Jets at weak and strong coupling

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YH, JHEP 0811:057 (2008)
Avsar, YH, Matsuo, JHEP 0903:011 (2009)
YH, Ueda, PRD 80:074018 (2009)

Outline

- Motivation: Energy flow away from jets
- Jets in QCD
- A curious similarity between jet physics and high energy scattering
    1. Nonglobal logarithms
    2. BFKL logarithms
- e+e- annihilation in AdS/CFT and its feedback on QCD

High-pt electroweak bosons at the LHC

Highly boosted EW bosons (W,Z) might be important for the discovery of physics beyond the SM. e.g., Agashe et al. (2007)

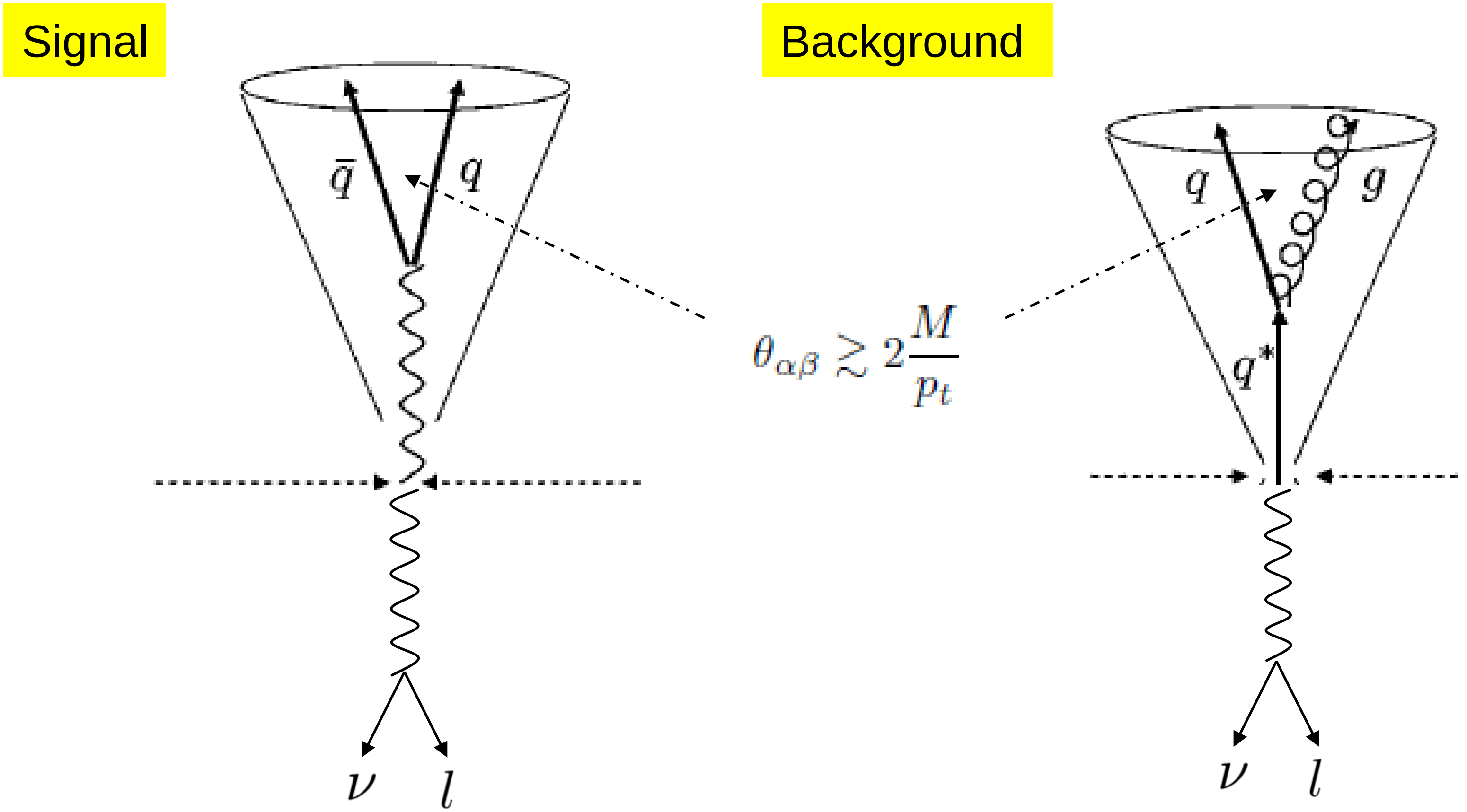
$$pp \longrightarrow Z' \longrightarrow WW, \quad W \to l\nu, \quad W \to jj$$

TeV scale new particles (KK exitations)

$p_t^W \sim 1\ \mathrm{TeV}$
$m_J \sim 100\ \mathrm{GeV}$ ⟶ QCD background from W+1-jet events

Find a method to distinguish them

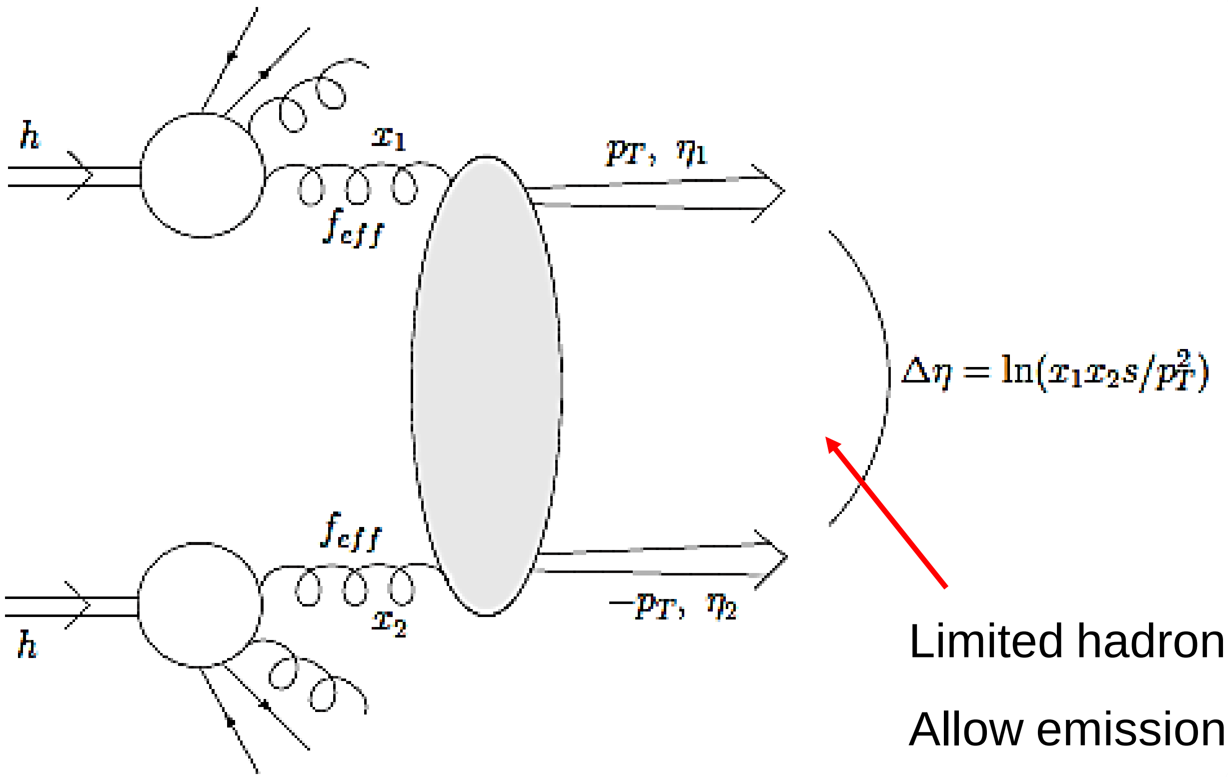
Energy flow



Very little radiation
(typically $\ll \alpha_s p_t$)
due to the QCD coherence

Sizable radiation $\lesssim \alpha_s p_t$ expected

Rapidity gap events at the Tevatron, HERA & LHC



Limited hadronic activity

Allow emission up to $\sum_{i\in gap} E_i \leq E_{out}$

Gap survival probability Oderda and Sterman (1998)

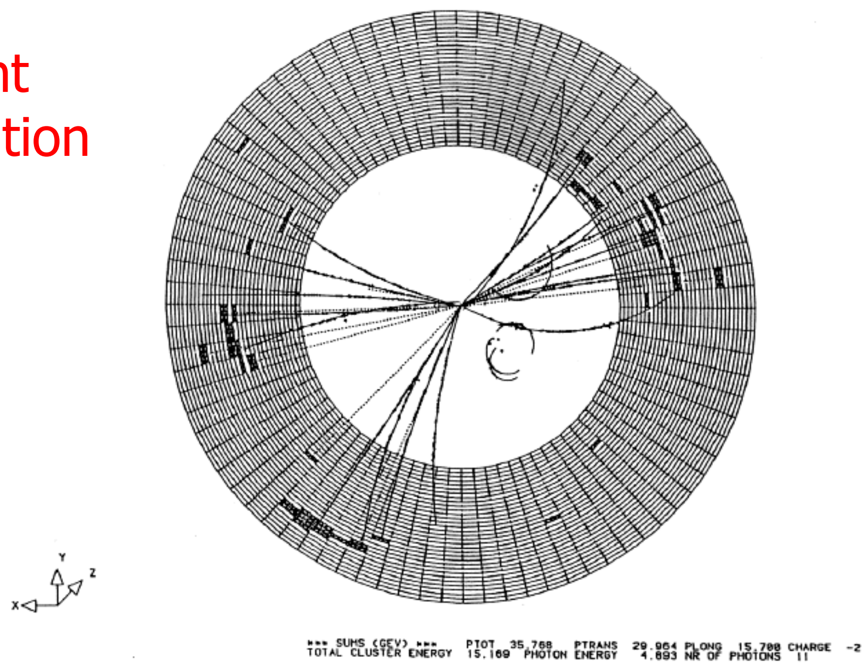
“Jet vetoing” in Higgs plus dijets events (gluon fusion vs. weak boson fusion)

Forshaw, et al. (2007~)

Jets in QCD

Observation of jets in 1975 has provided one of the most striking confirmations of QCD

A three-jet event
in e+e- annihilation



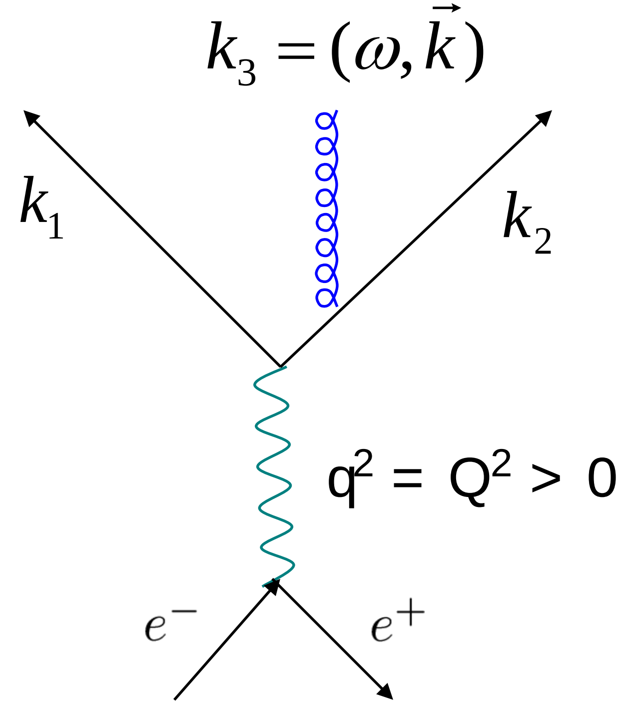
Average angular distribution of two jets $1 + \cos^2 \mu$
reflecting fermionic degrees of freedom (quarks)

