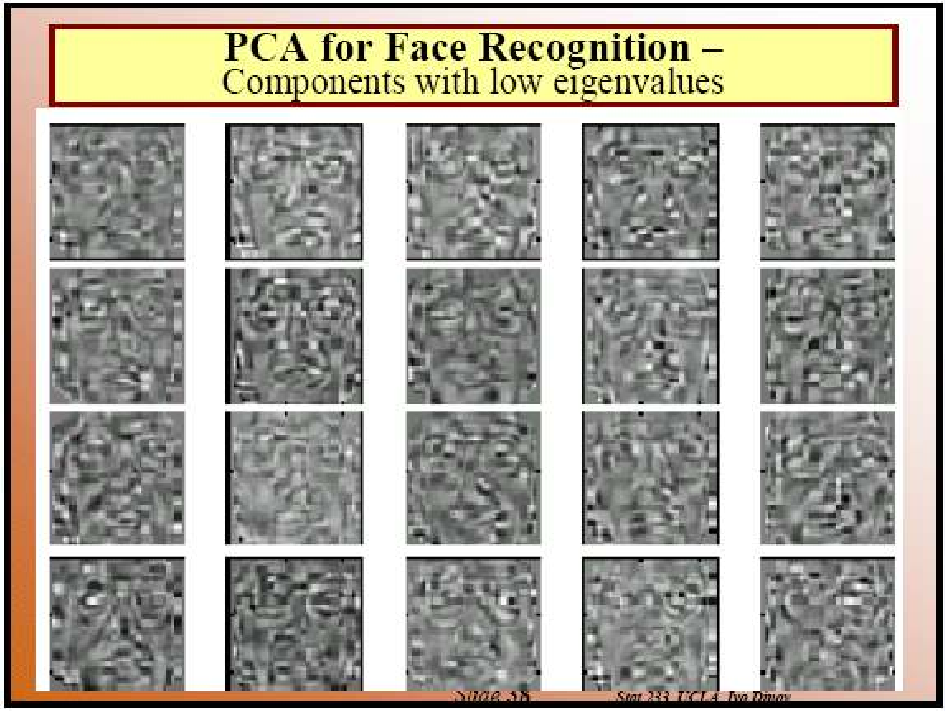
http://csnet.otago.ac.nz/cosc453/student_tutorials/principal_components.pdf

3. How? PCA



http://www.stat.ucla.edu/~dinov/





OBS:

How to obtain a covariance matrix if we have only 1 quantity observed?

It is possible to analytically obtain the covariance matrix using the Fisher matrix

4- Can you measure it?

Notation for Maximum Likelihood Method

sample $\rightarrow$ $\mathbf{x} = (x_1, \ldots, x_m)$

parameters $\rightarrow$ $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_n)$

Probability density function $\rightarrow$ $f(x_i; \boldsymbol{\theta})$

$\hat{\theta}$ $\rightarrow$ values of θ_i which maximize the likelihood

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{n} f(x_i; \boldsymbol{\theta}) \propto \exp(-\chi^2)$$

$$\chi^2(\Omega_m, \Omega_\Lambda) = \sum_{i=1}^{n} \frac{\left(D_i - D_C(z_i; \Omega_m, \Omega_\Lambda)\right)^2}{2\sigma_i^2}$$

4- Can you measure it?

Fisher Matrix

$$F_{ij} = \left(C^{-1}\right)_{ij} = \left\langle -\frac{\partial^2 \ln L}{\partial\theta_i \partial\theta_j}\right\rangle$$

We will focus our analysis in the Hubble parameter

4-Can you measure it?

$$F_{ij} = \left\langle -\frac{\partial^2 \ln L}{\partial\theta_i\,\partial\theta_j} \right\rangle$$

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{m} f(x_i;\boldsymbol{\theta}) \propto \exp(-\chi^2)$$

Magnitudes (distance modulus): what we measure from type Ia SN

$$\chi^2(\Omega_m,\Omega_\Lambda) = \sum_{i=1}^{n} \frac{\left(\mu_i - \mu\left(z_i;\Omega_m,\Omega_\Lambda\right)\right)^2}{2\sigma_i^2}$$

$$\mu(z) = 5\log_{10}\left[d_L(z)\right] + \mu_0,$$

$$d_L(z) \equiv (1+z)\int_0^z \frac{du}{H(u)},$$

To avoid other hypothesis:

$$H(z;\boldsymbol{\beta}) = \sum_{i=1}^{N_{bin}} \beta_i c_i(z),$$

$$c_i(z) = \begin{cases} 1 & \text{if} \quad (i-1)\Delta z < z \leq i\Delta z \\ 0 & \text{otherwise} \end{cases}$$

4-Can you measure it?

