GRAVITATIONAL LENSING IN A DARK MATTER FREE BRANEWORLD MODEL

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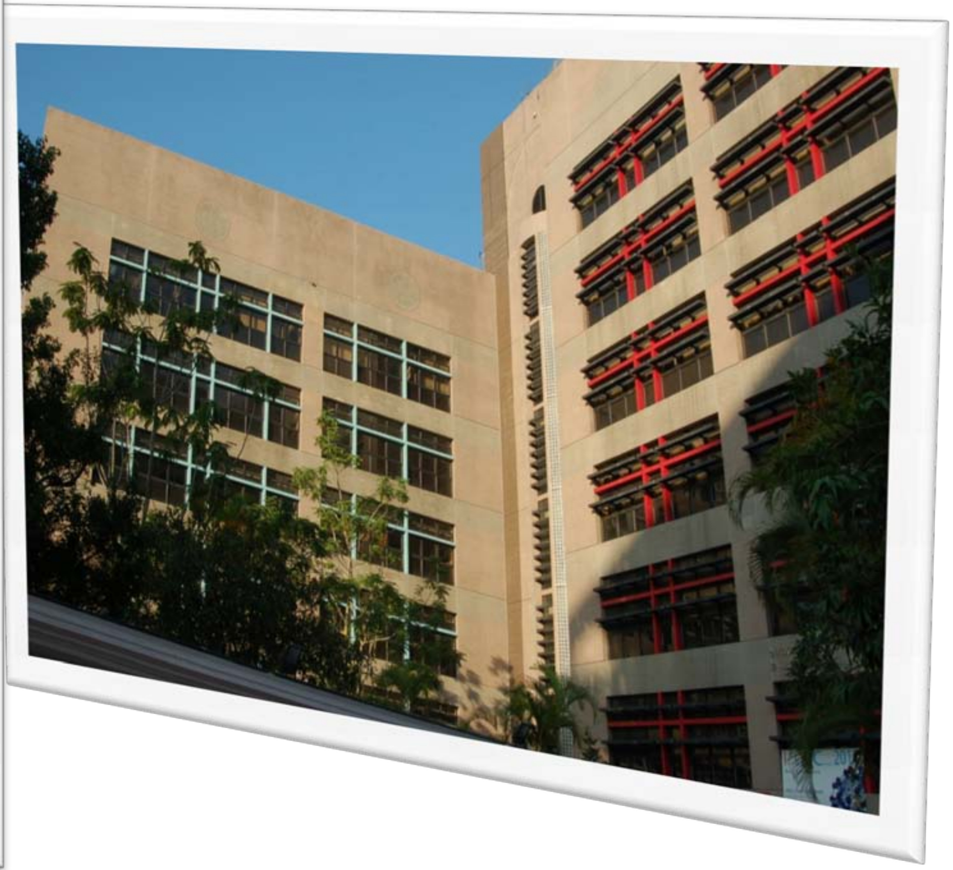
- All audience of this talk

- Dr. Lam Tsz Yan;
- Prof. Shinji Mukohyama; and
- Prof. Masamune Oguri

The University of Hong Kong

- Main building and physics building



Some Views from Physics building

Outline of presentation

- Background of this project
- Introduction to Randall Sundrum braneworld
- Motivations of this study
- Rotation Curves in braneworlds
- Gravitational Lensing in braneworlds

