Inflation in Gauge Mediation and Gravitino Dark Matter

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1. Introduction
Supersymmetry (SUSY)

is one of the most promising candidates beyond the standard model.

However, ...  

SUSY must be broken.

The allowed region of soft SUSY breaking parameters is very severely constrained.

For example, FCNC requires ...

\[ (m^2_Q)_{ij} = m^2_Q \delta_{ij} + (\Delta m^2_Q)_{ij} \]

\[ \Delta m^2 \ll m^2 \]

Moreover, ...

It is difficult to break SUSY spontaneously in the visible sector.

(Due to the existence of a light superpartner.)
Then, we take the following scenario ...

Gauge mediation

transmits the SUSY breaking in the hidden sector to the visible sector by the standard model gauge interactions.

Flavor blind ➔ FCNC is naturally suppressed!
The soft mass spectrum in gauge mediation


\[ W = \mathcal{M}_{ij}(X) \phi_i \tilde{\phi}_j = (\lambda_{ij} X + m_{ij}) \phi_i \tilde{\phi}_j \]

\[ \phi_i, \tilde{\phi}_i \ (i, j = 1, \ldots, N) \text{: messengers, } 5 \oplus \overline{5} \text{ under } SU(5) \supset G_{SM} \]

\[ \langle X \rangle = X + \theta^2 F, \]

**Gaugino mass:** \[ M_r = \frac{\alpha_r}{4\pi} \Lambda_G, \quad \Lambda_G = F \frac{\partial}{\partial X} \log \det \mathcal{M} \]

**Scalar mass:**

\[ m^2_{\tilde{f}} = 2 \sum_{r=1}^{3} C^r_f \left( \frac{\alpha_r}{4\pi} \right)^2 \Lambda^2_S, \quad \Lambda^2_S = \frac{1}{2} |F|^2 \frac{\partial^2}{\partial X \partial X^*} \sum_{i=1}^{N} (\log |\mathcal{M}_i|^2)^2 \]

\[ \mathcal{M}_i \text{: eigenvalues of } \mathcal{M} \]
Gravitino mass in gauge mediation

Promoted to supergravity \( \Rightarrow \quad m_{3/2} = \frac{F}{\sqrt{3} M_{pl}} \)

\( \sqrt{F} \gtrsim 100 \text{ TeV} \quad \Rightarrow \quad m_{3/2} \gtrsim 10 \text{ eV} \)

\( \Rightarrow \quad \text{Gravitino LSP!} \quad ( \text{Usually, a bino or stau is NLSP.} ) \)

Gravitino can be a candidate of the dark matter.

But, we must worry about the gravitino problem ...
Considering spontaneous SUSY breaking in the hidden sector, we will see a connection between ...

- Vacuum structure in the hidden sector \( \leftrightarrow \) Gaugino mass

Obtaining sizable gaugino masses is closely related with the structure of the SUSY breaking vacuum.

We will then consider a possibility which realizes ...

- Cosmological inflation
- Gauge mediation
- Gravitino dark matter with the correct abundance

in just one SUSY breaking model!
Contents

1. Introduction

*Keywords: supersymmetry, gauge mediation*

2. Gaugino mass and landscape of vacua

*Keywords: pseudomoduli, anomalously small gaugino mass*

3. Inflation in gauge mediation (main part)

*Keywords: inflation, gauge mediation, gravitino dark matter*

4. Summary
2. Gaugino mass and landscape of vacua
Case 1

\[ W = X_0(f + \lambda \varphi_1 \varphi_2) + m(X_1 \varphi_1 + X_2 \varphi_2) \text{ \ with \ canonical \ Kahler \ potential} \]

\[ X_0 : \text{SUSY breaking field}, \quad \varphi_1, \varphi_2, X_1, X_2 : \text{messengers} \]

SUSY breaking vacuum :

\[
\begin{align*}
\langle X_1 \rangle &= \langle X_2 \rangle = \langle \varphi_1 \rangle = \langle \varphi_2 \rangle = 0 \\
\langle X_0 \rangle &:\text{pseudomoduli field (tree-level flat direction)}
\end{align*}
\]

Gaugino mass: \( m_{\tilde{g}} \sim f \frac{\partial}{\partial X} \log \det \mathcal{M}_F \)

\( \mathcal{M}_F : \text{fermion mass matrix of messengers}, \quad \det \mathcal{M}_F = -m^2 \)

\[ m_{\tilde{g}} \sim 0 \quad \text{Vanishing leading order gaugino mass}! \]
Direct gauge mediation

K. I. Izawa, Y. Nomura, K. Tobe and T. Yanagida,

The global symmetry in the SUSY breaking sector is weakly gauged.

