Impact of Large θ_{13} on Flavor Physics

June 30, 2011

IPMU

Kashiwa, Japan

Morimitsu Tanimoto (Niigata University)

with Y. Shimizu and A. Watababe

Plan of my talk

1 Observation of Neutrino Mixing Angles

2 Toword observations for θ_{13}

3 Tri-bimaximal Paradigm

4 Breaking with Tri-bimaximal

5 Leptonic CP violation

6 Neutrinoless Double Beta Decays

7 Summary

1 Observation of Neutrino Mixing Angles

Neutrino Mixing Matrix

(P)MNS Matrix

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij},\ s_{ij} \equiv \sin\theta_{ij}$$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & e^{i\varphi_3} \end{pmatrix}$$

Neutrino Oscillation Probability

$P_{\nu_\alpha \to \nu_\beta}$ $(\alpha, \beta = e, \mu, \tau)$

$$= \delta_{\alpha\beta} - 4\mathrm{Re}\left\{U_{\alpha 1}U^*_{\beta 1}U_{\beta 2}U^*_{\alpha 2}\sin^2\frac{\Delta_{12}}{2} + U_{\alpha 2}U^*_{\beta 2}U_{\beta 3}U^*_{\alpha 3}\sin^2\frac{\Delta_{23}}{2} + U_{\alpha 3}U^*_{\beta 3}U_{\beta 1}U^*_{\alpha 1}\sin^2\frac{\Delta_{31}}{2}\right\}$$

$$+ 2\mathrm{Im}\left[U_{\alpha 1}U^*_{\beta 1}U_{\beta 2}U^*_{\alpha 2}\right]\left[\sin\Delta_{12} + \sin\Delta_{23} + \sin\Delta_{31}\right]$$

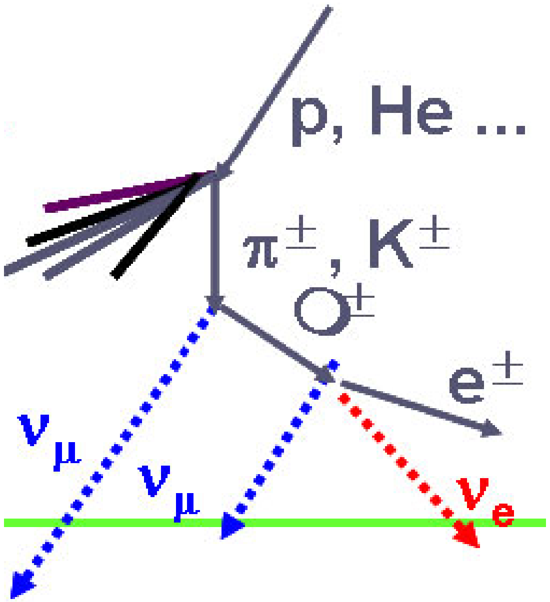
$$\Delta_{12} + \Delta_{23} + \Delta_{31} = 0$$

$$\Delta_{ij} \equiv \frac{\delta m^2_{ij}}{2E_\nu}L \simeq 2.534\frac{\delta m^2_{ij}(\mathrm{eV}^2)}{E_\nu(\mathrm{GeV})}L(\mathrm{km})$$

$$\delta m^2_{ij} = m^2_j - m^2_i$$ $(i, j = 1, 2, 3)$

Atmospheric Neutrinos θ_{23}

$$P_{\nu_\mu \to \nu_\mu} = 1 - 4\left|U_{\mu 3}\right|^2 \left(1 - \left|U_{\mu 3}\right|^2\right) \sin^2 \frac{\Delta_{13}}{2} + 2\left|U_{\mu 2}\right|^2 \left|U_{\mu 3}\right|^2 \Delta_{12} \sin \Delta_{13} + \mathcal{O}(\Delta_{12}^2)$$



First Evidence of neutrino oscillation in 1998

$$R = \frac{(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)\,|_{DATA}}{(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)\,|_{MC}} = 0.65 \pm 0.05 \pm 0.08$$

Multi-GeV

MC $(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)\,|_{MC} \approx 2$

Talk at Takayama 1998 by Kajita

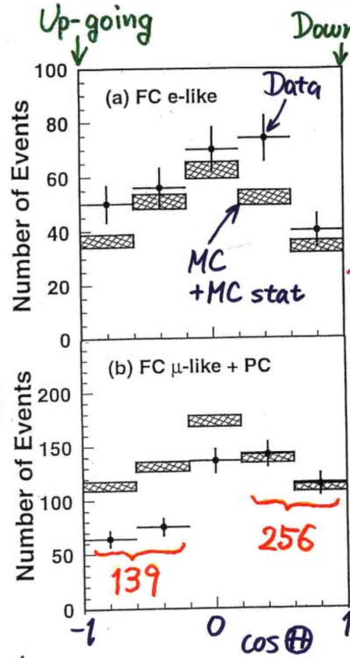
ν98, @Takayam
June 1998

Atmospheric neutrino results from Super-Kamiokande & Kamiokand

— Evidence for ν_μ oscillations —

T. Kajita
Kamioka observatory, Univ. of Tokyo
for the {Kamiokande, Super-Kamiokande} Collaborations

Zenith angle dependence (Multi-GeV)



e: χ^2(shape) = 2.8/4 dof

$\frac{Up}{Down}$ = 0.93 +0. −0.

μ: χ^2(shape) = 30/4 dof

$\frac{Up}{Down}$ = 0.54 +0. −0.

(6.2σ !!)

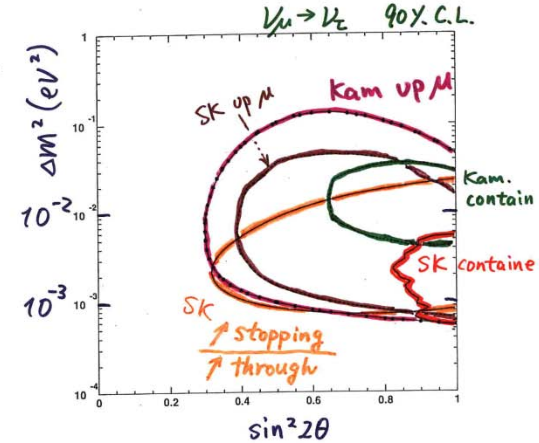
※ Up/Down syst. error for μ-like

Prediction (flux calculation ······ ≲ 1% ; 1km rock above SK ···· 1.5%) 1.8%

Data (Energy calib. for ↑↓ ···· 0.7% ; Non ν Background ·····< 2%) 2.1%

Summary

Evidence for ν_μ oscillations



- $\sin^2 2\theta > 0.8$, $\Delta m^2 \sim 10^{-3} \sim 10^{-2}$

(• $\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_s$?)

Recent Data

Atmospheric Neutrinos
Two Flavor Analysis

L/E Analysis: SK-I + SK-II + SK-III

S.Yamada, MORIOND EW201009

Zenith Physical Region (1σ)

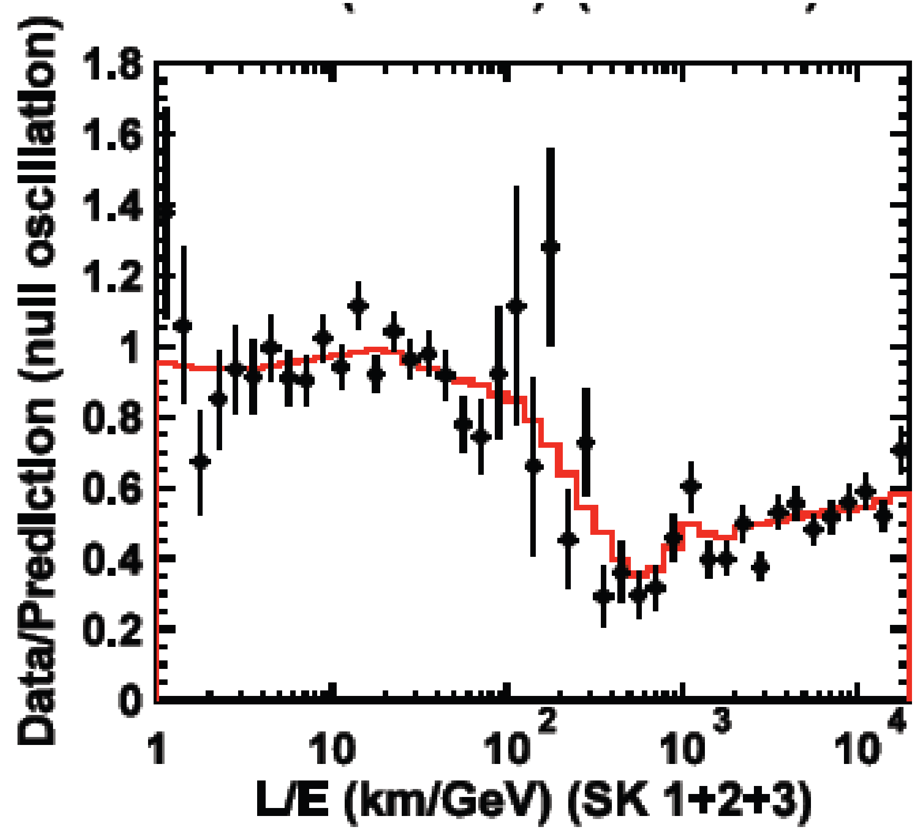
$$\Delta m^2_{23} = 2.11^{+0.11}_{-0.19} \times 10^{-3}$$

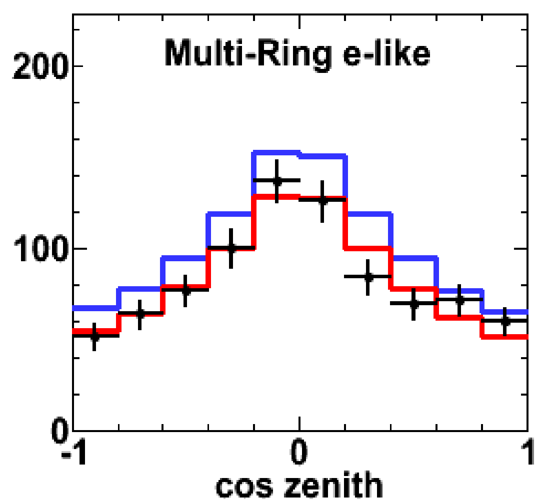
$$\sin^2 2\theta_{23} > 0.96 \;\; (90\% \;\; \text{C.L.})$$

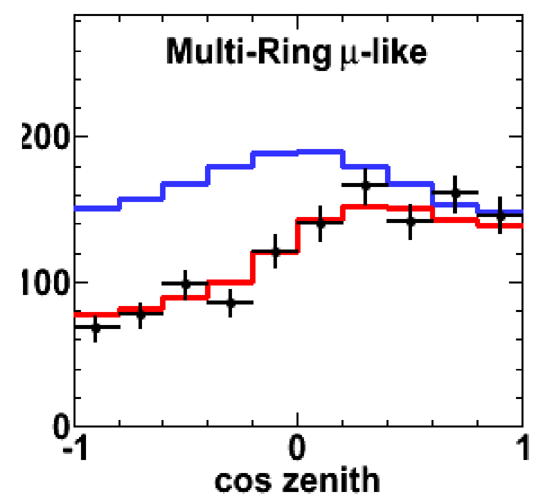
L/E Physical Region (1σ)