One-gluon emission

Peskin-Schroeder
"*Introduction to Quantum Field Theory*"
Part I, Final project

$$dP = \frac{2g^2 C_F}{q^2} \frac{d^3\vec{k}}{2\omega(2\pi)^3} \frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

$$x_i = \frac{2q\cdot k_i}{q^2} \qquad x_1 + x_2 + x_3 = 2$$



Collinear divergence

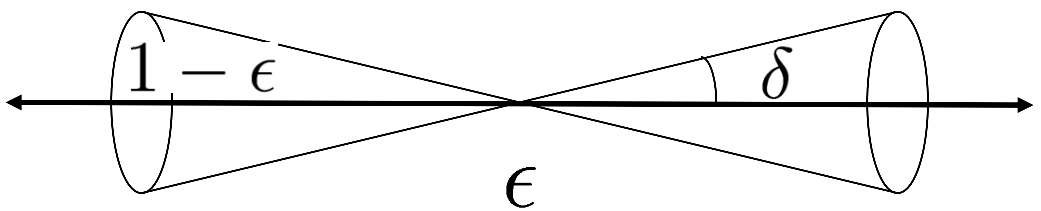
$x_1 \to 1$ and/or $x_2 \to 1$

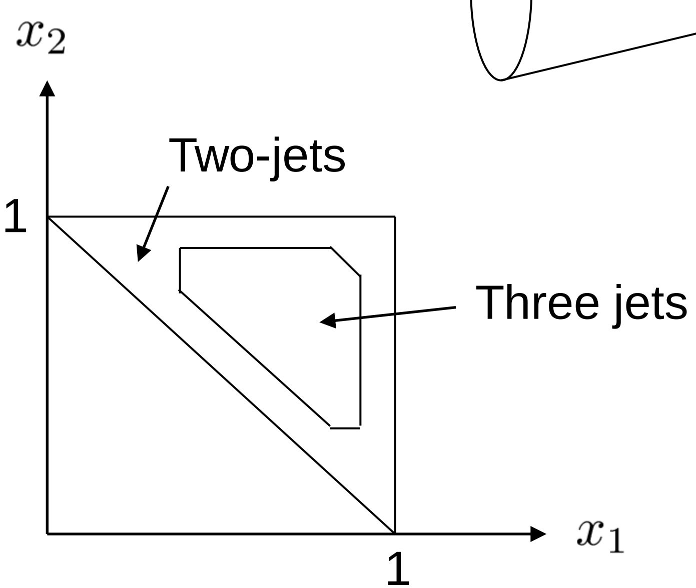
Soft divergence ! ! 0

cancel in the total cross section $\sigma_{tot} = \frac{4\pi\alpha_{em}^2}{3Q^2} \sum_f Q_f^2 \left(1 + \frac{\alpha_s}{\pi} + \cdots\right)$

The Sterman-Weinberg jet (1977)

Call it a two-jet event if the energy fraction emitted outside two cones of opening angle δ is less than ϵ





$$d\sigma = d\sigma_0 \left(1 - \frac{\alpha_s C_F}{\pi} \ln\delta \ln\epsilon + \cdots\right)$$

soft singularity

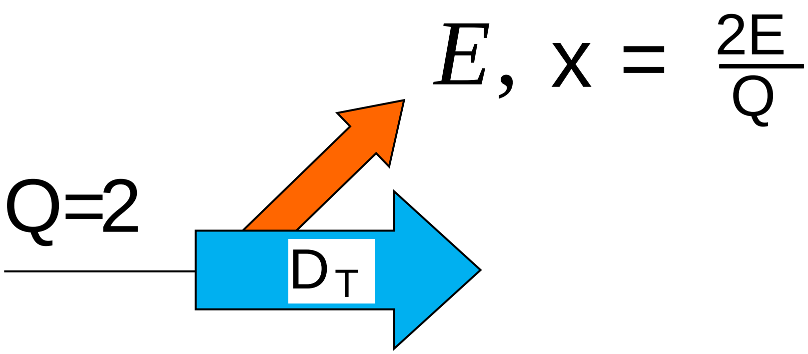
collinear singularity

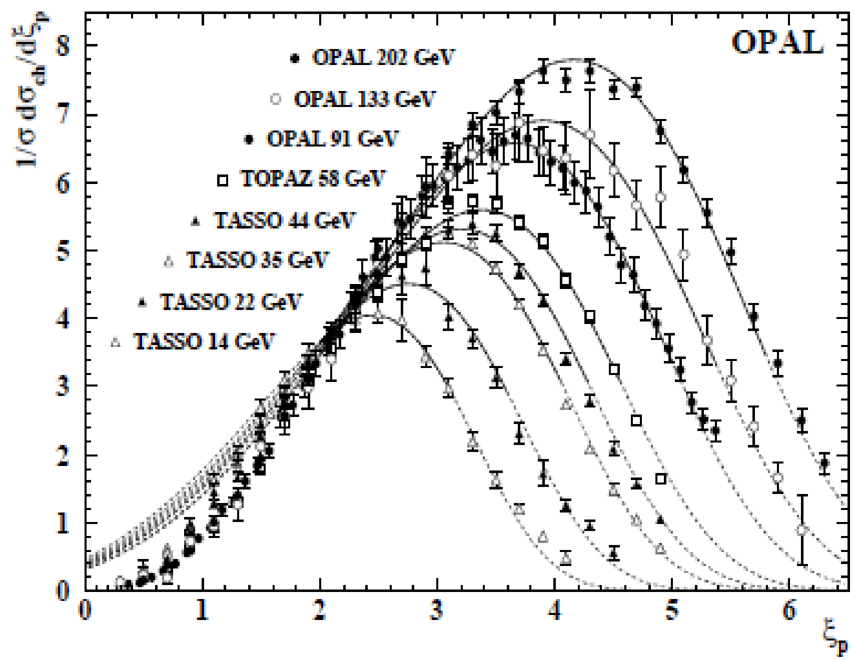
“Sudakov double-log”

Jet substructure

DGLAP equation for the fragmentation function

$$\frac{\partial}{\partial \ln Q^2} D_T(x,Q^2) = \int_x^1 \frac{dz}{x'} P_T(x') D_T\left(\frac{x}{x'},Q^2\right)$$



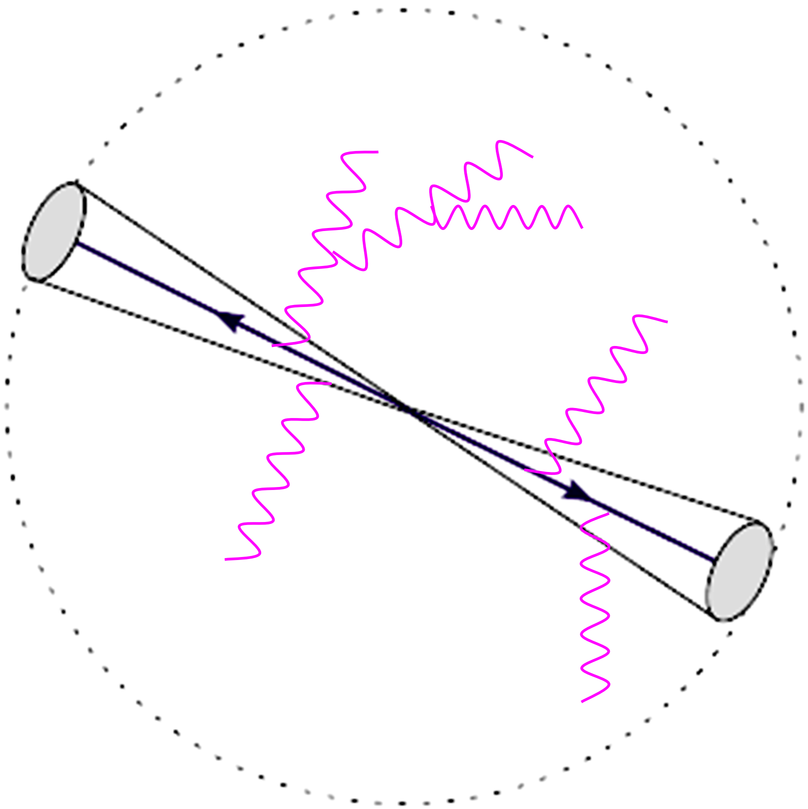


"hump-backed" distribution

Double-log approximation

$$P_T(x) \approx \frac{1}{x}$$

Away-from-jets region



Gluons emitted at large angle,
insensitive to the collinear singularity

Require that energy flow into the interjet region is less than E_{out} such that

$$p_t \gg E_{out} \gg \Lambda_{QCD}$$

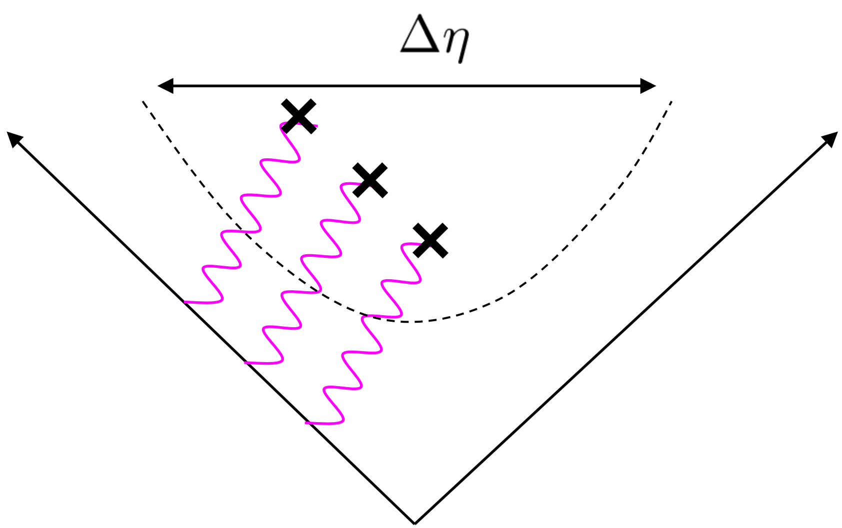
Resum only the soft logarithms

$$(\ln p_t / E_{out})^n$$

There are two types of soft logarithms.

Sudakov vs. non-global logs

Sudakov logarithm   e.g., Oderda and Sterman (1998)



Real emission forbidden, $k_i \leq E_{out}$
virtual emission allowed $k_i \leq p_t$

→ Miscancellation between the real and virtual contributions.

