Seminar@IPMU, 9th February 2009

Self-similar growth of black holes in the Friedmann universe

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based on

HM, Harada & Carr, arXiv:0901.1153, PRD accepted
Kyo, Harada & HM, PRD77, 124036 (2008)
HM, Harada & Carr, PRD77, 024023 (2008)
Harada, HM & Carr, PRD77, 024022 (2008)
Harada, HM & Carr, PRD74, 024024 (2006)

Contents

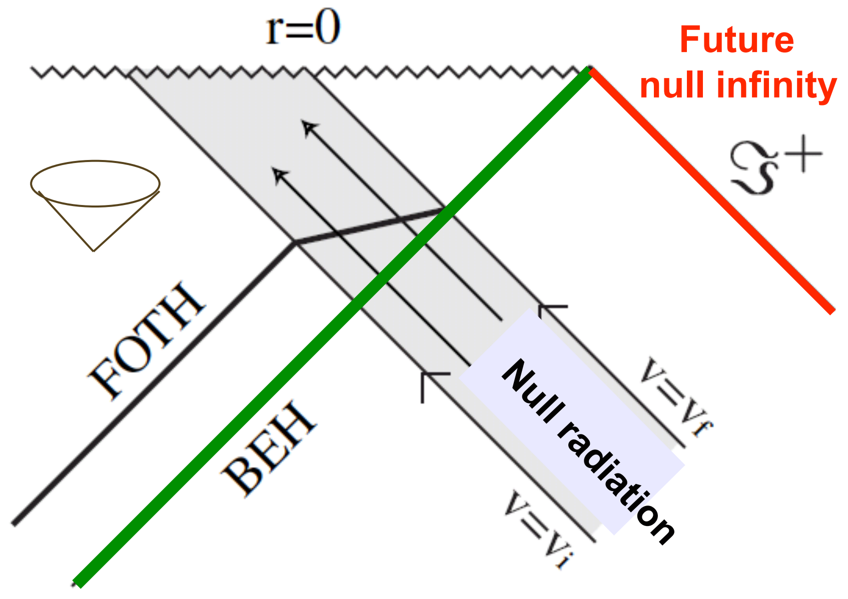
- (I) Introduction
- (II) Definitions and basic equations
- (III) Numerical results
- (IV) Summary and discussions

(I)

Introduction

Black holes

- Intuitively
  - ``Spacetime regions from which nothing can escape''
- A definition: a black-hole event horizon
  - Future boundary of the past causal set of the future null infinity
  - A global concept
    - Applicable both for stationary (static) and dynamical spacetimes



Stationary black holes

- Stationarity is defined by a timelike Killing vector field
- A Killing horizon
  - A null hypersurface on which a Killing vector becomes null
- Asymptotically flat and stationary black holes
  - A nice model of the final state of gravitational collapse
  - BH event horizon = Killing horizon
  - Strong results
    - BH uniqueness theorem
      - Asymptotically flat electromagnetic BH solution is the Kerr-Newman BH
    - BH thermodynamics
      - A similarity to the thermodynamical system

Cosmological black holes

- Black hole << Cosmological horizon
  - Asymptotically flat BH is a good model
- Black hole ≒ Cosmological horizon
  - Cosmic expansion is important
  - Cosmological black holes
    - Asymptotically expanding universe
    - Dynamical and inhomogeneous with matter (non-vacuum)
    - Properties have been not fully understood yet
      - Subject in this talk: the growth of cosmological black holes

How fast can a BH grow?

- Possibility of the self-similar growth
  - Growth of a primordial black holes as the same rate of the universe (Zel'dovich-Novikov `67)
  - Still under discussions for 40 years
- Primordial black holes (PBHs) (Hawking '71)
  - Formed in the very early universe
    - Formed with the particle horizon scale (Carr '75)
    - Large range of the initial mass
  - Candidate for dark matter and the source of Hawking radiation
- Self-similar growth of PBHs
  - An attractive scenario to explain the origin of giant black holes in galactic nuclei

Self-similar growth of PBHs

- Small BHs can be formed only in the early universe
  - $R_h=2GM/c^2 \sim 3(M/M_s)[km] \Rightarrow \rho_s \sim 10^{18}(M/M_s)^{-2}[g/cm^3]$
  - $\rho_{FRW}=1/(Gt)^2 \sim 10^6(t/s)^{-2}[g/cm^3]$
- PBHs form with the particle horizon scale (Carr `75)
  - $M_{PBH}=c^3t/G$
    - 10^{-5}[g] at 10^{-43}[s] (Planck)
    - 10^{15}[g] at 10^{-23}[s] (now evaporating)
    - $1M_s$ at 10^{-5} [s] (solar mass)
- Zel'dovich-Novikov's argument (`67)
  - PBHs grow in a self-similar manner in the radiation era
  - Final mass $\sim 10^{15}M_s$ >> galactic nuclei ($\sim 10^6$-10^9M_s)
  - No such giant BHs by observation=>No PBH formed

Zel'dovich-Novikov's argument (1)

- Bondi accretion with the density of the FRW universe

$$\frac{dM}{dt} = 4\pi r_{\mathrm{A}}^2 \left(\frac{\rho_{\mathrm{FRW}}}{c^2} \right) c_s \qquad r_{\mathrm{A}} := \frac{2GM}{c_s^2}$$

  - Sound velocity: $c_s = c/\sqrt{3}$
  - Energy density of the Friedmann universe: $\rho_{\mathrm{FRW}} = \frac{3c^2}{32\pi G t^2}$

- (Semi-Newtonian) basic equation for the accretion

$$\frac{dM}{dt} = \frac{KGM^2}{t^2}$$

where

$$K := \frac{9\sqrt{3}G}{2c^3}$$

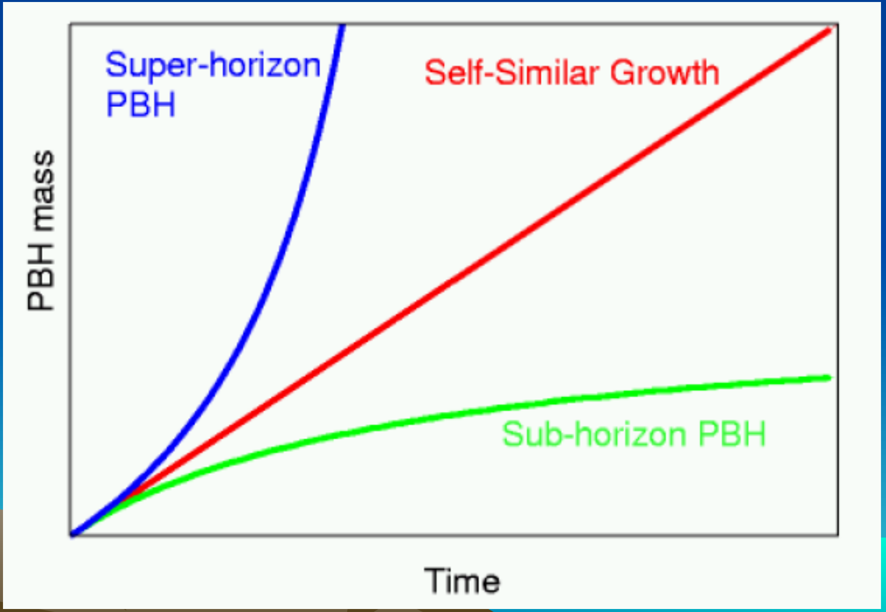
Zel'dovich-Novikov's argument (2)

- Accretion equation: $\frac{dM}{dt} = \frac{KGM^2}{t^2}$ where $K := \frac{9\sqrt{3}G}{2c^3}$
- Solution: $M(t) = \frac{M_0}{1 - \frac{KM_0}{t_0}\left(1 - \frac{t_0}{t}\right)}$ where $M(t_0) = M_0$
- Self-similar growth of PBHs with the horizon mass

$$M_{\mathrm{PBH}}(t) = K^{-1}t \simeq M_{\mathrm{PH}}(t)$$

for

$$M_0 = K^{-1}t_0 = \frac{2c^3t_0}{9\sqrt{3}G} \simeq M_{\mathrm{PH}}(t_0)$$



Not conclusive yet

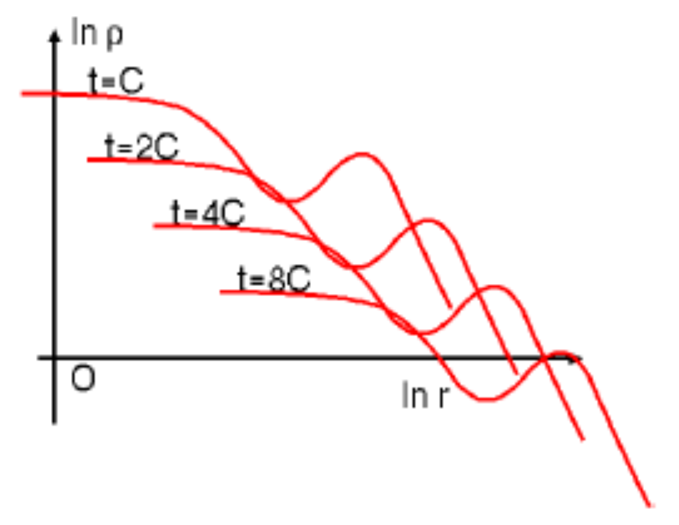
- Effect of the cosmic expansion is neglected in the Z-N argument
  - Full GR analysis is needed
  - Problem ``Is the self-similar black-hole solution which is asymptotically Friedmann universe?''
- 2 types of asymptotically Friedmann spacetime without a massive thin shell
  - 1) Smooth asymptotically Friedmann universe
  - 2) Attachment to the exterior exact Friedmann universe at some particular hypersurface
    - at a sonic horizon or a similarity horizon

Self-similarity in general relativity

- Defined by a homothetic Killing vector

$$\mathcal{L}_{\xi} g_{\mu\nu} = 2 g_{\mu\nu},$$

- Profiles of physical quantities are self-similar during the motion
  - Spherical system : $P(at,ar) \propto P(t,r)$
- Mathematically simple
  - Spherical systems: PDEs reduce to ODEs