$$\Delta m^2_{23} = 2.19^{+0.14}_{-0.13} \times 10^{-3}$$

$$\sin^2 2\theta_{23} > 0.96 \;\; (90\% \;\; \text{C.L.})$$

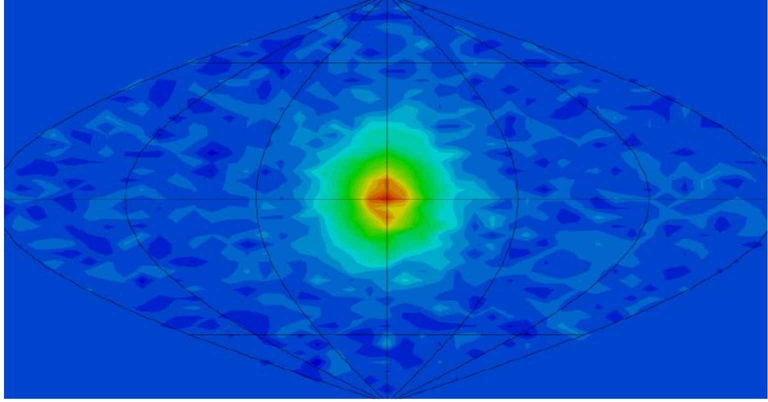




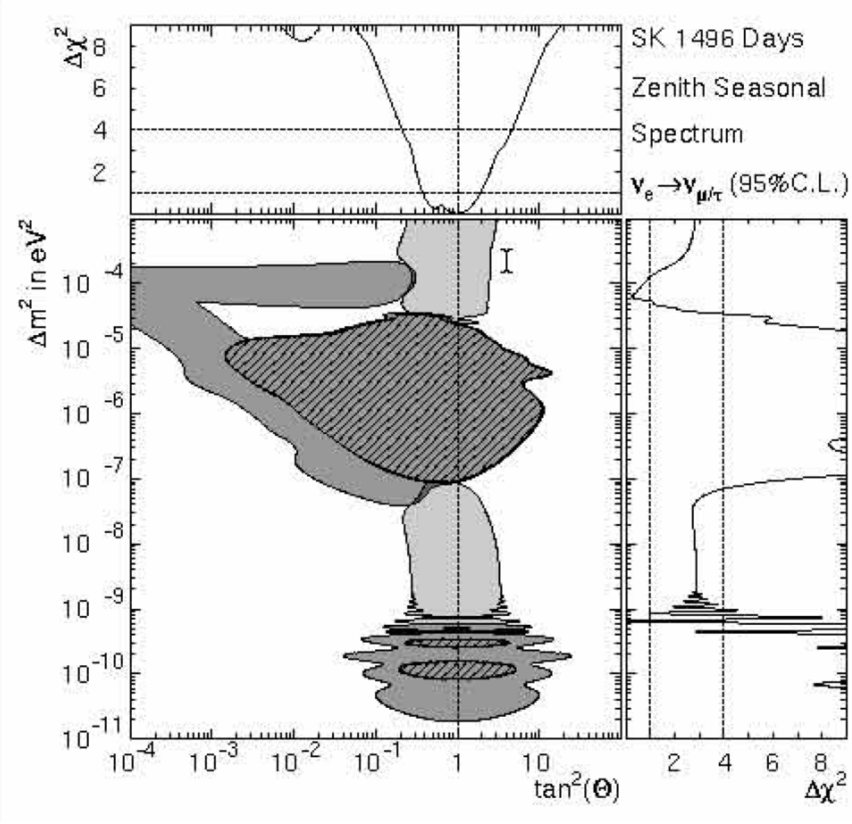


Solar Neutrinos

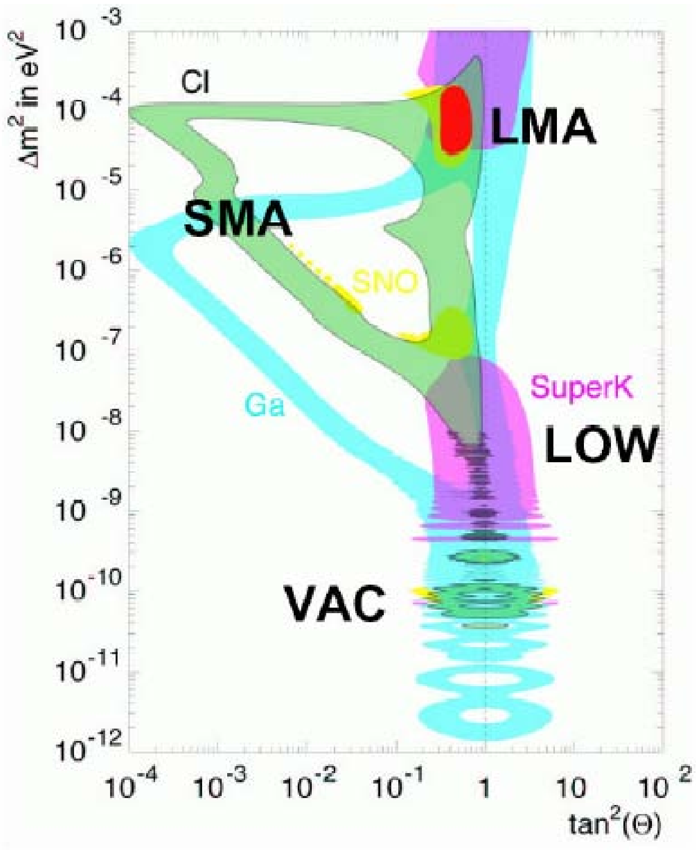
θ_{12}



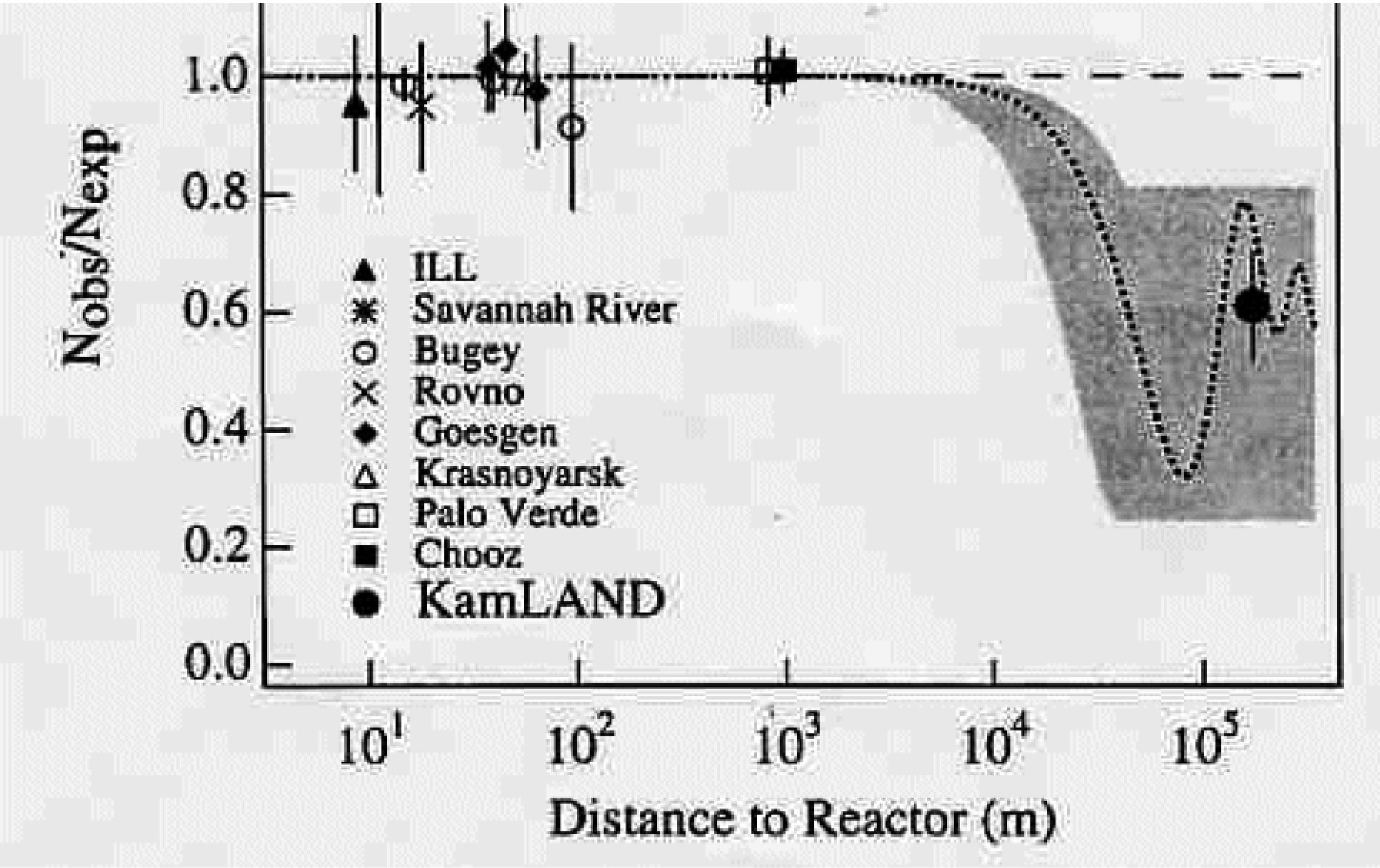
SuperK Ⅰ



combined



Kamaland 2003 θ_{12}



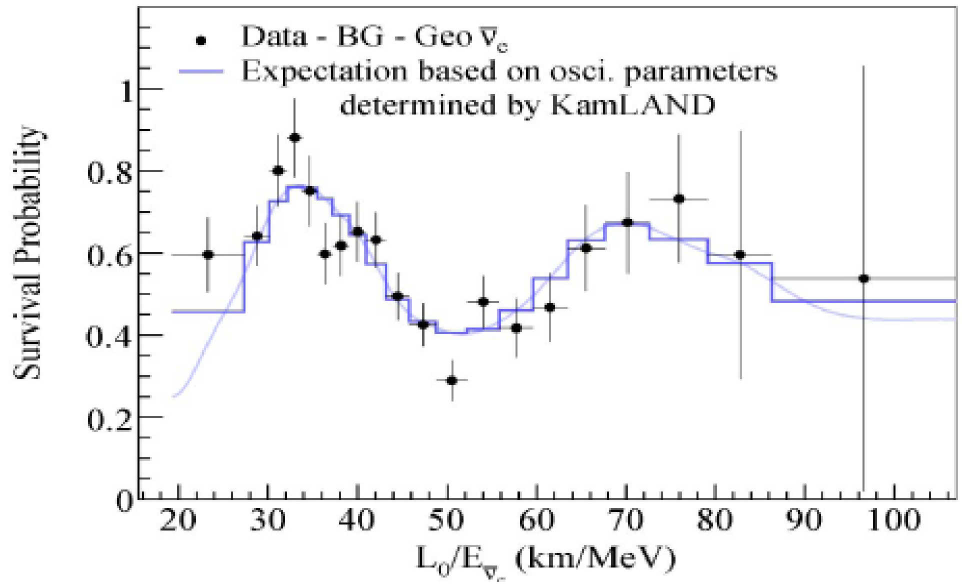
Evidence for reactor anti-neutrino disappearence

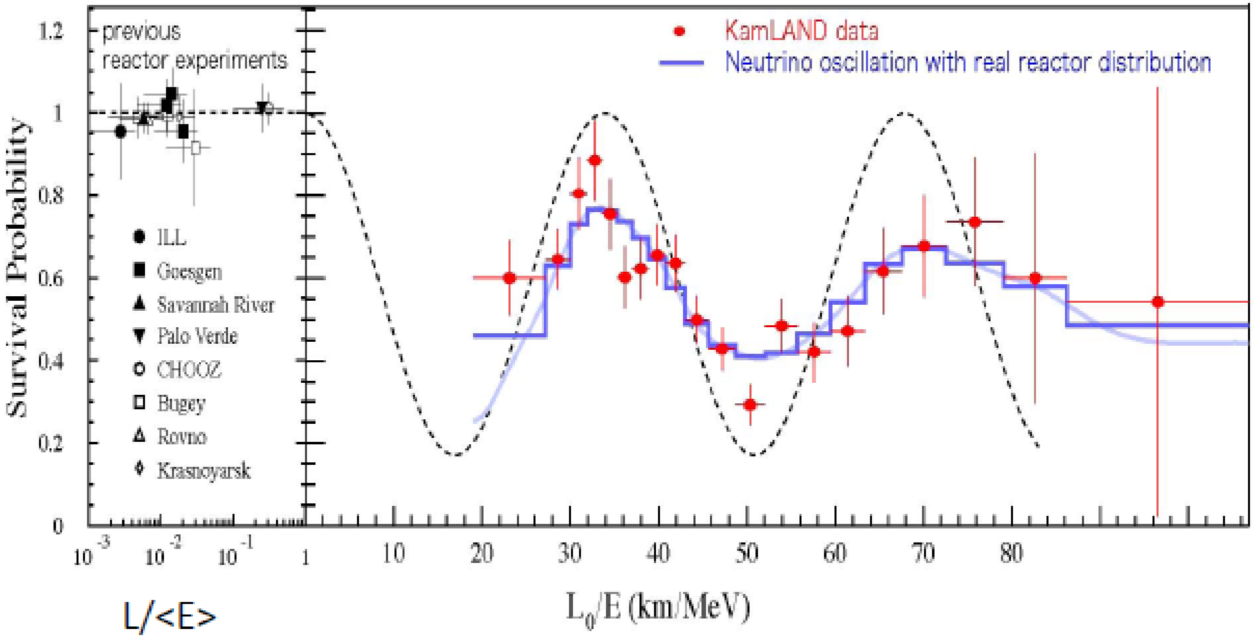
L_o/E Oscillatory Shape : L_o = 180 km

KL3

A. Suzuki,
Neutrino Telescopes09

KamLAND



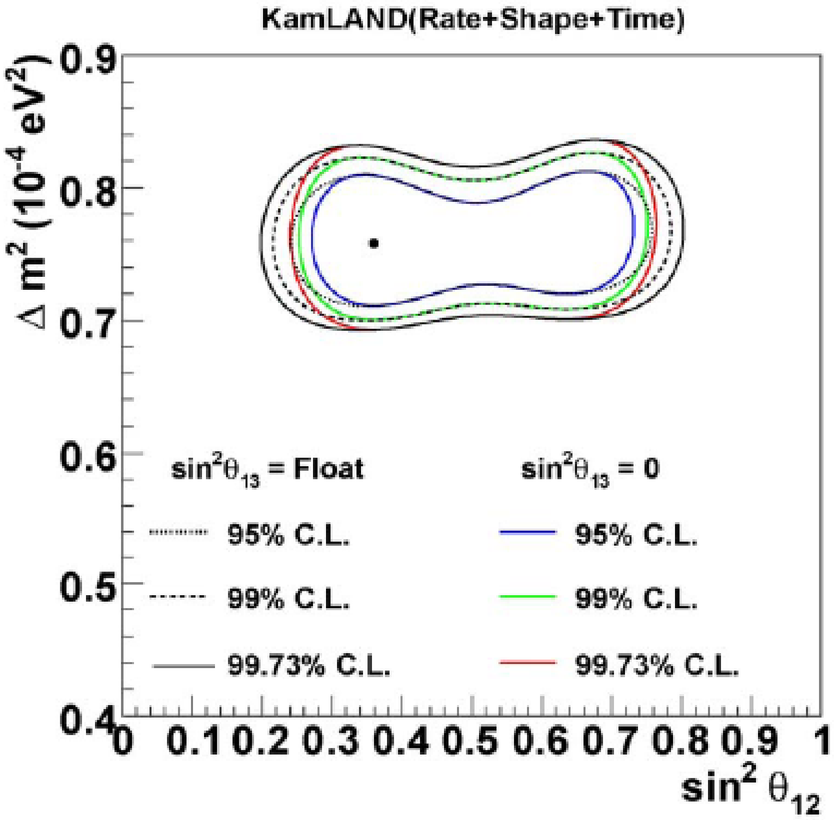


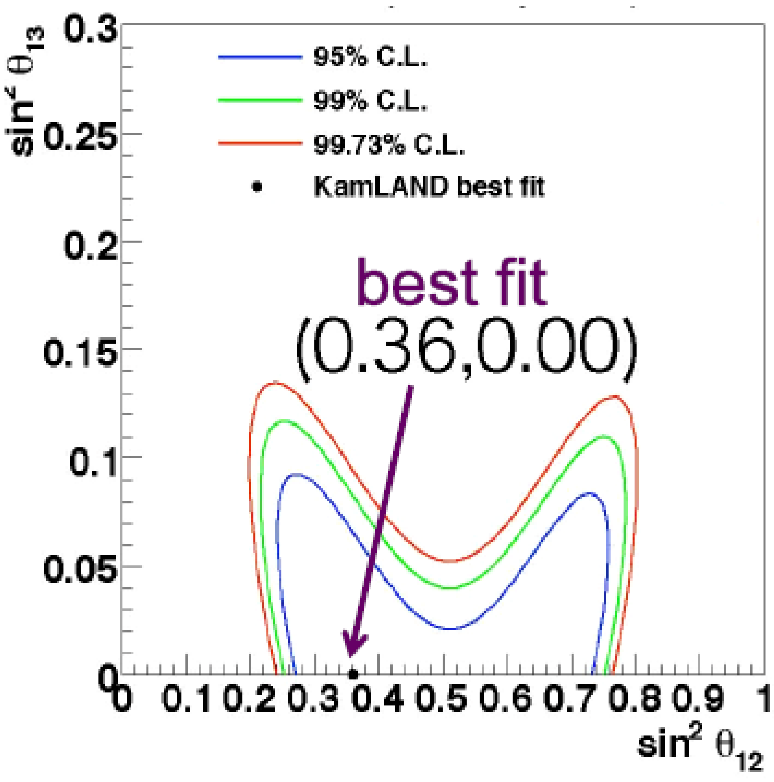
L/<E>

3-Flavor Oscillation Analysis

$$P_{ee}^{3\nu} = \cos^4\theta_{13} P_{ee}^{2\nu} + \sin^4\theta_{13}$$

KamLAND

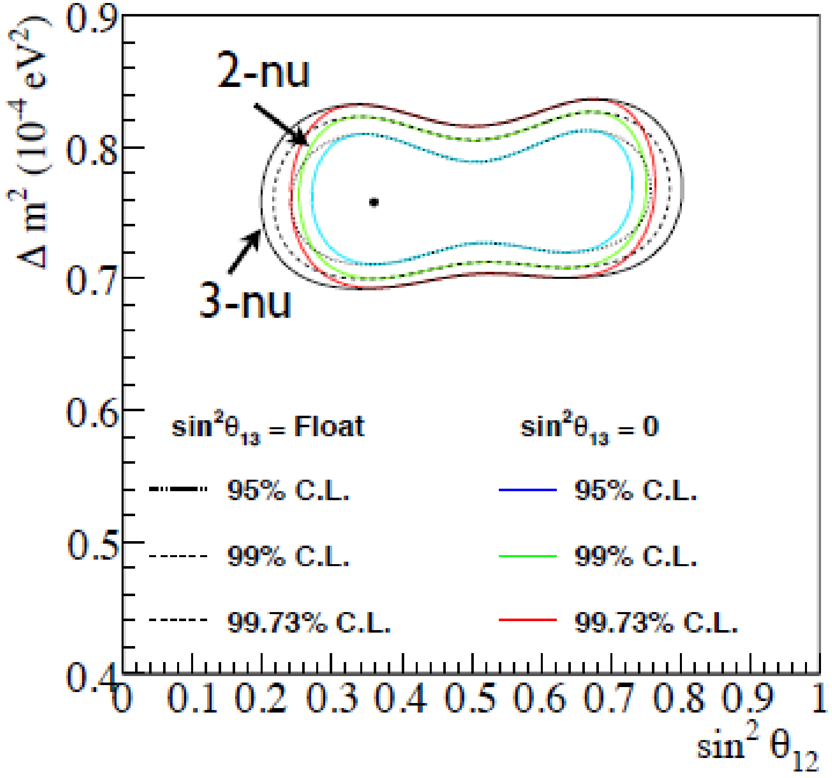




$$\Delta m^2 = 7.58^{+0.21}_{-0.20} \times 10^{-5} eV^2$$

$$\tan^2\theta = 0.56^{+0.14}_{-0.09}$$

3-flavor Analysis



Best-fit value does not change in 3-nu analysis

KamLAND has very little sensitivity to θ_{13}

Δm^2 stays the same in 3-flavor analysis

Global Analyses of 3 Flavors

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	$7.59^{+0.20}_{-0.18}$	7.24–7.99	7.09–8.19
Δm^2_{31} $[10^{-3}\text{eV}^2]$	2.45 ± 0.09 $-(2.34^{+0.10}_{-0.09})$	$2.28 - 2.64$ $-(2.17 - 2.54)$	$2.18 - 2.73$ $-(2.08 - 2.64)$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28–0.35	0.27–0.36
$\sin^2\theta_{23}$	0.51 ± 0.06 0.52 ± 0.06	0.41–0.61 0.42–0.61	0.39–0.64
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$ $0.013^{+0.009}_{-0.007}$	≤ 0.027 ≤ 0.031	≤ 0.035 ≤ 0.039

m3 > m2 > m1 Normal Hierarchy

m2>m1>m3 Inverted Hierarchy

T.Schwetz, M.Tortola and J.W.F.Valle,
New J. Phys. 13 (2011) 063004 [arXiv:1103.0734 [hep-ph]].

