Black holes
&
Blackfolds
in Higher Dimensions

Roberto Emparan
ICREA & U. Barcelona

Based on RE, T. Harmark, V. Niarchos, N. Obers *~~to appear~~*

Out today! 0902.0427

Why study D>4 gravity?

- Main motivation (at least for me!):
  - better understanding of gravity (i.e. of what *spacetime* can do)

- In General Relativity in vacuum

$$R_{\mu\nu}=0$$

$\exists$ only one parameter for tuning: D

  - BHs exhibit novel behavior for $D > 4$

Why study D>4 gravity?

- Also, for applications to:
  - String/M-theory
  - AdS/CFT

    (+its derivatives: AdS/QGP, AdS/cond-mat etc)
  - Math
  - TeV Gravity (bh's @ colliders…)
  - etc

- When first found, black hole solutions have always been "answers waiting for a question"!

- After the advances of the 1960's-70's, classical black holes have been widely regarded as essentially understood...
- ...so the only interesting open problems should concern quantum properties of black holes
- But recently (~8 yrs ago) we've fully realized how little we know about black holes – even classical ones! – and their dynamics in $D>4$

  E.g. we've learned that some properties of black holes are
  - 'intrinsic': Laws of bh mechanics ...
  - $D-$dependent: Uniqueness, Topology, Shape, Stability ...
- Black hole dynamics becomes richer in $D=5$, and much richer in $D \geqslant 6$

- Activity launched initially by two main results:
  - **Gregory-Laflamme (GL)-instability**, its endpoint, and inhomogeneous phases
  - **Black rings**, non-uniqueness, non-spherical topologies

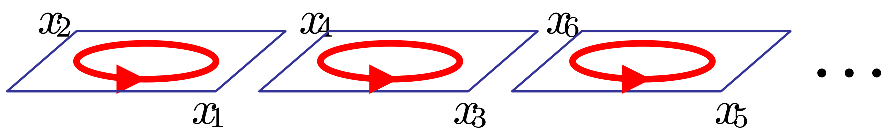
- I'll focus mostly on simplest set up: vacuum, $R_{\mu\nu} = 0$, asymptotically flat solutions

FAQ's

- Why is D>4 richer?
  - More degrees of freedom

$$O(D-1) \supset U(1)^{\lfloor (D-1)/2 \rfloor}$$

  - Rotation:



    - more rotation planes
    - gravitational attraction $\Leftrightarrow$ centrifugal repulsion

$$-\frac{GM}{r^{D-3}} + \frac{J^2}{M^2 r^2}$$

  - $\exists$ extended black objects: black p-branes
  - Both aspects combine to give rise to horizons w/ more than one length scale:

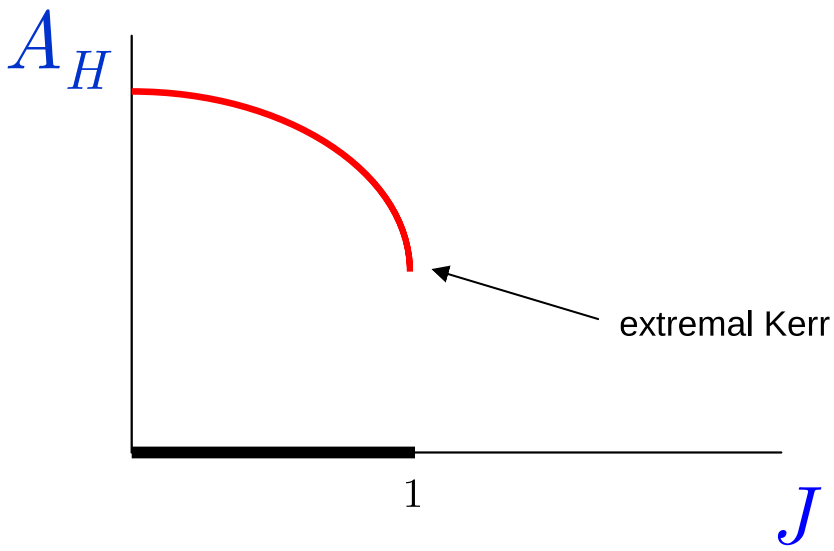
    new dynamics appears when two scales differ

FAQ's

- Why is D>4 harder?
  - More degrees of freedom
  - Axial symmetries:
    - U(1)'s at asymptotic infinity appear only every 2 more dimensions (asymptoticaly flat – or AdS – cases)
    - in D=4,5: stationarity+1,2 rotation symmetries is enough to reduce to integrable 2D σ-model (but only in vacuum, not in AdS)
    - if D>5 not enough symmetry to reduce to 2D σ-model
  - 4D approaches not tailored to deal with horizons with more than one length scale: need new tools

Phases of 4D black holes

- Just the Kerr black hole: Uniqueness thm



(fixed mass $M=1$)

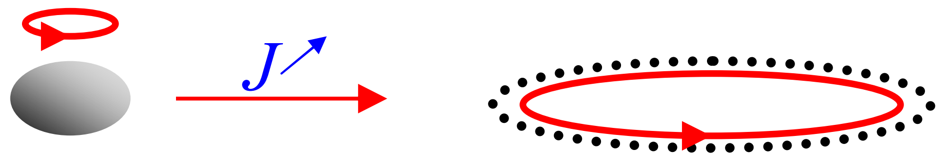
- **End of the story!**

Multi-bhs not rigorously ruled out, but physically unlikely to be stationary (eg multi-Kerr can't be balanced)

Myers-Perry black holes in D dimensions

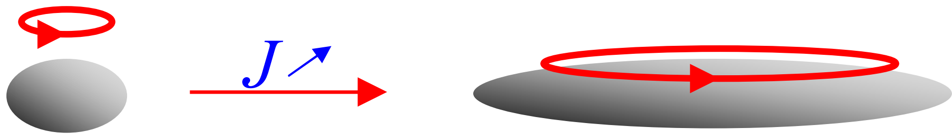
- They all have spherical topology S^{D-2}
- Consider a **single spin**:

