Cosmology, Scalar Fields and Hydrodynamics

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THIS TALK IS BASED ON WORK IN PROGRESS AND

- *Imperfect Dark Energy from Kinetic Gravity Braiding* **arXiv:1008.0048 [hep-th]**, JCAP 1010:026, 2010
- *The Imperfect Fluid behind Kinetic Gravity Braiding* **arXiv:1103.5360 [hep-th]**

IN COLLABORATION WITH

Cédric Deffayet, Oriol Pujolàs
& Ignacy Sawicki

- **Good news:** now cosmologists have some understanding about the processes in the very early universe when it was $10^{-36} - 10^{-32}$ seconds young! - ***Inflation***

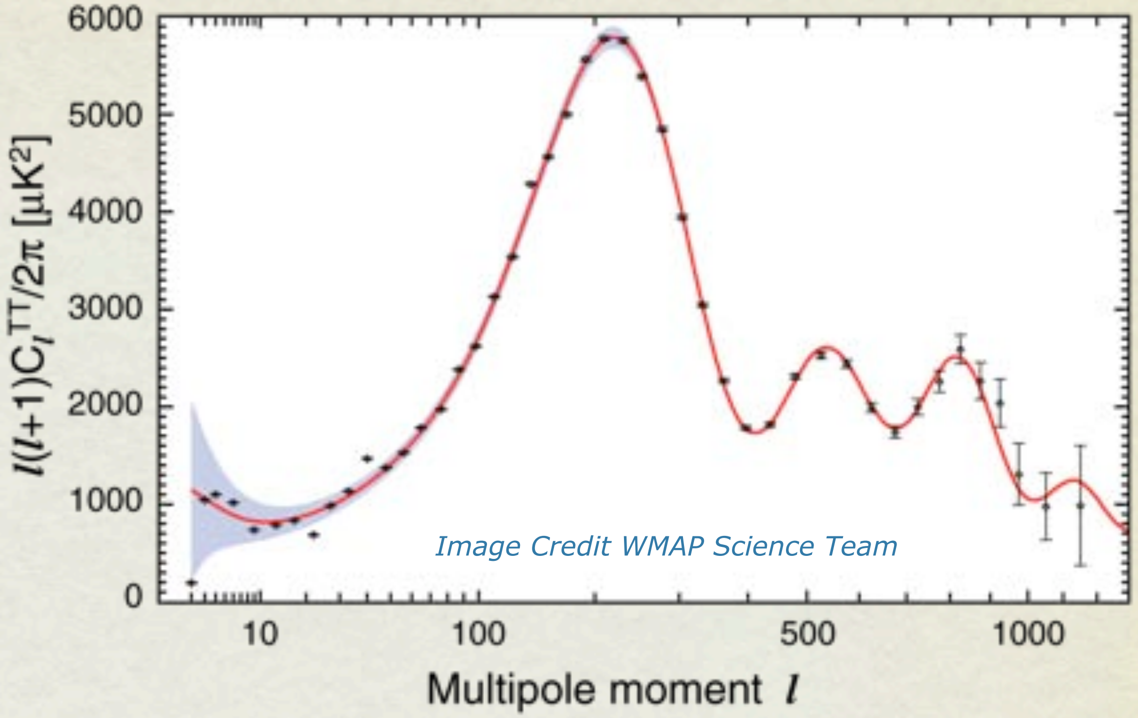


Image Credit WMAP Science Team

- **Bad news:** we do **not** know what the universe is made of now...
96 % is **Dark:**
Dark Matter (DM) & Dark Energy (DE)

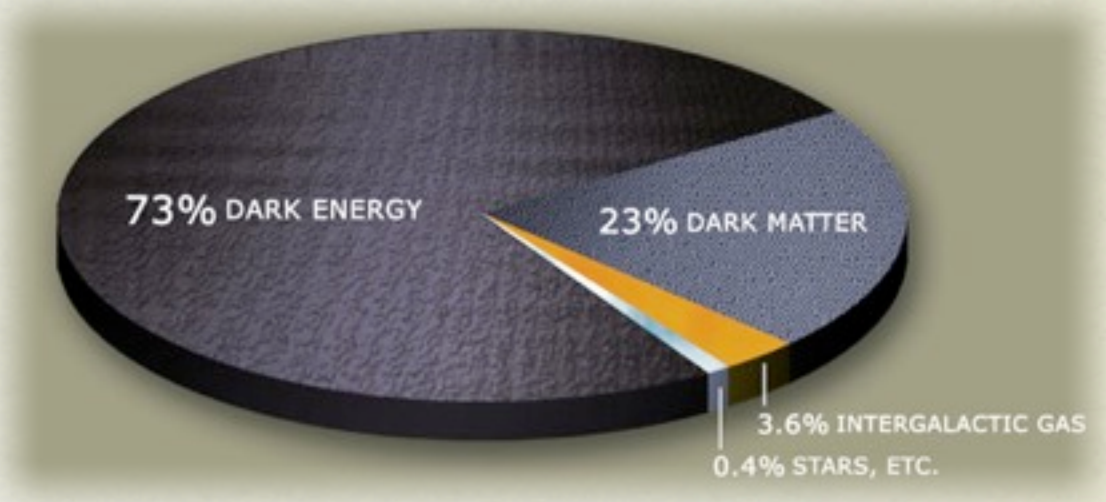


Image Credit: NASA

- **Excellent news - a lot of work to do for physicists !**

ISOTROPY AND HOMOGENEITY OF THE FRIEDMANN UNIVERSE

$$T_{\mu\nu} = (\mathcal{E} + P)\, u_\mu u_\nu - P g_{\mu\nu}$$

- One can "characterize" the universe by energy density $\mathcal{E}$ and pressure P (and "equation of state" $w_X = P/\mathcal{E}$)

- A perfect fluid or a "simple" scalar field have this Energy - Momentum Tensor (EMT) on "all" backgrounds

WHAT DO WE KNOW ABOUT
DARK ENERGY?

- **It is accelerating the universe and is Dark**
- **“Energy scale” 10^{-3} eV the same as for DM**
- **approximately 75% of the energy budget of the universe today**
- **equation of state $P \approx -\mathcal{E}$ up to ±5-10%**
- **Is it just Λ ?**

Cosmological Constant Problem...

MAY BE
DARK ENERGY
IS NOT JUST Λ
?

INDEED, THERE WAS
INFLATION-
ANOTHER STAGE OF THE
ACCELERATED EXPANSION
IN THE VERY EARLY UNIVERSE

WHAT DO WE KNOW ABOUT INFLATON?

- **It accelerated the universe (how Dark is it???)**
- **Mass scale** $10^{13}\ \mathrm{GeV}$
- **dominated the universe when it was** $10^{-36} - 10^{-32}$ **seconds young**
- **equation of state** $P \approx -\mathcal{E}$ **up to %**
- **lasted at least couple of dozens of cosmic times**

MAIN RESULTS OF THE SIMPLEST INFLATION

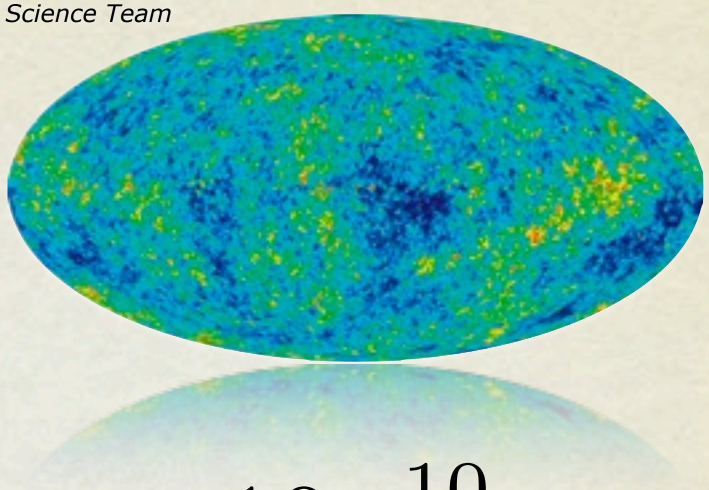


Image Credit WMAP Science Team

- the universe is huge and spatially flat
- the cosmological perturbations e.g. Newtonian potential Φ are Gaussian with the power spectrum

$$\delta^2_\Phi(k) \propto \left.\frac{\mathcal{E}}{c_{\mathrm{s}}(1+P/\mathcal{E})}\right|_{H=c_{\mathrm{s}}k} \sim 10^{-10}$$

Garriga & Mukhanov 99

speed of sound

$$c_{\mathrm{s}}^2 \equiv \left(\frac{\partial P}{\partial \mathcal{E}}\right)_\phi$$

$\delta_\Phi(k)$: fluctuation of on length scale $\ell = k^{-1}$

$$\delta^2_{h_{\mu\nu}}(k) \propto \left.\mathcal{E}\right|_{H=k}$$

for the Gravity Waves

$$n_s - 1 = \frac{d\ln\delta^2_\Phi}{d\ln k} = 0.96...$$

CAN ONE GO BEYOND THE PERFECT FLUID, BUT STILL KEEPING ONLY ONE SINGLE DEGREE OF FREEDOM

?