Why the Hubble parameter?

Because we can analitically determine the Fisher matrix

There is independent data to compare our results

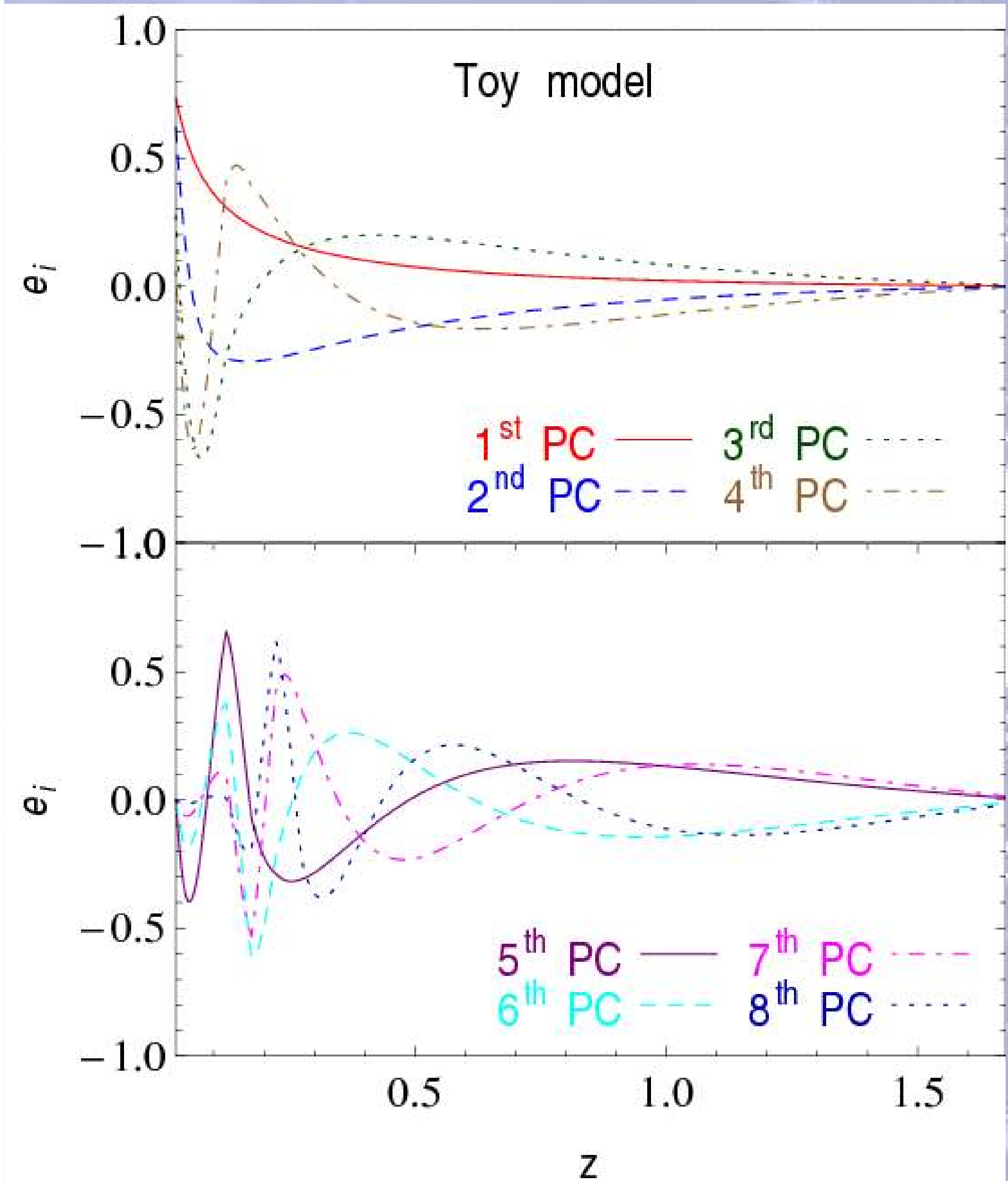
$$H_{rec}(z;\boldsymbol{\alpha}) = H_{base}(z) + \sum_{i=1}^{M} \alpha_i \boldsymbol{e_i}(z),$$

PCs

Initial model

$$\chi^2(\overline{\alpha}) = \sum_{i=1}^{M} \frac{(\mu_i - \mu_{rec}(z_i;\overline{\alpha}))^2}{2\sigma_i^2}$$

4- Can you measure it?