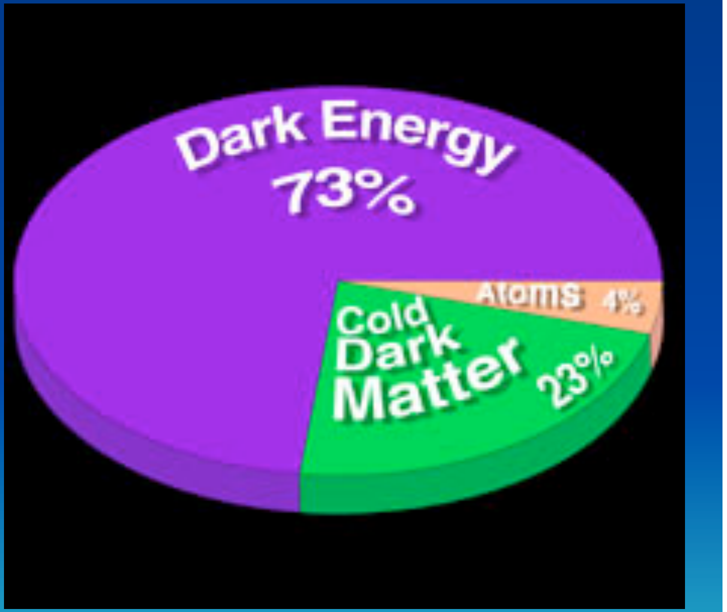


Past results

- Main issue
  - ``Are there asymptotically FRW self-similar BH solutions in general relativity?’’
- Past results in spherically symmetric spacetimes
  - A perfect fluid with non-negative pressure (Carr-Hawking `74, Bicknell-Henriksen `86)
    - Non-existence shown
    - Note: Negative pressure case is still open
  - A scalar field (Harada-HM-Carr `06)
    - Non-existence shown for the case of exact Friedmann exterior
    - Note: asymptotically Friedmann case is still open

The dark energy

- Recent observations suggest the accelerating expansion of the universe
- Strong energy condition is violated for matter fields
  - A quintessential scalar field
  - A perfect-fluid model with p=(γ-1)μ
    - Dark energy (0<γ<2/3)
    - Cosmological constant (γ=0)



Self-similar growth with dark energy?

- ``Self-similar growth of a BH is possible with a quintessential scalar field'' (Bean-Magueijo '02)
  - Essentially the same argument as Zel'dovich-Novikov
  - Full GR analysis is needed
- Our subject
  - We find the self-similar asymptotically Friedmann black-hole solutions with a perfect fluid model of the dark energy (quintessence)

(II)

Definitions and basic equations

Spherically symmetric self-similar perfect-fluid system

- Matter: $T_{\mu\nu} = pg_{\mu\nu} + (\mu + p)u_\mu u_\nu,$
- Self-similar metric: z=r/t（r is a comoving radius）

R:=rS: areal radius

$$ds^2 = -e^{2\Phi(z)}dt^2 + e^{2\Psi(z)}dr^2 + r^2S^2(z)d\Omega^2.$$

- Physical quantities

$$8\pi G\mu = \frac{W(z)}{r^2}, \quad 8\pi Gp = \frac{P(z)}{r^2}, \quad 2Gm = rM(z).$$

- Equation of state: $p = (\gamma - 1)\rho,$

Misner-Sharp mass

Basic equations

- Energy-momentum conservation equations give

$$e^{\Phi} = c_0 z^{2(\gamma-1)/\gamma} W^{-(\gamma-1)/\gamma}, \quad e^{\Psi} = c_1 S^{-2} W^{-1/\gamma},$$

  - c_0 and c_1 are constants
- One 1st order and one 2nd order ODEs of z:

$$\frac{V^2-(\gamma-1)}{\gamma}\frac{W'}{W} = \frac{\gamma c_1^2 W^{(\gamma-2)/\gamma}}{2S^4} - \frac{2(\gamma-1)}{\gamma} - 2V^2\frac{S'}{S},$$
$$S'' = S'\left(\frac{\gamma-2}{\gamma} - \frac{(\gamma-1)W'}{\gamma W}\right) - (S+S')\left(\frac{2S'}{S} + \frac{W'}{\gamma W}\right)$$

$' \equiv d/d\ln|z|$

$V = ze^{\Psi-\Phi}$

- Constraint equation: $M = WS^2(\gamma S' + S).$

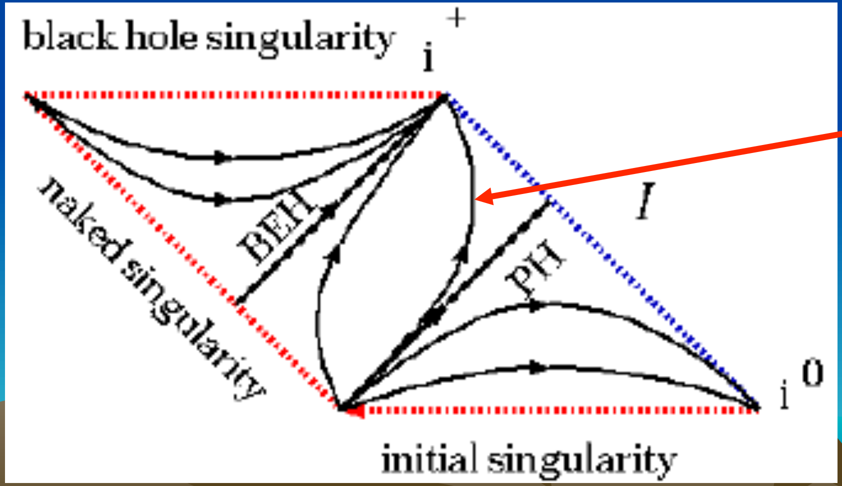
Similarity horizon

- A similarity horizon (=a homothetic Killing horizon)
  - A null hypersurface on which a homothetic Killing vector becomes null
- In our system, it is given by $e^{2\Phi}(1-V^2)=0$
  - A regular null hypersurface with $V^2=1$
  - Can be an event horizon or a particle horizon

$V = ze^{\Psi-\Phi}$

Velocity function

A typical
CBH spacetime



An orbit of
constant z

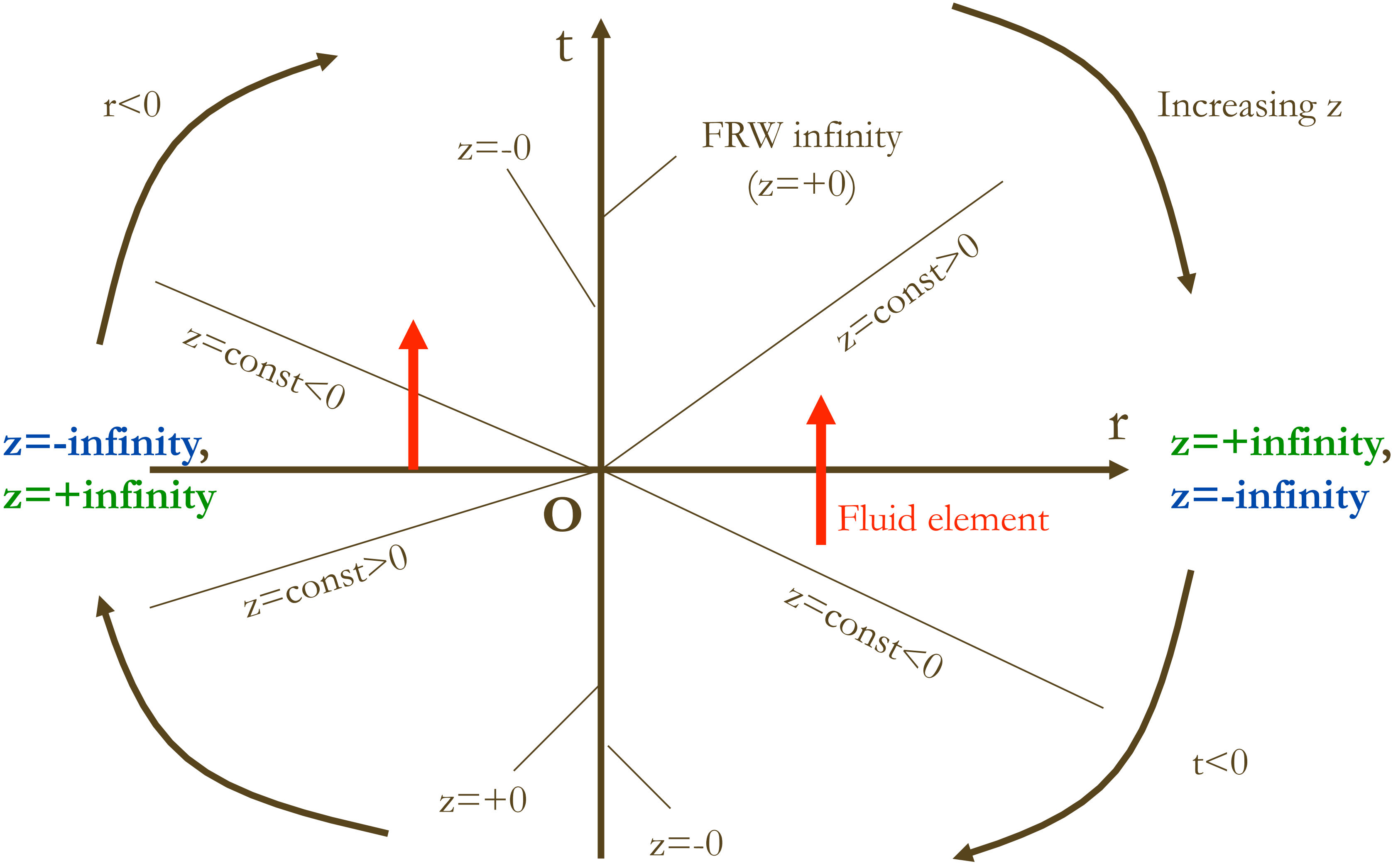
Asymptotically FRW solutions

- Asymptotically FRW at spatial infinity
  - Exist **only** for 0<γ<2/3 (dark energy)
    - Behavior near infinity: $W = W_{\mathrm{FRW}}(z)e^{A(z)}, \quad S = S_{\mathrm{FRW}}(z)e^{B(z)}$

$$A \approx A_0 z^{(2-\gamma)/\gamma}, \quad B \approx -\frac{1}{6\gamma} A_0 z^{(2-\gamma)/\gamma}$$

  - 1-parameter family : A_0
    - $A_0=0$ gives exact FRW solution
- We obtain numerical solutions by solving from infinity with different values of A_0
  - We set γ=1/3 and the gauge choice is $a_0=b_0=1$

