Fate of False Vacuum Revisited

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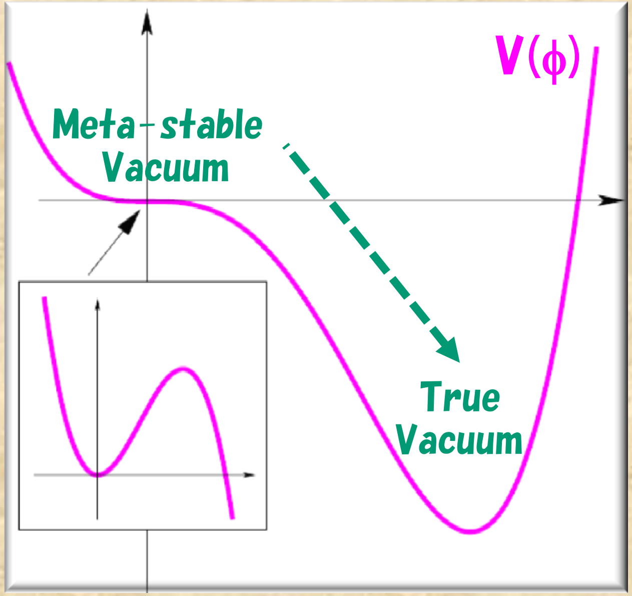
When the order parameter has interactions with light particles, the rate of the phase transition may be altered significantly due to the existence of non-local operators.

1. When we are living in a meta-stable vacuum ···



The most important physical quantity is

Rate of the phase transition
from false to true vacuum Γ/V!

The false vacuum should satisfies

$$(\Gamma/V)\, t_0^4 << 1$$

where $t_0 \sim 10^{10}$ yrs. (Age of Univ.).

The aim of the talk

When the order parameter couples to light particles, how the rate Γ/V is altered due to the interactions?

1. Corrections to the potential
2. Disturbance of the transition because of the lost of the coherence.

We will discuss the 2nd effect (Environment effect)!

2. Tunneling rate without Environment

~ The Bounce Method ~

The Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - V(\phi)$$

The Euclidean action

$$\mathcal{S}_{\rm E}[\phi] = \int d^4 x_{\rm E} \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\left(\nabla\phi\right)^2 + V(\phi)\right]$$

Partition Function Z & The rate Γ/V

$$Z = \int \mathcal{D}\phi \; e^{-\mathcal{S}_{\rm E}[\phi]}$$

with $\phi(\pm\infty, \vec{x}) = 0$

$$\Gamma/V \propto \mathrm{Im}(\log Z)$$

2. Tunneling rate without Environment

～ The Bounce Method ～

How to calculate Z & Γ/V?

Semi-Classical approximation

1. Find the classical path ϕ_B (Bounce solution)
2. Taking fluctuations around ϕ_B into account.

2. Tunneling rate without Environment

～ The Bounce Method ～

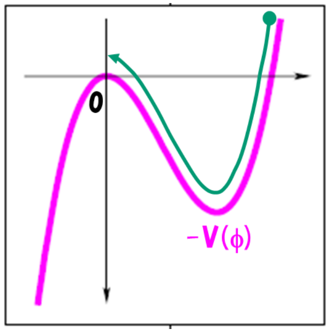
How to calculate Z & Γ/V?

Semi-Classical approximation

1. Find the classical path ϕ_B (Bounce solution)

2. Taking fluctuations around ϕ_B into account.

$$-\left(\frac{d^2}{d\tau^2}+\nabla^2\right)\phi_{\rm B}+V'(\phi_{\rm B})=0 \quad \textbf{with} \quad \phi(\pm\infty,\vec{x})=0$$



O(4) symmetry

$$\phi_B=\phi_B(r)$$
$$r=\sqrt{\tau^2+|\vec{x}|^2}$$

$$-\left(\frac{d^2}{dr^2}+\frac{3}{r}\frac{d}{dr}\right)\phi_{\rm B}+V'(\phi_{\rm B})=0 \quad \text{with} \quad \lim_{r\to\infty}\phi_{\rm B}(r)=0, \quad \dot{\phi}_{\rm B}(0)=0$$

2. Tunneling rate without Environment

~ The Bounce Method ~

How to calculate Z & Γ/V?

Semi-Classical approximation

1. Find the classical path ϕ_B (Bounce solution)
2. Taking fluctuations around ϕ_B into account.

We will find 4 zero modes & 1 negative mode.

