

# Testing the equivalence principle with fundamental constants

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- Some words on fundamental constants
- Links between constants and gravity [Equivalence principle]
- Links with cosmology

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### Constants

Physical theories involve constants

These parameters cannot be determined by the theory that introduces them.

These arbitrary parameters have to be assumed constant:

- experimental validation
- no evolution equation

By testing their constancy, we thus test the laws of physics in which they appear.

A physical measurement is always a comparison of two quantities, one can be thought as a unit

- it only gives access to dimensionless numbers

- we consider variation of dimensionless combinations of constants

JPU, Rev. Mod. Phys. 75, 403 (2003); Liv. Rev. Relat. 4, 2 (2011)
JPU, [astro-ph/0409424, arXiv:0907.3081]
R. Lehoucq, JPU, *Les constantes fondamentales* (Belin, 2005)
G.F.R. Ellis and JPU, Am. J. Phys. 73 (2005) 240
JPU, B. Leclercq, De *l'importance d'être une constante* (Dunod, 2005)
translated as "*The natural laws of the universe*" (Praxis, 2008).

### **Reference theoretical framework**

The number of physical constants depends on the level of description of the laws of nature.

In our present understanding [*General Relativity* + SU(3)xSU(2)xU(1)]:

- G : Newton constant (1)
- **6** Yukawa coupling for quarks
- **3** Yukawa coupling for leptons
- $\bullet$  mass and VEV of the Higgs boson:  ${\bf 2}$
- CKM matrix: **4** parameters
- Non-gravitational coupling constants: **3**
- • $\Lambda_{uv}$ : 1
- c, ħ : **2**

22 constants19 parameters

cosmological constant

### Number of constants may change

This number is « time-dependent ».

#### Neutrino masses

Add **3** Yukawa couplings + **4** MNS parameters = **7** more

#### Unification

$$lpha_i^{-1}(E) = lpha_{GUT}^{-1} + rac{b_i}{2\pi} {
m ln} rac{M_{GUT}}{E} \hspace{1cm} {
m SM:} \hspace{1cm} b_i = (41/10, -19/6, -7) \ {
m MSSM:} \hspace{1cm} b_i = (33/5, 1, -3)$$



### **Importance of unification**

### Unification $\alpha_i^{-1}(E) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi} \ln \frac{M_{GUT}}{E}$

Variation of  $\alpha$  is accompanied by variation of other coupling constants

**QCD scale** 
$$\Lambda_{\text{QCD}} = E\left(\frac{m_c m_b m_t}{E^3}\right)^{2/27} \exp\left[-\frac{2\pi}{9\alpha_s(E)}\right]$$

Variation of  $\Lambda_{QCD}$  from  $\alpha_s$  and from Yukawa coupling and Higgs VEV

Theories in which EW scale is derived by dimensional transmutation  $v \sim \exp\left[-\frac{8\pi^2}{h_t^2}\right]$ 

Variation of Yukawa and Higgs VEV are coupled

**String theory** All dimensionless constants are dynamical – their variations are all correlated.

#### These effects cannot be ignored in realistic models.

# New degrees of freedom

#### On the basis of general relativity

The equivalence principle takes much more importance in general relativity

It is based on **Einstein equivalence principle** 

universality of free fall local Lorentz invariance local position invariance



The outcome of any local non-gravitational experiment in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime.

If this principle holds then gravity is a consequence of the geometry of spacetime



This principle has been a driving idea in theories of gravity from Galileo to Einstein

### GR in a nutshell

Underlying hypothesis

Equivalence principle

- Universality of free fall
- Local lorentz invariance
- Local position invariance











Relativity

 $g_{\mu
u}=g^*_{\mu
u}$ 

### Equivalence principle and constants

Action of a test mass:

$$S = -\int mc \sqrt{-g_{\mu\nu}v^{\mu}v^{\nu}} dt \quad \text{with} \quad v^{\mu} = dx^{\mu}/dt$$
$$u^{\mu} = dx^{\mu}/d\tau$$
$$\delta S = 0$$
$$a^{\mu} \equiv u^{\nu} \nabla_{\nu} u^{\mu} = 0 \quad \text{(geodesic)}$$
$$g_{00} = -1 + 2\Phi_N/c^2 \quad \text{(Newtonian limit)}$$
$$\dot{\mathbf{v}} = \mathbf{a} = -\nabla \Phi_N = \mathbf{g}_N$$