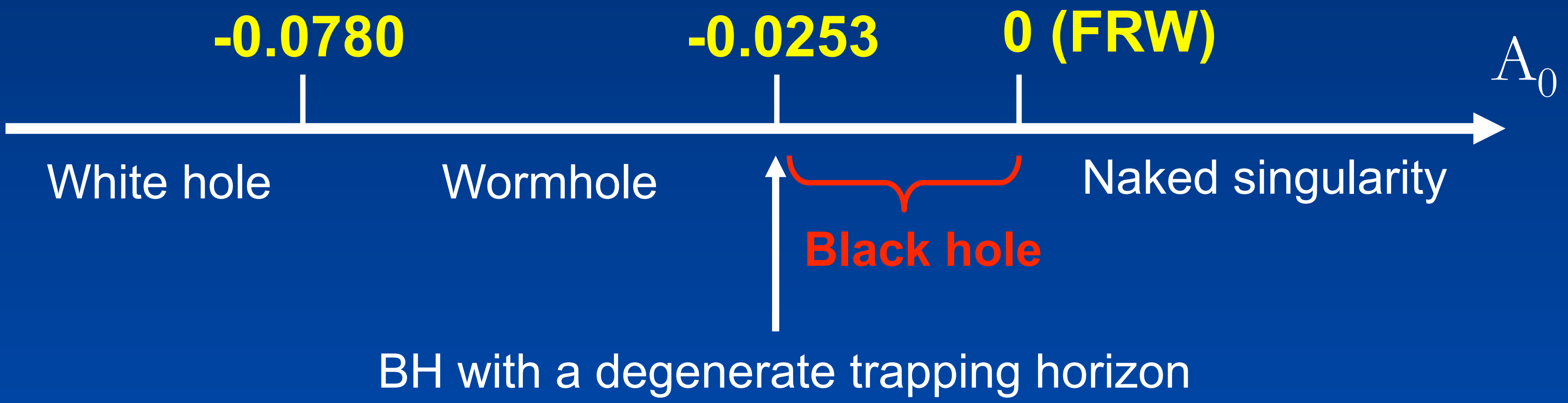
t-r plane


r<0
t
z=-0
FRW infinity
(z=+0)
Increasing z
z=const>0
z=const<0
r
z=-infinity,
z=+infinity
z=+infinity,
z=-infinity
O
Fluid element
z=const>0
z=const<0
t<0
z=+0
z=-0

(III)
Numerical results

Summary of numerical results

A one parameter-family of BH solutions exists



-0.0780

-0.0253

0 (FRW)

A_0

White hole

Wormhole

Naked singularity

Black hole

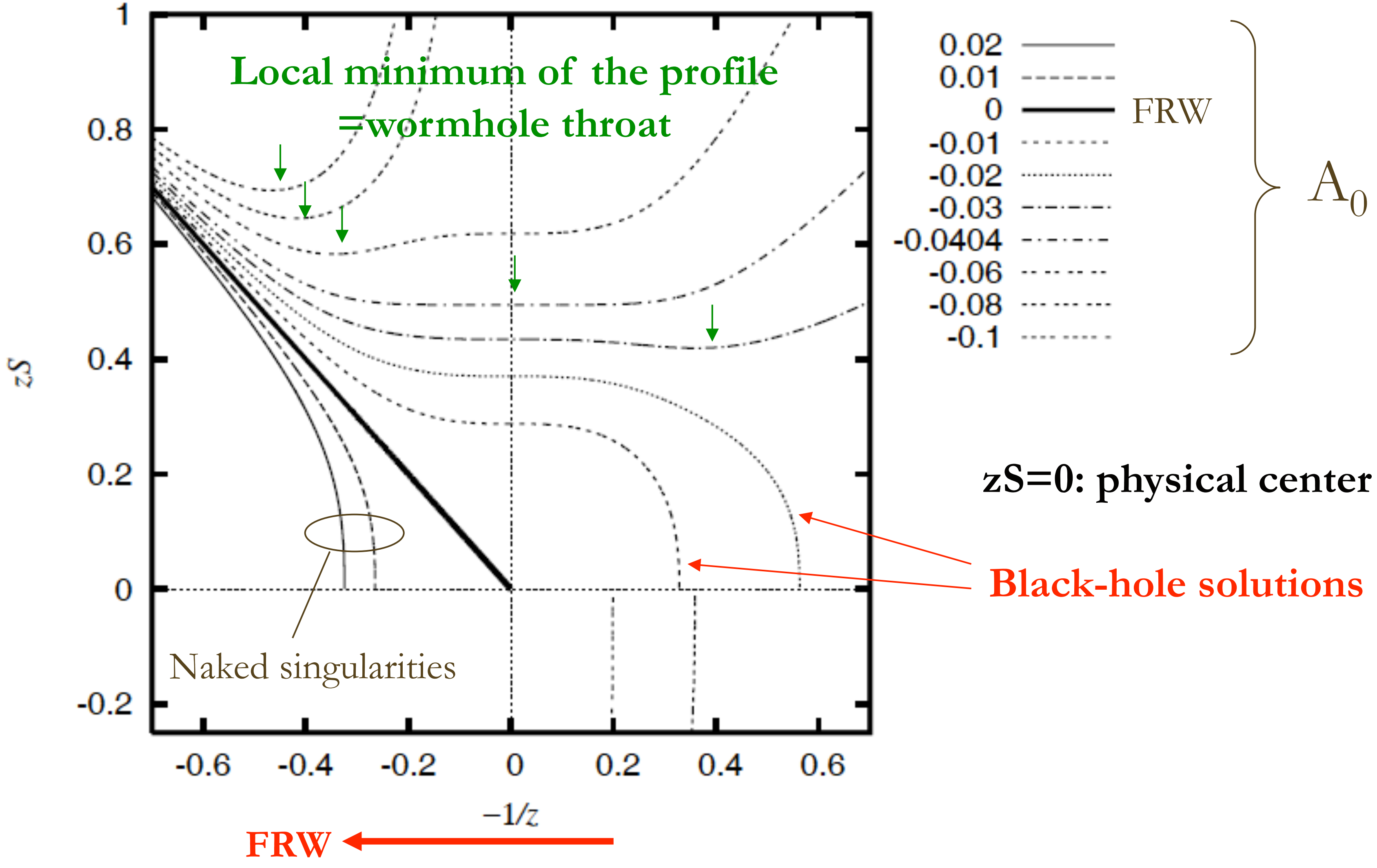
BH with a degenerate trapping horizon

Physical quantities to know the global structure

- Velocity function
  - To identify BH and/or cosmological event horizons
- Mass-area relation
  - A quasi-local definition of a BH (=a future outer TH)
- Energy density
  - Divergence = singularity
- Quasi-local mass
  - Negative mass singularity = naked singularity
- Areal radius and its time derivative
  - To identify spacelike infinity and wormhole throat
  - To know the expanding and collapsing region

