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# Statistical Symmetry Breaking in the CMB

Jiro Soda
Kyoto University

**Watanabe, Kanno, JS, arXiv:0902.2833; PRL 102, 191302, 2009.**

**Kanno, Watanabe, JS, arXiv:0908.3509; JCAP 0912:009, 2009.**

**Watanabe, Kanno, JS, arXiv:1003.0056; Prog. Theor. Phys. 123, 1041, 2010**

**Kanno, Watanabe, JS, arXiv:1010.5307; JCAP 1012:024, 2010.**

**Watanabe, Kanno, JS, arXiv:1011.3604; MNRAS 412:L83-L87, 2011.**

**Murata, JS, arXiv:1103.6164, JCAP 2011, to appear.**

# Introduction: Inflation and CMB fluctuations

- Horizon problem
- Flatness problem

→ de Sitter universe

$$ds^2 = -dt^2 + e^{2Ht}\left[dx^2 + dy^2 + dz^2\right]$$

The exponential expansion can be realized by the vacuum energy

$$S = \frac{M_p^2}{2}\int d^4x\sqrt{-g}\left[R - 6H^2\right]$$

- Origin of the large scale structure of the universe

→ Quantum fluctuations

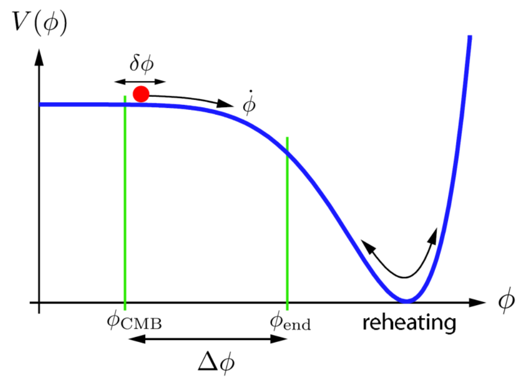
$$\delta\phi \approx H$$

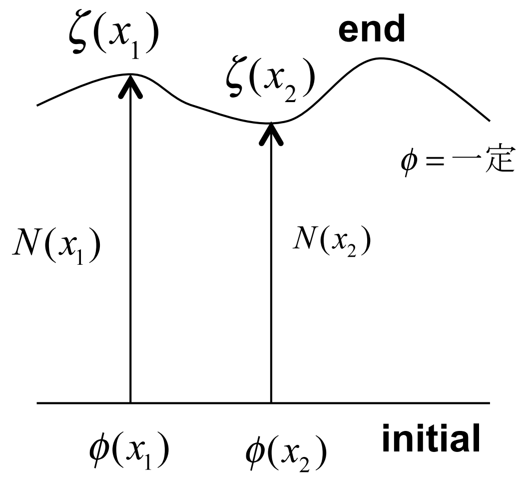
t

# What did COBE observe?

**Curvature perturbations**

$$\zeta = \delta N = H\delta t = H\frac{\delta\phi}{\dot\phi}$$



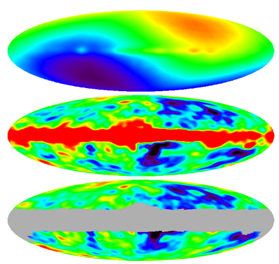



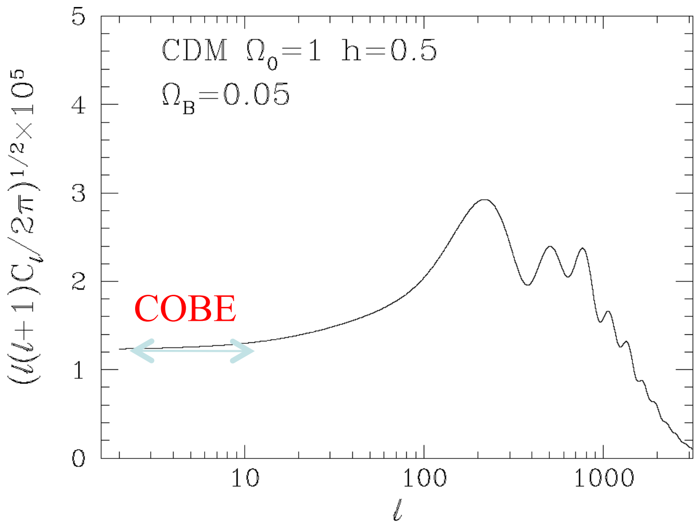

**CMB angular power spectrum**

**gravitational red shift** $\quad \frac{\delta T}{T} \sim \zeta$

$$\frac{\delta T}{T} = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi)$$

$$\langle a_{\ell m} a_{\ell' m'} \rangle = C_\ell \delta_{\ell\ell'} \delta_{mm'}$$





# Symmetry in inflation

**The nature of primordial fluctuations is determined by symmetry in inflation.**

- **First of all, in order to have inflation, we need to assume initial homogeneity.**

- **In addition to this initial condition, we need a sufficiently flat potential to realize the slow roll inflation. Hence, we have shift symmetry** $\phi \to \phi + c$

- **Once the slow roll inflation occurs, the cosmic no-hair conjecture suggests that the exponential expansion erases any classical anisotropy and leads to isotropic universe. This is nothing but the spatial de Sitter symmetry.**

- **de Sitter spacetime**

$$ds^2 = -dt^2 + e^{2Ht}\left[ dx^2 + dy^2 + dz^2 \right]$$

has the temporal de Sitter symmetry $t \to t + c, \quad x^i \to e^{-2Hc} x^i$

# The nature of primordial fluctuations

**Thus, approximately, we have the following predictions:**

**First of all, shift symmetry implies Gaussian statistics**

$$\left\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\right\rangle = P(\mathbf{k}_1,\mathbf{k}_2)$$