YES WE CAN


We Can Do It!
WAR PRODUCTION CO-ORDINATING COMMITTEE

BRAIDING METRIC WITH A SCALAR FIELD-
Kinetic Gravity Braiding

WHAT IS KINETIC GRAVITY BRAIDING?

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left[K\left(\phi, X\right) + G\left(\phi, X\right)\Box\phi\right]$$

k-inflation/essence, *Armendariz-Picon, Damour, Mukhanov, Steinhardt* 1999/2000

where $X \equiv \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$

Minimal coupling to gravity $S_{\mathrm{tot}} = S_\phi + S_{\mathrm{EH}}$

However, derivatives of the metric are coupled to the derivatives of the scalar, provided $G_X \neq 0$

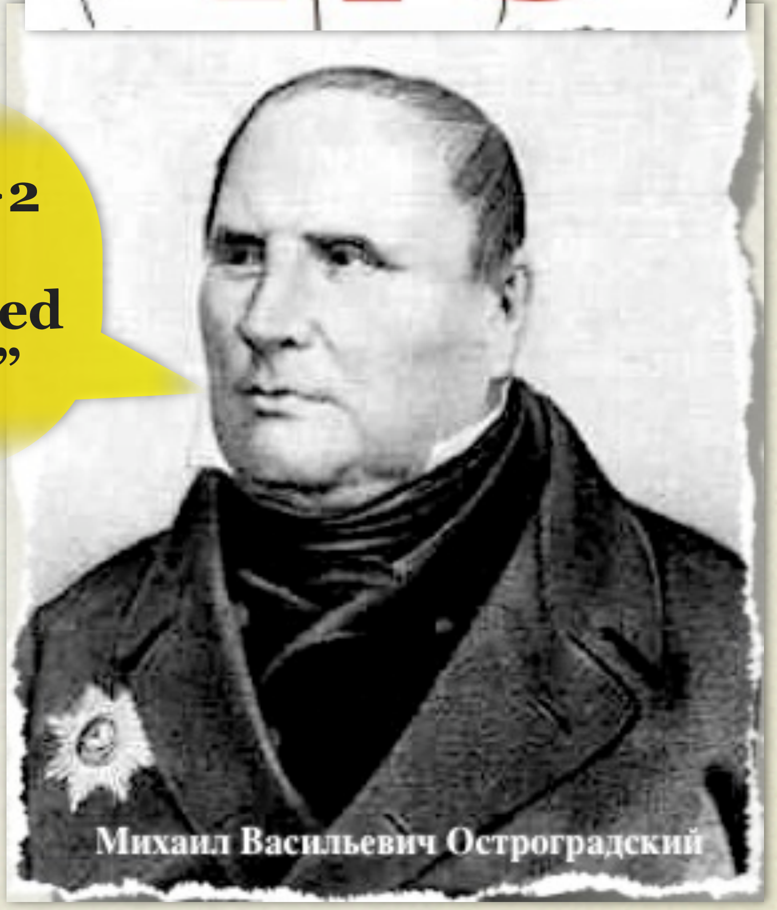
shift-symmetry: $\phi \longrightarrow \phi + c$

theory is **not** “parity” symmetric: $\phi \not\leftrightarrow -\phi$

ACTION FOR KINETIC GRAVITY BRAIDING IS SIMILAR TO EINSTEIN-HILBERT ACTION

- The second derivatives (higher derivative -HD) enter the action but only **linearly**
- One can eliminate the HD(in time) only by breaking the Lorentz-invariant formulation of the theory.
- Boundary terms are required!
- Despite the HD in the action, the equations of motion are still of the 2nd order: **NO new degrees of freedom - NO Ostrogradsky's ghosts**

order of equations of motion >2 implies Hamiltonian unbounded from below - "*ghosts*"



KINETIC GRAVITY BRAIDING IS SIMILAR TO
GALILEON

(©*Nicolis, Rattazzi, Trincherini 2008*)

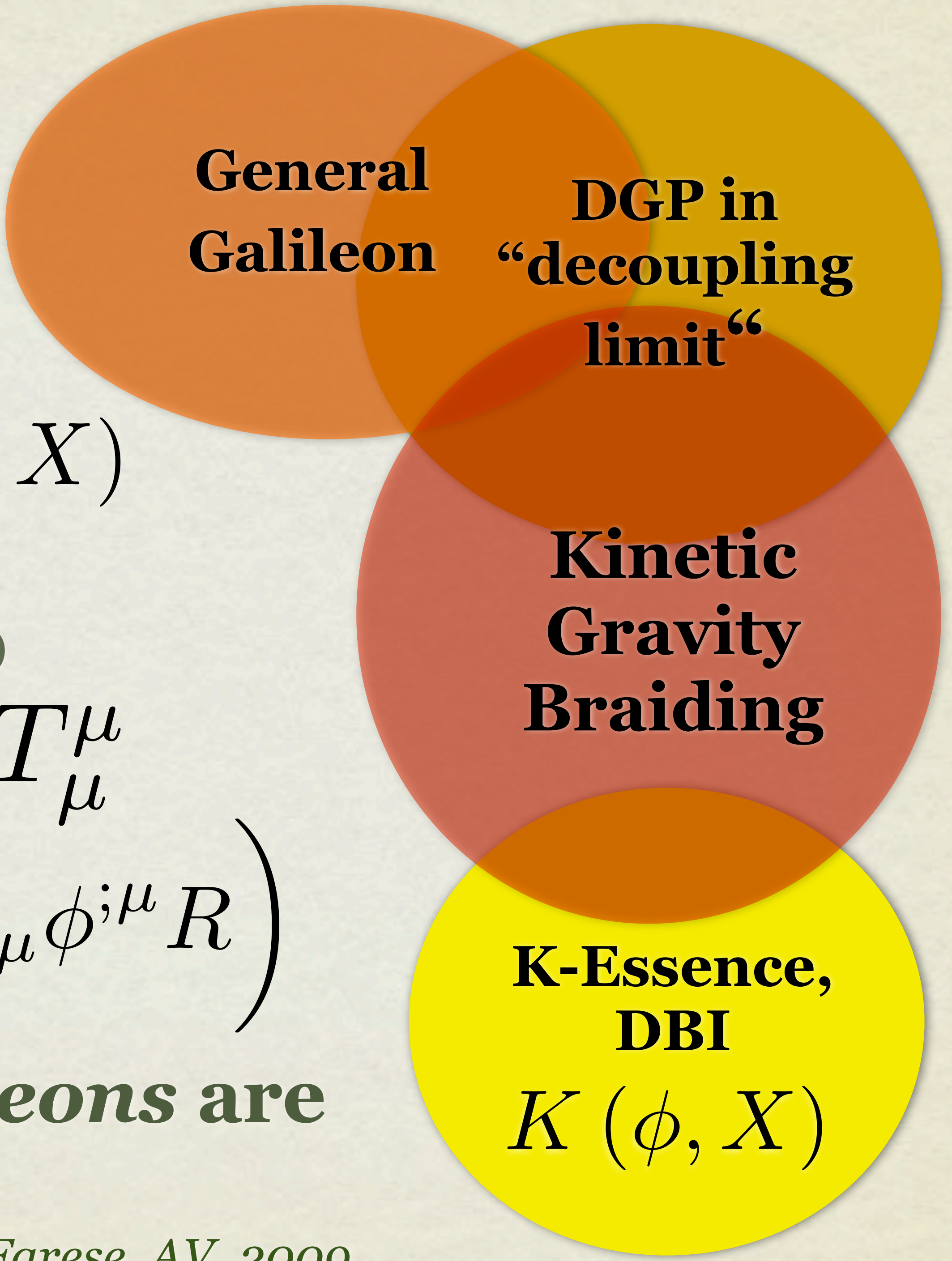
BUT

- Does **not require** the Galilean symmetry: $\partial_\mu \phi \rightarrow \partial_\mu \phi + c_\mu$ & $\phi \rightarrow \phi + c$
- General functions $K\left(\phi, X\right)$ & $G\left(\phi, X\right)$
- **Minimal coupling to gravity, NO NO higher order therms like** ϕT_μ^μ

$$\phi_{;\lambda}\phi^{;\lambda}\left(\left(\Box\phi\right)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu} - \frac{1}{4}\phi_{;\mu}\phi^{;\mu}R\right)$$