Physical quantities

zS(=R/t): areal radius at constant t



Local minimum of the profile
=wormhole throat

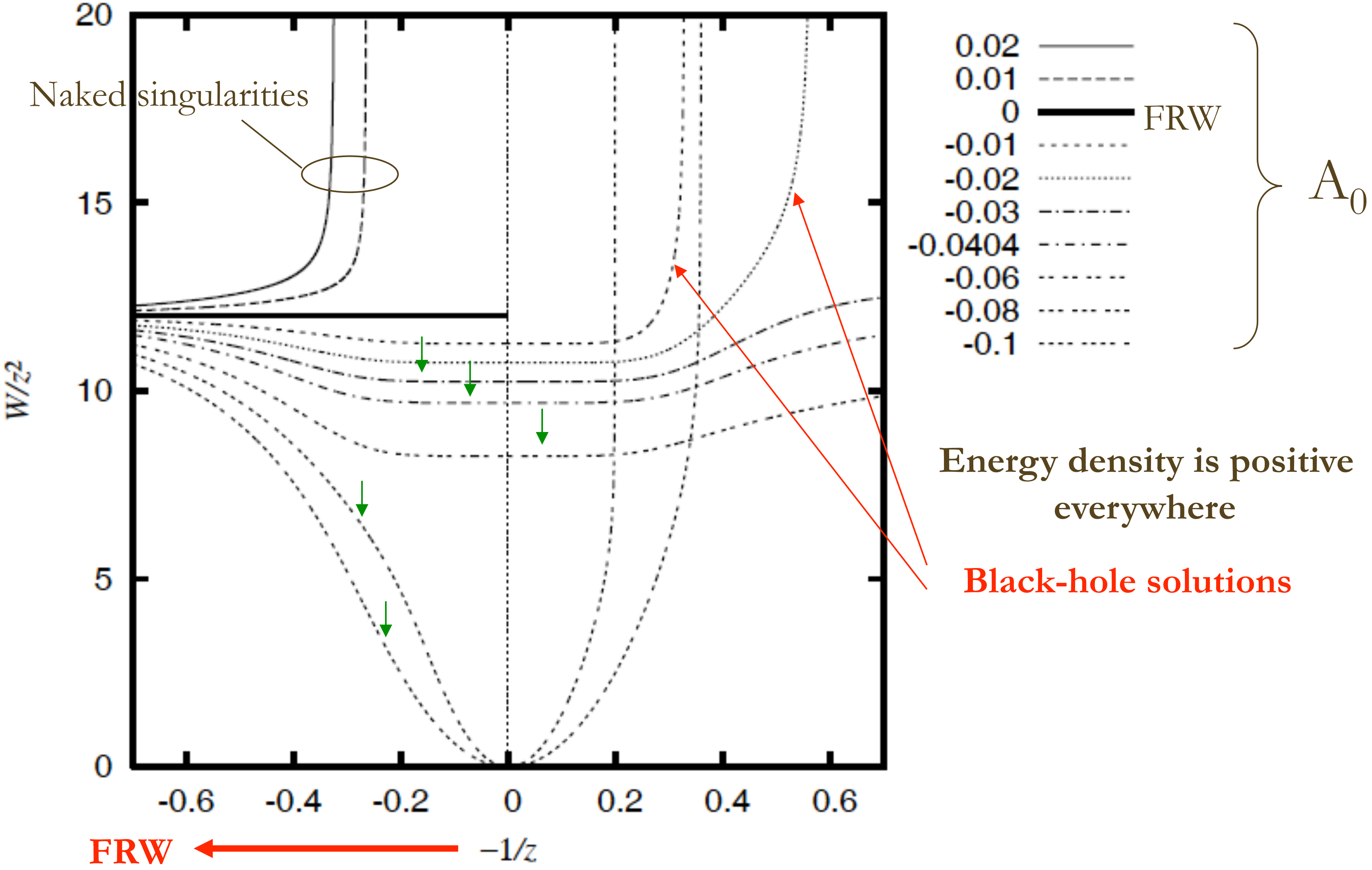
zS=0: physical center

Black-hole solutions

Naked singularities

FRW

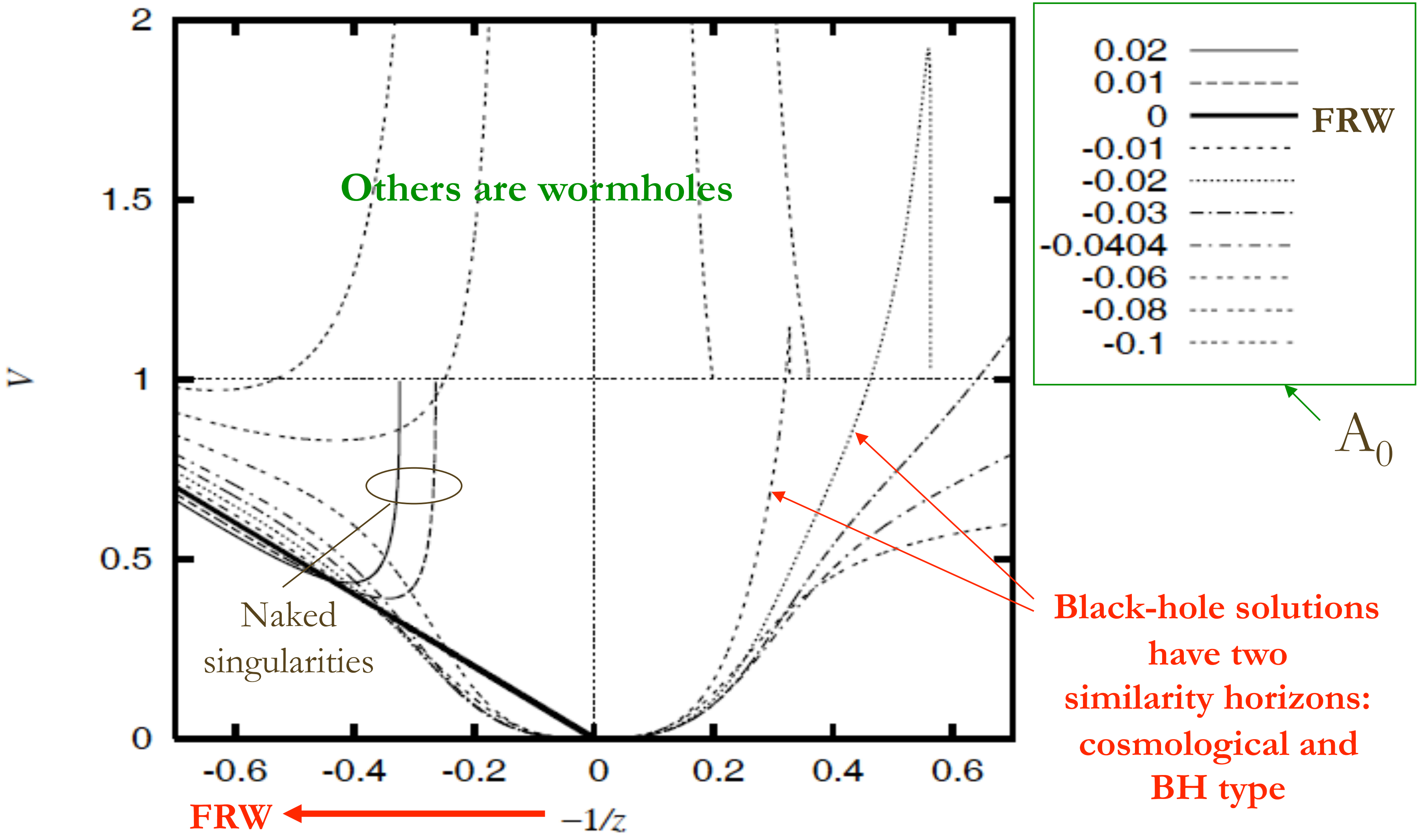
$W/z^2(=4\pi G\mu t^2)$



Energy density is positive everywhere

Black-hole solutions

Velocity function



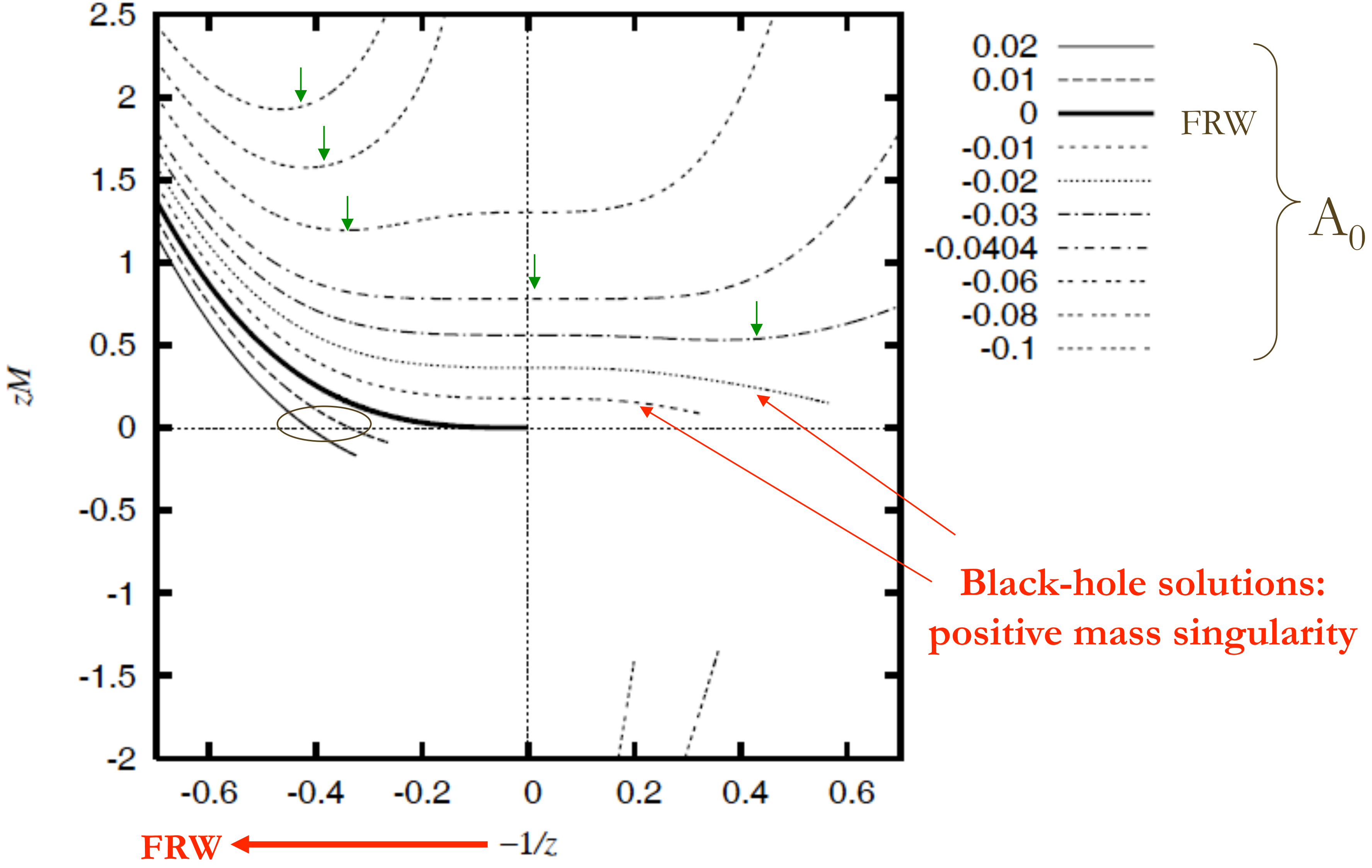
Others are wormholes

Naked singularities

Black-hole solutions have two similarity horizons: cosmological and BH type

FRW

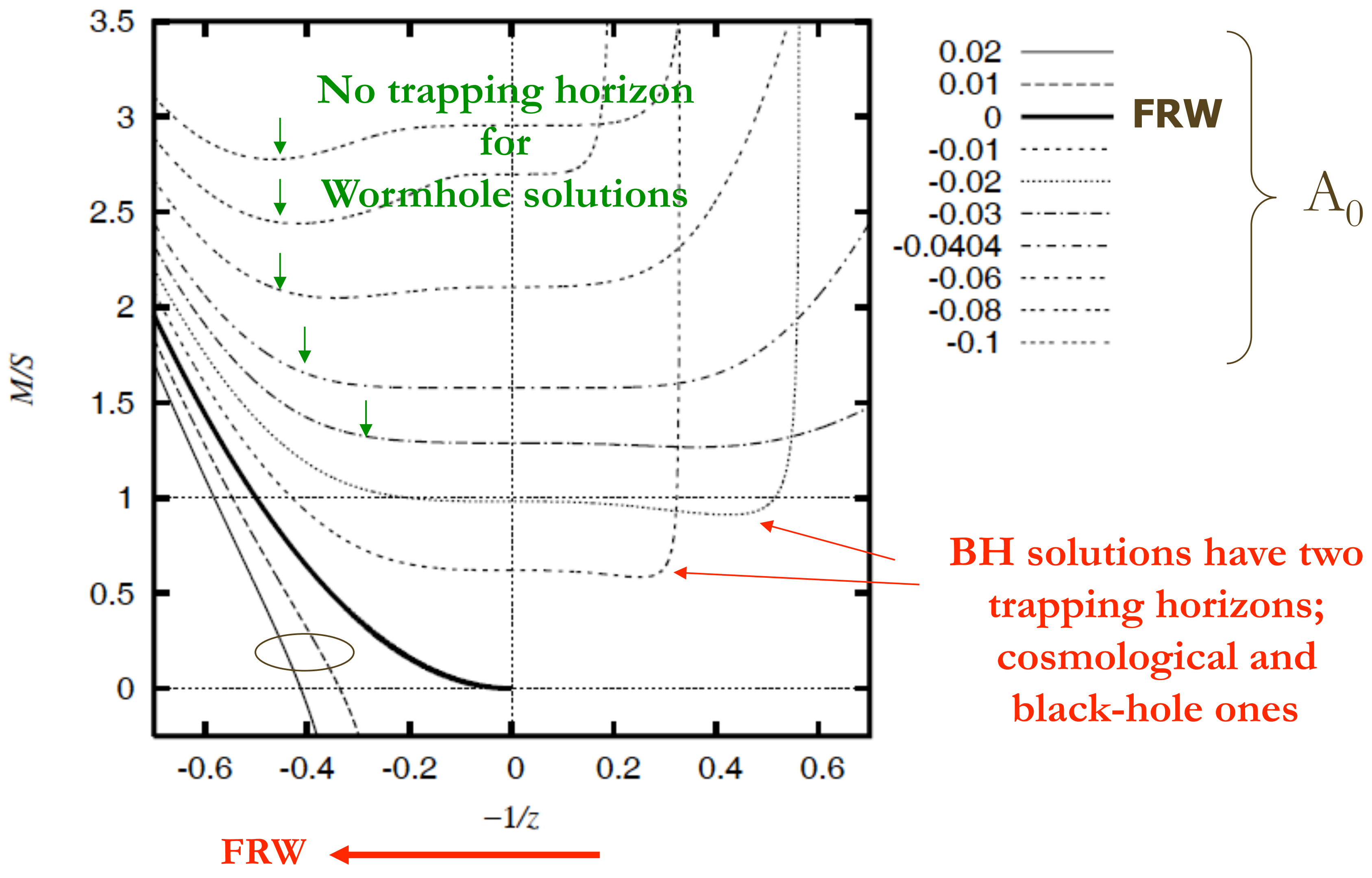
zM (=2Gm/t)