**Moreover, initial homogeneity implies statistical homogeneity**

$$\left\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\right\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)P(\mathbf{k}_1)$$

**And, spatial de Sitter symmetry accounts for statistical isotropy**

$$\left\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\right\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)P(k_1=|\mathbf{k}_1|)$$

**Finally, temporal deSitter symmetry yields scale invariant spectrum**

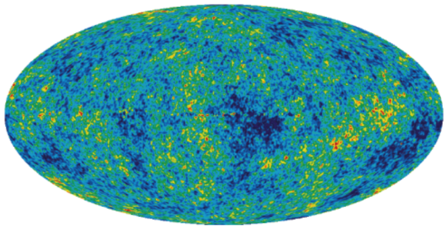
$$P(k) \approx const.$$

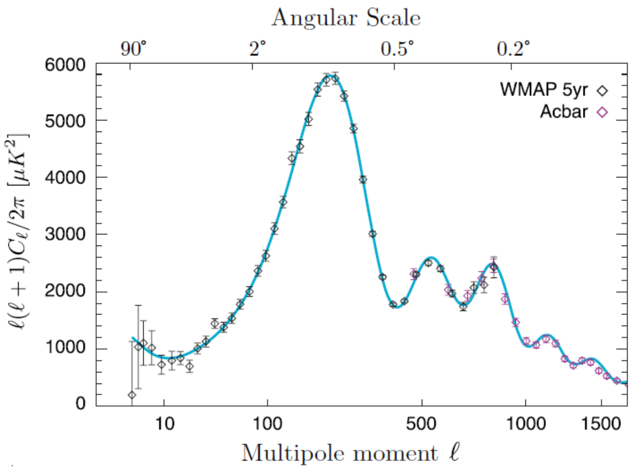
**The above predictions are model independent and robust.**

# More precise data are now available!

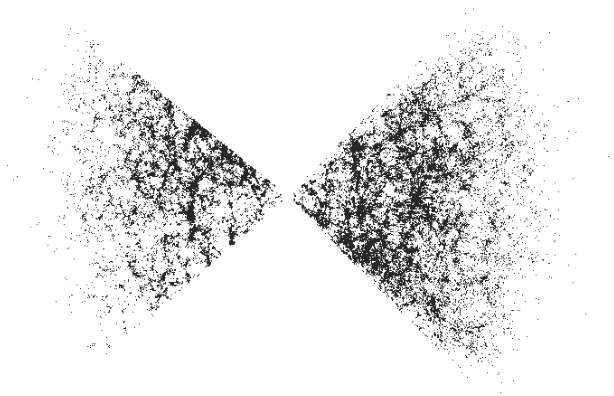
**We now have more precise data.**

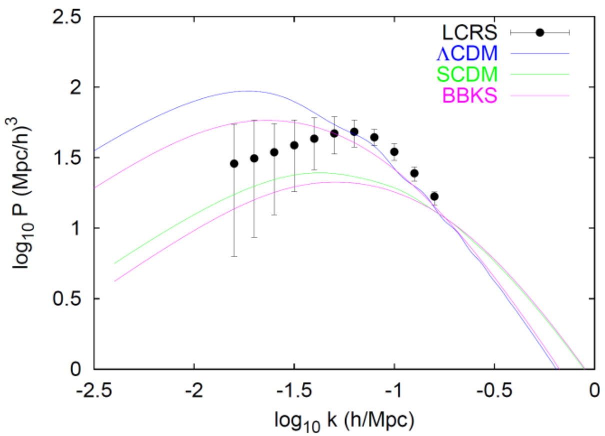
Cosmic microwave background radiation






galaxy distribution





# Statistical Symmetry Breaking

**Precision cosmology forces us to look at fine structures of fluctuations!**

**Violation of temporal de Sitter symmetry** -> **spectral tilt**

**There should be a slight tilt because the expansion is not exactly deSitter. The deviation from deSitter can be characterized by the slow roll parameter. Hence, the tilt should be of the order of the slow roll parameter.**

**Violation of shift symmetry** -> **non-Gaussianity**

**There should be small non-gaussianity of the order of the slow roll parameter because the shift symmetry is not exact.**

**Violation of spatial de Sitter symmetry ?**

**Along the line of this thought,**
**it is natural to study a deviation from the statistical isotropy.**

**In fact, as we will see,**
**the statistical anisotropy is ubiquitous in the framework of supergravity.**

**Watanabe, Kanno, Soda, PRL, 2009.**

# Gauge kinetic function in the sky

**Superstring theory** —low energy→ **Supergravity**

- Kahler potential K
- Superpotential W
- Gauge kinetic function f

$$g_{i\bar{j}} = \frac{\partial^2 K}{\partial \phi^i \partial \phi^{\bar{j}}}$$

$$D_i W = \frac{\partial W}{\partial \phi^i} + \kappa^2 \frac{\partial K}{\partial \phi^i} W$$

$$S = \int d^4x \left[ \sqrt{-g} R + g_{i\bar{j}} \partial^\mu \phi^i \partial_\mu \phi^{\bar{j}} - e^{\kappa^2 K} g^{i\bar{j}} \left( D_i W D_j \overline{W} - 3\kappa^2 |W|^2 \right) - \frac{1}{4} \mathrm{Re}\, f_{ab}(\phi^i) F^{a\mu\nu} F^b_{\mu\nu} + \cdots \right]$$

**Cosmological roles of K and W in inflation has been well discussed so far.**
**However, the role of gauge kinetic function f in inflation has been overlooked.**

**The main goal of this talk is to show that**

- **Anisotropic inflation is naturally realized due to gauge kinetic function.**
- **As a consequence, statistical anisotropy is produced.**
- **There arises cross correlation between temperature and B-mode polarization.**

**Namely, gauge kinetic function can be constrained by cosmological observations!**

# Plan of my talk

1. Anisotropic Inflation with a gauge kinetic function
2. Cosmological perturbation theory
   in a simple Bianchi universe
3. The nature of primordial fluctuations
   in anisotropic inflation
4. Summary

# Anisotropic Inflation
# with a gauge kinetic function

# A simple model

**Watanabe, Kanno, Soda, PRL, 2009.**

Action

**gauge kinetic function**

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{2}\left(\partial_\mu\phi\right)^2-V(\phi)-\frac{1}{4}f^2(\phi)\,F_{\mu\nu}F^{\mu\nu}\right]$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$$

For homogeneous background, the time component can be eliminated by gauge transformation.

