Khrono-metric model:

the low-energy limit of Horava-Lifshitz gravity

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Diego Blas, Oriol Pujolas, S.S., arXiv:0906.3046, 0909.3525, 0912.0550, 1007.3503

+ work in progress

Plan

- Renormalizing gravity: Why anisotropic scaling ?
- Horava-Lifshitz gravity
- Covariant (Stueckelberg) formulation: the khrono-metric model
- Phenomenology and observational constraints
- Conclusions & Outlook

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(alternative approaches: string theory, loop quantum gravity etc.)

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$$h \mapsto b\, h \quad \Rightarrow \quad \text{scaling dimension of } h \text{ is } 1$$

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We need to reduce *dim* h to 0

Can be achieved by including terms with higher derivatives: $R^2, \quad R_{\mu\nu}R^{\mu\nu}, \quad R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}$

$$\int (R^2 + M_P^2 R)\sqrt{-g}\, d^4x \quad \Rightarrow$$

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IR dynamics is determined by terms with 2 derivatives

But higher time derivatives $\Rightarrow$ ghosts

$$\langle h\, h \rangle = \frac{1}{M_P^2 p^2 - p^4} = \frac{1}{M_P^2}\left[\frac{1}{p^2} - \frac{1}{p^2 - M_P^2}\right] \quad \Rightarrow \quad \text{loss of unitarity}$$

The idea:

to get *dim h = 0* due to higher space derivatives,

keeping the e.o.m. second order in time

$\Longleftrightarrow$ no ghosts

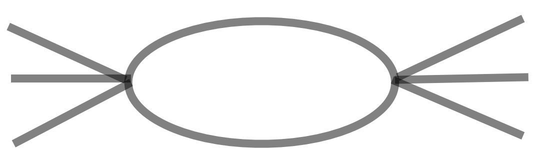
Anisotropic scaling: Lifshitz scalar

$$S = \int dt\, d^3x \left(\frac{\dot{\varphi}^2}{2} - \frac{\varphi(-\Delta)^z \varphi}{2M_*^{2(z-1)}} - V(\varphi) \right)$$

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}\ , \quad t \mapsto b^{-z} t\ , \quad \varphi \mapsto b^{(3-z)/2}\varphi$$

$z = 3$ $\Longrightarrow$ φ is dimensionless

Example: renormalizable φ^5 interaction



$$\sim \int \frac{d\omega d^3p}{(\omega^2 - p^6/M_*^4)^2} \sim M_*^6 \int \frac{d^3p}{p^9}$$

The most general renormalizable action includes all terms of $dim \leq 0$

$$S = \int dt d^3x \Big[\dot{\varphi}^2 + \left(A_1(\varphi)\Delta^3\varphi + A_2(\varphi)(\partial\varphi)^6 + \ldots\right) \\ + \left(B_1(\varphi)\Delta^2\varphi + B_2(\varphi)(\partial\varphi)^4 + \ldots\right) \\ + C^2(\varphi)\varphi\Delta\varphi - V_0(\varphi)\Big]$$

NB. The number of possible terms can be limited by symmetries

Example: the shift symmetry $\varphi \mapsto \varphi + const$

Second order in time derivatives ➡ no ghosts

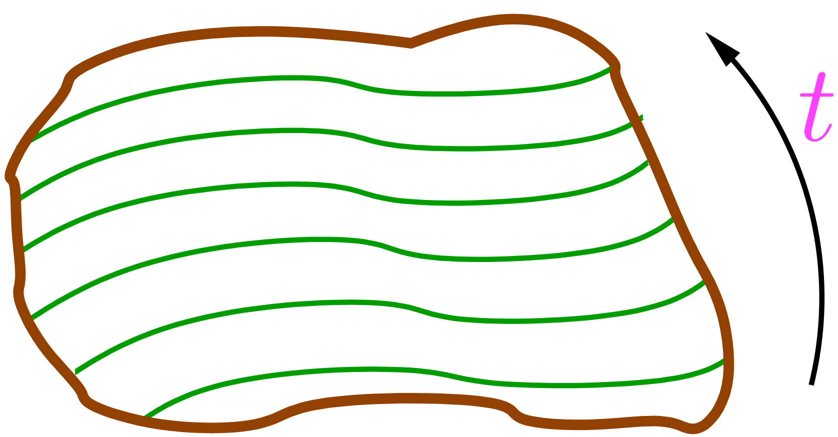
$C^2(0) = c^2$ ➡ "relativistic" dispersion relation $\omega^2 = c^2 p^2$ in IR