Indeed, manifestly healthy *Galileons* are NEVER *Galilean* symmetric !!!

Deffayet, Esposito-Farese, AV, 2009



General Galileon

DGP in "decoupling limit"

Kinetic Gravity Braiding

K-Essence, DBI

$K\left(\phi, X\right)$

EXPANSIONS IN GRADIENT TERMS

- K-Essence, DBI etc

$$K\left(\phi, X\right) \sim X\left(1 + c_1\left(\phi\right)X + c_2\left(\phi\right)X^2 + ...\right)$$

- Kinetic Gravity Braiding – integrate the canonical kinetic energy by parts

$$G\left(\phi, X\right)\Box\phi \sim -\phi\Box\phi\left(1 + \tilde{c}_1\left(\phi\right)X + \tilde{c}_2\left(\phi\right)X^2 + ...\right)$$

EQUATION OF MOTION I

$$L^{\mu\nu}\nabla_\mu\nabla_\nu\phi + (\nabla_\alpha\nabla_\beta\phi)\, Q^{\alpha\beta\mu\nu}\, (\nabla_\mu\nabla_\nu\phi) +$$

$$+Z - G_X R^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = 0$$

Braiding

EOM is of the second order: $L_{\mu\nu}\, , Q^{\alpha\beta\mu\nu}\, ,\ Z$

constructed from field and it's first derivatives

$Q^{\alpha\beta\mu\nu}$ **is such that EOM is a 4D Lorentzian generalization of the Monge-Ampère Equation,**

always *linear* in $\ddot{\phi}$

EQUATION OF MOTION II

- **Shift-Charge Current:** J_μ

$$J_\mu = \left(\mathcal{L}_X - 2G_\phi\right)\nabla_\mu\phi - G_X\nabla_\mu X$$

- **New Equivalent Lagrangian:** $\mathcal{P}$

$$\mathcal{P} = K - 2XG_\phi - G_X\nabla^\lambda\phi\nabla_\lambda X$$

- **Equation of motion is a “conservation law”:**

$$\boxed{\nabla_\mu J^\mu = \mathcal{P}_\phi}$$

BRAIDING

$$\begin{aligned} \text{Einstein Equations}\,(\phi, \partial\phi, \partial\partial\phi, g, \partial g, \partial\partial g) &= 0 \\ \phi\text{EoM}\,(\phi, \partial\phi, \partial\partial\phi, g, \partial g, \partial\partial g) &= 0 \end{aligned}$$

Cannot solve separately !!!!

characteristics (cones of propagation)

depend on external matter

IMPERFECT FLUID
FOR TIMELIKE GRADIENTS

- **Four velocity :** $u_\mu \equiv \dfrac{\nabla_\mu \phi}{\sqrt{2X}}$ → ϕ is an ***internal clock***

- **projector:** $\perp_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$

- **Time derivative:** $(\dot{\ }) \equiv \dfrac{\mathrm{d}}{\mathrm{d}\tau} \equiv u^\lambda \nabla_\lambda$

- **Expansion :** $\theta \equiv \perp^\lambda_\mu \nabla_\lambda u^\mu = \dot{V}/V$

comoving volume

Shift-symmetry
$\phi \longrightarrow \phi + c$
violates
$\phi \longrightarrow -\phi$
and introduces
arrow of time

EFFECTIVE MASS & CHEMICAL POTENTIAL

$$\kappa \equiv 2XG_X$$

- charge density: $n \equiv J^\mu u_\mu = n_0 + \kappa\theta$

“Braiding”

- energy density: $\mathcal{E} \equiv T^{\mu\nu} u_\mu u_\nu = \mathcal{E}_0 + \theta\dot{\phi}\kappa$
- effective mass per shift-charge / chemical potential:

$$m \equiv \left(\frac{\partial\mathcal{E}}{\partial n}\right)_{V,\phi} = \sqrt{2X} = \dot{\phi}$$

SHIFT-CURRENT AND DIFFUSION

"Diffusion"

$$J_\mu = n u_\mu - \frac{\kappa}{m} \perp_\mu^\lambda \nabla_\lambda m$$

§ 59, L&L, vol. 6

$$\kappa \equiv 2 X G_X$$

Is a "diffusivity"/ transport coefficient

IMPERFECT FLUID ENERGY-MOMENTUM TENSOR

- **Pressure** $$\mathcal{P} \equiv -\frac{1}{3} T^{\mu\nu} \perp_{\mu\nu} = P_0 - \kappa \dot{m}$$
- **Energy Flow** $$q_\mu \equiv \perp_{\mu\lambda} T^{\lambda}_{\nu} u^\nu = m \perp_{\mu\nu} J^\nu$$

$$q_\mu = -\kappa \perp^{\nu}_{\mu} \nabla_\nu m$$

No Heat Flux!

- **Energy Momentum Tensor**

$$T_{\mu\nu} = \mathcal{E} u_\mu u_\nu - \perp_{\mu\nu} \mathcal{P} + 2 u_{(\mu} q_{\nu)}$$

Solving for $\dot{m}$ *for small gradients or small* κ *one obtains "bulk viscosity"*

DIFFUSION OF CHARGE

For incompressible motion $\theta \equiv 0$

equation of motion is:

$$\dot{n} = -\overline{\boldsymbol{\nabla}}_\mu \left(\mathfrak{D} \overline{\boldsymbol{\nabla}}^\mu n \right) + \mathfrak{D} a^\mu \overline{\boldsymbol{\nabla}}_\mu n$$

where the diffusion constant: $\mathfrak{D} \equiv -\frac{\kappa}{n_m m}$

c.f. § *59, L&L, vol. 6, p 232*

4-acceleration: $a^\mu \equiv \dot{u}^\mu$

spatial gradient: $\overline{\boldsymbol{\nabla}}_\mu \equiv \perp^\nu_\mu \nabla_\nu$

ENERGY CONSERVATION IN COMOVING VOLUME

Energy conservation: $u_\nu \nabla_\mu T^{\mu\nu} = 0$

$$\mathrm{d}E = -\mathcal{P}\mathrm{d}V + m\mathrm{d}\mathcal{N}_{\mathrm{dif}}$$

Euler relation: $\mathcal{E} = mn - P_0$

Momentum conservation:

$$\perp_{\mu\nu} \nabla_\lambda T^{\lambda\nu} = 0$$

VACUUM-ATTRACTORS

Euler relation: $\mathcal{E} = mn - P_0$

for no particles: $n_* = 0$

$$\mathcal{E}_* = -\mathcal{P}_* - \kappa_* \dot{m}_*$$

almost dS!