Let the direction of the vector be $x$ - axis

$$A_\mu=(0,\ v(t),\ \ 0\ \ ,\ \ 0\ \ )\qquad\qquad \phi=\phi(t)$$

Then, the metric should be Bianchi Type-I

Anisotropy

$$ds^2=-dt^2+e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2+e^{2\sigma(t)}\left(dy^2+dz^2\right)\right]$$

Scale Factor

Plane Symmetry

The action reduces to

$$S=\int d^4x\ e^{3\alpha}\left[\frac{3}{\kappa^2}(-\dot{\alpha}^2+\dot{\sigma}^2)+\frac{1}{2}\dot{\phi}^2-V(\phi)+\frac{1}{2}f^2(\phi)e^{-2\alpha+4\sigma}\dot{v}^2\right]$$

$$\dot{v}=f^{-2}(\phi)e^{-\alpha-4\sigma}E$$

const. of integration

# Basic equations

Hamiltonian Constraint $\bullet = \partial_t$

$$\dot{\alpha}^2 = \dot{\sigma}^2 + \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{E^2}{2} f^{-2}(\phi) e^{-4\alpha-4\sigma}\right]$$

Scale factor

$$\ddot{\alpha} = -3\dot{\alpha}^2 + \kappa^2 V(\phi) + \frac{\kappa^2 E^2}{6} f^{-2}(\phi) e^{-4\alpha-4\sigma}$$

Anisotropy

$$\ddot{\sigma} = -3\dot{\alpha}\dot{\sigma} + \frac{\kappa^2 E^2}{3} f^{-2}(\phi) e^{-4\alpha-4\sigma}$$

Scalar field $' = \partial_\phi$

$$\ddot{\phi} = -3\dot{\alpha}\dot{\phi} - V_\phi(\phi) + E^2 f^{-3}(\phi) f_\phi(\phi) e^{-4\alpha-4\sigma}$$

# Isotropic Power-law Inflation

**Let us start with a natural choice for potential and gauge kinetic functions.**

$$V = V_0 e^{\lambda \frac{\phi}{M_p}} \qquad f = f_0 e^{\rho \frac{\phi}{M_p}}$$

**Cf. Primordial magnetic fields** **Ratra, 1992.**

**In this case, it is well known that there exists a simple solution**

$$ds^2 = -dt^2 + t^{4/\lambda^2}\left(dx^2 + dy^2 + dz^2\right)$$

$$\frac{\phi(t)}{M_p} = -\frac{2}{\lambda}\log t + \phi_0 \qquad \frac{V_0}{M_p^2} e^{\lambda\phi_0} = \frac{2(6-\lambda^2)}{\lambda^4}$$

For $\lambda \ll 1$, **this solution represents an isotropic power-law inflation.**

Here, the gauge kinetic function does not play any role.

**Is this a unique exact solution?**

# Anisotropic Power-law inflation

Kanno, Watanabe, Soda, JCAP, 2010.

**For the parameter region** $\lambda^2 + 2\rho\lambda - 4 > 0$ **, we found the following new solution**

$$ds^2 = -dt^2 + t^{2\omega}\left[ t^{-4\zeta} dx^2 + t^{2\zeta}\left(dy^2 + dz^2\right)\right]$$

$$\omega = \frac{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}{6\lambda(\lambda + 2\rho)} \qquad \zeta = \frac{\lambda^2 + 2\rho\lambda - 4}{3\lambda(\lambda + 2\rho)} \qquad \frac{\phi(t)}{M_p} = -\frac{2}{\lambda}\log t + \phi_0$$

$$\frac{V_0}{M_p^2} e^{\lambda\phi_0} = \frac{\left(\rho\lambda + 2\rho^2 + 2\right)(-\lambda^2 + 4\rho\lambda + 12\rho^2 + 8)}{2\lambda^2\left(\lambda + 2\rho\right)^2}$$

$$\frac{E^2 f_0^{-2}}{M_p^2} e^{-2\rho\phi_0} = \frac{\left(\lambda^2 + 2\rho\lambda - 4\right)(-\lambda^2 + 4\rho\lambda + 12\rho^2 + 8)}{2\lambda^2\left(\lambda + 2\rho\right)^2}$$