#### The equivalence principle in Newtonian physics

The deviation from the universality of free fall is characterized by

$$\eta \equiv 2 \frac{|a_1 - a_2|}{|a_1 + a_2|}$$
  
Second law:  $F = m_I a$   
Definition of weight  $F = m_G g$ 
$$= (m_G/m_I)g,$$

So that 
$$\eta = 2 \frac{|m_G^1/m_I^1 - m_G^2/m_I^2|}{m_G^1/m_I^1 + m_G^2/m_I^2}$$

Consider a pendumum of length *L* in a gravitational field *g*,

$$\ddot{ heta} + \omega^2 heta = 0$$
 où  $\omega \equiv \omega_0 \sqrt{rac{m_G}{m_I}}$  et  $\omega_0 \equiv \sqrt{rac{g}{L}}$ .  
 $\eta \approx 2 rac{|\omega_B - \omega_A|}{\omega_0}$ 

Then

#### Tests on the universality of free fall



### Testing general relativity on astrophysical scales

There is a growing need to test general relativity on astrophysical scales

dynamics of galaxies and **dark matter** 



acceleration of the universe and **dark energy** 



but also theoretical motivations...

Can we extend the test of the equivalence principle on astrophysical scales?

### **Universality classes of extensions**

Ordinary  $g_{\mu
u}$  $\phi$ matter

<u>Ex</u>: quintessence, ....

 $S_{\rm de}[{\rm de};g_{\mu\nu}]$ 

JDEM workshop, 2004

[JPU, Aghanim, Mellier, PRD 05] [JPU, GRG 2007]

### **Universality classes of extensions**



Ex: quintessence, ....

 $S_{\rm de}[{\rm de};g_{\mu\nu}]$ 



[JPU, Aghanim, Mellier, PRD 05] [JPU, GRG 2007]

#### **Famous example: Scalar-tensor theories**

$$S=rac{c^3}{16\pi G}\int\!\sqrt{-g}\{R-2(\partial_\mu\phi)^2-V(\phi)\}^{ ext{ spin 0}}+S_m\{ ext{matter}, ilde{g}_{\mu
u}=A^2(\phi)g_{\mu
u}\}$$

$$G_{ ext{cav}} = G(1+lpha^2) \qquad lpha = \mathrm{d}\ln A/\mathrm{d}\phi$$

Motion of massive bodies determines G<sub>cav</sub>M **not** GM.

G<sub>cav</sub> is a priori space-time dependent

### **Universality classes of extensions**



# Equivalence principle and fundamental constants

[Dicke 1964,...]

## Solar system

Test of Universality of free fall



Test of local position invariance

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln \left(\frac{\nu_{\mathrm{Al}}}{\nu_{\mathrm{Hg}}}\right) = (-5.3 \pm 7.9) \times 10^{-17} \,\mathrm{yr}^{-1}$$

[Rosenband, 2008]

### **Equivalence principle and constants**



[Dicke 1964,...]

### **Equivalence principle and constants**

Forget all this, and think Newtonian.

Mass of test body = mass of its constituants + binding energy



[Dicke 1964,...]

## **Varying constants**

The new fields can make the constants become dynamical.

The constant has to be replaced by a dynamical field or by a function of a dynamical field

This has 2 consequences:

1- the equations derived with this parameter constant will be modified

one cannot just make it vary in the equations

2- the theory will provide an equation of evolution for this new parameter

The field responsible for the time variation of the « constant » is also responsible for a long-range (composition-dependent) interaction *i.e.* at the origin of the deviation from General Relativity.

In most extensions of GR (e.g. string theory), one has varying constants.