$D = 5$

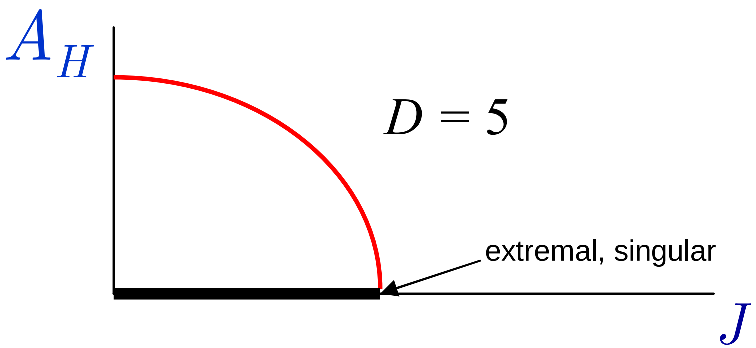


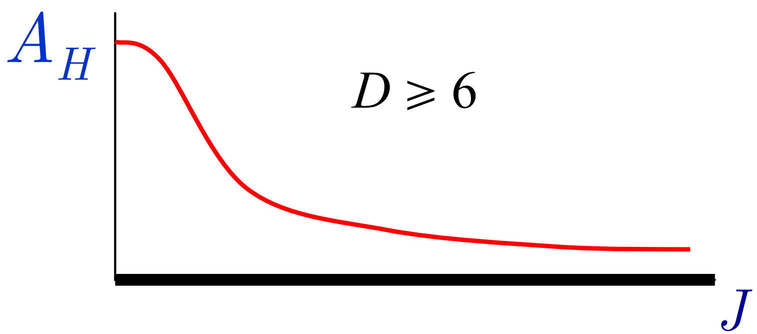
upper (extremal) limit on J is singular

$D \geqslant 6$



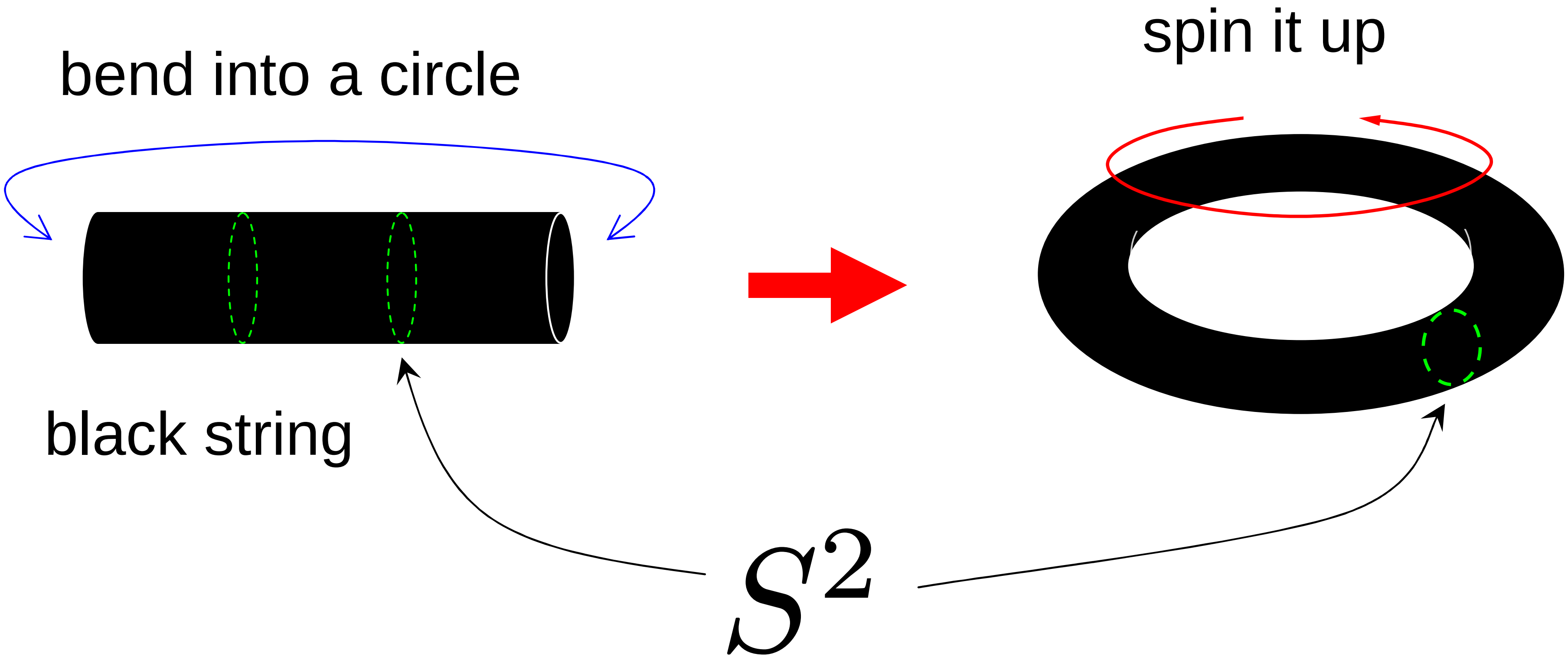
no upper limit on J for fixed M





But there's much more…

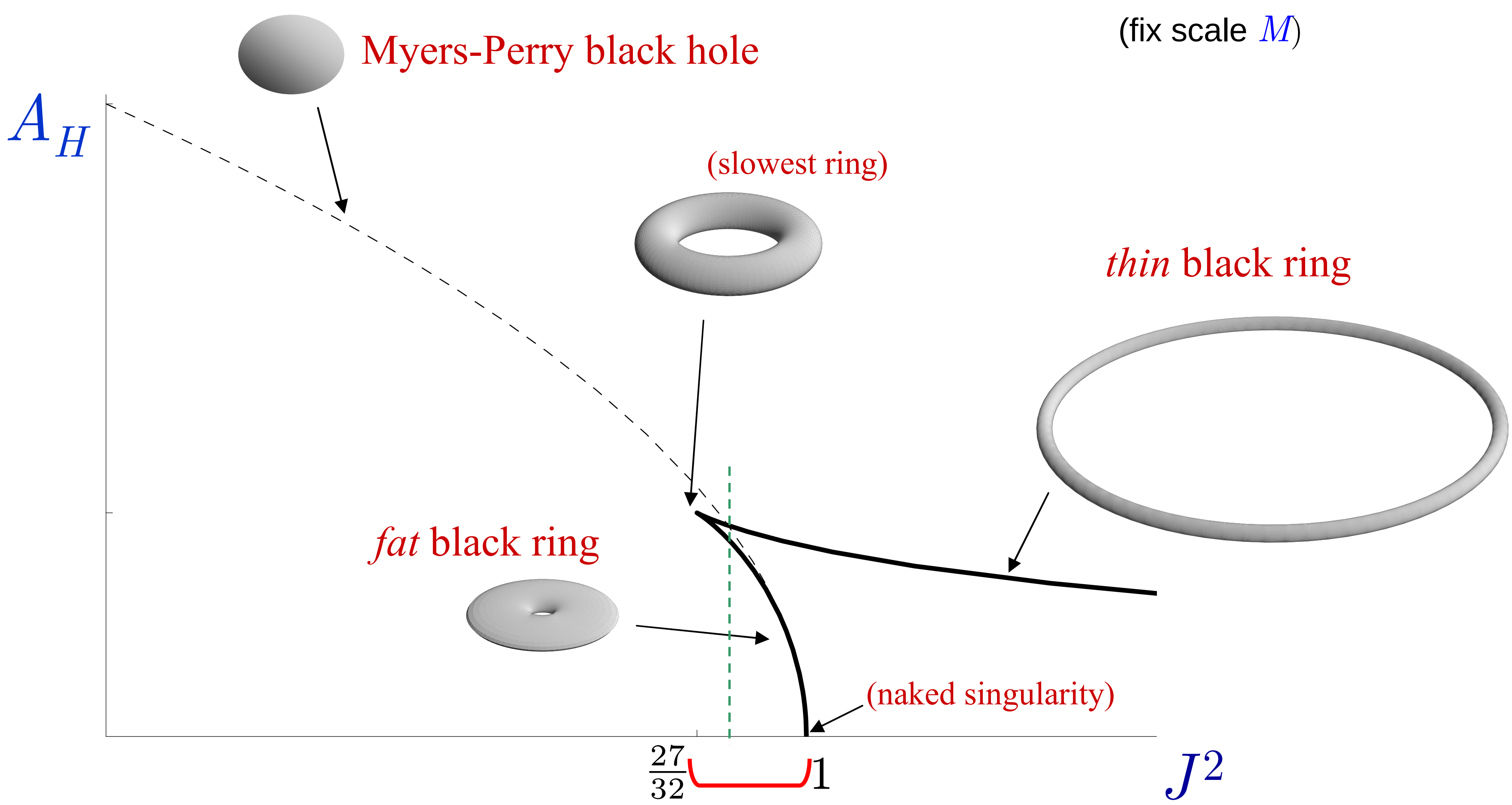
The forging of the ring (in $D=5$)



