Ghost of Massive graviton in de Sitter space

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Introduction

Motivation

- Current accelerated expansion of the universe
- Modification of cosmological model
  - Spin 0 sector

Cosmological constant,
quintessence, $f\,(R)$-gravity …

~Einstein + scalar field

Theoretically more challenging will be seeking for other possibilities.

  - Spin 2 sector

    Self-accelerating universe in DGP model

Modification of gravity theory

→ modification of spin-2 sector

Low energy effective theory

Massive gravity theory + interaction term

- Massive gravity theory **(**Fierz, Pauli, Proc. Roy. Soc., 173A,211 (1939))

To explain the accelerated expansion of the universe, $m^2 \approx H^2$.
However,
a spin-2 graviton with mass in the range $m^2 < 2H^2$
in the de Sitter background has the ghost excitation
in its helicity-0 component. (Higuchi Nucl. Phy. **B282**, 397 1986)

Normal → S = ⌜⌝ d^4x þ □ þ

Ghost → S = à ⌜⌝ d^4x □

Metric ; (-,+,+,+)

Dvali-Gabadadze-Porrati model

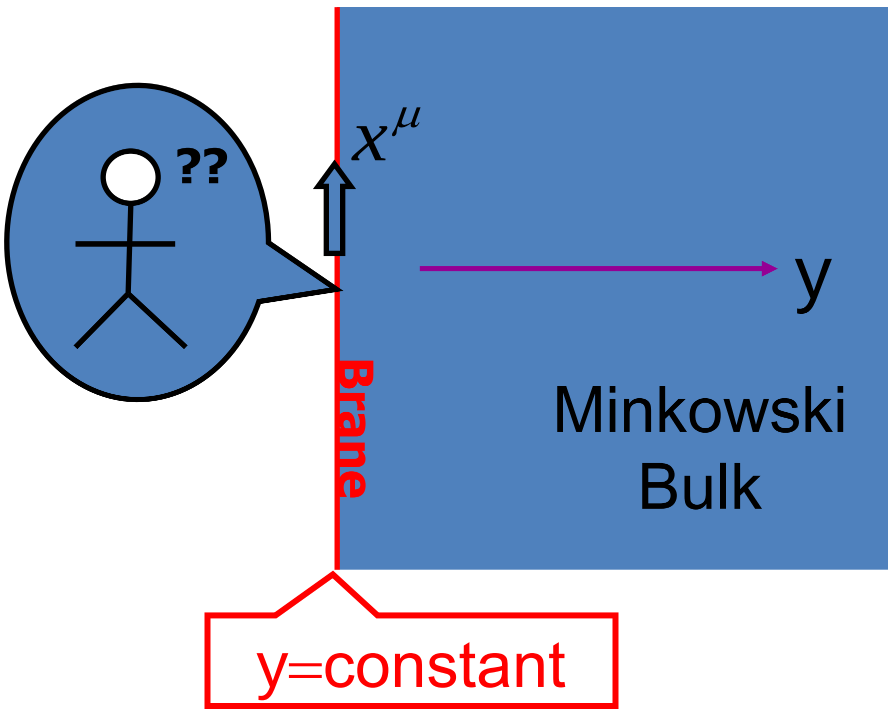
(Phys. Lett. **B485**, 208 (2000))

- model action

$$S = M_5^3 \int d^5x \sqrt{g}R + \int d^4x \sqrt{g^{(4)}} \left(M_4^2 R^{(4)} + L_{matt} \right)$$

$$M_5^3 = M_4^2 / 2r_c$$

Critical length scale



- For $r<r_c$, 4-D induced gravity term dominates?
- Extension is infinite, but 4-D GR seems to be recovered for $r < r_c$.

Cosmology in DGP model

- Flat Friedmann equation

$$\frac{1}{3M_4^2}\rho = H^2 - \varepsilon r_c H$$

- In early universe $H \gg r_c^{-1}$, cosmic expansion is normal.