Toy model

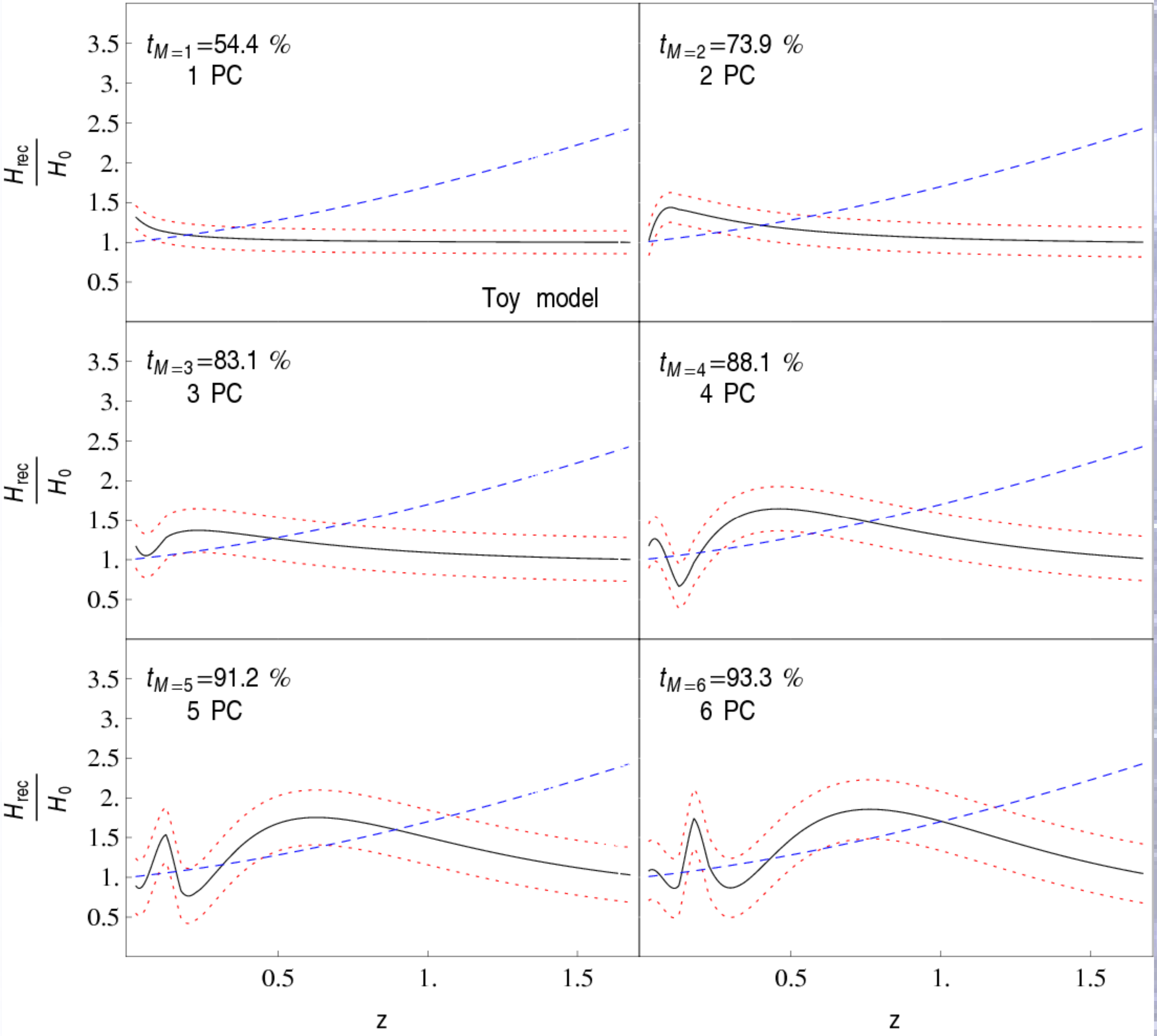
34 redshift bins

$\Delta z = 0.05 \ (0 \leq z \leq 1.7)$

50 supernovae.