Gaugino mass is often smaller than scalar mass.

When we take gaugino mass the 1TeV order, scalar mass typically becomes very heavy.

The hierarchy problem occurs again!

Why does the leading order gaugino mass vanish?

How can we take it nonzero?
Case 2

\[ W = \lambda X (\phi_1 \phi_1 + \phi_2 \phi_2) + m\phi_1 \phi_2 + f X \]  

with canonical Kahler potential

\[ X : \text{SUSY breaking field}, \quad \phi_1, \phi_1, \phi_2, \phi_2 : \text{messengers} \]

Metastable SUSY breaking vacuum:

\[ \langle \phi_1 \rangle = \langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_2 \rangle = 0 \]

\[ \langle X \rangle : \text{pseudomoduli field (tree-level flat direction)} \]

Gaugino mass:

\[ m_{\tilde{g}} \sim f \frac{\partial}{\partial X} \log \det \mathcal{M}_F \]

\[ \mathcal{M}_F : \text{fermion mass matrix of messengers}, \quad \det \mathcal{M}_F = \lambda^2 X^2 \]

\[ \Rightarrow m_{\tilde{g}} \sim \frac{f}{\langle X \rangle} \quad \text{Nonzero leading order gaugino mass!} \]

\[ \langle X \rangle = 0 \text{ in pseudomoduli space} \]

\[ \text{Eigenvalues of scalar mass matrix: } \left( m^2 \pm \sqrt{m^4 + 4\lambda^2f^2} \right)/2 \]

\[ \text{Tachyonic !!} \]

In fact, SUSY vacuum exists. \((X = 0, \phi, \bar{\phi} \neq 0)\)

\[ \text{The leading order gaugino mass is nonzero only when there is a tachyonic direction in the pseudomoduli space of the SUSY breaking vacuum.} \]


**Intuitive understanding**

Nonzero leading order gaugino mass \[ \Rightarrow \text{det} \mathcal{M}_F \text{ is a function of } X. \]

For example, \[ \mathcal{M}_F = \lambda X + m \Rightarrow \text{Zero point exists. } \left( X = -\frac{m}{\lambda} \right) \]

SUSY breaking mass splitting: \[ \pm F \Rightarrow \text{Tachyonic direction appears at zero point!} \]
A model with non-canonical Kahler potential


There is no pseudomoduli in general.

If there is a pseudomoduli space ...

Sizable gaugino mass can be obtained without tachyonic direction!

If there is no pseudomoduli space ...

How is the relation between gaugino mass and vacuum stability?

Leading order gaugino mass can be nonzero on the global minimum!

Return to the canonical case ...

**Is such a vacuum stable?**

*If messengers are not tachyonic at the stabilized point, ...*

→ The vacuum is metastable.

However, when we consider cosmology ...

**Why the higher vacuum is selected in the cosmic history?**
3. Inflation in gauge mediation
Inflation

A period of very rapid expansion of the universe.

*It solves many problems in standard cosmology!*

(Flatness, horizon, monopole)

*Quantum fluctuations of the inflaton can set the initial condition of structure formation.*

Inflation is now considered as the standard scenario of the early universe.

Then, a natural question is ...

How is inflation embedded in a particle physics model?
We here consider a possibility of ...

**Inflation in the SUSY breaking sector of gauge mediation**

SUSY breaking sector field is identified as the inflaton.

Higher vacuum is naturally selected after inflation.

The inflaton interacts with the visible sector fields through the messengers in gauge mediation.

Reheating process is calculable and predictable!

The SUSY breaking vacuum has a pseudomoduli.

Moduli oscillation and decay dilute gravitinos produced in the thermal bath!
**Cosmic history in our scenario**

*Inflation in the SUSY breaking sector*

- Moduli stabilizes at the origin.

*Inflation ends.*

*Inflaton decay*

- Many gravitinos are produced in the thermal bath.