- Late time behavior for $\varepsilon = +1$

$$H \to r_c^{-1} \text{ in the limit } \rho \to 0$$

self-acceleration

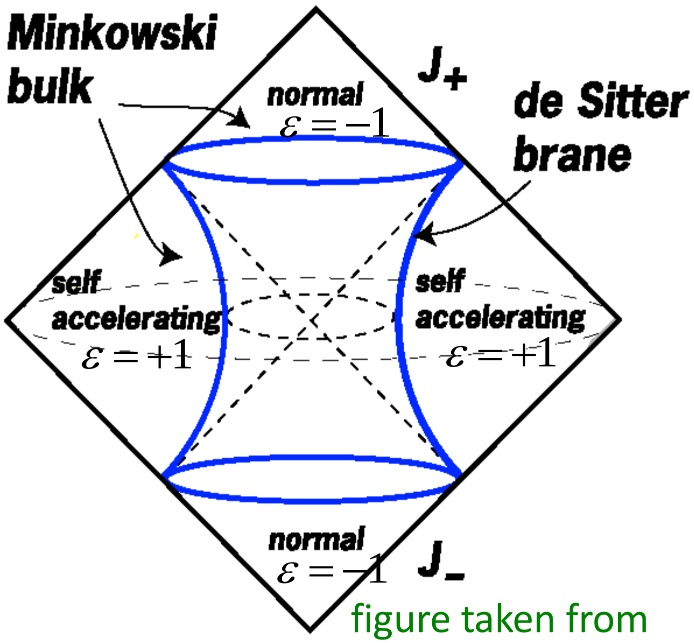


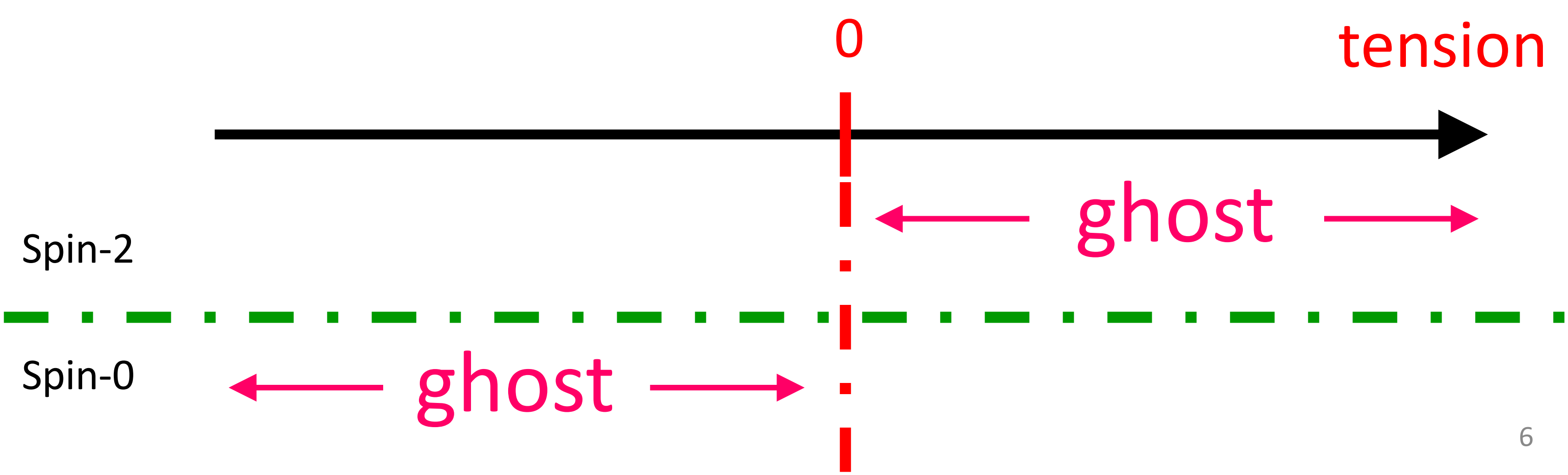
figure taken from Chamousis et.al. hep/th/0604086

Ghost
in self accelerating solution

(Koyama, Phys. Rev. **D72**, 123511 (2005))
(Gorbunov, Koyama and Sibiryakov, Phys. Rev. **D73,** 044016 (2006))

For positive tension, the lowest mass of KK graviton satisfies $m^2 < 2H^2$.
A helicity-0 excitation of graviton is a ghost.
For negative tension, a spin-0 mode is a ghost
which is a brane bending mode.