**Apparently, the expansion is anisotropic and its degree of anisotropy is given by**

$$\frac{\Sigma}{H} = \frac{\dot{\sigma}}{\dot{\alpha}} = \frac{1}{3} I\varepsilon \qquad I = \frac{\lambda^2 + 2\rho\lambda - 4}{\lambda^2 + 2\rho\lambda} \qquad \varepsilon = -\frac{\dot{H}}{H^2} = \frac{6\lambda(\lambda + 2\rho)}{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}$$

$$0 \le I < 1$$

**slow roll parameter**

# Dynamical system analysis

**To see which one is dynamically selected, we move on to**

**Autonomous system** $X = \frac{\dot{\sigma}}{\dot{\alpha}}$ $Y = \frac{1}{M_p}\frac{\dot{\phi}}{\dot{\alpha}}$ $Z = \frac{f(\phi)}{M_p} e^{-\alpha+2\sigma} \frac{\dot{v}}{\dot{\alpha}}$

$$\begin{cases} \dfrac{dX}{d\alpha} = \dfrac{1}{3} Z^2 (X+1) + X\left\{3\left(X^2-1\right) + \dfrac{1}{2}Y^2\right\} \\ \dfrac{dY}{d\alpha} = (Y+\lambda)\left\{3\left(X^2-1\right) + \dfrac{1}{2}Y^2\right\} + \dfrac{1}{3}YZ^2 + \left(\rho + \dfrac{\lambda}{2}\right)Z^2 \\ \dfrac{dZ}{d\alpha} = Z\left[3\left(X^2-1\right) + \dfrac{1}{2}Y^2 + \dfrac{1}{2}Y^2 - \rho Y + 1 - 2X + \dfrac{1}{3}Z^2\right] \end{cases}$$

**Isotropic fixed point** $(X, Y, Z) = (0, -\lambda, 0)$

**Anisotropic fixed point**

$$(X, Y, Z) = \frac{2}{A}\left(\lambda^2 + 2\rho\lambda - 4, -6(\lambda + 2\rho), \frac{3\sqrt{2}}{2}\sqrt{\left(\lambda^2 + 2\rho\lambda - 4\right)\left(A - 2\lambda^2 - 4\rho\lambda\right)}\right)$$

**This exists only for** $\lambda^2 + 2\rho\lambda - 4 > 0$

$A = \lambda^2 + 8\rho\lambda + 12\rho^2 + 8$

# Linear stability analysis

$$\lambda^2 + 2\rho\lambda - 4 < 0$$

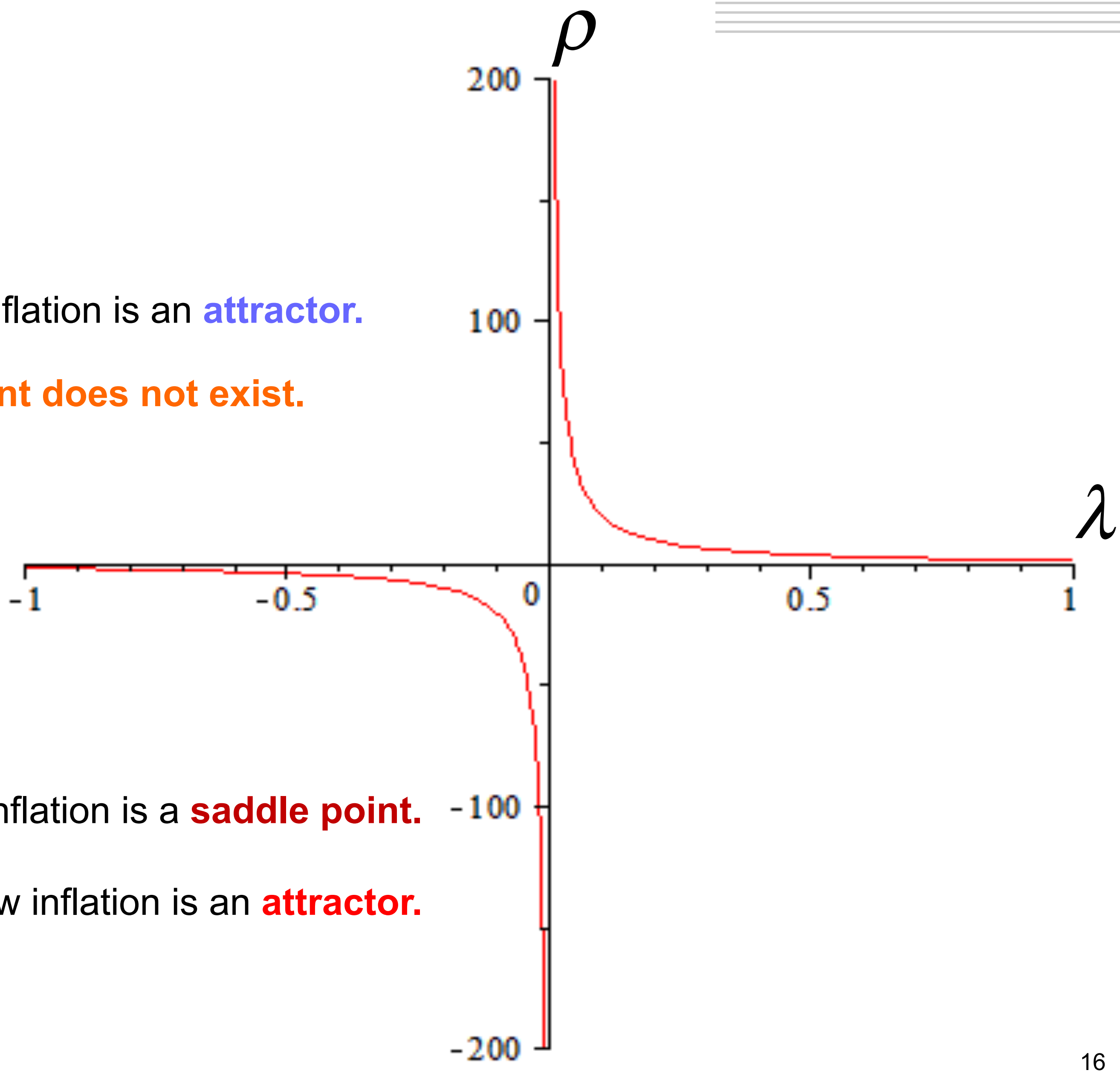
**Isotropic** power-law inflation is an **attractor.**

