Electroweak baryogenesis in the MSSM revisited

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in collaboration with
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PRD79, 115024 (2009) [arXiv:0905.2022] + ongoing work

Outline

- Motivation
  - ❐ Tensions in the MSSM baryogenesis
- Electroweak phase transition (EWPT) (1-loop)
  - ❐ Sphaleron decoupling condition
- 2-loop analysis
  - ❐ toy model (as an exercise)
- Summary

Motivation

☐ Universe is baryon asymmetric. (∵ cosmological data)

Baryon asymmetry of the Universe (BAU)

$$\boxed{\frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma} = (5.1 - 6.5) \times 10^{-10}, \quad (95\% \text{ C.L.})}$$

[PDG '09]

☐ How does the BAU arise dynamically from the B-symmetric Universe?

Conditions for the BAU Sakharov ('67)

(1) B violation (2) C and CP violation (3) out of equilibrium

SM:	B violation	sphaleron process
	C violation	chiral gauge interactions
	CP violation	CPV in the CKM matrix is not sufficient
	out of equilibrium	phase transition (PT) is not strong 1st order for $m_h > 114.4$ GeV. (see later)

☐ New physics is required to overcome these 2 issues.

☐ MSSM is one of the candidates for successful baryogenesis (BG).

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out of equilibrium: phase transition (PT) is not strong 1st order for $m_h > 114.4$ GeV. (see later)

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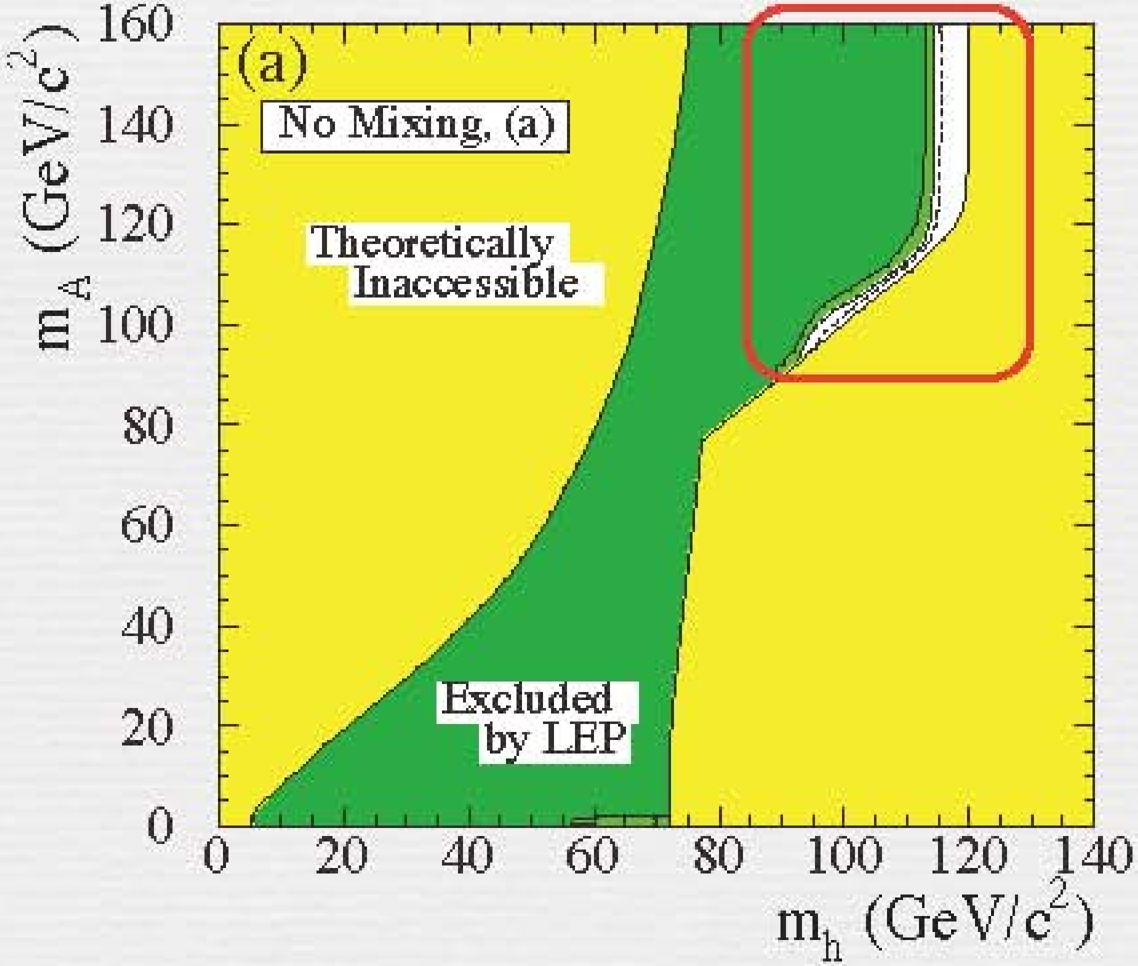
Tensions in the MSSM BG

Strong 1st order PT vs. LEP/B phys. data

❒ To have a strong 1st order PT, the light Higgs boson is needed. (see later)

❒ LEP/B phys. data can constrain such a light Higgs boson.



❒ MSSM BG is highly constrained, but there is still a viable window.

"viable" MSSM BG

[M. Carena, G. Nardini, M. Quiros, CEM. Wagner, NPB812, (2009) 243]

☐ Electroweak phase transition is strong 1st order if

$$m_H \lesssim 127 \text{ GeV}, \, m_{\tilde{t}_1} \lesssim 120 \text{ GeV}$$

viable case { EW vacuum: metastable (long-lived),
Charge-Color-Breaking (CCB) vacuum: global minimum

overlooked issues?

☐ Sphaleron decoupling condition $\Gamma^{(b)}_{\rm sph} < H \;\rightarrow\; v_C/T_C > \zeta$

Q.1 Is $v_C/T_C >$ 0.9 enough for the sphaleron decoupling?

☐ Effective potential at the 2-loop level

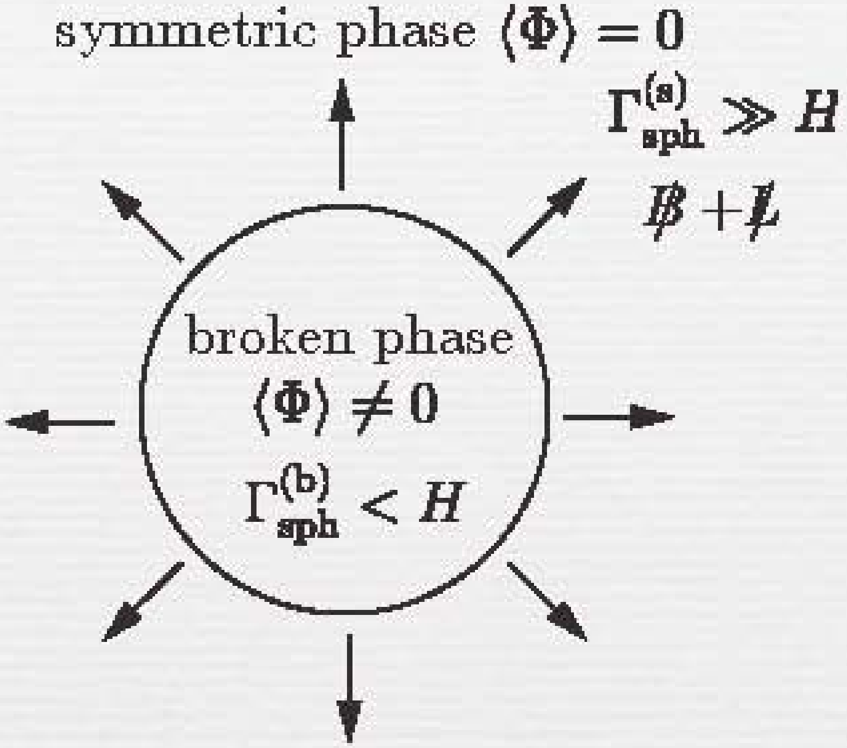
2-loop effects at finite temperature (T) can be sizable.
(based on the High-T expansion (HTE)) [P. Arnold, O. Espinosa, PRD47, ('93) 3546, J.R. Espinosa, NPB475, ('96) 273 etc]

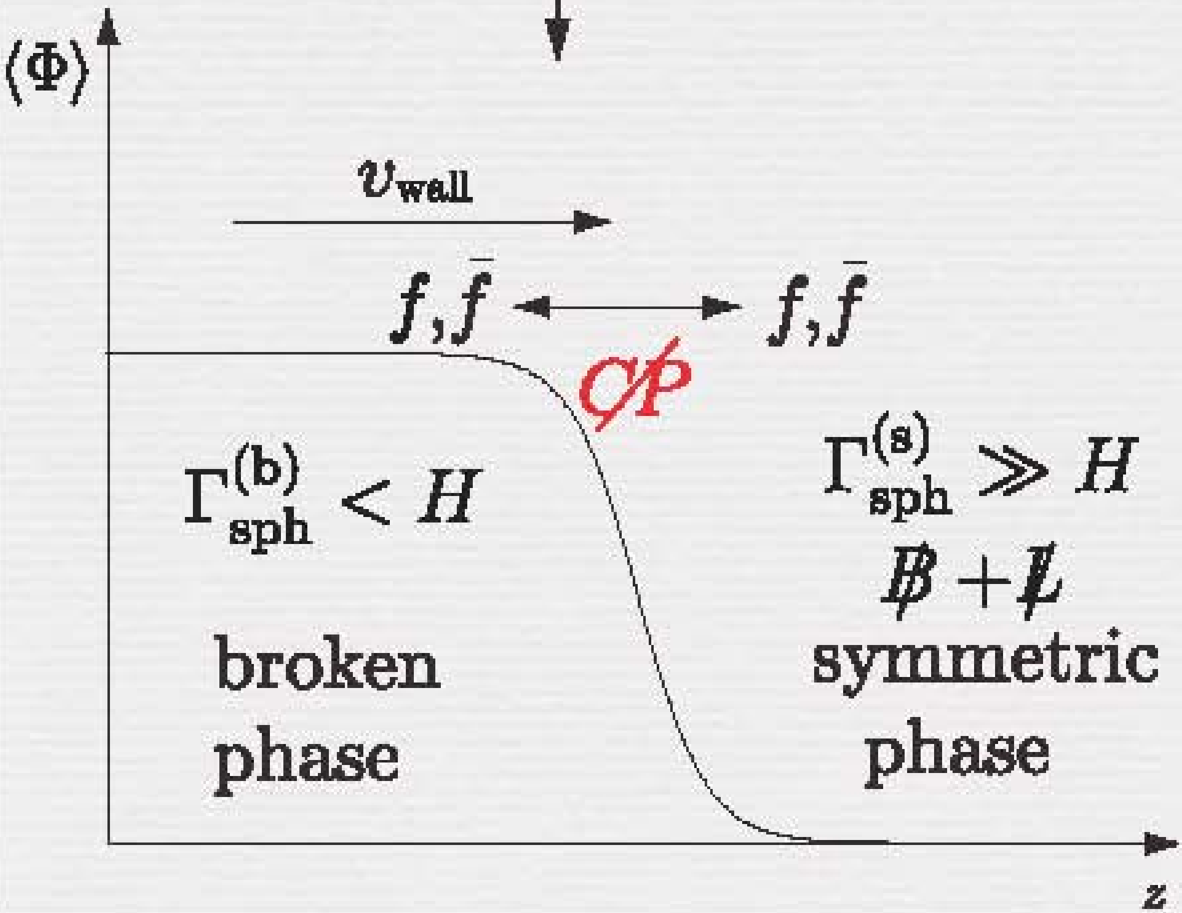
Q2. How reliable is the HTE at the 2-loop level?

Overview of EWBG

[Kuzmin, Rubakov, Shaposhnikov, PLB155,36 ('85)]

- EWPT must be a 1st order with expanding bubble wall.





outline

- CP violation at the bubble wall causes the chiral charge flux.
- Accumulation of the charges in the symmetric phase.
- Left-handed particle number densities are converted into B via sphaleron process.
- Sphaleron process is decoupled after the PT.
- B is frozen.

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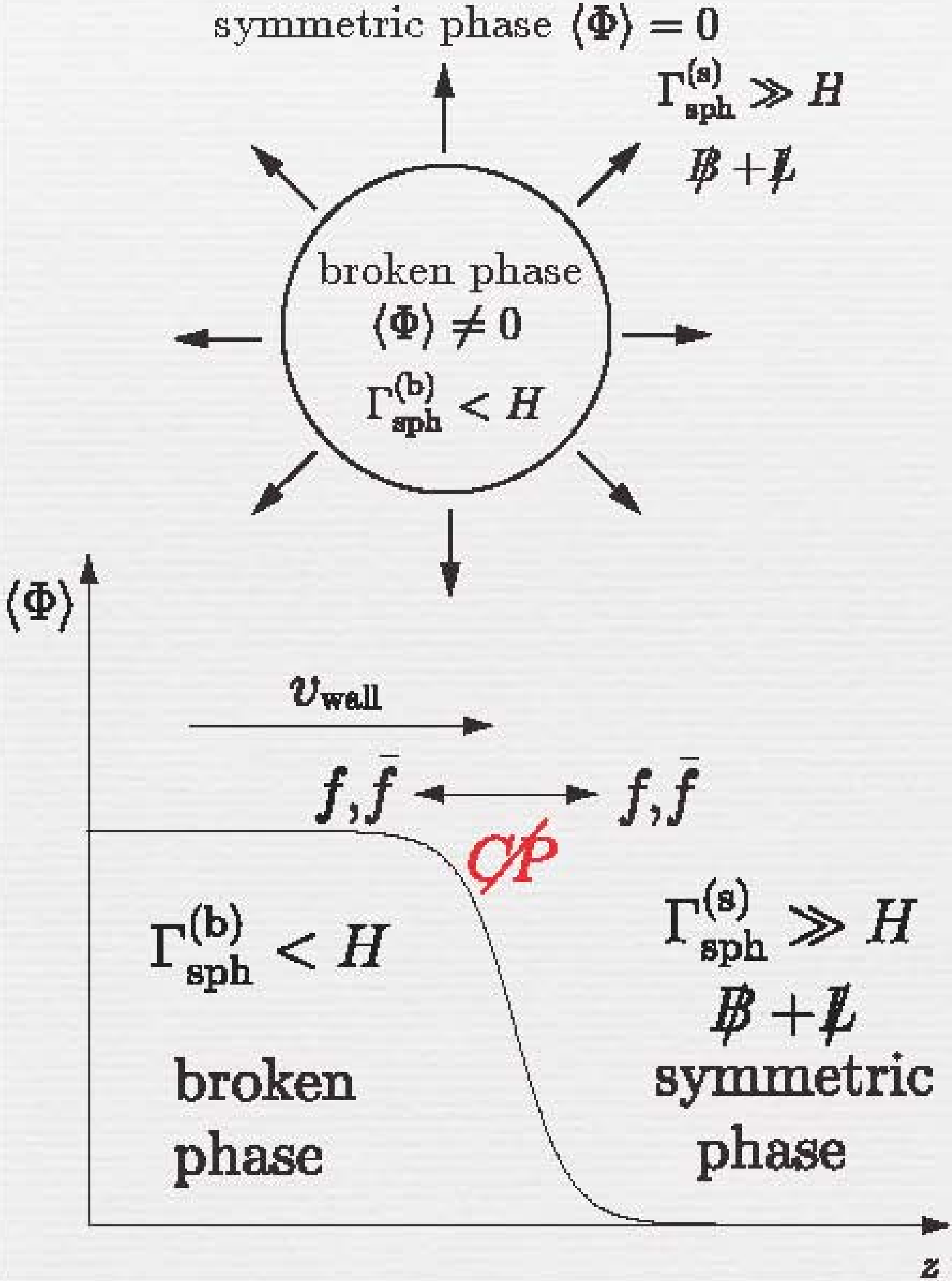

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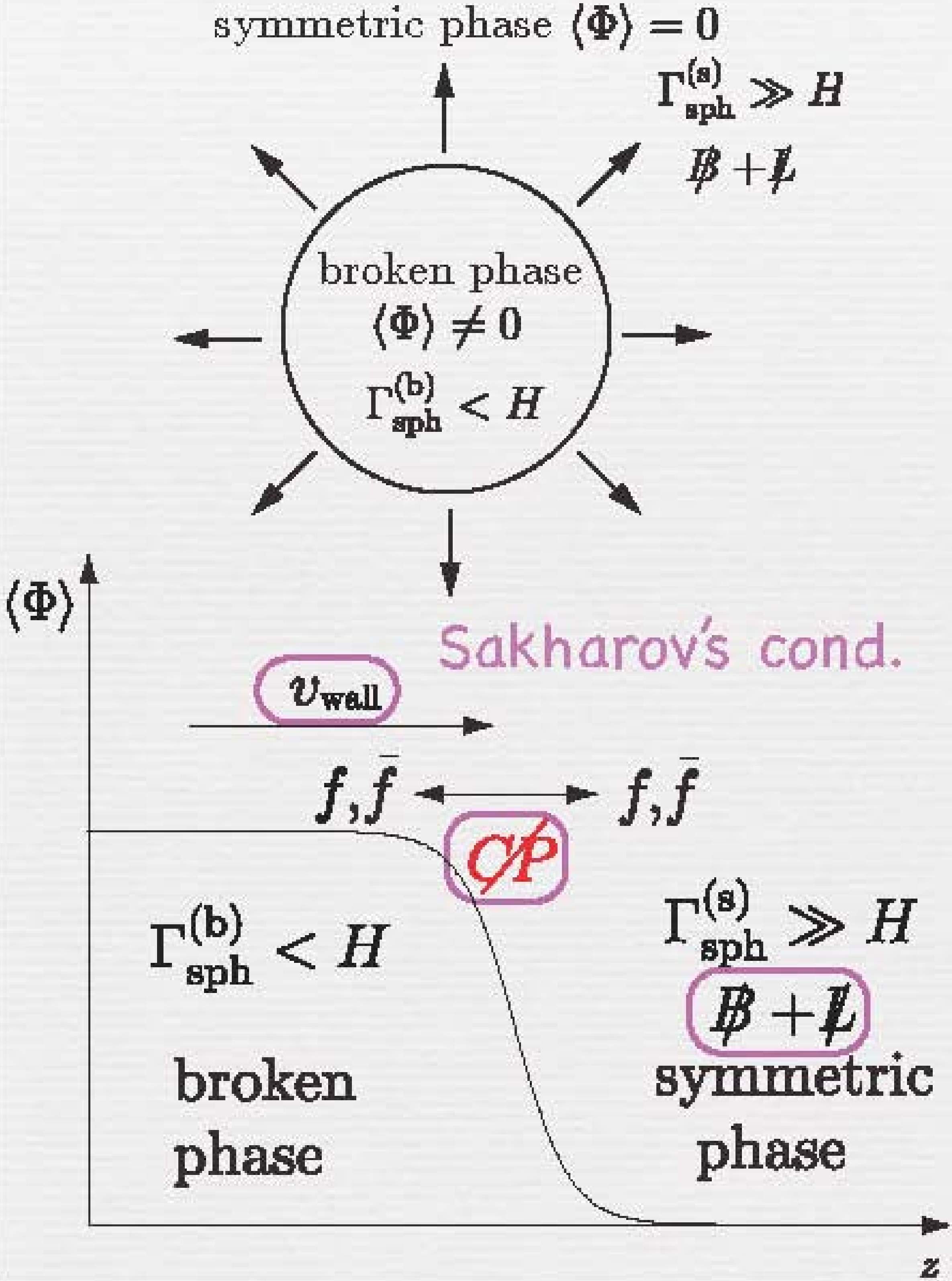