Horava gravity I

Split coordinates in space and time:
ADM decomposition of the metric (in GR -- a gauge choice)

$$ds^2 = (N^2 - N_i N^i)dt^2 - 2N_i dt dx^i - \gamma_{ij} dx^i dx^j$$

Think of the splitting as physical



spacetime =
Lorentzian manifold
equipped with folitation
by spacelike surfaces

Horava gravity II

$$S = \frac{M_P^2}{2}\int d^3x dt\,\sqrt{\gamma}N\Big(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}\Big)$$

$$K_{ij} = \frac{\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i}{2N}$$

$$\mathcal{V}_I = -R \quad + \quad M_*^{-2}(A_1 \Delta R + A_2 R_{ij}R^{ij} + \dots)$$
$$+ \quad M_*^{-4}(B_1 \Delta^2 R + B_2 R_{ij}R^{jk}R_k^i + \dots)$$

R_{ij} -- 3d Ricci tensor

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- collection of marginal and relevant operators under the scaling:
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???

Symmetry group

- FDiffs (maximally allowed)

$$\mathbf{x} \mapsto \tilde{\mathbf{x}}(\mathbf{x}, t) \ , \qquad t \mapsto \tilde{t}(t)$$

- Restricted FDiffs

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check for the absence of pathologies

VARIATIONS

Field content

- “projectable”: $N = N(t)$ (compatible with FDiffs)

  problematic: scalar graviton develops instability at finite spatial momenta

  Existence of inhomogeneous vacuum ??

- “non-projectable”: $N = N(t, \mathbf{x})$

VARIATIONS

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Extended non-projectable HL gravity

Not to overlook: A new object covariant under FDiffs

$$a_i \equiv N^{-1}\partial_i N$$

The potential must be extended by the terms with a_i

$$dim\ a_i = 1 \quad \Longrightarrow$$

$$\begin{aligned}\mathcal{V}_{II} =&\mathcal{V}_I - \alpha a_i a^i \\ &+ M_*^{-2}\left(C_1 a_i \Delta a^i + C_2 (a_i a^i)^2 + C_3 a_i a_j R^{ij} + \dots\right) \\ &+ M_*^{-4}\left(D_1 a_i \Delta^2 a^1 + D_2 (a_i a^i)^3 + D_3 a_i a^i a_j a_k R^{jk} + \dots\right)\end{aligned}$$

Scalar mode has a healthy dispersion relation:

$$\omega^2 = \frac{\lambda - 1}{2(3\lambda - 1)} \frac{P[-p^2/M_*^2]}{Q[-p^2/M_*^2]} p^2$$

$$\begin{aligned} P[x] =& (g_2^2 - g_1 g_3)x^4 - (g_1 f_3 + g_3 f_1 - 2 g_2 f_2)x^3 \\ &+ (f_2^2 - 4g_2 - f_1 f_3 - 2g_3 - g_1\alpha)x^2 \\ &- (2f_3 + f_1\alpha + 4f_2)x + (4 - 2\alpha) \\ Q[x] =& g_3 x^2 + f_3 x + \alpha \end{aligned}$$

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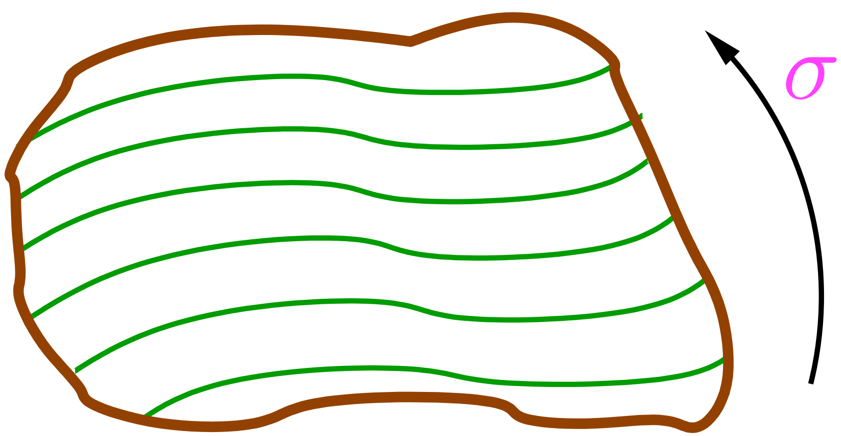