**Anisotropic fixed point does not exist.**

$$\lambda^2 + 2\rho\lambda - 4 > 0$$

**Isotropic** power-law inflation is a **saddle point.**

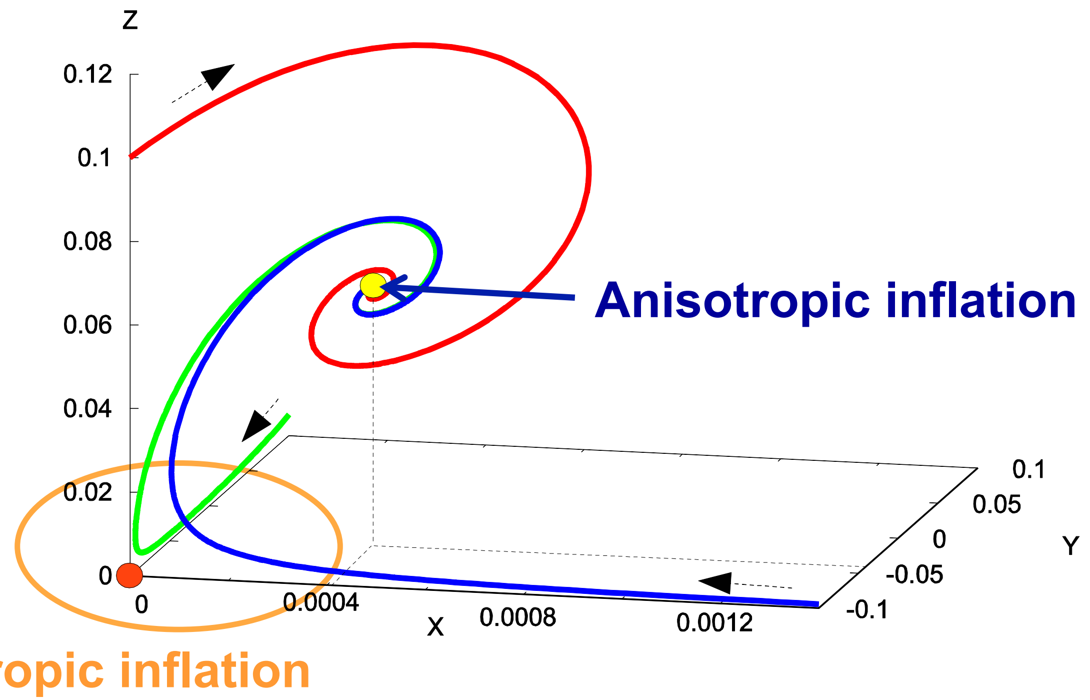
**Anisotropic** power-law inflation is an **attractor.**



# The whole picture

Kanno, Watanabe, Soda, JCAP, 2010.

$$\lambda^2 + 2\rho\lambda - 4 > 0$$




After a transient isotropic inflationary phase,
the universe enter into an anisotropic inflationary phase.

# Generality of anisotropic inflation

Consider the slow roll phase $\varepsilon \ll 1$

$$\dot{\alpha}^2 = \frac{\kappa^2}{3}\left[ V(\phi) + \frac{E^2}{2} f^{-2}(\phi) e^{-4\alpha-4\sigma} \right]$$

**Kanno, Watanabe, Soda, JCAP, 2009.**

In order for the vector contribution to increase, we need the condition $\frac{f_\phi}{\kappa f}\frac{V_\phi}{\kappa V} > 2$

**Once the vector contributes the dynamics of the inflaton field, the ratio does not increase any more**

$$\ddot{\phi} = -3\dot{\alpha}\dot{\phi} - V_\phi(\phi) + E^2 f^{-3}(\phi) f_\phi(\phi) e^{-4\alpha-4\sigma}$$

opposite to the potential force

The vector energy density saturates at $E^2 f^{-2}(\phi) e^{-4\alpha-4\sigma} = V_\phi \frac{f}{f_\phi}$

At this saturating point, Inflation continues $\frac{E^2 f^{-2}(\phi) e^{-4\alpha-4\sigma}}{2V} = \frac{1}{2}\frac{V_\phi}{V}\frac{f}{f_\phi} < 1$

Because of this vector contribution, we have anisotropy of the order of

$$\frac{\Sigma}{H} \approx \frac{E^2 f^{-2} e^{-4\alpha-4\sigma}}{V} \approx \frac{V_\phi}{V}\frac{f}{f_\phi} < \frac{1}{\kappa^2}\left(\frac{V_\phi}{V}\right)^2 \approx \varepsilon$$

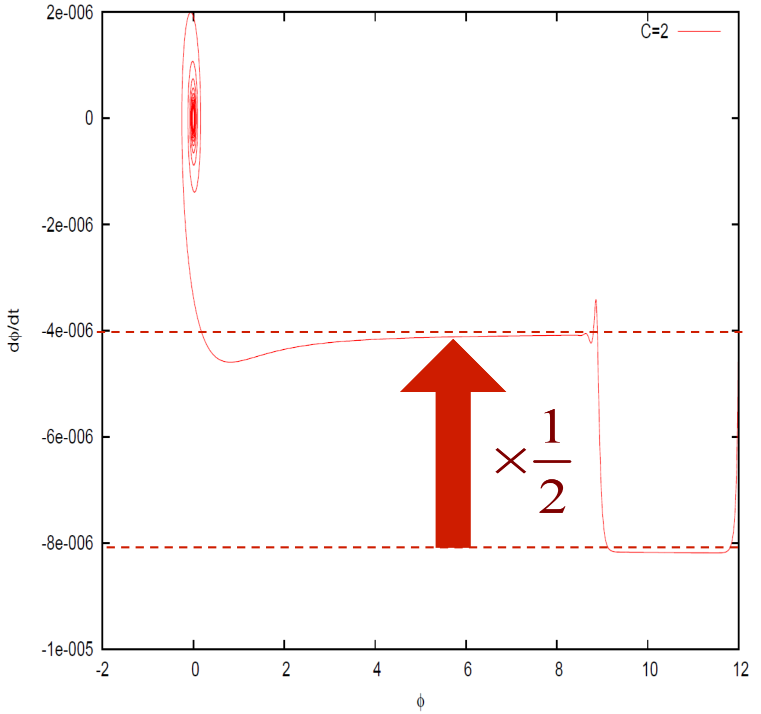
# Example : chaotic inflation $V = \frac{1}{2}m^2\phi^2$