COSMOLOGY

$q_\mu = 0$ **and** $\theta = 3H$

Friedmann Equation:

$$H^2 = \kappa m H + \frac{1}{3}\left(\mathcal{E}_0 + \rho_{\mathrm{ext}}\right)$$

$r_{\mathrm{c}}^{-1} = \kappa m$ **“crossover” scale in DGP**

CHARGE CONSERVATION

$$\dot{n} + 3Hn = \mathcal{P}_\phi$$

If there is shift-symmetry then

$$\mathcal{P}_\phi = 0$$

$$n \propto a^{-3}$$

INFLATION BRINGS THE SCALAR TO ATTRACTOR

$$n_* = 0$$

EXAMPLE:
SIMPLEST IMPERFECT DARK ENERGY

Only one free parameter μ

- Lagrangian

$$\mathcal{L} = X\left(-1 + \mu\Box\phi\right)$$

- shift-charge density

$$n = m\left(3\mu H m - 1\right)$$

NONTRIVIAL ATTRACTOR

No Particles: $n_* = 0$

$$m_* = (3\mu H)^{-1}$$

$$H_*^2 = \frac{1}{6}\rho_{\text{ext}}\left(1 + \sqrt{1 + \frac{2}{3}\left(\mu\rho_{\text{ext}}\right)^{-2}}\right)$$

STABLE

$$m_* = 0$$

$$H_*^2 = \frac{1}{3}\rho_{\text{ext}}$$

GHOSTY

DARK ENERGY

assume today $\sqrt{\frac{3}{2}\mu\rho_{\mathrm{ext}}} \ll 1$ → $H_*^2 \simeq \frac{1}{6}\sqrt{\frac{2}{3}}\mu^{-1}$

$\Lambda_* \simeq \frac{1}{2}\sqrt{\frac{2}{3}}\mu^{-1} \simeq 3\rho_{\mathrm{CDM}}$ → $\sqrt{\frac{3}{2}}\rho_{\mathrm{CDM}}\mu \simeq \frac{1}{6} \ll 1$

$$\text{Mass Scale} \sim \mu^{-1/3} \sim (H_0^2 M_{\mathrm{Pl}})^{1/3} \sim 10^{-13}\mathrm{eV}$$

Length Scale: **1000 km**

In Quintessence - the size of the universe

HIGH FREQUENCY STABILITY

Effective metric for perturbations

$$\mathcal{G}_{\mu\nu} = D u_\mu u_\nu + \Omega \perp_{\mu\nu} - \frac{2\kappa}{m}\mathcal{K}_{\mu\nu} - 2\kappa_m a_{(\mu}u_{\nu)}$$

Extrinsic curvature for $\phi = \mathrm{const}$

No Ghosts ⟺

$$D = \frac{\mathcal{E}_m - \kappa\theta}{m} + \frac{3}{2}\kappa^2$$

$$\Omega = \frac{n + \nabla_\lambda\left(\kappa u^\lambda\right)}{m} - \frac{1}{2}\kappa^2$$

In general propagation is anisotropic, but in cosmology:

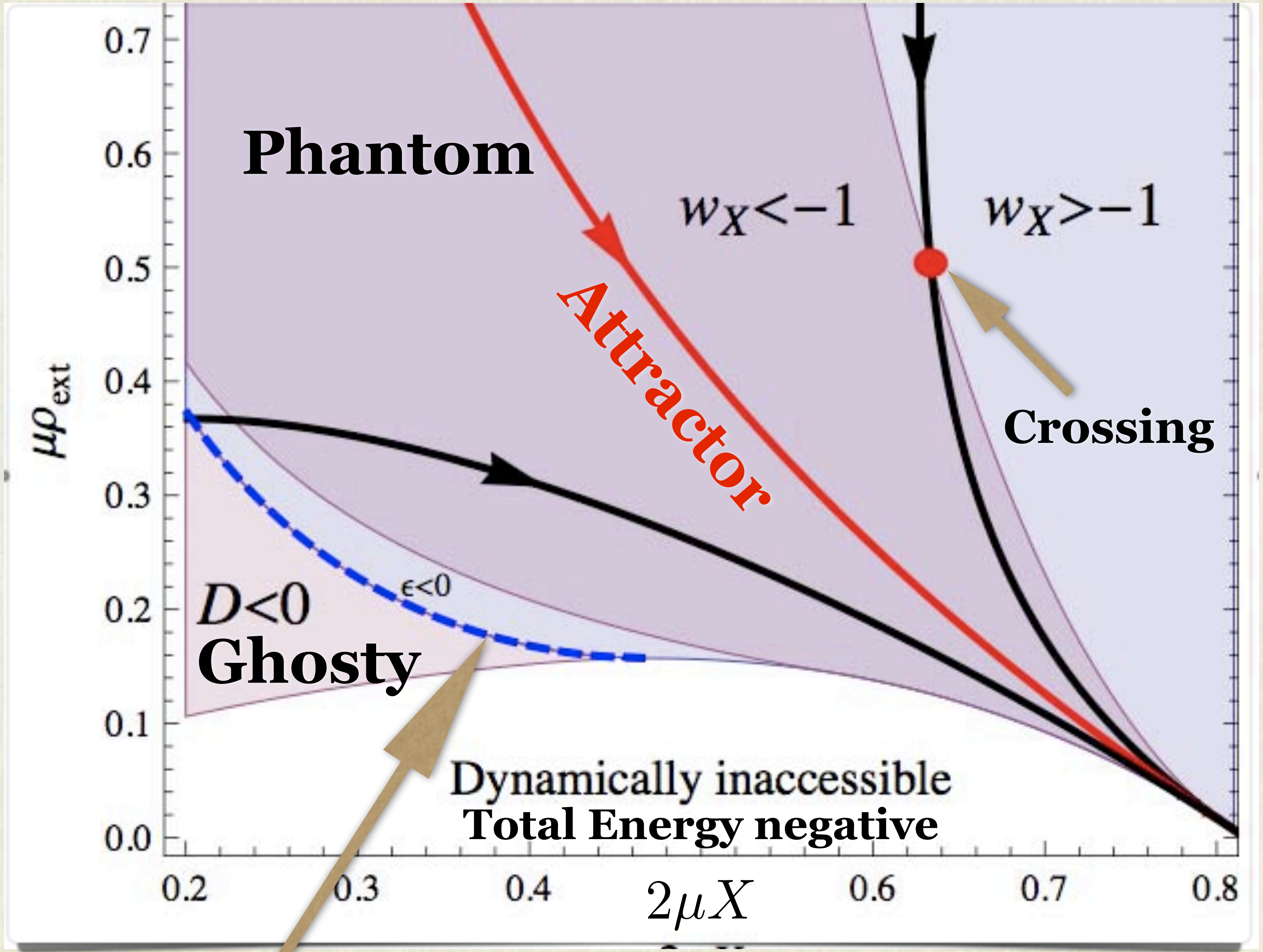
$$c_\mathrm{s}^2 = \frac{\Omega m - 2\kappa H}{mD}$$

SOUND SPEED

$$c_{\mathrm{s}}^2 = \frac{\mathcal{P}_m + 2\dot{\kappa} + \kappa\left(4H - \kappa m/2\right)}{\mathcal{E}_m - 3\kappa\left(H - \kappa m/2\right)} \neq \frac{\dot{\mathcal{P}}}{\dot{\mathcal{E}}}$$

