Ghost Bounce
鬼跳

A matter bounce by means of ghost condensation

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arXiv:1007.2654

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Outline

I. Alternative inflation models
   - Necessity
   - Matter bounce

II. Ghost condensation
   - Basic philosophy
   - Applications
   - Interesting features
   - Instability

III. Matter bounce by means of ghost condensation
   - Several advantages:
     ghost free, stable against radiation and anisotropic stress…
   - Perturbation
   - Cut off issue

Part I

Alternative inflation models

Look beyond…

- Inflation suffers from some conceptual problems

Flatness problem

Amplitude problem $\frac{V(\varphi)}{\Delta\varphi^4} \leq 10^{-12}$

Trans-Planckian problem

Singularity problem

……

- Some other attempts

Matter bounce, Ekpyrotic,

String gas, pre big bang

theory……

Matter Bounce

- Contracting universe before big bang
- Cold pressureless matter
- Scale invariant spectrum

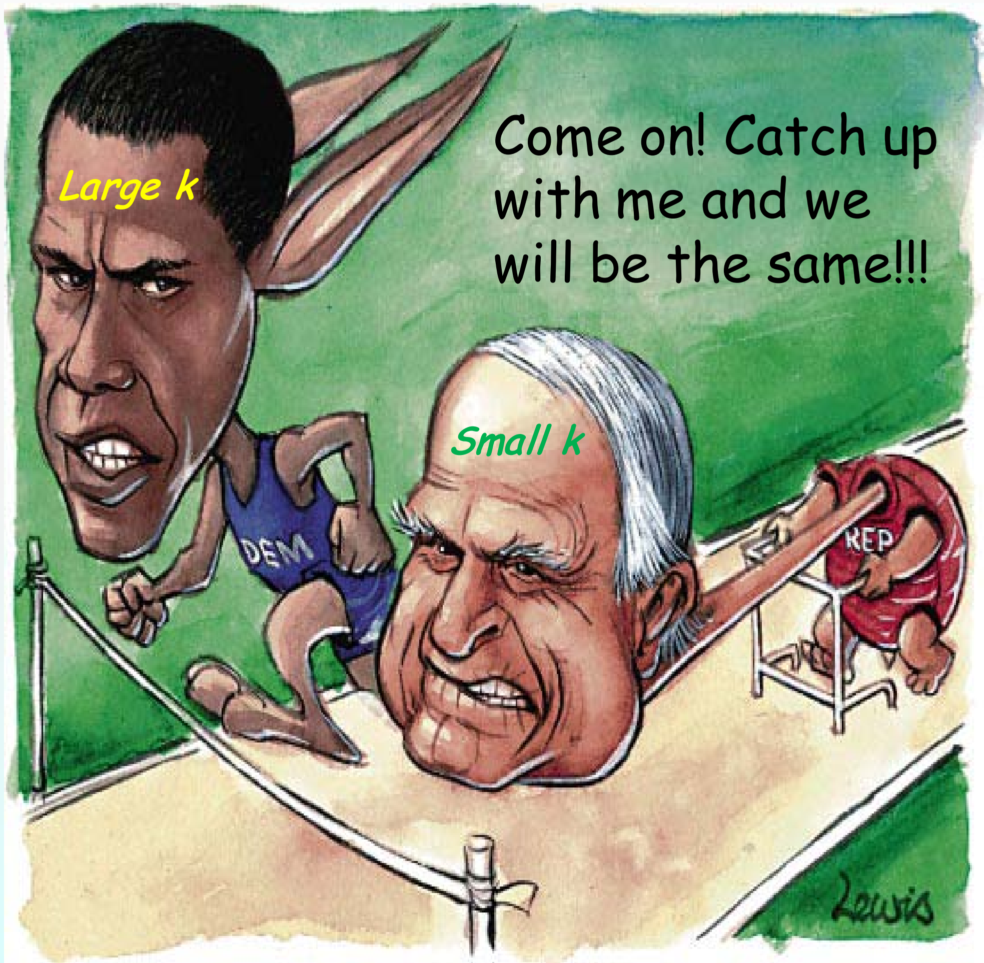
Horizon crossing

$$\delta\varphi_* \propto H_* \propto t_*^{-1}$$

super horizon growing

$$\varsigma(t) \propto t^{-1}$$

Amplitude of the larger scale perturbation mode will catch up with the smaller scale perturbation mode.



How to realize it?