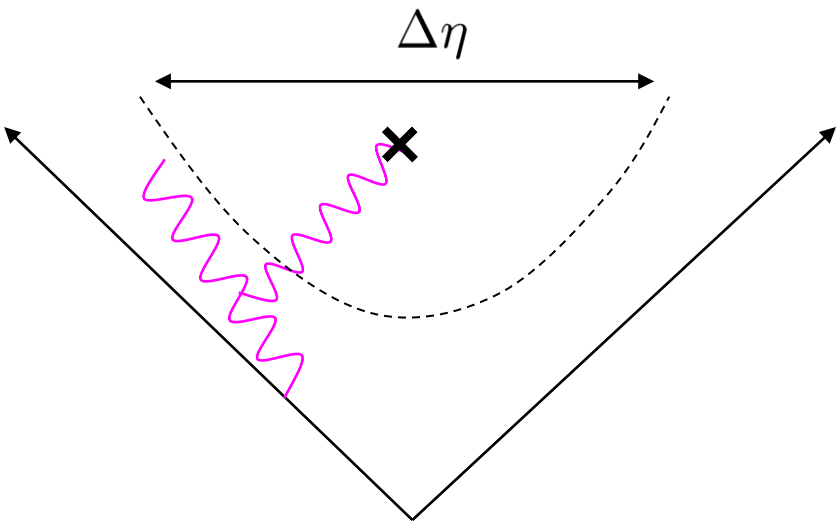
→ large logs $\left(\bar{\alpha}_s \ln \frac{p_t}{E_{out}}\right)^n$

⟶ exponentiate

$$P\left(\sum_{i\in gap} E_i \leq E_{out}\right) \sim \exp\left(-\bar{\alpha}_s \Delta\eta \ln \frac{p_t}{E_{out}}\right)$$

Sudakov vs. non-global logs

Nonglobal logarithm   Dasgupta and Salam (2001)



One should also forbid secondary emissions into the interjet region

Parametrically of the same order as the Sudakov logs.
Not easy to resum (does not exponentiate…)

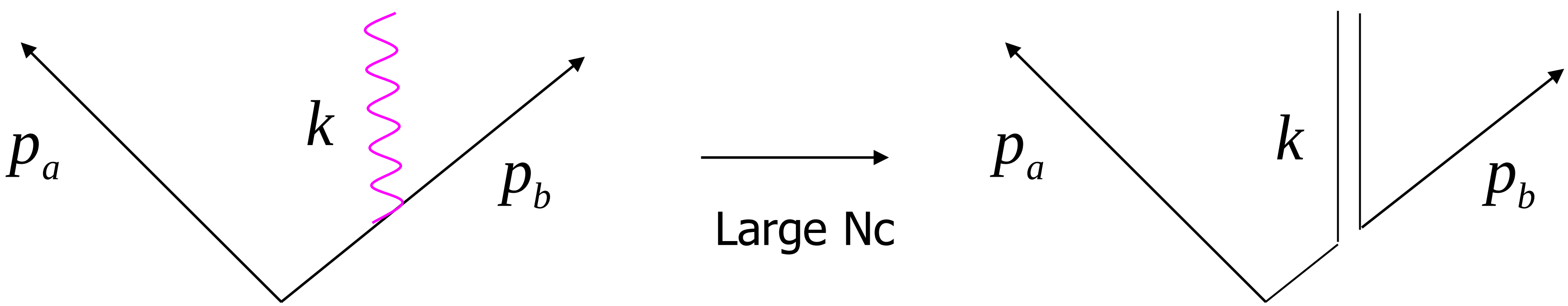
Sensitive to the complicated multi-gluon configuration in the interjet region.   → Monte Carlo simulation

@Large-Nc

Marchesini-Mueller equation (2003)

Differential probability for the soft gluon emission

$$dP = \bar{\alpha}_s \omega d\omega \frac{d\Omega_k}{4\pi} \frac{p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} \approx \bar{\alpha}_s \frac{d\omega}{\omega} \frac{d\Omega_k}{4\pi} \frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{bk})}$$



Evolution of the dipole ($q\bar{q}$ pair) distribution. Non-global logs included.

$$\partial_Y n(\theta_{ab}, \theta_{cd}, Y) = \bar{\alpha}_s \int \frac{d^2\Omega_k}{4\pi} \frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{bk})} \times \left(n(\theta_{ak}, \theta_{cd}, Y) + n(\theta_{bk}, \theta_{cd}, Y) - n(\theta_{ab}, \theta_{cd}, Y) \right).$$

$Y = \ln p_t / E_{out}$

"rapidity"

BMS equation

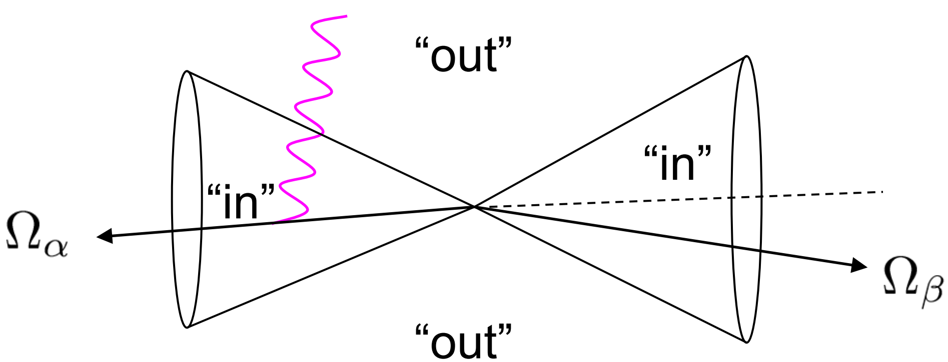
Banfi, Marchesini, Smye (2002)

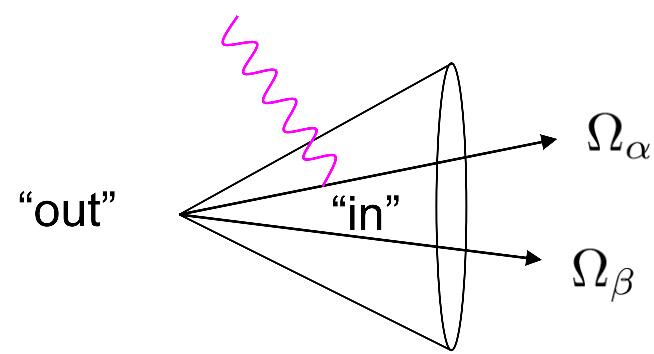
$P_{¿}(-_{a};-_{b})$

: Probability that the total energy emitted from a q¹q pair $(-_{a};-_{b})$ into the "out" region is less than E_{out}

$$\partial_\tau P_\tau(\Omega_\alpha,\Omega_\beta) = -\int_{C_{out}} \frac{d^2\Omega_\gamma}{4\pi}\frac{1-\cos\theta_{\alpha\beta}}{(1-\cos\theta_{\alpha\gamma})(1-\cos\theta_{\gamma\beta})}P_\tau(\Omega_\alpha,\Omega_\beta) + \int_{C_{in}} \frac{d^2\Omega_\gamma}{4\pi}\frac{1-\cos\theta_{\alpha\beta}}{(1-\cos\theta_{\alpha\gamma})(1-\cos\theta_{\gamma\beta})}\Big(P_\tau(\Omega_\alpha,\Omega_\gamma)P_\tau(\Omega_\gamma,\Omega_\beta)-P_\tau(\Omega_\alpha,\Omega_\beta)\Big)$$

$$\tau = \bar{\alpha}_s \ln \frac{p_t}{E_{out}}$$





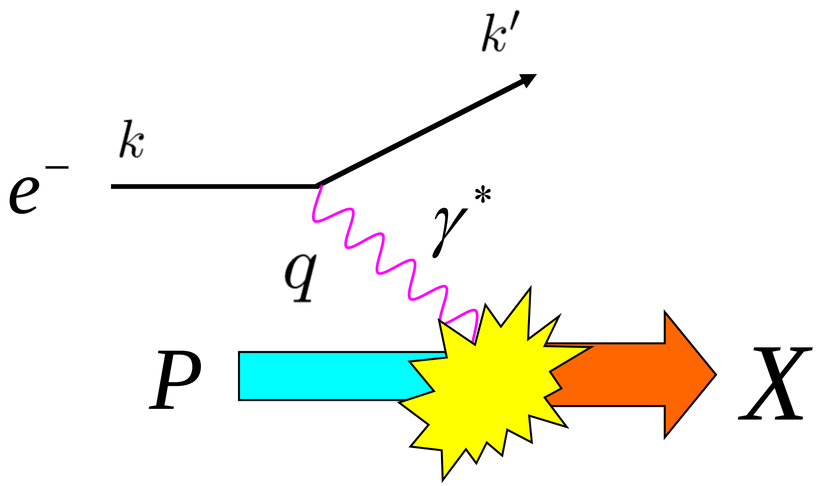
A puzzle

- The Marchesini-Mueller equation is very similar to the BFKL equation
- The BMS equation is very similar to the Balitsky-Kovchegov equation

Deep connection between jet physics and high energy (Regge) scattering?

Surprising because a jet has to do with the double-log resummation, while BFKL and BK are single-logarithmic.

Deep inelastic scattering



Photon virtuality

$$q^2 = -Q^2 < 0 \qquad \text{(spacelike)}$$

Bjorken variable

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{m_X^2 + Q^2 - m_p^2}$$

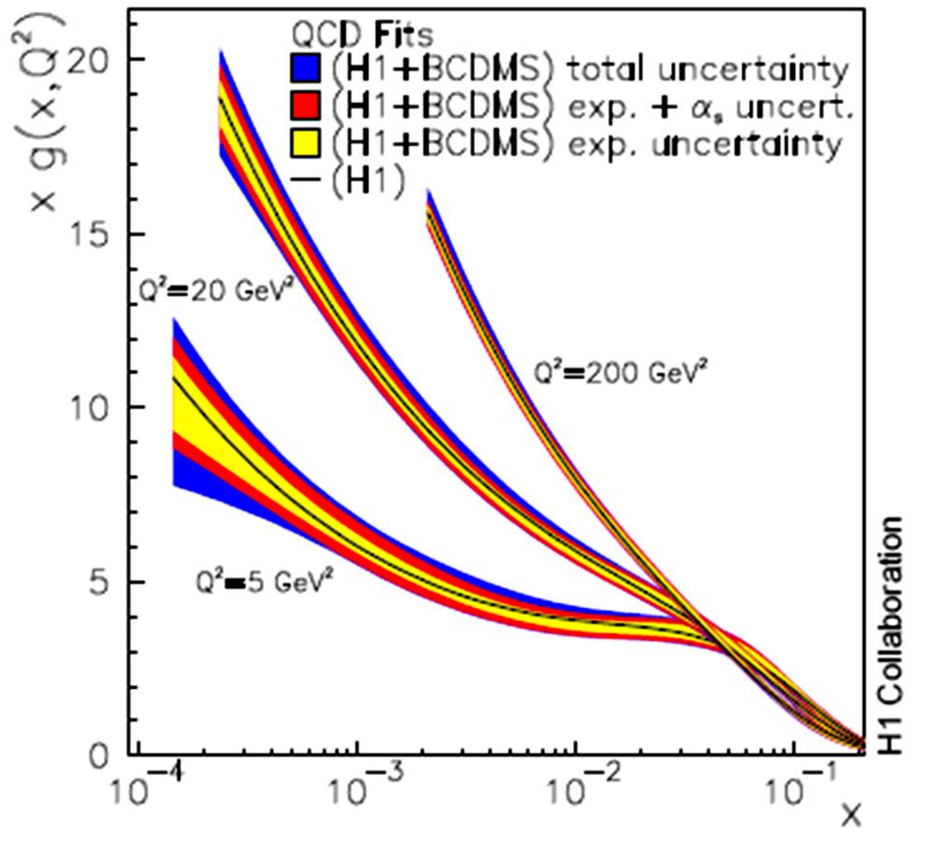
$$\approx \frac{p^+_{parton}}{P^+_{proton}}$$

(longitudinal energy fraction)