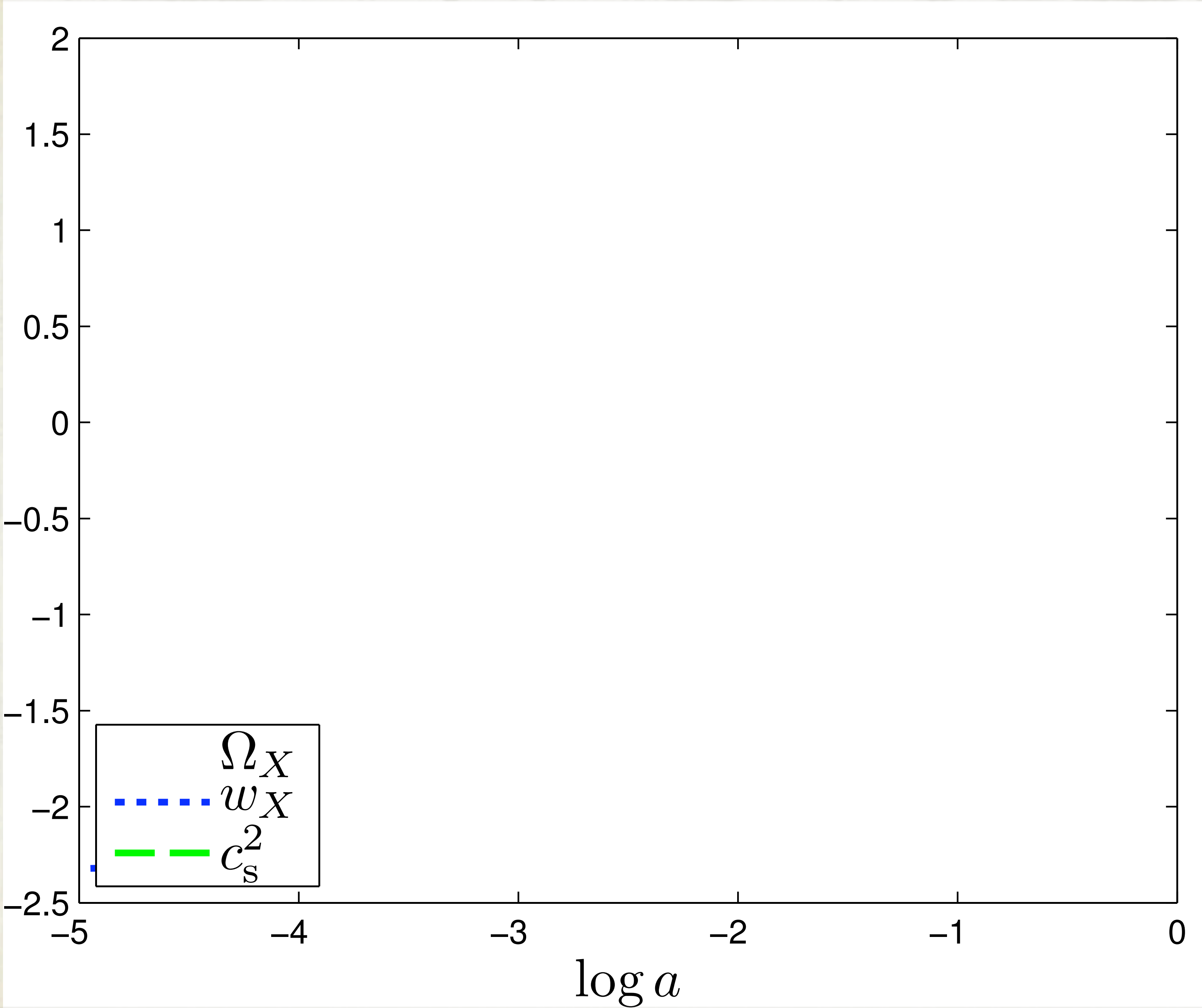
The relation between the equation of state, the sound speed and the presence of ghosts is very different from the k-*essence* & perfect fluid.

A manifestly stable *Phantom* ($w_X < -1$) is possible even with a *single* degree of freedom and *minimal* coupling to gravity