2 Toword observations for θ_{13}

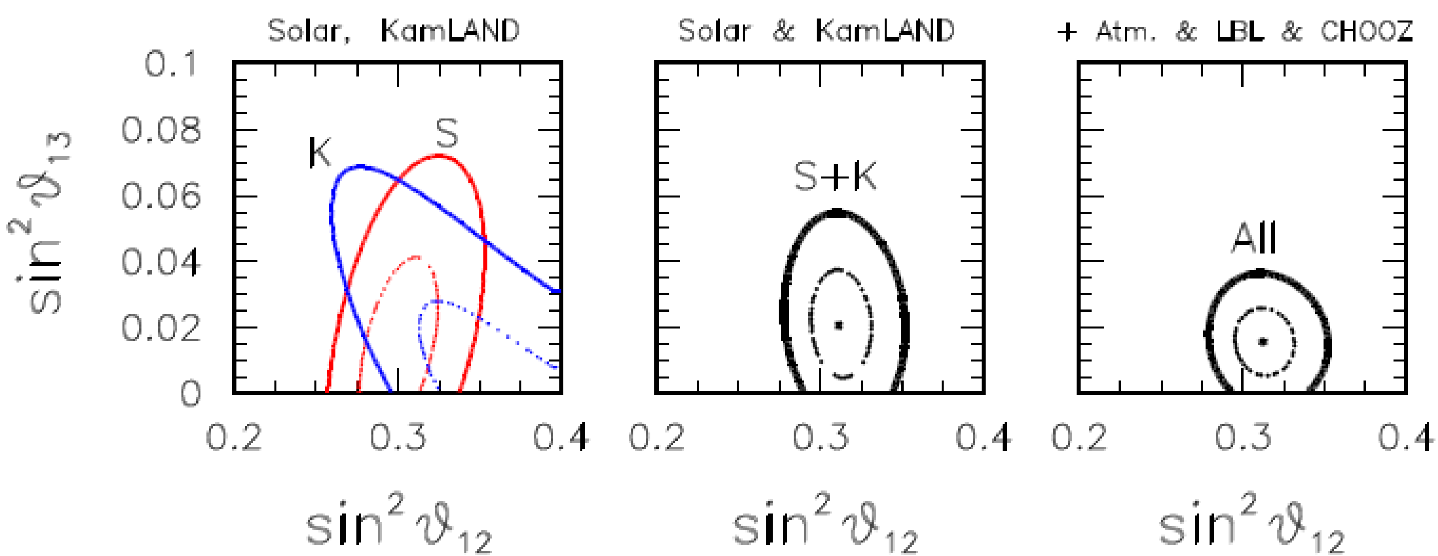
Hints of $\theta_{13} > 0$ from global neutrino data analysis

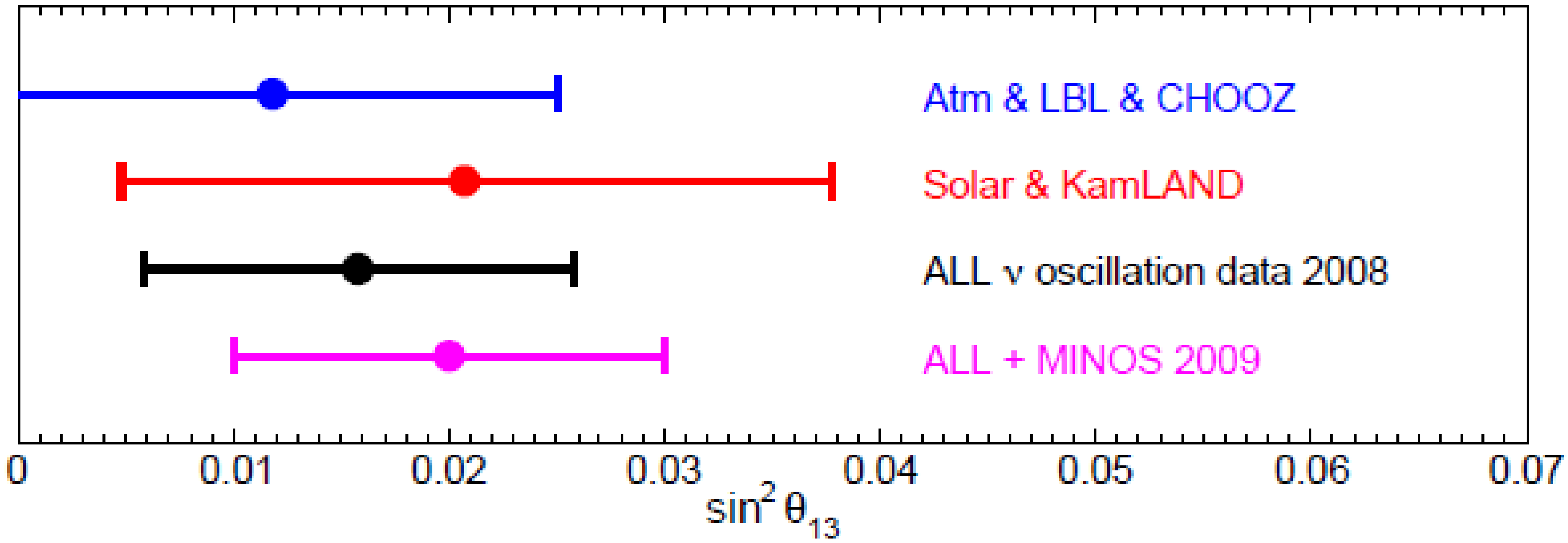
G.L. Fogli[1,2], E. Lisi[2], A. Marrone[1,2], A. Palazzo[3], and A.M. Rotunno[1,2]

Phys. Rev. Lett. 101:141801,2008 arXiv:0806.2649

Hint from atmospheric neutrino data

Hint from Solar and KamLAND data



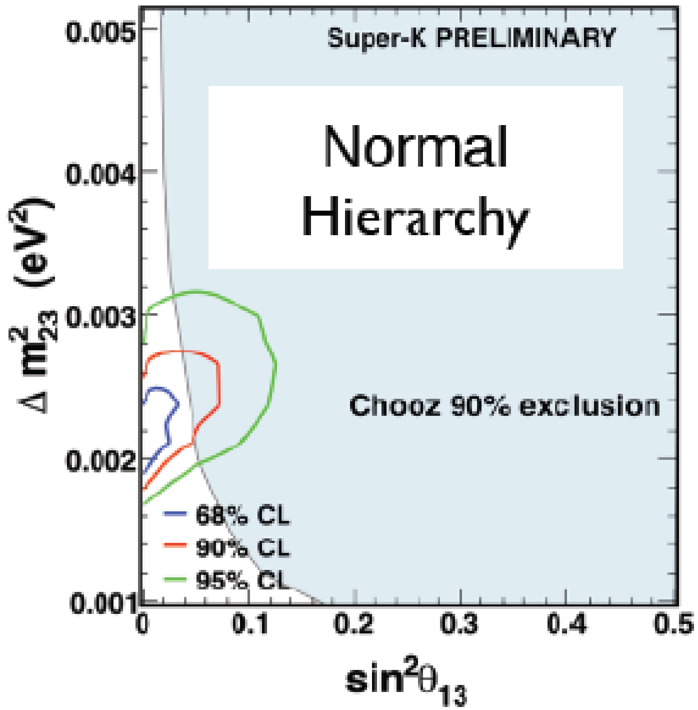


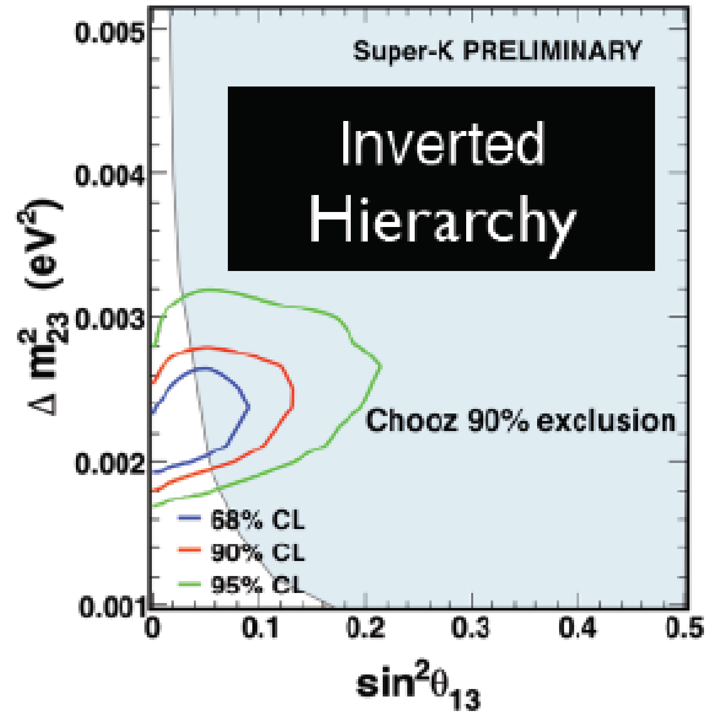
after inclusion of the MINOS result becomes [2],

$$\sin^2\theta_{13} = 0.02 \pm 0.01 \quad (1\sigma,\ \text{all data 2009})$$

G.L.Fogli, E.Lisi, A.Marrone, A.Palazzo, A.M.Rotunno,
Probing Theta(13) With Global Neutrino Data , Journal of Physics 203 (2010)

Allowed region for $\sin^2\theta_{13}$ and $|\Delta m^2_{23}|$





Best fitted value

	χ^2/dof	Δm^2_{23}	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$
Normal	469/417	$2.1\text{x}10^{-3}$	0.50	0
Inverted	468/417	$2.1\text{x}10^{-3}$	0.53	0.01

T2K (Tokai-to-Kamioka) experiment

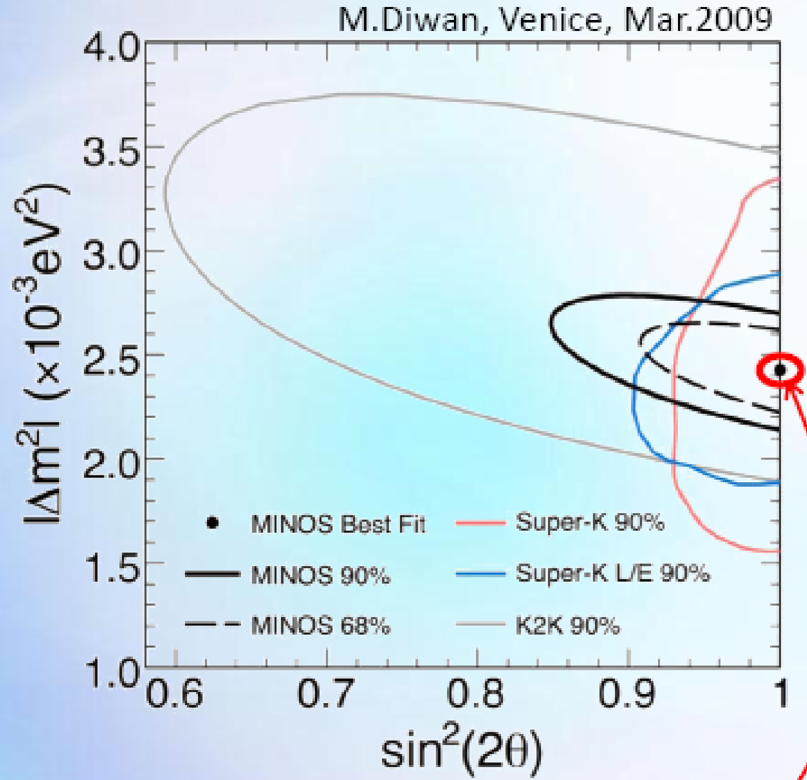
T2K Main Goals:

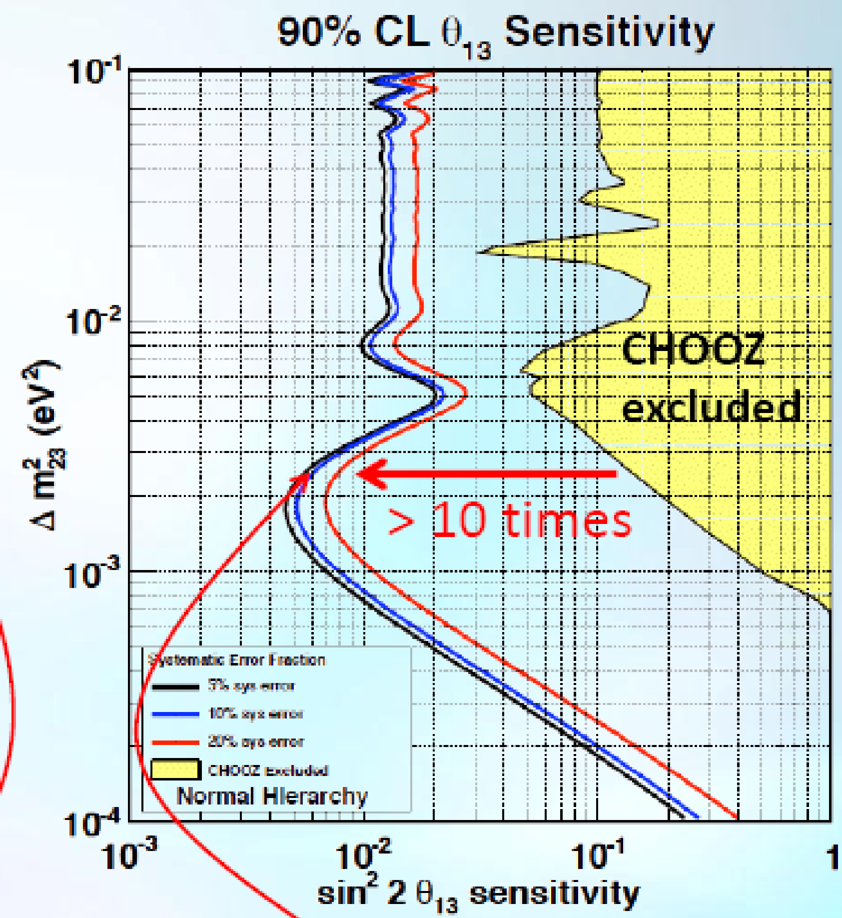
★ Discovery of $\nu_\mu \rightarrow \nu_e$ oscillation (ν_e appearance)