Small-x = high energy $\quad x \approx \frac{Q^2}{s} \ll 1$

Gluons at HERA

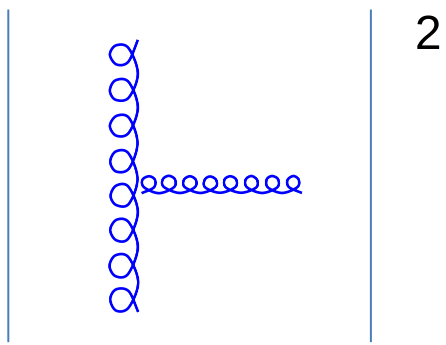
$$F_2(x,Q^2) \sim Q^2 \sigma_{tot}^{\gamma^* p}$$
$$\sim \alpha_s xg(x,Q^2)$$
$$\sim x^{-\lambda(Q^2)}$$



Cross sections at high energy are proportional to the number of small-x gluons

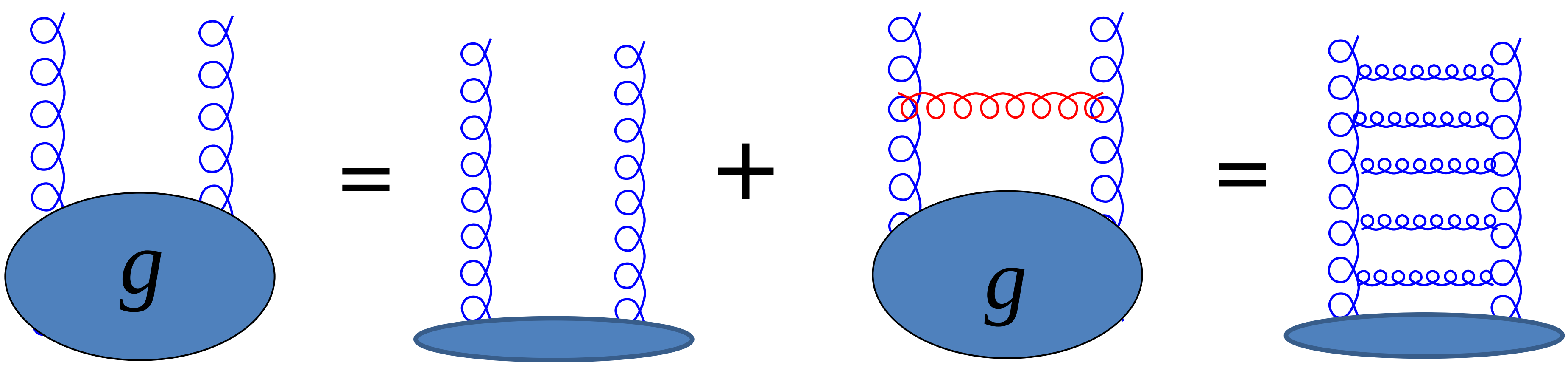
The BFKL resummation

Balitsky, Fadin, Kuraev & Lipatov, (1975~78)



$\sim \frac{1}{x}$

Soft singularity

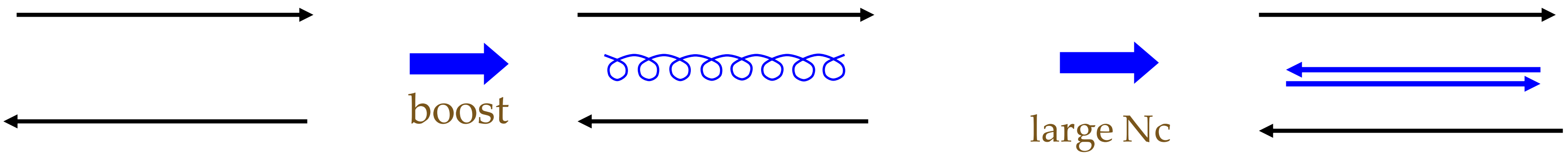


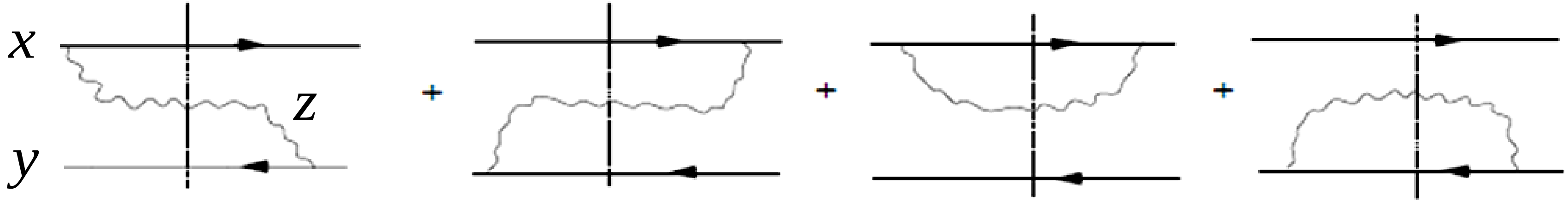
$$xg(x) \approx 1 + \lambda \ln\frac{1}{x} + \frac{\lambda^2}{2}\ln^2\frac{1}{x} + \cdots = \left(\frac{1}{x}\right)^{\lambda}$$

The QCD dipole model

Mueller, (1994)

2D Fourier transform to the transverse (impact parameter) space.

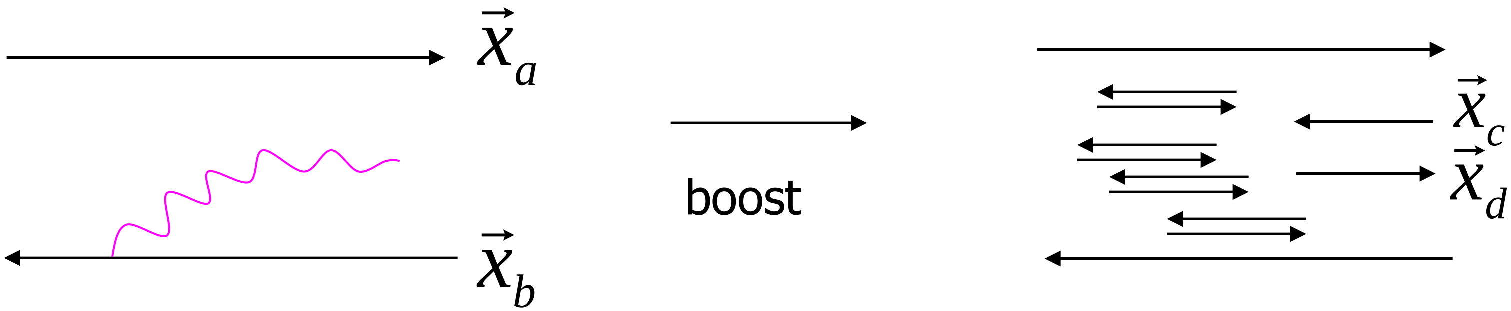




$$-2\frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2}+\frac{1}{(x-z)^2}+\frac{1}{(y-z)^2}=\frac{(x-y)^2}{(x-z)^2(y-z)^2}$$

$$dP=\bar{\alpha}_s\frac{(\vec{x}-\vec{y})^2}{(\vec{x}-\vec{z})^2(\vec{z}-\vec{y})^2}d^2\vec{z}\,\frac{dx}{x}$$

BFKL equation (dipole version)



$$\partial_Y n(x_{ab}, x_{cd}, Y) = \bar{\alpha}_s \int \frac{d^2\vec{x}_k}{2\pi} \frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2} \times \left(n(x_{ak}, x_{cd}, Y) + n(x_{bk}, x_{cd}, Y) - n(x_{ab}, x_{cd}, Y)\right)$$

$Y = \ln \frac{1}{x}$

Exact solution known thanks to 2D conformal SL(2,C) symmetry

$$x_1 + i x_2 = z \qquad z \to \frac{\alpha z + \beta}{\gamma z + \delta} \qquad (\alpha\delta - \beta\gamma = 1)$$

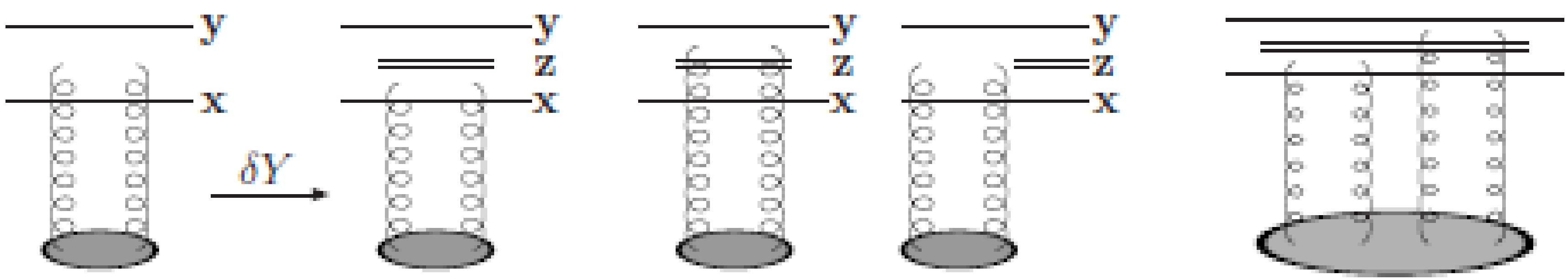
The Balitsky-Kovchegov equation

Forward scattering of a $q\bar{q}$ pair $(x; y)$ off a hadron.

$S_Y(x;y) = 1 + iT_Y(x;y)$ ¼ 1 ¡ $N_Y(x;y)$

$$\frac{\partial}{\partial Y} N_Y(x,y)$$

$$= \bar{\alpha}_s \int \frac{d^2 z}{2\pi} \frac{(x-y)^2}{(x-z)^2(z-y)^2} \Big(N_Y(x,z) + N_Y(z,y) - N_Y(x,y) - \langle N_Y(x,z) N_Y(z,y) \rangle \Big)$$



The nonlinear term represents gluon saturation.

Unitarity bound N_Y ! 1 $(Y$! 1 ; x ! $0)$

BFKL dynamics in jets

The BFKL equation and the Mueller-Marchesini equations become formally identical after the small angle approximation

$$1 - \cos\theta \approx \theta^2/2 \qquad d^2\Omega \approx d^2\vec{\theta}.$$

$$n(\mu_{ab};\mu_{cd};Y) \text{ » } n(x_{ab};x_{cd};Y) \text{ » } e^{4 \text{®}_s \ln 2\, Y} = \left(\frac{1}{x}\right)^{4 \text{®}_s \ln 2}$$

The interjet soft gluon number grows like the BFKL Pomeron !

Question : Is this just a coincidence, or is there any deep relationship between the two processes ?