*Moduli domination*

*Moduli decay*

- Gravitinos are diluted.

*Gravitinos are also produced by the decay process.*

*Big Bang Nucleosynthesis (BBN)*

*Gravitino dark matter*
Before we see a concrete realization of our scenario, ...

Caution!

Our model is the first step toward a viable SUSY breaking model with inflation.

Some observables may be already inconsistent with experiments.

Our model has some unattractive properties.

(Baryogenesis, ...)

We leave these problems to the future study ...
**Model**

Wess-Zumino model with $SU(N)$ global symmetry

Kahler potential is canonical.

<table>
<thead>
<tr>
<th>Waterfall fields</th>
<th>Model</th>
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<tbody>
<tr>
<td>$\chi$</td>
<td>$\bar{\chi}$</td>
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<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$\bar{\rho}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
<td>$Z$</td>
<td>$\bar{Z}$</td>
<td>-1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\bar{\chi}$</td>
<td>$\chi$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ W = m^2 Y + \mu^2 \Phi - h_Y \chi Y \bar{\chi} - h_\Phi \rho \bar{\rho} - h_Z (\chi Z \bar{\rho} + \rho \bar{Z} \bar{\chi}) - m_Z Z \bar{Z} \]

$m \gg \mu, h_Y, h_\Phi, h_Z$ : real coupling constants

**SUSY breaking vacuum**

\[ Y = \rho = \bar{\rho} = Z = \bar{Z} = 0, \quad \chi = \bar{\chi} = \frac{m}{\sqrt{h_Y}} \]

\[ V_0 = \mu^4 \]

Promoted to supergravity $\Rightarrow \mu \simeq 7.9 \times 10^9 \text{ GeV} \times \left( \frac{m_3/2}{15 \text{ GeV}} \right)^{1/2}$

**Promoters**

- Waterfall fields
- Messengers
- Inflaton $\rightarrow Y$
- Moduli $\rightarrow \Phi$
Mass spectrum

Pseudomoduli $\Phi$ is stabilized at 1-loop:

\[ |\Phi_0| \simeq \frac{1}{2} \frac{m_Z}{h_\Phi}, \quad \text{arg} \Phi_0 = 0, \]
\[ m^2_\Phi \simeq \frac{N}{64\pi^2} \frac{h_Y h_\Phi^4}{h_Z^2} \frac{\mu^4}{m^2} \equiv m^2_{\text{CW}} \]

Vacuum stability

SUSY vacuum also exists: 
\[ \chi \bar{\chi} = \frac{m^2}{h_Y}, \quad \rho \bar{\rho} = \frac{\mu^2}{h_\Phi}, \quad \Phi = \frac{h^2_Z}{h_Y h_\Phi m_Z}, \ldots \]

SUSY breaking vacuum is metastable. Decay rate: 
\[ \Gamma_{\text{vac}} \propto e^{-S}, \quad S \sim \left( \frac{m}{\mu} \right)^4 \]

Mass hierarchy: \[ m \gg \mu \]
Gauge mediation

\[ SU(N) \] global symmetry \:\Rightarrow \: \text{standard model gauge symmetry} \\
\[ Z, \ \bar{Z}, \ \rho, \ \bar{\rho} \: \text{: messengers} \]

\[
m_{\lambda_i} \simeq \frac{g_i^2}{16\pi^2} \frac{h_Y h_\Phi}{h_Z^2} \frac{\mu^2}{m} \frac{m_Z}{m},
\]

\[
m^2_{\tilde{f}} \simeq \sum_i C_2^i \left( \frac{g_i^2}{16\pi^2} \right)^2 \frac{h_Y h_\Phi^2}{h_Z^2} \frac{\mu^4}{m^2}
\]

\[ g_i \ (i = 1, 2, 3) : U(1) \times SU(2) \times SU(3) \: \text{standard model gauge coupling} \]

\[ C_2^i : \text{quadratic Casmir} \]

Gaugino-to-scalar mass ratio : \[ r_\tilde{g} \equiv m_{\tilde{g}} / m_{\tilde{e}} \]