Black-hole solutions: positive mass singularity

FRW

M/S(=2Gm/R)



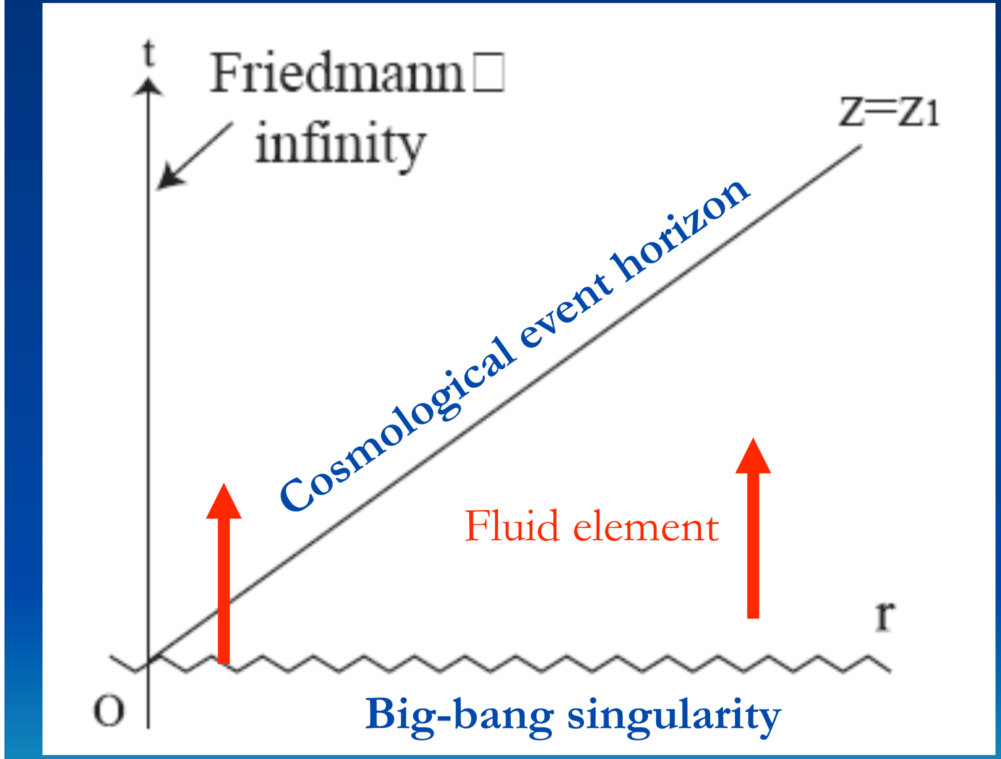
No trapping horizon for Wormhole solutions

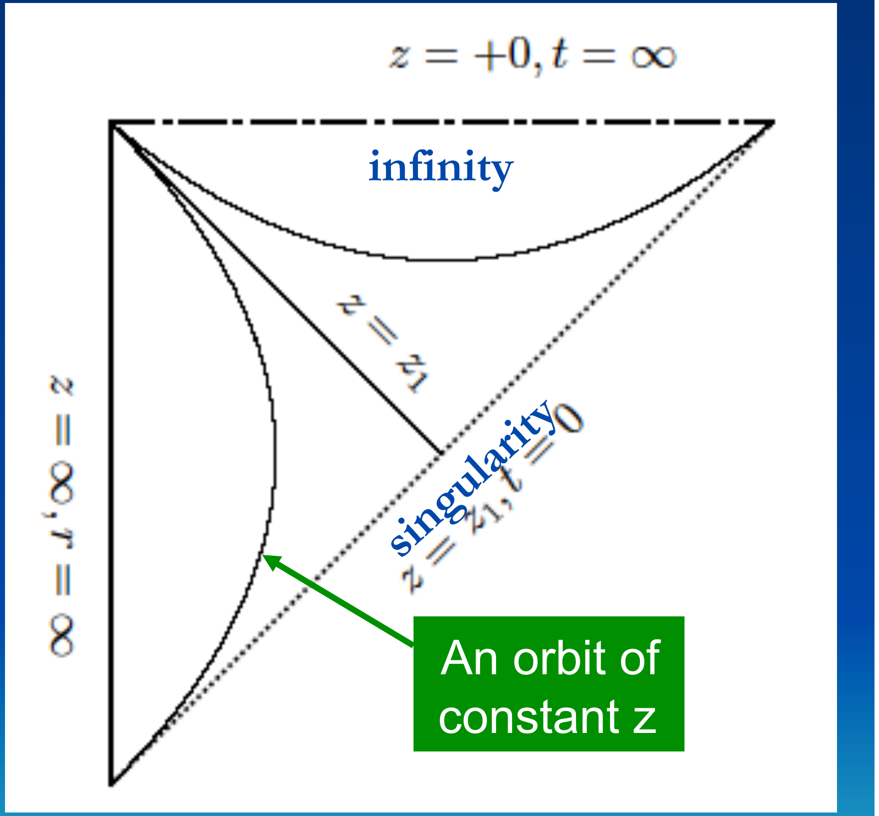
BH solutions have two trapping horizons; cosmological and black-hole ones