Dark Matter problem

- Missing mass at galactic scales and galactic cluster observations
- e.g. Rotation Curves of galaxies, X-ray clusters images, gravitational lensing; and
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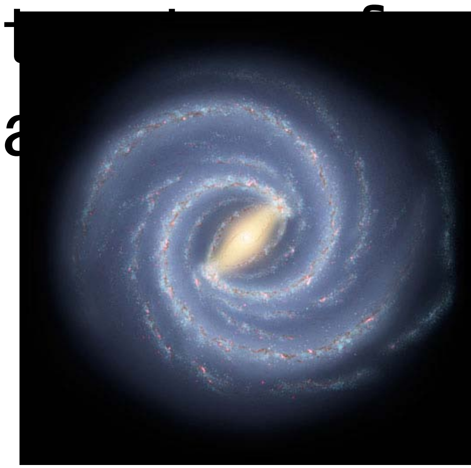
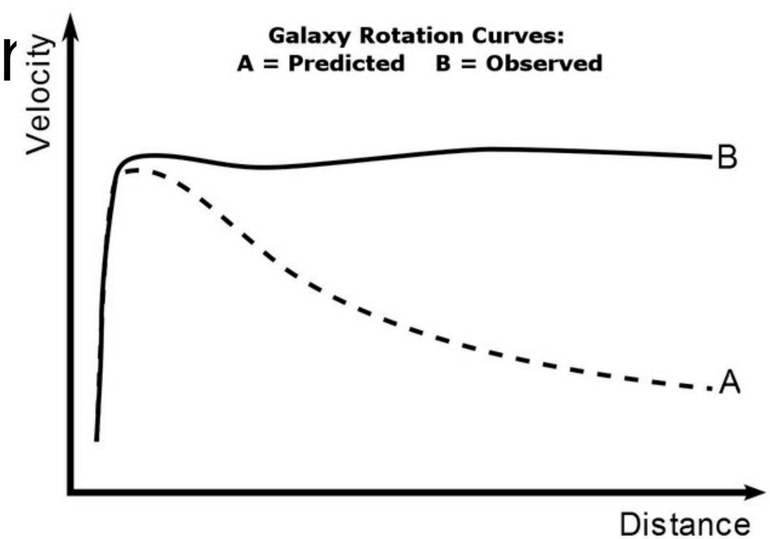


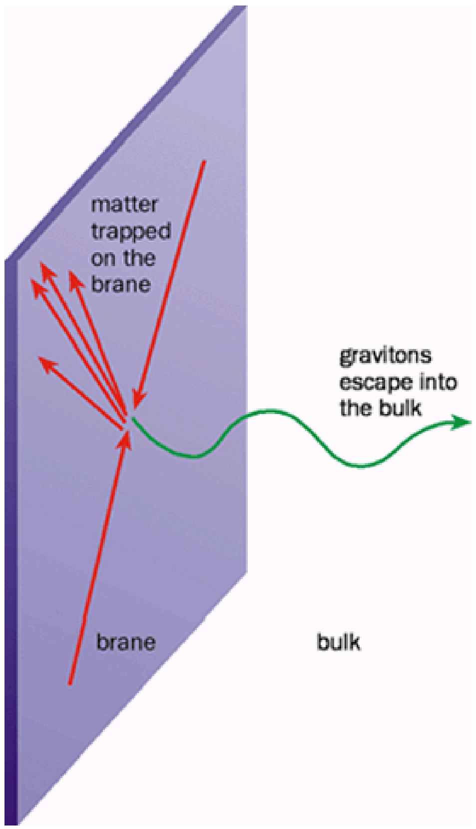


Braneworld as a solution

- 1. Evidences of Dark matter always inferred from gravitational motion
- 2. No simple theoretical framework for dark matter
- Supersymmetry offers a solution
- Detection of SUSY particle has never succeeded (yet?)
- Behavior of SUSY still not clear, alternative approach can not be ignored.