There's explicit solution, and fairly simple

RE + Reall 2001

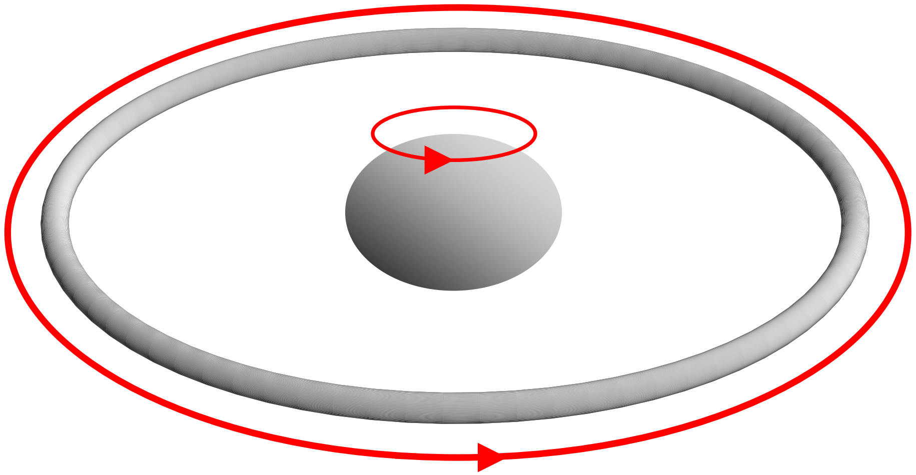
5D: one-black hole phases



3 different black holes with the same value of ***M,J***

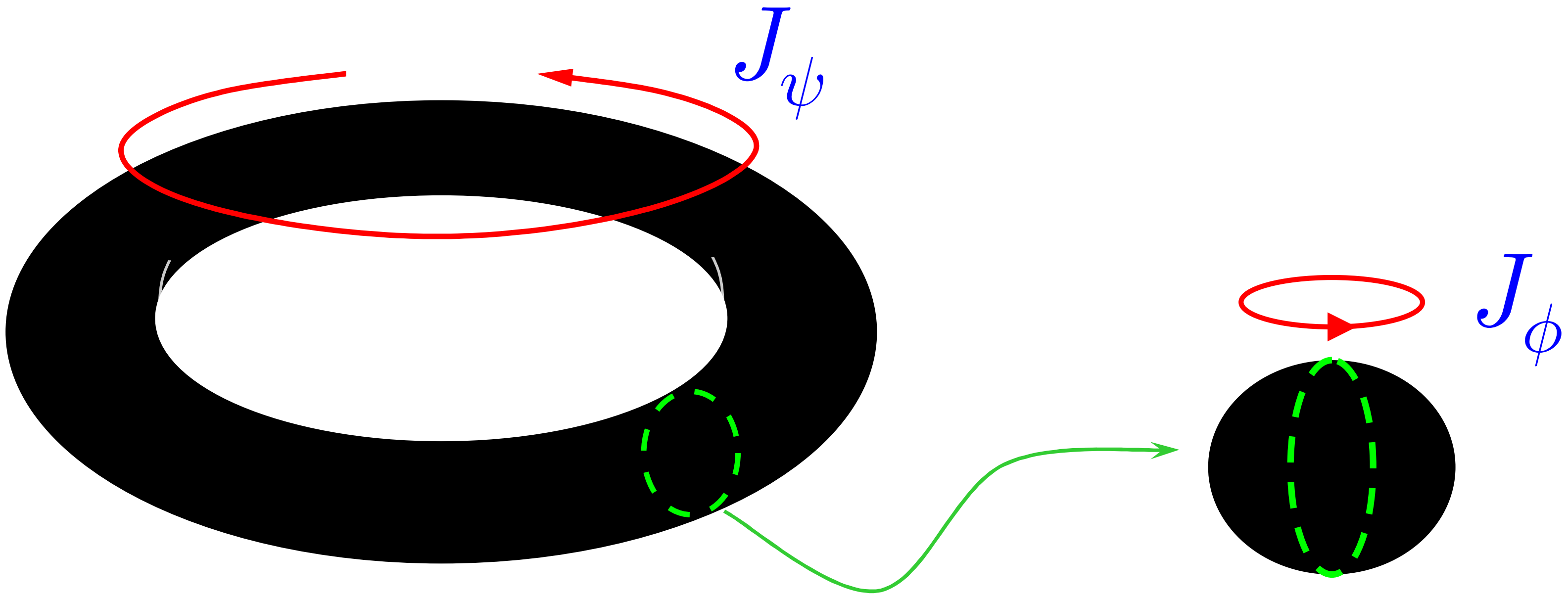
Multi-black holes

- *Black Saturn:*



- Exact solutions available (not simple!) *Elvang+Figueras*
- Co- & counter-rotating, rotational dragging…