FRW

t-r planes and global structures of the solutions

(a) Flat FRW universe（0<γ<2/3）

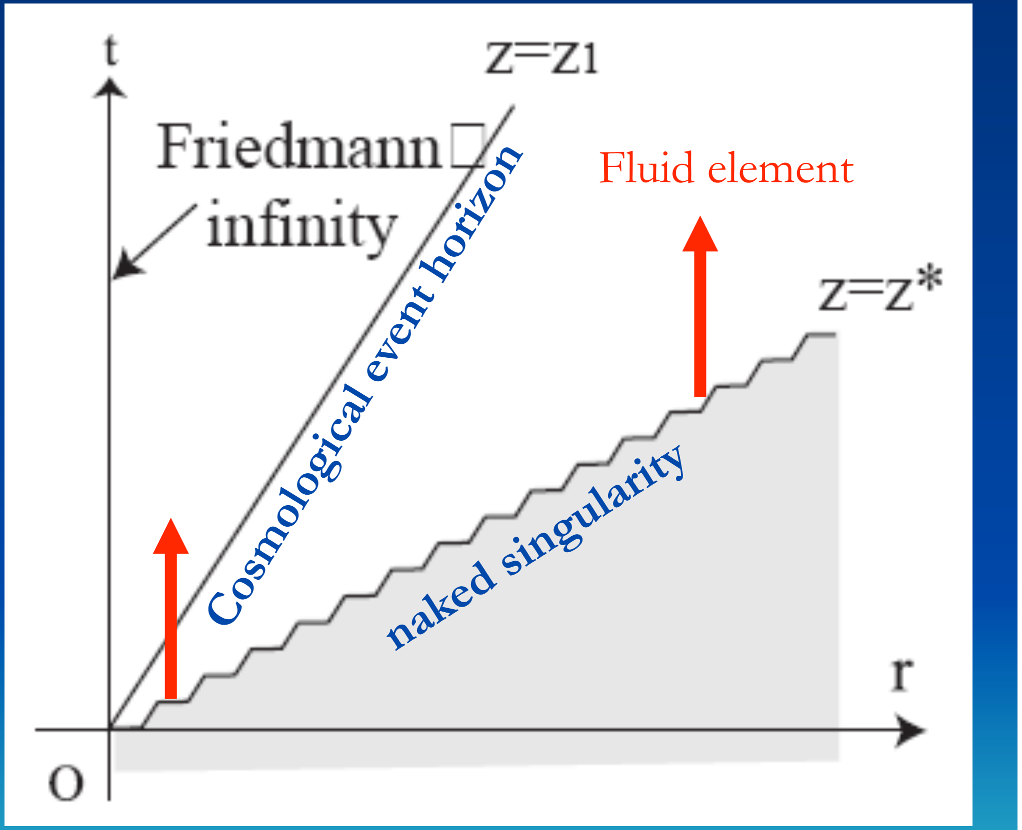


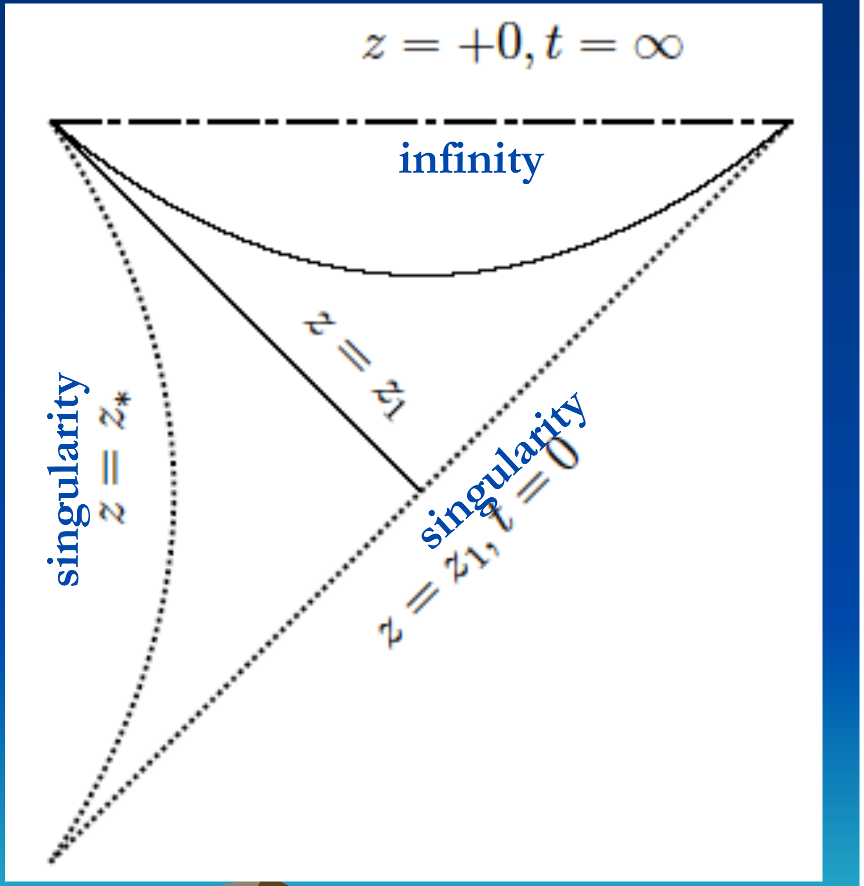


dot: singularity, dot-dashed: infinity

(b) Cosmological naked singularity

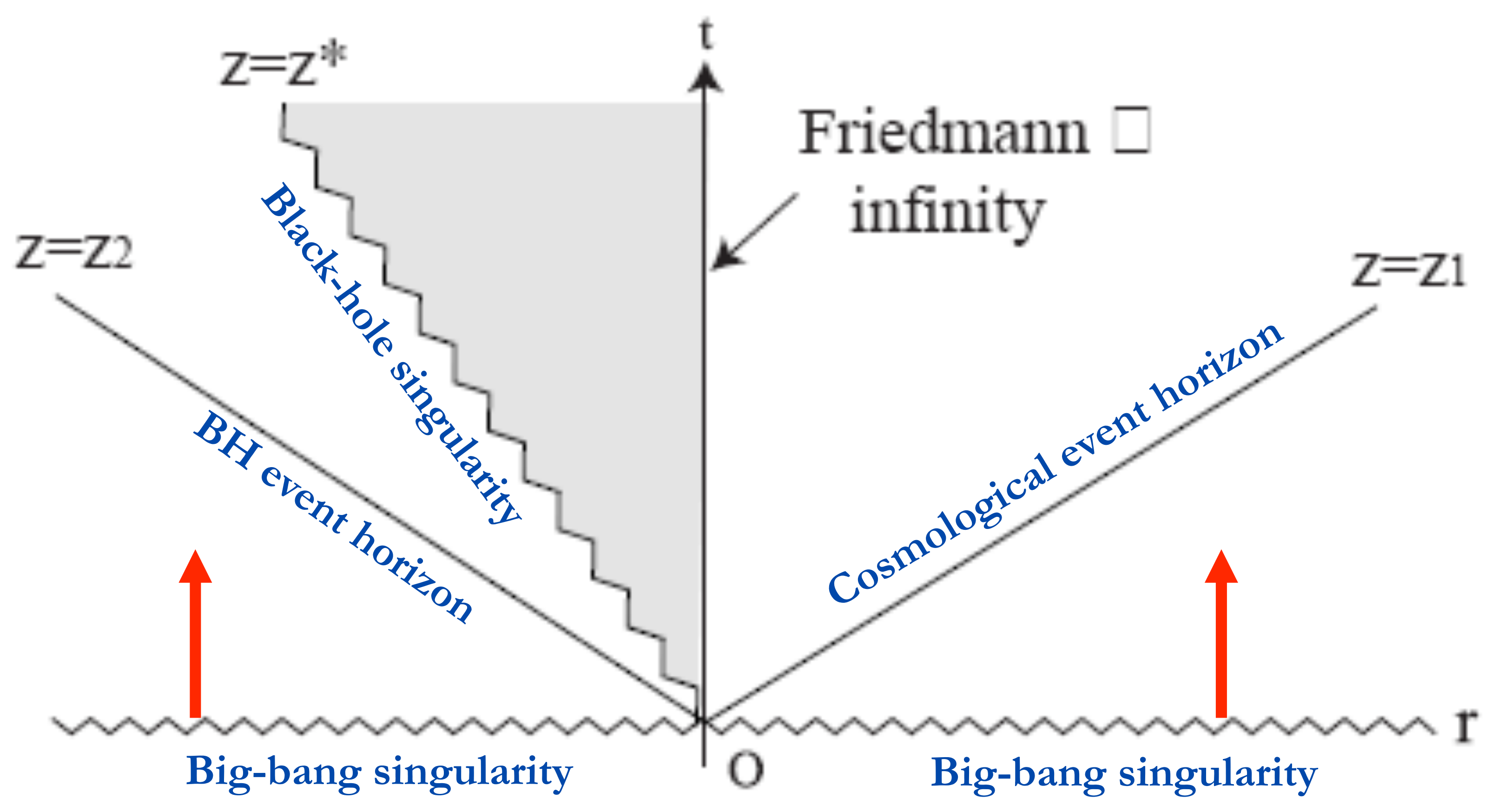
Timelike singularity with negative mass





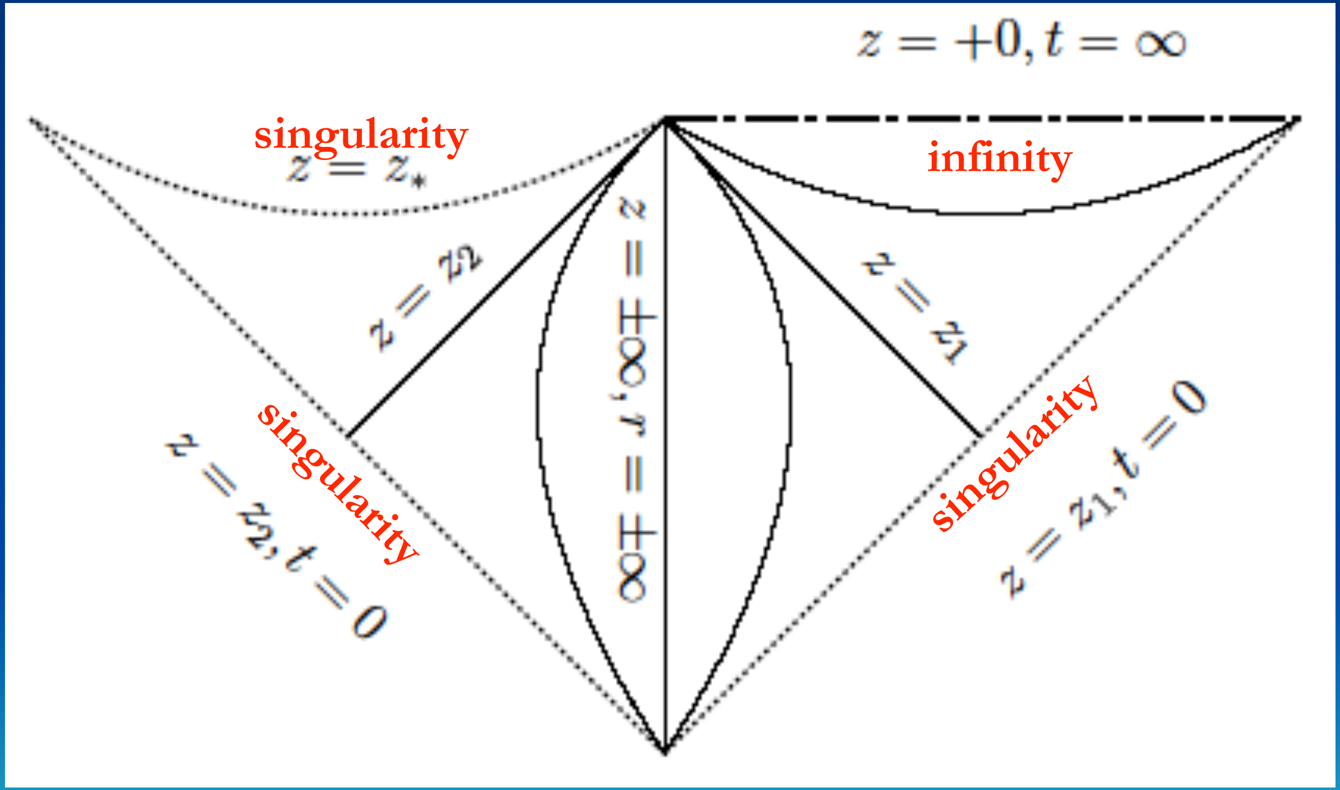
dot: singularity, dot-dashed: infinity