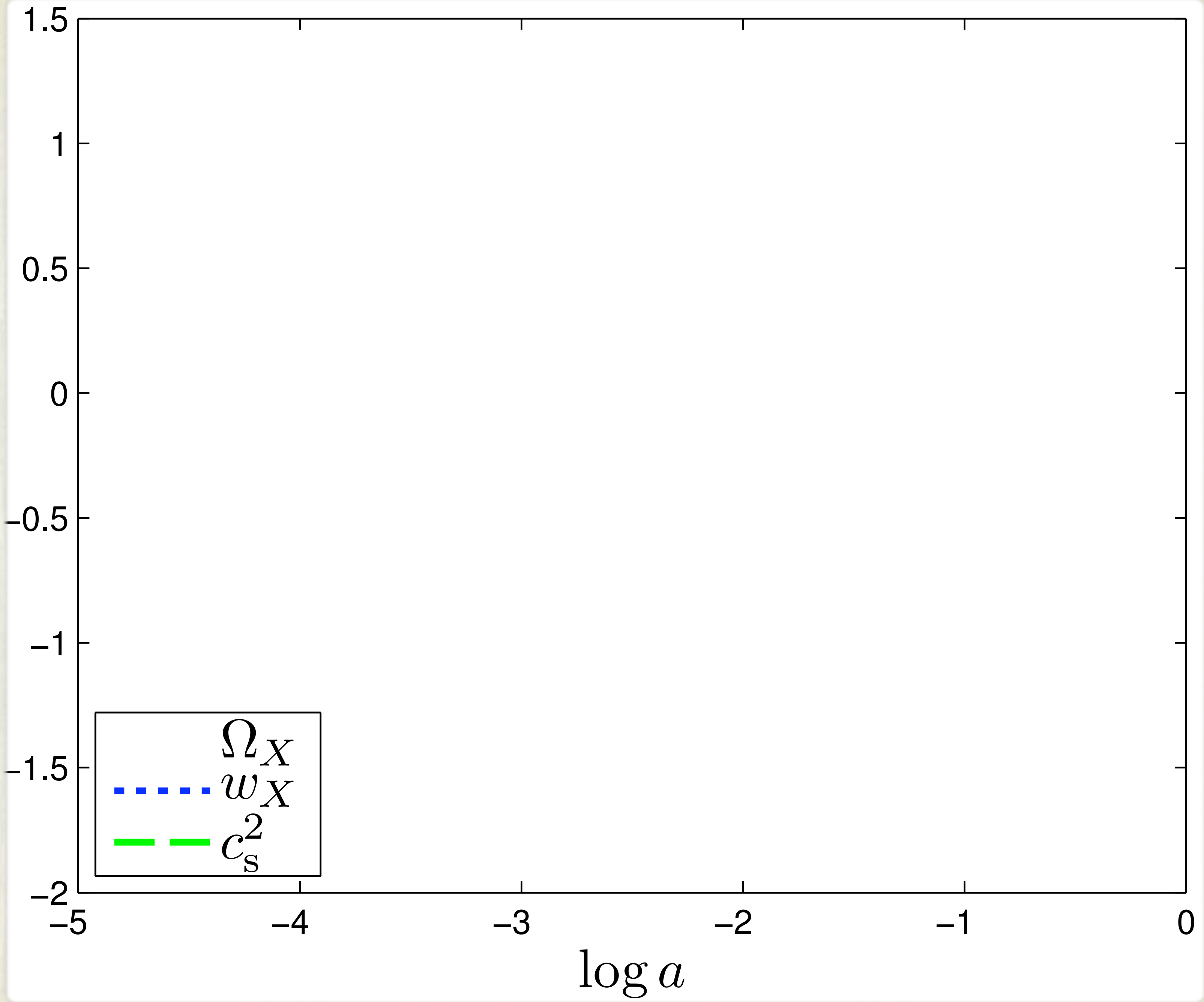


Pressure singularity

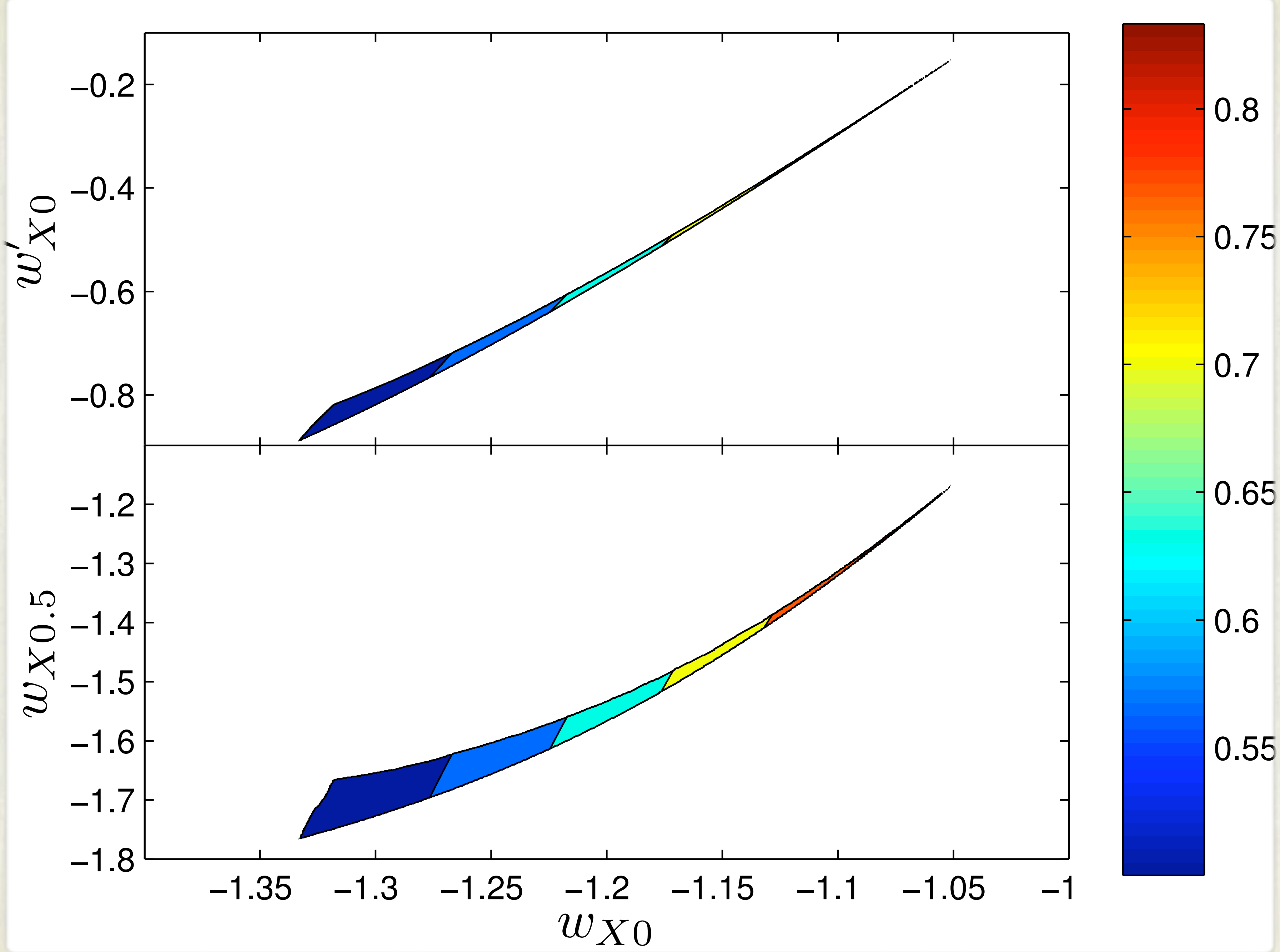
Phase portrait for scalar field & dust



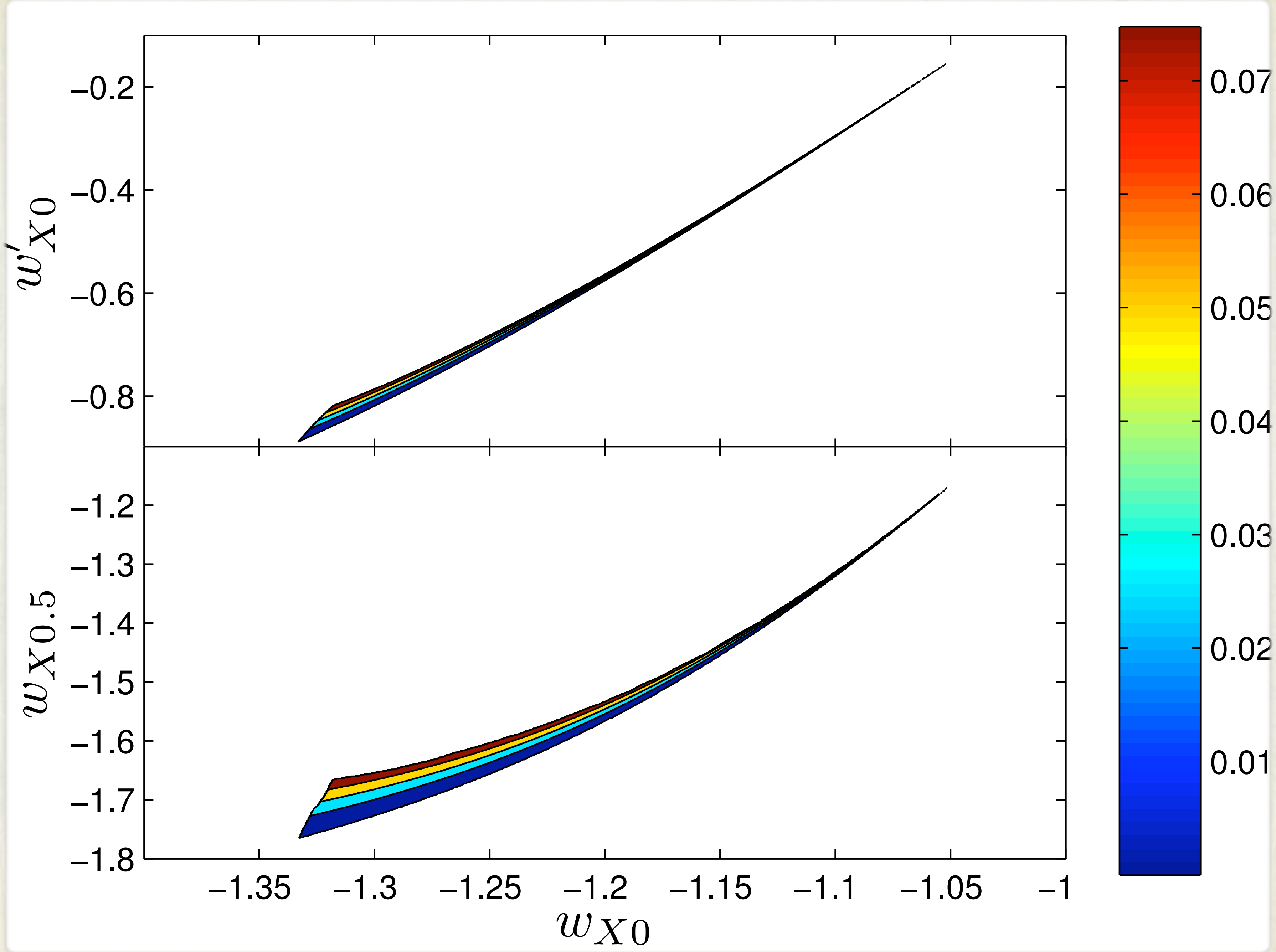
Evolution of dark energy properties in the Friedmann universe also containing dust and radiation. The scalar evolves on its attractor throughout the presented period. During matter domination $w_X = -2$, while $w_X = -7/3$ during radiation domination. The sound speed is superluminal when the scalar energy density is subdominant, becoming subluminal when $\Omega_X \approx 0.1$ and $w_X \approx -1.4$



Evolution of DE properties in the Friedmann universe which also contains dust and radiation. The energy density in the scalar is J-dominated (off attractor) until a transition during the matter domination epoch. This allows the scalar to increase its contribution to the total energy budget throughout radiation domination ($w_X = 1/6$) and provide an early DE peaked at matter-radiation equality, from whence it begins to decline with $w_X = 1/4$. The transition to the attractor behaviour is rapid. The equation of state crosses $w_X = -1$ and the scalar energy density begins to grow. The final stages of evolution are on the attractor and are similar to those presented in previous figure.



$0.1 < \Omega_\mathrm{m} < 0.5$ and $\Omega_{X\mathrm{eq}} < 0.1$. The shading contours correspond to the energy density of DE today Ω_{X0}. Two parameterisations of DE behaviour are shown: w_X and w'_X evaluated today, and w_X evaluated today and at $z = 1/2$. The requirement that the energy density in DE at matter-radiation equality be small, $\Omega_X^\mathrm{eq} < 0.1$ forces the value of the shift charge to be small today $Q_0 < 10^{-2}$. This means that in the most recent history, the evolution has effectively been on attractor or very close to it and the permitted value of w_X is very restricted and determined to all intents and purposes by Ω_X^0.