Zero modes: Corresponding to Location of the bounce solution → VT

Negative mode: Corresponding to Instability due to Tunneling → Im. Part

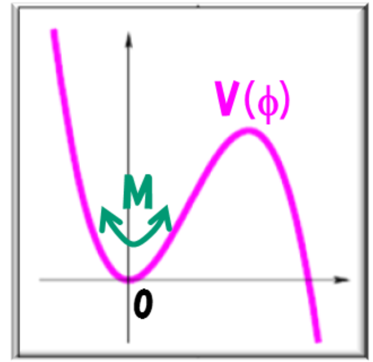
$$\Gamma/V \simeq K \exp\left(-S_{\mathrm{E}}[\phi_{\mathrm{B}}]\right)$$

$$K = \frac{S_{\mathrm{E}}[\phi_{\mathrm{B}}]^2}{4\pi^2}\sqrt{\left|\frac{\det[-\partial_\mu\partial_\mu + V''(0)]}{\det'[-\partial_\mu\partial_\mu + V''(\phi_{\mathrm{B}})]}\right|}$$

3. Possible Effect from Environment

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_{\rm Env} + \mathcal{L}_c$$

Counter terms
(On-Shell cond.)



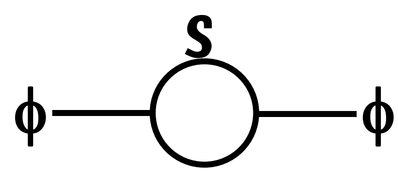
(Case S) $$\mathcal{L}^{(S)}_{\rm env} = \sum_{i=1}^{N}\left[|\partial S_i|^2 - m_S^2|S_i|^2 - y_S M\phi|S_i|^2\right]$$

$$S^{(\rm eff)}[\phi] \equiv -i\ln\left(\int \mathcal{D}S\mathcal{D}S^*\; e^{iS[\phi,S,S^*]}\right)$$

$$\mathcal{S}^{(\rm eff)}[\phi] = \int d^4x\left[\frac{1}{2}(\partial\phi)^2 - V(\phi)\right] + \frac{N}{2}\int d^4x d^4y\;\phi(x)F(x-y)\phi(y) + \mathcal{O}(N\overset{y_s^3}{\cancel{g^3}}\phi^3)$$

FT

$$\tilde{F}(p^2) = iy_S^2M^2\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m_S^2}\frac{i}{(p-k)^2-m_S^2} =$$



It has a imaginary (absorptive) part when M > 2m$_S$, which will disturb the phase transition (Tunneling)!

4. Tunneling with Environment

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_{\rm Env} + \mathcal{L}_c}$$

(Case S) $$\mathcal{L}_{\rm env}^{(S)} = \sum_{i=1}^{N}\left[|\partial S_i|^2 - m_S^2|S_i|^2 - y_S M\phi|S_i|^2\right]$$

(Case F) $$\mathcal{L}_{\rm env}^{(F)} = \sum_{i=1}^{N}\left[\bar{F}_i\left(i\partial\!\!\!/ - m_F\right)F_i - y_F\phi\bar{F}_iF_i\right]$$

$$Z = \int \mathcal{D}\phi\,\mathcal{D}S\,\mathcal{D}S^*\,e^{-\mathcal{S}_{\rm E}[\phi,S,S^*]} = \int \mathcal{D}\phi\,e^{-\mathcal{S}_{\rm E}^{(\rm eff)}[\phi]}$$

Integrating S & S* fields out!

$$\mathcal{S}_{\rm E}^{(\rm eff)}[\phi] = \int d^4x_{\rm E}\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)\right] + \int d^4x_{\rm E}\left[\frac{\delta Z_\phi}{2}\dot{\phi}^2 + \frac{\delta Z_\phi}{2}(\nabla\phi)^2 + \frac{\delta M^2}{2}\phi^2\right]$$
$$+\frac{1}{2}\int d^4x_{\rm E}\,d^4y_{\rm E}\,\phi(x_{\rm E})f_0(x_{\rm E}-y_{\rm E})\phi(y_{\rm E}) + \mathcal{O}(N\overset{y^3}{\cancel{g^3}}\phi^3)$$

