Inflation, reheating and flat direction preheating

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AEG, K. A. Olive, M. Peloso, M. Sexton [PRD78:063512 (2008)]

AEG [PRD80:123520 (2009)]

J.-F. Dufaux, AEG, M. Peloso [in progress]

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Outline

1. Inflation, Reheating and Role of Flat Directions
   - Theoretical uncertainties
   - Example: Gravitational decay of inflaton and reheating
   - Reheating in the presence of long lived flat directions
2. Nonperturbative Flat Direction Decay
   - Simple toy models for F.D. decay
   - MSSM example
3. Implications of the Nonperturbative Decay

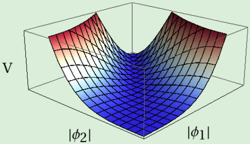
Flat directions in Cosmological Context

- Flat directions: directions in field space along which $V = 0$. $\Rightarrow$ generic feature of supersymmetric theories.

Example

$$V = e^2 \left(|\phi_1|^2 - |\phi_2|^2\right)^2$$

- Potential flat along $|\phi_1| = |\phi_2| = \Phi$.
- $+$ 2 phases:
  $U(1)$ gauge $\Rightarrow$ Only one combination physical
- Flat direction: $\Phi\, \mathrm{e}^{i\,\Sigma}$ $\Rightarrow$ Two real parameters



- May acquire large VEVs during inflation $\Rightarrow$ Interesting for cosmology:
  Generating baryon asymmetry `Affleck, Dine 1985 ···`
  Inflaton `Dvali 1996···`
  Curvaton `Enqvist, Kasuya, Mazumdar 2002···`
  Delayed reheating `Allahverdi, Mazumdar 2005`
- In this talk, inflaton $\rightarrow$ external field. Concentrate on reheating and the fate of the flat directions.

Thermal history and inflation

- Clear knowledge from BBN on ($z_{\rm BBN} \sim 10^{10}$).
- When radiation dominated era starts? ($z_{\rm RD} \gtrsim z_{\rm BBN}$)
- Before radiation: inflation. Both theoretical control and data
- Slow roll approximation $\Rightarrow$ Strict predictions within given model.

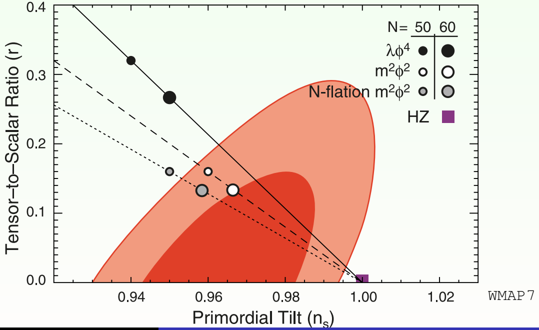
$$\epsilon = \frac{M_p^2}{16\,\pi}\left(\frac{V'}{V}\right)^2 \ll 1 \quad , \quad \eta = \frac{M_p^2}{8\,\pi}\left(\frac{V''}{V}\right) \ll 1$$

e.g. Chaotic inflation

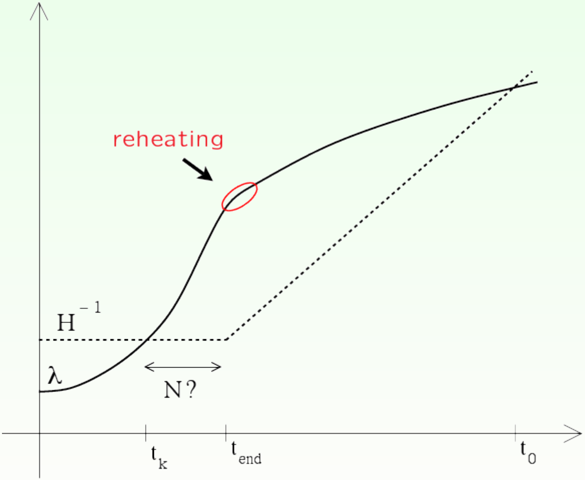
$V = \phi^\alpha$

$n_s - 1 = -\frac{2+\alpha}{2\,N}$

$r = \frac{4\,\alpha}{N}$



Uncertainty on N



- Predictions of inflation strongly depend on N.
- Physics at reheating $\Rightarrow$ Uncertainty on N
- Matching to when R.D. starts gives range of uncertainty on N: $\Delta N \sim 14$

Reheating

INFLATION —*REHEATING*→ **HOT BIG BANG**

We do not know:

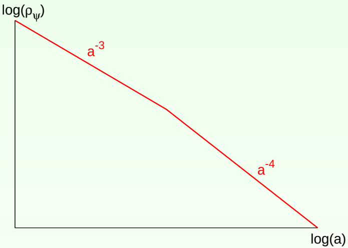
- Energy scale of inflation
- What is inflaton
- Coupling to SM fields

Bounds on RH

- BBN: $T_{\rm rh} \gtrsim$ MeV
- Single field slow roll inflation: $T_{\rm rh} \lesssim 10^{16}$ GeV
- Gravitino bound: $T_{\rm rh} \lesssim 10^5$–10^9 GeV   Kawasaki et al. 2008

Example of inflaton decay

- Chaotic inflation with massive inflaton ψ. Assume preheating negligible $\Rightarrow$ Perturbative decay.
- Inflaton oscillations after inflation: $\rho_\psi \sim a^{-3}$
- Gravitational decay: when $H = \Gamma \propto \frac{m_\psi^3}{M_p^2} \sim 10\,\mathrm{TeV}$, into relativistic particles, after $\sim 10^{13}$ oscillations.



- Gauge mediated $2 \to 3$ interactions lead to very rapid thermalization.

  Davidson, Sarkar '00

  At the time of inflaton decay: $\Gamma_{2\to3} \sim \alpha^3 \frac{M_p}{m_\psi} H$
- If $\alpha \gtrsim \left(\frac{m_\psi}{M_p}\right)^{1/3} \sim 10^{-2} \Rightarrow$ Immediate thermalization
- $T_{rh} \sim \left(\frac{m_\psi^3}{M_p}\right)^{1/2} \sim 10^8\,\mathrm{GeV}$

MSSM Flat Directions

MSSM Potential: $V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a g_a^2 \left(\phi^\dagger T^a \phi\right)^2$

$F_i \equiv \frac{\partial W_{\rm MSSM}}{\partial \phi_i}$, $W_{\rm MSSM} = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \mu H_u H_d$.

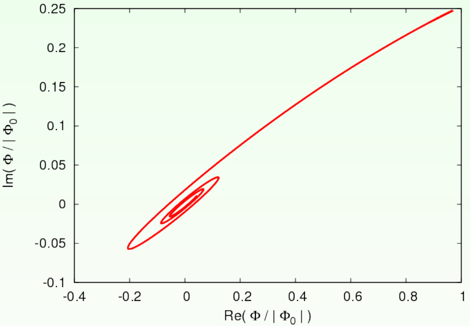
	$B-L$		$B-L$
$H_u H_d$	0	$QQQQ\bar{u}$	1
LH_u	-1	$QQ\bar{u}\bar{u}\bar{e}$	1
		$LL\bar{d}\bar{d}\bar{d}$	-3
$\bar{u}\bar{d}\bar{d}$	-1	$\bar{u}\bar{u}\bar{u}\bar{e}\bar{e}$	1
$QL\bar{d}$	-1		
$LL\bar{e}$	-1	$QLQL\bar{d}\bar{d}$	-2
		$QQLL\bar{d}\bar{d}$	-2
$QQ\bar{u}\bar{d}$	0	$\bar{u}\bar{u}\bar{d}\bar{d}\bar{d}\bar{d}$	-2
$QQQL$	0		
$QL\bar{u}\bar{e}$	0	$QQQQ\bar{d}LL$	-1
$\bar{u}\bar{u}\bar{d}\bar{e}$	0	$QLQLQL\bar{e}$	-1
		$QL\bar{u}QQ\bar{d}\bar{d}$	-1
		$\bar{u}\bar{u}\bar{u}\bar{d}\bar{d}\bar{d}\bar{e}$	-1

Dine, Randall, Thomas ’95
Gherghetta, Kolda, Martin ’95

Flat directions catalogued by gauge invariant monomials.
Plethora of flat directions along which $V = 0$.
However, in MSSM, SUSY is broken, so these directions are only approximately flat.

Evolution of Flat Directions

$$V = (m_\phi^2 - c H^2)|\phi|^2 + \frac{|\lambda|^2 |\phi|^{2d-2}}{M^{2d-6}} + \left(A \frac{\lambda \phi^d}{M^{d-3}} + \text{h.c} \right), \qquad d \geq 4$$



Dine, Randall, Thomas 1995

During inflation

- $c > 0 \Rightarrow$ VEV generation due to instability, up to global minimum
- $c < 0 \Rightarrow$ No growth.
- c model dependent. ($c < 0$ for minimal Kähler potential)

After inflation

- After $H \sim m_\phi \Rightarrow$ F.D. starts oscillating with initial magnitude
$$\langle \phi_0 \rangle \sim \left(\frac{m_\phi \, M^{d-3}}{|\lambda|} \right)^{1/(d-2)}$$
and $\langle \phi \rangle \sim a^{-3/2}$
- A terms provide initial angular momentum: Spiral motion towards origin

Role in reheating

- F.D. starts rotating before inflaton decays: $\frac{R_{d\psi}}{R_\phi} = \frac{M_p^{4/3} m_\phi^{2/3}}{m_\psi^2} > 1$
- In the presence of F.D., gauge fields acquire masses $\propto$ VEV, scatterings they mediate slow down.