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EWPT

Effective potential

- To discuss the phase transition, the effective potential is used.
- gauge bosons and 3rd generation of quarks/squarks are taken into account.

$$V_{\rm eff}(\Phi_d,\Phi_u)=V_0(\Phi_d,\Phi_u)+\Delta V(\Phi_d,\Phi_u;T),$$

Tree:

$$\begin{aligned}V_0(\Phi_d,\Phi_u) &= m_1^2\Phi_d^\dagger\Phi_d+m_2^2\Phi_u^\dagger\Phi_u-(m_3^2\epsilon_{ij}\Phi_d^i\Phi_u^j+\text{h.c.})\\ &\quad+\frac{g_2^2+g_1^2}{8}(\Phi_d^\dagger\Phi_d-\Phi_u^\dagger\Phi_u)^2+\frac{g_2^2}{2}(\Phi_d^\dagger\Phi_u)(\Phi_u^\dagger\Phi_d),\end{aligned}$$

1-loop:

$$\Delta V(\Phi_d,\Phi_u;T)=\sum_A c_A\left[F_0(\bar{m}_A^2)+\frac{T^4}{2\pi^2}I_{B,F}\left(\frac{\bar{m}_A^2}{T^2}\right)\right]$$

$$F_0(m^2)=\frac{m^4}{64\pi^2}\left(\ln\frac{m^2}{M^2}-\frac{3}{2}\right),\quad I_{B,F}(a^2)=\int_0^\infty dx\; x^2\ln\left(1\mp e^{-\sqrt{x^2+a^2}}\right)$$

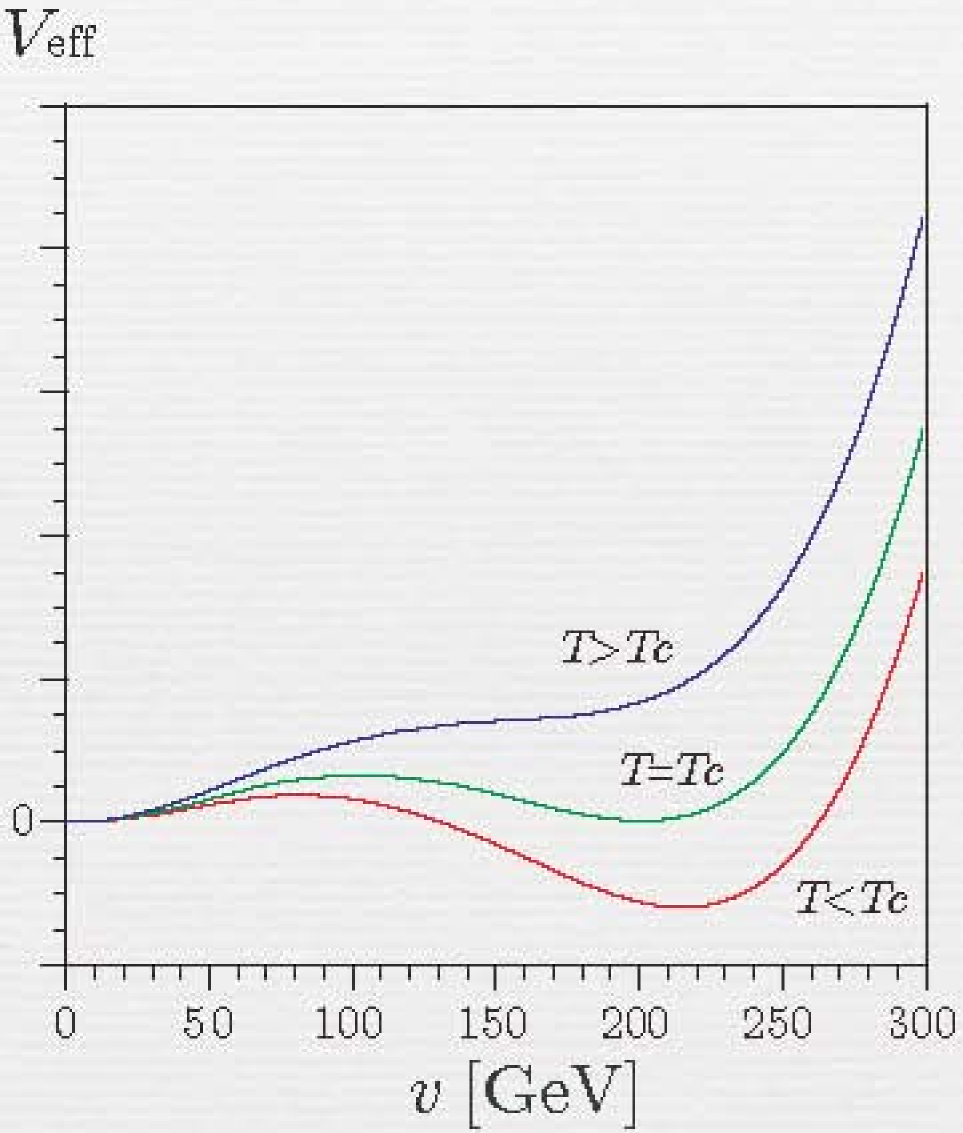
Numerical evaluation of $I_{B,F}(a^2)$ are extremely time-consuming.

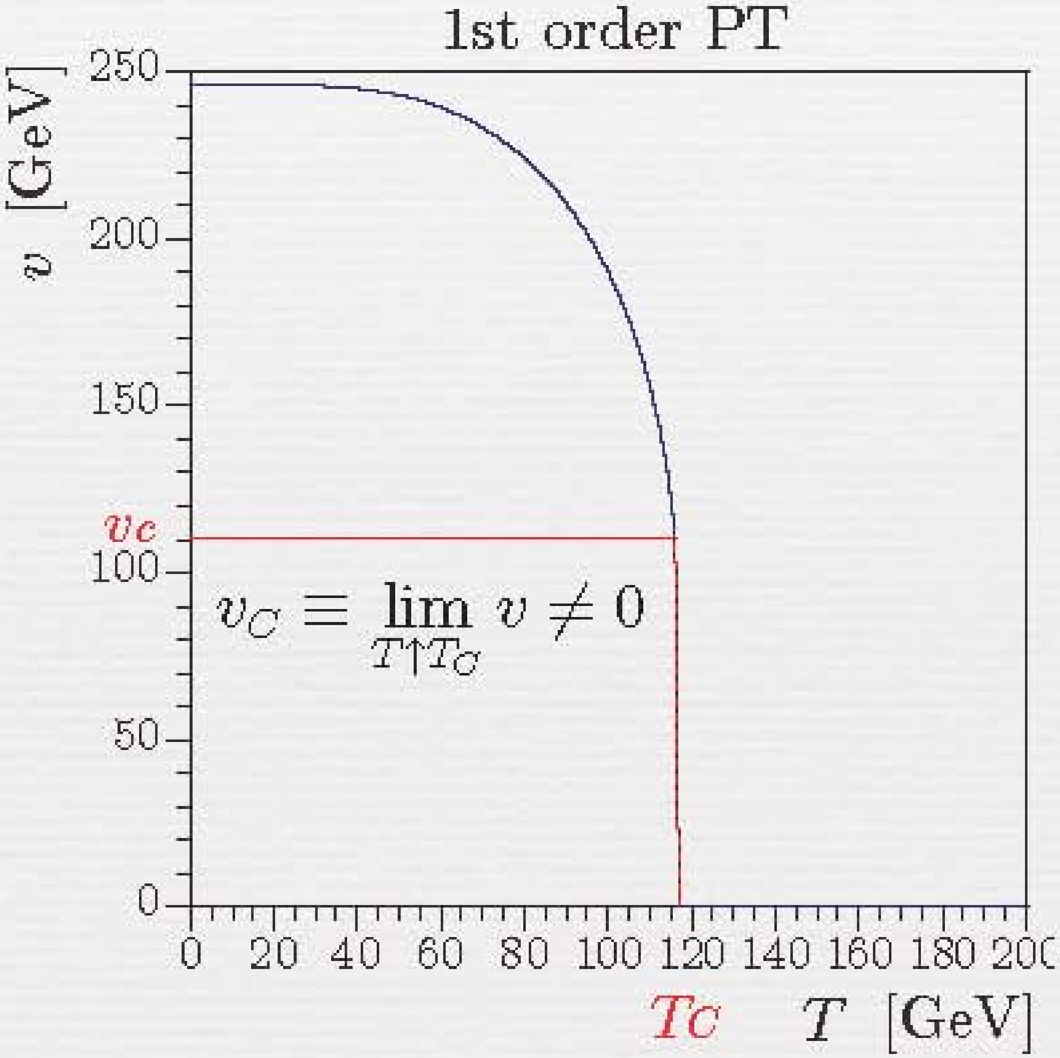
Fitting function: $\tilde{I}_{B,F}(a^2)=e^{-a}\sum_{n=0}^{N}c_n^{b,f}a^n,\quad |\tilde{I}_{B,F}-I_{B,F}|<10^{-6}\;(N=40).$

- The fitting function is used in our numerical analysis.

1^{st} order PT

- ❒ order parameter = Higgs VEVs





- ❒ At T_C, the Higgs potential has two degenerate minima.

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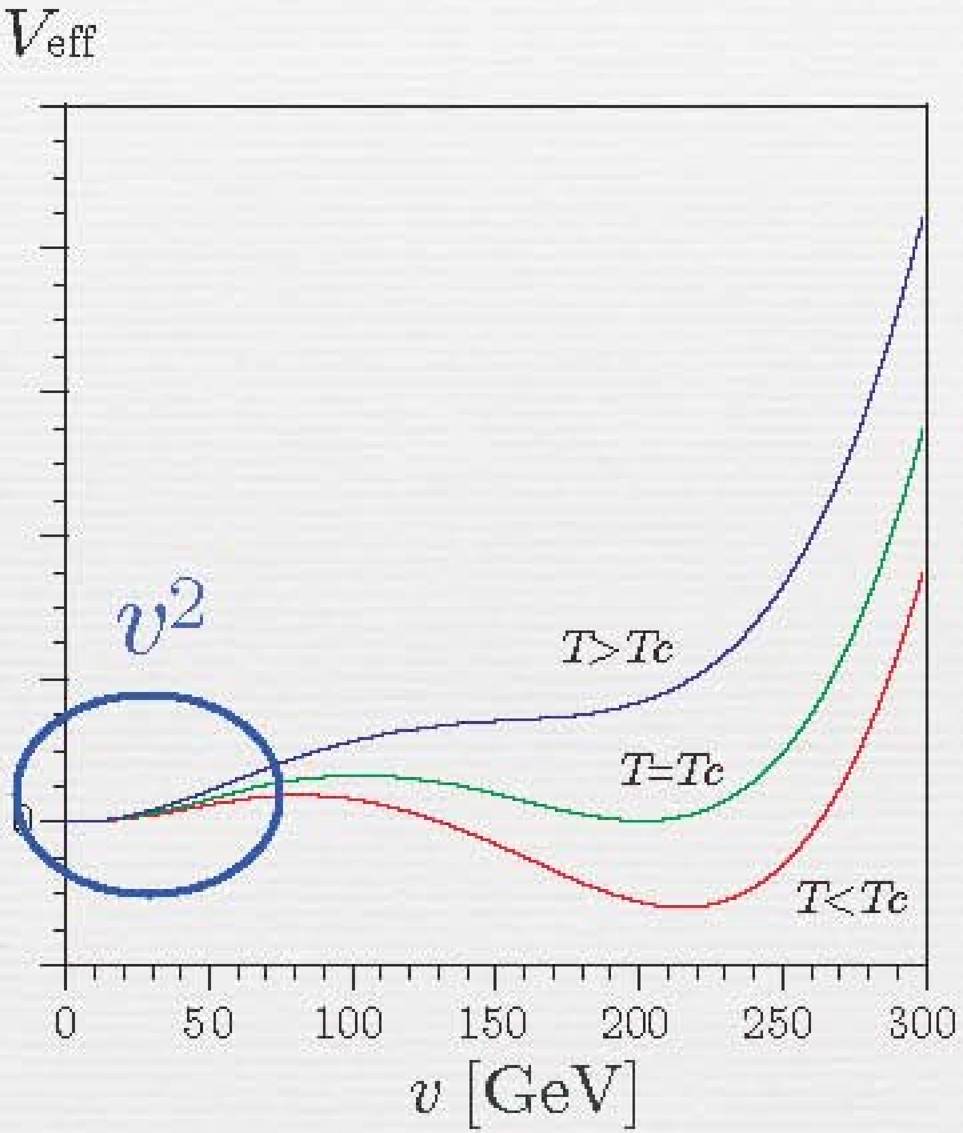

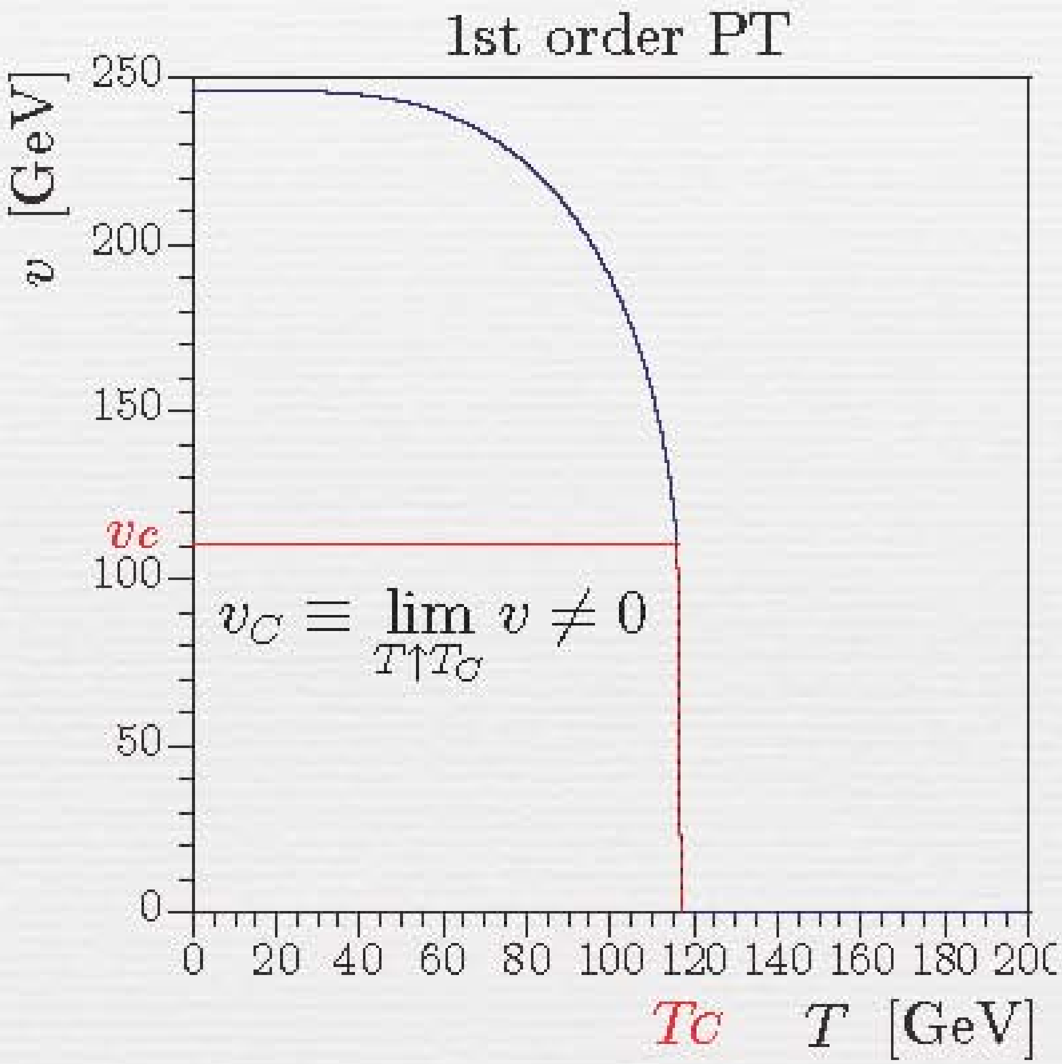

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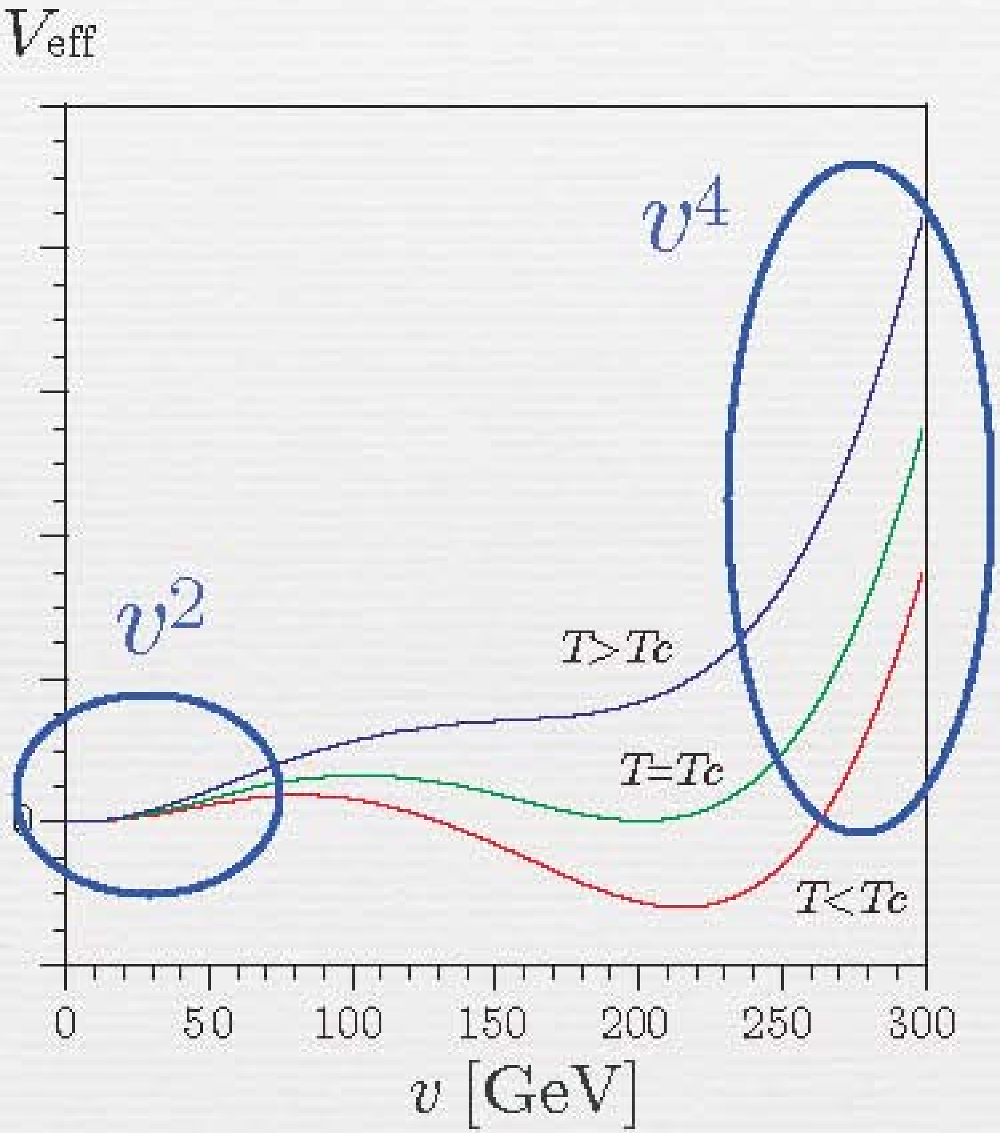