4. Tunneling with Environment

$$\mathcal{S}_{\rm E}^{(\rm eff)}[\phi]=\int d^4x_{\rm E}\left[\frac{1}{2}\dot{\phi}^2+\frac{1}{2}\left(\nabla\phi\right)^2+V(\phi)\right]+\int d^4x_{\rm E}\left[\frac{\delta Z_\phi}{2}\dot{\phi}^2+\frac{\delta Z_\phi}{2}\left(\nabla\phi\right)^2+\frac{\delta M^2}{2}\phi^2\right]$$

$$+\frac{1}{2}\int d^4x_{\rm E}\,d^4y_{\rm E}\,\phi(x_{\rm E})f_0(x_{\rm E}-y_{\rm E})\phi(y_{\rm E})+\mathcal{O}(N\overset{y^3}{\cancel{g^3}}\phi^3)$$

$$f_0(x_{\rm E})=\int\frac{d^4p}{(2\pi)^4}\tilde{f}_0(p^2)e^{-ip\cdot x_{\rm E}}=\underbrace{\int\frac{d^4p}{(2\pi)^4}\tilde{f}_0(0)e^{-ip\cdot x_{\rm E}}}_{=\;\tilde{f}_0(0)\delta^{(4)}(x_{\rm E})}+\int\frac{d^4p}{(2\pi)^4}\left\{\tilde{f}_0(p^2)-\tilde{f}_0(0)\right\}e^{-ip\cdot x_{\rm E}}$$

$$\mathcal{S}_{\rm E}^{(\rm eff)}[\phi]=\int d^4x_{\rm E}\left[\frac{1}{2}\dot{\phi}^2+\frac{1}{2}\left(\nabla\phi\right)^2+V(\phi)\right]+\frac{1}{2}\int d^4x_{\rm E}\,d^4y_{\rm E}\,\phi(x_{\rm E})f(x_{\rm E}-y_{\rm E})\phi(y_{\rm E})$$

$$f(x_{\rm E})=\int\frac{d^4p}{(2\pi)^4}\left\{\tilde{f}_0(p^2)-\tilde{f}_0(0)+\delta Z_\phi p^2\right\}e^{-ip\cdot x_{\rm E}}\equiv\int\frac{d^4p}{(2\pi)^4}\tilde{f}(p^2)e^{-ip\cdot x_{\rm E}}$$

If ϕ is constant, the term involving $f(x_{\rm E})$ vanishes!

4. Tunneling with Environment

$$\mathcal{S}_{\rm E}^{(\rm eff)}[\phi] = \int d^4x_{\rm E}\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)\right] + \frac{1}{2}\int d^4x_{\rm E}\, d^4y_{\rm E}\,\phi(x_{\rm E}) f(x_{\rm E}-y_{\rm E})\phi(y_{\rm E})$$

$$f(x_{\rm E}) = \int \frac{d^4p}{(2\pi^4)}\tilde{f}(p^2)e^{-ip\cdot x_{\rm E}} = \int \frac{d^4p}{(2\pi^4)}\left[\int_{2m}^{\infty} d\omega \frac{p^2\ r(\omega)}{\omega^2\ (\omega^2+p^2)} + p^2\ \delta Z_\phi\right]e^{-ip\cdot x_{\rm E}}$$

(Case S) $r_S(\omega) = N\dfrac{y_S^2}{8\pi^2}M^2\left(\omega^2-4m_S^2\right)^{1/2}$

(Case F) $r_F(\omega) = N\dfrac{y_F^2}{4\pi^2}\left(\omega^2-4m_F^2\right)^{3/2}$

The EOM for the O(4) symmetric bounce solution is

$$\boxed{-\left(\frac{d^2}{dr^2}+\frac{3}{r}\frac{d}{dr}\right)\phi_{\rm B} + V'(\phi_{\rm B}) + \int dr' \int dp\, \frac{pr'^2 J_1(pr)J_1(pr')}{r}\tilde{f}(p^2)\phi_{\rm B}(r') = 0}$$

$$\boxed{\Gamma/V \propto \exp\left[\overbrace{\quad}^{-1}\mathcal{S}_{\rm E}^{(\rm eff)}[\phi_B]\right]}$$

5. Environment Effect on Tunneling

$$-\left(\frac{d^2}{dr^2}+\frac{3}{r}\frac{d}{dr}\right)\phi_{\rm B}+V'(\phi_{\rm B})+\int dr'\int dp\,\frac{pr'^2J_1(pr)J_1(pr')}{r}\tilde{f}(p^2)\phi_{\rm B}(r')=0$$

$$\phi_{\rm B}(r)=\int_0^\infty dp\,\frac{p}{r}J_1(pr)\tilde{\phi}_{\rm B}(p)\longleftrightarrow\tilde{\phi}_{\rm B}(p)=\int_0^\infty dr\,r^2J_1(pr)\phi_{\rm B}(r)$$

This transformation satisfies the boundary conditions of $\phi_B(r)$.
We also assume the potential is approximated by the cubic form,
$V(\phi) = (1/2)M^2\phi^2(1 - \phi/\phi_0)$