- Modifying gravity

  - non-singular Universe

    R. H. Brandenberger, V. F. Mukhanov and A. Sornborger, gr-qc/9303001

  - higher derivative gravity action

    T. Biswas, A. Mazumdar and W. Siegel hep-th/0508194

  - mirage cosmology

    R. Brandenberger, H. Firouzjahi and O. Saremi, arXiv:0707.4181

  - Horava-Lifshitz gravity

    P. Horava, arXiv:0904.2835

    ⋮

- Modifying matter

  quintom bounce, Lee-wick bounce……

  - Ghost instability

    J.Cline, S.Jeon and G. Moore, hep-ph/0311312

  - Bounce may be unstable

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  - Anisotropic stress

    scales as a^{-6}

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Ghost condensation

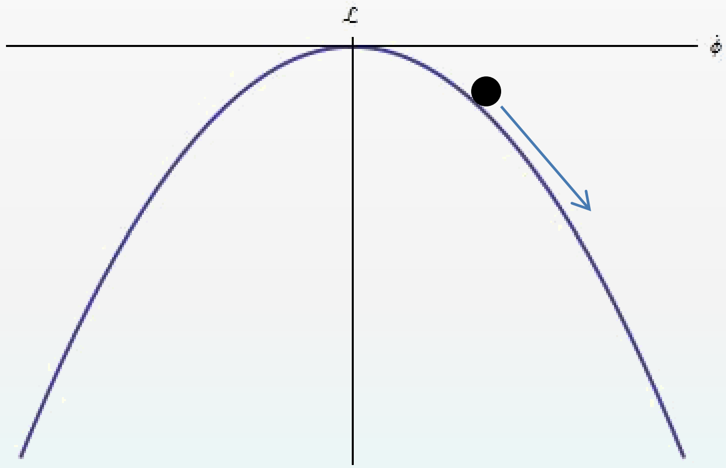
Part II

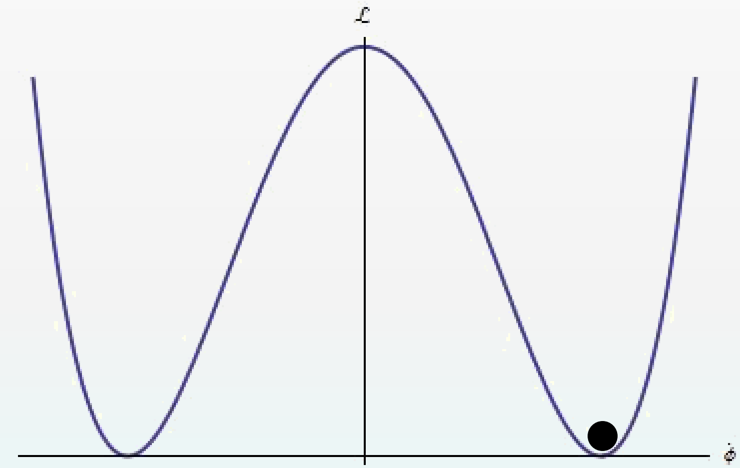
Ghost Condensation Theory

Ghost Condensation

$$L = -\frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi + \ldots$$

$$P = \frac{1}{8}(X - c^2)^2, \quad X = \partial^{\mu}\phi\partial_{\mu}\phi$$





It is similar as tachyon condensation

$$V_{tachyon} = -\frac{1}{2}m^2\phi^2 + \lambda\phi^4 + \ldots$$

Hep-th/0312099
Shinji, Nima, et.al…

Higher order potential (kinetic) terms stablize vacuum.

Applications

$$T_{\mu\nu} = -M^4 P(X) g_{\mu\nu} + 2M^4 P'(X) \partial_\mu \phi \partial_\nu \phi$$ where $P' \equiv \frac{\partial P}{\partial X}$

$P'(X) = 0, \quad \mathrm{P}(X) \neq 0$

$T_{\mu\nu} \to -g_{\mu\nu} M^4 P(c_*^2),$

$w = -1$

Inflation, Dark Energy

$P'(X) \neq 0, \quad \mathrm{P}(X) = 0$

$T_{00} \sim M^4 P' \sim a^{-3}, \quad T_{ij} = 0$

$w = 0$

Dark matter

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Dark matter

Matter Bounce

Interesting features

More generally,

$$\mathcal{L} = M^4 P(X) + M^2 S_1(X)(\Box\phi)^2 + M^2 S_2(X)\partial^\mu\partial^\nu\phi\partial_\mu\partial_\nu\phi + \cdots .$$