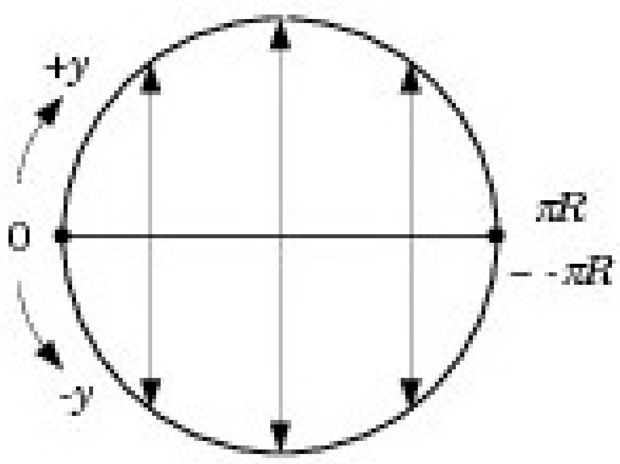
Braneworld

- Offers an explanation on why gravity is weak
- The idea is not new e.g. Kaluza-Klein theory in 1921
- RS models (Randall, Sundrum, 1999)
- Motivated by String theory/M theory



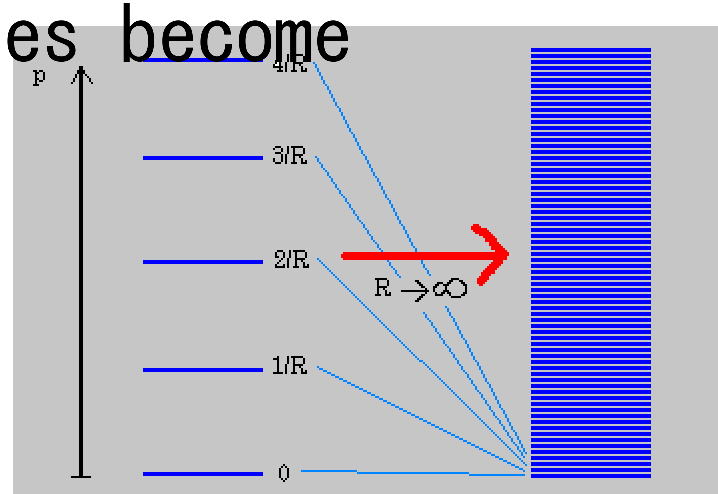
Randall-Sundrum models

- 2-brane model
- Suggested by Horava-Witten solution
- 1-brane model - one of the brane located at infinity
- Showing the possibility of infinite extra dimension



Major prediction of Braneworld

- Kaluza Klein (KK) Modes
- In 2-branes model
- It is like standing waves between two branes
- Predictions in TeV physics
- In 1-branes model KK modes become spectrum
- No predictions in TeV
- But also allows more study about it



Original RS 1-brane construction

- Gravity is a 5D interaction
- 5D spacetime (bulk) have large negative curvature, it is done by introducing a negative cosmological constant
- Standard Model fields confined on 4D brane, and there is a brane tension fine tuned with 5D cosmological constant
- Brane is mirror symmetric
- Construct action of 2-brane model and limit another brane to infinity

Braneworld as modified 4D Einstein theory

- The matter on membrane infer discontinuity of extrinsic curvature (Israel, 1966)
- Project 5D Einstein equation to 4D (Shiromizu, Maeda, Sasaki, 2000)

$$R_{MN} + \frac{1}{2} g_{MN} R = k_5^2 T_{MN} \text{ in 5D}$$

$$R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} + \frac{k_5^4 \lambda}{6} T_{\mu\nu} + k_5^4 S_{\mu\nu} - E_{\mu\nu}$$

$$S_{\mu\nu} = -\frac{1}{4} T_{\mu\alpha} T_{\nu}^{\alpha} + \frac{1}{12} T T_{\mu\nu} + \frac{1}{8} g_{\mu\nu} T_{\alpha\beta} T^{\alpha\beta} - \frac{1}{24} g_{\mu\nu} T^2$$