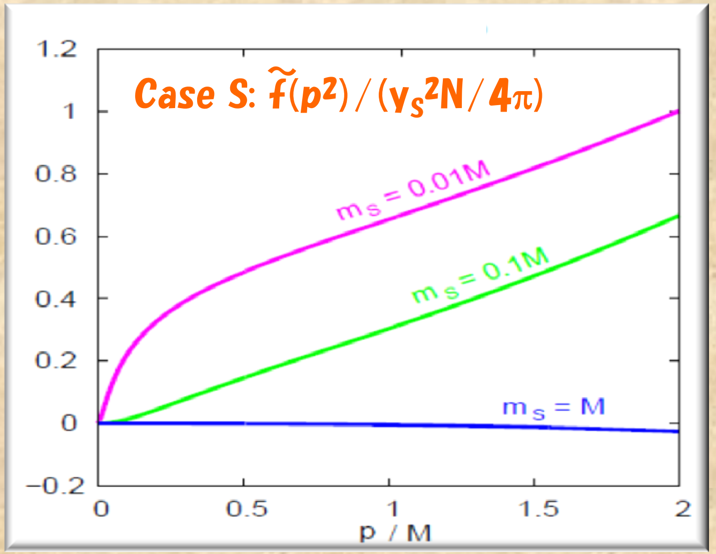
$$\left[p^2+M^2+\tilde{f}(p^2)\right]\tilde{\phi}_{\rm B}(p)=\frac{3M^2}{\pi\phi_0}\int_0^\infty dp'\int_{|p-p'|}^{p+p'}dp''\,\tilde{\phi}_{\rm B}(p')\tilde{\phi}_{\rm B}(p'')\frac{\Delta(p,p',p'')}{p}$$

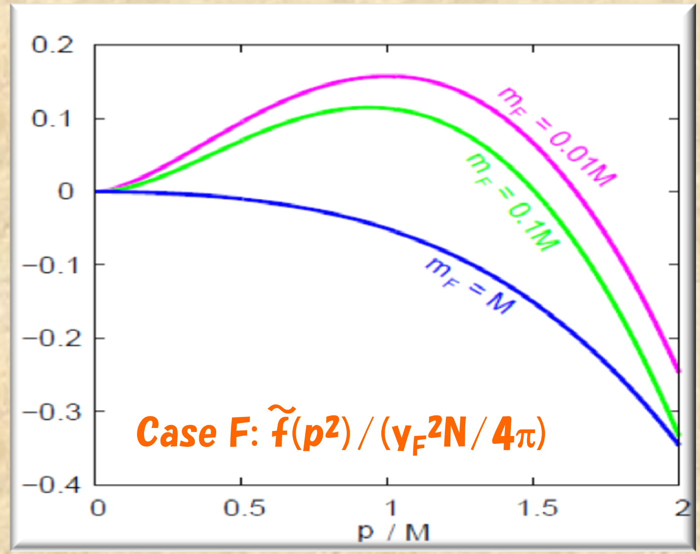
$$S_{\rm E}^{\rm (eff)}[\phi_{\rm B}]=\frac{\pi M^2}{\phi_0}\int dp\,dp'\,dp''\,\tilde{\phi}_{\rm B}(p)\tilde{\phi}_{\rm B}(p')\tilde{\phi}_{\rm B}(p'')\Delta(p,p',p'')$$

$\Delta(p, p', p'')$ is the area of the triangle with the sides p, p' and p''.

1. The environment effect contributes as a p^2-dependent mass correction.
2. When $\tilde{\phi}_B$ is exponentially decreasing, when p^2 is large enough.
3. When $\tilde{\phi}_B$ is larger, the action is larger, which leads to the suppression.

5. Environment Effect on Tunneling





(Case S)

1. When m_S is smaller, the environment effect becomes larger as expected.
2. When $m_S > M/2$, no effects can be seen, which is consistent with the picture that Tunneling is suppressed due to the loss of the coherence.