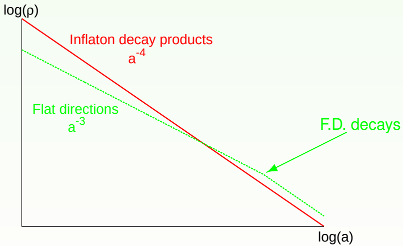
  Allahverdi, Mazumdar '05

  At the time of inflaton decay, $\Gamma_{2\to3} \sim \frac{\alpha^3 M_p^5 m_\phi^2}{m_\psi^5 \phi_0^2} H$
- Thermalization delayed if $\phi_0 \gtrsim \frac{\alpha^{3/2} M_p^{5/2} m_\phi}{m_\psi^{5/2}} \sim 10^{-2} M_p$
- F.D. dominates if

$$\phi_0 \gtrsim \frac{M_p^{4/3} m_\phi^{5/12}}{m_\psi^{3/4}} \sim 10^{-2} M_p$$

$$\Rightarrow T_{rh} \sim M_p^{1/6} m_\phi^{5/6} \sim 10^6 \mathrm{GeV}$$

Olive, Peloso 2006



Perturbative or nonperturbative decay?

- These consequences are valid only if flat directions are long lived.
- Standard lore: The fields coupled to flat direction acquire large mass

  $\Rightarrow$ very small perturbative decay rate: $\Gamma \sim \frac{m_\phi^3}{\phi^2}$. `Affleck, Dine 1985`
- Flat direction decays after $\sim 10^{11}$ rotations.

How about nonperturbative decay?

`Allahverdi, Shaw, Campbell 1999; Postma, Mazumdar 2003`

- Complex scalar field χ coupled to flat directions: $\Delta V = g^2\,|\phi|^2|\chi|^2$
- Eq. of motion of fluctuations: $\delta\chi'' + \left[p^2 + m_\chi^2 + g^2\,|\phi(t)|^2\right]\delta\chi = 0$.

rate of change in frequency

- Maximum ω'/ω^2 obtained when amplitude is minimum:

$$\left.\frac{\omega'}{\omega^2}\right|_{\max} \simeq \frac{m_\phi}{g\,\epsilon\,\phi_0} \sim \frac{10^{-14}}{\epsilon}$$

- Typically, $10^{-3} \lesssim \epsilon \lesssim 10^{-1}$ $\Rightarrow$ only perturbative decay?

Realistic example: $H_u H_d$ Flat Direction

- MSSM, with $H_u H_d$ F.D.:

$$H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} h_u \\ \phi + \xi_u \end{pmatrix} , \; H_d = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi + \xi_d \\ h_d \end{pmatrix} \quad , \quad VEV \to \phi = |\phi| \, \mathrm{e}^{i\,\sigma}$$

- Potential, quadratic in fluctuations:

F–terms:

$$V = \frac{y_u^2}{2} |\phi|^2 \left(|Q_u|^2 + |u|^2 \right) + \frac{y_d^2}{2} |\phi|^2 \left(|Q_d|^2 + |d|^2 \right) + \frac{y_e^2}{2} |\phi|^2 \left(|L_d|^2 + |e|^2 \right)$$

D–terms:

$$+ \frac{g^2 + g'^2}{16} |\phi|^2 \left(\mathrm{Re}[\xi_u - \xi_d] \, , \mathrm{Im}[\xi_u - \xi_d] \right) \mathcal{M}^2 \begin{pmatrix} \mathrm{Re}[\xi_u - \xi_d] \\ \mathrm{Im}[\xi_u - \xi_d] \end{pmatrix}$$

$$+ \frac{g^2}{8} |\phi|^2 \left(\mathrm{Re}[h_u + h_d] \, , \mathrm{Im}[h_u + h_d] \right) \mathcal{M}^2 \begin{pmatrix} \mathrm{Re}[h_u + h_d] \\ \mathrm{Im}[h_u + h_d] \end{pmatrix}$$

$$+ \frac{g^2}{8} |\phi|^2 \left(\mathrm{Im}[-h_u + h_d] \, , \mathrm{Re}[h_u - h_d] \right) \mathcal{M}^2 \begin{pmatrix} \mathrm{Im}[-h_u + h_d] \\ \mathrm{Re}[h_u - h_d] \end{pmatrix}$$