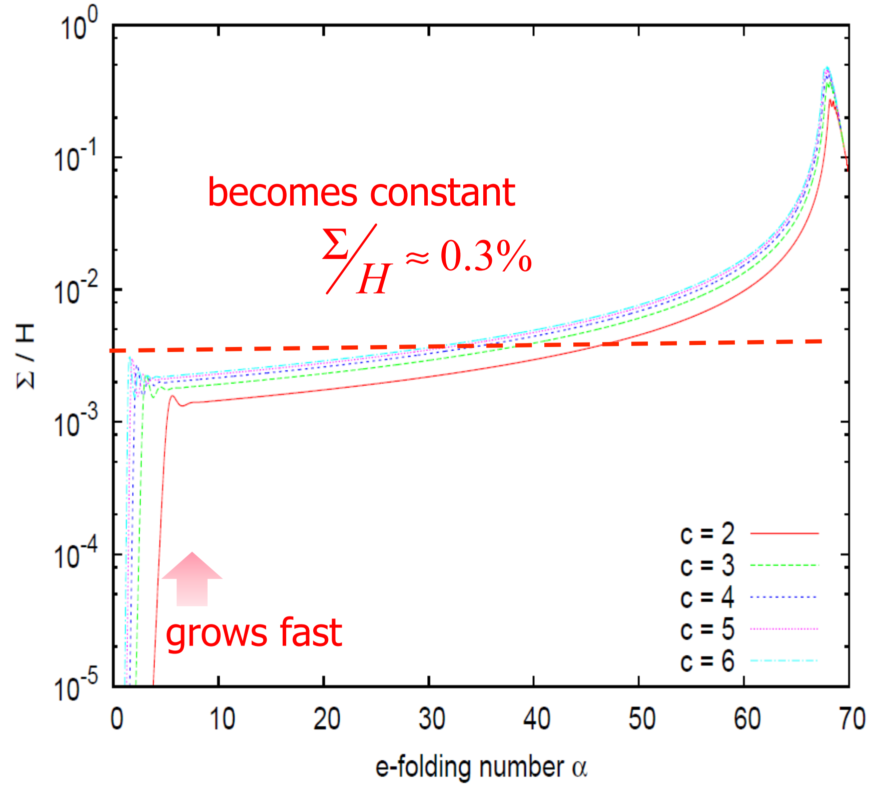
**A simple choice is** $f(\phi) = e^{c\kappa^2\phi^2/2}$ $c > 1$

**We find that the degree of anisotropy is written by the slow-roll parameter.**

$$\frac{\Sigma}{H} = \frac{1}{3} I \varepsilon_H$$

**: A universal relation** $I = \frac{c-1}{c}$




**Watanabe, Kanno, Soda, PRL, 2009.**

# COSMOLOGICAL PERTURBATION THEORY IN A SIMPLE BIANCHI UNIVERSE

Watanabe, Kanno, Soda, PTP, 2010.

Cf . Tomita, Den, 1986.

Dunsby, 1993.

Noh, Hwang 1995.

Gumrukcuoglu, Contaldi, Peloso, 2007.

Pitrou, Pereira, Uzan, 2007, 2008.

# Flat slicing gauge in anisotropic universe

In our case, we have only 2-dimensional rotational symmetry

$$ds^2 = a^2(\eta)\left[-d\eta^2 + dx^2\right] + b^2(\eta)\left[dy^2 + dz^2\right]$$

Perturbations are defined in a special frame $\vec{k}_{2D} = (k_y, 0)$

$$\sin\theta = \frac{k_y a}{kb} \qquad k = \sqrt{k_x^2 + k_y^2}$$

Here, theta is the angle between the wavenumber vector and the preferred direction x.

However, since the anisotropy is quite small $\frac{\Sigma}{H} = \frac{1}{3} I \varepsilon_H \ll 1$

we can treat the effect of anisotropy perturbatively

We take the flat slicing gauge:

graviton $\delta g_{\mu\nu} = \Gamma,\ G$

photon $\delta A_\mu = D,\ J$

inflaton $\delta\phi$

# Unconventional couplings

The main features of the action can be understood by looking at the following term

$$\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} f^2(\phi) F_{\mu\nu} F_{\alpha\beta}$$

Notice the following relations

$$\frac{f^2 v'^2}{a^2} \approx I\varepsilon_H \qquad \frac{f_\phi}{f} \approx \frac{\kappa^2 V}{V_\phi} \approx \frac{1}{\sqrt{\varepsilon_H}} \qquad -\frac{\dot{H}}{H^2} = \varepsilon_H$$

Now, we take variations

**vector-tensor** $\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} \underbrace{f^2(\phi) F_{\mu\nu}\, F_{\alpha\beta}}_{f^2 v'}$ $\qquad fv' \approx \sqrt{I\varepsilon_H}$

**vector-scalar** $\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} \underbrace{f^2(\phi)}_{ff_\phi \delta\phi} \underbrace{F_{\mu\nu}}_{v'} F_{\alpha\beta}$ $\qquad f_\phi v' \approx \frac{f_\phi}{f} fv' \approx \sqrt{I}$