Ghost field locate at the minima, with scalar excitation

$$\phi = c\,t + \pi$$

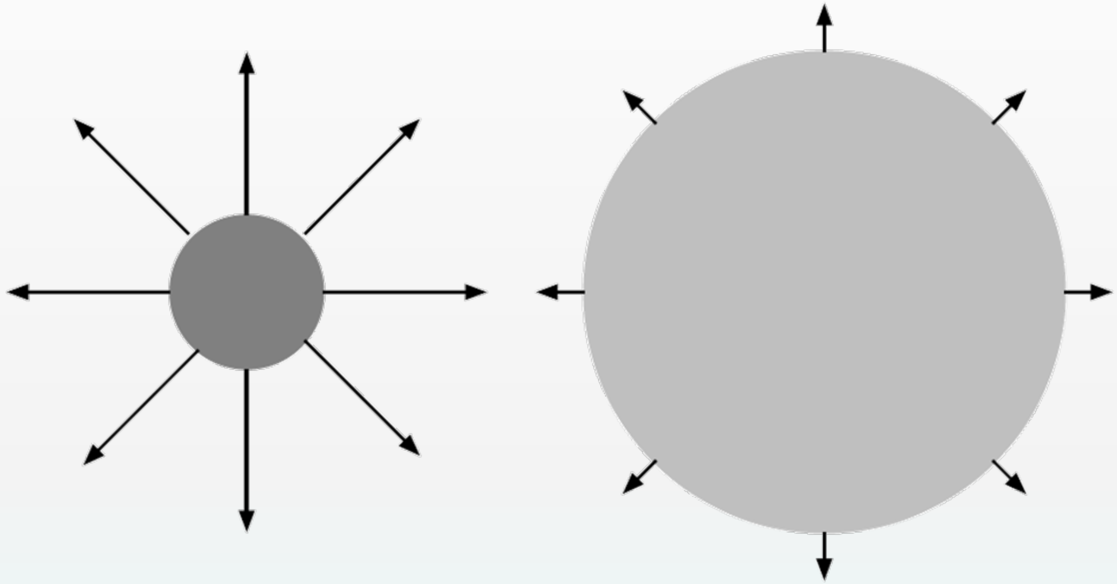
Low energy effective action for π is

$$S \sim \int d^4x \left[\frac{1}{2}\dot{\pi}^2 - \frac{1}{2M^2}(\nabla^2\pi)^2 + \cdots\right],$$

The dispersion relation $\omega^2 \sim \frac{k^4}{M^2}.$

Group velocity $v^2 \sim k^2/M^2,$

Interesting features



- Small lumps expand faster than larger lumps since $\omega^2 \sim \frac{k^4}{M^2}$.
- Small lumps also move faster than larger lumps since $v^2 \sim k^2/M^2$,

Lorentz invariance

Interesting Features

~~Equivalence principle~~

- "Particle physics" energy density

$$\mathcal{E}_{\rm pp} = \int d^3x\, T_{00} - c_* Q \sim \frac{1}{2}\dot{\pi}^2 + \frac{(\nabla^2\pi)^2}{2M^2} + \cdots$$

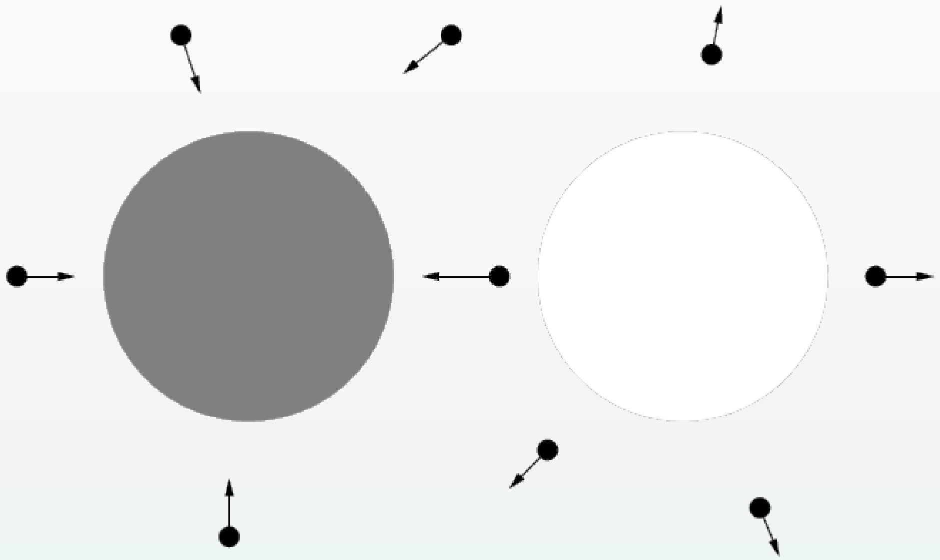
Inertial Mass!

- Gravitational energy density

$$\mathcal{E}_{\rm grav} = T_{00} \sim M^2 \dot{\pi} + \cdots$$

Gravitational Mass!

Interesting features



Lumps come from scalar excitation, its energy density always positive in terms of "particle physics", but the induced gravity can be either attractive or repulsive!