- $\mathcal{M}^2 = \begin{pmatrix} \cos^2\sigma & \cos\sigma \, \sin\sigma \\ \cos\sigma \, \sin\sigma & \sin^2\sigma \end{pmatrix} \Rightarrow$ Eigenmasses $(\frac{g}{2}|\phi|, 0)$, but eigenvectors have t dependence through $\sigma(t)$.

$$\left(\right.$$

Quantization of coupled bosons

Nilles, Peloso, Sorbo 2001

$$S = \frac{1}{2} \int d^3k \, d\eta \, \left(\phi'^{\,\dagger} \, \phi' - \phi^\dagger \, \Omega^2 \, \phi \right)$$

- Nondiagonal, time dependent frequency matrix:
  $\phi^\dagger \Omega^2 \phi = \underbrace{(\phi^\dagger \, C)}_{\tilde{\phi}^\dagger} \underbrace{(C^T \, \Omega^2 \, C)}_{\omega^2_{\rm diag}} \underbrace{(C^T \, \phi)}_{\tilde{\phi}}$
- Kinetic Mixing: $\phi'^{\,\dagger} \phi' = \tilde{\phi}'^{\,\dagger} \, \tilde{\phi}' + \tilde{\phi}'^{\,T} \, \Gamma \, \tilde{\phi} + \tilde{\phi} \, \Gamma^T \, \tilde{\phi}'^{\,\dagger} + \tilde{\phi}^\dagger \, C'^{\,T} \, C' \, \tilde{\phi}$
  $(\Gamma \equiv C^T \, C')$
- $\tilde{\phi}_i = \left[\frac{1}{\sqrt{2\,\omega}} \left(\underbrace{\mathrm{e}^{-i \int^t \omega \, dt} A}_{\alpha} + \underbrace{\mathrm{e}^{i \int^t \omega \, dt} B}_{\beta} \right) \right]_{ij} \hat{a}_j(\vec{k}) + [\cdots]^\star_{ij} \, \hat{a}^\dagger_j(-\vec{k})$

(Bogolyubov Matrices)

$$\alpha \, \alpha^\dagger - \beta^\star \, \beta^T = \mathbb{1} \,, \qquad \alpha \, \beta^\dagger - \beta^\star \, \alpha^T = 0$$

Quantization of coupled bosons

- $$\mathcal{H} = \frac{1}{2}\,(\hat{a}^\dagger\,,\,\hat{a})\begin{pmatrix}\alpha^\dagger & \beta^\dagger\\ \beta^T & \alpha^T\end{pmatrix}\begin{pmatrix}\omega & 0\\ 0 & \omega\end{pmatrix}\underbrace{\begin{pmatrix}\alpha & \beta^\star\\ \beta & \alpha^\star\end{pmatrix}\begin{pmatrix}\hat{a}\\ \hat{a}^\dagger\end{pmatrix}}$$
- Time dependent annihilation/creation operators: $\begin{pmatrix}\hat{b}\\ \hat{b}^\dagger\end{pmatrix}$

  $: \mathcal{H} := \omega_i\,\hat{b}_i^\dagger\,\hat{b}_i$.
- Occupation numbers: $N_i(t) = \langle \hat{b}_i^\dagger\,\hat{b}_i\rangle = \left(\beta^\star\,\beta^T\right)_{ii}$
- Equations of motion:

$$\alpha' = (-i\,\omega - I)\alpha + \left(\frac{\omega'}{2\,\omega} - J\right)\beta \qquad I \equiv \frac{1}{2}\left(\sqrt{\omega}\,\Gamma\,\frac{1}{\sqrt{\omega}} + \frac{1}{\sqrt{\omega}}\,\Gamma\,\sqrt{\omega}\right)$$

$$\beta' = (i\,\omega - I)\beta + \left(\frac{\omega'}{2\,\omega} - J\right)\alpha \qquad J \equiv \frac{1}{2}\left(\sqrt{\omega}\,\Gamma\,\frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}}\,\Gamma\,\sqrt{\omega}\right)$$

Anti-Hermitian: Rotate produced states. Preserves total $N(t)$

Hermitian: Particle production.