Black rings w/ two spins



Pomeransky+Sen'kov

Towards a complete classification of vacuum asymp flat 5D black holes

- Topology: S^3, S^1x S^2 (+possibly quotients & connected sums) *Galloway+Schoen*
- We *might* have identified essentially all bh solutions:
  - **MP bhs, black rings, multi-bhs** (saturns & multi-rings)
- …if two rotational symmetries *are* required:

  $\mathbb{R}_t$ x $U(1)_{\phi_1}$ x $U(1)_{\phi_2}$ ➔ complete integrability

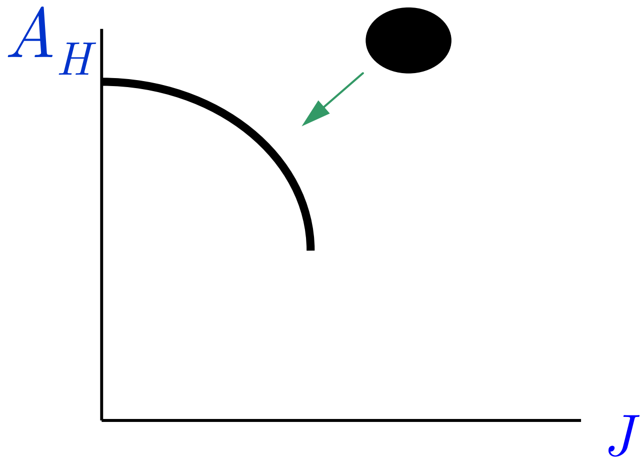
- But: stationarity → one axial $U(1)$, but not (yet?) necessarily two *Hollands+Ishibashi+Wald*

  (new solutions with broken $U(1)$? where/how?)

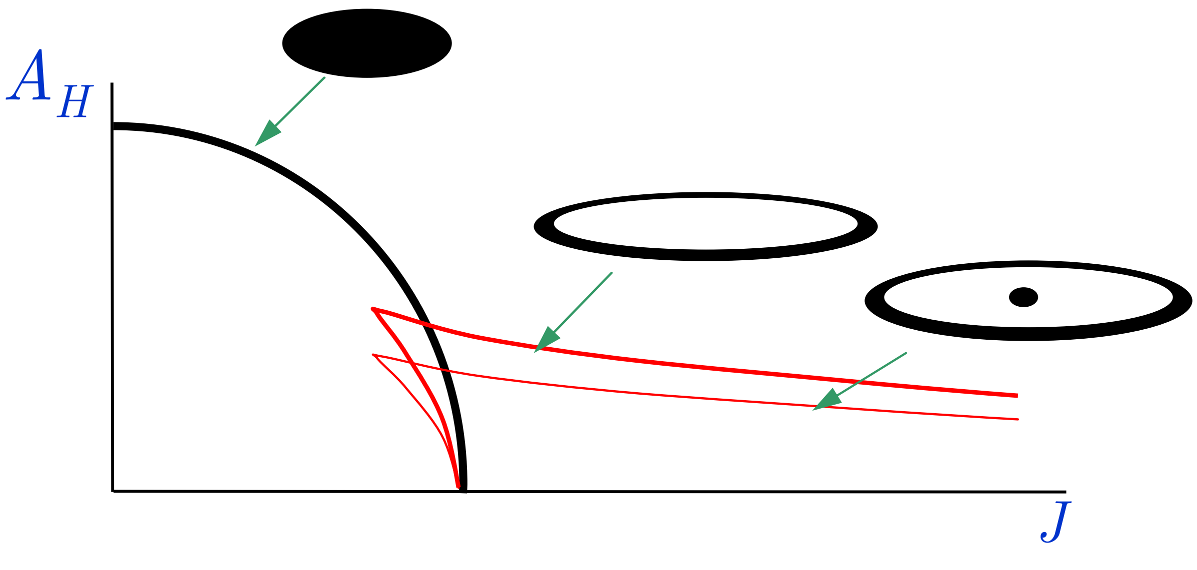
- This remains the **MAIN OPEN PROBLEM** in 5D

The plot thickens…

D=4



D=5



The story so far

Black Hole classification programme ($R_{\mu\nu} = 0$)

- $D{=}3$ has **no** black holes ✓
- $D{=}4$ has **one** black hole ✓
- $D{=}5$ has **three** black holes (two topologies) & infinitely many multi-bhs
- $D{\geqslant}6$ seem to have **infinitely many** black holes (many topologies, lumpy horizons…) & infinitely many more multi-bhs…

Should we still aim at "exact solutions & classification" or should we reassess our aims?


Blackfolds

4D vs hi-D Black Holes: Size matters

- 4D BHs: not only unique, spherical, and stable, but also dynamics depends on a single scale:

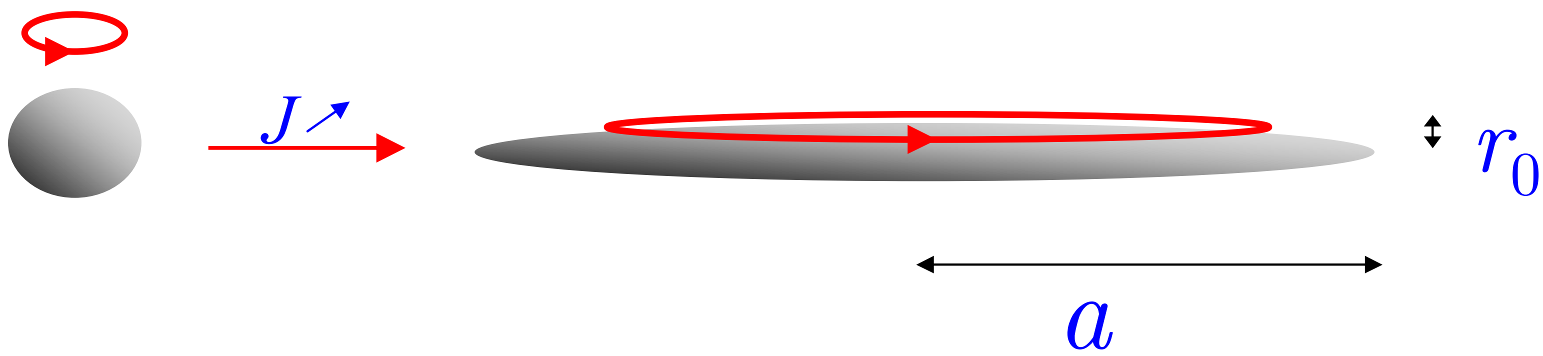
$$r_0 \sim GM$$

  - true even if rotating: Kerr bound $J \leqslant GM^2$

- Main novel feature of D>4 BHs: in some regimes they're characterized by two separate scales
  - No upper bound on J for given M in $D > 4$
  - → Length scales J/M and $(GM)^{1/(D-3)}$ can differ arbitrarily