$\dot{\pi} > 0$	attractive
$\dot{\pi} < 0$	repulsive

Jeans instability

- For a fluid with pressure p and energy density ρ,

$$\omega^2 = \frac{\delta p}{\delta \rho} k^2 - \omega_J{}^2, \quad \text{where} \quad \omega_J{}^2 = \frac{\rho}{2M_{Pl}{}^2}.$$

When $\omega^2 < 0$, Jeans collapse happens.

$$L_{\rm J} \sim \frac{M_{\rm Pl}}{M^2}, \qquad T_{\rm J} \sim \frac{M_{\rm Pl}^2}{M^3}$$

In linear regime, fluctuation with wavelength $\lambda \gtrsim L_{\rm J}$ grows on a time scale $\tau \sim T_{\rm J} \frac{\lambda}{L_{\rm J}}$,

So we need a very small M to protect the IR gravity.

e.g. $M \sim 10^{-3}{\rm eV}$ Ghost condensation plays the role of DE.

Jeans instability

- So we need a very small M to protect the IR gravity.

e.g. $M \sim 10^{-3}\mathrm{eV}$ Ghost condensation plays the role of DE.
The gravity is modified at length scale $r_J \sim H_0^{-1}$

But we need to wait $\tau >> H_0^{-1}$ to see this modification!

- An upper bound of M has been given in hep-ph/0507120,

(N.Arkani-Hamed, H.Cheng, M. Luty, S.Mukohyama and T.Wiseman)

Light lensing + Supernova time delay

$$M < 100 Gev$$

Our ghost bounce model is OK…

Gradient stability

- Up to 2nd order,

$$\mathcal{L} = M^4 \left[(P' + 2P''c^2)\dot{\pi}^2 - P'(\nabla\pi)^2 \right] + M^2(S_1 + S_2)(\nabla^2\pi)^2$$

the relevant dispersion relation

$$(P' + 2P''c^2)\omega^2 = -P'k^2 + \frac{\tilde{M}^2}{M^4}k^4 \quad \text{where} \quad \tilde{M}^2 = M^2(S_1 + S_2)$$

Ghost condensation locates at the minima of Lagrangian

$$P' = 0$$

$P' + 2P''\dot{\phi}^2 > 0$ is ghost free condition, so we get

$$\omega_{grad}^2 > 0$$

Ghost condensation stablize vacuum on background and perturbation level !