- Adiabaticity condition

$$\left[\frac{\omega'}{\omega^2} - 2\,\frac{1}{\sqrt{\omega}}\,J\,\frac{1}{\sqrt{\omega}}\right]_{ij} = \left[\frac{\omega'}{\omega^2} - \left(\Gamma\frac{1}{\omega} - \frac{1}{\omega}\Gamma\right)\right]_{ij} \ll 1$$

$$)$$

Toy Model (without gauge field)

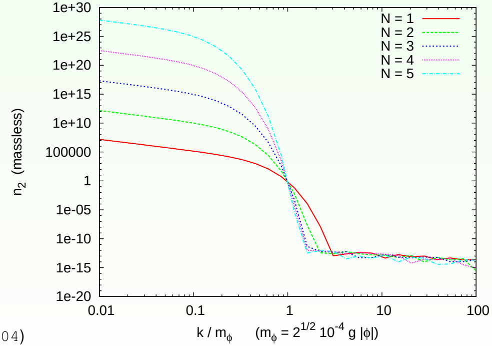
Olive, Peloso 2006

- Interaction that mimics the D-term potential: $\Delta V = 2\, g^2\, (\mathrm{Re}[\phi\, \chi^\star])^2$
- Coupling between Re and Im parts of χ through

$$\mathcal{M}^2 = 2\, g^2 |\phi|^2 \begin{pmatrix} \cos^2\sigma & \cos\sigma\, \sin\sigma \\ \cos\sigma\, \sin\sigma & \sin^2\sigma \end{pmatrix} + \begin{pmatrix} m_\chi^2 & 0 \\ 0 & m_\chi^2 \end{pmatrix}$$

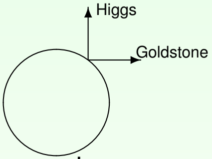
(Flat direction VEV: $\phi = |\phi| e^{i\,\sigma}$)

- Frequencies of physical states:
  $\omega_1 = \sqrt{k^2 + m_\chi^2 + 2\, g^2\, |\phi|^2}\,,$
  $\omega_2 = \sqrt{k^2 + m_\chi^2}.$
- Eigenfrequencies are adiabatically evolving, but the physical eigenstates are rotating nonadiabatically.
- Exponential particle production due to rotation of eigenstates, if
  $k < \sqrt{m_\phi^2 - m_\chi^2}.$



(Similar observation in Kawasaki, Takahashi 2004)

Toy Model $\rightarrow$ Gauged Computation



- No gauge field in previous toy model
- It actually applies to a global U(1) broken by F.D. VEV. The two modes are Higgs+Goldstone
- In a local theory, the Goldstone boson removed $\Rightarrow$ No rotation $\Rightarrow$ No decay
- Olive-Peloso 2006 argued: "The quick rotation of the mass matrix in field space, and the corresponding preheating effect, takes place if two or more flat directions are excited".
- Flat directions that are mutually exclusive exist. But in general, expect a set of compatible flat directions to be excited during inflation.
- No production if hierarchical VEVs . Allahverdi, Mazumdar '07
- "How much hierarchy for production?" $\Rightarrow$ Calculation

Gauged toy model with two FD

- Computation in a model with 2 FD + U(1) gauge field

AEG, Olive, Peloso, Sexton 2008

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi_i|^2 - V\,, \\
V &= m^2 \left(|\Phi_1|^2 + |\Phi_2|^2\right) + \tilde{m}^2 \left(|\Phi_3|^2 + |\Phi_4|^2\right) + \lambda \left(\Phi_1^2 \Phi_2^2 + \mathrm{h.c.}\right) \\
&\quad + \tilde{\lambda} \left(\Phi_3^2 \Phi_4^2 + \mathrm{h.c.}\right) + \frac{g^2}{8} \left(q\,|\Phi_1|^2 - q\,|\Phi_2|^2 + q'\,|\Phi_3|^2 - q'\,|\Phi_4|^2\right)^2
\end{aligned}
$$

with $\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \frac{F}{a}\, \mathrm{e}^{i\,\Sigma}\,, \quad \langle \Phi_3 \rangle = \langle \Phi_4 \rangle = \frac{G}{a}\, \mathrm{e}^{i\,\tilde{\Sigma}}$

- Nonrelativistic matter (oscillating massive inflaton) dominates expansion.
- Degrees of freedom in unitary gauge:

$$
\begin{array}{ccccccccc}
\{\Phi_i & , & A_\mu\} & \rightarrow & \left\{ A_\mu^T \right. & , & \begin{array}{c} \delta\Phi_1 + \delta\Phi_2 \\ \delta\Phi_3 + \delta\Phi_4 \end{array} & , & \left. \begin{array}{c} \text{Higgs} \\ A_\mu^L \\ 2 \text{ light fields} \end{array} \right\} \\
8 & + & 2 & = & 2 & + & 4 & + & 4
\end{array}
$$

Mixing

- In contrast, for the 1 FD + U(1) counterpart
$\Rightarrow$ no extra light degrees $\Rightarrow$ No nonadiabatic mixing.