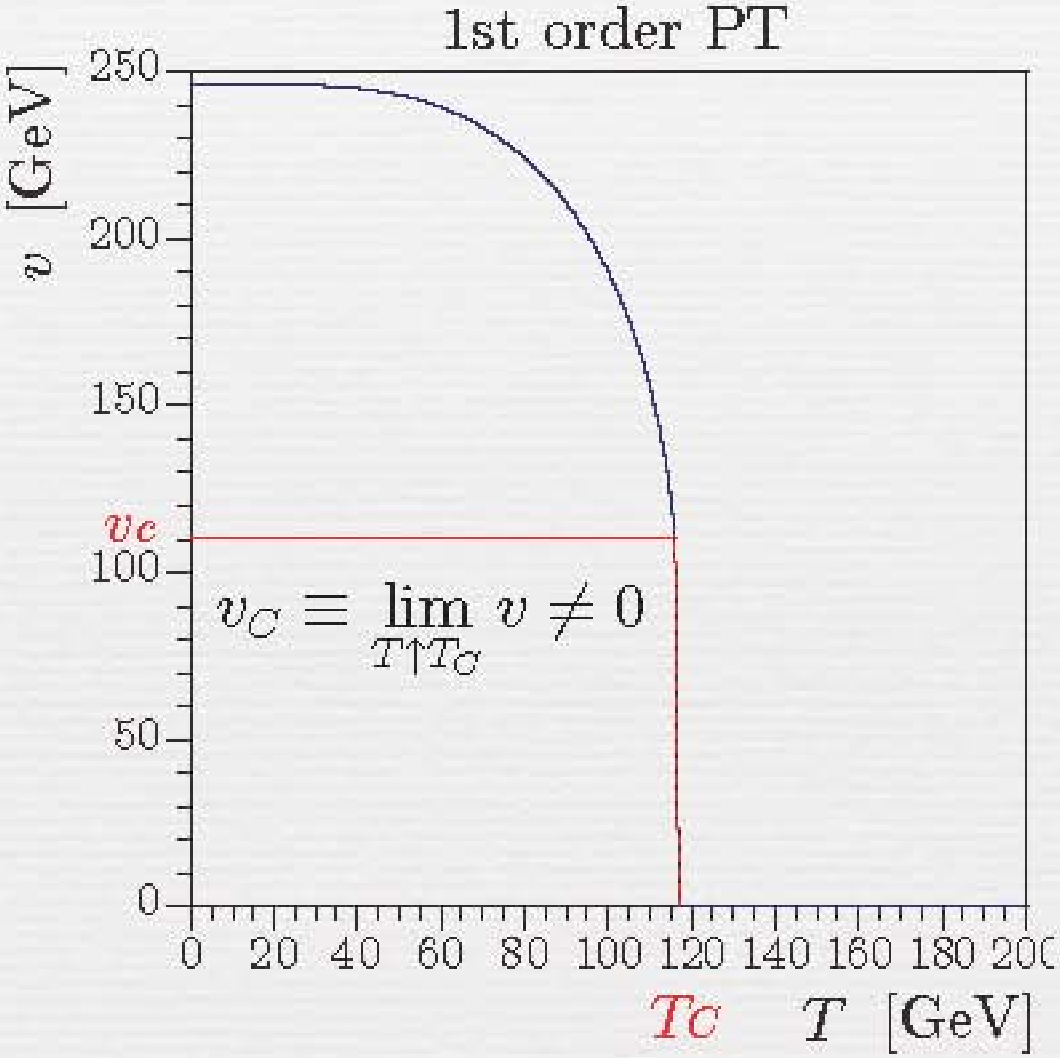

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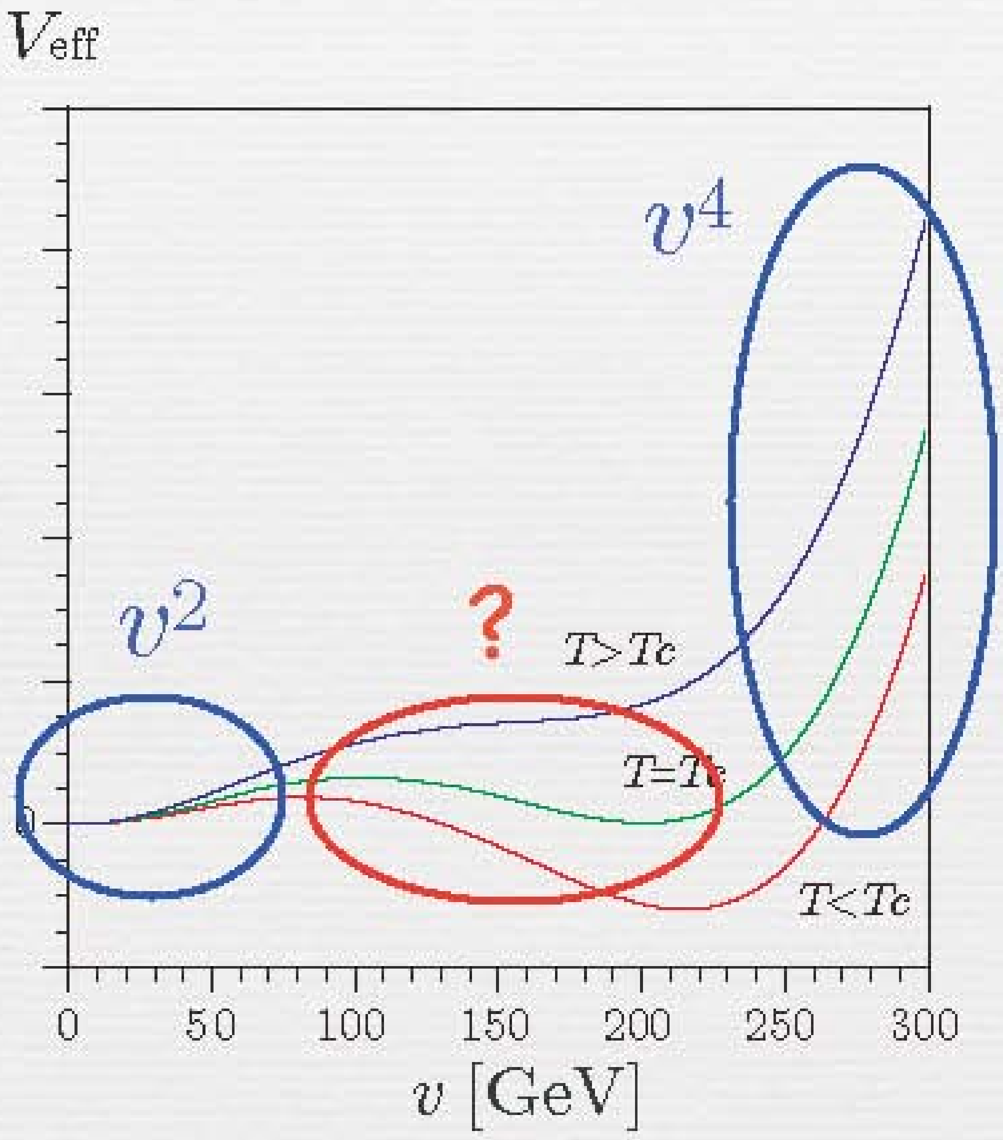
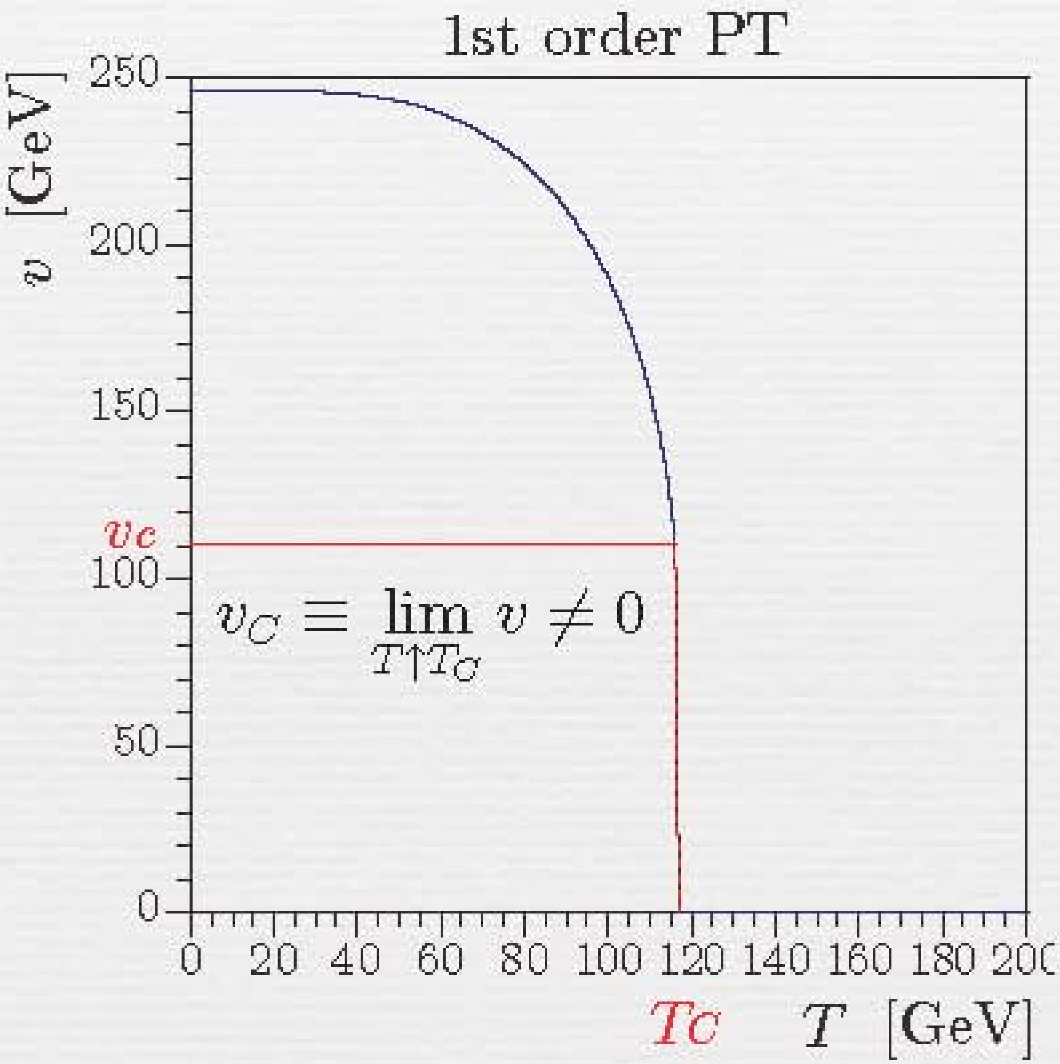




❒ At T_C, the Higgs potential has two degenerate minima.

❒ How do we get the negative contribution in the Higgs potential?

High-T expansion

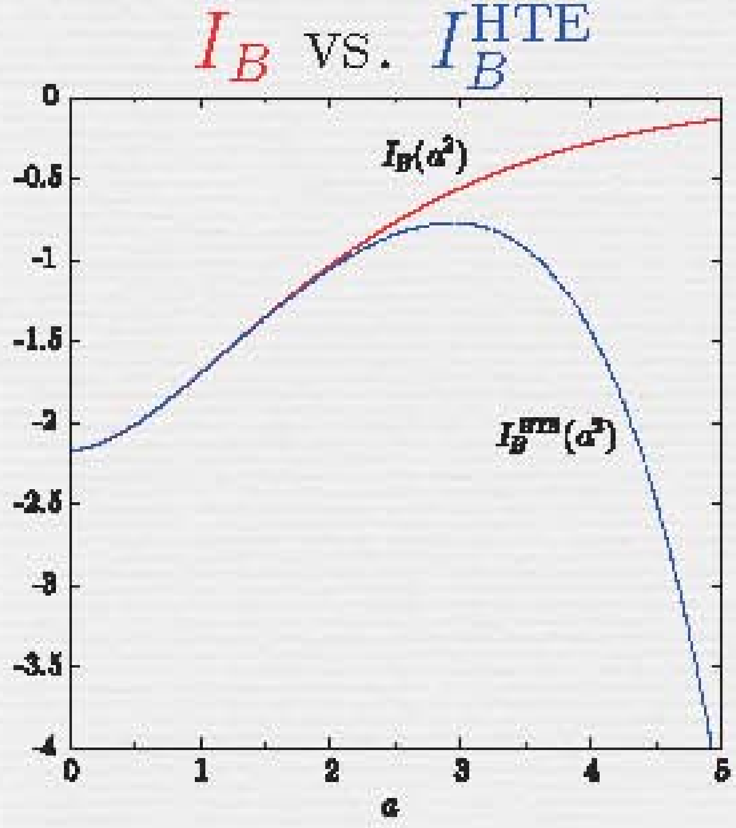
❒ For a small $a = m/T$, $I_{BF}(a^2)$ can be expanded in powers of a^2.

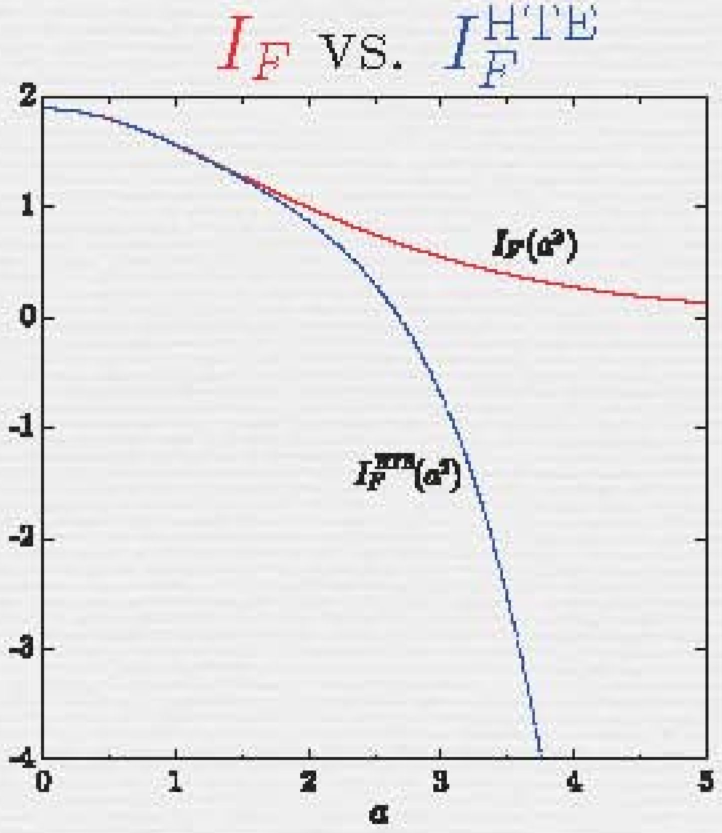
$$I_B^{\mathrm{HTE}}(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{32}\left(\log\frac{a^2}{\alpha_B} - \frac{3}{2}\right) + \mathcal{O}(a^6),$$

$$I_F^{\mathrm{HTE}}(a^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{32}\left(\log\frac{a^2}{\alpha_F} - \frac{3}{2}\right) + \mathcal{O}(a^6).$$

❒ Bosonic loop gives a cubic term with a negative coefficient, which comes from the zero frequency mode.

$$\omega_n = 2n\pi T \text{ for boson} \qquad cf.\ \omega_n = (2n+1)\pi T \text{ for fermion}$$





validity of the HTE

$$|I_B - I_B^{\mathrm{HTE}}| \lesssim 0.05 \text{ if } a \lesssim 2.3$$

$$|I_F - I_F^{\mathrm{HTE}}| \lesssim 0.05 \text{ if } a \lesssim 1.7$$

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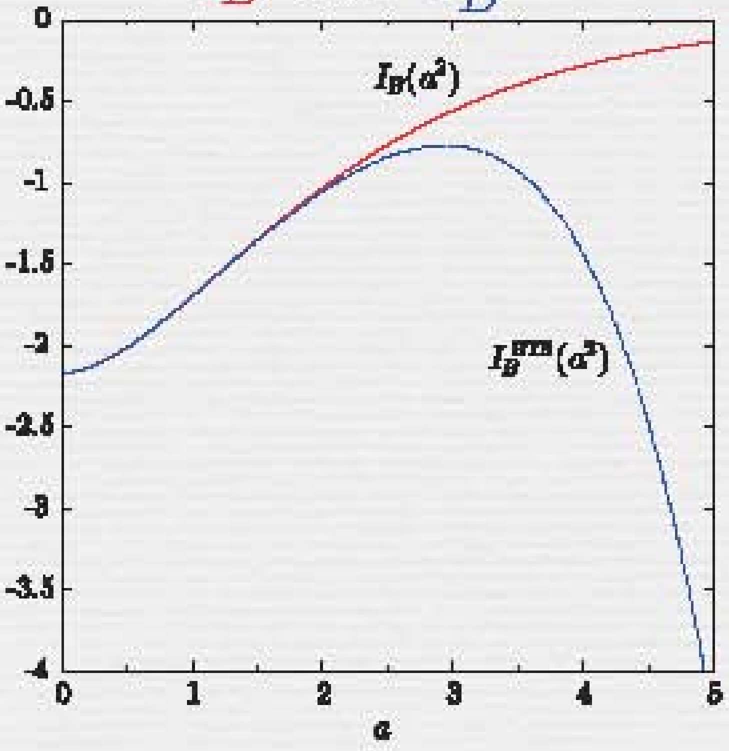
I_B vs. I_B^{HTE}



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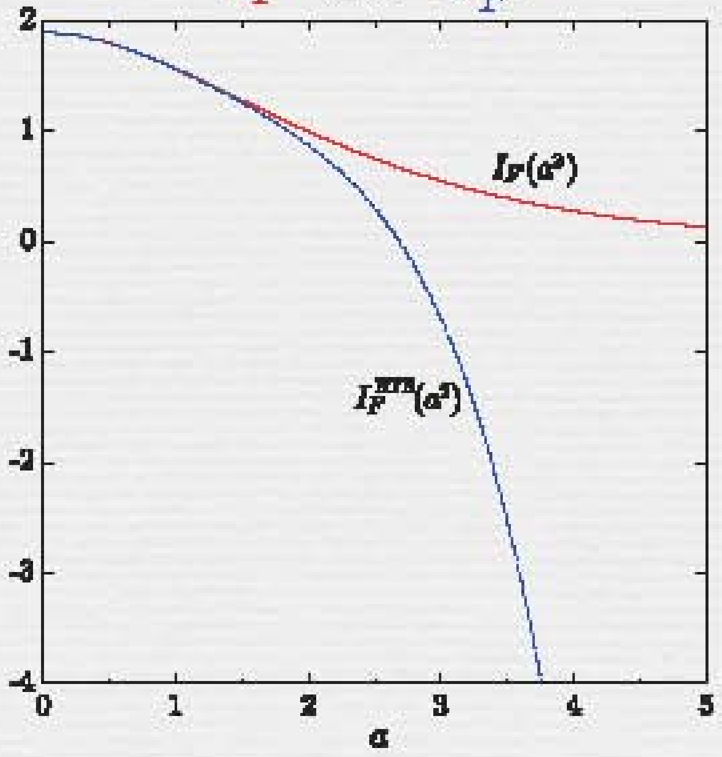