### **Example of varying fine structure constant**

It is a priori « **easy** » to design a theory with varying fundamental constants Consider

$$S = \int \{ rac{1}{16\pi G} R - 2(\partial_\mu \phi)^2 - V(\phi) - rac{1}{4} B(\phi) F_{\mu
u}^2 \} \sqrt{-g} \, d^4x$$

But that may have dramatic implications.

$$m_A(\phi) \supset 98.25 lpha rac{Z(Z-1)}{A^{1/3}} \mathrm{MeV} \quad \longrightarrow \quad f_i = \partial_\phi \ln m_i \sim 10^{-2} rac{Z(Z-1)}{A^{4/3}} lpha'(\phi)$$

Violation of UFF is quantified by

$$\eta_{12} = 2 \frac{|\vec{a}_1 - \vec{a}_2|}{|\vec{a}_1 + \vec{a}_2|} = \frac{f_{\text{ext}}|f_1 - f_2|}{1 + f_{\text{ext}}(f_1 + f_2)/2}$$

It is of the order of

$$\eta_{12} \sim 10^{-9} \underbrace{\mathrm{X}_{1,2,\mathrm{ext}}(A,Z)}_{\mathcal{O}(0.1-10)} imes \left( \partial_{\phi} \ln B 
ight)_{0}^{2}$$

Requires to be close to the minimum



### **Screening & decoupling mechanisms**

To avoid large effects, one has various options:

- *Least coupling principle*: all coupling functions have the same minimum and the theory can be attracted toward GR



$$V\!=\!\mathrm{cst}, \quad A=\exp\left(\! rac{1}{2}eta\phi^2
ight)$$

### **Screening & decoupling mechanisms**

To avoid large effects, one has various options:

- *Least coupling principle*: all coupling functions have the same minimum and the theory can be attracted toward GR [Damour, Nordtvedt & Damour, Polyakov]

- *Chameleon mechanism*: Potential and coupling functions have different minima.



$$m_{\min}^2 = V_{,\phi\phi}(\phi_{\min}) + A_{,\phi\phi}(\phi_{\min})\rho$$

The field can become massive enough to evade existing constraints.

[Khoury, Weltmann, 2004] [Ellis et al., 1989] To avoid large effects, one has various options:

- *Least coupling principle*: all coupling functions have the same minimum and the theory can be attracted toward GR [Damour, Nordtvedt & Damour, Polyakov]

- *Chameleon mechanism*: Potential and coupling functions have different minima.

[Khoury, Weltmann, 2004]

- *Symmetron mechanism*: similar to chameleon but VEV depends on the local density. [Pietroni 2005; Hinterbichler,Khoury, 2010]

$$V(\phi) = -\frac{1}{2}\mu^2 \phi^2 + \frac{1}{4}\lambda \phi^4$$
$$A(\phi) = 1 + \frac{1}{2M^2}\phi^2 + \mathcal{O}(\phi^4/M^4)$$
$$V_{\text{eff}}(\phi) = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{1}{4}\lambda \phi^4$$

Symmetry is restored at high density.

Environmental dependence

# Atomic clocks and astrophysical systems

### **Physical systems**



### **Atomic clocks**

Based the comparison of atomic clocks using different transitions and atoms

<i>e.g.</i>	hfs Cs vs fs Mg :
	hfs Cs vs hfs H:

 ${g_p \mu}$  ; ( ${g_p / g_I} lpha$ 

Examples

$$\frac{\nu_{Cs}}{\nu_{Rb}} \propto \frac{g_{Cs}}{g_{Rb}} \alpha^{0.49}$$

$$rac{
u_{Cs}}{
u_{H}} \propto g_{Cs} \mu lpha^{2.83}$$

#### High precision / redshift o (local)

Clock 1	Clock 2	Constraint $(yr^{-1})$	Constants dependence	Reference
	$rac{\mathrm{d}}{\mathrm{d}t}\ln\left(rac{ u_{\mathrm{clock}_1}}{ u_{\mathrm{clock}_2}} ight)$			
$^{87}$ Rb	$^{133}Cs$	$(0.2 \pm 7.0) \times 10^{-16}$	$\frac{g_{\rm Cs}}{q_{\rm Pb}} \alpha_{\rm EM}^{0.49}$	
$^{87}$ Rb	$^{133}Cs$	$(-0.5 \pm 5.3) \times 10^{-16}$	310	Bize (2003)
$^{1}\mathrm{H}$	$^{133}Cs$	$(-32\pm 63)\times 10^{-16}$	$g_{Cs}\mu\alpha_{EM}^{2.83}$	Fischer (2004)
$^{199}Hg^{+}$	$^{133}Cs$	$(0.2 \pm 7) \times 10^{-15}$	$g_{C_8}\mu\alpha_{C_8}^{6.05}$	Bize (2005)
$^{199}Hg^{+}$	$^{133}Cs$	$(3.7 \pm 3.9) \times 10^{-16}$	EIM	Fortier (2007)
$^{171}\mathrm{Yb^{+}}$	$^{133}Cs$	$(-1.2 \pm 4.4) \times 10^{-15}$	$g_{\rm Cs}\mu\alpha_{\rm TM}^{1.93}$	Peik (2004)
$^{171}\mathrm{Yb^{+}}$	$^{133}Cs$	$(-0.78 \pm 1.40) \times 10^{-15}$	B - T EM	Peik (2006)
$^{87}Sr$	$^{133}Cs$	$(-1.0 \pm 1.8) \times 10^{-15}$	$q_{C_{s}}\mu\alpha^{2.77}_{}$	Blatt (2008)
$^{87}$ Dy	$^{87}$ Dy		5 Car EM	Cingöz (2008)
<sup>27</sup> Al <sup>+</sup>	$^{199}\mathrm{Hg^{+}}$	$(-5.3\pm7.9)\times10^{-17}$	$\alpha_{\rm EM}^{-3.208}$	Blatt (2008)

### **Atomic clocks**

The gyromagnetic factors can be expressed in terms of  $g_p$  and  $g_n$  (shell model).

$$\frac{\delta g_{\rm Cs}}{g_{\rm Cs}} \sim -1.266 \frac{\delta g_p}{g_p} \qquad \frac{\delta g_{\rm Rb}}{g_{\rm Rb}} \sim 0.736 \frac{\delta g_p}{g_p}$$

All atomic clock constraints take the form

$$\frac{\dot{\nu}_{AB}}{\nu_{AB}} = \lambda_{g_{\rm p}} \frac{\dot{g}_{\rm p}}{g_{\rm p}} + \lambda_{\mu} \frac{\dot{\mu}}{\mu} + \lambda_{\alpha} \frac{\dot{\alpha}}{\alpha}$$

Using Al-Hg to constrain  $\alpha$ , the combination of other clocks allows to constraint  $\{\mu, g_p\}$ .

Note: one actually needs to include the effects of the polarization of the non-valence nucleons and spin-spin interaction.

[Flambaum, 0302015,...



### **Atomic clocks**

One then needs to express  $m_p$  and  $g_p$  in terms of the quark masses and  $\Lambda_{QCD}$  as

$$\begin{split} \frac{\delta g_{\rm p}}{g_{\rm p}} &= \kappa_{\rm u} \frac{\delta m_{\rm u}}{m_{\rm u}} + \kappa_{\rm d} \frac{\delta m_{\rm d}}{m_{\rm d}} + \kappa_{\rm s} \frac{\delta m_{\rm s}}{m_{\rm s}} + \kappa_{\rm QCD} \frac{\delta \Lambda_{\rm QCD}}{\Lambda_{\rm QCD}} \\ \frac{\delta m_{\rm p}}{m_{\rm p}} &= f_{T_{\rm u}} \frac{\delta m_{\rm u}}{m_{\rm u}} + f_{T_{\rm d}} \frac{\delta m_{\rm d}}{m_{\rm d}} + f_{T_{\rm s}} \frac{\delta m_{\rm s}}{m_{\rm s}} + f_{T_{\rm g}} \frac{\delta \Lambda_{\rm QCD}}{\Lambda_{\rm QCD}} \\ m_i &= h_i v \end{split}$$

Assuming unification.