What is $E_{\mu\nu}$

- A traceless tensor
- Projection of 5D Weyl tensor to the brane

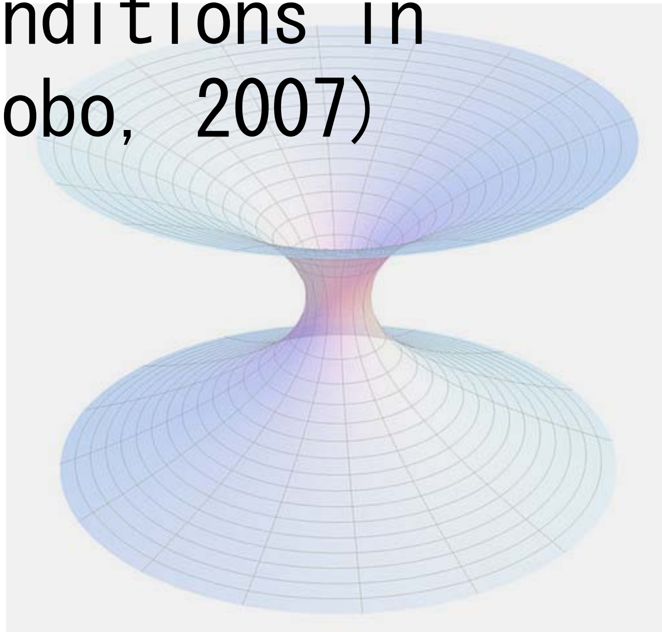
$$E_{\mu\nu} = {}^{(5)}C_{ABCD} n^C n^D g_\mu^A g_\nu^B$$

- Weyl tensor = component of Curvature that is not governed by Einstein’s equations
- It contains freedom for 5D Gravitational wave, therefore KK spectrum
- KK spectrum could “source” by the brane via $$\nabla^\mu S_{\mu\nu} = k_5^4 \nabla^\mu E_{\mu\nu}$$

- Is it possible to be dark matter?

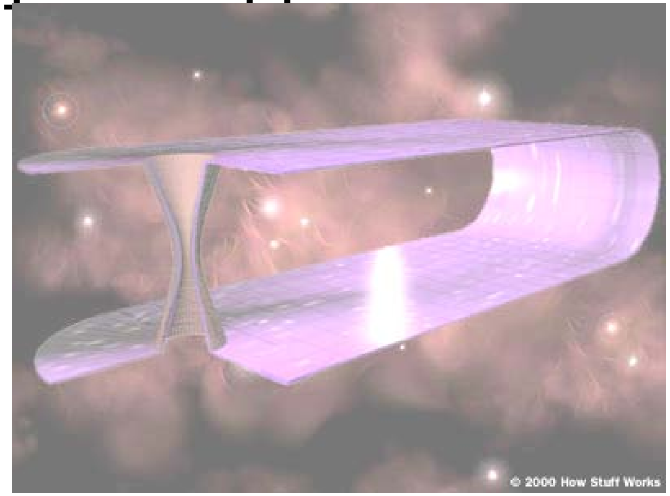
Motivations from wormhole

- Origins of Dark Matter
- Galactic scale evidence of extra dimension
- Inter Galactic wormhole may be possible
- Wormholes violate energy conditions in GR but not in"braneworld (Lobo, 2007)
- "Dark Matter" that we see everywhere is the key to sustain it.



A little advertisement on wormhole

- Wong, Harko, Cheng, 2011
- Braneworld wormhole could evolve with the Universe, and expand with the Universe
- Wormholes that exist before inflation collapse
- Expanding wormholes exis
  inflation could be still
  size



Testify or Falsify

- Explaining dark matter phenomenology
- The braneworld corrections depend on geometry
- More empirical results are required to determine the true structure of spacetime
- Predict different observations from the same object
- Galactic rotation curves (Gergely et al, 2011)

HSB and LSB

- Low Surface Brightness galaxies - low visible mass content, diffuse gas rich $\mu_B \geq 23\mathrm{mag}/\mathrm{arsec}^2$
- LSB galaxies are conventionally dark matter rich
- HSB very different from LSB