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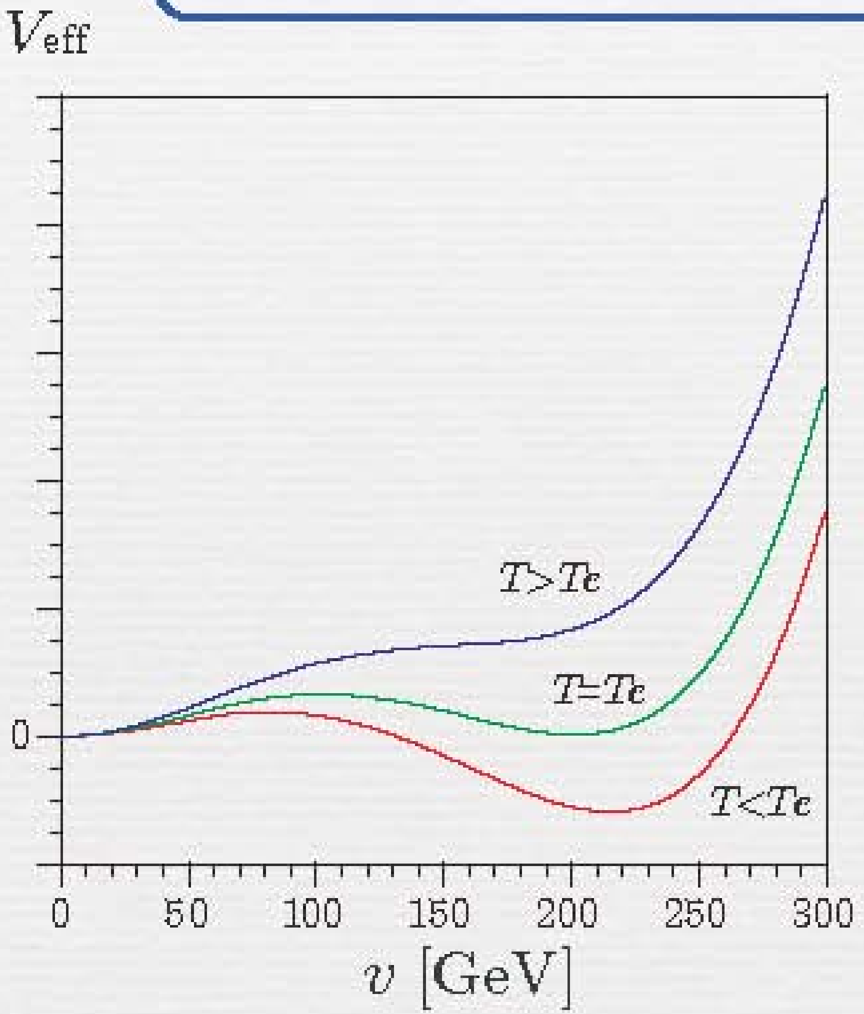
SM EWPT

$$V_{\rm eff} \simeq D(T^2 - T_0^2)v^2 - ETv^3 + \frac{\lambda_T}{4}v^4 \xrightarrow[T=T_C]{} \frac{\lambda_{T_C}}{4}v^2(v-v_C)^2.$$



$$E_{\rm SM} \simeq \frac{1}{4\pi v^3}(2m_W^3 + m_Z^3) \simeq 0.01$$

$$\lambda_{T_C} \simeq \lambda = m_{h^{\rm SM}}^2/(2v^2)$$

$$v_C = \frac{2ET_C}{\lambda_{T_C}} \qquad \frac{v_C}{T_C} \sim \frac{\text{cubic coeff.}}{\text{quartic coeff.}}$$

$$\Gamma_{\rm sph}^{(b)} < H \;\Rightarrow\; v_C/T_C > \zeta \quad \xrightarrow{\zeta=1} \quad m_{h^{\rm SM}} \lesssim 48\ {\rm GeV}$$

(discuss later)

SM EWBG is ruled out.

- ❒ Light Higgs boson (small λ) is favored.
- ❒ Additional bosons (ΔE) can rescue this situation.

Caveat

"Bosons do not always play a role."

Suppose that a boson mass is given by

$$M^2 = m^2 + g^2 v^2,$$

m^2: gauge invariant mass
g: coupling constant

If $m^2 \ll g^2 v^2$ $\quad V_{\text{eff}} \ni -g^3 T v^3 \left(1 + \frac{m^2}{g^2 v^2}\right)^{3/2}$ helpful boson

If $m^2 \gg g^2 v^2$ $\quad V_{\text{eff}} \ni -|m|^3 T \left(1 + \frac{g^2 v^2}{m^2}\right)^{3/2}$ helpless boson

Requirements: 1. large coupling g, 2. small m^2

In the MSSM → top Yukawa coupling: y_t → small $m^2_{\tilde{t}_R}$

stop loop effects

[Carena, Quiros, Wagner, PLB380 ('96) 81]

- LEP bound on m_H;
- ρ parameter

→ $m_{\tilde{q}}^2 \gg m_{\tilde{t}_R}^2, X_t^2, \quad X_t = A_t - \mu \cot\beta$

(6.5 TeV)^2 >

Stop masses

$$\boxed{\begin{aligned}\bar{m}_{\tilde{t}_2}^2 &= m_{\tilde{q}}^2 + \frac{y_t^2 \sin^2\beta}{2}\left(1 + \frac{X_t^2}{m_{\tilde{q}}^2}\right) v^2 + \mathcal{O}(g^2) \simeq m_{\tilde{q}}^2. \\ \bar{m}_{\tilde{t}_1}^2 &= m_{\tilde{t}_R}^2 + \frac{y_t^2 \sin^2\beta}{2}\left(1 - \frac{X_t^2}{m_{\tilde{q}}^2}\right) v^2 + \mathcal{O}(g^2)\end{aligned}}$$

At finite T, there is a thermal correction, $\boxed{\Delta_T m_{\tilde{t}_R}^2} \sim \mathcal{O}(T^2)$.

To have a large loop effect, $m_{\tilde{t}_R}^2 + \Delta_T m_{\tilde{t}_R}^2$ must be small.

$$m_{\tilde{t}_R}^2 + \Delta_T m_{\tilde{t}_R}^2 = 0, \quad \therefore \quad m_{\tilde{t}_R}^2 < 0 \quad \Rightarrow \quad \text{CCB vacuum}$$

$$\boxed{m_{\tilde{t}_1} < m_t}$$

light stop is needed!!

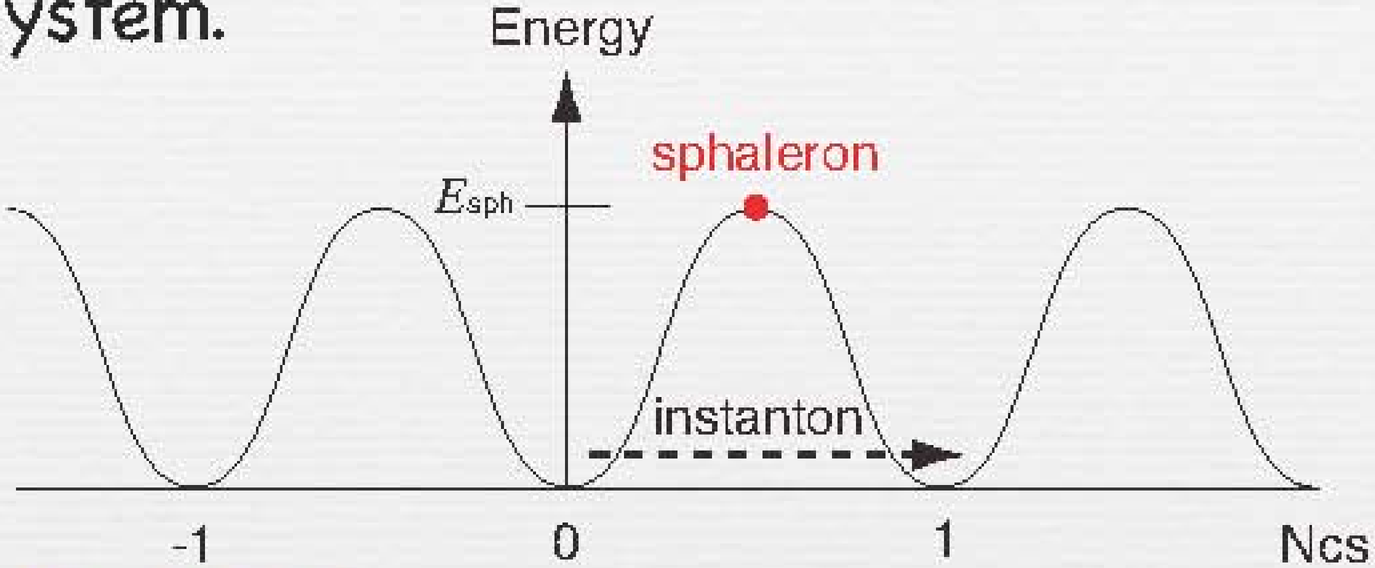
Also, $X_t = 0$ (no-mixing) can maximize the loop effect.

Sphaleron

Sphaleron

❒ Static saddle point solution w/ finite energy of the gauge-Higgs system.

[N.S. Manton, PRD28 ('83) 2019]



$\underline{\Delta B \neq 0}$

Instanton: quantum tunneling

Sphaleron: thermal fluctuation

B violation:

N_f: number of generations

$$\Delta B = N_f \Delta N_{CS}$$

$$N_{CS} = \frac{g_2^2}{32\pi^2}\int d^3x\; \epsilon_{ijk}\mathrm{Tr}\left[F_{ij}A_k - \frac{2}{3}g_2 A_i A_j A_k\right]$$

vacuum transition rates:

symmetric phase: $\Gamma^{(s)}_{\rm sph} \simeq \kappa\alpha_W^4 T, \quad \alpha_W = g_2^2/(4\pi),\ \kappa = \mathcal{O}(1)$

broken phase: $\Gamma^{(b)}_{\rm sph} \simeq T e^{-E_{\rm sph}/T}$

At $T = 0:\ \Gamma \simeq e^{-2S_{\rm instanton}} = e^{-16\pi^2/g_2^2} \simeq 10^{-161}$

❒ B violating process is active at finite T but is suppressed at T=0.

➔ no proton decay problem

Sphaleron solution

❒ gauge-Higgs system in the MSSM $g_1 = 0$ for simplicity

$$\mathcal{L}_{\text{gauge-Higgs}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + (D_\mu\Phi_d)^\dagger D^\mu\Phi_d + (D_\mu\Phi_u)^\dagger D^\mu\Phi_u - V_0(\Phi)$$

$$F_{\mu\nu} = \frac{\tau^a}{2}F^a_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_2[A_\mu, A_\nu], \quad D_\mu\Phi_{d,u} = (\partial_\mu - ig_2A_\mu)\Phi_{d,u}$$

❒ Ansatz for a noncontractible loop $\mu \in [0, \pi],\ (\mu, \theta, \phi) \in S^3,\ \pi_3(SU(2)) \simeq \mathbb{Z}.$

$$A_i(\mu, r, \theta, \phi) = -\frac{i}{g_2}f(r)\partial_i U(\mu, \theta, \phi)U^{-1}(\mu, \theta, \phi) \qquad r = \sqrt{\boldsymbol{x}^2}$$

$$\Phi_d(\mu, r, \theta, \phi) = \frac{v_d}{\sqrt{2}}\left\{(1 - h_1(r))\begin{pmatrix} e^{i\mu}\cos\mu \\ 0 \end{pmatrix} + h_1(r)U(\mu, \theta, \phi)\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\},$$

$$\Phi_u(\mu, r, \theta, \phi) = \frac{v_u e^{i\rho}}{\sqrt{2}}\left\{(1 - h_2(r))\begin{pmatrix} 0 \\ e^{-i\mu}\cos\mu \end{pmatrix} + h_2(r)U(\mu, \theta, \phi)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\},$$

$$U(\mu, \theta, \phi) = \begin{pmatrix} e^{i\mu}(\cos\mu - i\sin\mu\cos\theta) & e^{i\phi}\sin\mu\sin\theta \\ -e^{-i\phi}\sin\mu\sin\theta & e^{-i\mu}(\cos\mu + i\sin\mu\cos\theta) \end{pmatrix}$$

$$\delta E[f, h_1, h_2](\mu = \pi/2) = 0 \longrightarrow \text{EOM of sphaleron}$$

w/ b.c. $f(0) = h_{1,2}(0) = 0, \quad f(\infty) = h_{1,2}(\infty) = 1.$

Sphaleron decoupling condition

❒ To avoid the washout of the generated BAU, the sphaleron process must be decoupled after the PT.

[Arnold, McLerran, PRD36,581 ('87)]

Hubble constant

$$\boxed{-\frac{1}{B}\frac{dB}{dt}} \simeq \frac{13\cdot 3}{4\cdot 32\pi^2}\frac{\omega_-}{\alpha_W^3}\kappa\mathcal{N}_{\rm tr}\mathcal{N}_{\rm rot}e^{-E_{\rm sph}/T} < \boxed{H(T)} \simeq 1.66\sqrt{g_*}T^2/m_{\rm P}$$

$\mathcal{N}_{\rm tr}$: translational zero modes,
$\mathcal{N}_{\rm rot}$: rotational zero modes.

If we denote $E_{\rm sph} = 4\pi v\mathcal{E}/g_2$

$$\boxed{\frac{v}{T} > \frac{g_2}{4\pi\mathcal{E}}\left[42.97 + \ln(\kappa\mathcal{N}_{\rm tr}\mathcal{N}_{\rm rot}) + \ln\left(\frac{\omega_-}{m_W}\right) - \frac{1}{2}\ln\left(\frac{g_*}{106.75}\right) - 2\ln\left(\frac{T}{100\ {\rm GeV}}\right)\right]}$$

In the SM: [Klinkhamer, Manton, PRD30,2212 ('84); Carson, McLerran, PRD41,647 ('90); Akiba, Kikuchi, Yanagida, PRD40,647 ('89), etc (list is not complete)]

$$\mathcal{E} = 2.00,\ \mathcal{N}_{\rm tr}\mathcal{N}_{\rm rot} = 80.13,\ \omega_-^2 = 2.3m_W^2,\ \kappa = 1,\ T = 100\ {\rm GeV},\ \lambda = g_2^2.$$

$$\frac{v}{T} > 0.026 \times (42.97 + \underset{\text{10\% correction}}{4.38} + 0.416) = 1.24$$

MSSM case

❐ We investigate the effects of T and zero modes factors on the sphaleron decoupling condition.

Consider 3 cases

I: based on $V_{\rm eff}(T=0)$ *without* the zero modes
II: based on $V_{\rm eff}(T=0)$ *with* the zero modes
III: based on $V_{\rm eff}(T\neq 0)$ with the zero modes

For the typical parameter set

$$\tan\beta = 10.1,\ m_{H^\pm} = 127.4\ \text{GeV},$$
$$A_t = A_b = -300\ \text{GeV},\ \mu = 100\ \text{GeV}.$$

	I	II	III
$\mathcal{E}$	1.89	1.89	1.77
$\mathcal{N}_{\rm tr}$	—	7.36	6.65
$\mathcal{N}_{\rm rot}$	—	10.84	12.27
$v_N/T_N >$	1.17	1.29	1.38

❐ Zero mode factors cannot be neglected.

❐ T-dependence must be taken into account.

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$\mathcal{E}$	1.89	1.89	1.77
$\mathcal{N}_{\text{tr}}$	—	7.36	6.65
$\mathcal{N}_{\text{rot}}$	—	10.84	12.27
$v_N/T_N >$	1.17	1.29	1.38

❐ Zero mode factors cannot be neglected.

❐ T-dependence must be taken into account.

Q.1 Is $v_C/T_C > 0.9$ enough for the sphaleron decoupling?

A. No, $v_C/T_C \gtrsim 1.4$ is needed.

2-loop analysis

Toy model

❒ As a first step towards the complete analysis, I discuss the PT in the toy model at the 2-loop level.

$$\mathcal{L} = \mathcal{L}_{\text{Abelian}-\text{Higgs}} + \Delta\mathcal{L}$$

$$\mathcal{L}_{\text{Abelian}-\text{Higgs}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)^* D^\mu\Phi - V_0(|\Phi|^2),$$

$$D_\mu\Phi = (\partial_\mu - ieA_\mu)\Phi, \quad V_0(|\Phi|^2) = -\nu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4. \qquad \Phi = \frac{1}{\sqrt{2}}(v + h + ia)$$

$$\Delta\mathcal{L} = -\frac{1}{4}(\partial_\mu G_\nu - \partial_\nu G_\mu)^2 + |(\partial_\mu - ig_3 G_\mu)\tilde{t}|^2 - m_0^2|\tilde{t}|^2 - \frac{\tilde{\lambda}}{4}|\tilde{t}|^4 - y_t^2|\Phi|^2|\tilde{t}|^2.$$

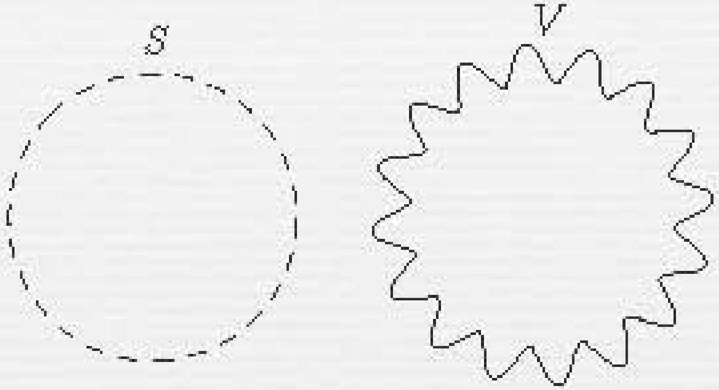
❒ Landau gauge ξ=0, MS-bar scheme

Field-dependent masses

$$\bar{m}_h^2 = -\nu^2 + \frac{3\lambda\mu^\varepsilon}{4}v^2, \quad \bar{m}_a^2 = -\nu^2 + \frac{\lambda\mu^\varepsilon}{4}v^2, \quad \bar{m}_A^2 = e^2\mu^\varepsilon v^2, \quad \bar{m}_{\tilde{t}}^2 = m_0^2 + \frac{y^2\mu^\varepsilon}{2}v^2.$$

Effective potential

1-loop diagrams



$$V^{(1)}(v;T) = \sum_i n_i \left[\underset{T=0}{\boxed{F_0(\bar{m}_i^2)}} + \underset{T\neq 0}{\boxed{\frac{T^4}{2\pi^2} I_B\left(\frac{\bar{m}_i^2}{T^2}\right)}} \right]$$

$$F_0(\bar{m}_i^2) = \frac{\bar{m}_i^4}{64\pi^2}\left(\ln\frac{\bar{m}_i^2}{\bar{\mu}^2} - C_i\right), \quad I_B(a^2) = \int_0^\infty dx\; x^2 \ln\left(1 - e^{-\sqrt{x^2+a^2}}\right)$$