★ Precision measurement of ν_μ disappearance

T2K sensitivity

30 GeV, 8.3 x 10^{21} POT, $\delta_{CP} = 0$

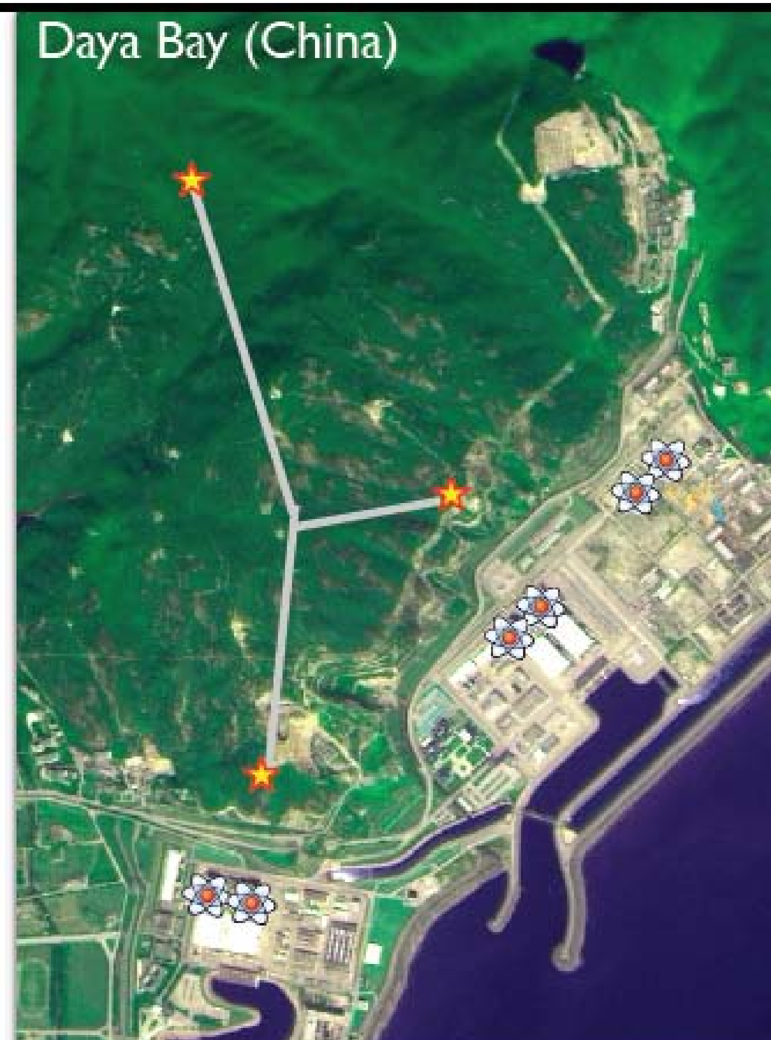




$\delta(\sin^2 2\theta_{23}) \approx 0.01$, $\delta(\Delta m_{23}^2) < 10^{-4}$ eV^2

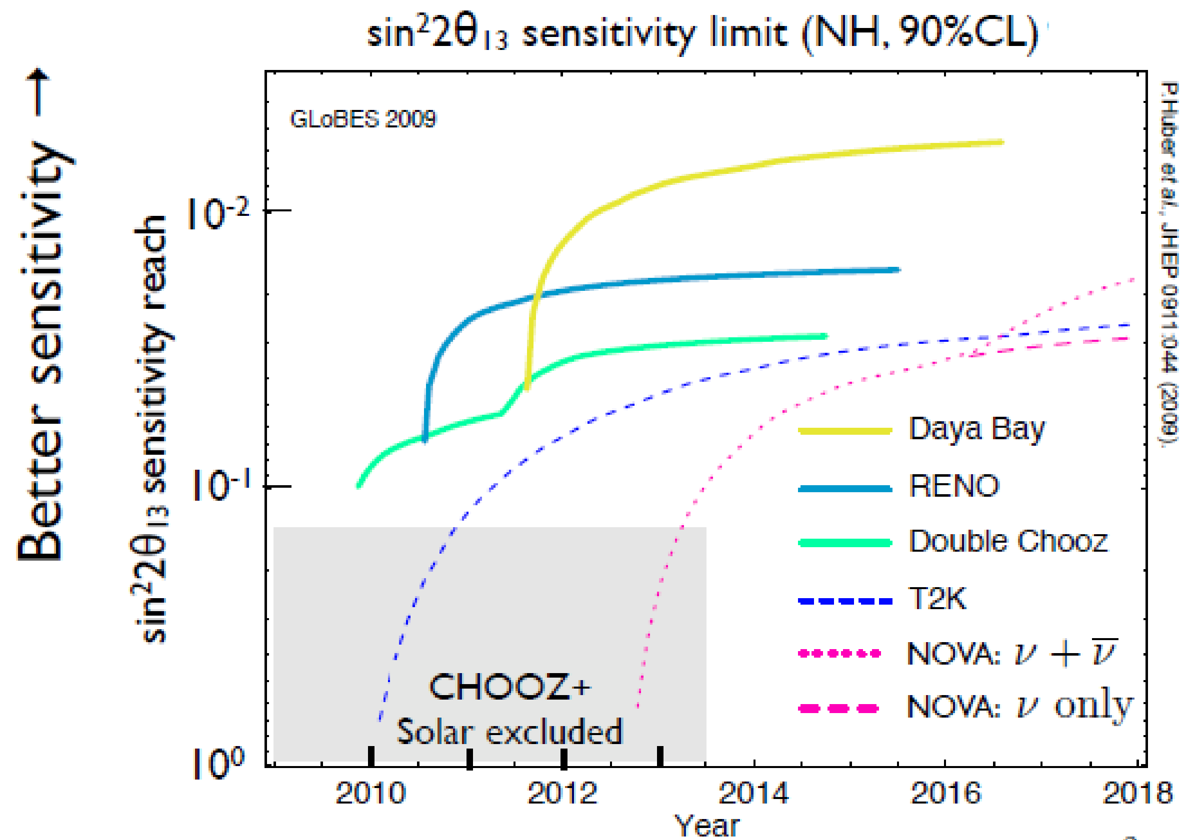
0.0060 @ $\Delta m_{23}^2 = 2.4 \times 10^{-3}$ eV^2
(for a 10% sys. error)


Double Chooz (France)
~0.4km
1.05km


Daya Bay (China)


Yellow Sea
Near Detector
Reactors
RENO (South Korea)
1 km
Far Detector

Sensitivity Limits



$$\sin^2 \theta_{13} \approx \frac{1}{4} \sin^2 2\theta_{13}$$

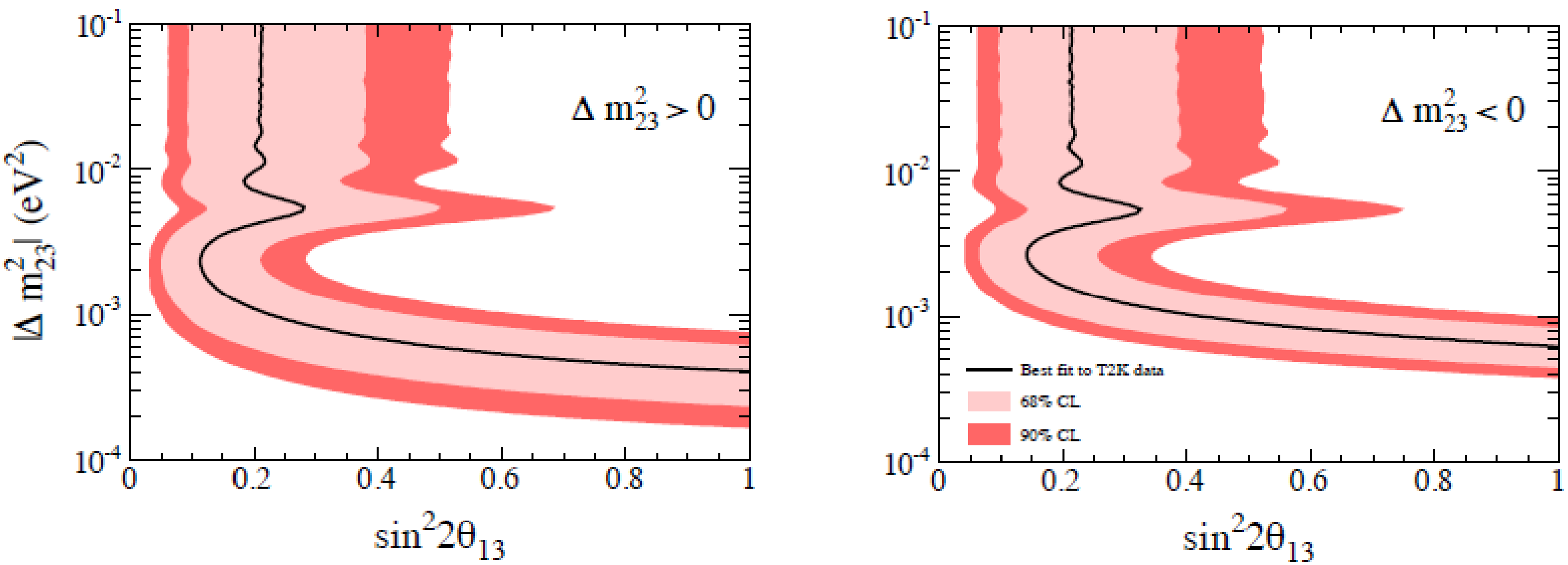
Reactor experiments will find or put best limit on θ_{13}

T2K June 15, 2011

1.43×10^{20} p.o.t. (2010 Jan. - 2011 Mar.)

Allowed region of $\sin^2 2\theta_{13}$ for each Δm^2_{23}

(assuming $\delta_{CP}=0$)



Feldman-Cousins method was used

Including matter effect of the earth,

$$P(\nu_\mu \to \nu_e) = 4c_{13}^2 s_{13}^2 s_{23}^2 \left[1 \pm \frac{2a}{|\delta m_{31}^2|}(1-2s_{13}^2) - \frac{\delta m_{21}^2 L}{2E}\frac{c_{23}c_{12}s_{12}}{s_{13}s_{23}}\sin\delta_{\rm CP}\right]$$
$$\times \sin^2\left[\frac{|\delta m_{31}^2|L}{4E} \mp \frac{aL}{4E}(1-2s_{13}^2) \mp \frac{\delta m_{21}^2 L}{4E}\left(s_{12}^2 - \frac{c_{23}c_{12}s_{12}}{s_{13}s_{23}}\cos\delta_{\rm CP}\right)\right]$$

$$a = 2\sqrt{2}G_F E n_e = 7.56\times 10^{-5}{\rm eV}^2\left(\frac{\rho}{\rm g/cm^3}\right)\left(\frac{E}{\rm GeV}\right)$$

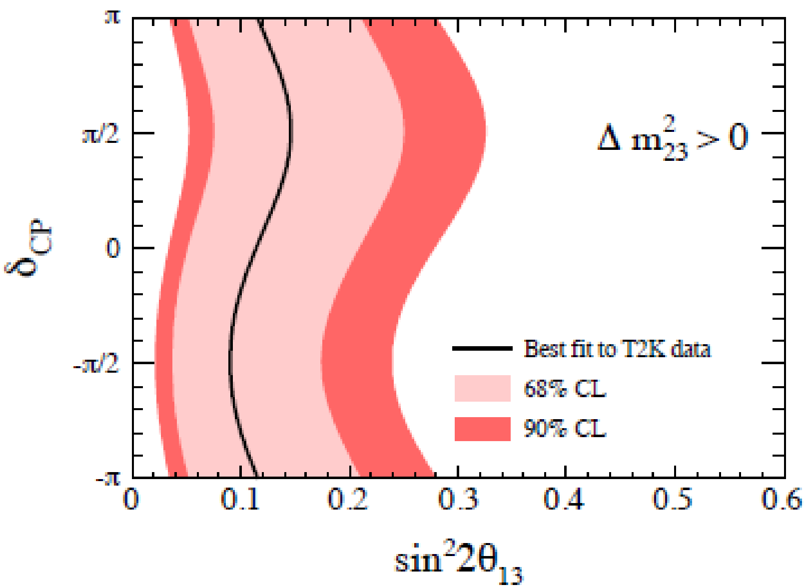
$aL/2E$ is about 0.17 at SK

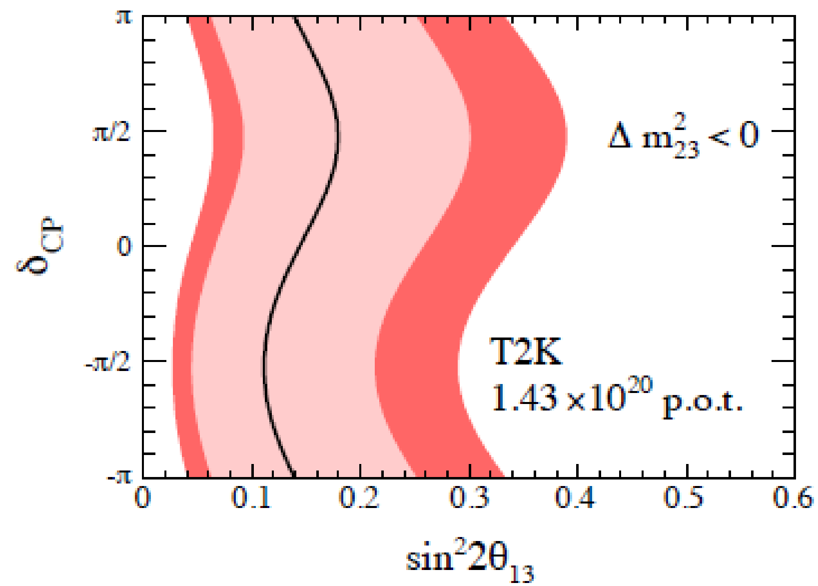
Next leading terms depend on the mass hierarchy and CP violating phase !