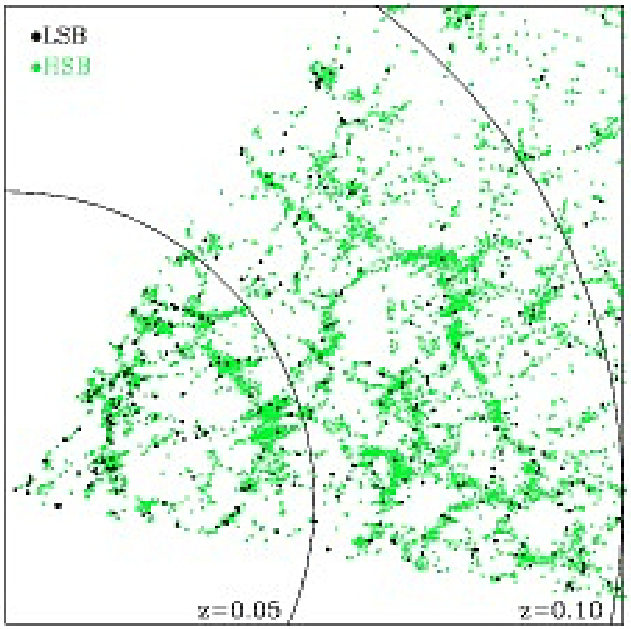


Fig. 1. One of the two analysed pie slices. The right ascension range 354° ≤ α ≤ 53° containing ~400 LSB galaxies and ~8000 HSB galaxies is plotted.

Rosenbaum and Bomans, 2004

Bulk and brane

- The geometry of bulk leaves an imprint on the brane.
- The brane spherical geometry

$$ds^2 = g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + r^2 d\Omega^2$$

- Possible bulk configuration

$${}^{(5)}ds^2 = -M(r,y)^2 dt^2 + N(r,y)^2 dr^2 + Q(r,y)^2 d\Omega^2 + dy^2$$

- Calculate 5D Weyl tensor
- Project it to the brane
- Simplify with 5D Einstein equati

The form of $E_{\mu\nu}$

$$E^t_t = \frac{N_{,y,y}}{N} + \frac{2Q_{,y,y}}{Q} - \frac{\Lambda_5}{2}$$

$$E^r_r = -\frac{N_{,y,y}}{N} + \frac{\Lambda_5}{6}$$

$$E^\theta_\theta = E^\phi_\phi = -\frac{Q_{,y,y}}{Q} + \frac{\Lambda_5}{6}$$

- 2nd derivative of metric along extra dimension
- Additional constraint from the conservation equation $\nabla_\mu E^\mu_\nu = 0$
- Only the r component is non-trivial

Energy and Pressure of $E_{\mu\nu}$

- 1st approach: Hypothesize high energy events that could source E in the formation history. Based on the physics we can model the remnant form of E, and test it with observations
- Alternative approach in Gergely, et al., 2011
- Components of E view as energy U and pressure P of some fluid
- Guess equation of state based on Schwarzschild case in 2+1+1 decomposition