2-loop diagrams

Sunset:

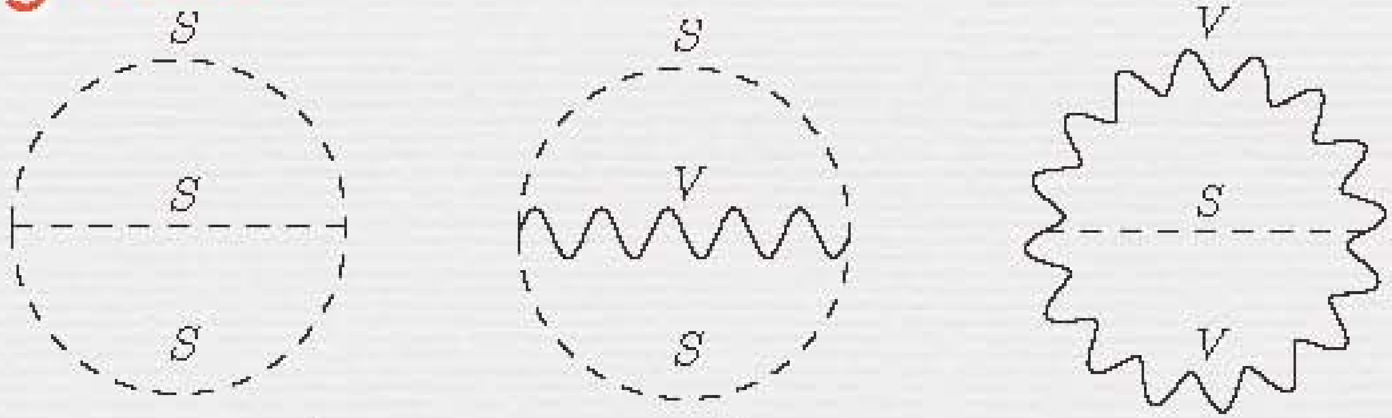
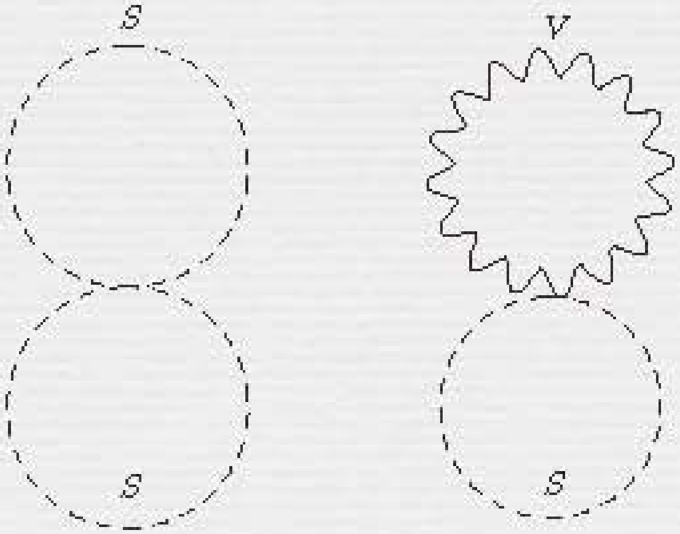
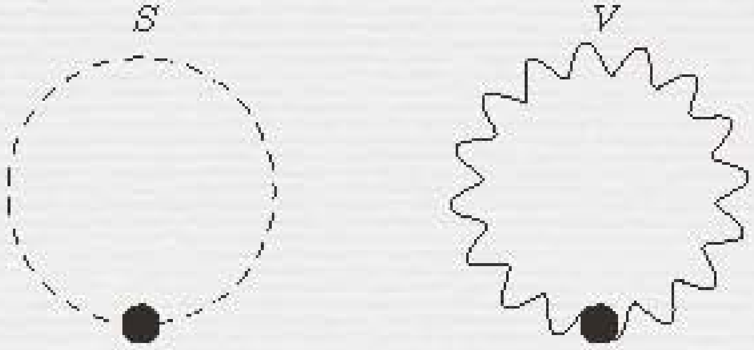


Figure-eight:



counter terms



1-loop resummed $V_{\rm eff}$

❒ At high T, thermal corrections to the masses must be taken into account.

Thermally corrected masses

$$\bar{m}_h^2(T) = \bar{m}_h^2 + \boxed{\frac{T^2}{12}(3e^2+\lambda+y^2),} \quad \bar{m}_a^2(T) = \bar{m}_a^2 + \boxed{\frac{T^2}{12}(3e^2+\lambda+y^2),}$$

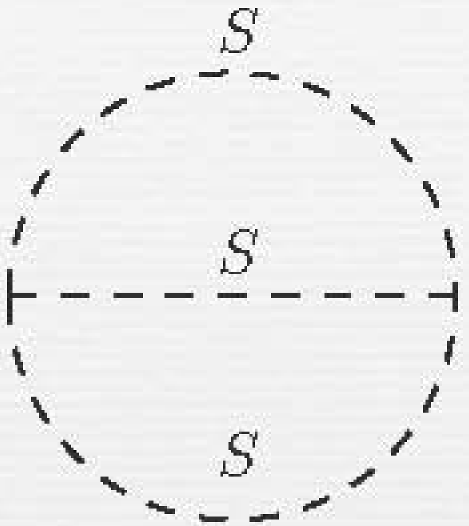
$$\bar{m}_T^2(T) = \bar{m}_A^2, \quad \bar{m}_L^2(T) = \bar{m}_A^2 + \boxed{\frac{e^2}{3}T^2,} \quad \bar{m}_{\tilde{t}}^2(T) = \bar{m}_{\tilde{t}}^2 + \boxed{\frac{T^2}{12}(3g_3^2+\tilde{\lambda}+y^2)}$$

$$\bar{m}_{G,L}^2(T) = 0, \quad \bar{m}_{G,T}^2(T) = \boxed{\frac{g_3^2}{3}T^2.}$$

Resummed potential

$$\boxed{\begin{aligned} V_R^{(1)}(v;T) = {} & \frac{\bar{m}_h^4(T)}{64\pi^2}\left(\ln\frac{\bar{m}_h^2(T)}{\bar{\mu}^2} - \frac{3}{2}\right) + \frac{\bar{m}_a^4(T)}{64\pi^2}\left(\ln\frac{\bar{m}_a^2(T)}{\bar{\mu}^2} - \frac{3}{2}\right) \\ & + \frac{\bar{m}_L^4(T)}{64\pi^2}\left(\ln\frac{\bar{m}_L^2(T)}{\bar{\mu}^2} - \frac{3}{2}\right) + \frac{2\bar{m}_T^4(T)}{64\pi^2}\left(\ln\frac{\bar{m}_T^2(T)}{\bar{\mu}^2} - \frac{1}{2}\right) + \frac{2\bar{m}_{\tilde{t}}^4(T)}{64\pi^2}\left(\ln\frac{\bar{m}_{\tilde{t}}^2(T)}{\bar{\mu}^2} - \frac{3}{2}\right) \\ & + \frac{T^4}{2\pi^2}\left[I_B\left(\frac{\bar{m}_h^2(T)}{T^2}\right) + I_B\left(\frac{\bar{m}_a^2(T)}{T^2}\right) + I_B\left(\frac{\bar{m}_L^2(T)}{T^2}\right) + 2I_B\left(\frac{\bar{m}_T^2(T)}{T^2}\right) + 2I_B\left(\frac{\bar{m}_{\tilde{t}}^2(T)}{T^2}\right)\right], \end{aligned}}$$

Scalar sunset



$$H(m) = \int_P \int_Q \frac{1}{(P^2+m^2)(Q^2+m^2)[(P+Q)^2+m^2]}$$

$$= \underbrace{H^{\text{div}}(m)}_{\text{divergent}} + \underbrace{\tilde{H}(m)}_{\text{finite}}$$

$$\int_P \equiv \mu^\epsilon T \sum_m \int \frac{d^{d-1}\boldsymbol{P}}{(2\pi)^{d-1}}, \quad P^2 = (2m\pi T)^2 + \boldsymbol{P}^2$$

Finite part:

$$\tilde{H}(m) = -\frac{3m^2}{(4\pi)^4}\left[\ln^2\frac{m^2}{\bar{\mu}^2} - 3\ln\frac{m^2}{\bar{\mu}^2} + \frac{7}{2} + \frac{\pi^2}{12} + \frac{2}{3}f_2\right] \quad T\text{=0}$$

$$+\frac{3T^2}{(2\pi)^4}\left[-\left(\ln\frac{m^2T^2}{\bar{\mu}^4} + \ln 4 - 4 + \frac{\pi}{\sqrt{3}}\right)I'_B(a^2) - j_-(a^2) + \frac{1}{4}\boxed{K(a)}\right] \quad T\text{≠0}$$

where $a = m/T$, $f_2 = -1.7579$.

❐ I focus on $K(a)$ which is more relevant than others.

2-loop function

$$K_{--}(a_1,a_2,a_3)=\int_0^1\frac{ds}{e^{a_1}-s}\int_0^1\frac{dt}{e^{a_2}-t}\ln\left|\frac{\bar{Y}_+^{(0)}(s,t;a_1,a_2,a_3)}{\bar{Y}_-^{(0)}(s,t;a_1,a_2,a_3)}\right|,$$

where

$$\begin{aligned}\bar{Y}_\pm^{(0)}(s,t;a_1,a_2,a_3)=16\Big[&-\frac{1}{4}\{(a_1+a_2)^2-a_3^2\}\{(a_1-a_2)^2-a_3^2\}\\&+a_1^2\ln t(\ln t-2a_2)+a_2^2\ln s(\ln s-2a_1)\\&\pm(a_1^2+a_2^2-a_3^2)\sqrt{\ln s(\ln s-2a_1)\ln t(\ln t-2a_2)}\Big]^2.\end{aligned}$$

$$K(a)=K_{--}(a,a,a)$$

For $a{=}m/T{<}1$

HTE of $K(a)$

[R.R.Parwani, PRD45, 4695 (1992)]

$$K^{\rm HTE}(a)=-\frac{\pi^2}{3}\left(\ln a^2+3.48871+\cdots\right)$$

This formula is exclusively used for the EWPT at 2-loop level in the literature.

HTE

The sunset diagram is expressed as

$$\tilde{H}(m) = -\frac{3m^2}{(4\pi)^4}\left[\ln^2\frac{m^2}{\bar{\mu}^2} - 3\ln\frac{m^2}{\bar{\mu}^2} + \frac{7}{2} + \frac{\pi^2}{12} + \frac{2}{3}f_2\right] + \frac{T^2}{64\pi^2}\left[-2\left(\ln\frac{T^2}{\bar{\mu}^2} + \ln 2 - 2\right) - \frac{12}{\pi^2}j(0) - c_H - 2\ln a^2\right] + \mathcal{O}(a).$$

where $j(0) = \zeta(2)(1-\gamma_E) + \zeta'(2)$ and $c_H = 5.3025$.

$$m^2\tilde{H}(m) \ni -\frac{m^2T^2}{16\pi^2}\ln\frac{m}{T} \underset{m\simeq T}{\simeq} +\frac{1}{16\pi^2}\left(m^2T^2 - m^3T\right)$$

If $m \simeq gv$

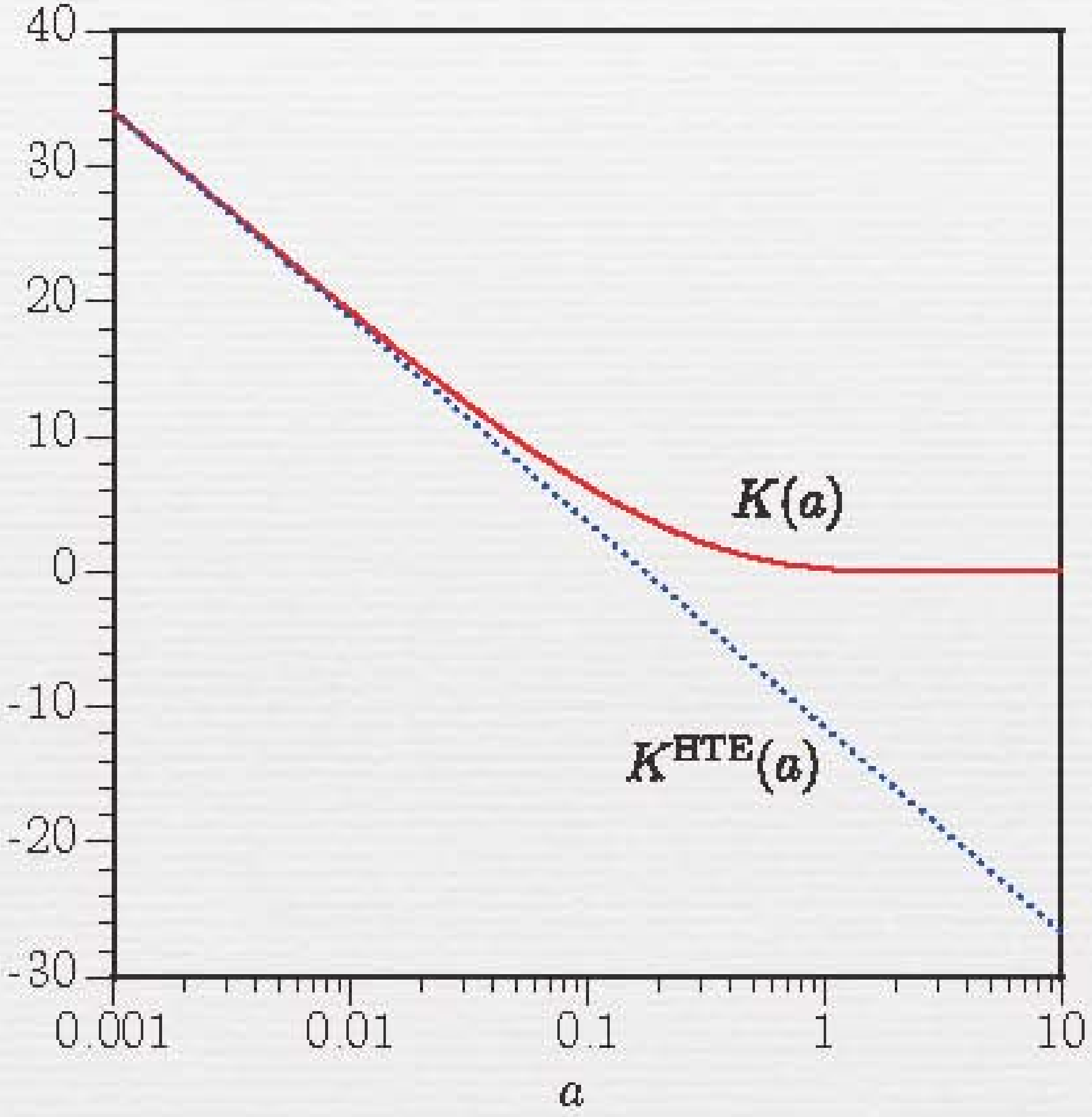
positive quadratic coefficient

-> enhance the curvature at $v=0$

negative cubic coefficient -> enhance v_C/T_C

NOTE: If $-m^2\tilde{H}(m)$, v_C/T_C gets smaller.

Validity of the HTE



❐ Numerical evaluation (red) vs. HTE (blue)

❐ HTE of $K(a)$ is valid only for small a ($\lesssim$0.01).

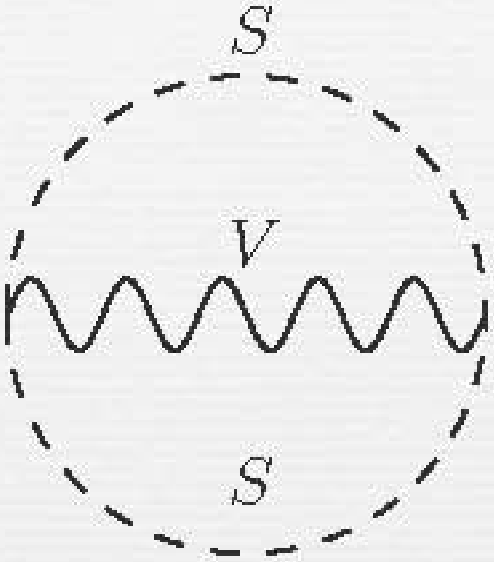
If $T \simeq 100$ GeV, $m < 1$ GeV

❐ For a strong 1st order EWPT, $a=O(1)$.

❐ This result is consistent with the previous study.