Part III

Realization of Matter Bounce

Ghost bounce

- Matter sector + ghost condensation

$$\rho_m(t) \sim a(t)^{-3(1+w_m)} \qquad \rho_X \sim a(t)^{-p}$$

minimal requirement $p > 3$

against radiation $p > 4$

against anisotropic stress $p > 6$

- Lagrangian of GC takes the following general form

$$\mathcal{L} = M^4 P(X) - V(\phi)$$

$P(X)$ takes the prototypical form

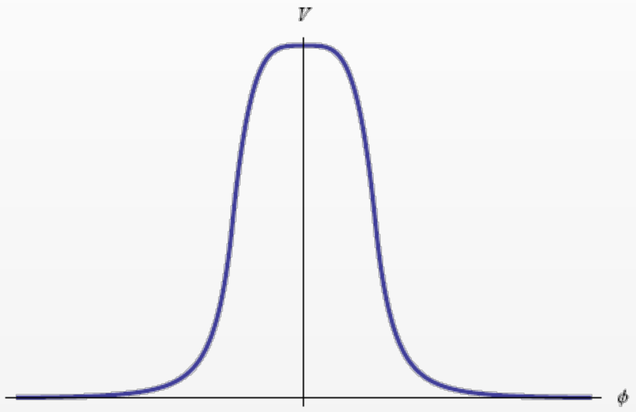
$$P(X) = \frac{1}{8}(X - c^2)^2$$

Ghost bounce

- Ansatz for potential

$$V(\phi) = V_0 M^{-\alpha} \phi^{-\alpha}$$

Divergence is cut off at M^4



- Ghost field changes as

$$\phi(t) = ct + \pi(t)$$

$\pi(t)$ is the small deviation from minima, its EoM

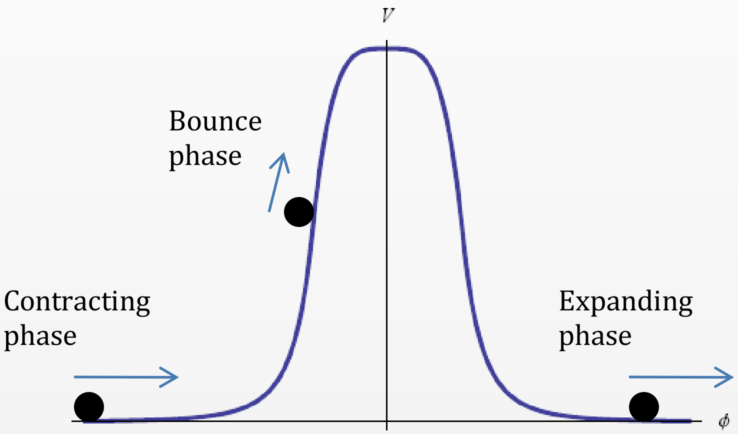
$$\ddot{\pi} + 3H\dot{\pi} = 2c^{-2} V_0 M^{-4-\alpha} \alpha (ct)^{-(\alpha+1)}$$

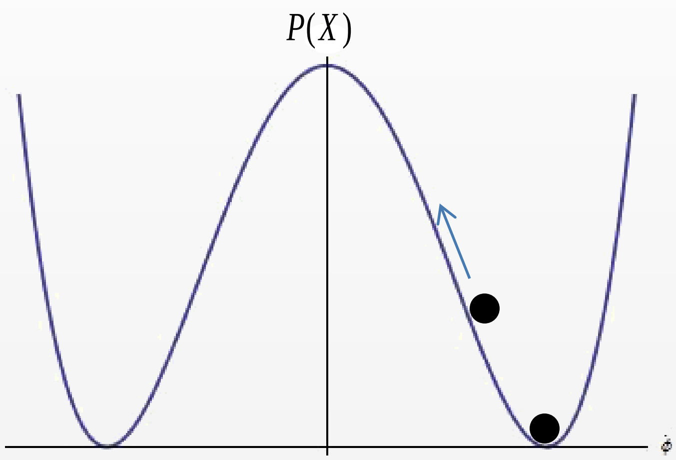
It yields $\rho_X \sim \dot{\pi} \sim t^{-\alpha}$.

$\alpha = 4$ Marginally stable against anisotropic stress

$\alpha = 6$ stable

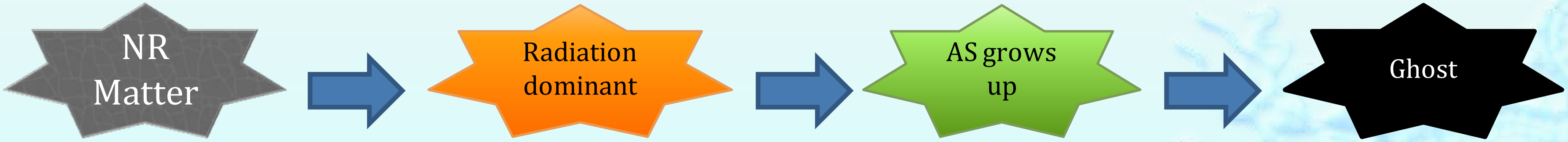
Ghost bounce



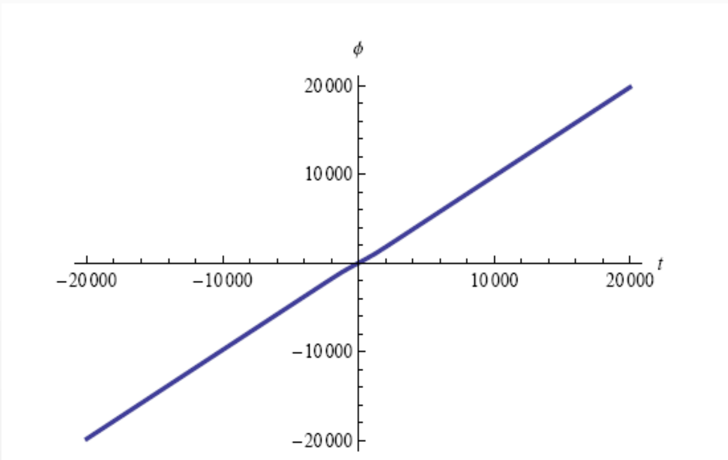


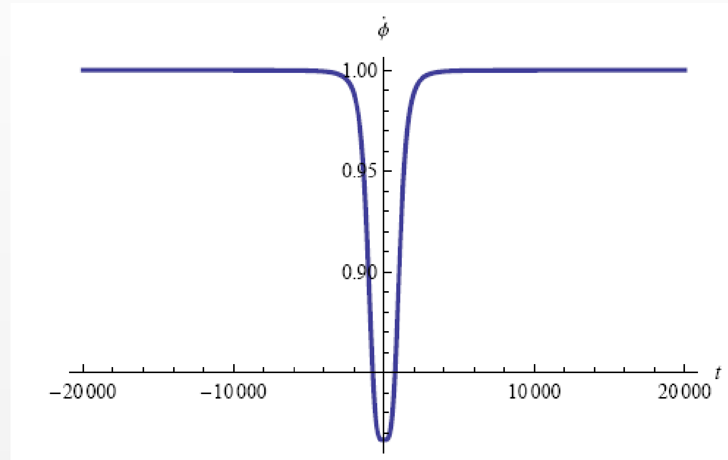
$$2M_p^2\dot{H} = -2M^4 X P' - (1+w_m)\rho_m$$