For tensionless, the lowest mass of the KK gravitons conforms to $m^2 = 2H^2$
Though this is marginal case, there is ghost excitation.



1, Introduction

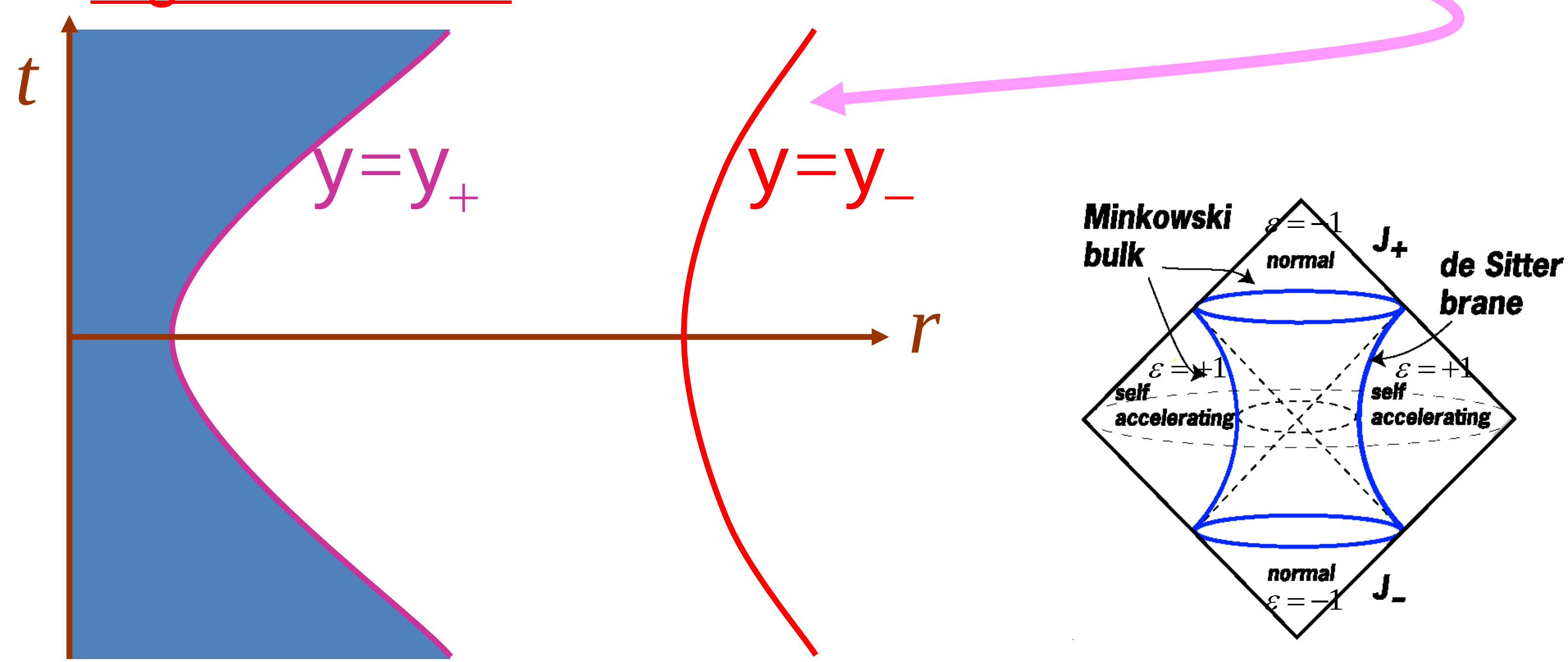
2, Modification of DGP braneworld
twobranes model
stabilization of brane

3, Is ghost really harmful??