[M.Laine, M.Losada,NPB,582 (2000) 277]

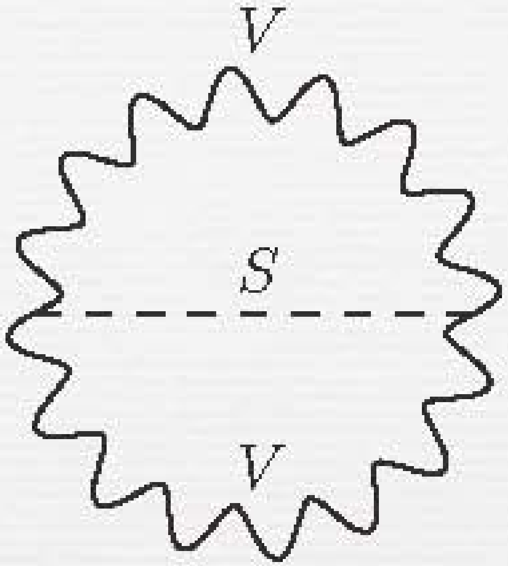
SSV sunset



$$
\begin{aligned}
\mathcal{D}_{SSV}(m_1,m_2,M) &= \int_P \int_Q \frac{4Q^2 - 4(P\cdot Q)^2/P^2}{(P^2+M^2)(Q^2+m_1^2)((P+Q)^2+m_2^2)} \\
&= \mathcal{D}^{II}_{SSV}(m_1,m_2,M) + \mathcal{D}^{I}_{SSV}(m_1,m_2,M) + \underbrace{\tilde{\mathcal{D}}_{SSV}(m_1,m_2,M)}_{\text{finite part}},
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\mathcal{D}}_{SSV}(m_1,m_2,M) = {} & \bar{I}_-(M)\Big(\bar{I}_-(m_1)+\bar{I}_-(m_2)\Big) - \bar{I}_-(m_1)\bar{I}_-(m_2) \\
& + \frac{m_1^2-m_2^2}{M^2}\Big(\bar{I}_-(M)-\bar{I}_-(0)\Big)\Big(\bar{I}_-(m_1)-\bar{I}_-(m_2)\Big) \\
& - \frac{1}{8\pi^2}\Bigg[(m_1^2+m_2^2)i_\epsilon^-(M) + M^2\Big(i_\epsilon^-(m_1)+i_\epsilon^-(m_2)\Big) - i_\epsilon^-(m_1)m_2^2 - i_\epsilon^-(m_2)m_1^2 \\
& \qquad + \frac{m_1^2-m_2^2}{M^2}\Big\{(m_1^2-m_2^2)\Big(i_\epsilon^-(M)-i_\epsilon^-(0)\Big) + M^2\Big(i_\epsilon^-(m_1)-i_\epsilon^-(m_2)\Big)\Big\}\Bigg] \\
& \boxed{\begin{aligned} & + \left[M^2 - 2(m_1^2+m_2^2) + \frac{(m_1^2-m_2^2)^2}{M^2}\right]\tilde{H}(m_1,m_2,M) \\ & - \frac{(m_1^2-m_2^2)^2}{M^2}\tilde{H}(m_1,m_2,0). \end{aligned}} \quad \ni K(a)
\end{aligned}
$$

SVV sunset



$$\mathcal{D}_{SVV}(m,M_1,M_2) = \int_P\int_Q \frac{4(D-2)-4(P\cdot Q)^2/(P^2Q^2)}{(P^2+M_1^2)(Q^2+M_2^2)((P+Q)^2+m^2)}$$

$$= \mathcal{D}^{II}_{SVV}(M_1,M_2,m)+\mathcal{D}^{I}_{SVV}(M_1,M_2,m)+\underbrace{\tilde{\mathcal{D}}_{SVV}(M_1,M_2,m)}_{\text{finite part}},$$

$$\tilde{\mathcal{D}}_{SVV}(M_1,M_2,m)$$

$$= \frac{4}{(4\pi)^4}(m^2+M_1^2+M_2^2)-\frac{1}{2\pi^2}\Big(\bar{I}_-(m)+\bar{I}_-(M_1)+\bar{I}_-(M_2)\Big)$$

$$+\frac{1}{M_1^2}\Big(\bar{I}_-(M_1)-\bar{I}_-(0)\Big)\Big(\bar{I}_-(M_2)-\bar{I}_-(m)\Big)+\frac{1}{M_2^2}\Big(\bar{I}_-(M_2)-\bar{I}_-(0)\Big)\Big(\bar{I}_-(M_1)-\bar{I}_-(m)\Big)$$

$$-\frac{m^2}{M_1^2M_2^2}\Big(\bar{I}_-(M_1)-\bar{I}_-(0)\Big)\Big(\bar{I}_-(M_2)-\bar{I}_-(0)\Big)$$

$$-\frac{1}{8\pi^2}\Bigg[\frac{M_1^2+M_2^2-2m^2}{M_1^2}\Big(i_\varepsilon^-(M_1)-i_\varepsilon^-(0)\Big)$$

$$+\frac{M_1^2+M_2^2-2m^2}{M_2^2}\Big(i_\varepsilon^-(M_2)-i_\varepsilon^-(0)\Big)-2\Big(i_\varepsilon^-(m)-i_\varepsilon^-(0)\Big)\Bigg]$$

$$\boxed{\begin{aligned}&+\frac{1}{M_1^2M_2^2}\Bigg[\Big\{(M_1^2+M_2^2-m^2)^2+8M_1^2M_2^2\Big\}\tilde{H}(m,M_1,M_2)\\&\qquad-(M_1^2-m^2)^2\tilde{H}(m,M_1,0)-(M_2^2-m^2)^2\tilde{H}(m,M_2,0)+m^4\tilde{H}(m,0,0)\Bigg].\end{aligned}} \ni K(a)$$

Figure-eight diagrams

- ❒ Putting all figure-eight diagrams together, one gets

$$\mu^{\varepsilon}V_{\not{R}}^{(\text{fig-8})}(v;T) = \frac{\lambda}{16}\Big[3\big(I_-^2(\bar{m}_h)+I_-^2(\bar{m}_a)\big)+2I_-(\bar{m}_h)I_-(\bar{m}_a)\Big]$$

$$+\frac{e^2}{2}(D-1)I_-(\bar{m}_A)\big(I_-(\bar{m}_h)+I_-(\bar{m}_a)\big),$$

$$+\frac{y^2}{2}I_-(\bar{m}_{\tilde{t}})\Big[I_-(\bar{m}_h)+I_-(\bar{m}_a)\Big]+\frac{\tilde{\lambda}}{2}I_-^2(\bar{m}_{\tilde{t}})+(D-1)g_3^2I_-(0)I_-(\bar{m}_{\tilde{t}})$$

where $D = 4-\epsilon$

$$I_-(m^2) = -\frac{m^2}{16\pi^2}\frac{2}{\epsilon}+\bar{I}_-(m^2)+\epsilon i_{\varepsilon}^-(m^2)+\mathcal{O}(\epsilon^2),$$

$$\bar{I}_-(m^2) = \frac{m^2}{16\pi^2}\left(\ln\frac{m^2}{\bar{\mu}^2}-1\right)+\frac{T^2}{\pi^2}I_B'(a^2),$$

$$i_{\varepsilon}^-(m^2) = -\frac{m^2}{64\pi^2}\left[\left(\ln\frac{m^2}{\bar{\mu}^2}-1\right)^2+1+\frac{\pi^2}{6}\right]-\frac{T^2}{2\pi^2}\left[\left(\ln\frac{T^2}{\bar{\mu}^2}+\ln 4-2\right)I_B'(a^2)+j(a^2)\right].$$

- ❒ No $K(a)$ in the figure-eight diagrams.
- ❒ Corrections to quartic term. (v_C/T_C gets smaller.)

2-loop V_{eff}

Unresummed potential

$$
\begin{aligned}
\mu^{\varepsilon} V_{\not{R}}^{(2)}(v;T) = & -\frac{\lambda^2\mu^{\varepsilon}}{16} v^2 \left[3\tilde{H}(\bar{m}_h) + \tilde{H}(\bar{m}_h, \bar{m}_a, \bar{m}_a)\right] - \frac{e^2}{2}\tilde{\mathcal{D}}_{SSV}(\bar{m}_h, \bar{m}_a, \bar{m}_A) \\
& -\frac{e^4\mu^{\varepsilon}v^2}{4}\tilde{\mathcal{D}}_{SVV}(\bar{m}_h, \bar{m}_A, \bar{m}_A) - \frac{g_3^2}{2}\tilde{\mathcal{D}}_{SSV}(\bar{m}_{\tilde{t}}, \bar{m}_{\tilde{t}}, 0) - \frac{y^4\mu^{\varepsilon}}{2} v^2 \tilde{H}(\bar{m}_h, \bar{m}_{\tilde{t}}, \bar{m}_{\tilde{t}}) \\
& + \frac{\lambda}{16}\left[3\Big(\bar{I}_-^2(\bar{m}_h) + \bar{I}_-^2(\bar{m}_a)\Big) + 2\bar{I}_-(\bar{m}_h)\bar{I}_-(\bar{m}_a)\right] + \frac{3}{2}e^2\bar{I}_-(\bar{m}_A)\Big(\bar{I}_-(\bar{m}_h) + \bar{I}_-(\bar{m}_a)\Big) \\
& + \frac{e^2}{16}\left[\bar{m}_A^2\Big(\bar{I}_-(\bar{m}_h) + \bar{I}_-(\bar{m}_a)\Big) + \left(\bar{m}_h^2 + \bar{m}_a^2 - \frac{10}{3}\bar{m}_A^2\right)\right] \\
& + \frac{y^2}{2}\bar{I}_-(\bar{m}_{\tilde{t}}^2)\Big(\bar{I}_-(\bar{m}_h) + \bar{I}_-(\bar{m}_a)\Big) + \frac{\tilde{\lambda}}{2}\bar{I}_-^2(\bar{m}_{\tilde{t}}) + g_3^2\bar{I}_-(0)\left(3\bar{I}_-(\bar{m}_{\tilde{t}}^2) + \frac{\bar{m}_{\tilde{t}}^2}{8\pi^2}\right) \\
& + \frac{1}{16\pi^2}\left[3e^2\nu^2 + \left(6e^4 + \frac{5}{4}\lambda^2 - \frac{9}{4}\lambda e^2 + y^4\right)\mu^{\varepsilon}v^2\right] i_{\varepsilon}^{(-)}(\bar{m}_h) \\
& + \frac{1}{16\pi^2}\left[3e^2\nu^2 + \left(\frac{1}{4}\lambda^2 - \frac{3}{4}\lambda e^2\right)\mu^{\varepsilon}v^2\right] i_{\varepsilon}^{(-)}(\bar{m}_a) \\
& + \frac{1}{16\pi^2}\left[6e^2\nu^2 + (-3\lambda e^2 + 10e^4)\mu^{\varepsilon}v^2\right] i_{\varepsilon}^{(-)}(\bar{m}_A) \\
& + \frac{1}{8\pi^2}\left[-3g_3^2 m_0^2 + y^2\left(-\frac{3}{2}g_3^2 + y^2\right)\mu^{\varepsilon}v^2\right] i_{\varepsilon}^{(-)}(\bar{m}_{\tilde{t}}) - \frac{3g_3^2}{8\pi^2}\bar{m}_{\tilde{t}}^2 i_{\varepsilon}^{(-)}(0).
\end{aligned}
$$

sunset

Assumptions

- ☹ Sunset diagram with the unequal masses

$$H(m_1, m_2, m_3) = \int_P \int_Q \frac{1}{(P^2 + m_1^2)(Q^2 + m_2^2)[(P+Q)^2 + m_3^2]},$$

which is approximated by

$$H(m_1, m_2, m_3) = H\left(\frac{m_1 + m_2 + m_3}{3}\right) + \mathcal{O}(m^2).$$

[P. Arnold and O. Espinosa, PRD47, (1993) 3546]

- ☹ Resummation

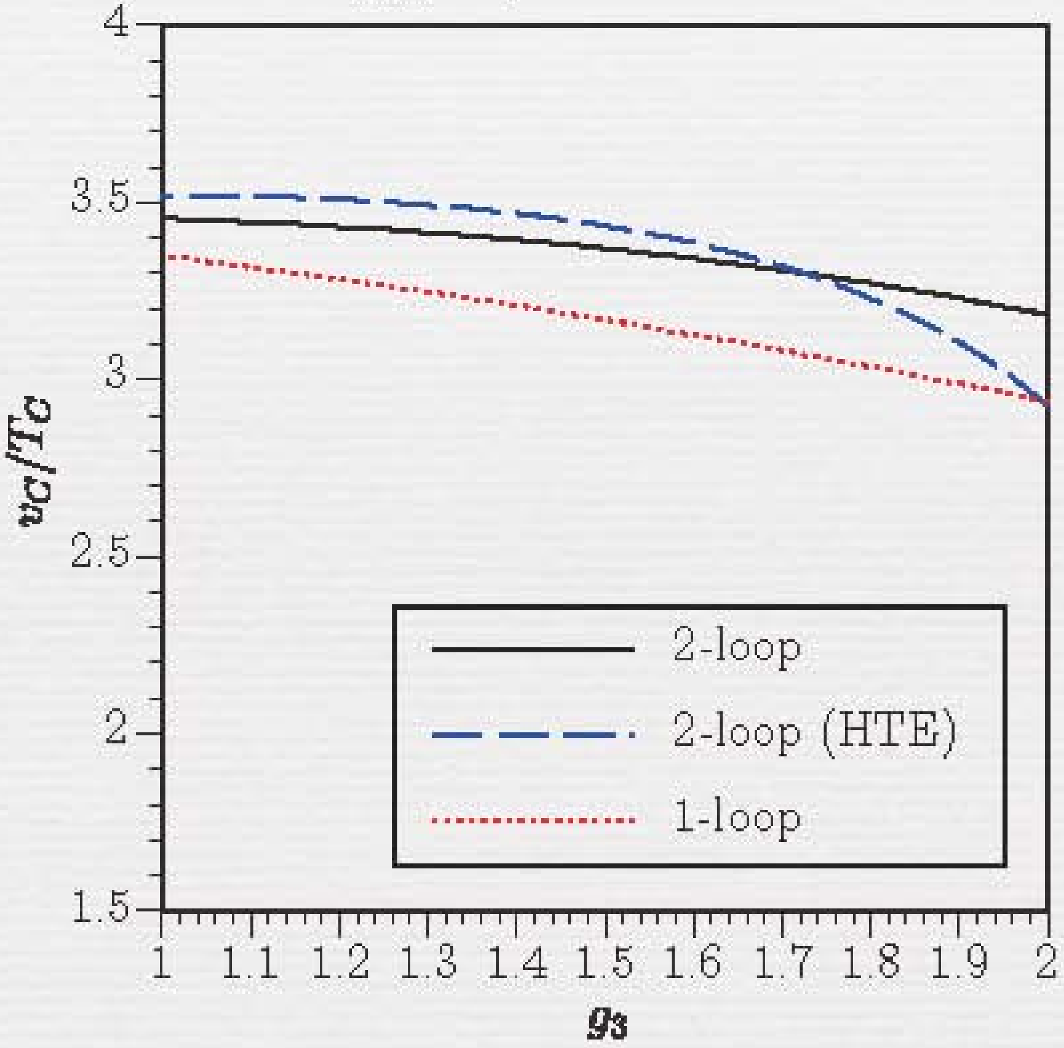
I replace the masses with the thermally corrected masses under the assumption $\bar{m}_T^2(T) = \bar{m}_L^2(T)$ (theoretically unjustifiable, though)

v_C/T_C

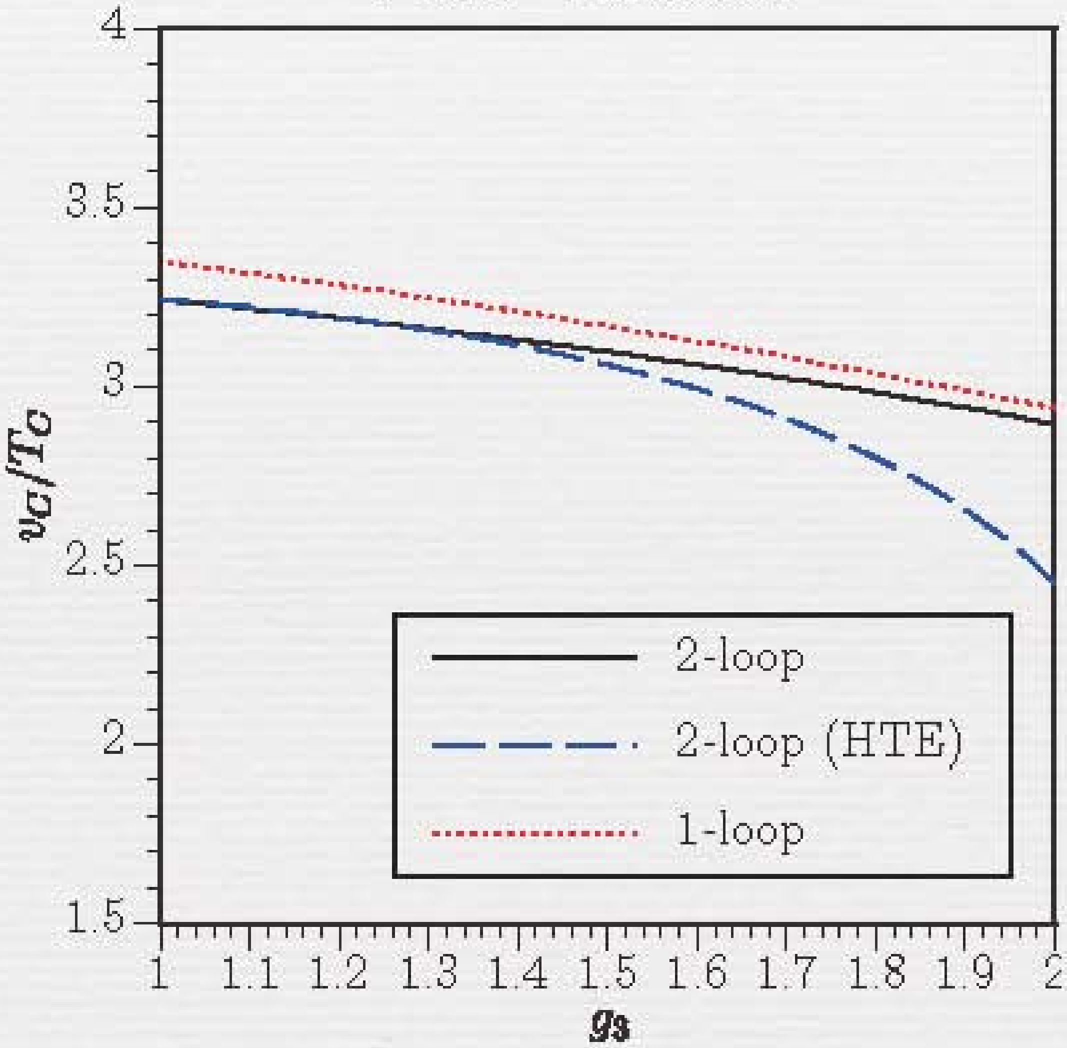
(preliminary)

$$v_0 = 200,\ e = 0.5,\ y = 1.0,\ \lambda = \tilde{\lambda} = 0.03,\ m_0^2 = 0.$$

Abelian-Higgs part (λ,e) + $\tilde{t}$-$\tilde{t}$-G_μ



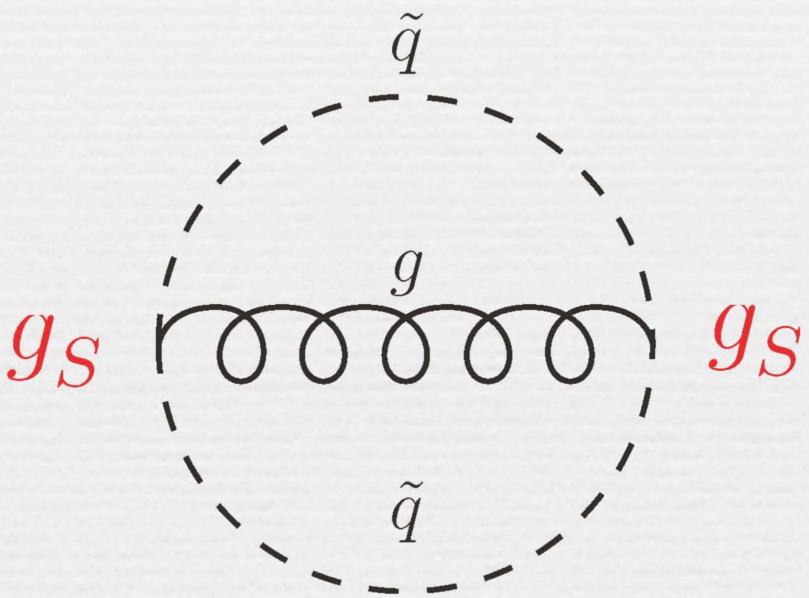
full result

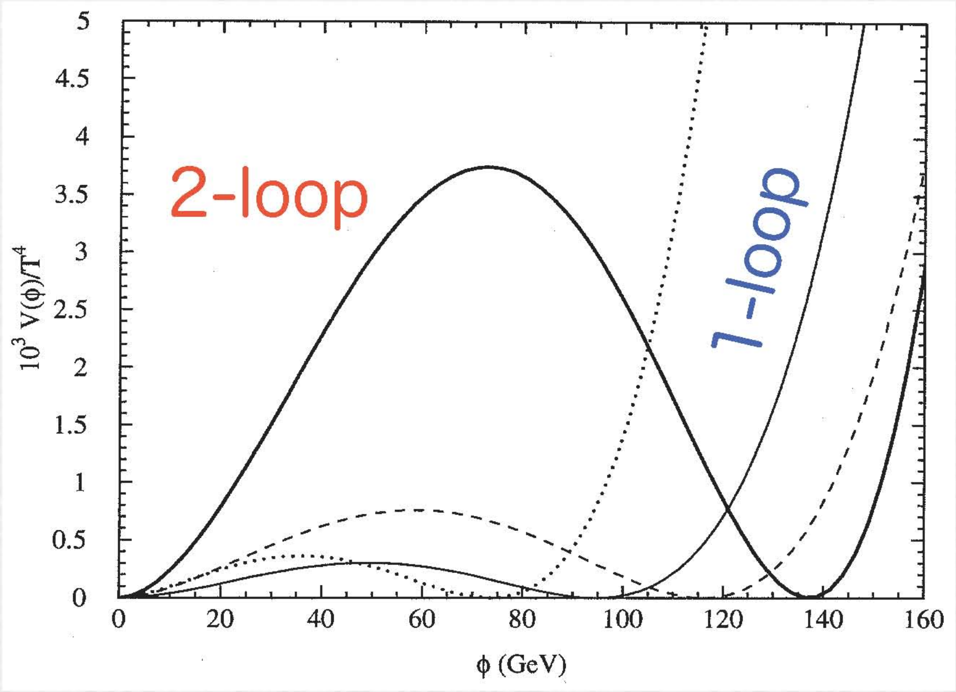


- 2-loop effect can play a role.
- v_C/T_C can be enhanced if $\tilde{t}$-$\tilde{t}$-G_μ diagram is dominant.
- But, v_C/T_C is overestimated if the HTE of $K(a)$ is used.