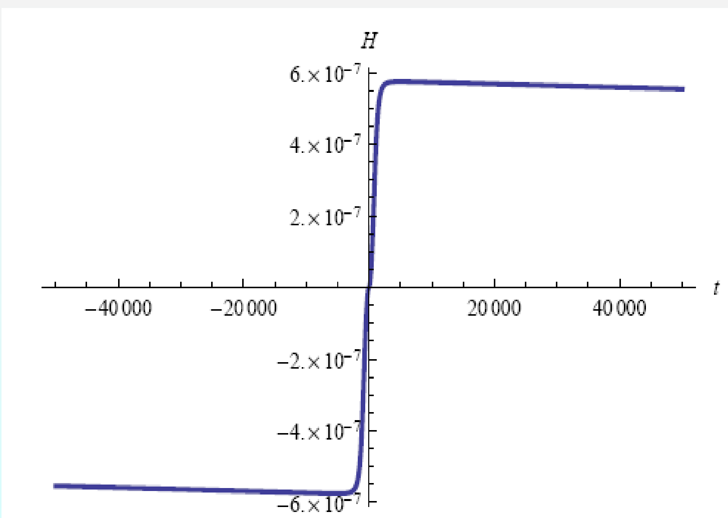
Realization of NEC violation!



Numerical results







Initial condition of numerical calculation

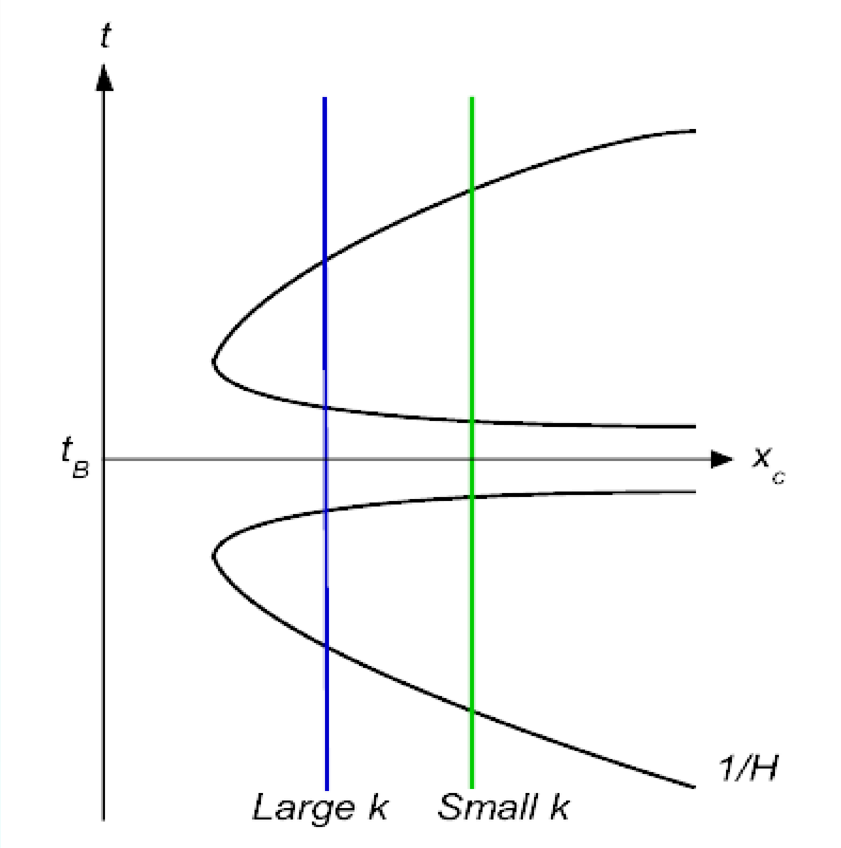
$$\phi(0) = 0 \text{ and } \dot{\phi}(0) = \sqrt{2/3}$$

$$M = 2 \times 10^{-3}$$

$$\rho_m(0) = 10^{-12}$$

$$\alpha = 4$$

Matter perturbation



The metric in Newtonian gauge

$$ds^2 = -(1+2\Phi)dt^2 + a(t)^2(1-2\Psi)d\mathrm{x}^2$$

$$\varphi(\eta,\mathrm{x}) = \varphi_0(\eta) + \delta\varphi(\eta,\mathrm{x})$$

Introduce M-S Variable

$$v = a\left[\delta\varphi + \frac{z}{a}\Phi\right]$$

$$S^{(2)} = \frac{1}{2}\int d^4x\left[v'^2 - v_{,i}v_{,i} + \frac{z''}{z}v^2\right]$$