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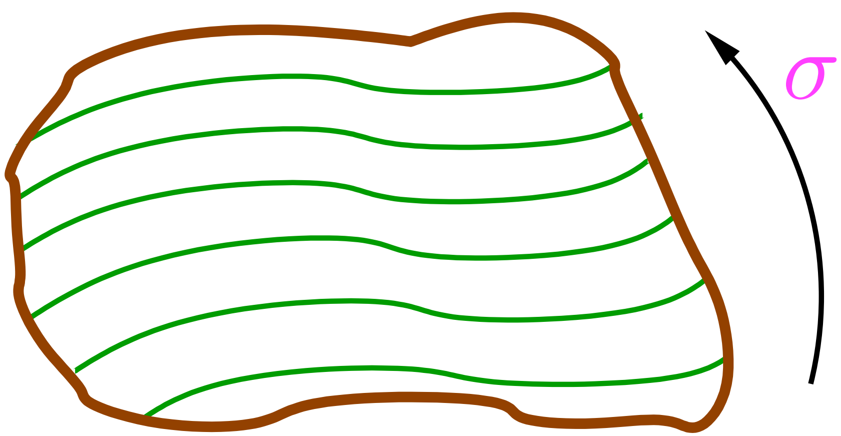

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KHRONON

Constructing KHRONO-METRIC action

- Time reparameterizations in ADM frame

  ➡ symmetry $\sigma \mapsto \tilde{\sigma} = f(\sigma)$

  Invariant object -- unit normal to the foliation surfaces:

$$u_\mu = \frac{\partial_\mu \sigma}{\sqrt{(\partial\sigma)^2}}$$

- low-energy limit = Lagrangian with lowest number of derivatives

$$S_{kh\text{-}m} = -\frac{M_P^2}{2} \int d^4x \sqrt{-g} \Big[{}^{(4)}R + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu + \lambda' (\nabla_\mu u^\mu)^2 + \alpha u^\mu u^\nu \nabla_\mu u_\rho \nabla_\nu u^\rho \Big]$$

cf. with Einstein-aether theory (*Jacobson & Mattingly, 2001*): a LV theory of a unit vector

Alternative way:

identify covariant objects in the ADM frame

$$\gamma_{ij} \mapsto P_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu, \quad K_{ij} \mapsto K_{\mu\nu} = P_\mu^\lambda \nabla_\lambda u_\nu$$

$a_i \mapsto a_\mu = u^\nu \nabla_\nu u_\mu$, etc.

$$S_{HL} = S_{kh-m} + \text{higher derivatives}$$

$$\lambda' = \lambda - 1 - \beta$$

No-ghost theorem

Action contains higher derivatives

$$(\nabla_\mu u^\mu)^2 = \frac{1}{(\partial\sigma)^2}\left[\Box\sigma - \frac{\nabla^\mu\sigma\nabla^\nu\sigma}{(\partial\sigma)^2}\nabla_\mu\nabla_\nu\sigma\right]^2$$

No problem: there is a preferred frame where e.o.m. are second order in time

Theorem Consider linear perturbations

$$\sigma = \bar{\sigma} + \chi$$

In the frame where background is in ADM gauge,

$$\bar{\sigma} = t$$

e.o.m. for χ is second order in time

Around Minkowski background

$$S_{kh} = \frac{M_P^2}{2} \int d^4x \left[\alpha (\partial_i \dot{\chi})^2 - (\beta + \lambda')(\Delta\chi)^2 \right]$$

$$\Rightarrow \quad \Delta \left[\alpha \ddot{\chi} - (\beta + \lambda')\Delta\chi \right] = 0$$