$$P = (a-2)U + \frac{B}{r^2}$$

LSB metric and Visible mass matching

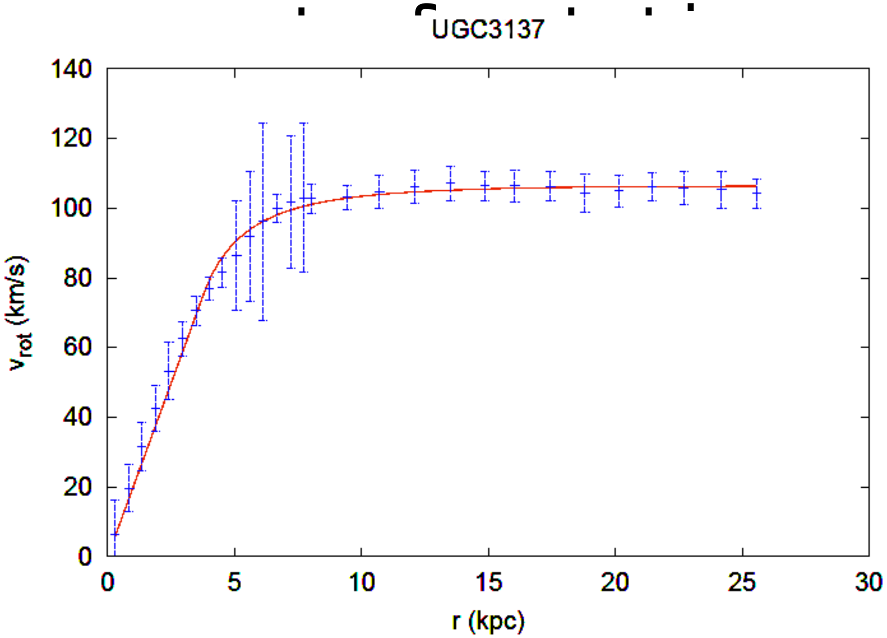
$$-g_{tt} \approx 1 - \frac{2GM}{c^2 r} + \frac{2\gamma}{1-\alpha}\left(\frac{r}{r_c}\right)^{\alpha-1}$$

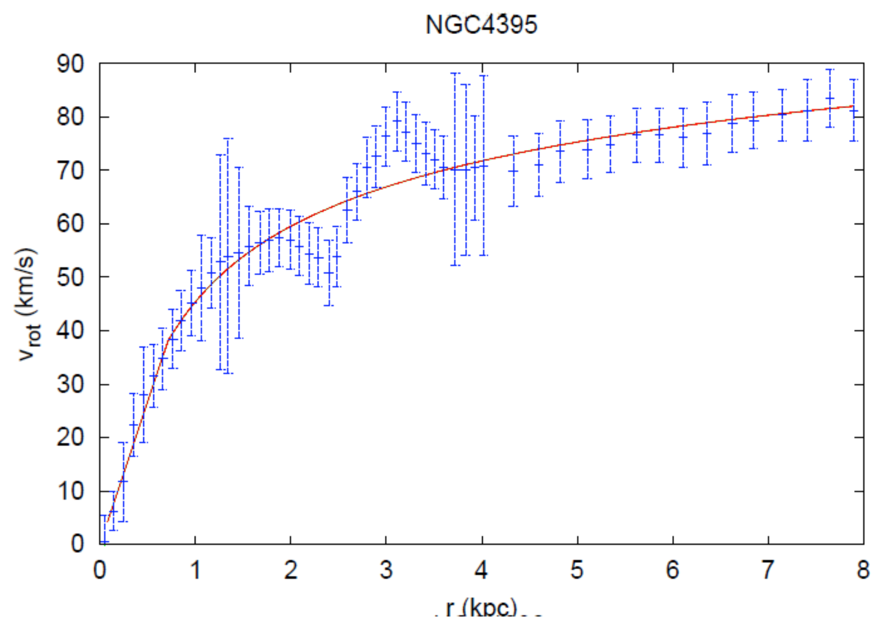
$$g_{rr} \approx \left\{1 - \frac{2G(M+M_b)}{c^2 r} + \gamma\left[\left(\frac{r}{r_c}\right)^{\alpha-1} - 1\right]\right\}^{-1}$$

- α and γ depend on a and B
- Mb is a degenerate parameter in rotation curve
- rc is obtained from matching Baryonic mass

Rotation Curve studies

- Data from Blok, 2001
- Gergely, et al., 2011 – approximate Newtonian in linear part of rotation curve and braneworld dominant in outer