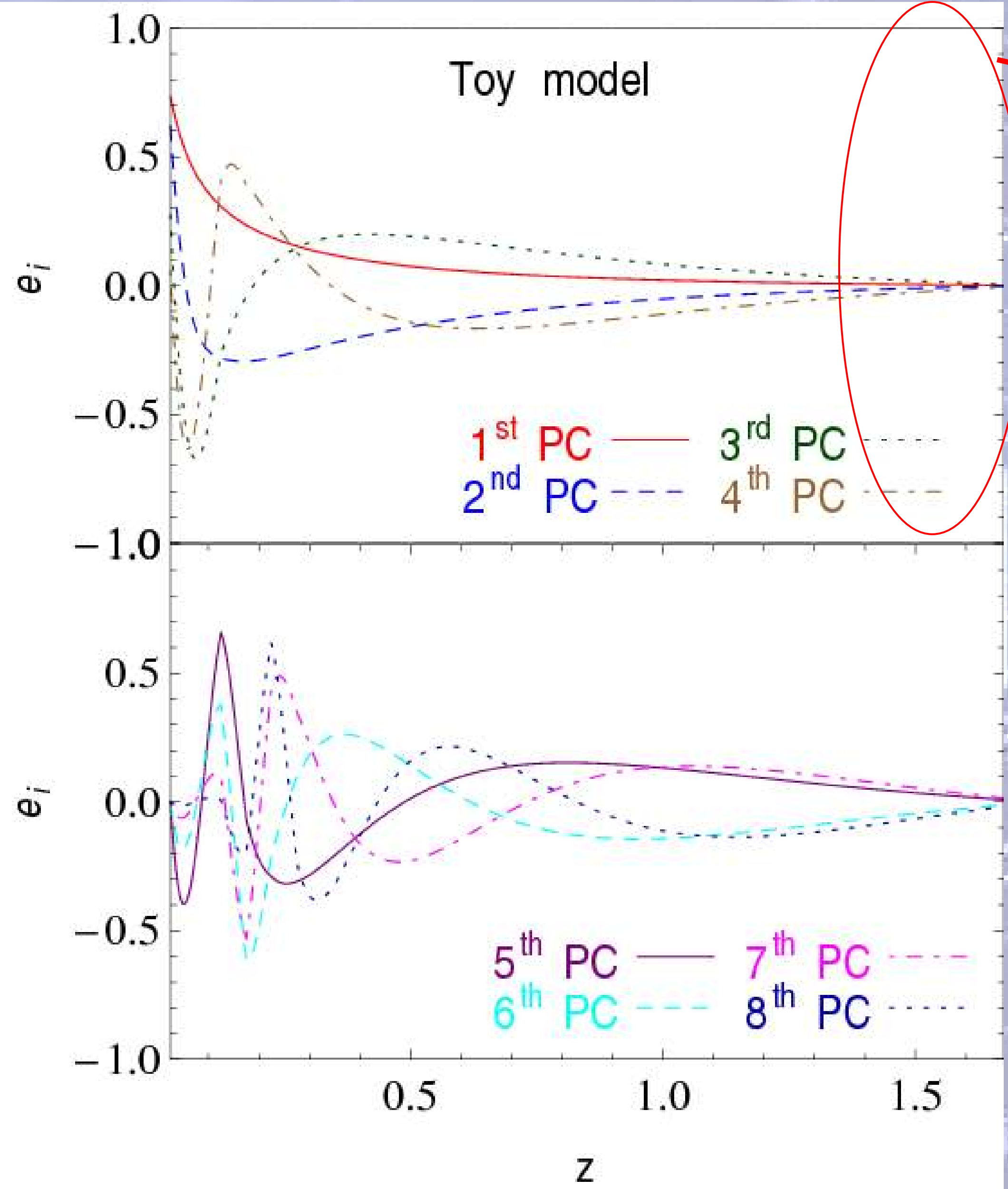
$H_{base}(z) \ (\beta_i = 1, \forall i)$

4- Can you measure it?



Obviously, it didn't work as well as expected...

4- Can you measure it?



What went wrong?

$$H_{rec}(z;\boldsymbol{\alpha}) = H_{base}(z) + \sum_{i=1}^{M} \alpha_i \boldsymbol{e_i}(z),$$

$$(\beta_i = 1, \forall i)$$

4- Can you measure it?

Statistical tricks...