Ultra-spinning regime: two scales and black brane limit

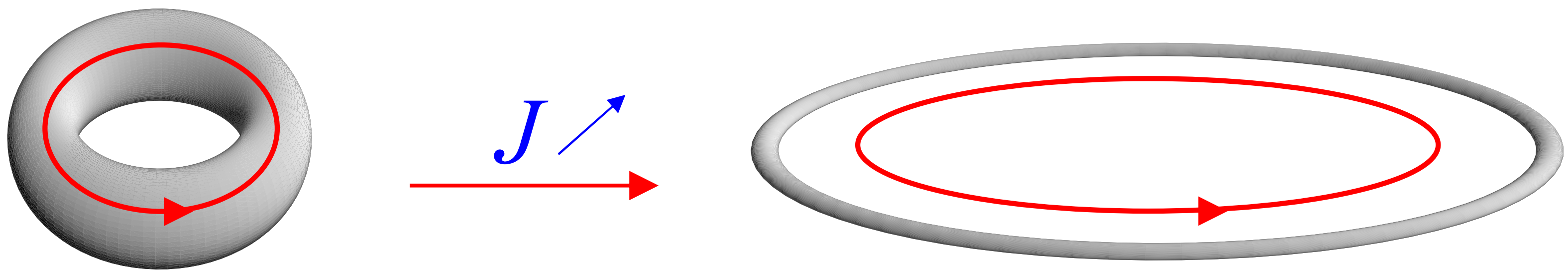
- $D \geqslant 6$: take $a \gg r_0$



- Limit $a \to \infty$, r_0 finite, close to axis:

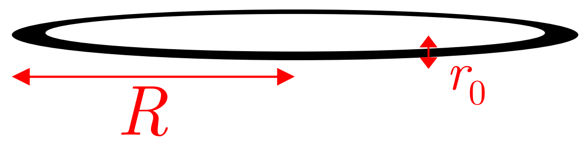
$\Rightarrow$ black 2-brane along rotation plane

Black Ring in $D\!=\!5$



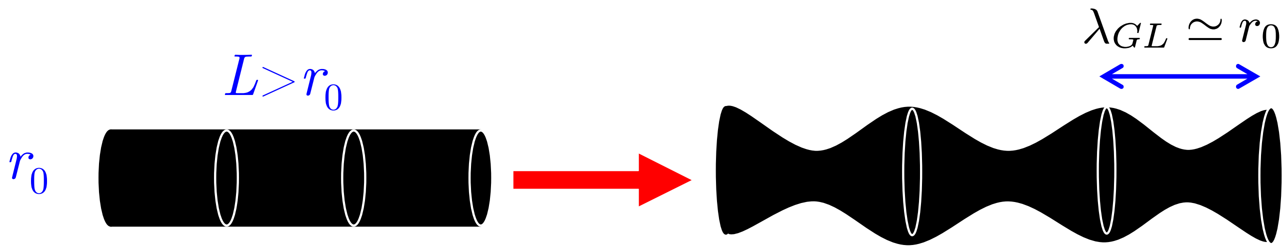
- **Two scales:** S^2-radius r_0, S^1-radius R
- **Limit:** $r_0 \ll R$ (ultra-spinning)

  → black string (boosted)



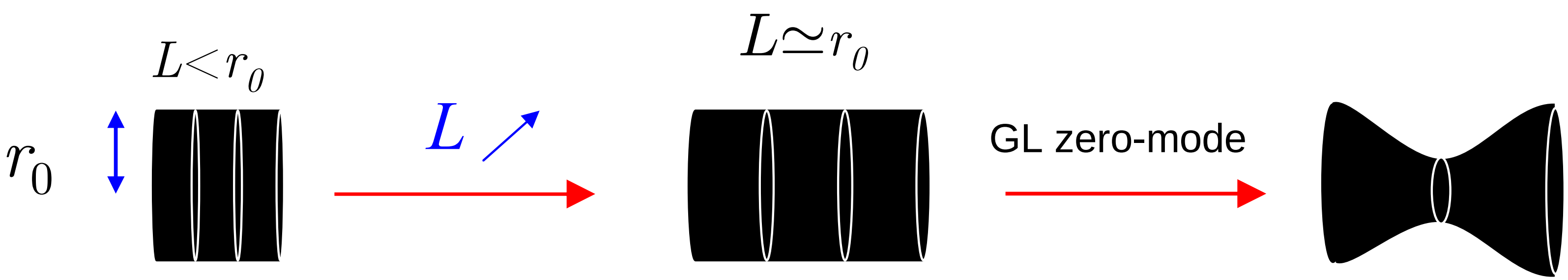
Also: Novel dynamics at short scale r_0

- Gregory-Laflamme instability of black brane when the two scales r_0, L begin to differ



- GL zero mode = static perturbation

  → **inhomogeneous** static black strings



- All this has led to realize that hi-D bhs have qualitatively new dynamics that was unsuspected from experience with 4D bhs

- 4D bhs only possess short-scale ($\sim r_0$) dynamics

- Hi-D bhs: need new tools to deal with long-distance ($\sim R \gg r_0$) dynamics

- Natural approach: integrate out short-distance physics, find long-distance effective theory

- Two different (but essentially equivalent) techniques for this:
  - Matched Asymptotic Expansion (Mae)
    - GR-minded approach, well-developed, produces explicit perturbative metric, but cumbersome
  - Classical Effective Field Theory (Cleft)