4, Summary

Can we erase the ghost?

- Can we erase the ghost simply by putting the second regulator brane in the bulk?

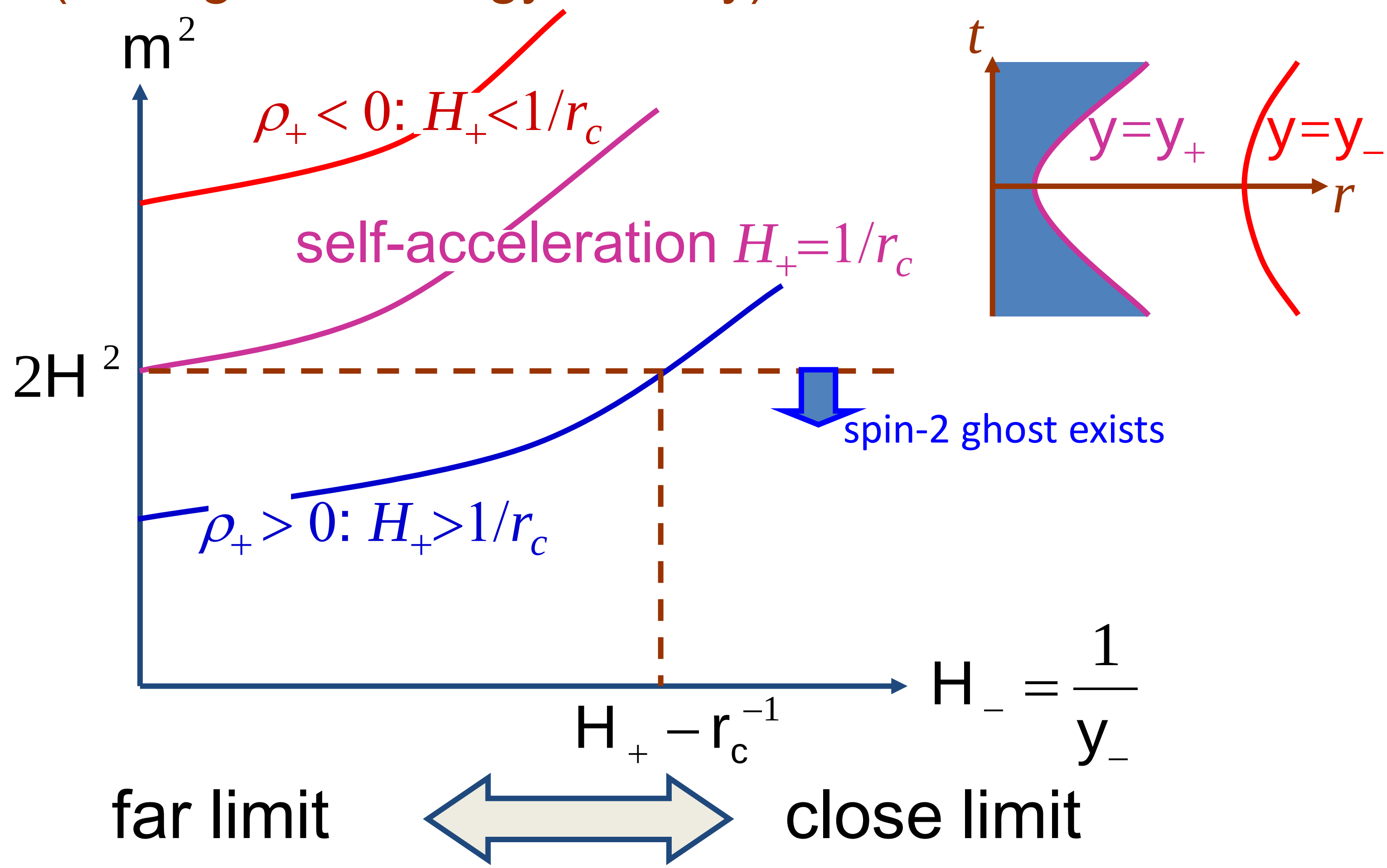


- The idea is: if the distance between two branes becomes closer, the KK mass will increase.

$$\Rightarrow \quad m^2 > 2H^2$$

  - The ghost will disappear.