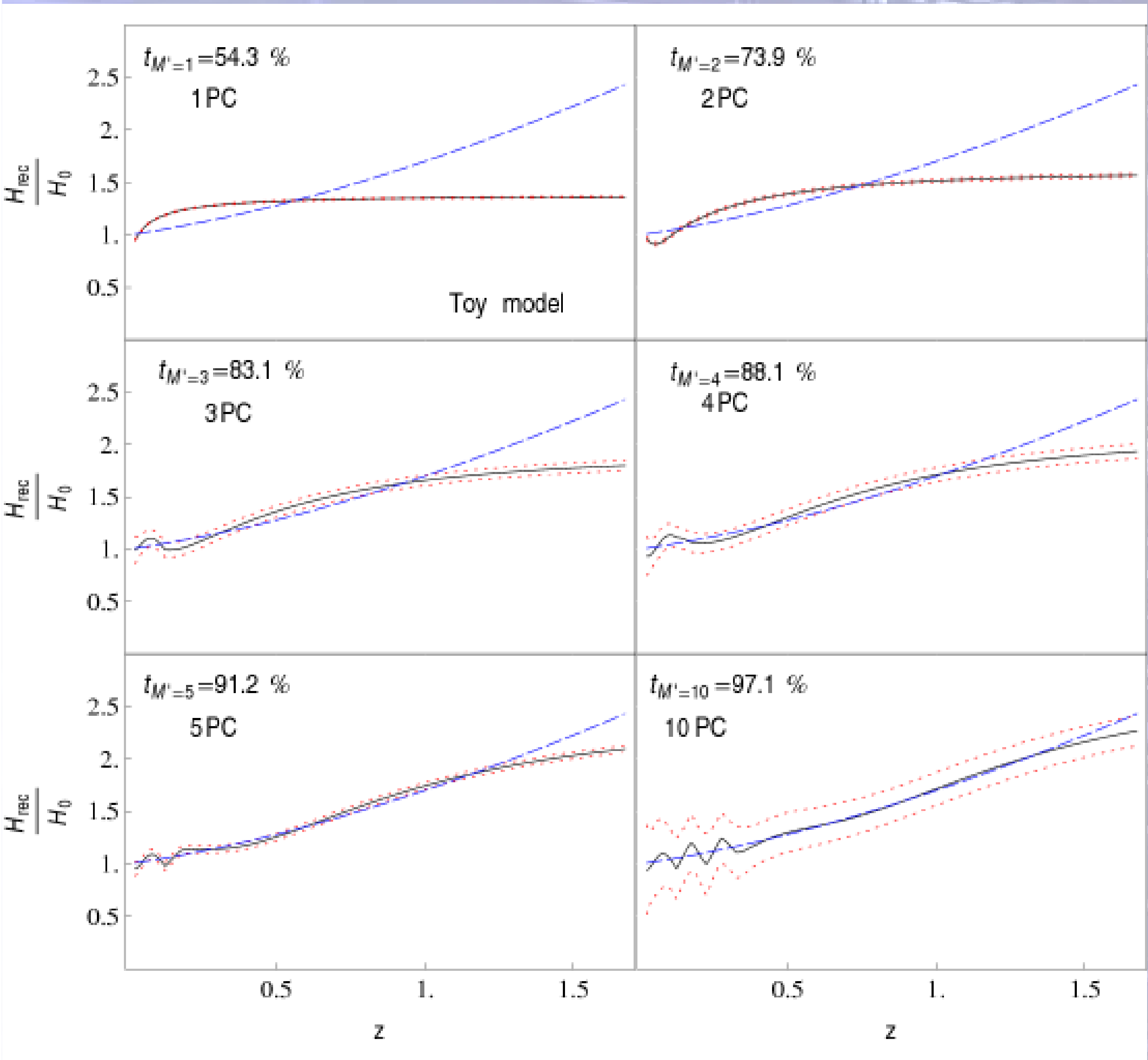
$$H_{rec}(z) = h_{fid} + H_{base}(z) + \sum_{i=1}^{N} \alpha_i \boldsymbol{e_i}(z).$$

$$\chi^2(h_{fid}, \boldsymbol{\alpha}) = \sum_{i=1}^{N} \frac{(\mu_i - \mu_{rec}(z_i, h_{fid}, \boldsymbol{\alpha}))^2}{2\sigma_i^2},$$

Does it worth?

Remember that main point is dimensionality reduction...

4- Can you measure it?