EoM:

$$v_k'' + k^2 v_k - \frac{z''}{z}v_k = 0$$

In matter contracting phase $z \sim a$

$$v(t) \sim t^{-1/3}$$

Matter perturbation

- M-S variable corresponds to

$$\zeta = a^{-1} v \sim t^{-1}$$

So the relation between late time ζ and ζ at horizon crossing

$$\zeta(k,t) \sim \zeta(k,t_H) \cdot \frac{t_H}{t} \sim a(k,t_H)^{-1} v(k,t_H) \cdot \frac{t_H}{t}$$

where $v(k,t_H) \sim k^{-1/2}$, $t_H(k) = k^{-1} a(t_H(k))$

So we get

$$\zeta \sim t_H(k)^{1/3} v(k,t_H) \sim k^{-3/2}$$

$$P_\zeta(k,t) \sim k^3 |\zeta(k,t)|^2 \sim k^0$$

Scale invariant spectrum!

Ghost perturbation

- We need to prove
  a) In matter contracting phase, Ghost perturbation does NOT grow faster than matter perturbation;
  b) The spectrum of ghost perturbation can NOT be red;
  c) There is NO large amplification of ghost perturbation around bounce phase;
- Focus on matter dominant contracting background

$$M_{pl}^2 G_{\mu\nu} = T_{\mu\nu}^{(\phi)} + T_{\mu\nu}$$

Linear decompose Newtonian potential

$$\Phi = \Phi_m + \Phi_g$$

Ghost perturbation

The Eom of Φ is [hep-th/0607181, S.Mukohyama]

$$\partial_t^2\Phi_g + 3H\partial_t\Phi_g + (2H^2+\dot{H})\Phi_g + \frac{\alpha}{M^2}\left(\frac{\mathbf{k}^2}{a^2}\right)^2\Phi_g - \frac{\alpha M^2}{2M_{pl}^2}\frac{\mathbf{k}^2}{a^2}\Phi_g = \frac{\alpha}{2}\frac{M^2}{M_{pl}^2}\frac{\mathbf{k}^2}{a^2}\Phi_m$$

Define a new variable $\tilde{\Phi} = a(t)\Phi$,

$$\tilde{\Phi}_g'' + \frac{81\alpha}{M^2}\frac{\mathbf{k}^4 t_0^4}{\tau^4}\tilde{\Phi}_g - \frac{\alpha M^2}{2M_{pl}^2}\mathbf{k}^2\tilde{\Phi}_g = \frac{\alpha}{2}\frac{M^2}{M_{pl}^2}\mathbf{k}^2\tilde{\Phi}_m$$

The solution of the above EoM

$$\tilde{\Phi}_g^p(\tau) \simeq k^2\gamma\tau\left(\frac{D}{3}\tau^3 - \frac{S}{2}\tau^{-2}\right)$$

Compare $\tilde{\Phi}_m = a(t)\Phi_m = D\tau^2 + \frac{S}{\tau^3}$

It is blue, and grows slower.

Ghost perturbation

- Since the duration of bounce phase is short

$$H = \theta \cdot (t - t_B)$$

Where $\theta \gg H_c^2$ we interested in large scale perturbation

$$\partial_t^2 \Phi_g + \theta \Phi_g = 0$$

the solution is

$$\Phi_g = d_1 e^{i\sqrt{\theta}t} + d_2 e^{-i\sqrt{\theta}t}$$

Since the bounce phase is very short

$$\Phi_g(t) \simeq \Phi_g^c$$

This result is still true near bounce point since $k^2 \ll \theta$ is still true.