Sizable gaugino mass \:\Leftarrow \: \text{The existence of the lower vacuum} \\

**Inflationary scenario**

Hybrid inflation in the SUSY breaking sector

\[ Y : \text{inflaton}, \quad \chi, \quad \bar{\chi} : \text{waterfall fields} \quad (\rho = \bar{\rho} = Z = \bar{Z} = \Phi = 0) \]

stabilized by Hubble induced mass during inflation.

\[ V_g \simeq e^{\frac{|\psi|^2}{M_{P1}^2}} (3H^2 M_{P1}^2) \simeq 3H^2 |\psi|^2 + \cdots \]

\[ V_{\text{tree}} \simeq \left| m^2 - h_Y \chi \bar{\chi} \right|^2 + h_Y^2 |Y|^2 (|\chi|^2 + |\bar{\chi}|^2) \]

\[
\begin{align*}
|Y| > Y_c & \equiv m / \sqrt{h_Y} \quad \chi = \bar{\chi} = 0 \quad \Rightarrow \text{Inflation!} \\
|Y| < Y_c & \quad \chi = \bar{\chi} = \frac{m}{\sqrt{h_Y}}
\end{align*}
\]

\[ H \simeq \sqrt{\frac{1}{3 M_{P1}^2}} \]

\[ Y, \quad \chi, \quad \bar{\chi} \]
**Inflaton motion**

Loop correction due to the waterfall fields

\[ \text{The flat inflaton potential is lifted.} \]

The inflaton rolls off to the critical point. \[ \Rightarrow \text{Inflation ends.} \]

*Moduli stabilizes at the origin during inflation by the Hubble effect.*

\[ (x \equiv Y/Y_c) \]

\[ V_{\text{rad}}/((k^2 m^4)/(32\pi^2)) \]

During inflation

<table>
<thead>
<tr>
<th>V</th>
<th>0</th>
</tr>
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<tbody>
<tr>
<td>Φ</td>
<td>0</td>
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</table>
Cosmological perturbation

COBE/WMAP normalization, \( P_R^{1/2} \approx 4.9 \times 10^{-5} \)

\[
\frac{m}{h_Y^{1/2}} \approx 5.9 \times 10^{15} \text{GeV} \times \left\{ \begin{array}{ll}
\left( \frac{h_Y}{3 \times 10^{-3}} \right)^{1/3} & \text{for } h_Y < 3 \times 10^{-3} \\
\left( \frac{N_{\text{COBE}}}{51} \right)^{-1/4} & \text{for } h_Y > 3 \times 10^{-3}
\end{array} \right.
\]

Spectral tilt:

\( n_s = 1 - 6\epsilon + 2\eta \approx \left\{ \begin{array}{ll}
1 - \frac{h_Y^3 M_{\text{pl}}^2}{2\pi^2 m^2} & \text{for } h_Y < 3 \times 10^{-3} \\
1 - \frac{1}{N_{\text{COBE}}} & \text{for } h_Y > 3 \times 10^{-3}
\end{array} \right. \approx 0.98,

Scalar-to-tensor ratio:

\( r = 16\epsilon \approx \left\{ \begin{array}{ll}
\frac{h_Y^{10/3}}{16\pi^4} \left( \frac{h_Y^{5/6} M_{\text{pl}}}{m} \right)^2 & \text{for } h_Y < 3 \times 10^{-3} \\
\frac{h_Y^2}{2\pi^2 N_{\text{COBE}}} & \text{for } h_Y > 3 \times 10^{-3}
\end{array} \right.

Hereafter, \( h_Y < 3 \times 10^{-3} \)
Reheating after inflation

The decays of the inflaton and the waterfall field $\bar{X} \equiv \chi + \bar{\chi}$

$\mathcal{O}(\sqrt{h_Y m})$ mass

They dominantly decay into an SSM gaugino pair.

$T_R \simeq \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \times \sqrt{\Gamma_R M_{Pl}}$

$\simeq 0.45 \times \frac{N^2}{(4\pi)^2} \left( \frac{\sqrt{h_Y}}{8\pi} \right)^{1/2} \frac{h_Y g_3^2}{h_Z^3} (m M_{Pl})^{1/2}$

Gravitinos are produced in the thermal bath.