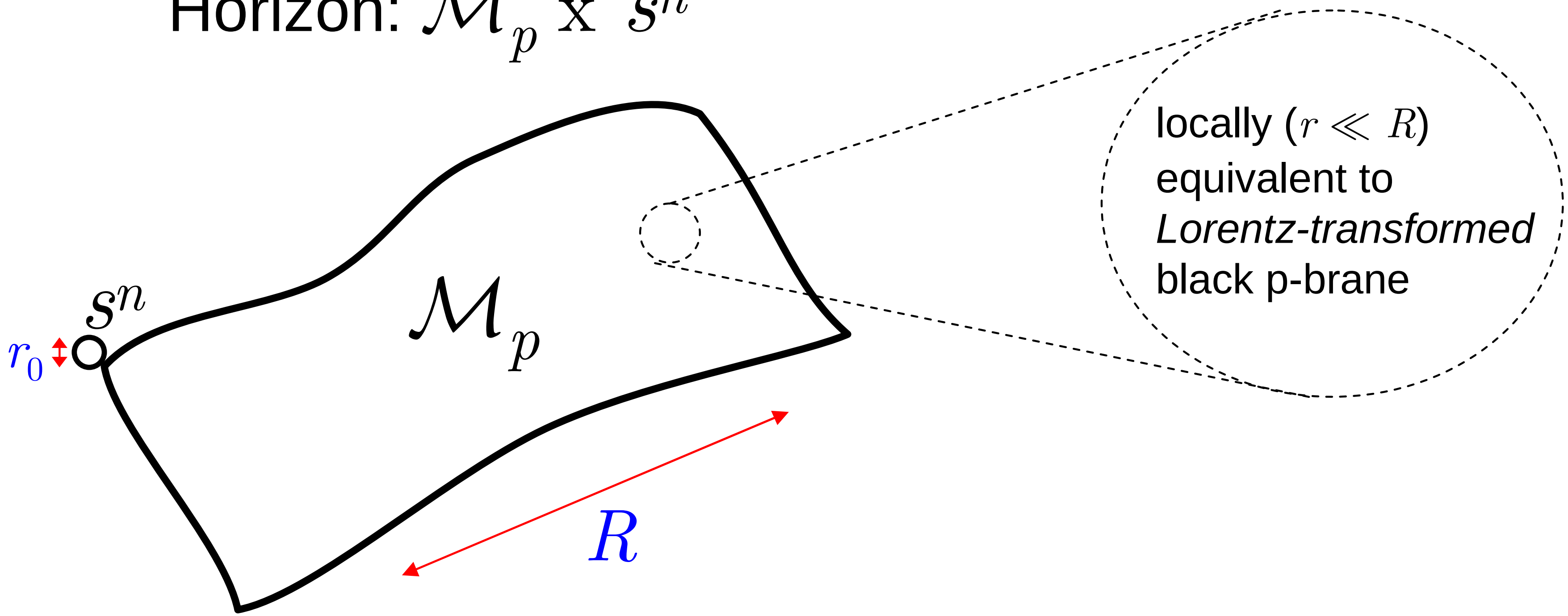
    Goldberger+Rothstein
    Kol

    - QFT-inspired (Feynman diagrams), efficient for calculating physical magnitudes, but not yet developed for extended objects
- To leading order, no difference
- Leading order = blackfold approach

Blackfolds: long-distance effective dynamics of hi-d black holes

- **Black** p-branes w/ worldvolume = curved submani**fold** of spacetime

Horizon: $\mathcal{M}_p$ x s^n



Long-distance theory

- Long-distance dynamics:

  find brane spacetime embedding $x^{\mu}(\sigma^{\alpha})$

- General *Classical Brane Dynamics*: Carter

Given a source of energy-momentum, in probe approx,

$$\nabla_{\mu} T^{\mu\nu} = 0 \quad \Rightarrow \quad T^{\mu\nu} K_{\mu\nu}{}^{\rho} = 0$$

extrinsic curvature

or, with external force: $F^{\rho} = T^{\mu\nu} K_{\mu\nu}{}^{\rho}$

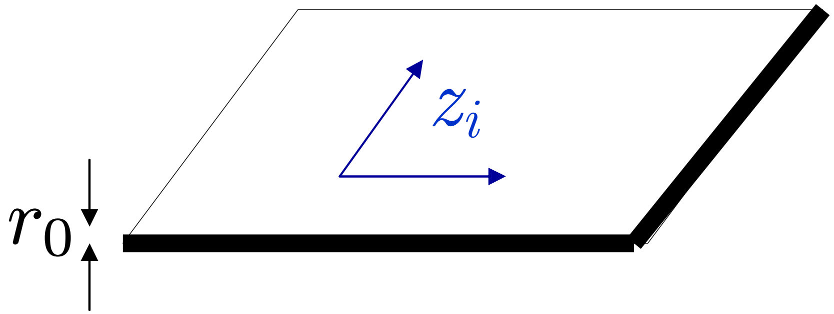
- Newton's force law: $F = ma$
- Nambu-Goto-Dirac eqns: $T_{\mu\nu} = T g_{\mu\nu}$ → $K^{\rho} = 0$: minimal surface

- But, what is $T_{\mu\nu}$ for a blackfold?
- Short-distance physics determines effective stress-energy tensor:
  - blackfold locally Lorentz-equivalent to black p-brane of thickness (s^n-size) r_0
  - In region $r_0 \ll r$ field linearizes $\Rightarrow$ approximate brane by equivalent distributional source $T_{\mu\nu}(\sigma^\alpha)$

- Black p-brane

$$ds^2 = -\left(1 - \frac{r_0^n}{r^n}\right) dt^2 + \sum_{i=1}^{p} dz_i^2 + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}^2$$

$$\begin{aligned} T_{tt} &= r_0^n (n+1) \\ T_{ii} &= -r_0^n \end{aligned}$$



Lorentz-transform:

$$(t, z_i) = \sigma^\mu \,, \quad \sigma^\mu \to \Lambda^\mu{}_\nu \sigma^\nu \,, \quad \Lambda^\mu{}_\nu \in O(1,p)$$

$$T_{\mu\nu} \to T_{\mu\nu} = r_0^n \left[(n+1) \Lambda_\mu{}^t \Lambda^t{}_\nu - \textstyle\sum_{i=1}^{p} \Lambda_\mu{}^i \Lambda^i{}_\nu \right]$$

(e.g., black string → boosted black string)

- Position-dependent thickness $r_0(\sigma^\alpha)$ and "boosts" $\Lambda^\mu{}_\nu(\sigma^\alpha) \Rightarrow T_{\mu\nu}(\sigma^\alpha)$

- Impose global *blackness* condition:
  - Uniform surf gravity κ & angular velocities Ω_i

  → eliminate thickness and boost: only geometry

- Solve $K_{\mu\nu}{}^\rho(\sigma^\alpha)\,T^{\mu\nu}(\sigma^\alpha;\kappa,\Omega_i)=0$

$\Rightarrow$ determine $x^\mu(\sigma^\alpha;\kappa,\Omega_i)$

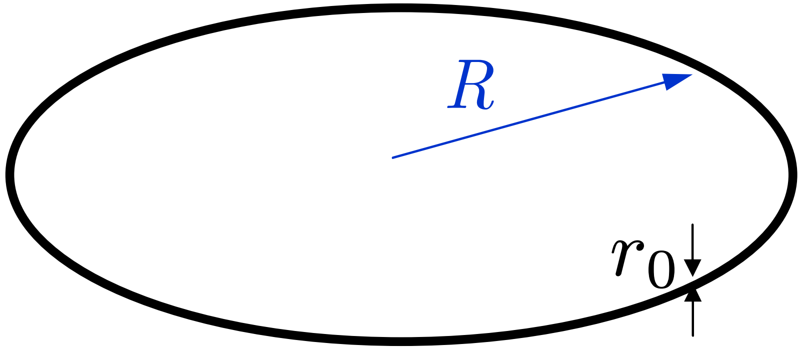
A Blackfold Bestiary

"**Bestiary**: a descriptive or anecdotal treatise on various real or mythical kinds of animals, esp. a medieval work with a moralizing tone."