⟶ Hint from AdS/CFT

The AdS/CFT correspondence

Maldacena, `97

N=4 SYM at strong `t Hooft coupling $\lambda = g_{YM}^2 N_C >> 1$ and large Nc is dual to weak coupling type IIB on

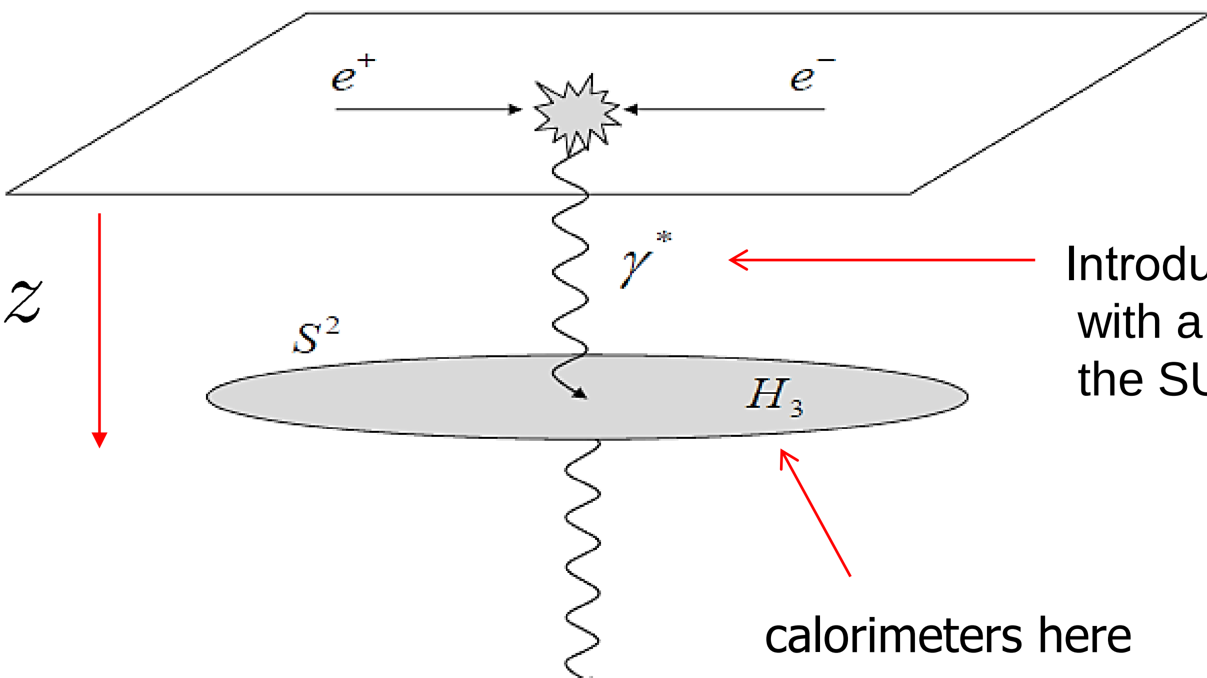
AdS$_5$ £ S^5

CFT		string
(anomalous) dimension	⟷	mass
`t Hooft parameter λ	⟷	curvature radius R^4/α'^2
number of colors λ/N_C	⟷	string coupling constant $4\pi g_s$

e+e- annihilation in N=4 SYM at strong coupling

Hofman & Maldacena, 0803.1467; YH, Iancu & Mueller, 0803.2481;
YH & Matsuo, 0804.4733, 0807.0098; YH, 0810.0889.

AdS metric in the Poincare coordinates $$ds^2 = \frac{dx^1\, dx_1 + dz^2}{z^2}$$



Introduce a “photon” associated with a U(1) subgroup of the SU(4) R-symmetry.

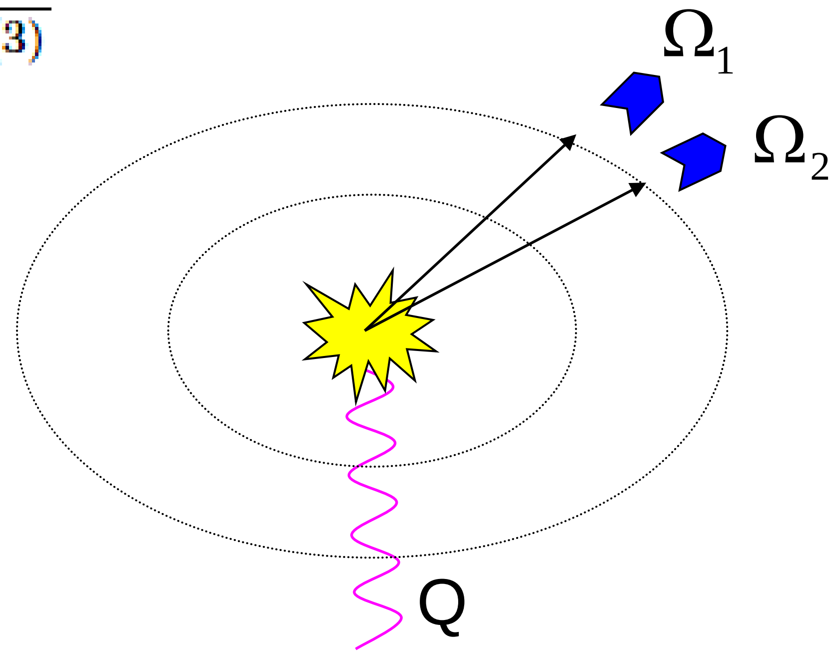
calorimeters here

Jets at strong coupling?

Energy correlation function nonsingular

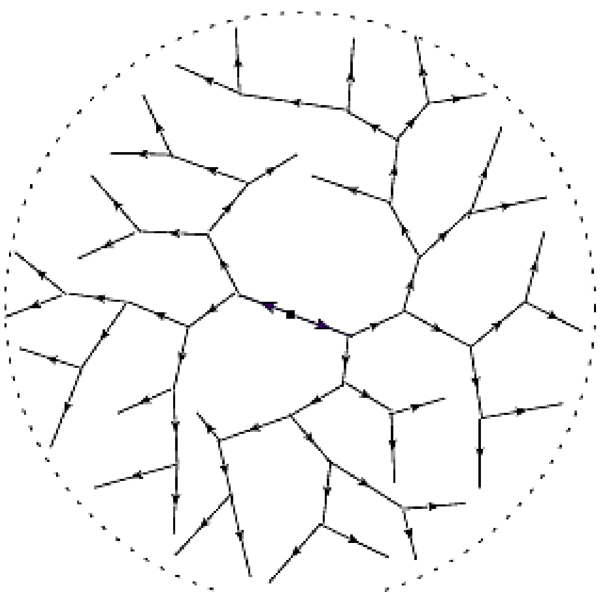
Hofman, Maldacena (2008)

$$\langle \mathcal{E}(\Omega_1)\mathcal{E}(\Omega_2)\rangle \sim \frac{1}{|\theta_{12}|^{2+2\gamma_S(3)}}$$



Collinear singularity absent.
"Democratic branching"

YH, Iancu, Mueller (2008)



Energy distribution is spherical.
The entire solid angle looks
like an interjet region!

Jets at strong coupling?

DGLAP equation

$$\frac{\partial}{\partial \ln Q^2} D_T(x,Q^2) = \int_x^1 \frac{dz}{z} P_T(z) D_T\left(\frac{x}{z}, Q^2\right)$$

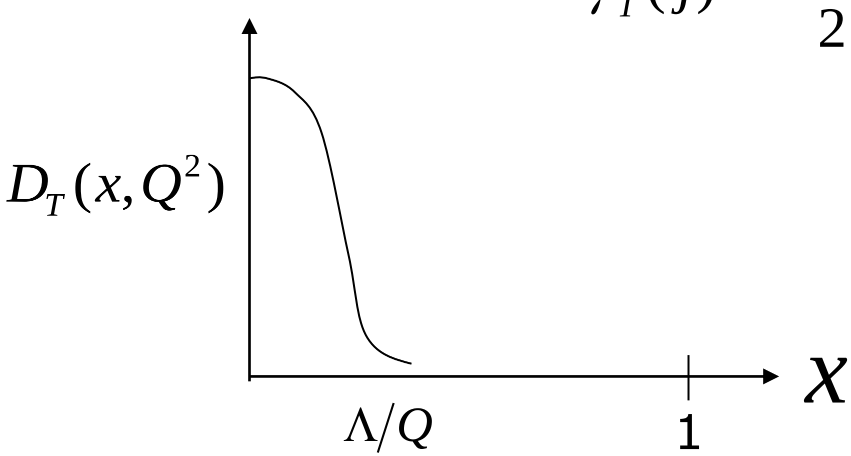
Mellin transform

$$\frac{\partial}{\partial \ln Q^2} D_T(j,Q^2) = \gamma_T(j) D_T(j,Q^2)$$

Timelike anomalous dimension at strong coupling

$$\gamma_T(j) = -\frac{1}{2}\left(j - j_0 - \frac{j^2}{2\sqrt{\lambda}}\right)$$

YH, Matsuo (2008)



Fragmentation function peaked at the kinematical lower limit. There are no hard particles.

Two Poincare coordinates

AdS_5 as a hypersurface in 6D

$$X_0^2 - X_1^2 - X_2^2 - X_3^2 - X_4^2 + X_5^2 = 1$$

Introduce two Poincare coordinate systems Cornalba (2007)

Our universe

Poincare 1 : $X_4 + X_5 = \frac{1}{z}, \quad X_\mu = \frac{x_\mu}{z} \quad (\mu = 0, 1, 2, 3)$

Poincare 2 : $X_0 + X_3 = \frac{1}{y_5}, \quad X_5 = -\frac{y^0}{y_5}, \quad X_4 = -\frac{y^3}{y_5}, \quad X_{1,2} = \frac{y^{1,2}}{y_5}$

Related via a conformal transformation on the boundary

$$y^+ = -\frac{1}{2x^+}, \quad y^- = x^- - \frac{x_1^2 + x_2^2}{2x^+}, \quad \vec{y}_T = \frac{\vec{x}_T}{\sqrt{2}x^+}$$

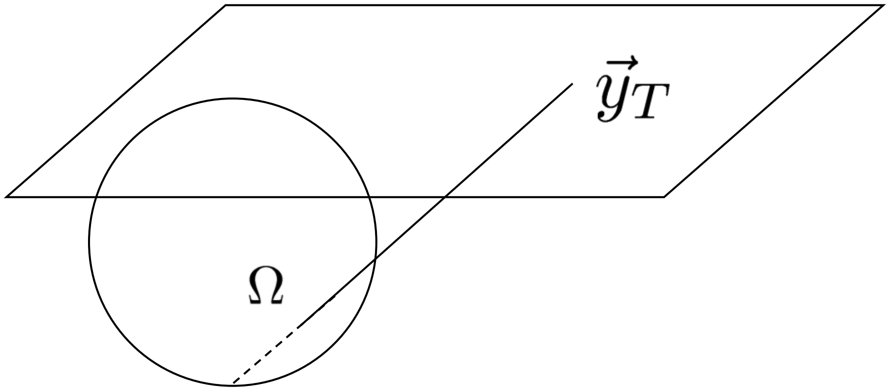
Energy flow operator

Energy operator in Poincare 1

Sveshnikov, Tkachov, (1996)
Korchemsky, Oderda, Sterman (1997)

$$\mathcal{E}(\Omega) \equiv \lim_{r\to\infty} r^2 \int_0^\infty dx^0\, n_i T^{0i}(x^0, r\vec{n})$$

The sphere can be mapped onto the transverse plane $\vec{y}_T$ of Poincare 2 via the stereographic projection $\Omega \to \vec{y}_T$



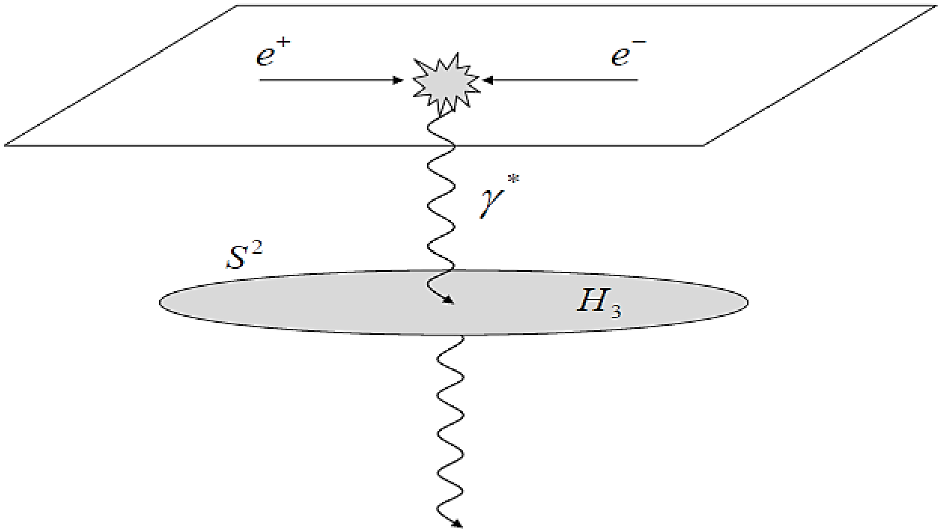
$$y^1 = \tan\frac{\theta}{2}\cos\phi$$
$$y^2 = \tan\frac{\theta}{2}\sin\phi$$