$\frac{\rho_{3/2}^{(th)}}{s} \simeq 9.5 \times 10^{-8} \text{ GeV} \times \left( \frac{m_{\tilde{g}}}{1.5 \text{ TeV}} \right)^2 \left( \frac{m_{3/2}}{15 \text{ GeV}} \right)^{-1} \left( \frac{T_R}{10^{10} \text{ GeV}} \right)$

$s$ : entropy density

overproduced ! $\frac{\rho_{3/2}}{s} < \frac{\rho_{DM}}{s} \simeq 4.1 \times 10^{-10} \text{ GeV}$
Moduli oscillation

Moduli stabilizes at the origin during inflation by the Hubble effect.

Moduli stabilizes with a nonzero vev on the SUSY breaking vacuum.

\[ |\Phi_0| \simeq 1.1 \times 10^{14} \text{ GeV} \times \left( \frac{r_g}{3.5} \right)^2 \left( \frac{m_3/2}{15 \text{ GeV}} \right) \left( \frac{m_\tilde{g}}{1.5 \text{ TeV}} \right)^{-1} \]

\[ H < m_\Phi \quad \text{The oscillation starts around } \Phi_0 \]

\[ T_{osc} \simeq \left( \frac{90}{\pi^2 g_*^{osc}} \right)^{1/4} \times \sqrt{M_{Pl} m_\Phi} \]

\[ \simeq 1.2 \times 10^{10} \text{ GeV} \times \left( \frac{m_\Phi}{300 \text{ GeV}} \right)^{1/2} \quad g_*^{osc} \simeq 220 \]

There is a tachyonic direction in the pseudomoduli space.

The stability of oscillation

\[ r_g \lesssim 4.5 \]
Entropy production

The long lifetime of moduli $\Rightarrow$ *The oscillation dominates the energy density of the universe.*

The decay of moduli $\Rightarrow$ *Entropy production*

$\Rightarrow$ *Thermally produced gravitinos are diluted.*

\[ 3M_P^2 m_\Phi^2 \left( \frac{T_{\text{dom}}}{T_{\text{osc}}} \right)^4 = m_\Phi^2 |\Phi_0|^2 \left( \frac{T_{\text{dom}}}{T_{\text{osc}}} \right)^3 \]

$T_d$ : moduli decay temperature

Dilution factor :

\[ \Delta^{-1} \approx \frac{T_d}{T_{\text{dom}}} \]

\[ \approx \frac{T_d}{T_{\text{osc}}} \left( \frac{|\Phi_0|}{\sqrt{3}M_P} \right)^{-2} \]
Moduli decay

M. Ibe and R. Kitano, Phys. Rev. D75, 055003 (2007) , ... (many other works)

Dominant decay process : \( \Phi \rightarrow hh \ (m_\Phi > 2m_h) \)

Interaction Lagrangian : 
\[
\mathcal{L}_f = \frac{\partial m_f^2(\Phi)}{\partial \Phi} \Phi \bar{f} f^\dagger + \text{h.c.}
\]
\[
\approx \frac{3}{4} \sum_i C_i^2 \left( \frac{g_i^2}{16\pi^2} \right)^2 \frac{h_Y^2 h_{\Phi}^3}{h_Z^4} \frac{\mu^4 m_Z}{m^4} \Phi \bar{f} f^\dagger + \text{h.c.}
\]

\[ T_d \sim \sqrt{\Gamma_H M_{Pl}} \]
\[
\approx 4.4 \text{ MeV} \times \left( \frac{r_g}{3.5} \right)^{-2} \left( \frac{m_g}{1.5 \text{ TeV}} \right)^3 \left( \frac{m_{3/2}}{15 \text{ GeV}} \right)^{-1} \left( \frac{m_\Phi}{300 \text{ GeV}} \right)^{-1/2}
\]

\( \Gamma_H \) : decay width

The temperature is required to be above \( \sim 2 \text{ MeV} \) so that the BBN properly occurs.
Gravitino abundance

Moduli decay: $\Phi \rightarrow \psi_{3/2} \psi_{3/2}$ (longitudinal mode)

Interaction Lagrangian: $\mathcal{L}_{3/2} \simeq -\frac{N}{(16\pi)^2} \frac{h_Y h^4_{\Phi}}{h^2_Z} \left( \frac{\mu}{m} \right)^2 \Phi^\dagger \bar{\psi}_{3/2} \psi_{3/2} + c.c.$