Gauged toy model with two FD : Spectrum

- Squared-Eigenmasses:

$$m_1^2 = \frac{g^2}{4\,a^2}\left(F^2+G^2\right)\,,$$

$$m_2^2 = \frac{g^2}{4\,a^2}\left(F^2+G^2\right)+\frac{\left(F^2\,m^2+G^2\,\tilde{m}^2\right)}{F^2+G^2}+\frac{3\left(F^2\,\Sigma'+G^2\,\tilde{\Sigma}'\right)^2}{a^2\,\left(F^2+G^2\right)^2}\,,$$

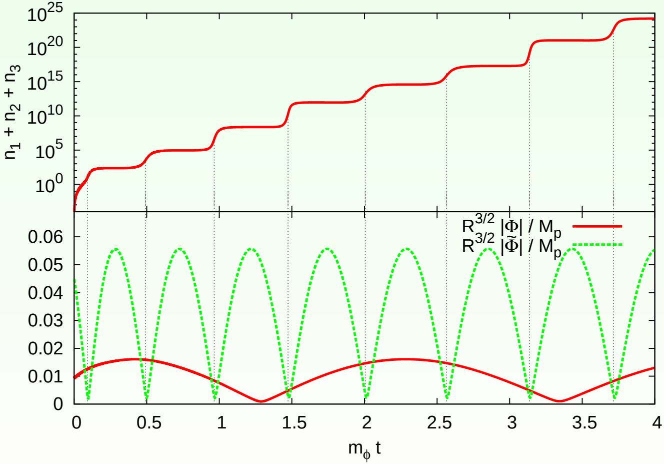
$$m_3^2 = \frac{\left(F^2\,\tilde{m}^2+G^2\,m^2\right)}{F^2+G^2}+\frac{3\,\left(F\,G'-G\,F'\right)^2}{a^2\,\left(F^2+G^2\right)^2}+\frac{3\,F^2\,G^2\,\left(\Sigma'-\tilde{\Sigma}'\right)^2}{a^2\,\left(F^2+G^2\right)^2}\,,$$

$$m_4^2 = \frac{\left(F^2\,\tilde{m}^2+G^2\,m^2\right)}{F^2+G^2}\,.$$

- $m_1, m_2 \sim \mathrm{VEV}$ (Higgs + Longitudinal vector field)
- $m_3, m_4 \sim \mathrm{TeV}$ (Light fields)
- Quick rotation of eigenstates present. (Light/Light, Light/Higgs)
- $|m'_{\mathrm{light}}| > m^2_{\mathrm{light}} \rightarrow$ Also have nonadiabatic eigenvalue evolution.
    - Light modes produced
    - Produced light modes rotated into Higgs.

Gauged 4 Field Toy Model : Occupation numbers

- Production when instantaneous vevs comparable, not the initial vevs. ⇐ Elliptic orbits.



$\tilde{m}/m = 3.7, \quad G_0/F_0 = 30, \quad k/m = 1/10$

Ellipticity

- Degree of ellipticity determines how much initial hierarchy between VEVs can be allowed for production. In this toy model it is provided by quartic terms:

$$\lambda\left(\Phi_1^2\Phi_2^2+\mathrm{h.c.}\right)\quad,\quad\tilde{\lambda}\left(\Phi_3^2\Phi_4^2+\mathrm{h.c.}\right)$$

- Choice of λ is highly model dependent. We chose $\lambda=\frac{m^2}{10\,|\Phi_0|^2}\Rightarrow\epsilon\sim\mathcal{O}(10^{-2})$, compatible with Affleck-Dine.
- eg. if $W=0$, no CP violating term $\Rightarrow$ Radial motion. (Dine, Randall, Thomas '95 ; Giudice, Mether, Riotto, Riva '08)
  $\Rightarrow$ Maximal hierarchy allowed : Single F.D. can also decay
- Circular motion $\Rightarrow$ Requires comparable VEVs for production

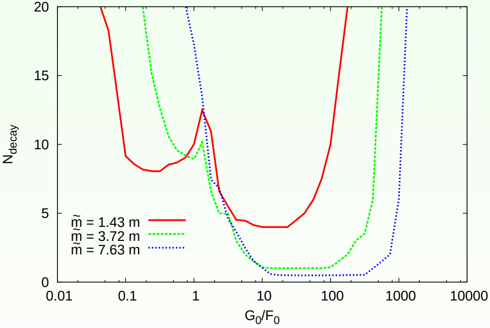
Gauged toy model with two FD : Final result