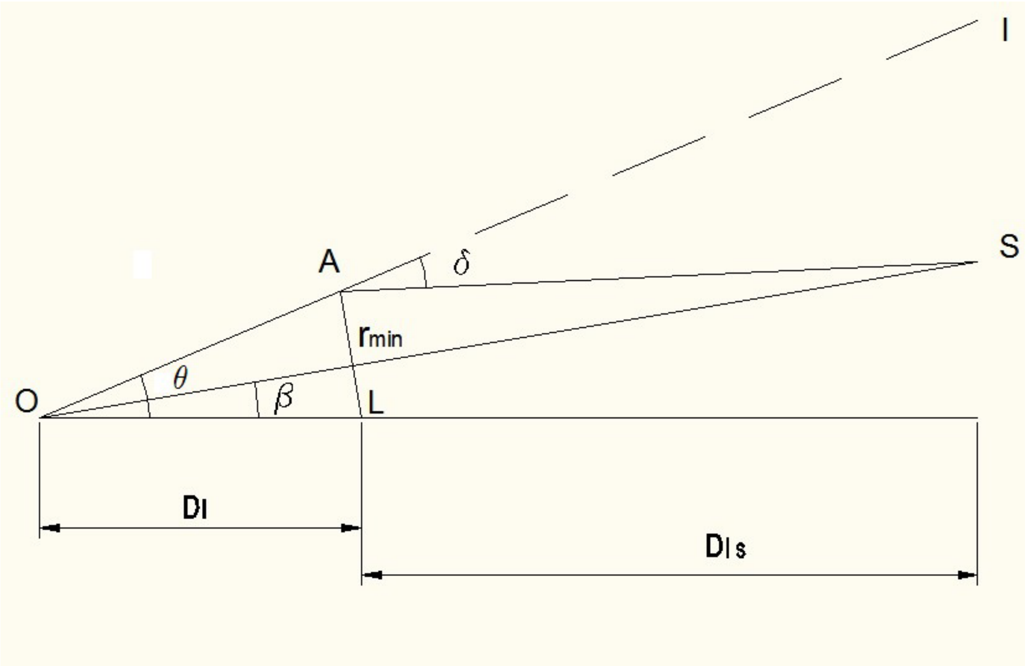


List of Candidate galaxies

- k is introduced in step function to switch from baryonic mass dominant region

Galaxy	k	M_0	r_c	α	γ	$\chi^2_{\rm min}$
	kpc^{-1}	$\odot$	kpc			
DDO 189	57.5	4.05×10^8	1.25	0.3	6.43×10^{-8}	0.742
NGC 2366	46.0	1.05×10^9	1.47	0.8	1.12×10^{-7}	2.538
NGC 3274	138.1	4.38×10^8	0.69	-0.4	6.73×10^{-8}	18.099
NGC 4395	30.0	2.37×10^8	0.71	0.9	3.43×10^{-7}	27.98
NGC 4455	99.7	2.26×10^8	1.03	0.9	2.72×10^{-7}	7.129
NGC 5023	86.3	2.69×10^8	0.74	0.9	4.53×10^{-7}	10.614
UGC 10310	36.4	1.28×10^9	2.6	0.4	1.12×10^{-7}	0.729
UGC 1230	15.3	3.87×10^9	3.22	-1.7	1.12×10^{-7}	0.539
UGC 3137	34.5	5.32×10^9	3.87	-0.5	1.23×10^{-7}	4.877

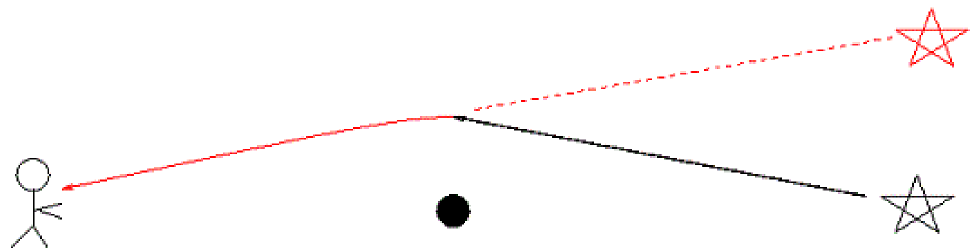
Simple 1D lensing



- Lens equation

$$\tan|\theta| - \tan(s\beta) - \frac{D_{ls}}{D_s}\left[\tan|\theta| + \tan(\delta - |\theta|)\right] = 0 \qquad (1)$$