Stability during bounce

the dispersion relation

$$\omega^2 = \frac{-(\tilde{M}^2 M^4 + 4M_{pl}^4 \dot{H})k^2 + 2M_{pl}^2 \tilde{M}^2 k^4}{2M_{pl}^2 M^4}$$

the typical instability rate

$$\omega_c = \frac{1}{4}\frac{\tilde{M} M^2}{M_{pl}^2} + \dot{H}\frac{M_{pl}^2}{M^2 \tilde{M}}$$

Its growing rate during bounce phase

$$\Delta t \omega_c \sim (\frac{V_0}{M^4})^{1/\alpha}[\frac{1}{4}\frac{\tilde{M} M}{M_{pl}^2} + \dot{H}\frac{M_{pl}^2}{M^3 \tilde{M}}]$$

Since $\dot{H} \sim \frac{M^4 \dot{\pi}}{M_{pl}^2}$ if $V_0 \ll M^4$ We get

$$\Delta t \omega_c \ll 1$$

Cut-off issue

- For a fluid with pressure p and energy density ρ,

$$\omega^2 = \frac{\delta p}{\delta \rho} k^2 - \omega_J{}^2, \quad \text{where} \quad \omega_J{}^2 = \frac{\rho}{2M_{Pl}{}^2}.$$

When $\omega^2 < 0$, Jeans collapse happens.

$$L_{\rm J} \sim \frac{M_{\rm Pl}}{M^2}, \qquad T_{\rm J} \sim \frac{M_{\rm Pl}^2}{M^3}$$

So we need a very small M to protect the IR gravity.

A constraint condition has been given in hep-ph/0507120, where

$$M < 100Gev$$

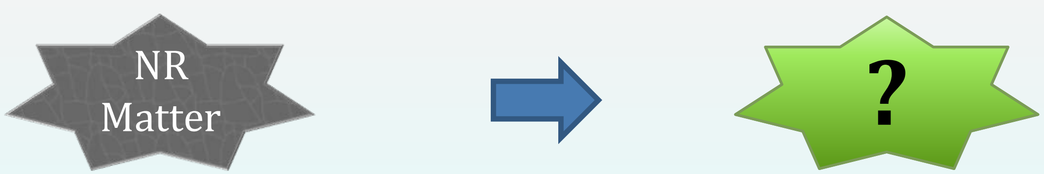
Cut-off issue

- However, in matter bounce

$$\zeta_m \sim H \sim 10^{-5}$$

Much greater than 100Gev!

- The problem can be solved by assuming…



$$v'' + \left(k^2 - \frac{p\,(2p-1)}{(p-1)^2}\frac{1}{\tau^2}\right) v = 0$$

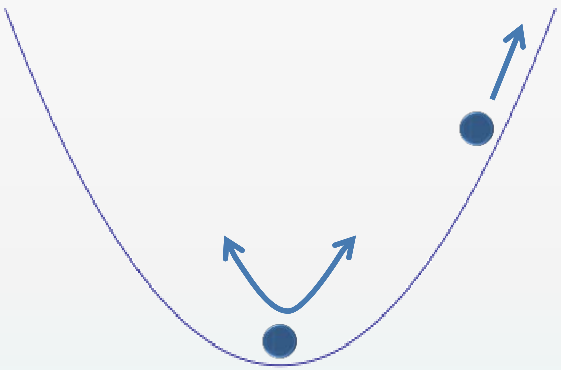
$$\zeta_m = a^{-1} v \sim t^0 \quad or \quad \sim t^{1-3p}\,,$$

We need $\zeta_m > H$

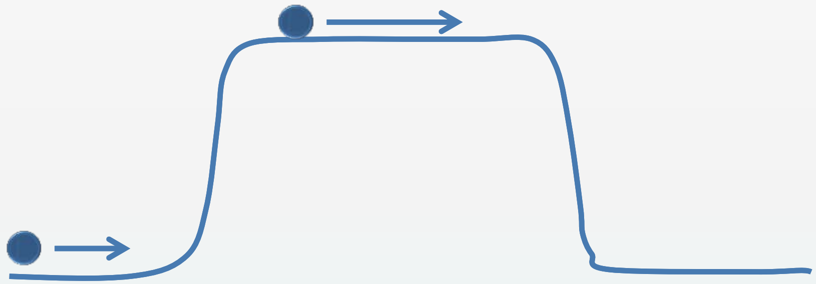
$-1 < w < 0$

Cut-off issue

- E.g. w=-1 Deflation $\zeta_m \sim \tau^3 \sim a^{-3}$



Matter scalar field $m^2\phi^2$



Ghost field

Assume matter contracting phase end up with H~100Gev~10^{-17}

We need e-folding $\frac{1}{3}\log\frac{10^{-17}}{10^{-5}} \simeq -10$

Conclusion & Discussion

- Ghost condensation theory
  - Stablize vacuum
  - Interesting feature
  - Jeans instability → low energy scale 100Gev
- We realize matter bounce by means of ghost condensation

Advantages:

1. No ghost;
2. Background is stable against radiation and anisotropic stress;

Preserve scale invariant spectrum:

1. Grows slower than matter perturbation;
2. Blue spectrum;
3. No large amplification during bounce phase;
4. The gradient instability during bounce phase has no enough time to develop.

Thank you!

ONE WORLD, ONE GHOST!