M. Koike, N.Okamura, M. Saito, T.Takeuchi, Phys. Rev D 73 (2006) 053010

Allowed region of $\sin^2 2\theta_{13}$ for each δ_{CP}

(assuming $\Delta m^2_{23}=2.4 \times 10^{-3}\ eV^2$)





90% C.L. interval (assuming $\Delta m^2_{23}=2.4 \times 10^{-3}\ eV^2$, $\delta_{CP}=0$)

$0.03 < \sin^2 2\theta_{13} < 0.28$

$0.04 < \sin^2 2\theta_{13} < 0.34$

MINOS reported recent result of ν_e appearance on 24th June 2011

MINOS

- **MINOS (Main Injector Neutrino Oscillation Search)**
  - long baseline (735 km) oscillation experiment
  - neutrinos produced using the 120 GeV proton beam at the Fermilab Main Injector (NuMI beam)
- **Concept**
  - Near Detector at Fermilab to measure the beam composition and the energy spectrum
  - Far Detector in Minnesota to look for and study oscillations

Results on appearance of electron-neutrinos with 8.2×10^{20} POT

For LEM>0.7

Expected background events:
49.5 ± 2.8 (syst) ± 7.0 (stat)

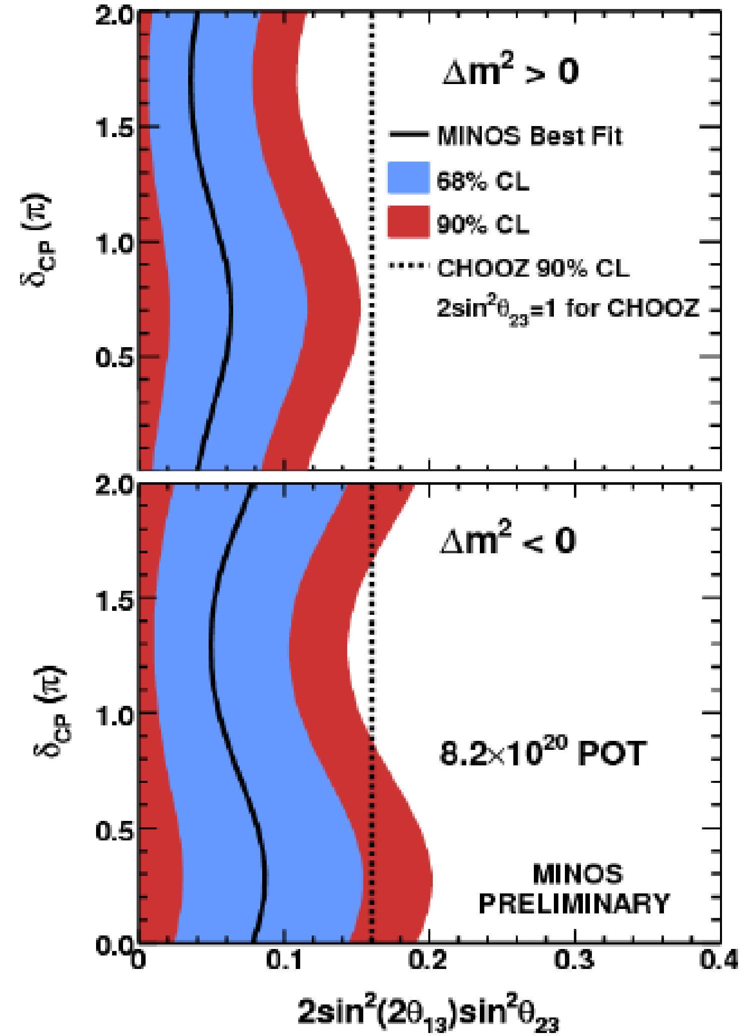
Observed events in FD data:
62

1.7σ excess above background

Results on appearance of electron-neutrinos with 8.2×10^{20} POT

For $\delta_{CP} = 0$ the allowed values of $2\sin^2(2\theta_{13})\sin^2(\theta_{23})$ at 90% CL are:

0 to 0.12 (normal) central value: 0.04
0 to 0.19 (inverted) central value: 0.08



Exclusion limits based on the selected ν_e candidate event distribution.
Allowed values are in the colored regions

3 Tri-bimaximal Paradigm

Three Flavor analysis suggested
Tri-bimaximal Mixing of Neutrinos

Harrison, Perkins, Scott (2002)

$$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0,$$

$$U_{\rm tri-bimaximal} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Neutrino Parameters

M.C. G-Garcia, M. Maltoni, J. Salvado, arXiv:1001.4524

parameter	best fit	1σ	3σ	tri-bi
θ_{12}	34.4°	$33.4^\circ - 35.4^\circ$	$31.5^\circ - 37.6^\circ$	35.3°
θ_{23}	42.3°	$39.5^\circ - 47.6^\circ$	$35.2^\circ - 53.7^\circ$	45°
θ_{13}	6.8°	$3.2^\circ - 9.4^\circ$	$< 13.2^\circ$	0°
$\Delta m^2_{\rm sol}$ $[10^{-5}{\rm eV}^2]$	7.59	7.39-7.79	6.90-8.20	*
$\Delta m^2_{\rm atm}$ $[10^{-3}{\rm eV}^2]_N$	2.51	2.39-2.63	2.15-2.90	*

$$|\nu_\alpha\rangle = U_{\alpha i}\,|\nu_i\rangle\,, \qquad \alpha = e, \mu, \tau, \quad i = 1, 2, 3,$$

$$U_{\rm MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Tri-bi maximal Mixing

Harrison, Perkins, Scott (2002)

$\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$

$$V_{\text{tri-bi}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

ν_2 = tri-maximal mixture of ν_e, ν_μ, ν_τ

ν_3 = bi-maximal mixture of ν_μ, ν_τ

Consider the structure of Neutrino Mass Matrix, which gives Tri-bi maximal mixing

$$
\begin{aligned}
M_\nu^{\rm exp} &\simeq V_{\rm tri-bi}^* \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} V_{\rm tri-bi}^\dagger \\
&= \frac{m_1+m_3}{2}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}
\end{aligned}
$$

- integer (inter-family related) matrix elements

  ⟺ non-abelian discrete flavor sym

Mixing angles are independent of mass eigenvalues

$$\left(\theta_{ij} \not\sim \sqrt{\frac{m_i}{m_j}}\right)$$

Different from quark mixing angles

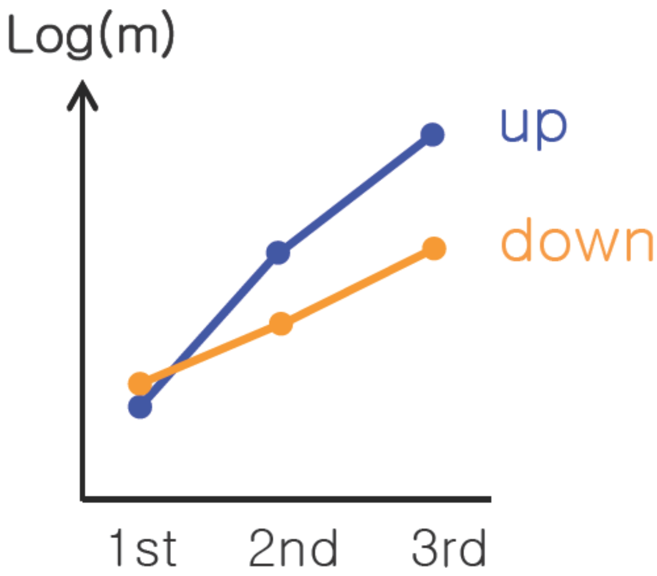
Suppose A_4 triplet $(\nu_e, \nu_\mu, \nu_\tau)_L$

$$M_\nu^{\rm exp} = \frac{m_1+m_3}{2}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}$$

The third matrix is A_4 symmetric !

The first and second matrices could be derived from S_3 symmetry.

Quark Sector



For example :

$$M_{\rm up} \sim \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^0 \end{pmatrix}$$

$$M_{\rm down} \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon^1 & \epsilon^0 \end{pmatrix}$$

- large mass hierarchy
- small mixing

i.e. "separate" generations

$\not\Leftrightarrow$ non-abelian flavor sym ....

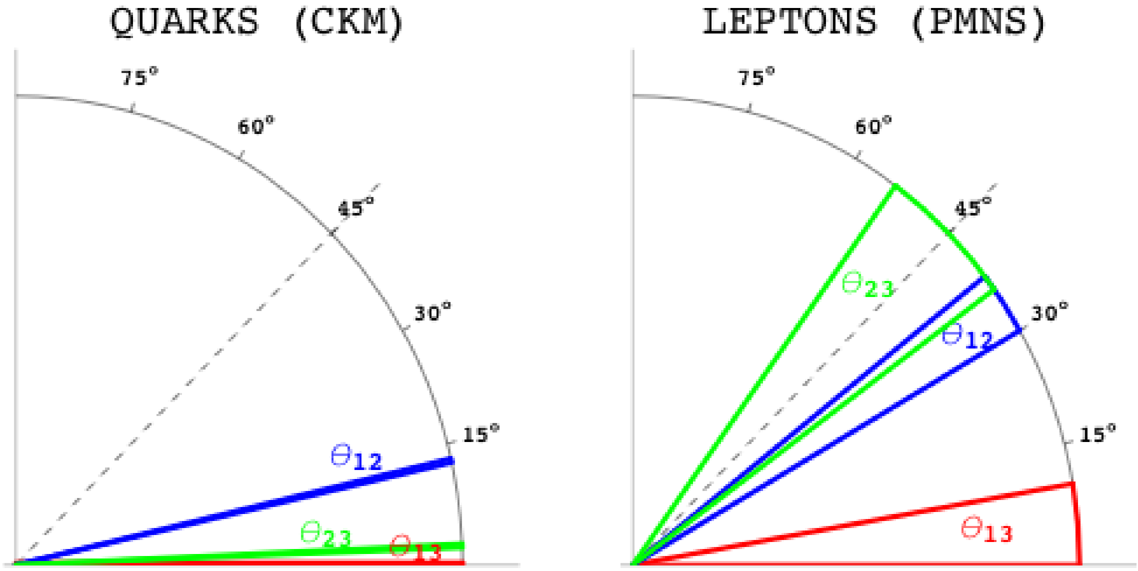
U(1) Froggatt-Nielsen mechanism ○?

Quark/Lepton mixing

~10° ?

Lepton : $\theta_{12} = 30\sim35^\circ$, $\theta_{23} = 38\sim52^\circ$, $\theta_{13} < 12^\circ$

Quark : $\theta_{12} \sim 13^\circ$, $\theta_{23} \sim 2.3^\circ$, $\theta_{13} \sim 0.2^\circ$ (90% C.L.)



by M.Frigerio

Quark ⇔ Lepton :

- Comparable in 1-2 and 1-3 mixing.
- Large hierarchy in 2-3 mixing.

(Maximal 2-3 mixing in Lepton sector ?)

Tri-Bi maximal mixing ?

4 Breaking with tri-bimaximal

T2K Collaboration suggest us breaking with tri-bimaximal paradigm !

T2K Collaboration

Tri-bimaximal Mixing realized in

$$M_{\rm TBM} = \frac{m_1+m_3}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

Additional Matrices break Tri-bimaximal

$$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}, \quad \begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}, \quad \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix} = \begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} - \begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix} - \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

which could appear in A_4, S_4, $\Delta(27)$ flavor symmetries.

Modified Neutrino Mass Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

Tri-bimaximal if d=0

$$M_\nu = V_{\text{tri-bi}}\begin{pmatrix}a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d\\ 0 & a+3b+c+d & 0\\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2}\end{pmatrix} V_{\text{tri-bi}}^T$$

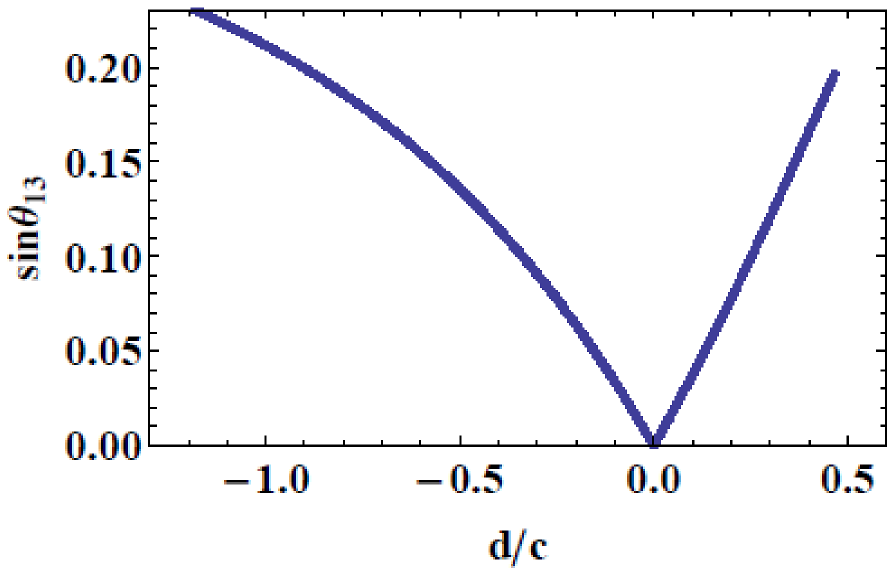
$$V_{\text{tri-bi}} = \begin{pmatrix}\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

Neutrino Mass Eigenvalues in terms of 4 parameters

$$a+\sqrt{c^2+d^2-cd}, \qquad a+3b+c+d, \qquad a-\sqrt{c^2+d^2-cd}$$

Suppose 4 parameters to be real,

$$\Delta m^2_{31} = -4a\sqrt{c^2+d^2-cd}\,, \qquad \Delta m^2_{21} = (a+3b+c+d)^2 - (a+\sqrt{c^2+d^2-cd})^2$$



$$\tan 2\theta = \frac{\sqrt{3}d}{-2c+d}$$

$$U_{\rm MNS} = V_{\text{tri-bi}} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$|U_{e2}| = \frac{1}{\sqrt{3}}\,, \quad |U_{e3}| = \frac{2}{\sqrt{6}}|\sin\theta|\,, \quad |U_{\mu 3}| = \left|-\frac{1}{\sqrt{6}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta\right|$$

$\sin\theta_{13}$

$$|U_{e2}| = |U_{\mu 2}| = |U_{\tau 2}| = \frac{1}{\sqrt{3}}$$

Tri-maximal mixing

A_4 Symmetry

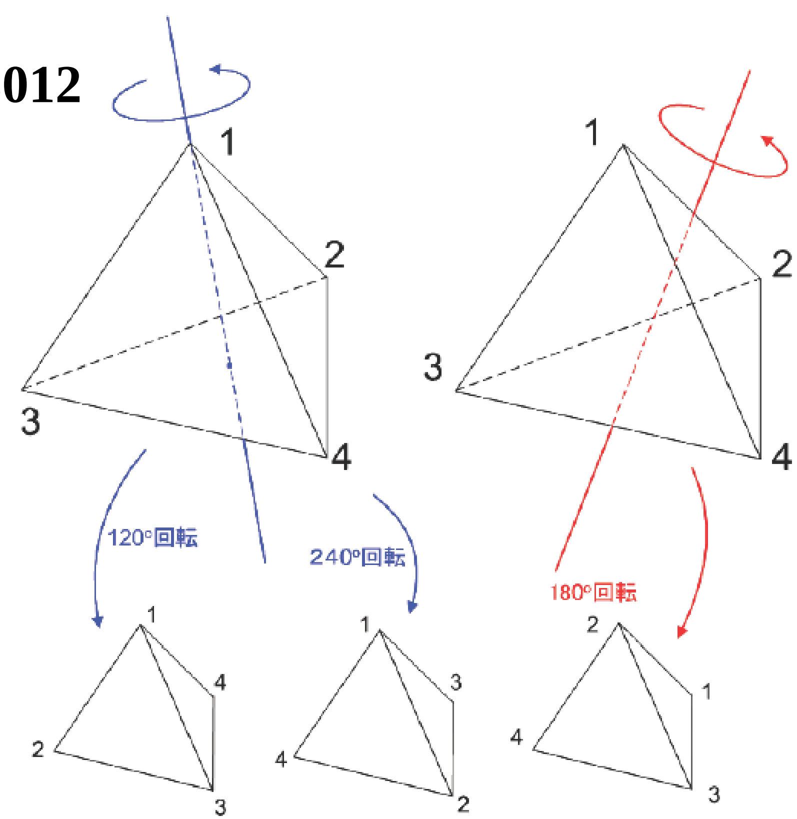
Tetrahedral Symmetry

Four irreducible representations in A_4 symmetry ***1 1' 1" 3***

E. Ma and G. Rajasekaran, PRD64(2001)113012

the even permutation of 4 objects

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	1	3
C_2	4	3	1	ω	ω^2	0
C_3	4	3	1	ω^2	ω	0
C_4	3	2	1	1	1	-1



At first

let us understand how to get the tri-bimaximal mixing in the example of A_4 flavor model.