(c) Cosmological BH: t-r plane



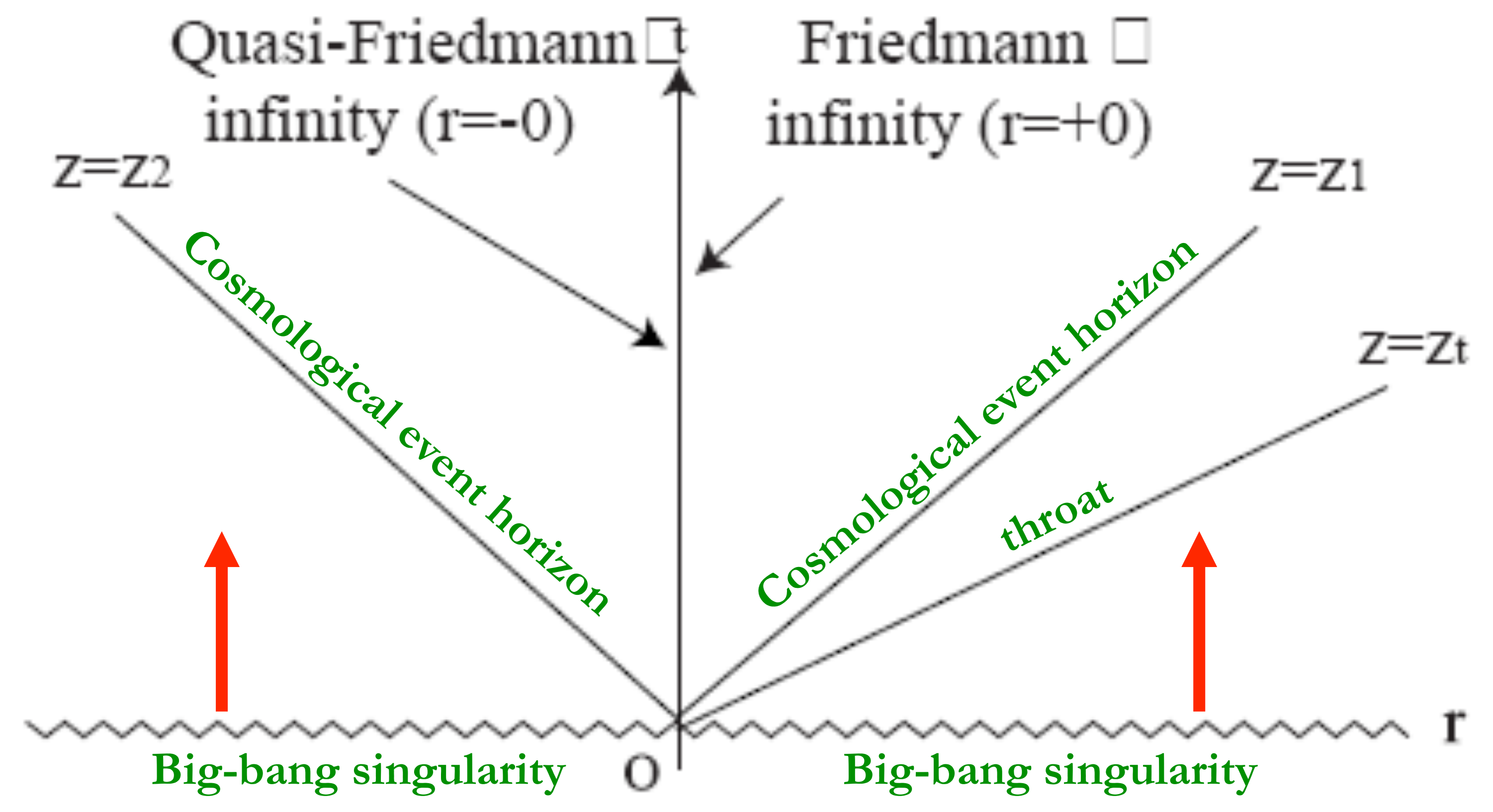
(c) Cosmological BH: Penrose diagram

Collapsing region exists inside the BH event horizon



dot: singularity, dot-dashed: infinity

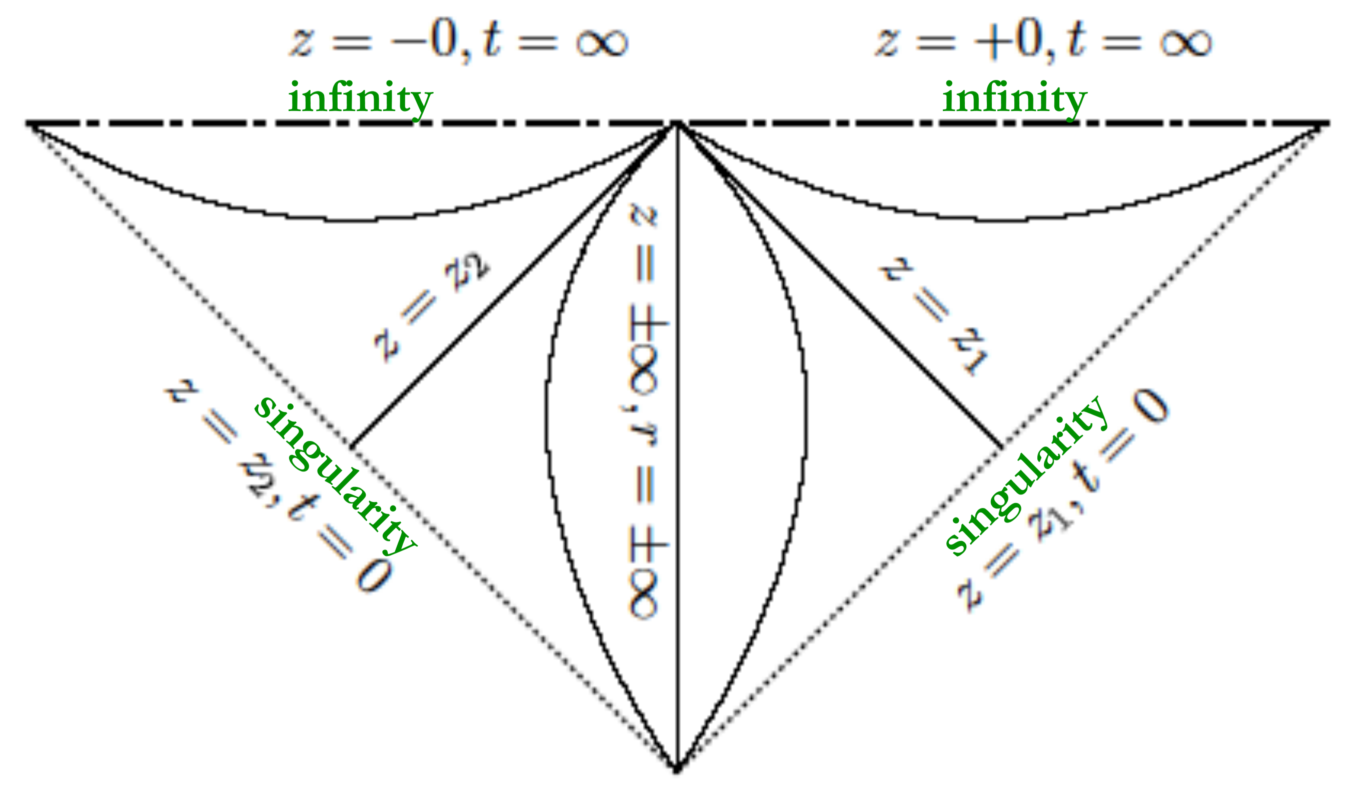
(d) FRW-(quasi-)FRW wormhole : t-r plane



The areal radius R is finite at (t,r)=(const, ∞)

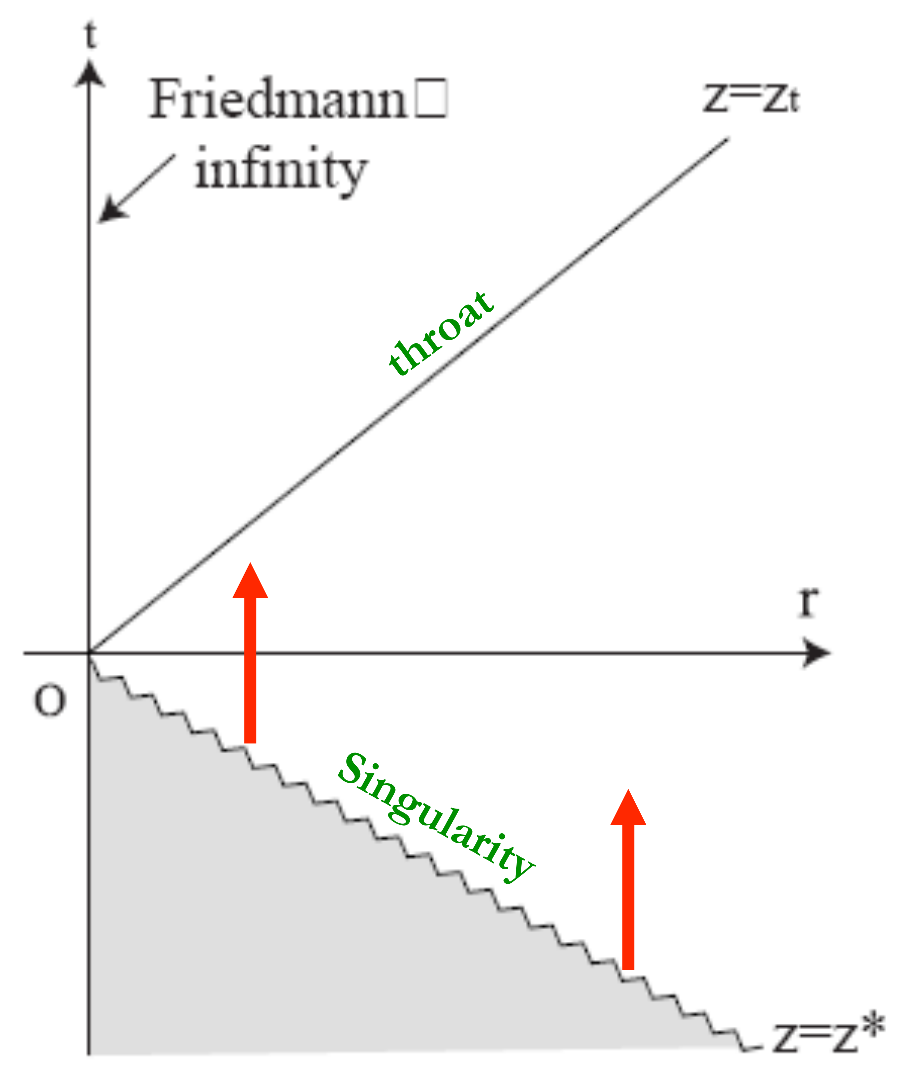
(d) FRW-(quasi-)FRW wormhole: Penrose diargam