- Hierarchy between heavy and light modes: VEV/TeV $\sim 10^{14}$. Need to control both scales in numerical analysis. Fortunately, the ratio of energy densities have the scaling property

$$\frac{\rho_{\rm prod}}{\rho_{\rm flat}} \simeq C \, \frac{m \, \tilde{m}}{F_0 \, G_0} \, 10^{\sigma \, N_{\rm rot}}$$

$$C, \sigma = f\left(\frac{G_0}{F_0}, \frac{\tilde{m}}{m}\right)$$

- "Decay", when $\rho_{\rm prod} / \rho_{\rm flat} \sim 1$



$m = {\rm TeV}\,, \quad \sqrt{F_0 \, G_0} = 10^{-2} M_p$

Realistic MSSM example with 2 FD: $udd + QLd$

	Field	Y (hypercharge)	degree of freedom
ϕ_1	u^c	$-\frac{4}{3}$	6
ϕ_2	s^c	$\frac{2}{3}$	6
ϕ_3	b^c	$\frac{2}{3}$	6
ϕ_4	d^c	$\frac{2}{3}$	6
ϕ_5	L_e	-1	4
ϕ_6	$Q_{c/s}$	$\frac{1}{3}$	12

AEG '09

- VEV configuration:

$$\langle u_1^c \rangle = \langle s_2^c \rangle = \langle b_3^c \rangle = \Phi$$
$$\langle d_1^c \rangle = \langle \nu_e \rangle = \langle s_1 \rangle = \tilde{\Phi}$$

- Breaks all SM symmetries
- The two flat directions decoupled at background level.

Potential: $V = |F|^2 + \frac{1}{2} D^2 + V_{\rm soft}$

$$|F|^2 = |y_d\, \phi_6\, \phi_2|^2 \,,$$

$$\frac{1}{2} D^2 = \frac{1}{8}\, g_1^2 \left| \sum_i \phi_i^\dagger\, Y \phi_i \right|^2 + \frac{1}{8}\, g_2^2 \sum_{a=1}^{3} \left| \sum_i \phi_i^\dagger \sigma_a \phi_i \right|^2 + \frac{1}{8}\, g_3^2 \sum_{a=1}^{8} \left| \sum_i \phi_i^\dagger \lambda_a \phi_i \right|^2 \,,$$

$$V_{\rm soft} = m^2 \sum_{i=1}^{3} \phi_i^\dagger \phi_i + \tilde{m}^2 \sum_{i=1}^{3} \phi_i^\dagger \phi_i \,,$$

$udd + QLd$: Spectrum

- Quadratic action can be separated into 9 decoupled subsystems

	$\perp$ Vector	$\parallel$ Vector	Higgs	Flat Dir.	Other heavy	Other light	Total
S_1	16	–	–	–	–	–	16
S_2	8	–	–	–	–	–	8
S_3	–	–	–	–	2	–	2
S_4	–	–	–	–	–	2	2
S_5	–	2	2	–	–	–	4
S_6	–	2	2	–	–	–	4
S_7	–	4	4	4	–	–	12
S_8	–	2	2	–	2	2	8
S_9	–	2	2	–	–	4	8
Total	24	12	12	4	4	8	64

- Only S_8 and S_9 may contribute to nonperturbative production
- S_8 gives a system where nonadiabatic rotation of eigenvectors may occur.
- S_9 gives two copies of the coupled system from $U(1)$ toy model with 2 FD
  $\Rightarrow$ The numerical analysis is also valid here. F.D. decay after $\mathcal{O}(10)$ rotations.

What do we know now?

- We know, through semi-analytical studies in a linearized setup, the flat directions start decaying nonperturbatively through the D term, in the following models:
  1. Toy model with D–term like potential, 1 FD — Olive, Peloso '06
  2. 2 FD + $SU(N)$ toy model — AEG, Olive, Peloso, Sexton '08
  3. Simultaneous excitation of $udd + QLd$ directions in MSSM — AEG '09

  All these cases involve "independent" flat directions, where the two condensates are decoupled from each other at leading order.
- Similar works show nonadiabatic rotation of eigenstates for "overlapping" flat directions — Basbøll, Maybury, Riva, West '07 --Basbøll '08
- Non-perturbative decay starts for an initial VEV ratio range of 3 orders $\Rightarrow$ decay in $\mathcal{O}(10)$ rotations. (cf. 10^{11} rotations in perturbative decay)

The effect of reheating?

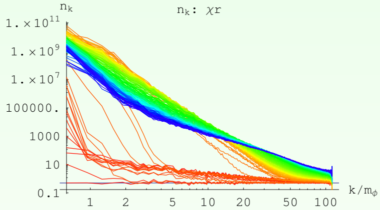
The nonperturbative decay necessarily implies that the thermalization is no longer delayed?