Towards MSSM baryogenesis

❒ stop-stop-gluon sunset diagram is the dominant 2-loop contribution to v_C/T_C. [J.R. Espinosa, NPB475, ('96) 273 etc]





Q2. How reliable is the HTE at the 2-loop level?

My answer: numerical evaluation w/o the HTE is necessary.

Summary

- ❒ We have discussed the sphaleron decoupling condition in the MSSM.
- ❒ $v_C/T_C \gtrsim$ 1.4 is needed for the sphaleron decoupling.
- ❒ We also examined the validity of the high-T expansion used in the 2-loop effective potential using the toy model.
- ❒ HTE of $K(a)$ is valid only for small a<0.01.
- ❒ $\tilde{t}$-$\tilde{t}$-G_μ diagram can enhance v_C/T_C.
- ❒ HTE of K(a) can lead to the overestimated v_C/T_C.

Analysis of the EWPT at the 2-loop level w/o the HTE is necessary to check the feasibility of the MSSM baryogenesis.

Back Matter

Review papers on EWBG

- A.G. Cohen, D.B. Kaplan, A.E. Nelson, hep-ph/9302210
- M. Quiros, Helv.Phys.Acta 67 ('94)
- V.A. Rubakov, M.E. Shaposhnikov, hep-ph/9603208
- K. Funakubo, hep-ph/9608358
- M. Trodden, hep-ph/9803479
- A. Riotto, hep-ph/9807454
- W. Bernreuther, hep-ph/0205279

Experimental constraints

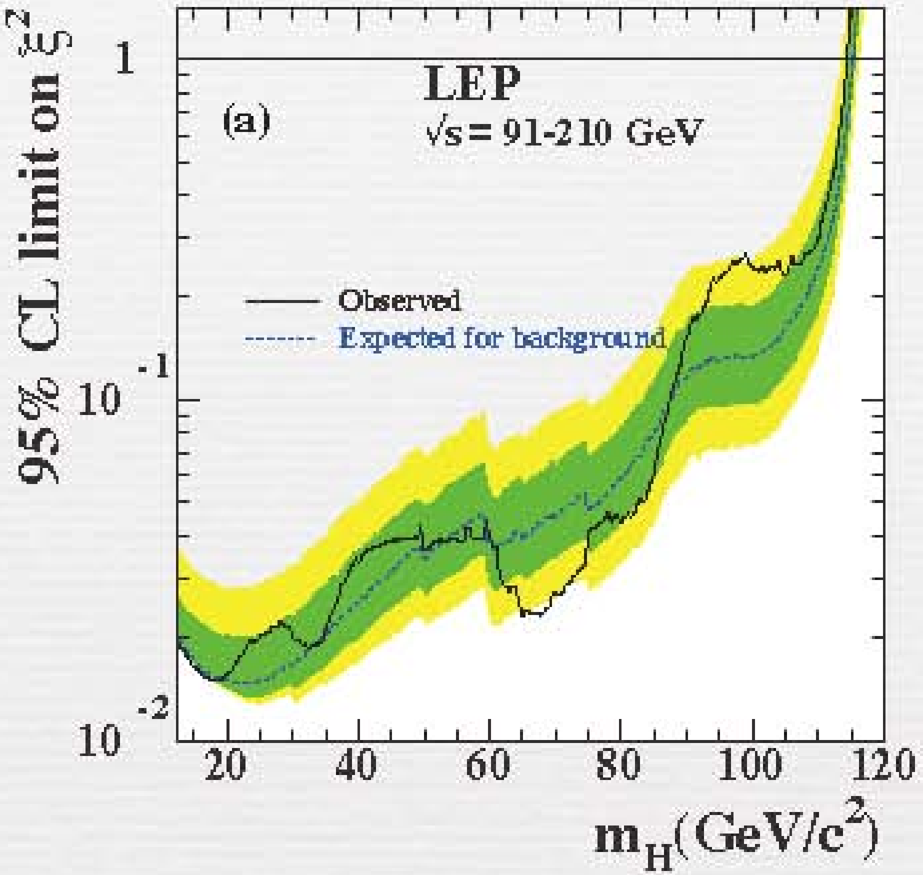
- **Higgs bounds@LEP** [PLB565, 61 (2003)]

$$g^2_{H_iZZ} \times \mathrm{Br}(H_i \to f\bar{f}) < \mathcal{F}_{H_iZ}(m_{H_i}),$$

$$g^2_{H_iH_jZ} \times \mathrm{Br}(H_i \to f\bar{f}) \times \mathrm{Br}(H_j \to f\bar{f}) < \mathcal{F}_{H_iH_j}(m_{H_i} + m_{H_j}),$$

where $f = b, \tau$. $\mathcal{F}_{H_iZ}$ and $\mathcal{F}_{H_iH_j}$ are the 95% C.L. upper limits

e.g. Higgsstrahlung



- **Lower bounds for SUSY particles:**

e.g. $m_{\tilde{t}_1} > 95.7$ GeV, $m_{\chi_1^\pm} > 94$ GeV, $m_{\chi_1^0} > 46$ GeV

- **ρ-parameter:**

$$\Delta\rho < 0.002$$

- **B physics observables:**

If, (Higgs mass)<140 GeV

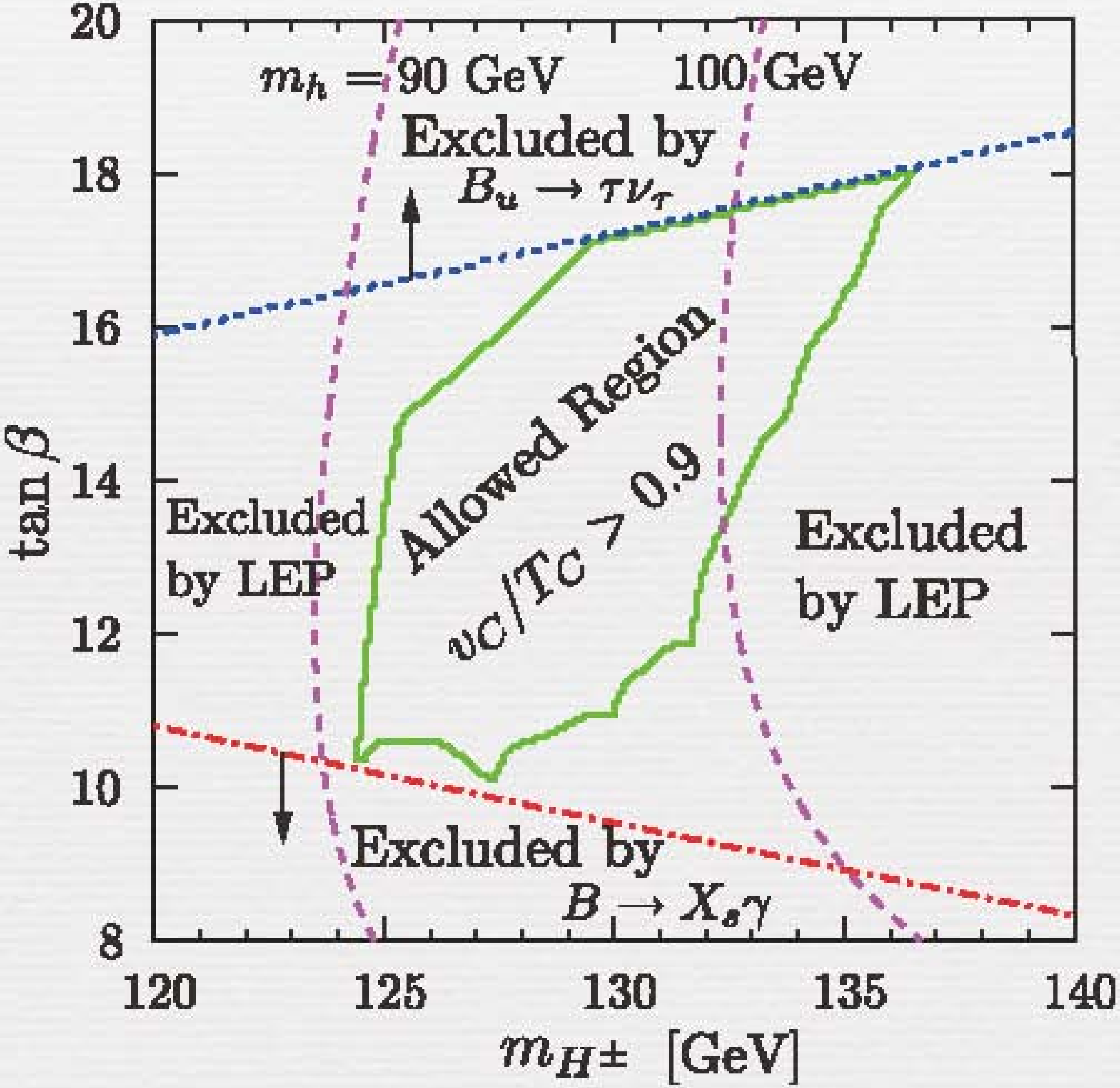
constraints on

$$\left.\begin{array}{rcl} \mathrm{Br}(B_u \to \tau\nu_\tau)_{\mathrm{exp}} & = & 1.41^{+0.43}_{-0.42} \times 10^{-4}, \\ \mathrm{Br}(\overline{B} \to X_s\gamma)_{\mathrm{exp}} & = & (3.52 \pm 0.23 \pm 0.09) \times 10^{-4}, \end{array}\right\} \text{light charged Higgs bosons}$$

$$\mathrm{Br}(B_s \to \mu^+\mu^-)_{\mathrm{exp}} < 0.23 \times 10^{-7}. \quad \Longrightarrow \quad \text{light neutral Higgs bosons}$$

v_C/T_C in the allowed region

$m_{\tilde{q}} = 1200$ GeV, $m_{\tilde{t}_R} = 10^{-4}$ GeV, $m_{\tilde{b}_R} = 1000$ GeV, $A_t = A_b = -300$ GeV.



- ☐ $v_C/T_C > 0.9$
- ☐ max of v_C/T_C

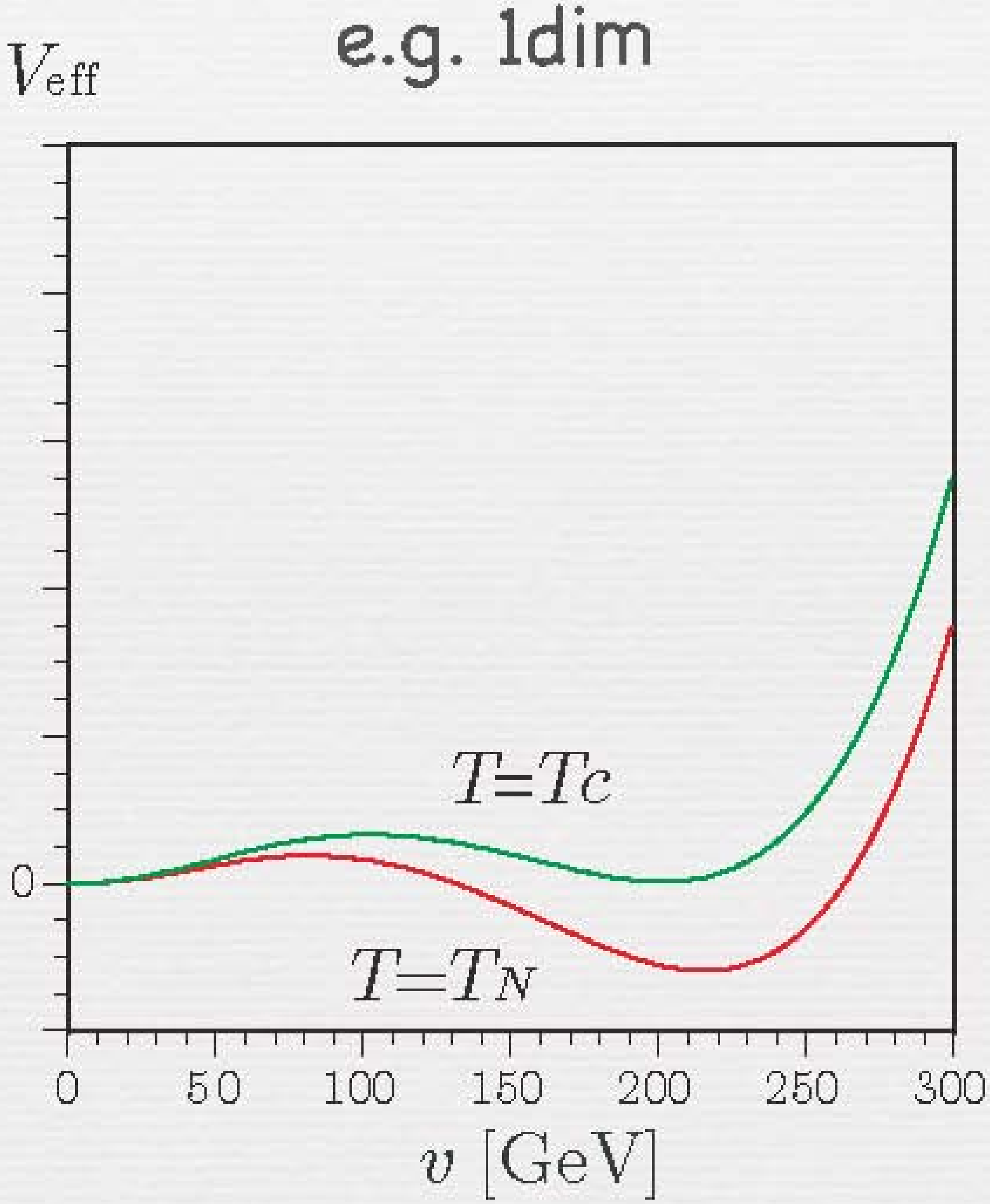
$\tan\beta = 10.1$, $m_{H^\pm} = 127.4$ GeV

$$\boxed{\frac{v_C}{T_C} = \frac{107.10\ \text{GeV}}{116.27\ \text{GeV}} = 0.92}$$

- ☐ Sphaleron process is not decoupled at T_C.
- ☐ loophole: "supercooling"

⇒ PT begins to proceed with bubble wall at below T_C.

Below T_C



We evaluate the nucleation temperature T_N.

Critical bubble

❒ EWPT develops with the expanding bubbles.

"Not all bubbles can grow"

☹ Bubble which is smaller than a critical size

Surface energy dominates ⇒ shrink by the surface tension

☺ Bubble which is larger than a critical size

Volume energy dominates ⇒ grow

Critical bubble = bubble which has the critical size

Critical bubble

Energy functional in the temporal gauge:

$$E = \int d^3\boldsymbol{x}\left[\frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}B_{ij}B_{ij} + (D_i\Phi_d)^\dagger D_i\Phi_d + (D_i\Phi_u)^\dagger D_i\Phi_u + V_{\rm eff}(\Phi_d,\Phi_u;T)\right]$$

Here we assume that the least energy has the pure-gauge config. for $A^a(x)$ and $B(x)$, $F_{ij} = B_{ij} = 0$.

Higgs fields:

$$\Phi_d = \frac{1}{\sqrt{2}}\begin{pmatrix}\rho_d\\ 0\end{pmatrix}, \quad \Phi_u = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \rho_u\end{pmatrix},$$

Equation of motion (EOM):

b.c.

$$\begin{aligned} -\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\rho_d}{dr}\right) + \frac{\partial V_{\rm eff}}{\partial\rho_d} &= 0, \qquad \lim_{r\to\infty}\rho_d(r) = 0, \ \lim_{r\to\infty}\rho_u(r) = 0, \\ -\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\rho_u}{dr}\right) + \frac{\partial V_{\rm eff}}{\partial\rho_u} &= 0. \qquad \left.\frac{d\rho_d(r)}{dr}\right|_{r=0} = 0, \ \left.\frac{d\rho_u(r)}{dr}\right|_{r=0} = 0. \end{aligned}$$

$$r = \sqrt{\boldsymbol{x}^2}$$

☐ Solutions can exist only for $T_0 < T < T_C$.

At T_0, $V_{\rm eff}$ at the origin is destabilized. ⇒ not a local min.

Bubble nucleation

- ❐ Nucleation rate per unit time per unit volume

$$\Gamma_N(T) \simeq T^4 \left(\frac{E_{\rm cb}(T)}{2\pi T} \right)^{3/2} e^{-E_{\rm cb}(T)/T}$$

[A.D. Linde, NPB216 ('82) 421]

$E_{\rm cb}(T)$: energy of the critical bubble at T

- ❐ Definition of nucleation temperature (T_N)

horizon scale $\simeq H^{-1}(T)$

$$\boxed{\Gamma_N(T_N) H(T_N)^{-3} = H(T_N)}$$

$$\frac{E_{\rm cb}(T_N)}{T_N} - \frac{3}{2} \ln \left(\frac{E_{\rm cb}(T_N)}{T_N} \right) = 152.59 - 2 \ln g_*(T_N) - 4 \ln \left(\frac{T_N}{100 \text{ GeV}} \right)$$

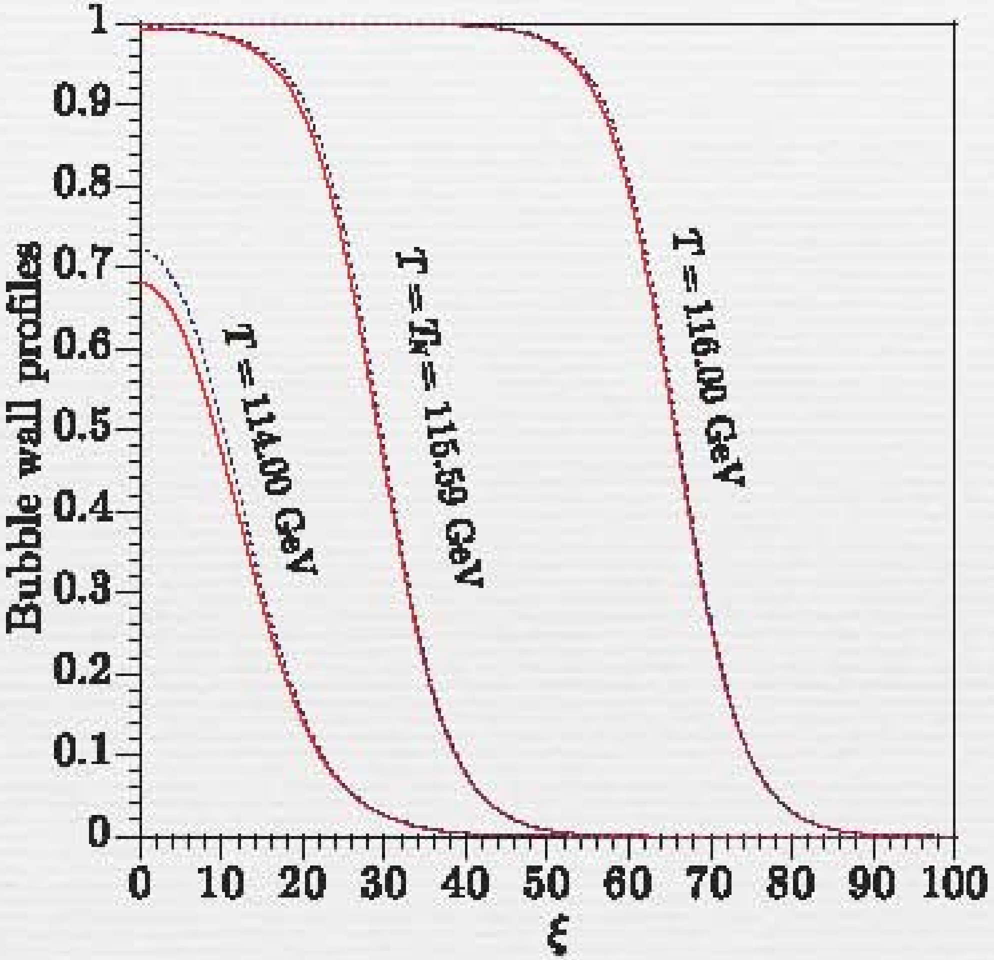
Roughly, $E_{\rm cb}/T \lesssim 150$ is necessary for the development of the EWPT.