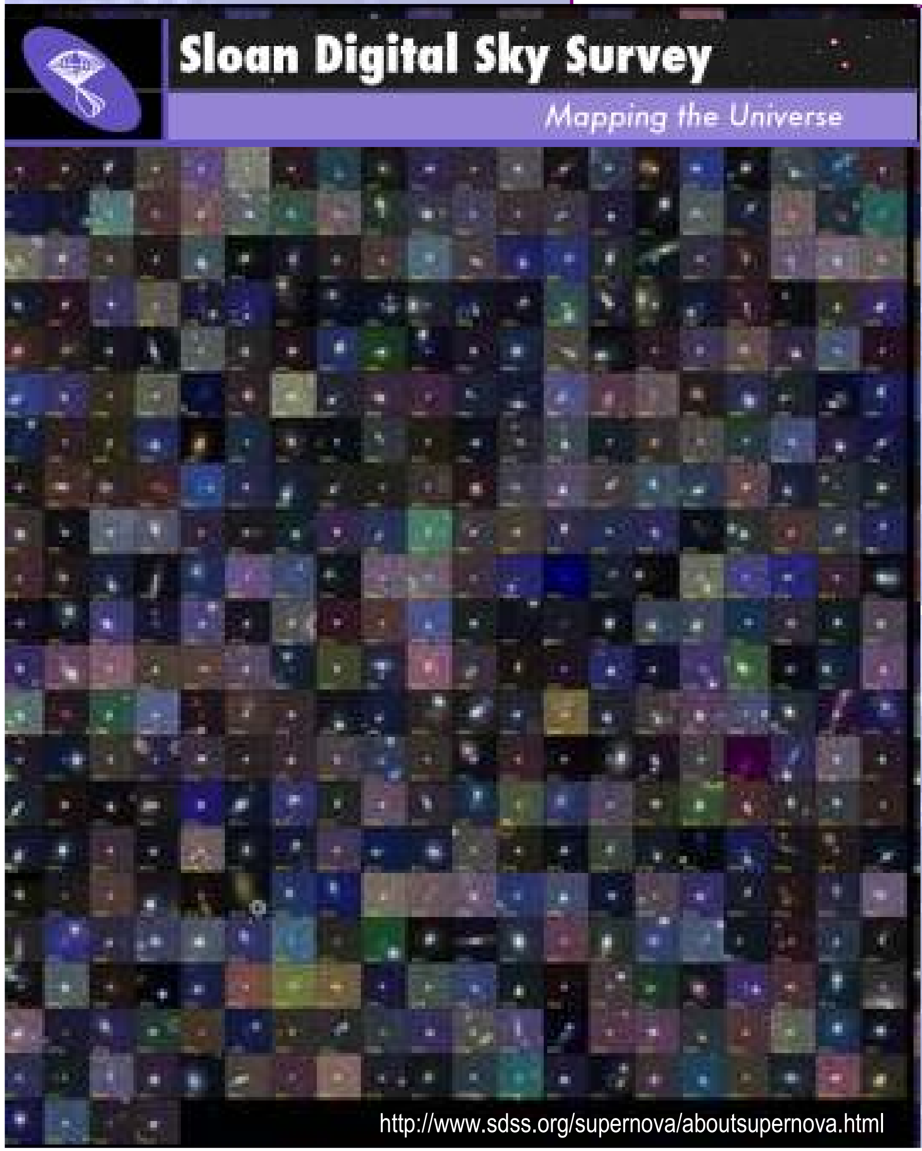
Cumulative percentage of total variance

$$t_{M'} = 100 \frac{\sum_{i=1}^{M'} \lambda_i}{\sum_{j=1}^{M} \lambda_j}.$$

Initially we had 34 parameters!

4- Can you measure it?

What about the real world?