NB. The r.h.s. does not vanish when khronon is coupled to sources

$\Rightarrow$ instantaneous interaction

Towards phenomenology: coupling to matter

Experimental fact: matter sector is Lorentz invariant at low energies

➡ direct coupling of the khronon to SM fields is forbidden

a mechanism to suppress these couplings is required, see below

Observational constraints

Exploit known bounds for Einstein - aether (beware: in our case there are no helicity-1 modes)

- Absence of gravitational Cherenkov losses by UHECR

$$\Rightarrow \quad c_g\ ,\ c_\sigma \geq 1 \quad \Rightarrow \quad \boxed{\beta \geq 0\,, \quad \frac{\lambda' + \beta}{\alpha} \geq 1}$$

- Newton law vs Friedman equation

$$G_N = \frac{1}{8\pi M_P^2(1-\alpha/2)} \quad \neq \quad G_{cosm} = \frac{1}{8\pi M_P^2(1+\beta/2+3\lambda'/2)}$$

$$H^2 = \frac{8\pi}{3} G_{cosm}\, \rho$$

BBN bound:

$$|G_{cosm}/G_N - 1| \leq 0.13$$

$$\Rightarrow \quad \boxed{\alpha\ ,\ \beta\ ,\ \lambda' \lesssim 0.1}$$

PPN parameters

Spherically symmetric solutions the same as in Einstein-aether

all PPN parameters the same as in GR

except $\underbrace{\alpha_1^{PPN}, \alpha_2^{PPN}}_{\text{measure preferred frame effects}}$

measure preferred
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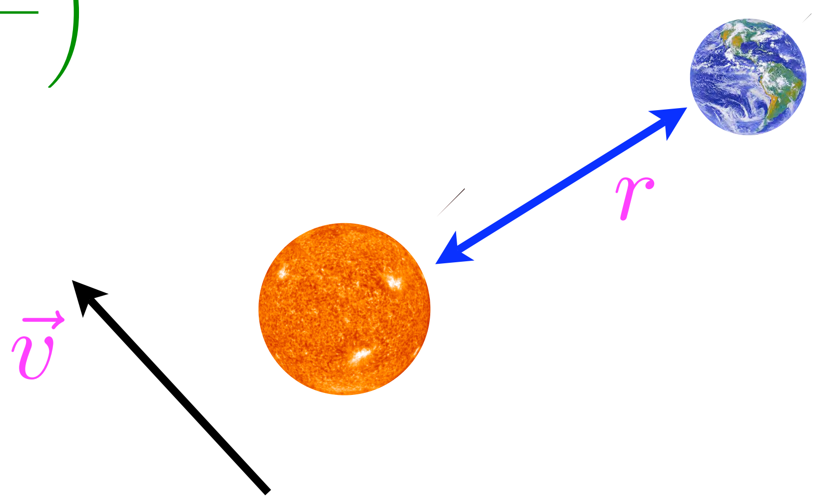
Definition of $\alpha_{1,2}^{PPN}$

$$h_{00} = -2G_N \frac{m}{r}\left(1 - \frac{\alpha_2^{PPN}}{2}\frac{(x^i v^i)^2}{r^2}\right)$$

$$h_{0i} = \frac{\alpha_1^{PPN}}{2} G_N \frac{m}{r} v^i$$



Solar system bounds:

$$|\alpha_1^{PPN}| \lesssim 10^{-4} \,, \qquad |\alpha_2^{PPN}| \lesssim 10^{-7}$$

$$\alpha_1^{PPN} = -4(\alpha - 2\beta)$$

$$\alpha_2^{PPN} = \frac{(\alpha - 2\beta)(\alpha - \lambda' - 3\beta)}{2(\lambda' + \beta)}$$

- vanish if $\alpha = 2\beta$ (hidden symmetry ?)
- α_2^{PPN} vanishes when $\beta = 0$, $\lambda' = \alpha$ ($c_g = c_\sigma = 1$)
- barring cancellations