- In fact, there is no ghost in spin-2 sector once the second brane (or negative energy density) is introduced.

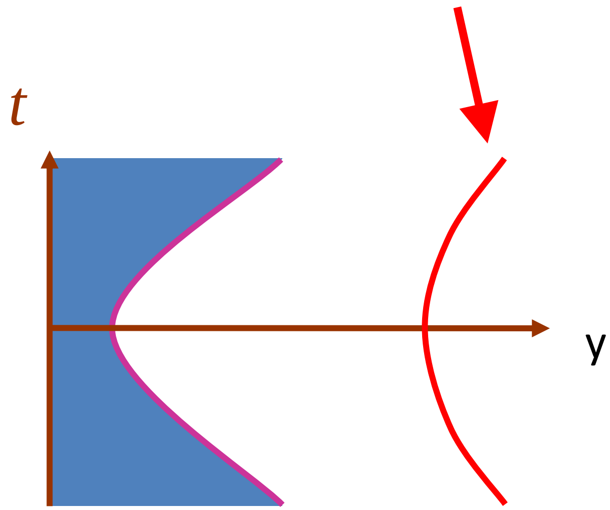


- However, at the point where the spin 2 ghost disappears, spin 0 (brane bending mode) ghost appears instead.

Two-branes model with stabilization

(K.I., K. Koyama and T. Tanaka 2007)

We put another brane in the bulk in order to make the KK mass increase.



In order to stabilize this radion mode,
we introduce the bulk scalar field.

Stabilization of brane distance
by a bulk scalar field .
(Goldberger-Wise mechanism)

When spin-2 mode has no ghost excitation,
spin-0 mode, which is derived from bulk scalar field, becomes ghost.

$m^2 < -4H^2$

Ghost cannot be removed in this mechanism.

- But why these two issues of different types of ghost can couple with each other?

- Strange, strange, strange….

- Usually, spin 2 mode and spin 0 mode are completely decoupled.
- However,

$$Y \quad \text{:scalar}$$

$$\left(\nabla_\mu \nabla_\nu - \frac{1}{4}\gamma_{\mu\nu}\Box^{(4)}\right)Y \quad \text{:traceless tensor}$$

$$\nabla^\mu\left(\nabla_\mu \nabla_\nu - \frac{1}{4}\gamma_{\mu\nu}\Box^{(4)}\right)Y = \frac{3}{4}\nabla^\mu\left(\Box^{(4)} + 4H^2\right)Y$$

$$\Box^{(4)} + 4H^2 = 0 \text{ , i.e., } m^2 + 4H^2 = 0 \text{ is quite special}$$

- Namely, when $m^2 = -4H^2$, we can construct a Transverse-Traceless tensor from a scalar function.
  (spin 2) (spin 0)

- And

$$\left(\Box^{(4)} - 4H^2\right)\left(\nabla_\mu \nabla_\nu - \frac{1}{4}\gamma_{\mu\nu}\Box^{(4)}\right)Y = \left(\nabla_\mu \nabla_\nu - \frac{1}{4}\gamma_{\mu\nu}\Box^{(4)}\right)\left(\Box^{(4)} - \left(-4H^2\right)\right)Y$$

$= m_{s=2}^2 + 2H^2$

$= m_{s=0}^2$

$m_{s=2}^2 = 2H^2$ ⟺ $m_{s=0}^2 = -4H^2$

"the mass at which spin 0 – spin 2 coupling occurs"
= "the critical mass for spin 2 ghost"

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Is ghost harmful?