287 data points

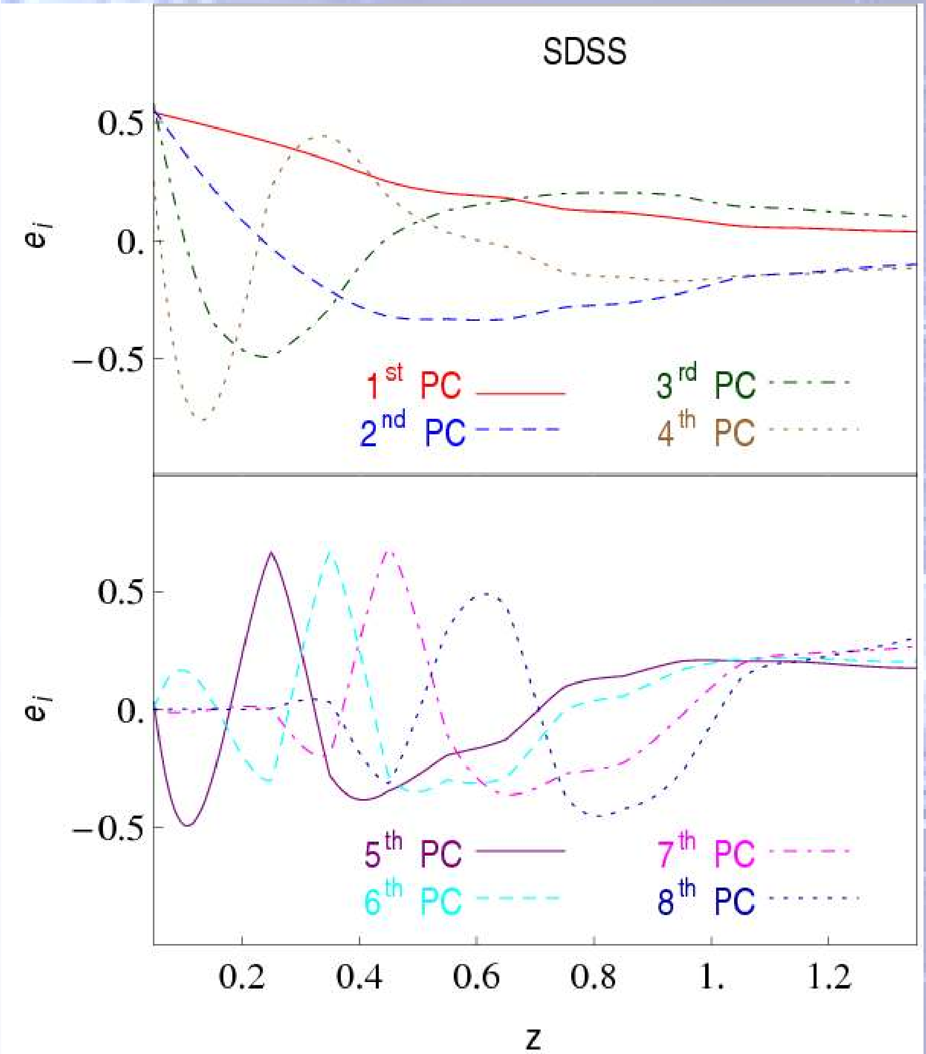
$z \leq 1.4$

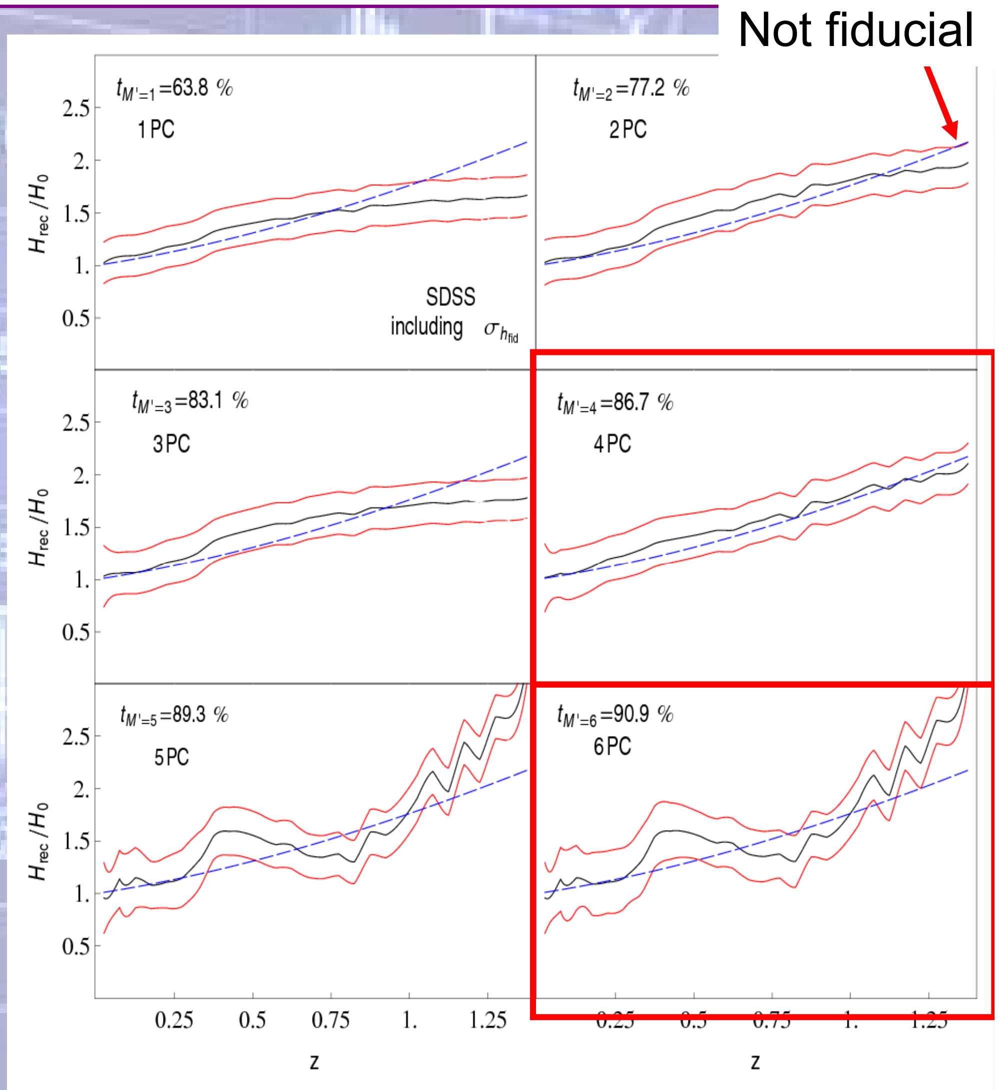
28 redshift bins

$\Delta z = 0.05$

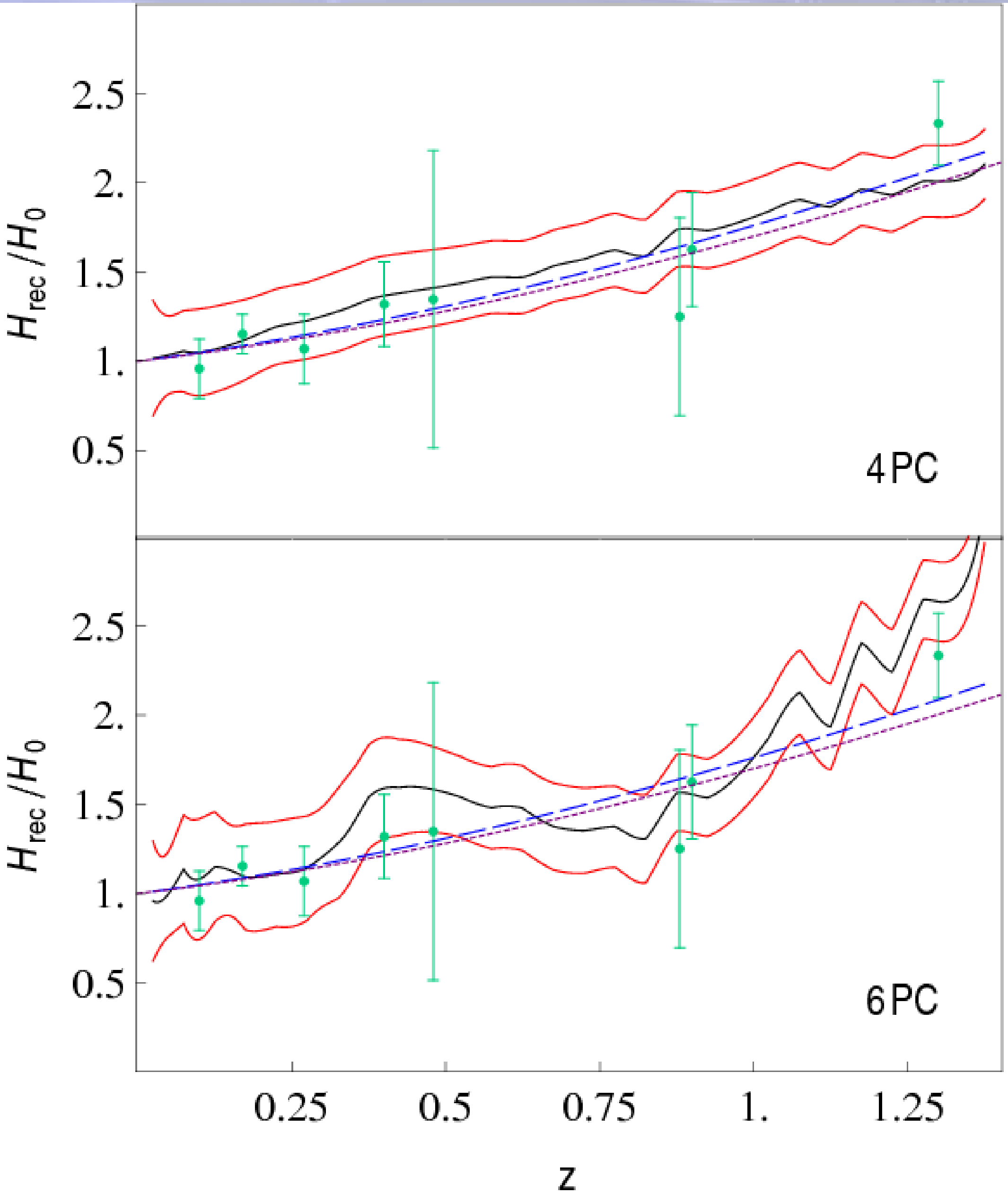
4- Can you measure it?

SDSS SN data





4- Can you measure it?



SDSS SN data

+

Measurements of H(z) from red-envelope galaxies

Stern et al., 2010

Is this a coincidence?

Final Remarks

PCA is not completely free of hypothesis...

... but it can be a viable way to determine features of cosmological quantities that might be present in the data, but not in our theoretical models...

...although, some crucial issues must be solved (how many PCs?).

With the current error bars, the agreement between red-envelope galaxies and our reconstructions is nothing more than a happy coincidence...

... but can this situation change in the near future?

Can we thing about something interesting in those redshifts that would deviate from our smooth models?