G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

$A_4 \times Z_3$ charge assignment

A_4 Flavor model

	(L_e, L_μ, L_τ)	R_e^c	R_μ^c	R_τ^c	$H_{u,d}$	χ_ℓ	χ_ν	χ
A_4	**3**	**1**	**1′**	**1″**	**1**	**3**	**3**	**1**
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω

χ_ℓ, χ_ν, χ are new scalars of gauge singlets.

A_4 invariant superpotential can be written by:

for charged leptons

$$1' \times 1'' \rightarrow 1$$

$$\begin{aligned} W_L &= \frac{y_e}{\Lambda}(L_e\chi_{\ell_1} + L_\mu\chi_{\ell_3} + L_\tau\chi_{\ell_2})R_e H_d && \mathbf{3_L} \times \mathbf{3_{flavon}} \rightarrow \mathbf{1} \\ &+ \frac{y_\mu}{\Lambda}(L_e\chi_{\ell_2} + L_\mu\chi_{\ell_1} + L_\tau\chi_{\ell_3})R_\mu H_d && \mathbf{3_L} \times \mathbf{3_{flavon}} \rightarrow \mathbf{1''} \\ &+ \frac{y_\tau}{\Lambda}(L_e\chi_{\ell_3} + L_\mu\chi_{\ell_2} + L_\tau\chi_{\ell_1})R_\tau H_d + h.c., && \mathbf{3_L} \times \mathbf{3_{flavon}} \rightarrow \mathbf{1'} \end{aligned}$$

for neutrinos

$$\begin{aligned} W_\nu &= \frac{y_1}{\Lambda^2}(L_e L_e + L_\mu L_\tau + L_\tau L_\mu)H_u H_u \chi && \mathbf{3_L} \times \mathbf{3_L} \times \mathbf{1_{flavon}} \rightarrow \mathbf{1} \\ &+ \frac{y_2}{3\Lambda^2}[(2L_e L_e - L_\mu L_\tau - L_\tau L_\mu)\chi_{\nu_1} \\ &+ (-L_e L_\tau + 2L_\mu L_\mu - L_\tau L_e)\chi_{\nu_2} && \mathbf{3_L} \times \mathbf{3_L} \times \mathbf{3_{flavon}} \rightarrow \mathbf{1} \\ &+ (-L_e L_\mu - L_\mu L_e + 2L_\tau L_\tau)\chi_{\nu_3}]H_u H_u + h.c., \end{aligned}$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$

After $A_4 \times Z_3$ symmetry is spontaneously broken by VEVs of χ_ℓ, χ_ν, and χ, mass matrices are obtained as

$$M_l = \frac{v_d}{\Lambda}\begin{pmatrix} y_e \langle \chi_{\ell_1} \rangle & y_e \langle \chi_{\ell_3} \rangle & y_e \langle \chi_{\ell_2} \rangle \\ y_\mu \langle \chi_{\ell_2} \rangle & y_\mu \langle \chi_{\ell_1} \rangle & y_\mu \langle \chi_{\ell_3} \rangle \\ y_\tau \langle \chi_{\ell_3} \rangle & y_\tau \langle \chi_{\ell_2} \rangle & y_\tau \langle \chi_{\ell_1} \rangle \end{pmatrix}$$

$$M_\nu = \frac{v_u^2}{3\Lambda}\begin{pmatrix} 3y_1 \langle \chi \rangle + 2y_2 \langle \chi_{\nu_1} \rangle & -y_2 \langle \chi_{\nu_3} \rangle & -y_2 \langle \chi_{\nu_2} \rangle \\ -y_2 \langle \chi_{\nu_3} \rangle & 2y_2 \langle \chi_{\nu_2} \rangle & 3y_1 \langle \chi \rangle - y_2 \langle \chi_{\nu_1} \rangle \\ -y_2 \langle \chi_{\nu_2} \rangle & 3y_1 \langle \chi \rangle - y_2 \langle \chi_{\nu_1} \rangle & 2y_2 \langle \chi_{\nu_3} \rangle \end{pmatrix}$$

where $v_d = \langle H_d \rangle$, $v_u = \langle H_u \rangle$.

The mass matrices do not yet predict tri-bimaximal mixing !

Can one get Desired Vacuum

in Spontaneous Symmetry Breaking ?

We need Scalar Potential Analysis.

If vacuum expectation values are aligned,
$\langle\chi_\ell\rangle = (V_\ell, 0, 0)$ and $\langle\chi_\nu\rangle = (V_\nu, V_\nu, V_\nu)$,
which are obtained by potential analysis, then

$$M_l = \frac{v_d v_T}{\Lambda}\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$M_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} a+2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a-b/3 \\ -b/3 & a-b/3 & 2b/3 \end{pmatrix}.$$

where $a = y_1 V/\Lambda$, $b = y_2 V_\nu/\Lambda$.

$3_L \times 3_L \times 3_{flavon}$ $3_L \times 3_L \times 1_{flavon}$

$$M_\nu = \frac{v_u^2 b}{\Lambda}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{v_u^2 b}{3\Lambda}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{v_u^2 a}{\Lambda}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Therefore, mixing matrix is tri-bimaximal matrix, and masses are

$$m_1 = \frac{v_u^2(a+b)}{\Lambda}, \quad m_2 = \frac{v_u^2 a}{\Lambda}, \quad m_3 = -\frac{v_u^2(a-b)}{\Lambda}.$$

Let us consider Modified A_4 Model

	(l_e, l_μ, l_τ)	e^c	μ^c	τ^c	$h_{u,d}$	ϕ_l	ϕ_ν	ξ	ξ'
$SU(2)$	2	1	1	1	2	1	1	1	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1''}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1'}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} \quad = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1'} \quad = a_1 * b_2 + a_2 * b_1 + a_3 * b_3$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1''} \quad = a_1 * b_3 + a_2 * b_2 + a_3 * b_1$$

ξ $\qquad\qquad\qquad$ ξ'

$$\mathbf{1} \times \mathbf{1} \Rightarrow \mathbf{1} \quad , \qquad \mathbf{1''} \times \mathbf{1'} \Rightarrow \mathbf{1}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \qquad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \qquad c = \frac{y^\nu_{\xi}\alpha_\xi v_u^2}{\Lambda}, \qquad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda}$$

There is one relation $a = -3b$

Neglecting phases of three parameters a, c, d,
we can predict $\sin\theta_{13}$ **versus** Σm_i **.**

$$m_1 = a + \sqrt{c^2 + d^2 - cd}$$
$$m_2 = c + d$$
$$m_3 = -a + \sqrt{c^2 + d^2 - cd}$$

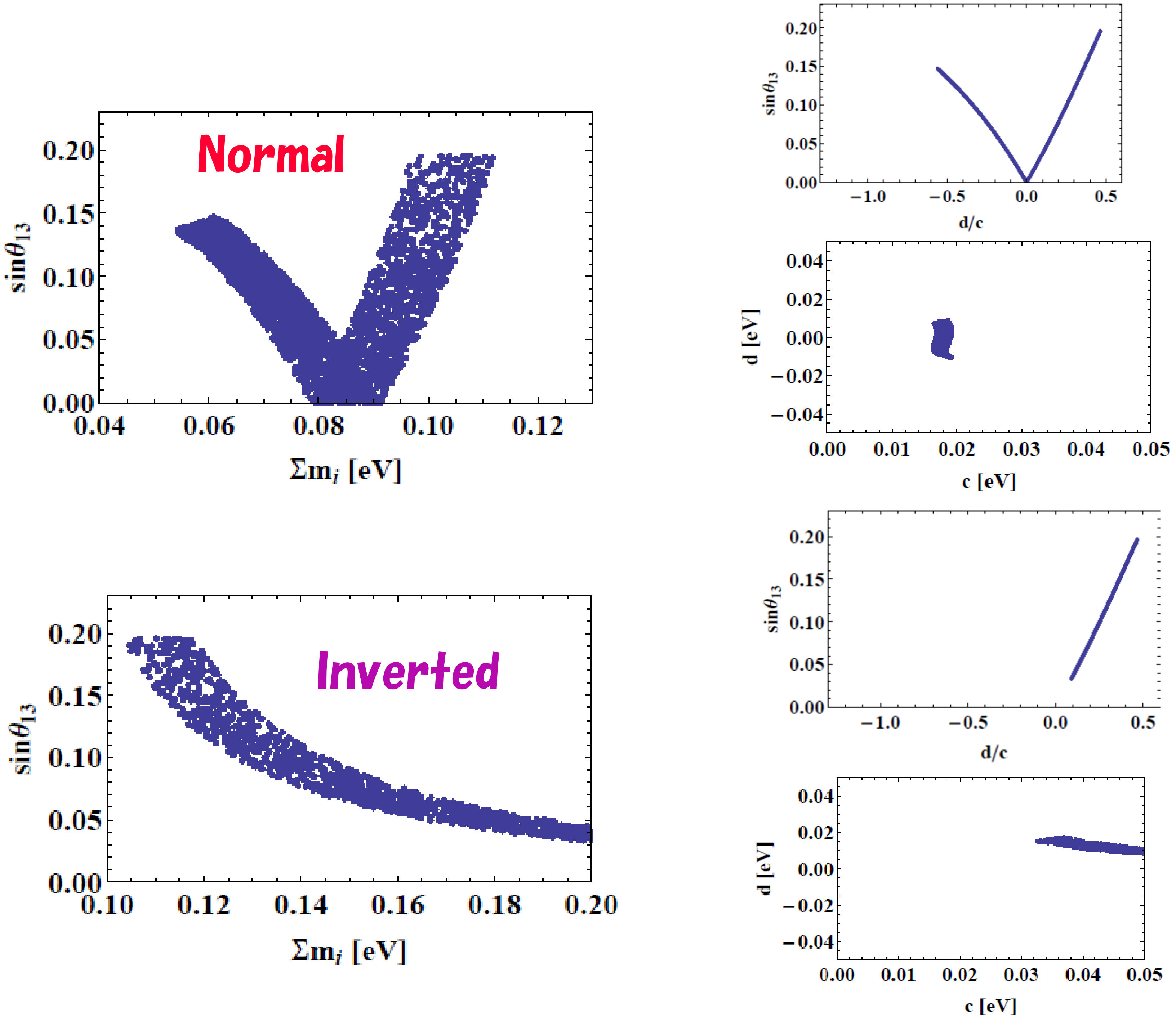
Rough estimate for the normal hierarchy

$$m_1 \ll m_3 \quad \Rightarrow \quad a \simeq -\sqrt{c^2 + d^2 - cd} < 0$$

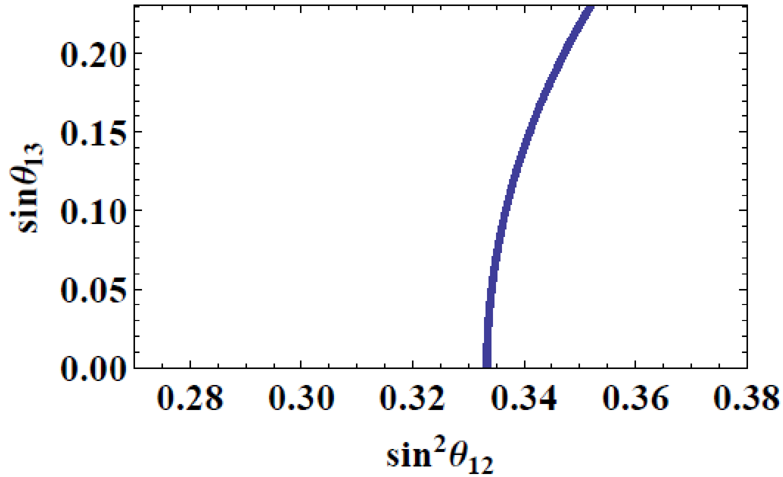
We get approximately

$$\Delta m^2_{\rm atm} \simeq 4(c^2 + d^2 - cd), \quad \Delta m^2_{\rm sol} \simeq (c + d)^2$$

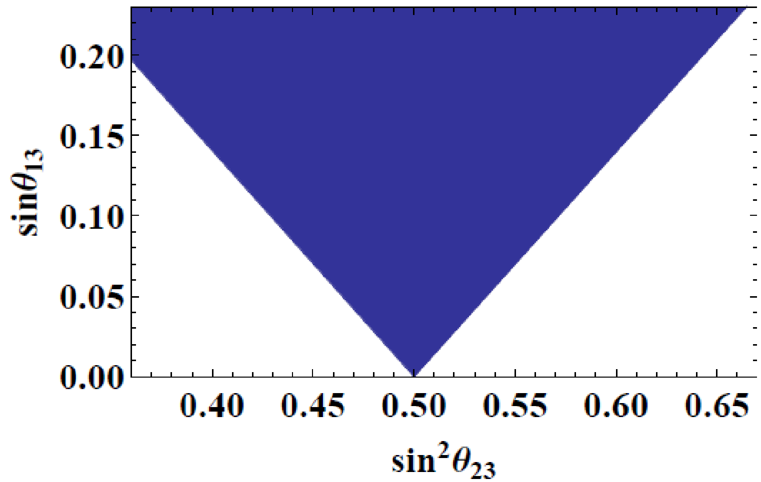
$$\tan 2\theta = \frac{\sqrt{3}d}{-2c + d} \quad \Rightarrow \quad \sin\theta_{13} = \frac{2}{\sqrt{6}} \sin\theta \simeq 0.14$$



Normal
sinθ13
Σmi [eV]
0.00
0.05
0.10
0.15
0.20
0.04
0.06
0.08
0.10
0.12
Inverted
sinθ13
Σmi [eV]
0.00
0.05
0.10
0.15
0.20
0.10
0.12
0.14
0.16
0.18
0.20
sinθ13
d/c
−1.0
−0.5
0.0
0.5
d [eV]
c [eV]
0.04
0.02
0.00
−0.02
−0.04
0.00
0.01
0.02
0.03
0.04
0.05
sinθ13
d/c
d [eV]
c [eV]



Independent of phases



Including phases

relative phase could be inserted

$c_{13}\, s_{12}$ $\quad$ s_{13} $\quad$ $c_{13}\, s_{23}$

$$|U_{e2}| = \frac{1}{\sqrt{3}} \,, \quad |U_{e3}| = \frac{2}{\sqrt{6}} |\sin\theta| \,, \quad |U_{\mu 3}| = \left| -\frac{1}{\sqrt{6}} \sin\theta - \frac{1}{\sqrt{2}} \cos\theta \right|$$

$$d\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

breaks the tri-bimaximal mixing.