dot: singularity, dot-dashed: infinity



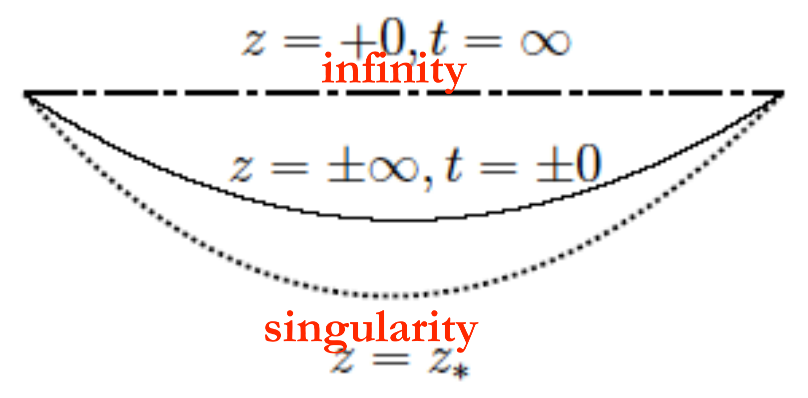
Enegy density is positive everywhere

(e) Cosmological white hole : t-r plane

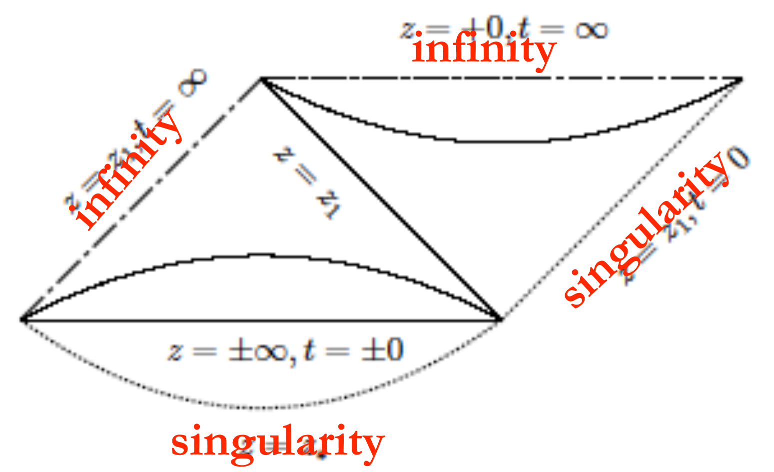


(e) Cosmological white hole: Penrose diagram

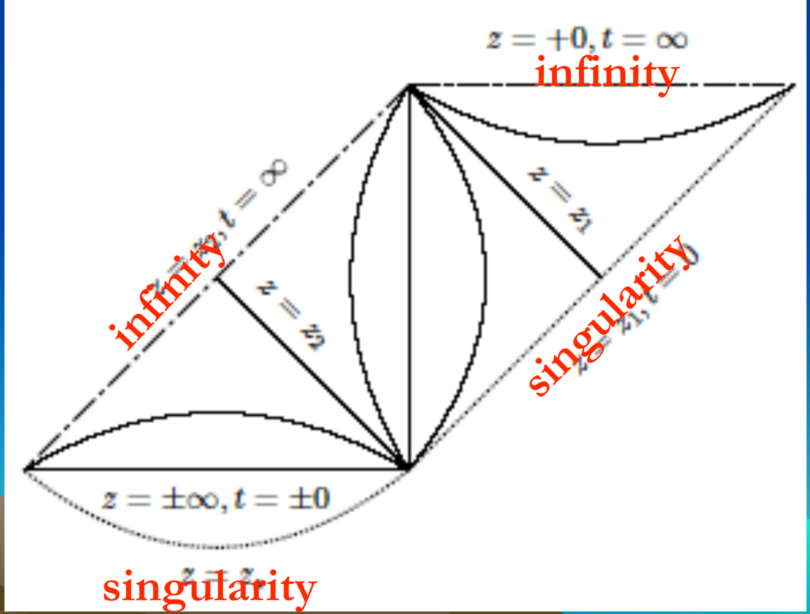
Extreme case



With a
degenerate horizon



With cosmological
and
white-hole horizons



dot: singularity, dot-dashed: infinity

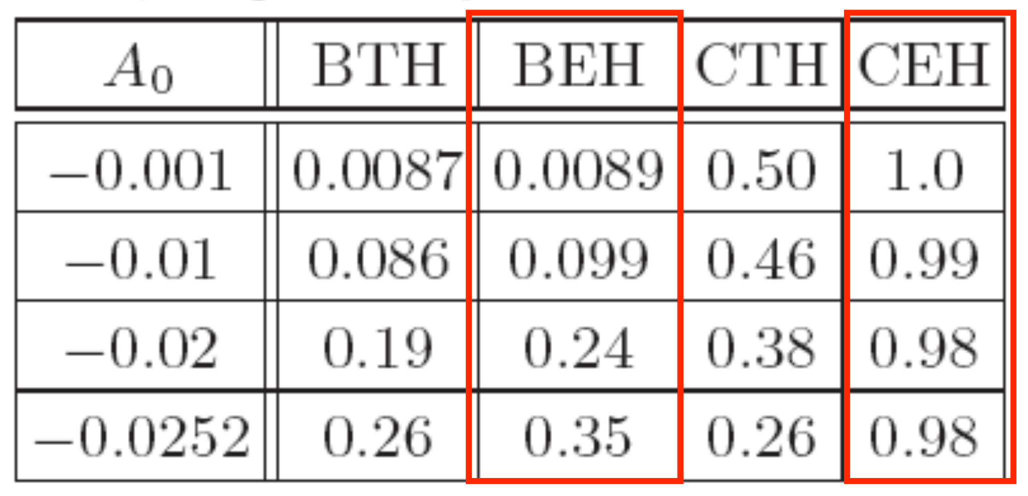
(IV)
Summary and discussions

Interpretation of the solutions: Summary

- Exact Friedmann universe (A_0=0)
- Cosmological naked singularity (A_0>0)
- Cosmological black hole (-0.0253<A_0<0)
  - First example of self-similar BH solutions in the Friedmann universe in a physical situation
- FRW-(quasi-)FRW cosmological wormhole (-0.0780<A_0<-0.0253)
  - FRW-FRW cosmological wormhole (A_0=-0.0404)
- Cosmological white hole (A_0<-0.0780)

Size of a self-similar BH

The value of z for the location of different horizons



A_0	BTH	BEH	CTH	CEH
-0.001	0.0087	0.0089	0.50	1.0
-0.01	0.086	0.099	0.46	0.99
-0.02	0.19	0.24	0.38	0.98
-0.0252	0.26	0.35	0.26	0.98

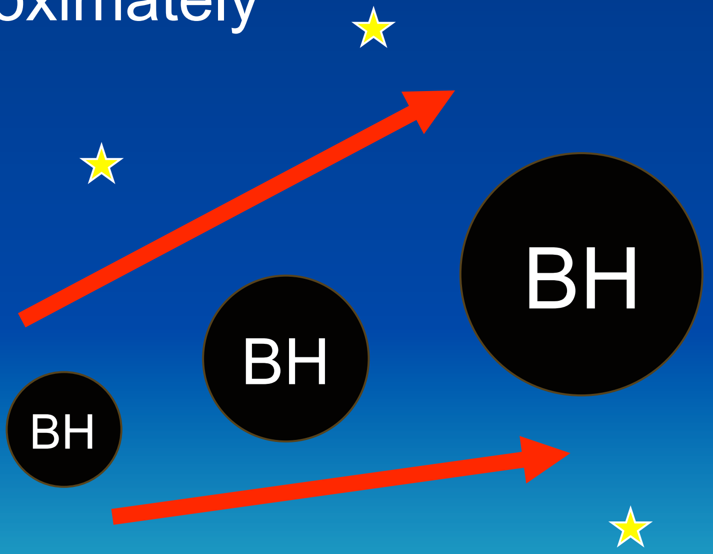
Black-hole event horizon **Cosmological event horizon**

The ratio BEH/CEH is from 0 to about 0.36

Relatively small BHs can grow in a self-similar manner

An application

- If the dark-energy dominant era started from a 10 billion years ago (as observations suggest so), which is about 2/3 of the age of the universe, the mass of a black hole which exists at the beginning of this era has been tripled up to now approximately



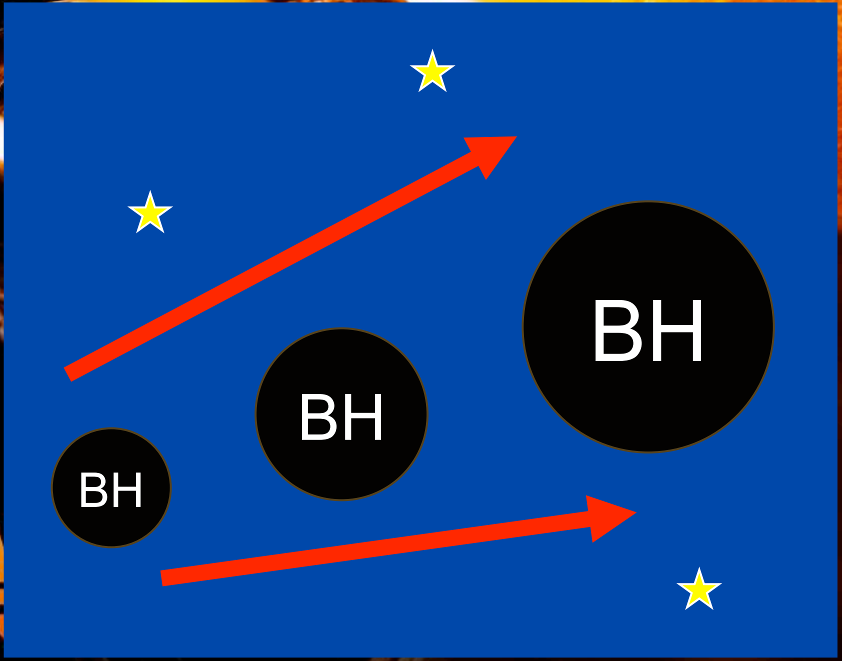
Summary of this talk

- Asymptotically FRW self-similar solution with dark energy
  - $\gamma=1/3$
  - FRW universe is accelerated
  - Numerical solutions
- Black hole solutions exist
  - Self-similar growth of a BH is possible for relatively small BHs
- A bonus: non-trivial solutions
  - Cosmological black (white) holes
  - Cosmological wormholes
  - Cosmological naked singularities

A recent result with a quintessence

- Two dark-energy models
  - Scalar field model
  - Perfect fluid model
- Are they equivalent in the spherical case?
  - Cosmology (homogeneous): Yes
  - Black hole in the universe (inhomogeneous): No (in general)
- Problem: Cosmological BH solutions with a scalar field?
  - Partial answer (Kyo-Harada-HM `08) : **asymptotically FRW self-similar solutions exists**
  - A scalar field with a quintessential potential
    - An exponential potential (FRW universe is accelerated)
- **An important problem: Black hole solutions exist?**
  - An important future work

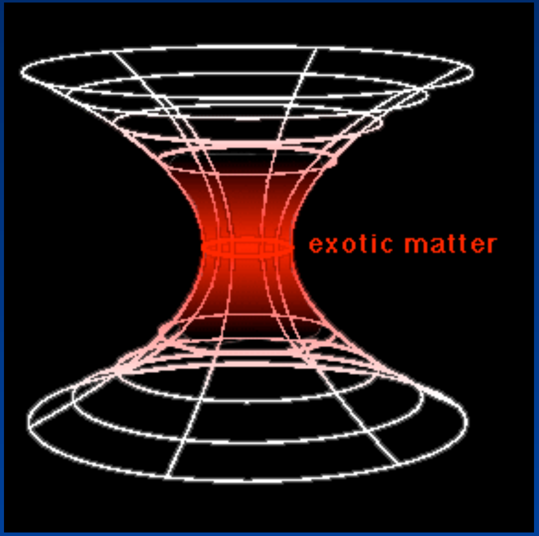
Dark energy makes
cosmological BHs
as well as
wormholes
in the universe!



FIN

Appendix:
More on wormhole solutions

Wormholes



- Spacetime structure with two (or more) different infinities
  - Characterized by a ``throat''
    - Spacelike 2-surface with minimum area on a spacelike hypersurface
- Static case
  - A natural time-slicing exists (associated with a timelike Killing vector)
  - Theorem
    - Null energy condition is violated at the wormhole throat
- How about the dynamical case?

Generalization of the concept of a wormhole throat is not unique

Several definitions of a dynamical wormhole throat

- Definition by Hochberg & Visser(1997)
  - $\theta+=0$ and $\partial\theta+/\ \partial\ +>0$
- Definition by Hayward(1999)
  - $\theta+=0$ and $\partial\theta+/\ \partial-<0$
    - Outer temporal (timelike) trapping horizon
- Definition in terms of a trapping horizon
  - Define a throat on a **null** (not spacelike) hypersurfaces
- Theorem
  - Null energy condition is violated at the wormhole throat

Our definition of a wormhole

- Spherical metric : diag(g_{AB},$r^2\gamma_{ab}$)
- Definition: **a wormhole throat**

$$\begin{aligned} A_{,\mu}\zeta^\mu &= 0, \\ (A_{,\mu}\zeta^\mu)_{,\nu}\zeta^\nu &> 0, \end{aligned}$$

$$A \equiv 4\pi r^2$$

  - ζ is a radial spacelike vector
  - Area of S^2 is minimum on a spacelike hypersurface
  - Slice dependent
    - Comoving slice is adopted in our analysis
- Our wormholes are not Hayward or Hochberg-Visser wormholes