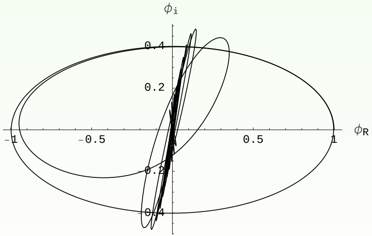
- All the previous examples involve actions up to quadratic in perturbations. Particle production inevitably gives rise to breakdown of the linear approximation.
- Is the condensate completely depleted or do high order terms oppose the resonant decay?
- Produced particles are nonrelativistic ($k \leq m_\phi$), variances high $\Rightarrow$ gauge fields still massive?
- Showed: Condensate $\rightarrow$ Non thermal distribution $\Rightarrow \mathcal{O}(10)$ rotations
- Non thermal distribution $\rightarrow \left[E \sim T\,,\ N \sim E^3\right]$
  $\Rightarrow$ How slow/fast?
- "How fast produced quanta thermalize?": Maybe not in 10 rotations, but we expect $\ll 10^{11}$ rotations.

⇒ Nonlinear study of the decay

Toy model $V = \frac{1}{2}\, m_\phi^2\, |\phi|^2 + \frac{1}{2}\, m_\chi^2\, |\chi|^2 + g^2\, (\phi\, \chi^\star + \phi^\star\, \chi)^2$,
with $\langle\phi\rangle = \phi_0 e^{i\,\sigma}$.

- Occupation numbers for Re[χ] for 30 rotations. Thermalization already proceeding. In this toy model, no gauge interactions, no expansion, initially circular orbits.





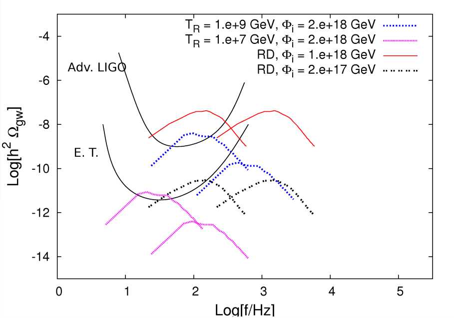
- Same model, with expansion, elliptical orbits, shows the depletion of the background condensate (Axes multiplied by $R^{3/2}$ (Dufaux 2009))

Gravity waves from flat direction decay

Dufaux 2009

- The resonant decay of flat directions also gives rise to gravity wave production. Present-day peak frequency and amplitude of the emergent gravity waves:

$$f_* \sim \left(\frac{a_i}{a_r}\right)^{1/4} \sqrt{\frac{m}{\mathrm{TeV}}} \left(5 \times 10^2 \mathrm{Hz}\right), \quad h^2 \Omega^*_{\mathrm{gw}} \sim 10^{-4} \left(\frac{\Phi_i}{M_p}\right)^4 \left(\frac{a_i}{a_r}\right)$$



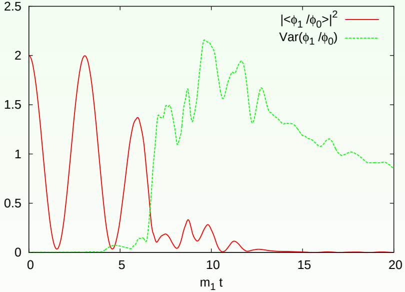
$m = 100$ GeV (left), $m = 10$ TeV (right)

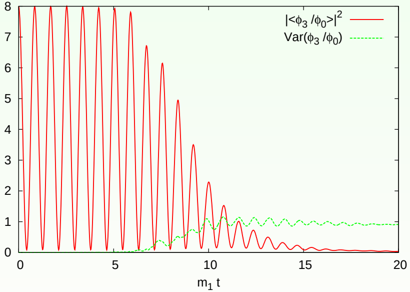
At $a = a_i$, FD oscillates.
At $a = a_r$, RD.

c.f. Inflaton preheating: GW observable only if coupling constants very small.

Nonlinear study of gauged 2 FD model

- Ongoing project: Solving the nonlinear (classical) equations of motion for 4 complex scalar + U(1) gauge field model (with Dufaux and Peloso)
- U(1) gauge fields implemented to ClusterEasy (Felder 2007) complete for $\dot{R} = 0$.
- Preliminary runs show that the condensates decay almost completely.





$\tilde{\Phi}_0/\Phi_0 = 2$, $\tilde{m}/m = 3.72$, $\frac{m}{e\,\Phi_0} = .25$

Nonlinear study of gauged 2 FD model