The shading representing the contribution of DE to energy density at matter-radiation equality. We choose to cut the parameters such that the contribution to this early DE at that time is no larger than 10%. It can clearly be seen that values of w_X closer to -1 are obtained when the shift charge is larger, but this leads to more early DE, eventually disagreeing with current constraints

FURTHER DEVELOPMENT

Kinetic Gravity Braiding with $G \propto X^n$

arXiv:1011.2006v2 [astro-ph.CO], *Rampei Kimura, Kazuhiro Yamamoto*

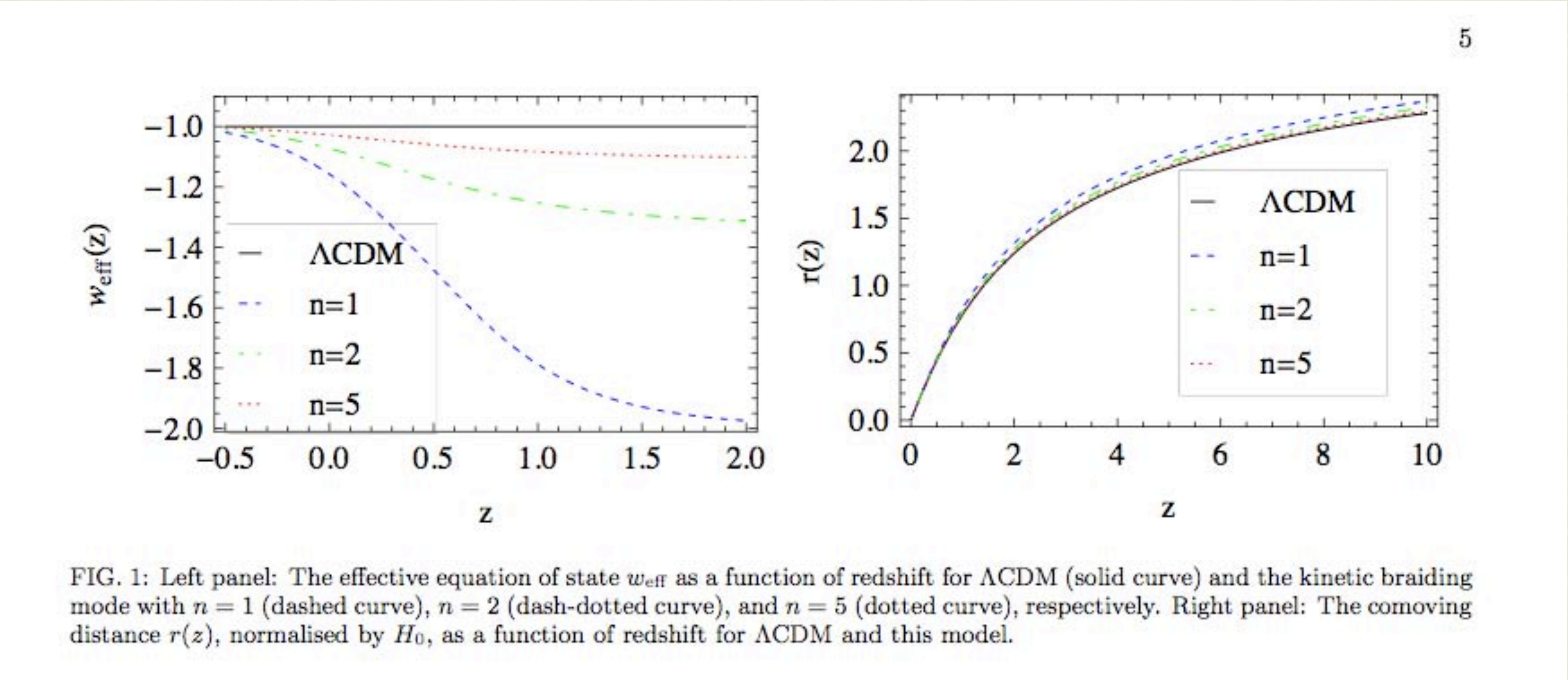


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FIG. 1: Left panel: The effective equation of state $w_{\rm eff}$ as a function of redshift for ΛCDM (solid curve) and the kinetic braiding mode with $n = 1$ (dashed curve), $n = 2$ (dash-dotted curve), and $n = 5$ (dotted curve), respectively. Right panel: The comoving distance $r(z)$, normalised by H_0, as a function of redshift for ΛCDM and this model.

CONSTRAINTS FROM CMB AND SN IA

Kinetic Gravity Braiding with $G \propto X^n$

arXiv:1011.2006v2 [astro-ph.CO], *Rampei Kimura, Kazuhiro Yamamoto*

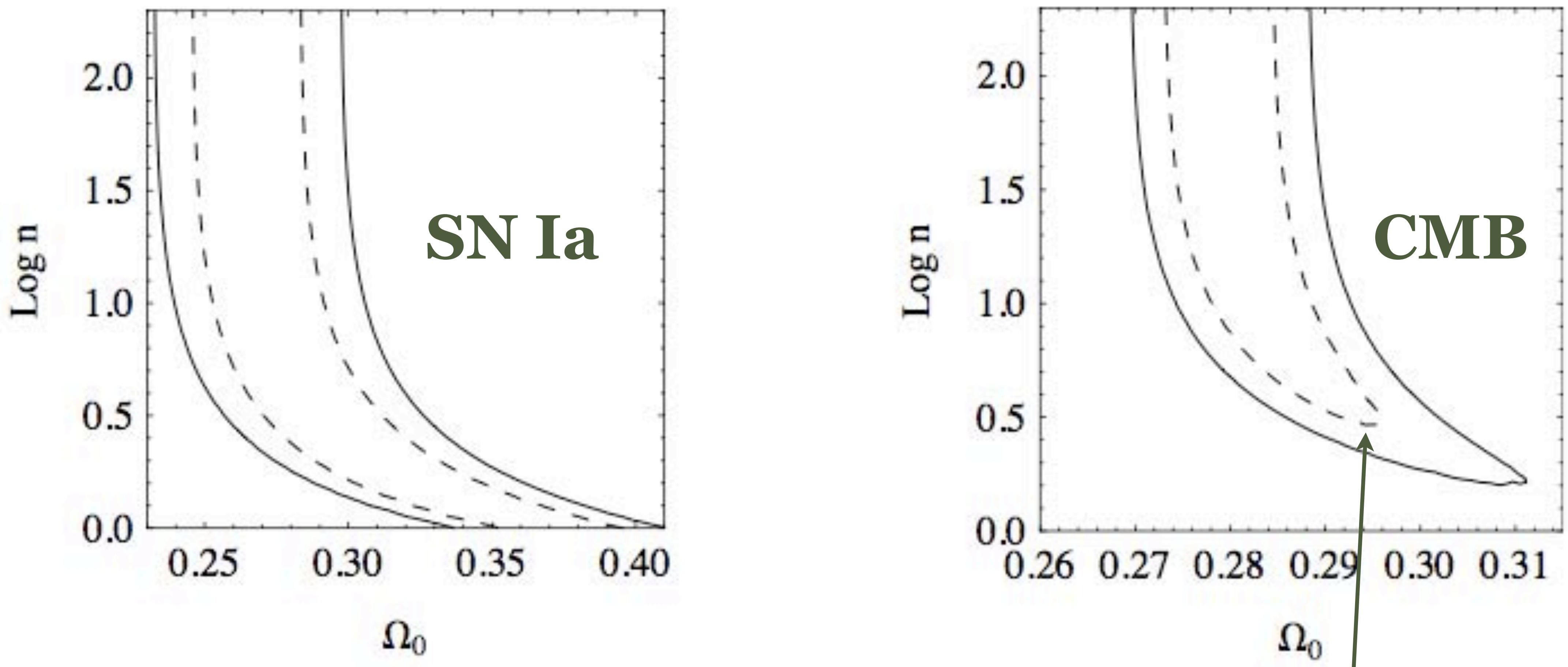


FIG. 3: The left panel is the contour of $\chi^2_{\rm SN}$ on the plane Ω_0 and n for the kinetic braiding model. The dashed curve and the solid curve are the 1 σ and 2 σ contours, respectively. The right panel is the same but of $\chi^2_{\rm CMB}$.