Spontaneous pair production of ghost and usual particles

➡ Vacuum becomes unstable.

In flat background, since, due to the Lorentz symmetry,
the coupling is the same value in any flame
instantaneously the particle production is infinity.

Even if theory has cutoff scale,
cutoff is 4D one, not 3D one.

However, in de Sitter background of the massive gravity theory with ghost
Lorentz symmetry is broken in vacuum state.

Let's estimate the particle production in the de Sitter background
of the massive gravity theory.

When ghost is in Spin 0 sector, is de Sitter invariance maintained?

It is known that a scalar field with $m^2 < 0$ does not have de Sitter invariant vacuum.

$$\phi = \sum_{lmn} \chi_{lmn}(\eta) Y_{lmn}(\Omega)$$

de Sitter invariant +ve frequency function

$$\chi_{lmn}(\eta) = u(\eta)\hat{a} + \bar{u}(\eta)\hat{a}^\dagger \qquad u(\eta) = A_l (\sin\eta)^{3/2} Q^{\mu}_{l+1/2}(-\cos\eta - i\varepsilon)$$

$$A_l = \frac{\sqrt{\pi}}{2} H e^{i\mu\pi/2} \left[\frac{\Gamma(l-\mu+3/2)}{\Gamma(l+\mu+3/2)}\right] \qquad \mu = \left(\frac{9}{4} - \frac{m^2}{H^2}\right)^{1/2}$$

$u(\eta)$ is Klein-Gordon normalized: $u(\eta)\bar{u}'(\eta) - u'(\eta)\bar{u}(\eta) = i(H\sin\eta)^2$

$u(\eta)$ ~ real $\Longrightarrow$ A_l diverges. $l = \mu - 3/2 - j \quad j = 0,1,2,\cdots$

At this point wave fn. $\Psi(\chi_{lmn}) \propto \frac{1}{\sqrt{\bar{u}}} \exp\left(i \frac{\bar{u}'}{2\bar{u}} \chi^2_{lmn}\right)$ becomes unnormalizable.

De Sitter invariant vacuum disappears.

Special cases

$m^2 = 0 : l = -j \Longrightarrow$ $l = 0$ mode is special.

∃ Kirsten-Garriga de Sitter invariant vacuum

$l = 0$ mode = shift of the origin of ϕ.

$m^2 = -4H^2 : l = 1 - j \Longrightarrow$ $l = 0,1$ modes are special.

Violation of de Sitter invariance is subtle.

Vachaspati-Vilenkin

$l = 0,1$ modes = translation = gauge in single brane case

Spin-2 ghost sector

Quantization of a ghost)

$$L = \frac{\alpha}{2}\left(\dot{x}^2 - \omega^2 x^2\right) \qquad \hat{H} = \frac{1}{2\alpha}\hat{p}^2 + \frac{\alpha}{2}\omega^2\hat{x}^2$$

$$\left.\begin{aligned} \hat{x} &= \frac{1}{\sqrt{2\omega}}\left(e^{-i\omega t}\hat{a} + e^{i\omega t}\hat{a}^\dagger\right) \\ \hat{p} = \alpha\dot{\hat{x}} &= \frac{i\alpha\sqrt{\omega}}{\sqrt{2}}\left(-e^{-i\omega t}\hat{a} + e^{i\omega t}\hat{a}^\dagger\right) \end{aligned}\right\} \Longrightarrow \sqrt{\frac{2}{\omega}}e^{-i\omega t}\hat{a} = \hat{x} - \frac{1}{i\alpha\omega}\hat{p}$$