**scalar-tensor** $\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} \underbrace{f^2(\phi)}_{ff_\phi \delta\phi} \underbrace{F_{\mu\nu} F_{\alpha\beta}}_{v'^2}$ $\qquad f_\phi v'^2 \approx I\sqrt{\varepsilon_H}$

# Reduced Quadratic Action: Slow roll Approximation

$$S = S_{free}\left(\Gamma, D\right) + \int d\eta d^3k \left[ \frac{\sqrt{6I\varepsilon_H}}{2}\left(-\eta\right)^{-1}\sin\theta\left(\Gamma' D^* + \Gamma'^* D\right) - \frac{\sqrt{6I\varepsilon_H}}{2}\left(-\eta\right)^{-2}\sin\theta\left(\Gamma D^* + \Gamma^* D\right) \right]$$

**vector-tensor**

$$+S_{free}\left(G, J, \delta\phi\right)$$

$$+\int d\eta d^3k \left[ -3I\sqrt{\varepsilon_H}\left(-\eta\right)^{-2}\sin^2\theta\left(G\delta\phi^* + G^*\delta\phi\right) \right.$$

**scalar-tensor**

$$+\sqrt{6I}\left(-\eta\right)^{-1}\sin\theta\left(\delta\phi'^* J + \delta\phi'^* J\right) - \sqrt{6I}\left(-\eta\right)^{-2}\sin\theta\left(\delta\phi^* J + \delta\phi J^*\right)$$

**vector-scalar**

$$\left. -\frac{\sqrt{6I\varepsilon_H}}{2}\left(-\eta\right)^{-1}\sin\theta\left(G'^* J + G' J^*\right) + \frac{\sqrt{6I\varepsilon_H}}{2}\left(-\eta\right)^{-2}\sin\theta\left(G^* J + G J^*\right) \right]$$

# The nature of primordial fluctuations in anisotropic inflation

# Perturbative estimation of statistical anisotropy

**In the isotropic limit, we have**

**Mode functions** $\delta\phi = u(\eta)a_k + u(\eta)^* a_k^\dagger$ $u(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta}\left(1 - \frac{i}{k\eta}\right)$

**Interaction Hamiltonian** $H_I = \int d^3k \left[ -\sqrt{\frac{6I}{1-I}}(-\eta)^{-1} \sin\theta \left(\delta\phi'^\dagger J + \delta\phi J\right) + \cdots \right]$

**Assuming that I is small, we can calculate corrections to the power spectrum**

$$\frac{\delta\langle 0|\delta\phi_k(\eta)\delta\phi_p(\eta)|0\rangle}{\langle 0|\delta\phi_k(\eta)\delta\phi_p(\eta)|0\rangle} = \frac{i^2}{\langle 0|\delta\phi_k(\eta)\delta\phi_p(\eta)|0\rangle}\int_{\eta_{in}}^{\eta} d\eta_2 \int_{\eta_{in}}^{\eta_2} d\eta_1 \langle 0|\left[H_I(\eta_1),\left[H_I(\eta_2),\delta\phi_k(\eta)\delta\phi_p(\eta)\right]\right]|0\rangle$$

$$\approx \frac{24I}{1-I}\sin^2\theta N^2(k)$$

Here, N(k) is the e-folding number from the horizon exit of the mode with wavenumber k to the end of the inflation.

# Predictions of anisotropic inflation

Dulaney, Gresham, PRD, 2010.
Gumrukcuoglu, et al., PRD, 2010.

**Thus, we found the following nature of primodordial fluctuations in anisotropic inflation.**

Watanabe, Kanno, Soda, PTP, 2010.

**statistical anisotropy in curvature perturbations**

$$P_s(\mathbf{k}) = P_s(k)\left[1 + g_s \sin^2\theta\right] \qquad g_s = 24\, I\, N^2(k)$$

Ackerman et al, 2007.

**statistical anisotropy in primordial GWs**

$$P_t(\mathbf{k}) = P_t(k)\left[1 + g_t \sin^2\theta\right] \qquad g_t = 6\, I\, \varepsilon_H\, N^2(k)$$

**cross correlation between curvature perturbations and primordial GWs**

$$r_c = \frac{\langle \zeta G \rangle}{\langle \zeta\zeta \rangle} = -24\, I\, \varepsilon_H N^2(k)$$

**TB correlation in CMB**

**small linear polarization in primordial GWs**

**These results give consistency relations between observables.**

$$4 g_t = \varepsilon_H g_s \qquad r_c = -4 g_t$$

# How to test the anisotropic inflation?

**The current observational constraint is given by**

**WMAP constraint** Pullen & Kamionkowski 2007 $g_s = 24\, I\, N^2(k) \leq 0.3$

Now, suppose we detected $g_s = 24\, I\, N^2(k) = 0.3$ $\varepsilon_H = 0.02$

Then we could expect

- **statistical anisotropy in GWs** $g_t = 1.5 \times 10^{-3}$
- **cross correlation between curvature perturbations and GWs** $\frac{\langle \zeta G \rangle}{\langle \zeta \zeta \rangle} = -6 \times 10^{-3}$

**If these predictions are proved, it must be an evidence of anisotropic inflation!**

# How does the anisotropy appear in the CMB spectrum?

**Angular power spectrum of X and Y reads**

$$C_{\ell\ell'}^{XY} \propto \int d\Omega_k P_{XY}(\mathbf{k})\, {}_{-s}Y_{\ell m}^{*}(\hat{\mathbf{k}})\, {}_{-s}Y_{\ell' m'}(\mathbf{k})$$