Energy operator in Poincare 2

$$\mathcal{E}(\Omega) = \frac{\sqrt{2}}{(1+\cos\theta)^3}\int dy^- T_{--}(y^+=0, y^-, \vec{y}_T) \equiv \frac{1}{(1+\cos\theta)^3}\mathcal{E}(\vec{y}_T)$$

Shock wave picture of e+e- annihilation

YH (2008)



center of AdS (infinite future)
= hyperbolic space H_3

$y^+ = X_4 + X_5 = 0 \quad (x^+ = 1)$

$$X_0^2 - X_1^2 - X_2^2 - X_3^2 = 1$$

$$\longrightarrow \quad y_5 = 1, \ \vec{y}_T = 0$$

Treat the photon as a shock wave in Poincare 2. Solve the 5D Einstein equation

$$T_{--} = q^+ \delta(y_5 - 1)\delta^{(2)}(\vec{y}_T)\delta(y^-)$$

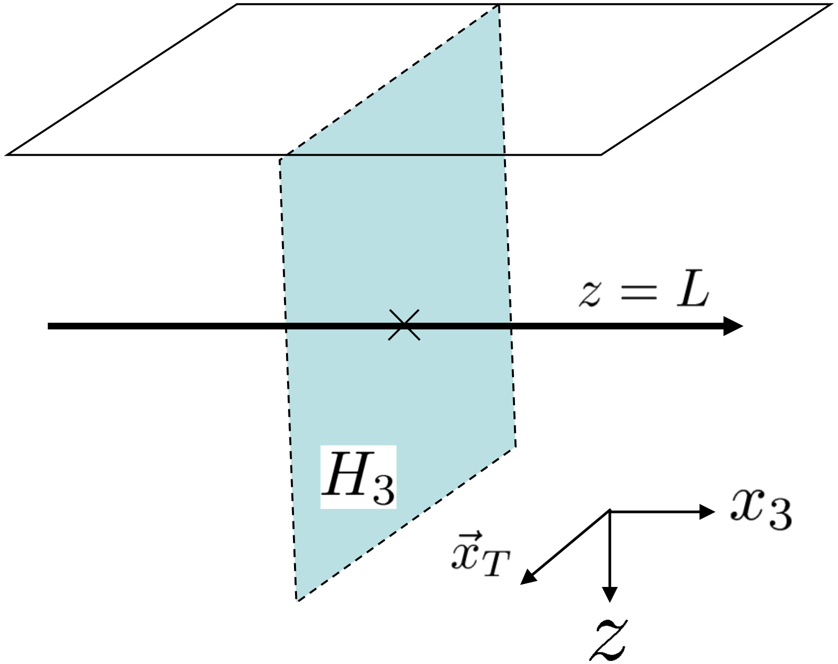
Energy density on the boundary from the holographic renormalization

$$\langle \mathcal{E}(\Omega) \rangle = \frac{Q}{4\pi}$$

in agreement with Hofman, Maldacena (2008)

Shock wave picture of a high energy "hadron"

A color singlet state lives in the bulk.
At high energy, it is a shock wave
in Poincare 1.



$$T^{++} = z^7 p^+ \delta(z - L)\delta^{(2)}(\vec{x}_T)\delta(x^-)$$

Energy distribution on the boundary transverse plane

$$\langle \mathcal{E}(\vec{x}_T) \rangle = \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} dx^- \langle T_{--}(x^+ = 0, x^-, \vec{x}_T) \rangle = \frac{2EL^4}{\pi(L^2 + (\vec{x}_T)^2)^3}$$

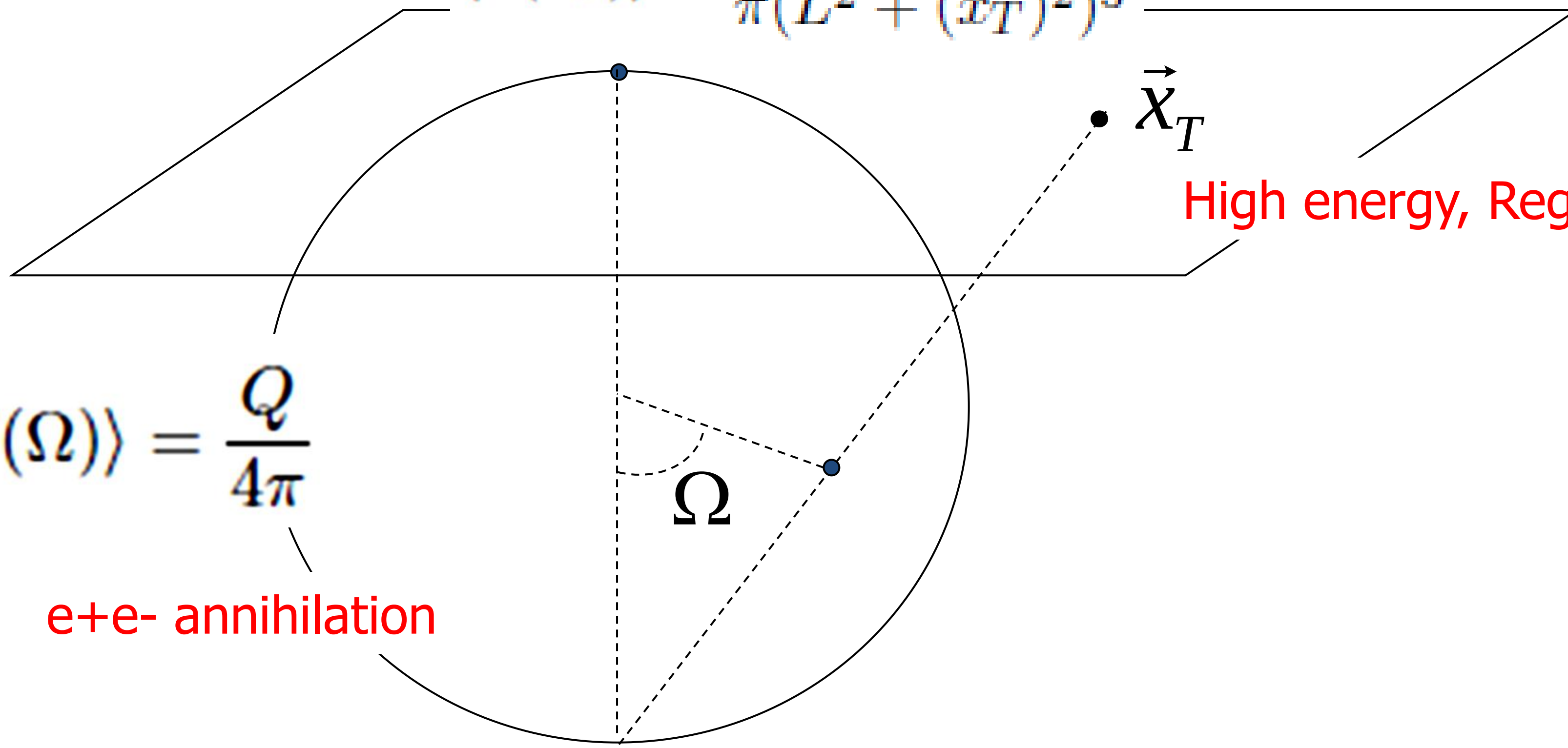
Gubser, Pufu & Yarom (2008)

Exact map at strong coupling

YH (2008)

$$\langle \mathcal{E}(\vec{x}_T)\rangle = \frac{2EL^4}{\pi(L^2+(\vec{x}_T)^2)^3}$$

$$\langle \mathcal{E}(\Omega)\rangle = \frac{Q}{4\pi}$$

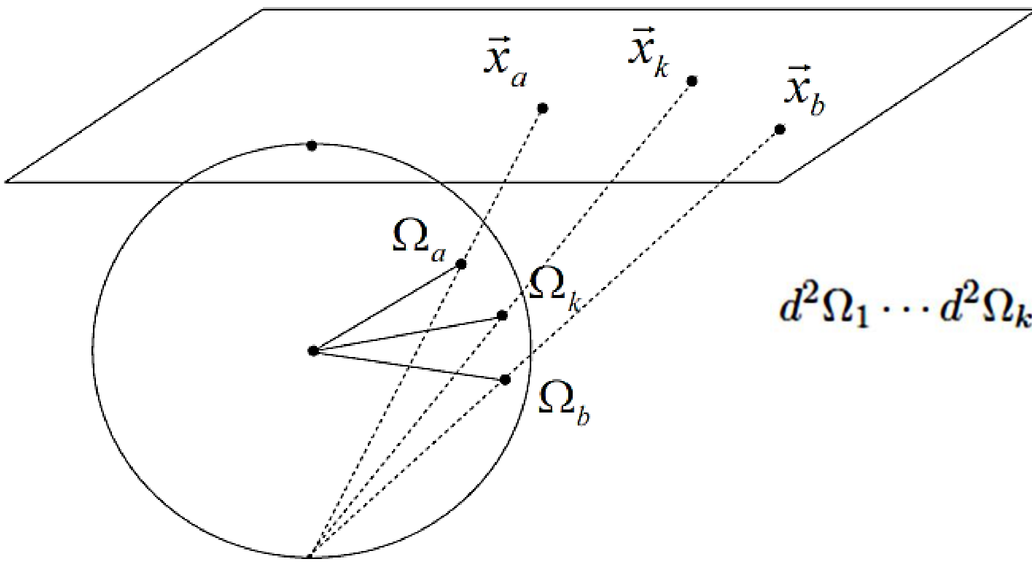


The two processes are mathematically identical.
The only difference is the choice of the coordinate system in AdS !

Exact map at weak coupling

Apply the stereographic projection to the gluon emission kernel

$$\frac{d^2\Omega_k}{4\pi}\frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{bk})}=\frac{d^2\vec{x}_k}{2\pi}\frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2}$$



k-gluon emission probability

$$d^2\Omega_1\cdots d^2\Omega_k\frac{1-\cos\theta_{ab}}{(1-\cos\theta_{a1})(1-\cos\theta_{12})\cdots(1-\cos\theta_{kb})}$$

$$=d^2x_1\cdots d^2x_k\frac{x_{ab}^2}{x_{a1}^2x_{12}^2\cdots x_{kb}^2}$$

Stereographic projection works both in the weak and strong coupling limits.
Valid to all orders !?