- Simplest example: black rings in $D \geqslant 5$

$$K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0$$

$$\Rightarrow \quad \frac{T_{11}}{R} = 0$$



$$T_{11} = r_0^{D-4} \left[(D-4) \sinh^2 \sigma - 1 \right]$$

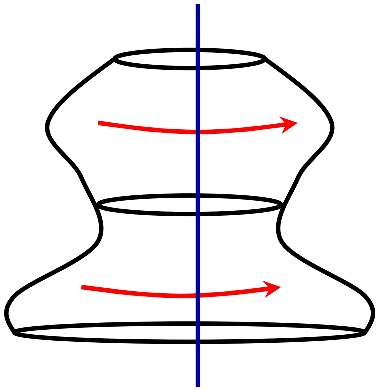
Tune boost to equilibrium $\Rightarrow \sinh^2 \sigma = \dfrac{1}{D-4}$

(in D=5 reproduces value from exact soln)

Horizon S^1 x s^{D-3}

"small" transverse sphere $\sim r_0$

- Axisymmetric blackfolds



(possibly rotations along all axes)

- Simple analytic solutions:
  - even p: ultraspinning MP bh, with $p/2$ ultraspins
  - odd p : round S^p, with all $(p+1)/2$ rotations equal

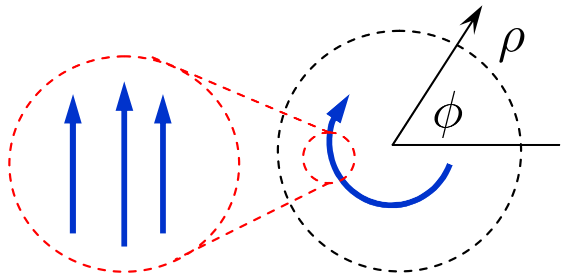
- I'll illustrate two simple cases of each

• Ultra-spinning 6D MP bh as blackfold

- Embed black two-brane in 3-space

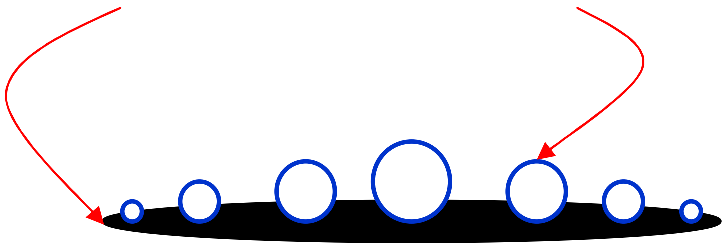
  $ds^2 = dz^2 + d\rho^2 + \rho^2 d\phi^2$, as $z = \text{const}$

  to obtain planar blackfold $\mathcal{P}_2 \times s^2$



locally equiv to ***boosted black 2-brane***

- Find size $r_0(\rho)$ of $s^{\,2}$ & boost $\alpha(\rho)$ of locally-equiv 2-brane:
  - Soln: $\alpha(\rho) \to \infty,\ \ r_0(\rho) \to 0$ at $\rho_{max} = 1/\Omega$
- Disk D_2 fibered by $s^{\,2}$: topology S^4: like 6D MP bh!

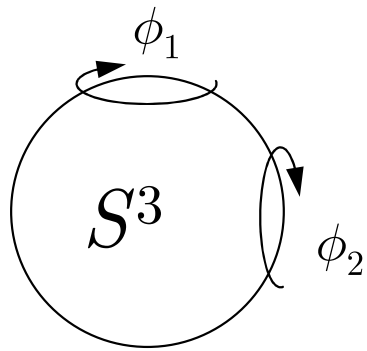


- *All physical magnitudes* match those of the ultraspinning 6D MP bh

- S^3 x s^{n+1} black hole as blackfold

$(n \geqslant 1)$

- Embed three-brane in a space containing



$$ds^2 = dr^2 + r^2 d\Omega^2{}_{(3)}$$

as $r = R$

- Solution exists if $|\Omega_1| = |\Omega_2| = (3/(3+n))^{1/2} R^{-1}$

  size of s^{n+1} $r_0 = \text{const}$

- If $|\Omega_1| > |\Omega_2|$ then numerical solution for

  $r = R(\theta)$: non-round S^3

- Other beasts
- Sphere products:

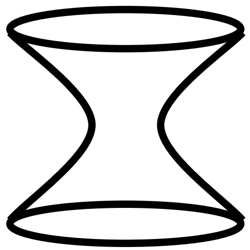
$$\prod_{p_i \in \mathrm{odd}} S^{p_i} \times s^{n+1} \qquad D = \sum_i p_i + n + 3$$

"small" transverse sphere $\sim r_0$

- Static minimal blackfolds:

If no boost (no local Lorentz transf)