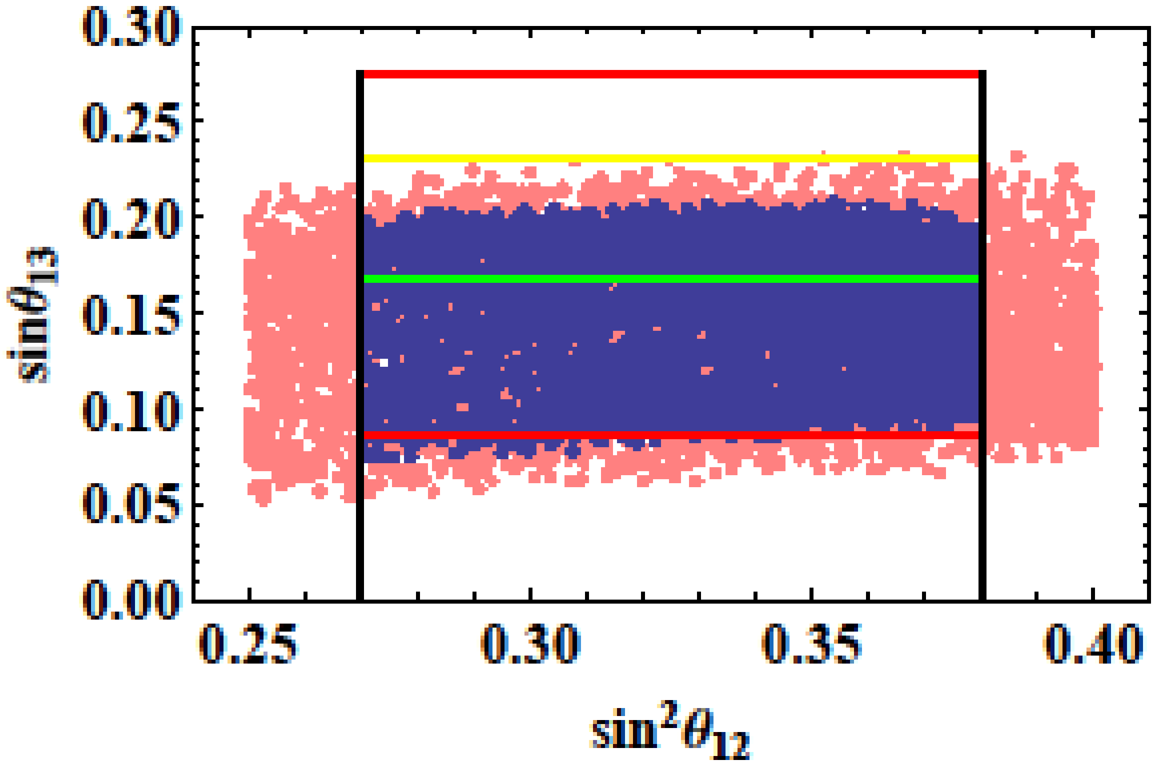
There are some models with non-vanishing d ;

☆ **Δ(27) flavor model** $d = e^{i\pi/3}c$ **Grimus-Laboura**

☆ **S_4 flavor model with flavor twisting in the five dimensional framework.** $\sin\theta_{13} \simeq 0.18$ **for normal hierarchy.**

Ishimori, Shimizu, Tanimoto, Watanabe

There are other possibility to get large θ_{13}.



M.Fukugita, M.T, T. Yanagida PLB 526 (2003) 273

$$\begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix}$$

for Charged Leptons
Dirac Neutrinos

MR: Unit Matrix

5 Leptonic CP violation

Possible origin of Baryon Asymmetery in the Universe

$$\Delta P_{CP} \equiv P(\overline{\nu}_\mu \to \overline{\nu}_e) - P(\nu_\mu \to \nu_e) = 4J^\nu_{CP} f_{CP}$$

$$= -4 \sin\frac{\Delta_{21}}{2} \sin\frac{\Delta_{32}}{2} \sin\frac{\Delta_{13}}{2}$$

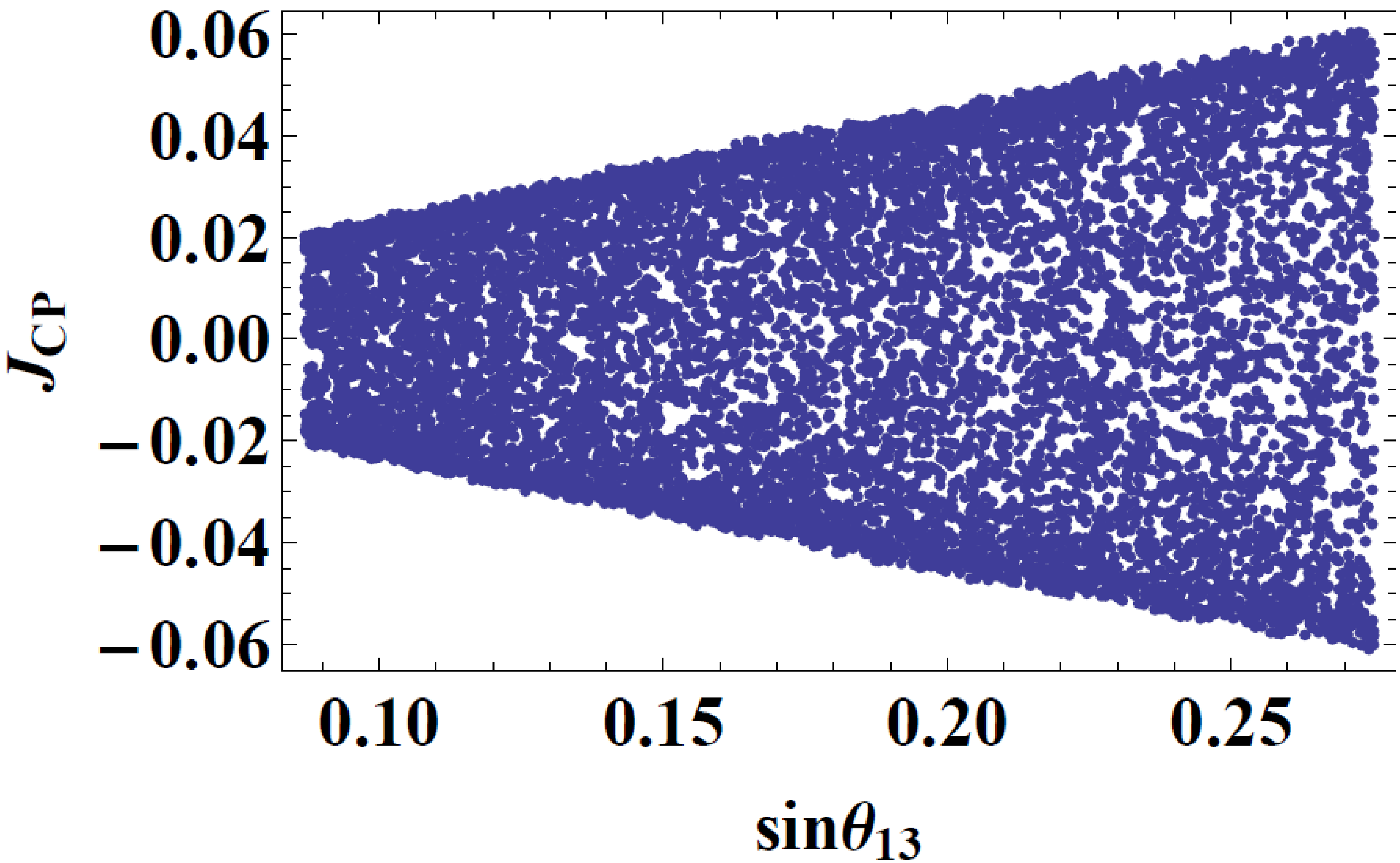
$$f_{CP} \equiv \sin\Delta_{12} + \sin\Delta_{23} + \sin\Delta_{31}$$

$$\Delta_{ij} \equiv \delta m^2_{ij}\frac{L}{2E} = 2.54\frac{(\delta m^2_{ij}/10^{-2}\mathrm{eV}^2)}{(E/\mathrm{GeV})}(L/100\mathrm{km})$$

$$\Delta_{12} + \Delta_{23} + \Delta_{31} = 0$$

$$J^\nu_{CP} = s_{12}s_{23}s_{13}c_{12}c_{23}c^2_{13}\sin\phi$$

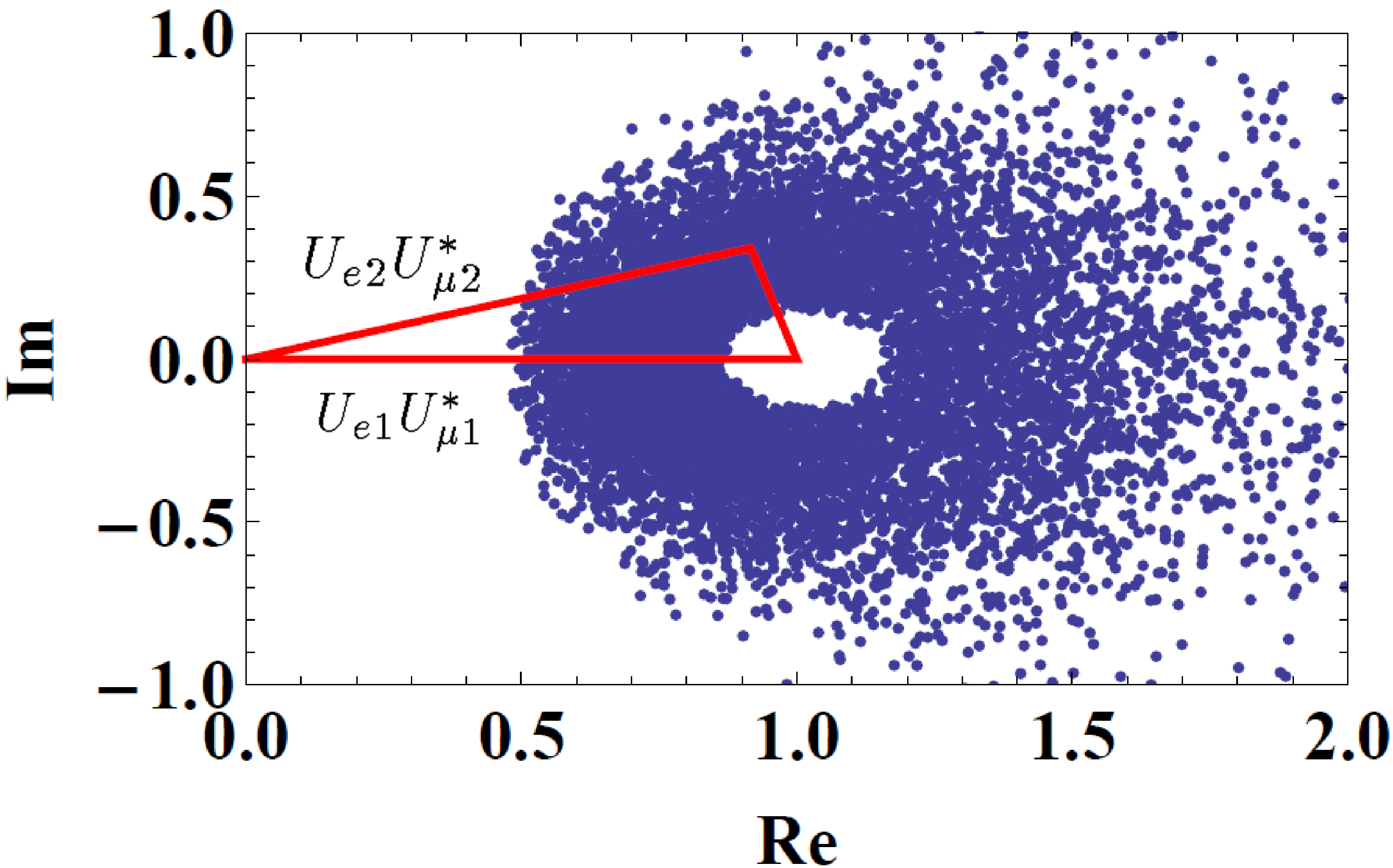
$$J_{CP}^{\nu} = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin\phi$$



$J_{CP}^{quark} \fallingdotseq 3\times10^{-5}$

Lepton Unitarity Triangle

$$U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$$



CP Violation in Neutrinos

$\sin\theta_{13}$ ***is large*** ~ 0.1- 0.2

Measurement of CP Violation is possible ?

T2KK **L~1000 Km**

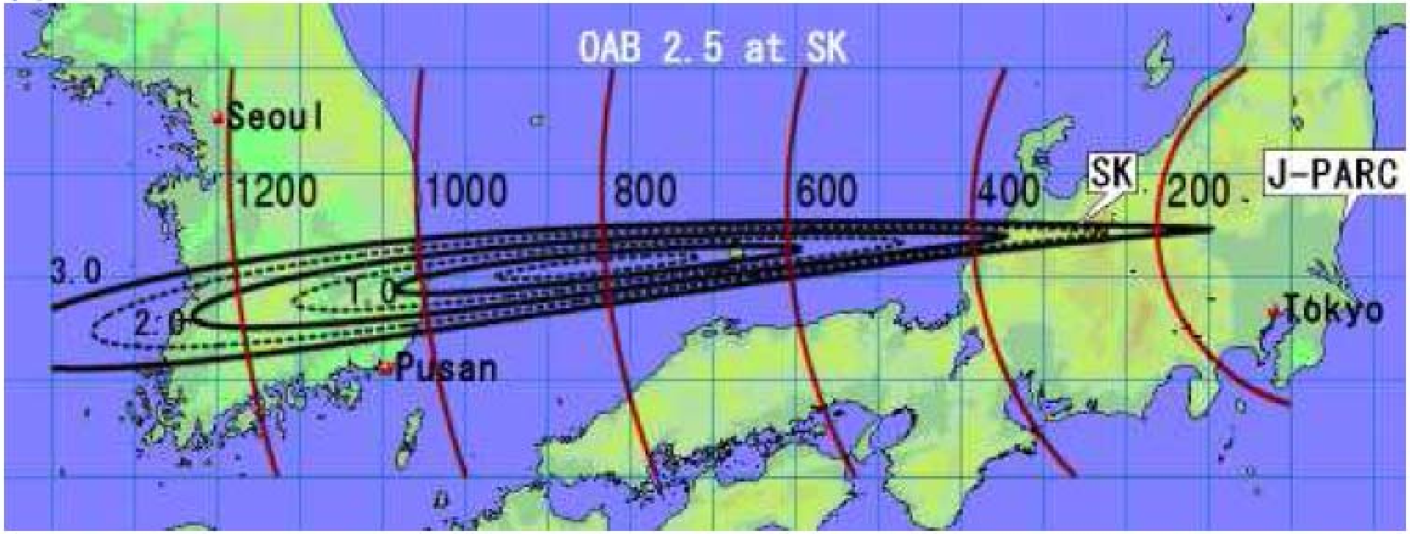
$$P(\nu_\mu \to \nu_e) = 4c_{13}^2 s_{13}^2 s_{23}^2 \left[1 \pm \frac{2a}{|\delta m_{31}^2|}(1-2s_{13}^2) - \frac{\delta m_{21}^2 L}{2E}\frac{c_{23}c_{12}s_{12}}{s_{13}s_{23}}\sin\delta_{\rm CP}\right]$$
$$\times \sin^2\left[\frac{|\delta m_{31}^2|L}{4E} \mp \frac{aL}{4E}(1-2s_{13}^2) \mp \frac{\delta m_{21}^2 L}{4E}\left(s_{12}^2 - \frac{c_{23}c_{12}s_{12}}{s_{13}s_{23}}\cos\delta_{\rm CP}\right)\right]$$