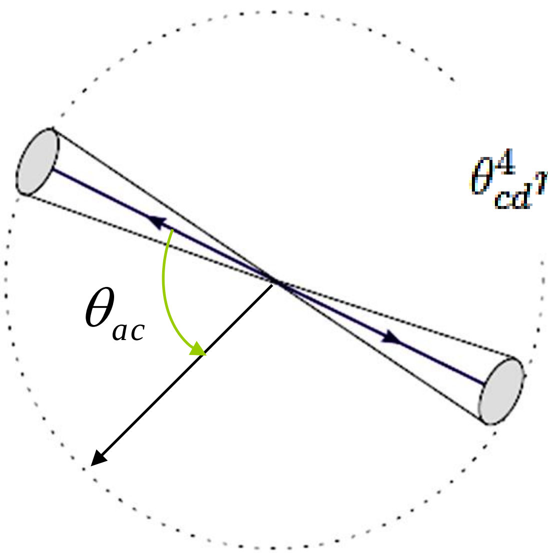
Solution to the Marchesini-Mueller equation

BFKL kernel invariant under the 2D conformal group SL(2,C)

$$z \to \frac{\alpha z+\beta}{\gamma z+\delta} \qquad (\alpha\delta-\beta\gamma=1)$$

Exact solution to the BFKL equation known. Due to conformal symmetry, it is a function only of the anharmonic ratio.

$$|\rho|^2 = \frac{x_{ab}^2 x_{cd}^2}{x_{ac}^2 x_{bd}^2} = \frac{(1-\cos\theta_{ab})(1-\cos\theta_{cd})}{(1-\cos\theta_{ac})(1-\cos\theta_{bd})}$$



$$\theta_{cd}^4 n(\theta_{ab},\theta_{cd},Y) \sim \frac{|\rho|}{(D\bar{\alpha}_s Y)^{3/2}} \ln\left(\frac{16}{|\rho|}\right) e^{4\ln 2\bar{\alpha}_s Y} e^{-\frac{2\ln^2(|\rho|/16)}{D\bar{\alpha}_s Y}}$$

Analytically calculate the distribution and correlation of gluons in the interjet region.

Avsar, YH, Matsuo (2009)

Marchesini-Mueller equation at NLO in N=4 SYM

Avsar, YH, Matsuo (2009)

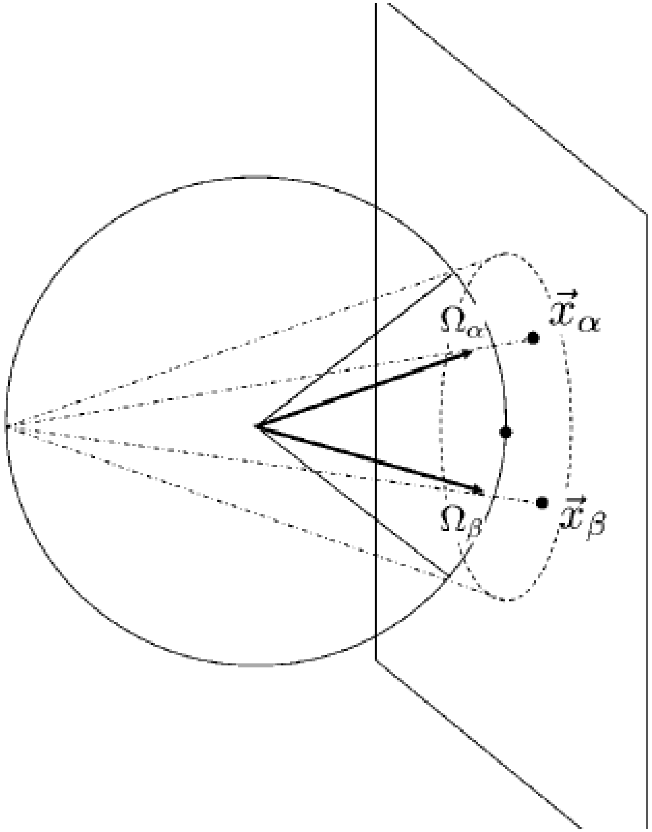
Apply the stereographic projection to the NLO BFKL equation by Balitsky & Chirilli (2008).

$$\partial_Y n_Y(\Omega_{ab}) = \bar{\alpha}_s \left(1 - \bar{\alpha}_s \frac{\pi^2}{12}\right) \int d^2\Omega_c\, K_{ab}(\Omega_c) \left[n_Y(\Omega_{ac}) + n_Y(\Omega_{cb}) - n_Y(\Omega_{ab})\right]$$
$$+\bar{\alpha}_s^2 \int d^2\Omega_c d^2\Omega_d K'_{ab}(\Omega_c, \Omega_d) n_Y(\Omega_{cd})\,,$$

$$K'_{ab}(\Omega_c, \Omega_d) = \frac{1}{8\pi^2} \left\{ \frac{(1-\cos\theta_{ab})}{(1-\cos\theta_{ac})(1-\cos\theta_{cd})(1-\cos\theta_{db})} \right.$$
$$\times \left[\left(1 + \frac{(1-\cos\theta_{ab})(1-\cos\theta_{cd})}{(1-\cos\theta_{ac})(1-\cos\theta_{bd}) - (1-\cos\theta_{ad})(1-\cos\theta_{bc})} \right) \right.$$
$$\left. \times \ln \frac{(1-\cos\theta_{ac})(1-\cos\theta_{bd})}{(1-\cos\theta_{ad})(1-\cos\theta_{bc})} + 2\ln \frac{(1-\cos\theta_{ab})(1-\cos\theta_{cd})}{(1-\cos\theta_{ad})(1-\cos\theta_{bc})} \right]$$
$$\left. + 12\pi^2 \zeta(3) \delta^{(2)}(\Omega_{ac}) \delta^{(2)}(\Omega_{bd}) \right\}.$$

Hidden symmetry of the BMS equation

YH & Ueda (2009)



Jet cone breaks the SL(2,C) conformal symmetry down to the subgroup

SL(2,R)=SU(1,1)=SO(1,2)

→ Poincare disk.

$P_\tau(\Omega_\alpha, \Omega_\beta)$ depends only of the chordal distance

$$d^2(\vec{x}_\alpha, \vec{x}_\beta) = \frac{(\vec{x}_\alpha - \vec{x}_\beta)^2}{(1-\vec{x}_\alpha^2)(1-\vec{x}_\beta^2)} = \frac{\sin^2\theta_{in}(1-\cos\theta_{\alpha\beta})}{2(\cos\theta_\alpha - \cos\theta_{in})(\cos\theta_\beta - \cos\theta_{in})}$$

Hidden symmetry of the BK equation?

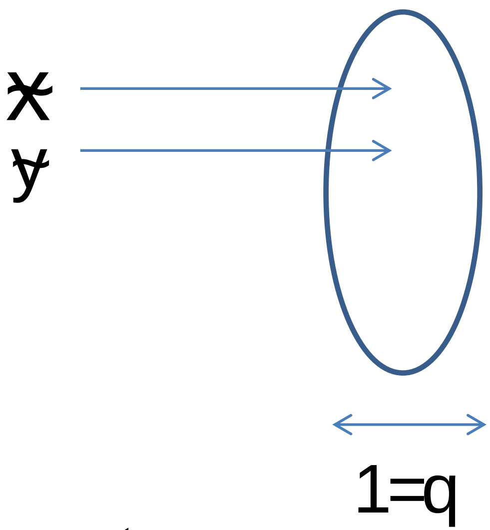
Gubser, arXiv:1102.4040

The target size breaks conformal symmetry, but not completely.

SL(2,C) → SO(3)

$$N_Y(\vec{x};\vec{y}) = N_Y(d(\vec{x};\vec{y}))$$

$$d(\vec{x};\vec{y}) = \frac{(\vec{x}-\vec{y})^2}{(1+q^2\vec{x}^2)(1+q^2\vec{y}^2)}$$



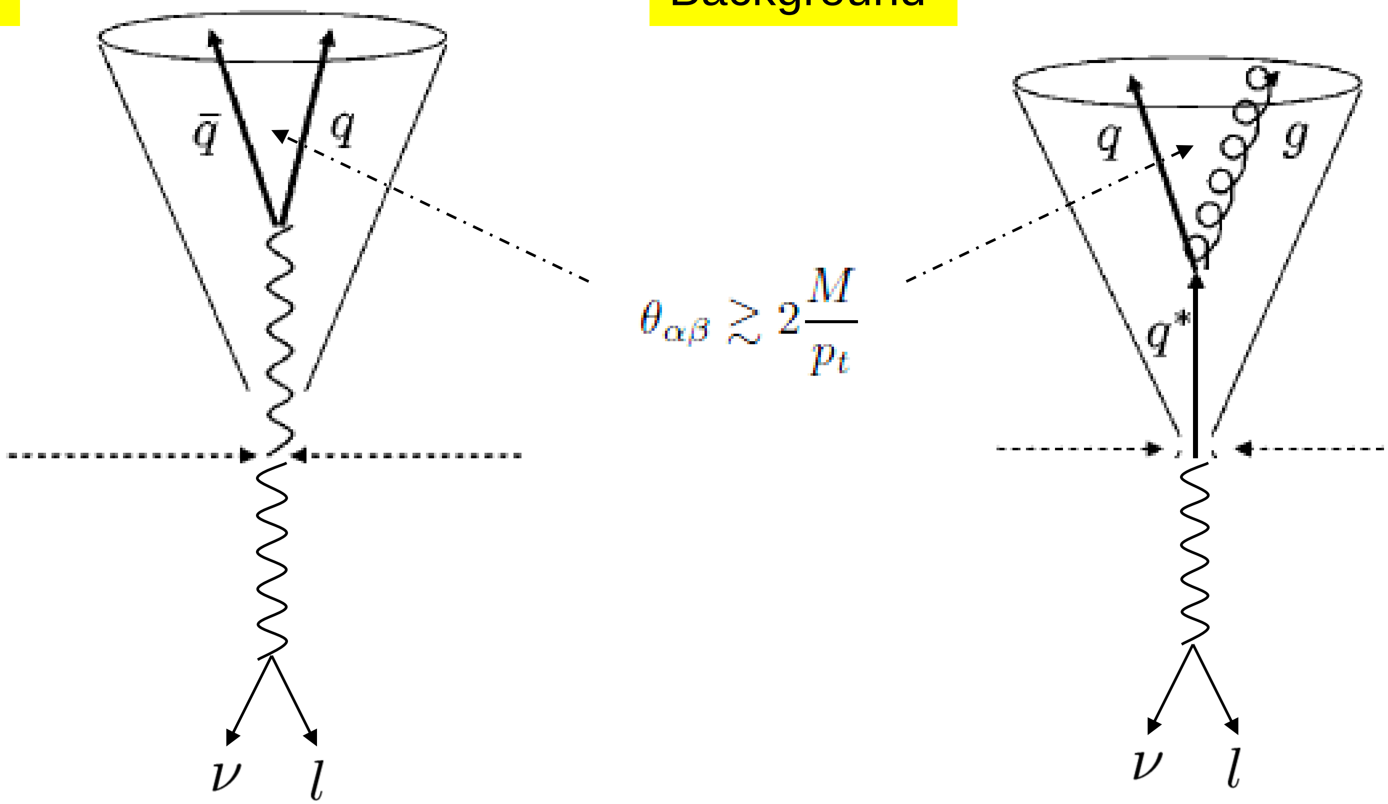
Need to check if the initial condition has this symmetry….