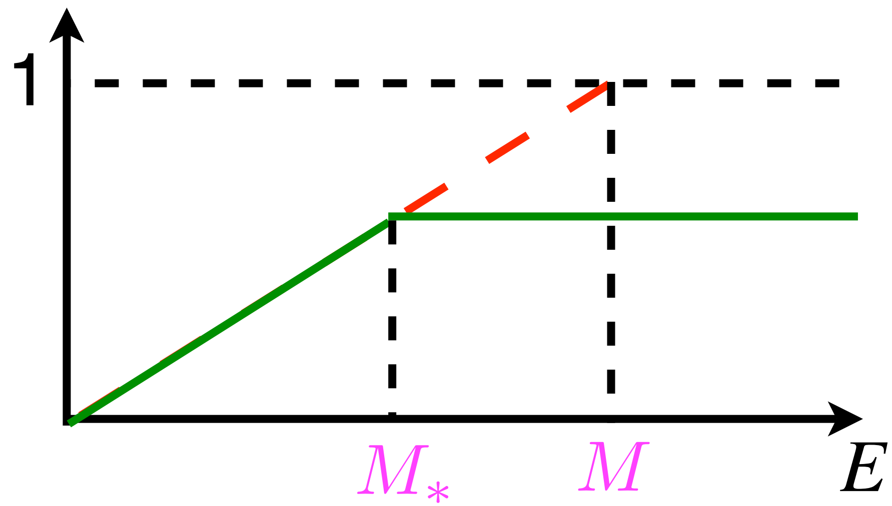
$$\boxed{\alpha\ ,\ \beta\ ,\ \lambda' \lesssim 10^{-7} \div 10^{-6}}$$

Upper bound on the scale of quantum gravity

Khrono-metric theory -- a sigma model with the scale

$$M \sim M_P \sqrt{\alpha} \sim 10^{16} \mathrm{GeV}$$

would enter into strong coupling unless cut off by higher derivatives



UPPER bound: $\boxed{M_* \lesssim 10^{15} \div 10^{16} \mathrm{GeV}}$

NB. Lower bound can be taken from the constraints on UV modifications of photon dispersion relation: $M_* \gtrsim 10^{11} \mathrm{GeV}$

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SUSY breaking generates dim 4 LV operators suppressed by

$$(m_{soft}/M_*)^2$$

LI from SUSY: example of SQED

- SUSY algebra without boosts is closed:

$$[Q_\alpha, \bar{Q}_{\dot{\alpha}}]_+ = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu$$

  Enough to generate superspace

- field content: $\Phi_+ \ , \ \Phi_- \ , \ V$

  Kahler potential: $\int d^4\theta \, \bar{\Phi}_+ \mathrm{e}^{2eV} \Phi_+$

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  $\cancel{\int d^2\theta \, \Phi_+ \partial_\mu \Phi_-}$ - not gauge invariant

Prospects for cosmology

Can we address the dark energy / cosmological constant problem ?

Yes! Add a scalar with shift symmetry $\Phi \mapsto \Phi + const$
with dim 2 coupling to the khronon

$$S_\Phi = \int d^4x\sqrt{-g}\left(\frac{(\partial_\nu\Phi)^2}{2} + \mu^2 u^\nu \partial_\nu \Phi\right)$$

stable under RC: breaks $\Phi \mapsto -\Phi$

$$\Rightarrow \quad H^2 = \frac{8\pi G}{3}\left(\frac{\dot\Phi^2}{2} + \rho_{mat}\right)$$

$$\frac{d}{dt}\left(a^3(\dot\Phi + \mu^2)\right) = 0 \quad \Rightarrow \quad \dot\Phi = -\mu^2 + \frac{C}{a^3}$$

NB. There is a Minkowski solution $\rho_{mat} = 0\,,\ \dot\Phi = 0$

unstable $\Rightarrow$ "self-acceleration"

Observational signatures: growth of perturbations ?

DE-DM interaction ?

CONCLUSIONS

* Extended non-projectable Horava--Lifshitz model provides a power counting renormalizable setup for quantum gravity without obvious pathologies
* Its low-energy limit is described by the khrono-metric model: a theory of GR interacting with a scalar having time-dependent VEV
* The model possesses predictive power and leads to interesting phenomenology. Existing data constrain the parameters of the model but do not rule it out
* A simple modification of the model provides a technically natural explanation of dark energy giving rise to self-accelerated cosmology

OUTLOOK

- Actual proof of renormalizability. Complications: gauge invariance, instantaneous modes
- Emergence of Lorentz invariance at low energies
- Constraints from strongly bound gravitational systems (binary pulsars, black holes)
- Implications for CMB and LSS
- Phenomenology of instantaneous interaction
- Inflation