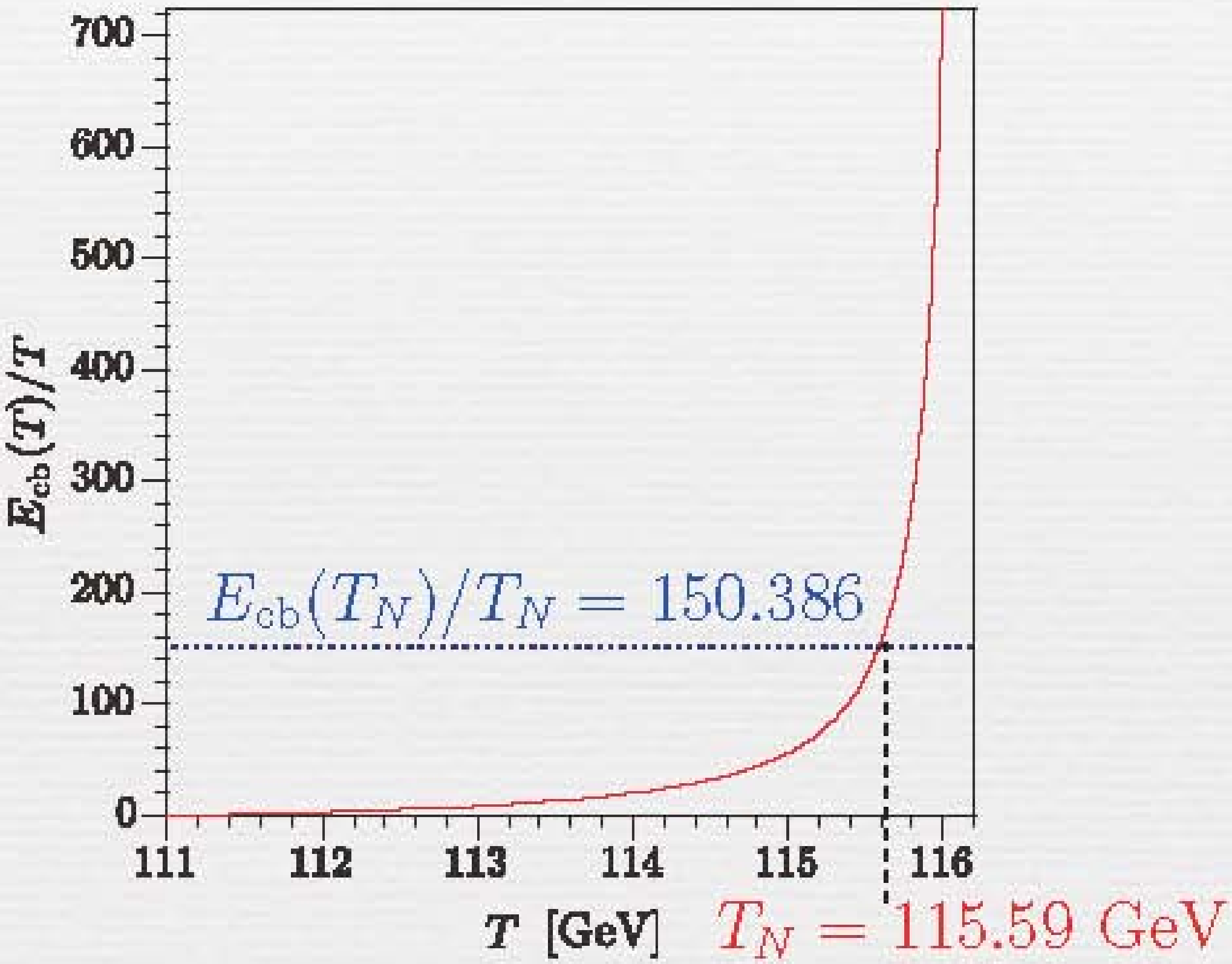
Bubble nucleation

$$m_{\tilde{q}} = 1200\ \mathrm{GeV},\ m_{\tilde{t}_R} = 10^{-4}\ \mathrm{GeV},\ m_{\tilde{b}_R} = 1000\ \mathrm{GeV},\ A_t = A_b = -300\ \mathrm{GeV}.$$

$$\tan\beta = 10.1,\ m_{H^\pm} = 127.4\ \mathrm{GeV}$$

$$\xi = vr,\ h_1(\xi) = \frac{\rho_d(r)}{v\cos\beta},\ h_2(\xi) = \frac{\rho_u(r)}{v\sin\beta}$$





$$\boxed{\frac{v_N}{T_N} = \frac{116.73\ \mathrm{GeV}}{115.59\ \mathrm{GeV}} = 1.01}$$

10% enhancement! But, $\boxed{\frac{v_N}{T_N} > 1.38}$

Sphaleron process is not decoupled at T_N either.

Light Higgs scenario

$$m_h < 114.4 \text{ GeV}, \quad m_Z \sim m_A$$

$A_t = A_b = -300$ GeV, $m_{\tilde{t}_R} = 10^{-4}$ GeV, $m_{\tilde{b}_R} = 1000$ GeV,
$|\mu| = 100$ GeV.

$m_{\tilde{q}}$ (GeV)	1200	1300	1400	1500
$\tan\beta$	10.11	9.87	9.75	9.57
$m_{H^\pm}$ (GeV)	127.40	127.40	127.50	127.50
v_C/T_C	$\frac{107.096}{116.274} = 0.921$	$\frac{107.512}{116.496} = 0.923$	$\frac{107.769}{116.770} = 0.923$	$\frac{107.915}{117.045} = 0.922$
$\tan\beta_C$	13.803	13.640	13.597	13.455
v_N/T_N	$\frac{116.727}{115.585} = 1.010$	$\frac{117.155}{115.798} = 1.012$	$\frac{117.404}{116.067} = 1.012$	$\frac{117.531}{116.339} = 1.010$
$\tan\beta_N$	13.676	13.503	13.453	13.307
$E_{\text{cb}}(T_N)/T_N$	150.386	150.379	150.370	150.360
$\mathcal{E}$	1.769	1.770	1.770	1.771
$\mathcal{N}_{\text{tr}}$	6.652	6.658	6.662	6.667
$\mathcal{N}_{\text{rot}}$	12.266	12.253	12.240	12.229
$v_N/T_N >$	1.383	1.382	1.382	1.380

Typically, $v_N/T_N > 1.38$ is needed for the sphaleron decoupling.

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Decoupling limit

$$m_h > 114.4 \text{ GeV} \quad m_Z \ll m_A$$

- No strong B physics constraints.

$|A_t| = |A_b| = |\mu|/\tan\beta$, $m_{\tilde{t}_R} = 10^{-4}$ GeV, $m_{\tilde{b}_R} = 1000$ GeV,
$|\mu| = 100$ GeV.

$m_{\tilde{q}}$ (GeV)	1700	1800	1900	2000
$\tan\beta$	42.62	15.10	10.97	9.35
$m_{H^\pm}$ (GeV)	1000.00	1000.00	1000.00	1000.00
v_C/T_C	$\frac{111.461}{116.993} = 0.953$	$\frac{111.460}{117.007} = 0.953$	$\frac{111.483}{116.994} = 0.953$	$\frac{111.440}{117.060} = 0.952$
$\tan\beta_C$	42.966	15.171	11.022	9.394
v_N/T_N	$\frac{121.454}{116.221} = 1.045$	$\frac{121.452}{116.236} = 1.045$	$\frac{121.478}{116.222} = 1.045$	$\frac{121.424}{116.288} = 1.044$
$\tan\beta_N$	42.955	15.168	11.019	9.392
$E_{\text{cb}}(T_N)/T_N$	150.366	150.370	150.364	150.360
$\mathcal{E}$	1.773	1.773	1.773	1.773
$\mathcal{N}_{\text{tr}}$	6.677	6.677	6.678	6.678
$\mathcal{N}_{\text{rot}}$	12.211	12.210	12.210	12.209
$v_N/T_N >$	1.379	1.379	1.379	1.379

- No region satisfying the sphaleron decoupling condition either.

Decoupling limit

$$m_h > 114.4 \text{ GeV} \quad m_Z \ll m_A$$

- No strong B physics constraints.

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- No region satisfying the sphaleron decoupling condition either.

Loop integrals

$$I'_B(a^2) = \frac{1}{2}\int_0^\infty dx \frac{x^2}{\sqrt{x^2+a^2}} \frac{1}{e^{\sqrt{x^2+a^2}}-1},$$

$$j(a^2) = \int_0^\infty dx \frac{x^2 \ln x}{\sqrt{x^2+a^2}} \frac{1}{e^{\sqrt{x^2+a^2}}-1}.$$

For $a = m/T < 1$ HTE of $I'_B(a^2)$ and $j(a^2)$

$$I_B^{\prime\mathrm{HTE}}(a^2) = \frac{\pi^2}{12} - \frac{\pi}{4}(a^2)^{1/2} - \frac{a^2}{16}\left(\log\frac{a^2}{\alpha_B} - 1\right) + \mathcal{O}(a^4),$$

$$j^{\mathrm{HTE}}(a^2) = j(0) - \frac{\pi\sqrt{a^2}}{4}\ln a^2 - \frac{a^2}{8}\left(\ln^2\frac{\sqrt{a^2}}{2} + \ln\frac{\sqrt{a^2}}{2}\right)$$

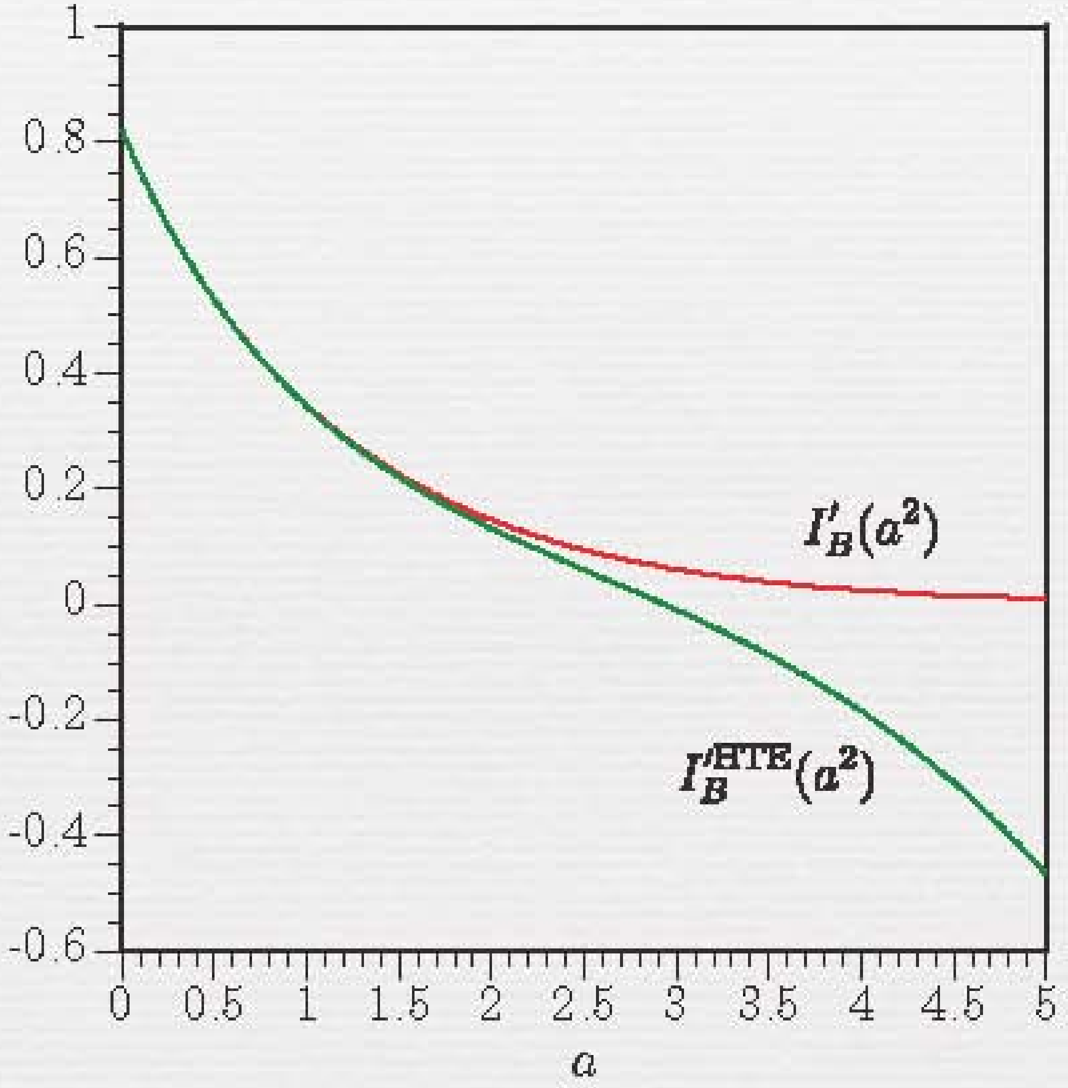
$$+\frac{a^2}{4}\left[\ln 2\pi + \frac{1}{2}\ln^2 2\pi + \frac{1}{4} - (\gamma_E + \gamma_E \ln 2\pi + \gamma_1) + \frac{\pi^2}{24}\right]$$

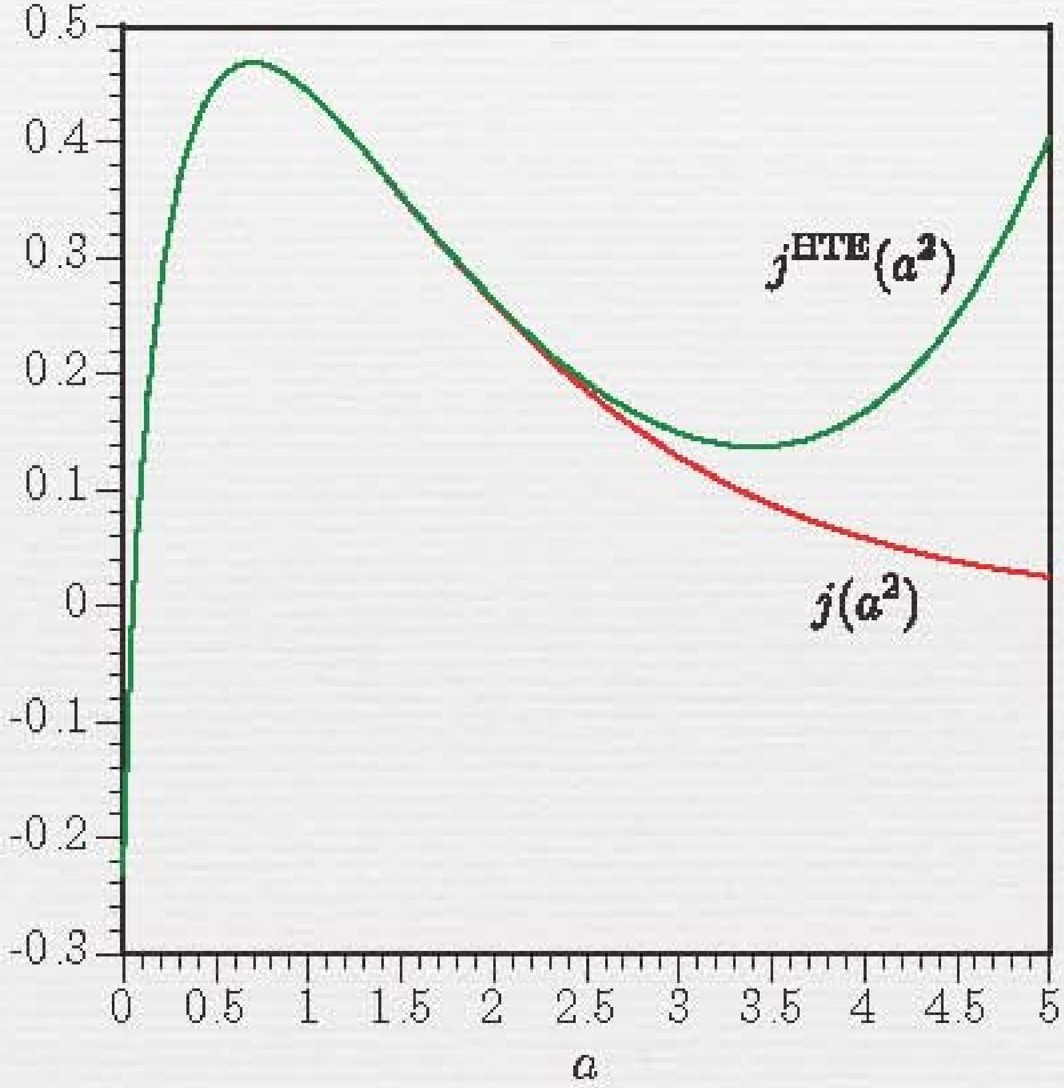
$$+\frac{a^4}{64\pi^2}(\zeta(3)\ln 2\pi - \zeta'(3)) + \mathcal{O}(a^6).$$

where $j(0) = \zeta(2)(1-\gamma_E) + \zeta'(2)$

Validity of HTE

Numerical integration vs. HTE





$|I'_B(a^2) - I'^{\rm HTE}_B(a^2)| \lesssim 0.01$ for $a \lesssim 1.8$, $|j(a^2) - j^{\rm HTE}(a^2)| \lesssim 0.01$ for $a \lesssim 2.6$