Application: boosted W boson at the LHC

YH, Ueda (2009)

Signal

Background



Very little radiation
due to the QCD coherence

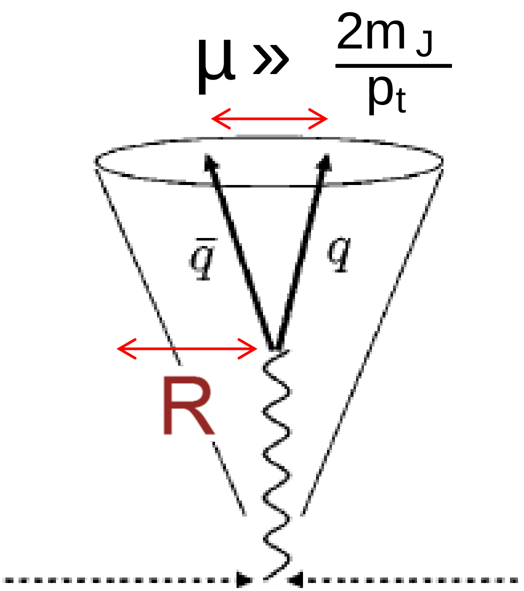
Sizable radiation expected

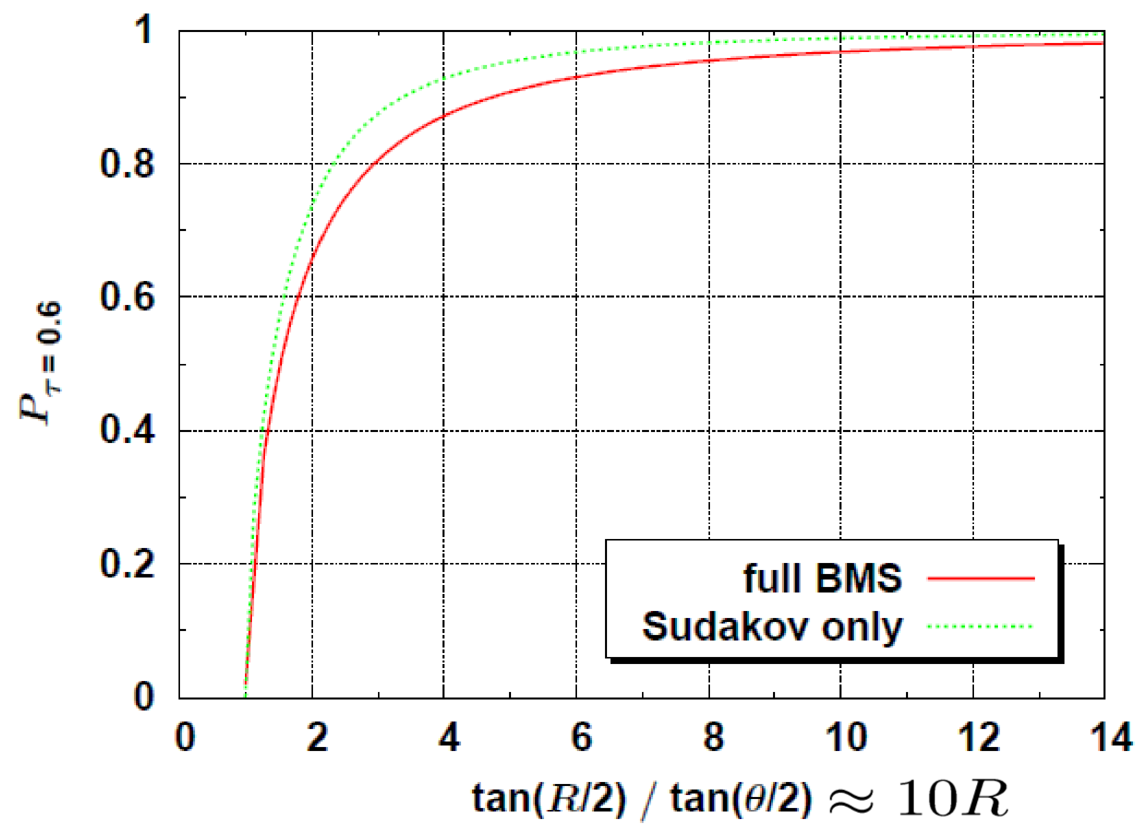
Weak boson jet

$$p_t \sim 1 \ \mathrm{TeV}$$

$$m_J \sim 100 \, \mathrm{GeV}$$

$$E_{out} = 10 \ \mathrm{GeV}$$

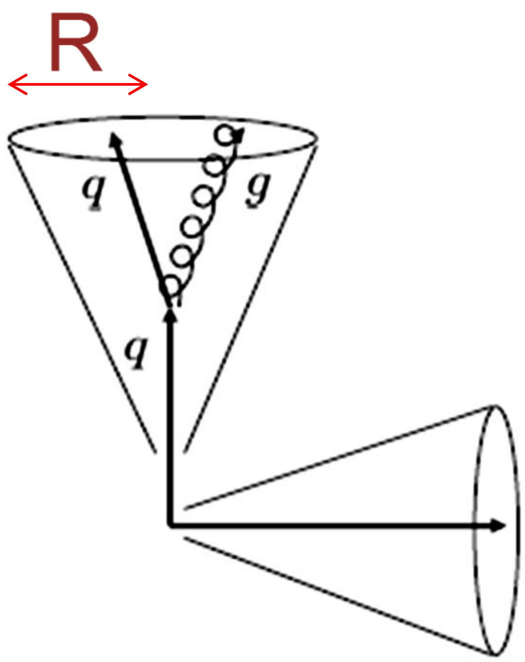


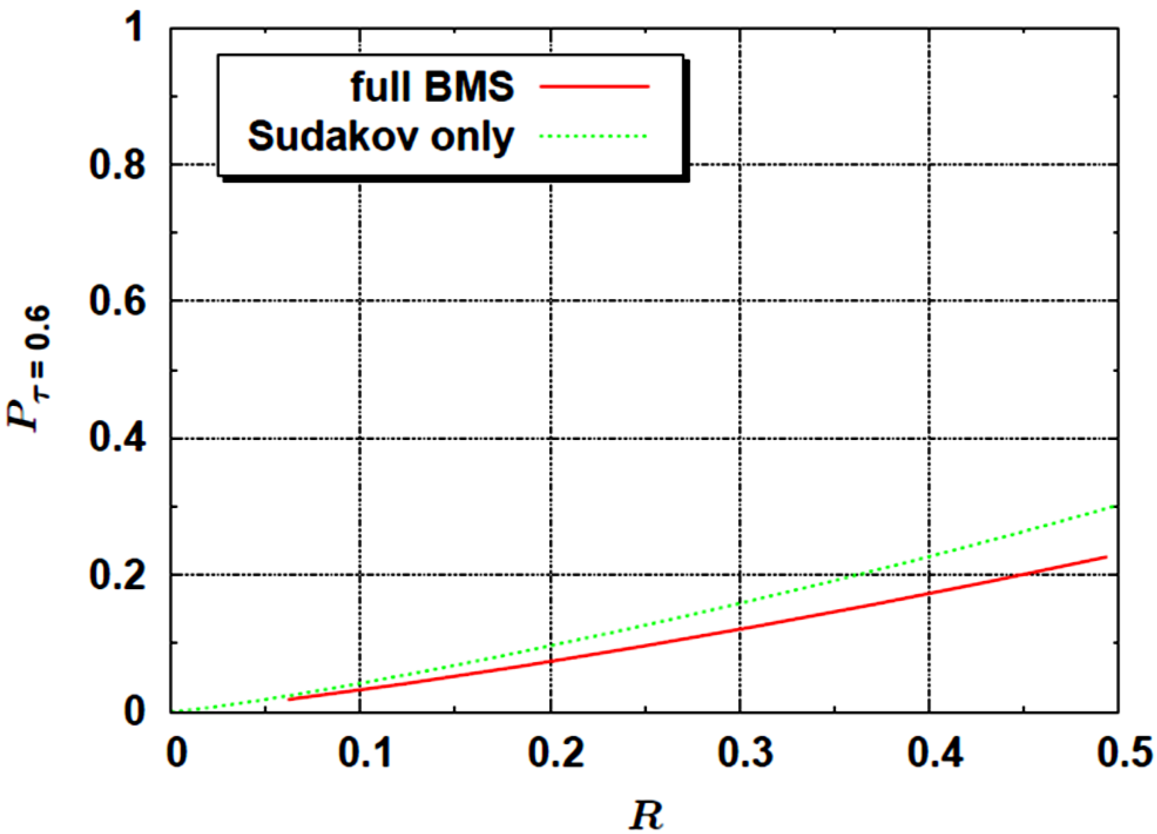


With 80~90% probability, energy radiated outside the jet cone is less than 10 GeV (only 1%)

Typical energy radiated $\ll \alpha_s p_t$

QCD jet

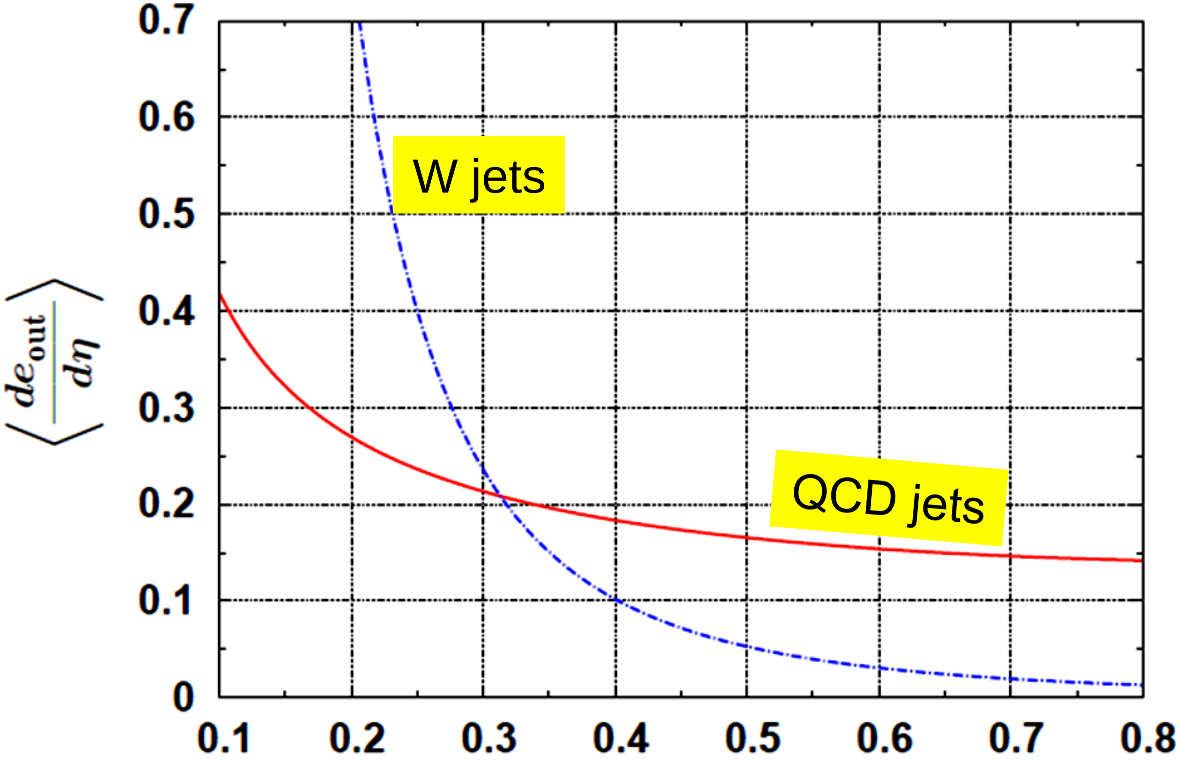




With 80~90% probability, energy radiated outside the jet cone is larger than 10 GeV

Typical energy radiated $\lesssim \alpha_s p_t$

Rapidity distribution of energy



Fully included in Ariadne, partially in Herwig & Pythia. Initial state radiation and the underlying event tend to diminish the difference...

Conclusions

- Physics of the interjet region deeply related to high energy scattering. There is an exact conformal mapping in the leading-log approximation in QCD.

- In N=4 SYM, the correspondence probably holds to all orders. AdS/CFT provides a vivid geometrical picture of the equivalence.

- Phenomenological application of energy flow at collider experiments.