$\alpha \Rightarrow +$ (normal case)

$$\hat{a}|0\rangle = 0 \Longrightarrow \left(x + \frac{1}{\alpha\omega}\partial_x\right)\Psi_0(x,t) = 0 \Longrightarrow \Psi_0(x,t) \propto \exp\left(-\frac{\alpha\omega}{2}x^2\right)$$

$$\hat{H} = \omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \quad \& \quad |n\rangle = \left(\hat{a}^\dagger\right)^n|0\rangle \Longrightarrow E_n = \left(n + \frac{1}{2}\right)\omega$$

$\alpha \Rightarrow -$ (ghost case)

To make the ground state wave function normalizable,

$$\Longrightarrow \Psi_0(x,t) \propto \exp\left(+\frac{\alpha\omega}{2}x^2\right) \Longrightarrow \hat{a}^\dagger|0\rangle = 0$$

$$|n\rangle = \hat{a}^n|0\rangle \Longrightarrow E_n = -\left(n + \frac{1}{2}\right)\omega \qquad \text{negative energy states}$$

de-Sitter symmetry breaking in vacuum

(K. I. and T. Tanaka 2009)

Simple example of symmetry breaking

1

$$L = \frac{1}{2}\left(\dot{x}_1^2 - \omega^2 x_1^2\right) - \left(\dot{y}_1^2 - \omega^2 y_1^2\right)$$

Ghost??

$$x_1 = a_{x,1}\exp(-i\omega t) + a_{x,1}^\dagger \exp(i\omega t)$$
$$y_1 = a_{y,1}^\dagger \exp(-i\omega t) + a_{y,1}\exp(i\omega t)$$

$$x_1 = x_2\cosh\theta + y_2\sinh\theta$$
$$y_1 = y_2\cosh\theta + x_2\sinh\theta$$

2

$$L = \frac{1}{2}\left(\dot{x}_2^2 - \omega^2 x_2^2\right) - \left(\dot{y}_2^2 - \omega^2 y_2^2\right)$$

Ghost??

$$x_2 = a_{x,2}\exp(-i\omega t) + a_{x,2}^\dagger \exp(i\omega t)$$
$$y_2 = a_{y,2}^\dagger \exp(-i\omega t) + a_{y,2}\exp(i\omega t)$$

"1"-vacuum state

$$a_{x,1}|0\rangle_1 = 0$$
$$a_{y,1}|0\rangle_1 = 0$$

$$a_{x,2} = a_{x,1}\cosh\theta + a_{y,2}^\dagger \sinh\theta$$

$$a_{x,2}|0\rangle_1 = a_{y,2}^\dagger \sinh\theta |0\rangle_1 \neq 0$$

Symmetry breaking !!

In the de-Sitter background of the massive gravity theory,
helicity-0 mode of graviton is ghost.
Since the different helicities can be mixed by the rotation of the de Sitter group,
the helicity-0 is different from the one of the different frame.

There are no de-Sitter symmetry vacuum.

Simple toy model: conformally coupled scalar field

Action for spin 2, helicity-0 mode

$$S_{\rm ghost} = \sum_k \int \frac{M_4^2 m^2 \left(m^2 - 2H^2\right)}{k^4 \eta^2} s_k \left[\Box_{\rm flat} + \frac{2}{\eta}\partial_\eta - \frac{m^2}{(H\eta)^2} \right] s_k \, d\eta$$