 $C_{AB}$  coefficients range from 70 to 0.6 typically.

Model-dependence remains quite large.

### **Quasar absorption spectra**



### Generalities

The method was introduced by Savedoff in 1956, using Alkali doublet

Most studies are based on <u>optical techniques</u> due to the profusion of strong UV transitions that are redshifted into the optical band *e.g.* SiIV @ z>1.3, FeII $\lambda$ 1608 @ z>1

<u>Radio observations</u> are also very important

e.g. hyperfine splitting (HI21cm), molecular rotation, lambda doubling, ...

- offer high spectral resolution (<1km/s)

- higher sensitivity to variation of constants

- isotopic lines observed separately (while blending in optical observations)

Shift to be detected are small

e.g. a change of  $\alpha$  of 10<sup>-5</sup> corresponds to

- a shift of 20 mÅ (i.e. of 0.5 km/s) at  $z\sim2$ 

- <sup>1</sup>/<sub>3</sub> of a pixel at R=40000 (Keck/HIRES, VLT/UVES)

Many sources of uncertainty

- absorption lines have complex profiles (inhomogeneous cloud)
- fitted by Voigt profile (usually not unique: require lines not to be saturated)
- each component depends on z, column density, width

## **QSO** absorption spectra

#### 3 main methods: Si IV alkali doublet P<sub>3/2</sub> Alkali doublet (AD) Savedoff 1956 P<sub>1/2</sub> $\Delta\lambda/\lambda\propto \alpha^2$ Fine structure doublet, 1393.8Å ~ 1402.8Å Single atom S1/2 Rather weak limit VLT/UVES: Si IV in 15 systems, 1.6<z<3 HIRES/Keck: Si IV in 21 systems, 2 < z < 3 $\frac{\Delta \alpha}{\alpha} = (-0.5 \pm 1.3) \times 10^{-5}$ $\frac{\Delta \alpha}{\alpha} = (0.15 \pm 0.43) \times 10^{-5}$ Chand et al. 2004 Murphy et al. 2001

<u>Many multiplet (MM)</u>

Webb et al. 1999

Compares transitions from multiplet and/or atoms s-p vs d-p transitions in heavy elements Better sensitivity



<u>Single Ion Differential α Measurement (SIDAM)</u> Analog to MM but with a single atom / FeII

Levshakov et al. 1999

# **QSO: many multiplets**



HIRES-Keck, 143 systems, *0.2*<*z*<*4.2* 

 $\frac{\Delta \alpha}{\alpha} = (-0.57 \pm 0.11) \times 10^{-5}$ Murphy et al. 2004



## **QSO: VLT/UVES analysis**

Selection of the absorption spectra:

- lines with similar ionization potentials
  - most likely to originate from similar regions in the cloud
- avoid lines contaminated by atmospheric lines
- at least one anchor line is not saturated
  - redshift measurement is robust
- reject strongly saturated systems

Only 23 systems

lower statistics / better controlled systematics R>44000, S/N per pixel between 50 & 80

VLT/UVES

 $\Delta \alpha_{\rm EM} / \alpha_{\rm EM} = (0.01 \pm 0.15) \times 10^{-5}$ 

Srianand et al. 2007

#### **DOES NOT CONFIRM HIRES/Keck DETECTION**

### To vary or not to vary



<u>Claim</u>: Dipole in the fine structure constant [« Australian dipole »]

Indeed, this is a logical possibility to reconcile VLT constraints and Keck claims of a variation.