Gravitino number density: $\frac{n_{3/2}}{s} = \frac{3}{4} \frac{T_d}{m_\Phi} B_{3/2} \times 2 \quad B_{3/2} \equiv \Gamma_{3/2}/\Gamma_H$

Density parameter: $\Omega_{3/2}^{(d)} h^2 \simeq 0.033 \times \left( \frac{r_g}{3.5} \right)^2 \left( \frac{m_\Phi}{300 \text{ GeV}} \right)^{9/2} \left( \frac{m_\tilde{g}}{1.5 \text{ TeV}} \right)^{-3}$

Gravitino abundance produced in thermal bath ($T_R \approx T_{osc}$)

Dilution factor $\Delta^{-1}$

$\Omega_{3/2}^{(th)} h^2 \simeq 0.016 \times \left( \frac{r_g}{3.5} \right)^{-6} \left( \frac{m_\Phi}{300 \text{ GeV}} \right)^{-1/2} \left( \frac{m_\tilde{g}}{1.5 \text{ TeV}} \right)^7 \left( \frac{m_{3/2}}{15 \text{ GeV}} \right)^{-4}$
$2h_Z > h_Y$

$\Omega_{3/2}^{(th)} h^2 = 0.11$

$r_{3/2} = 5$

$T_d > 2\text{MeV}$

$\Omega_{3/2}^{(tot)} h^2 = 0.11$

$\Omega_{3/2}^{(d)} h^2 = 0.11$

$0.2$

$1$

$2.5$

$3.0$

$m_{\tilde{g}/\text{TeV}}$

$m_{3/2}/\text{GeV}$

Moduli mass: $300\text{GeV}$, $r_g = 3.5$

$r_{3/2} \equiv \Omega_{3/2}^{(th)}/\Omega_{3/2}^{(d)}$
$T_d > 2\text{MeV}$

$\Omega_{3/2}^{(d)} h^2 = 0.11$

$\Omega_{3/2}^{(\text{tot})} h^2 = 0.11$

$m_{\rho(Z)} > 0$

$r_{3/2} = 0.1$

$2h_Z > h_Y$

$m_{3/2}/\text{GeV}$

Moduli mass : 300 GeV, Gluino mass : 1.5 TeV
Model parameters

\[
    h_{\Phi} \simeq 0.036 \times \frac{1}{\sqrt{N}} \left( \frac{r_g}{3.5} \right) \left( \frac{m_{\Phi}}{300 \text{ GeV}} \right) \left( \frac{m_{\tilde{g}}}{1.5 \text{ TeV}} \right)^{-1}
\]

\[
    h_{Z} \simeq 1.8 \times 10^{-3} \times \frac{1}{\sqrt{N}} \left( \frac{r_g}{3.5} \right)^2 \left( \frac{m_{3/2}}{15 \text{ GeV}} \right) \left( \frac{m_{\Phi}}{300 \text{ GeV}} \right) \left( \frac{m_{\tilde{g}}}{1.5 \text{ TeV}} \right)^{-2} \left( \frac{h_Y}{3 \times 10^{-3}} \right)^{-1/3}
\]

\[
    h_{Y} \simeq 2.2 \times 10^{-3} \times \frac{1}{N^{21/34}} \times \left( \frac{r_g}{3.5} \right)^{18/17} \left( \frac{m_{3/2}}{15 \text{ GeV}} \right)^{9/17} \left( \frac{m_{\Phi}}{300 \text{ GeV}} \right)^{21/34} \left( \frac{m_{\tilde{g}}}{1.5 \text{ TeV}} \right)^{-18/17}
\]

\[
    \mu \simeq 7.9 \times 10^9 \text{ GeV} \times \left( \frac{m_{3/2}}{15 \text{ GeV}} \right)^{1/2}
\]

\[
    \frac{m}{h_Y^{1/2}} \simeq 5.9 \times 10^{15} \text{ GeV} \times \begin{cases}
    \left( \frac{h_Y}{3 \times 10^{-3}} \right)^{1/3} & \text{for } h_Y < 3 \times 10^{-3} \\
    \left( \frac{N_{\text{COBE}}}{51} \right)^{-1/4} & \text{for } h_Y > 3 \times 10^{-3}
\end{cases}
\]

\[
    m_{Z} \simeq 8.2 \times 10^{12} \text{ GeV} \times \frac{1}{\sqrt{N}} \left( \frac{r_g}{3.5} \right)^3 \left( \frac{m_{3/2}}{15 \text{ GeV}} \right) \left( \frac{m_{\Phi}}{300 \text{ GeV}} \right) \left( \frac{m_{\tilde{g}}}{1.5 \text{ TeV}} \right)^{-2}
\]
4. Summary
Inflation in the SUSY breaking sector of gauge mediation

Metastable vacuum is naturally selected after inflation.