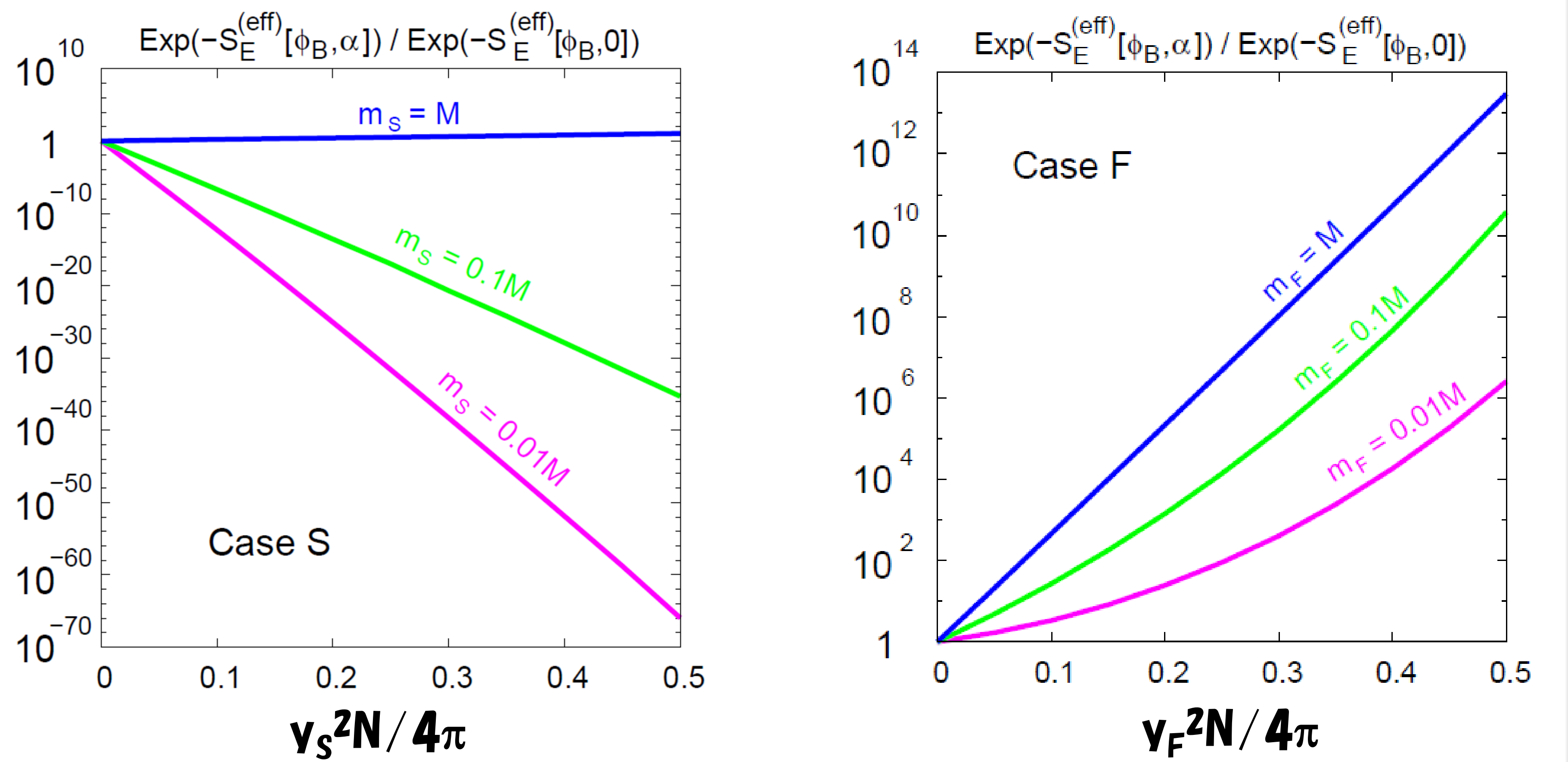
(Case F)

1. We can find the positive contribution m_S is smaller $M/2$, however it is not so efficient compared to the Case S because of P-wave decays.
2. We can also find the negative contribution, which is coming from the scale dependence of the two point function (anomalous dimension).

$$\tilde{f}(p^2) = (\alpha_F)/(2\pi)p^2[1 - \ln(p^2/M^2)] \Rightarrow p^2 - (\alpha_F)/(2\pi)p^2 \ln(p^2/M^2) \simeq M^2(p/M)^{2-\alpha_F/\pi}$$

5. Environment Effect on Tunneling

The ratio of the tunneling rate between α = finite and α = 0 cases.



The parameter in the potential ϕ_0/M is fixed so that

$$\mathrm{S}_E^{(\mathrm{eff})}[\phi_B(x_{\mathrm{E}}), \alpha = 0] = 400$$

Summary

- When the order parameter describing the 1st order phase transition has interactions with light particles, the tunneling rate will be altered due to the effect of environment which is (usually) not included in the effective potential.
- The origin of the environment effects comes from the loss of the coherence due to the interactions, which suppress the tunneling rate significantly.
- If we consider the scenario on a meta-stable vacuum, it is important to take the effect into account.
- What' s happen beyond the semi-classical approximation especially when the coupling is large?
- It may also be important to the first order phase transition in the early universe.