### Can it make sense?

- Dipole is not aligned with the CMB dipole
- With such a dipole, CMB fluctuations must be modulated

[Moss et al., 2010]

$$D_{\ell m}^{(0)} \equiv \langle a_{\ell m}^{\text{obs}} a_{\ell+1m}^{\text{obs}*} \rangle = \varepsilon_0 \sqrt{\frac{3}{4\pi}} \frac{\sqrt{(\ell+1)^2 - m^2}}{\sqrt{(2\ell+1)(2\ell+3)}} (C_\ell + C_{\ell+1})$$
$$D_{\ell m}^{(1)} \equiv \langle a_{\ell m}^{\text{obs}} a_{\ell+1m+1}^{\text{obs}*} \rangle = \sqrt{\frac{3}{4\pi}} \sqrt{\frac{(\ell+2+m)(\ell+1+m)}{(2\ell+1)(2\ell+3)}} [C_\ell + C_{\ell+1}] \frac{\varepsilon^*}{\sqrt{2}}$$
[Prunet, JPU, Bernardeau, Brunier, 2005]

- Theoretically: in all existing models, time variation is larger than spatial variation

Is it possible to design a model compatible with such a claim?

## Wall of fundamental constant

[Olive, Peloso, JPU, 2010]

**Idea:** Spatial discontinuity in the fundamental constant due to a domain wall crossing our Hubble volume.



### Wall of fundamental constants

$$S = \int \left[ \frac{1}{2} M_p^2 R - \frac{1}{2} (\partial_\mu \phi)^2 + V(\phi) + \frac{1}{4} B_F(\phi) F_{\mu\nu}^2 - \sum_j i \bar{\psi}_j \not{D} \psi_j - B_j(\phi) m_j \bar{\psi}_j \psi_j \right] \sqrt{-g} d^4 x,$$
$$B_i(\phi) = \exp\left(\xi_i \frac{\phi}{M_*}\right) \simeq 1 + \xi_i \frac{\phi}{M_*}$$
$$V(\phi) = \frac{1}{4} \lambda (\phi^2 - \eta^2)^2$$

-Parameters  $(\lambda, M_*, \eta, \xi_F, \xi_i)$ 

- We assume only  $\xi_F$  is non-vanishing BUT the scalar field couples radiatively to nucleons  $\xi_N = m_N^{-1} \langle N | (\xi_F/4) F_{\mu\nu}^2 | N \rangle$ 

$$\xi_p = -0.0007\xi_F \quad \xi_n = 0.00015\xi_F$$
$$V_{\text{eff}} = V(\phi) + \xi_N \frac{\phi}{M_*} \rho_{\text{baryon}}$$

### Constraints

-Constraints from atomic clocks / Oklo / Meteorite dating are trivially satisfied

- To reproduce the «observations»

$$\frac{\Delta \alpha}{\alpha} \simeq 2\xi_F \frac{\eta}{M_*} \sim \text{few} \times 10^{-6}.$$

- The contribution of the walls to the background energy is

$$\Omega_{\text{wall}} = \frac{U_{\text{wall}} H_0}{\rho_0} \simeq \left(\frac{\eta}{100 \text{ MeV}}\right)^3, \qquad \text{Assume } \eta = \mathcal{O} \text{ (MeV)},$$

- CMB constraints 
$$\left(\frac{\delta T}{T}\right)_{\text{CMB}} \sim 10^{-6} \left(\frac{\eta}{1 \text{ MeV}}\right)^3$$

-Valid field theory up to an energy scale  $M_*/\xi_F \sim 10^6 \text{ MeV}$ 

- Astrophysical constraints
- Tunelling to the true vacuum
- Walls form at a redshift of order 8x109

### **Physical systems: new and future**



# **Conclusions**

The constancy of fundamental constants is a **test of the equivalence principle**. The variation of the constants, violation of the universality of free fall and other deviations from GR are of the same order.

« Dynamical constants » are **generic** in most extensions of GR (extra-dimensions, string inspired model.

Need for a stabilisation mechanism (least coupling principle/chameleon) Why are the constants so constant? Variations are expected to be larger in the past (cosmology) All constants are expected to vary (unification)

Observational developments allow to set **strong constraints** on their variation *New systems [Stellar physics] / new observations* 

They offer tests of GR independently of LSS and may offer signature in regimes where LSS cannot

e.g. can set constraints of time variation of a scalar field even if it does not dominate the matter content of the universe. [see/listen e.g. C. Matrins' talk]

Question concerning their actual values and a possible fine-tuning.