**For isotropic spectrum,** $P(\mathbf{k}) = P(k)$ **, we have** $C_{\ell\ell'}^{XY} \propto \delta_{\ell\ell'}$

**For anisotropic spectrum, there are off-diagonal components.**

**For example,**

$$C_{\ell\ell'}^{TB} \propto \int d\Omega_k P_{TB}(\mathbf{k}) Y_{\ell m}^{*}(\hat{\mathbf{k}})\, {}_{-2}Y_{\ell' m'}(\mathbf{k})$$

$$\propto \delta_{\ell,\ell'\pm 1}$$

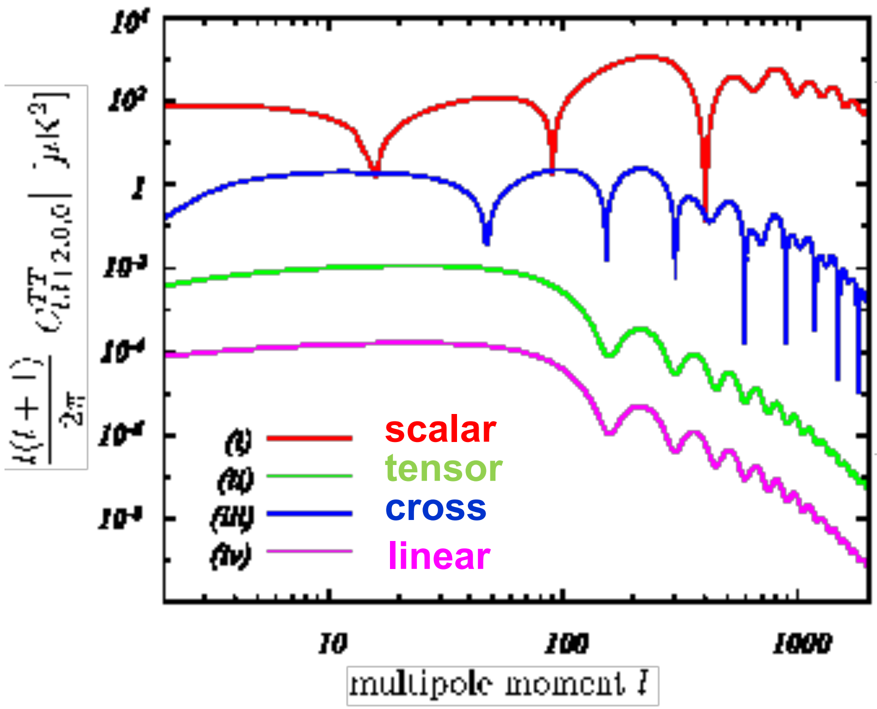
**The off-diagonal part of the angular power spectrum tells us if the gauge kinetic function plays a role in inflation.**

# We should look for the following signals in PLANCK data!

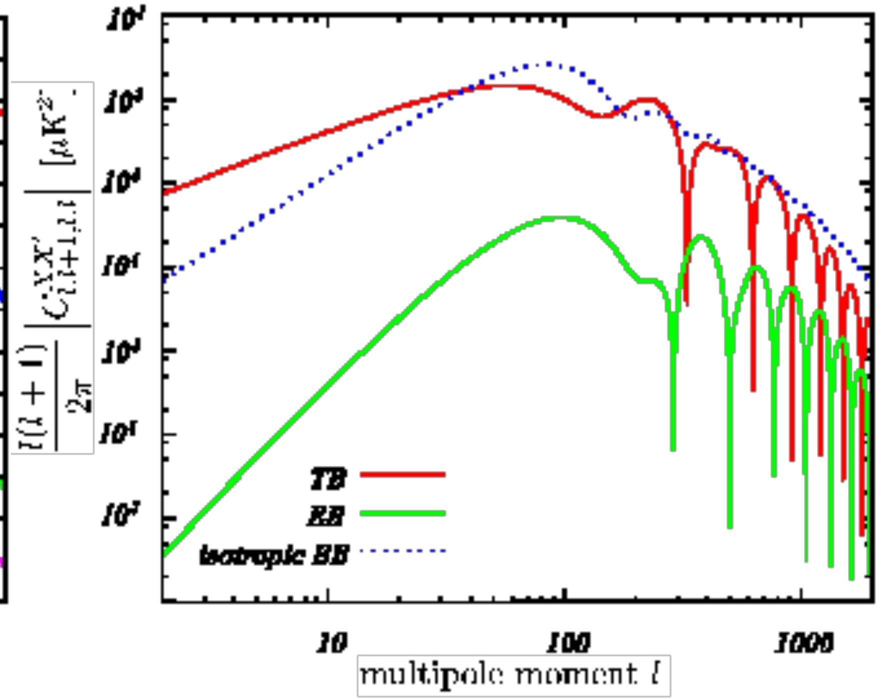
When we assume the tensor to the scalar ratio $r = 0.3$

and scalar anisotropy $g_s = 0.3$

The off-diagonal spectrum becomes **Watanabe, Kanno, Soda, MNRAS Letters, 2011.**




$C^{TT}_{\ell,\ell+2}$




$C_{\ell,\ell+1}$

The anisotropic inflation can be tested through the CMB observation!

# Summary

- We have shown that **anisotropic inflation** can be realized once we take into account a **gauge kinetic function.**

  **We have given the predictions:**

  ✓the statistical anisotropy in scalar and tensor fluctuations
  ✓the cross correlation between scalar and tensor
  ✓the linear polarization of tensor fluctuations

- **Off-diagonal** angular power spectrum can be used to prove or disprove our scenario.

- Our analysis gives a first cosmological constraint on gauge kinetic functions.

- As a by-product, we found a counter example to **the cosmic no-hair conjecture.**

The main message of this talk is that
the statistical isotropy needs a serious observational check!