Reactor anti-neutrinos **Double Chooz**

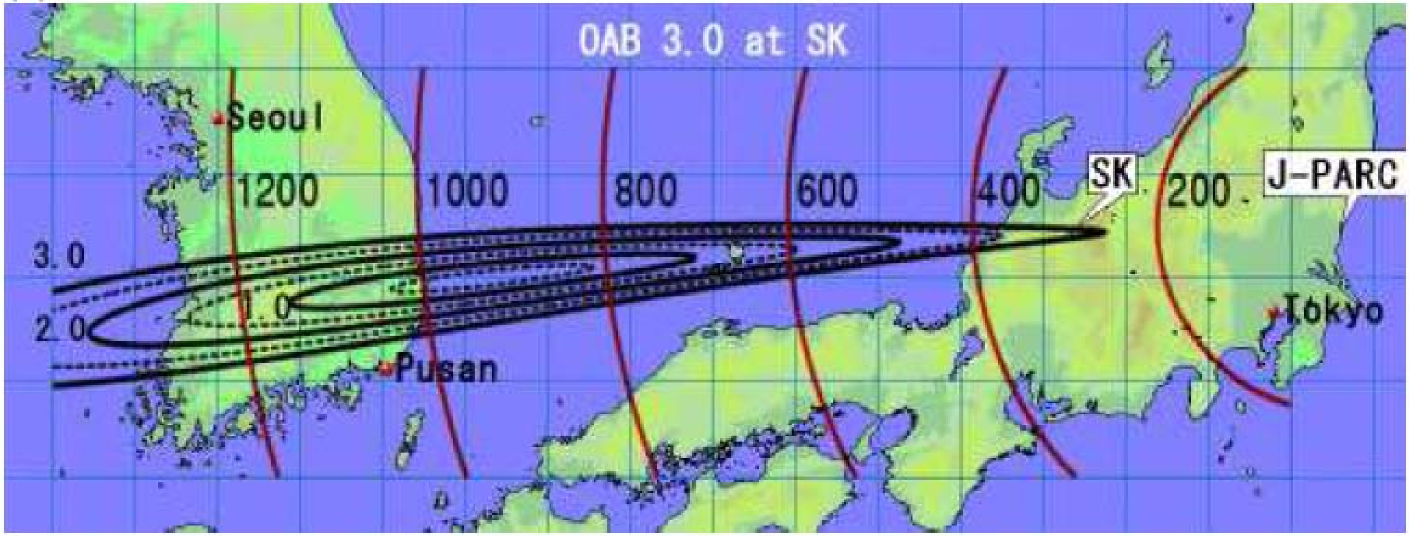
$$P(\bar\nu_e \to \bar\nu_e) = 1 - 4c_{13}^2 s_{13}^2 \sin^2\frac{\delta m_{31}^2 L}{4E}$$

Hagiwara, Okamura, Senda, PRD76(2007)

(a)



(b)



Re-evaluation of the T2KK physics potential with simulations including backgrounds

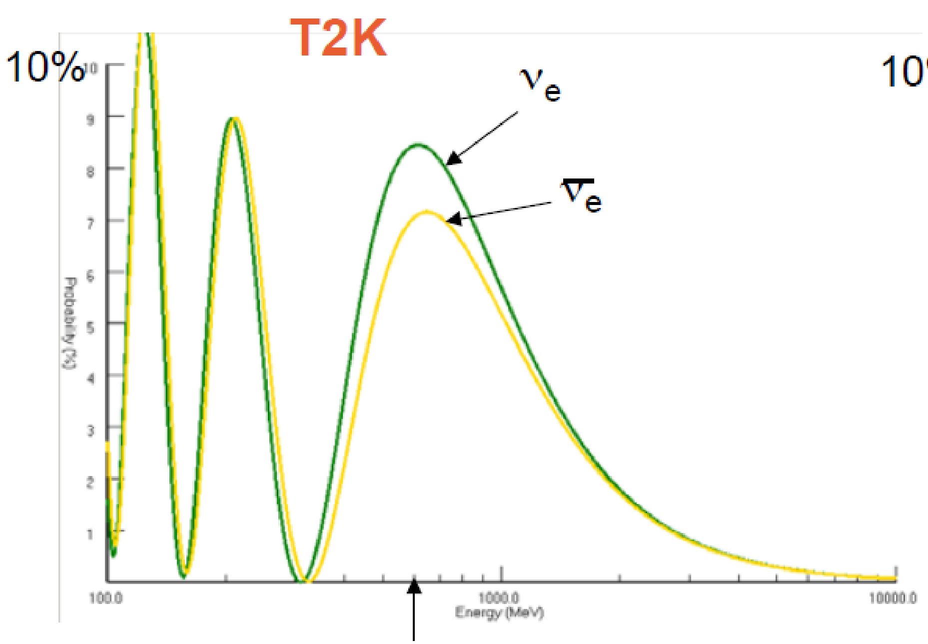
Kaoru Hagiwara[a,b] and Naotoshi Okamura[a]

JHEP07(2009)031

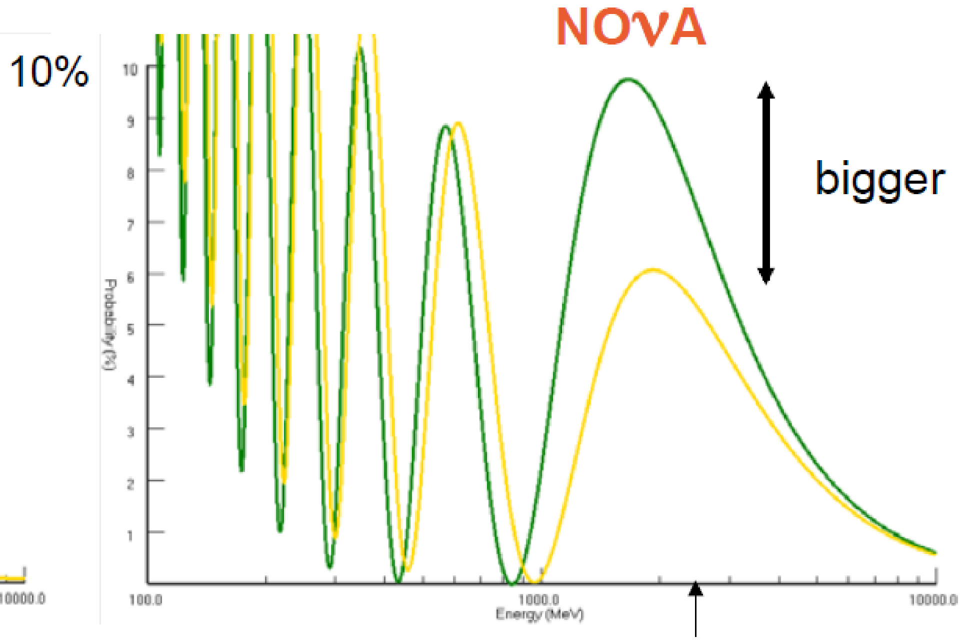
ABSTRACT: The Tokai-to-Kamioka-and-Korea (T2KK) neutrino oscillation experiment under examination can have a high sensitivity to determine the neutrino mass hierarchy for a combination of relatively large ($\sim 3.0^\circ$) off-axis angle beam at Super-Kamiokande (SK) and small ($\sim 0.5^\circ$) off-axis angle at $L \sim 1,000$ km in Korea. We elaborate previous studies by taking into account smearing of reconstructed neutrino energy due to finite resolution of electron or muon energies, nuclear Fermi motion and resonance production, as well as the neutral current π^0 production background to the $\nu_\mu \to \nu_e$ oscillation signal. It is found that the mass hierarchy pattern can still be determined at 3σ level if $\sin^2 2\theta_{\rm RCT} \equiv 4|U_{e3}|^2(1-|U_{e3}|^2) \gtrsim 0.08$ (0.09) when the hierarchy is normal (inverted) with 5×10^{21} POT (protons on target) exposure, or 5 years of the T2K experiment, if a 100 kton water Čerenkov detector is placed in Korea. The π^0 backgrounds deteriorate the capability of the mass hierarchy determination, whereas the events from CC nuclear resonance productions contribute positively to the hierarchy discrimination power. We also find that the π^0 backgrounds seriously affect the CP phase measurement. Although $\delta_{\rm MNS}$ can still be constrained with an accuracy of $\sim \pm 45^\circ$ ($\pm 60^\circ$) at 1σ level for the normal (inverted) hierarchy with the above exposure if $\sin^2 2\theta_{\rm RCT} \gtrsim 0.04$, CP violation can no longer be established at 3σ level even for $\delta_{\rm MNS} = \pm 90^\circ$ and $\sin^2 2\theta_{\rm RCT} = 0.1$. About four times higher exposure will be needed to measure $\delta_{\rm MNS}$ with $\pm 30^\circ$ accuracy.

Resolving the Mass Hierarchy

Matter effect enhances ν_e appearance for normal hierarchy
Effect is reversed (enhanced anti-ν_e) for inverted hierarchy



600 MeV
290 km



2.3 GeV
820 km

δm^2_{23}=2.5x10^{-3} eV^2 $\sin^2 2\theta_{23}$=1.0

δm^2_{12}=7.1x10^{-5} eV^2 $\sin^2 2\theta_{12}$=0.81

$\sin^2 2\theta_{13}$=0.16

6 Neutrinoless Double Beta Decay

$\langle m_\nu \rangle$ and ν spectrum

Neutrinos mix, thus:

For normal hierarchy

$$\langle m_\nu \rangle = \sum_j U_{ej}^2 m_j \qquad s_{12}^2 c_{13}^2 m_2 \sim s_{13}^2 m_3 \simeq 2 \text{ meV}$$

$$= c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2 + s_{13}^2 e^{i\beta} m_3$$

A priori seven unknown quantities:

⇒ 3 masses: m_i

⇒ 2 angles: θ_{12} and θ_{13}

⇒ 2 CP violating phases: α and β

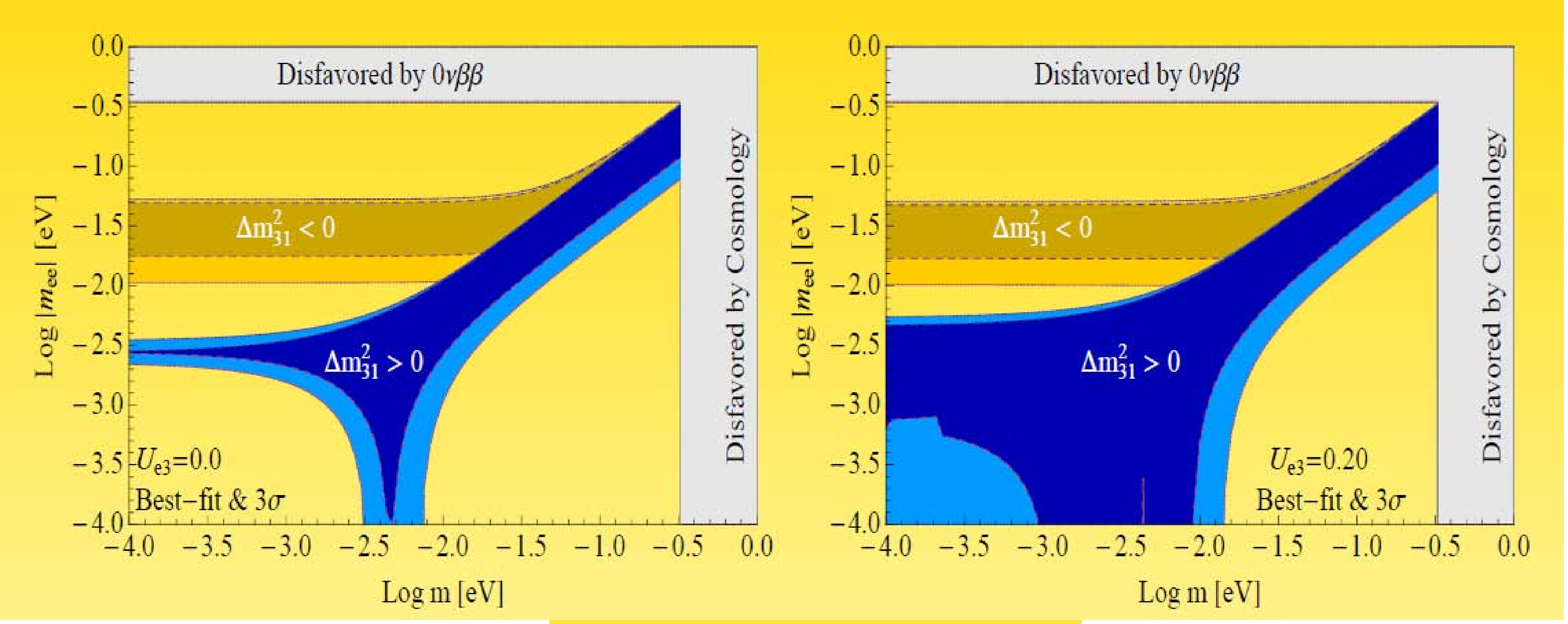
Lower limit - inverse hierarchy

Recall:

$$
\begin{aligned}
\langle m_\nu \rangle &= \sum_j U_{ej}^2 m_j \\
&\simeq c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2 \\
&\sim (c_\odot^2 - s_\odot^2)\sqrt{\Delta m_{Atm}^2} \\
&\simeq 0.4 \cdot \sqrt{2.2 \cdot 10^{-3}}\ \text{eV} \quad \simeq 19\ \text{meV}
\end{aligned}
$$

$\Rightarrow$ Lower limit exists, if $\theta_\odot$ non-maximal

Ue3=0 Ue3=0.2



Werner Rodejohann

Future experiments

Currently under construction / comissioning:

	EXO-200	GERDA-I/II	CUORE	SNO+
A^Z	^{136}Xe	^{76}Ge	^{130}Te	^{150}Nd
Mass	160 kg	35 kg	200 kg *	56 kg
Method	liquid TPC	ionization	bolometer	scintillation
Location	WIPP	LNGS	LNGS	SNOlab
Starts	2009	2009	2012	2011
$T_{1/2}^{0\nu\beta\beta}$ (est.)	6.4 $\times 10^{25}$	3$\times 10^{25}$ - 1.5 $\times 10^{26}$	(2 - 6.5) $\times 10^{26}$	? 1.5 $\times 10^{24}$?
$\langle m_\nu \rangle^{(est.)}$ eV	0.19	0.28-0.12 **	0.050-0.027 **	0.15 ***

Assumptions:

* - All towers

** - Background level 10^{-2} - 10^{-3} $e/(\mathrm{y} \cdot \mathrm{kg} \cdot \mathrm{keV})$, i.e. improvement $\simeq 20 - 200$

*** - Optimistic nuclear matrix element

7 Summary

Large θ_{13} suggests

☆breaking with tri-bimaximal mixing.

Modified flavor model A4, S4, $\Delta(27)$?

New flavor model building ?

☆Neutrino Mass Hierarchy ?

$$\frac{\Delta m_{32}^2}{\Delta m_{21}^2} = 0.026 - 0.040 \sim \mathcal{O}(\lambda^2)$$

Normal mass hierarchy $m_3 \gg m_2 \geq m_1$

Inverted mass hierarchy $m_2 \geq m_1 \gg m_3$

T2K(K) and NOνA combined with Reactor neutrinos

☆Leptonic CP violation ?