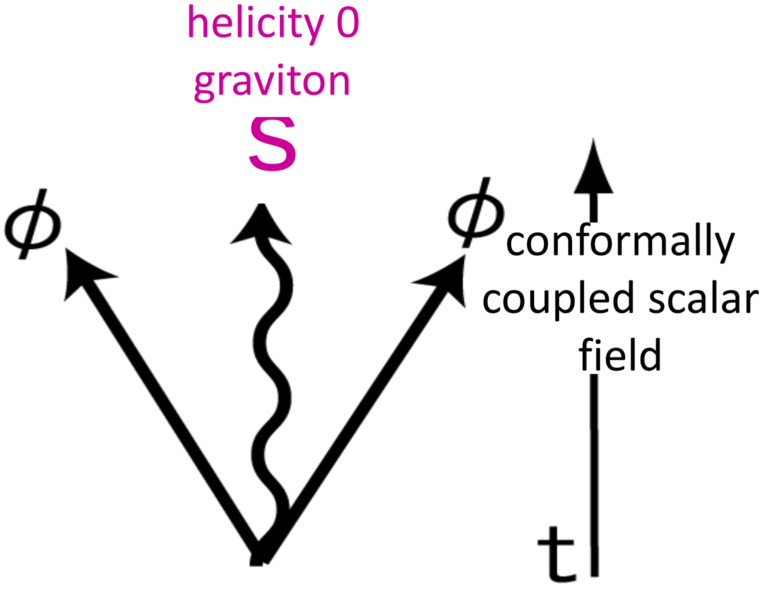
For $m^2 < 2H^2$, the signature of the action flips.

For large k, $< s >$ becomes large ⇒ strong coupling

a conformally coupled scalar field

interaction term

$$S_{\rm int} = -\frac{1}{2}\int d^4x \sqrt{-g}\, h^{\mu\nu} T_{\mu\nu}$$

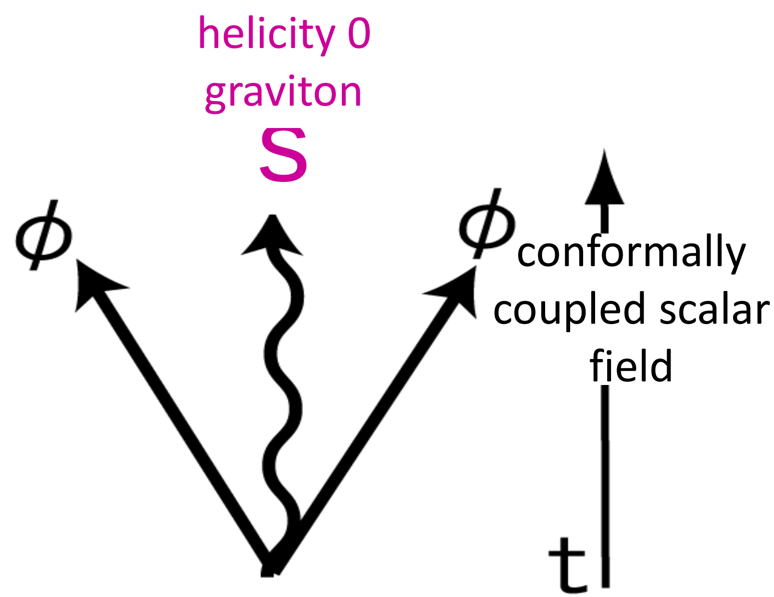


Particle creation in de Sitter space

total energy density of created scalar particles

$$\rho \approx \frac{H^3}{M_4^2}\Lambda^3$$

Λ : 3D cutoff scale



Result is divergent, but
if we set $\Lambda = \mathrm{M}_4 \Rightarrow \rho \approx H^3 M_4 << \rho_{crit}$

Generally strong coupling scale is much lower.
If we set cutoff at this energy scale,
particle creation is extremely suppressed.

4, Summary

①To built the ghost-free theory in which spin-2 sector accelerate the expansion of universe, we improve the DGP braneworld model

Two-branes model

When there are no spin-2 ghost,
spin-0 ghost appears.

Stabilization

If the mass of spin-2 mode approaches the critical mass the mass of spin-0 goes to the critical mass.
So the property of the ghost can transfer from spin-2 to spin-0

②We investigate if ghost is harmful or not

de-Sitter breaking vacuum

Since we must select ghost mode, which is helicity-0 mode of graviton, de-Sitter symmetry is broken.

Pair creation

We calculate the particle production from de Sitter vacuum in the massive gravity theory with conformally coupling scalar field.
Result is UV divergent, but if we set that the 3D cutoff equals to Planck mass, energy density becomes much smaller than critical density of universe.