Reheating process  Messenger loop

Moduli oscillation & decay

Thermally produced gravitinos are diluted.

Non-thermally produced gravitino

Gravitino dark matter

Model parameters are severely constrained.
Future work

Baryogenesis

Dilution factor $\Delta^{-1} \approx 10^{-3}$

Sufficient baryon asymmetry is required before moduli domination.

Various inflation models

- Cosmic string problem ,
- $\eta$ problem ,
- Small coupling constants , ...

Thank you for your attention!
Extra slides
1-loop lifting of pseudomoduli

1-loop effective potential (Coleman-Weinberg potential):

\[ V_{\text{eff}}^{(1)} = \frac{1}{64\pi^2} \text{STr} \left( \mathcal{M}^4 \log \frac{\mathcal{M}^2}{M_{\text{cutoff}}^2} \right) \]

\[ \equiv \frac{1}{64\pi^2} \left[ \text{Tr} \left( m_B^4 \log \frac{m_B^2}{M_{\text{cutoff}}^2} \right) - \text{Tr} \left( m_F^4 \log \frac{m_F^2}{M_{\text{cutoff}}^2} \right) \right] \]

\[ m_B^2, m_F^2 \]: tree-level boson and fermion masses

(functions of pseudomoduli vev)

\[ M_{\text{cutoff}} \]: UV cutoff

\[ \Phi^a \]: k chiral superfields, \[ K = \Phi^a \bar{\Phi}^a \], \[ W(\Phi^a) \]

\[ m_0^2 = \left( \begin{array}{cc} W_{ac}W_{cb} & W_{abc}W_c \cr W_{abc}W_{\bar{c}b} & W_{a\bar{c}}W_{\bar{c}b} \end{array} \right), \quad m_{1/2}^2 = \left( \begin{array}{cc} W_{ac}W_{cb} & 0 \cr 0 & W_{a\bar{c}}W_{\bar{c}b} \end{array} \right) \]

\[ W_c \equiv \partial W/\partial Q^c \], \[ m_0^2, m_{1/2}^2 : 2k \times 2k \text{ matrix} \]

\[ W = \mathcal{M}_F(X)_{ab} \tilde{\phi}^a \phi^b + f(X) \quad \text{with non-canonical Kahler potential} \]

\[ X : \text{SUSY breaking field}, \quad \phi, \tilde{\phi} : \text{messengers}, \quad \mathcal{M}_F : \text{messenger mass matrix} \]

There is no pseudomoduli in general.

\[ \text{Preserving the flat direction of } X \]

\[ \partial_X g^{X \bar{X}} \bigg|_0 = 0 \]

\[ \langle \phi^a \rangle = \langle \tilde{\phi}^a \rangle = 0 \]

\[
\begin{align*}
g_{a \bar{a}} &= \partial_a \partial_{\bar{a}} K \\
\mathcal{L}_{scalar} &= g_{a \bar{a}} \partial_\mu \Phi^a \partial^\mu \Phi^{\bar{a}} - V(\Phi, \bar{\Phi}) \\
V &= g^{a \bar{a}} \partial_a W \partial_{\bar{a}} \bar{W}
\end{align*}
\]
A model with non-canonical Kahler potential

\[ W = \lambda X (\phi_1 \tilde{\phi}_1 + \phi_2 \tilde{\phi}_2) + m \phi_1 \tilde{\phi}_2 + f X \]

\[ K = |X|^2 + \left( 1 + \frac{|X|^2}{M^2} \right) \left( |\phi_1|^2 + |\tilde{\phi}_2|^2 \right) + \left( 1 - \frac{|X|^2}{M^2} \right) \left( |\phi_1|^2 + |\phi_2|^2 \right) \]

\[ \partial_X g^{XX} \big|_0 = 0 \quad \Rightarrow \quad \text{The flat direction of } X \]