$T_{ij} = -P g_{ij}$ (spatial) $\Rightarrow$ $K^\rho = 0$: minimal submanifold

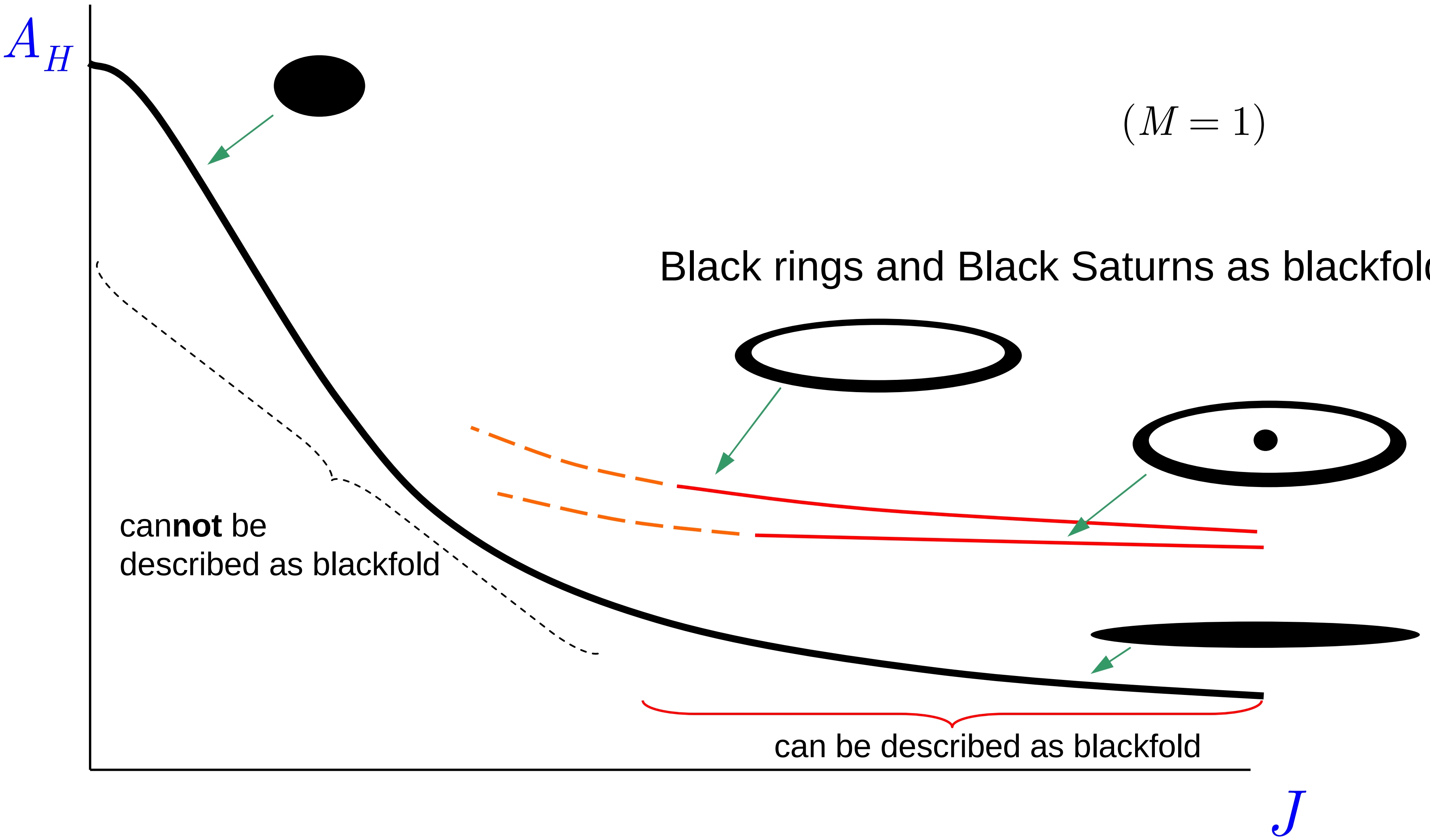


e.g.: hyperboloid

→ non-compact static blackfold

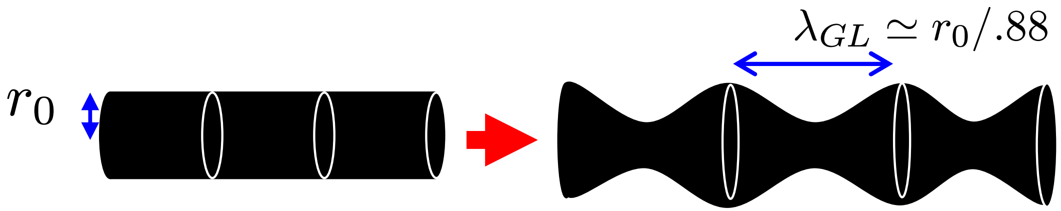
Blackfolds in $D \geqslant 6$ phase diagram

(w/ one spin)

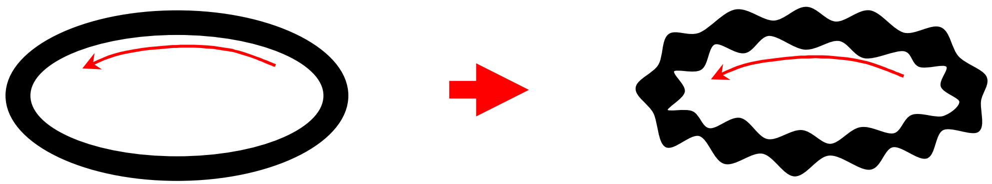


Instabilities

- Stability of blackfolds for long wavelength ($\lambda \gg r_0$) perturbations can be analyzed within blackfold approximation
- But black branes have short-wavelength G-L instabilities



- Blackfolds can be unstable – on quick time scales, $\Gamma \sim 1/r_0$



Other blackfold applications

- Curved strings & branes in gravitational potentials:
  - black rings in AdS: ∃ w/ arbitrarily large radius
  - black rings in dS: ∃ static ones
  - black rings in Taub-NUT (D0-D6 brane interactions)
  - black saturns, charged and susy blackfolds
  - Probe D-branes at finite temperature
- Dynamical evolution
  - Time-dependent evolution
  - Coupling to gravitational radiation (quadrupole formula)

Limitations

- Stationarity not guaranteed at higher orders (i.e. w/ backreaction)
- Misses 'threshold' regime of connections and mergers of phases, where scales meet $r_0 \sim R$

→resort to extrapolations, numerical analysis, and other information

A conjecture about uniqueness and stability of hi-d black holes

- Non-uniqueness (=more than one solution for given asymptotic conserved charges) and instabilities appear only as the two horizon scales begin to differ, ie when

$$J_i / M \gtrsim (GM)^{1/(D-3)}$$

(for at least one J_i)

- For smaller spins $\exists$ only MP bhs (as far as we know), with properties similar to Kerr, and there is no suggestion of their instability
- It really looks like it's the appearance of more than one scale that brings in **all** that differs from 4D

A conjecture about uniqueness and stability of hi-d black holes

- **Conjecture:**

  MP black holes are unique iff $\forall i = 1,\ldots,\lfloor (D-1)/2 \rfloor$

  $$|J_i| < \alpha_D M (GM)^{1/(D-3)},$$

  and dynamically stable iff $|J_i| < \beta_D M (GM)^{1/(D-3)}$

  w/ $\alpha_D \leqslant \beta_D, \ \alpha_D, \beta_D = O(1)$ (to be determined, presumably numerically)

- Applies to stationary black hole solutions w/ connected non-degenerate horizons, and possibly to multi-black hole solutions in thermal equilibrium

Moral: Blackfolds as organizing framework for hi-d black holes

- Long-distance dynamics of hi-d black holes efficiently captured by blackfold approach (extends & includes DBI approach to D-branes) – black branes are very elastic!

- Black hole dynamics splits into three regimes:
  - $0 \leqslant |J_i| < M(GM)^{1/(D-3)}$: single scale, Kerr-like – not much new expected
  - $|J_i| \sim M(GM)^{1/(D-3)}$: threshold of separating scales: instabilities and phase mergers – this is the most difficult to analyze!
  - $|J_i| \gg M(GM)^{1/(D-3)}$: separated scales: blackfold dynamics – we have the tools to study it

Moral: Blackfolds as organizing framework for hi-d black holes

- Change focus:
  - search for all D$\geqslant$6 black hole solutions in closed analytic form is futile (some may still show up: $p=D-4$)
  - classification becomes increasingly harder at higher D, but also less interesting

➔ Investigate novel dynamical possibilities