The SCP Union2 Compilation is a collection of 557 type Ia supernovae data whose range of the redshift is 0.015 < z < 1.4

Thus $n \gtrsim 3$ mass scale $\sim 10^{-3}\mathrm{eV}$

Length Scale: **1/10 mm**

GROWTH FACTOR

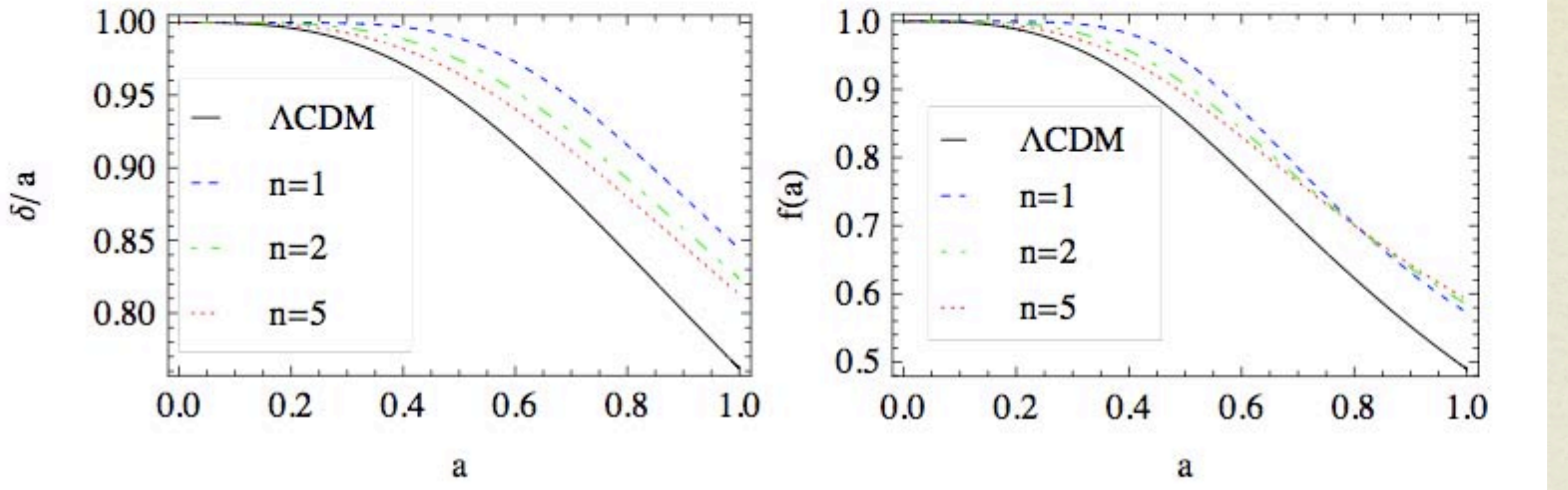
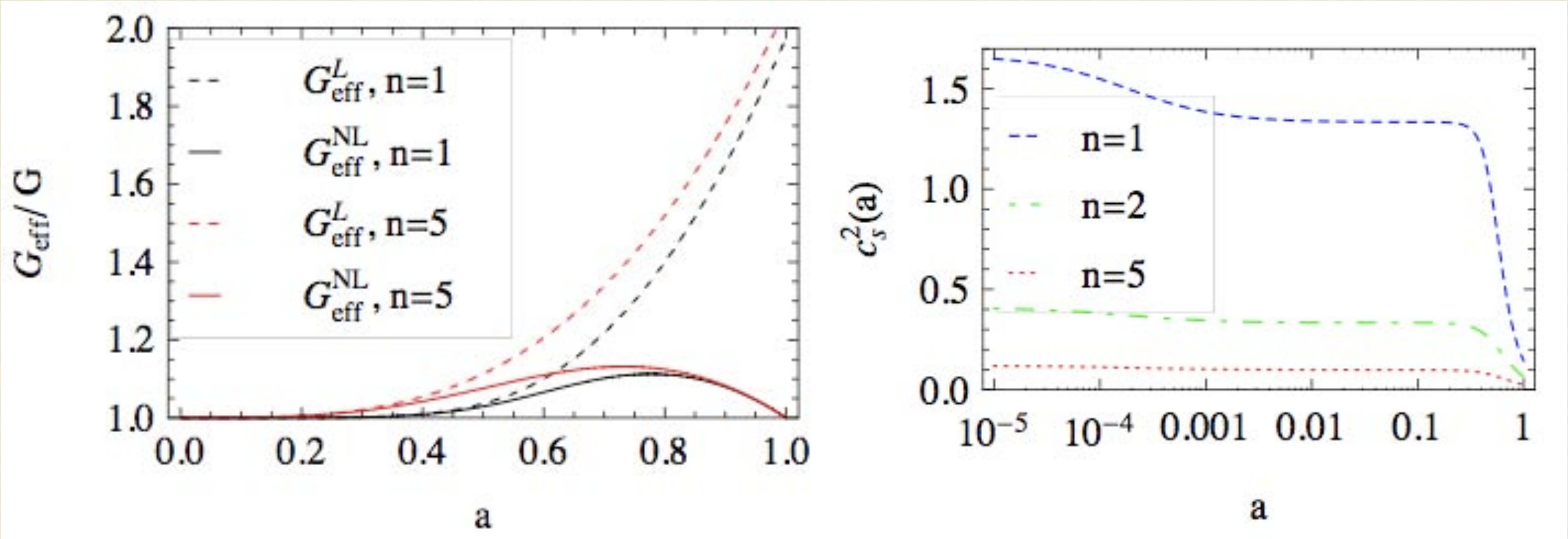


FIG. 4: Left panel: The growth factor divided by scale factor as a function of scale factor for the Λ CDM model (solid curve) and the kinetic braiding model $n = 1$ (dashed curve), $n = 2$ (dash-dotted curve), and $n = 5$ (dotted curve), respectively. Right panel: The linear growth rate as a function of scale factor.

Kinetic Gravity Braiding with $G \propto X^n$

arXiv:1011.2006v2 [astro-ph.CO], *Rampei Kimura, Kazuhiro Yamamoto*

EFFECTIVE NEWTON CONSTANT FOR PERTURBATIONS AND THE SOUND SPEED



Kinetic Gravity Braiding with $G \propto X^n$

arXiv:1011.2006v2 [astro-ph.CO], *Rampei Kimura, Kazuhiro Yamamoto*

FOR INFLATIONARY MODELS WITH BRAIDING SEE G-(& *GENERALIZED G)-INFLATION*

T. Kobayashi , M. Yamaguchi, J. Yokoyama
arXiv:1008.0603 [hep-th]
arXiv:1105.5723 [hep-th]

CONCLUSIONS I

- **Scalar field ϕ with a non-canonical *action* can "behave" like imperfect fluid: on general (not exactly isotropic and homogeneous FRW) background:**

$$T_{\mu\nu} \neq \mathcal{E} u_{\mu} u_{\nu} - \perp_{\mu\nu} \mathcal{P}$$

- **Fluid elements (particles) are shift-charges: charges with respect to:** $\phi \longrightarrow \phi + c$

- ϕ **kinetically mixes / "braids" with the metric:**

$$(\partial\phi)^2 \, \partial_{\mu} \left(\sqrt{-g} g^{\mu\nu} \partial_{\nu}\phi\right) \text{ c.f. } F^{(1)}_{\mu\nu}\left(A^{\alpha}\right) F^{(2)\mu\nu}\left(B_{\beta}\right)$$

CONCLUSIONS II

- **Manifestly stable (*no ghosts* and *no gradient instabilities*) and *large* violation of the Null Energy Condition (NEC) is possible even in theories *minimally coupled to gravity*: *healthy Phantom* with** $w < -1$
- **Vanishing shift-charge,** $n = 0$ **, corresponds to cosmological attractors similar to *Ghost Condensate* / “bad” *k-Inflation*. But here these attractors can be manifestly stable (*no ghosts* and *no gradient instabilities*) and their exact properties depend on external matter. Through Euler relation** $\mathcal{E} = mn - P_0$ **these attractors generically evolve to de Sitter in late time asymptotic.**
  Interesting for DE & Inflation!

THANKS A LOT FOR YOUR ATTENTION!