Canonical Kahler potential \quad \Rightarrow \quad A tachyonic direction around \quad \langle X \rangle = 0

Now ... \quad The eigenvalues of messenger boson mass-squared matrix:

\[ \frac{1}{2} \left( m^2 \pm \sqrt{m^4 + 4 \lambda^2 f^2 - 4 \left( \frac{f}{M} \right)^2 m^2 + 4 \left( \frac{f}{M} \right)^4} \right) \]

\[ \lambda^2 f^2 - \left( \frac{f}{M} \right)^2 m^2 + \left( \frac{f}{M} \right)^4 < 0 \quad \Rightarrow \quad \text{Sizable gaugino mass without tachyonic direction!} \]
Sizable gaugino mass on the global minimum

If there is no pseudomoduli space ...

How is the relation between gaugino mass and vacuum stability?

Leading order gaugino mass can be nonzero on the global minimum!


Example (SUSY breaking sector + Messenger sector + Visible sector)

SUSY breaking sector: \(U(1)\) gauge theory ↔ Messenger gauge interaction

\[
W = X_0(f + \lambda \varphi_1 \varphi_2) + m(X_1 \varphi_1 + X_2 \varphi_2), \quad f \ll m^2
\]

\(U(1)\) charge of \(X_0, X_1, X_2, \varphi_1\) and \(\varphi_2\): 0, -1, 1, 1 and -1

SUSY breaking vacuum: \(\langle X_1 \rangle = \langle X_2 \rangle = \langle \varphi_1 \rangle = \langle \varphi_2 \rangle = 0\)

\(X_0\) has a nonzero F-term.
**Messenger sector**: 

\[ W_{mess} = y_q S q \bar{q} + y_E S E \bar{E} + \frac{k}{3} S^3 \]

\( q \) and \( \bar{q} \) : messengers, \( S, E, \bar{E} \) : standard model gauge singlet

\[ U(1) \text{ charge : } 1, -1 \]

Integrating out the SUSY breaking sector

\[ m_E^2 \approx m_{\bar{E}}^2 \sim \left( \frac{g_{mess}^2}{16\pi^2} \right)^2 \left( \frac{\lambda f}{m} \right)^2 \]

\( g_{mess} \) : U(1) gauge coupling

\[ \rightarrow 1\text{-loop effect of } E, \bar{E} \rightarrow \text{Negative mass of } S : \]

\[ -m_S^2 \approx \frac{4}{16\pi^2} y_E^2 m_E^2 \ln \frac{\Lambda}{m_E} \]

\( \Lambda \) : Cut-off scale

\[ y_E \ll 1 \rightarrow m_E^2 \gg |m_S^2| \]
Effective scalar potential of the messenger sector:

\[ V_{mess} = |y_E S \tilde{E}|^2 + |y_E S E|^2 + |y_q S \tilde{q}|^2 + |y_q S q|^2 + |y_E E \tilde{E} + y_q q \tilde{q} + \kappa S|^2 \]
\[ + m_E^2 |E|^2 + m_{\tilde{E}}^2 |\tilde{E}|^2 + m_S^2 |S|^2. \]

SUSY breaking global minimum:

\[ \langle |S|^2 \rangle = \frac{|m_S^2|}{2\kappa^2}, \quad \langle q \rangle = \langle \tilde{q} \rangle = \langle E \rangle = \langle \tilde{E} \rangle = 0 \]
\[ V_0 = -\frac{m_S^4}{4\kappa^2} \]

\textit{S is determined uniquely and pseudomoduli space does not exist in the messenger sector.}

Gaugino mass:

\[ m_{\tilde{g}} \sim \frac{\langle |F_S| \rangle}{\langle S \rangle} = \frac{|m_S|}{\sqrt{2}} \]

Leading order gaugino mass is nonzero on the global minimum!
$2h_Z > h_Y$

$\Omega^{(th)}_{3/2} h^2 = 0.11$

$\Omega^{(tot)}_{3/2} h^2 = 0.11$

$r_{3/2} = 5$

1

0.2

$T_d > 2\text{MeV}$

$m_{3/2}/\text{GeV}$

Moduli mass: 500 GeV, $r_g = 3.5$
Moduli mass : 500 GeV, Gluino mass : 1.5 TeV