Minimal approach radius



- Null geodesic

$$g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = g_{tt}(r)t'^2 + g_r r(r)r'^2 + r^2\phi'^2 = 0$$

- Minimum radius on trajectory $r' = 0$
- Identify the constants of motion with lensing geometry

$$g_{tt}(r_{\min})D_l^2 \sin(\theta) + r_{\min}^2 = 0 \qquad (2)$$

Deflection angle expanded

- Compare angle between asymptotes

$$\delta(r_{\min}) = 2\int_{r_{\min}}^{\infty} \mathcal{I} - \pi$$

$$\mathcal{I}(r) = \frac{1}{r}\left\{\frac{g_{rr}(r)}{\left[g_{tt}(r_{\min})/g_{tt}(r)\right](r/r_{\min})^2 - 1}\right\}^{1/2} \qquad (3)$$

- Deflection angle can be studied by treating minimal approach radius like impact parameter

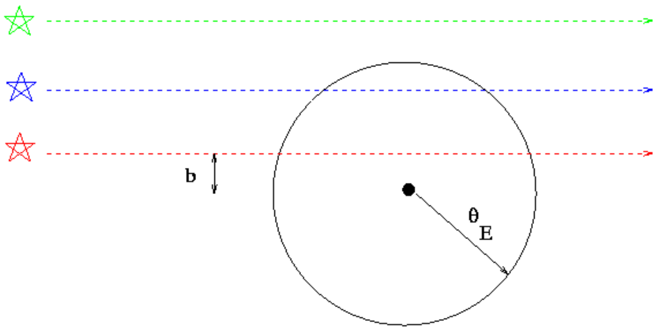


Image angle and remarks

- Lens equation approximated for small angle

$$\theta^+ = \beta + \frac{D_{ls}}{D_s}\delta(r^+_{\min}, M_b)$$

$$\theta^- = \beta - \frac{D_{ls}}{D_s}\delta(r^-_{\min}, M_b)$$

- With equation (2)

$$\theta^\pm = \left(\frac{r^\pm_{\min}}{D_l}\right)^2 \frac{1}{g_{tt}(r^\pm_{\min})}$$

- A single lensing observation from a known source should able to rule out the model

Post Newtonian Expansion

- Focus on the deflection angle
- Expand the integral as series of small parameter $\delta = \delta_{\rm GR} + \delta_{\rm BW}$

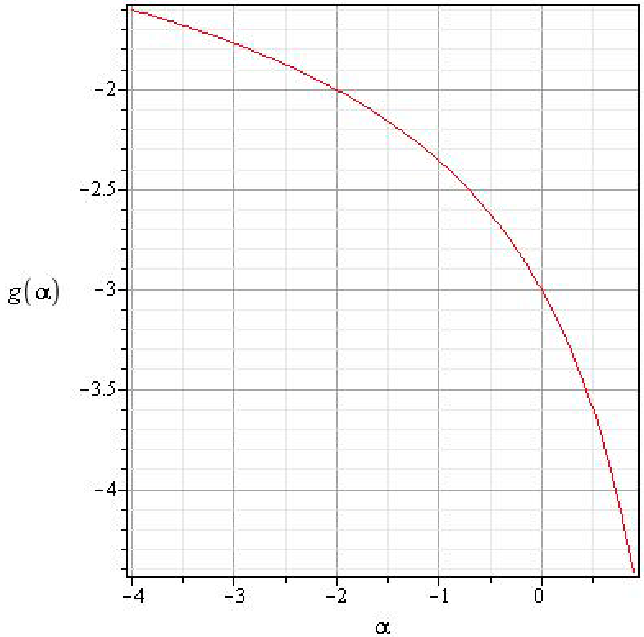
$$\delta_{\rm GR} = \frac{4GM}{c^2 r_{\rm min}}$$

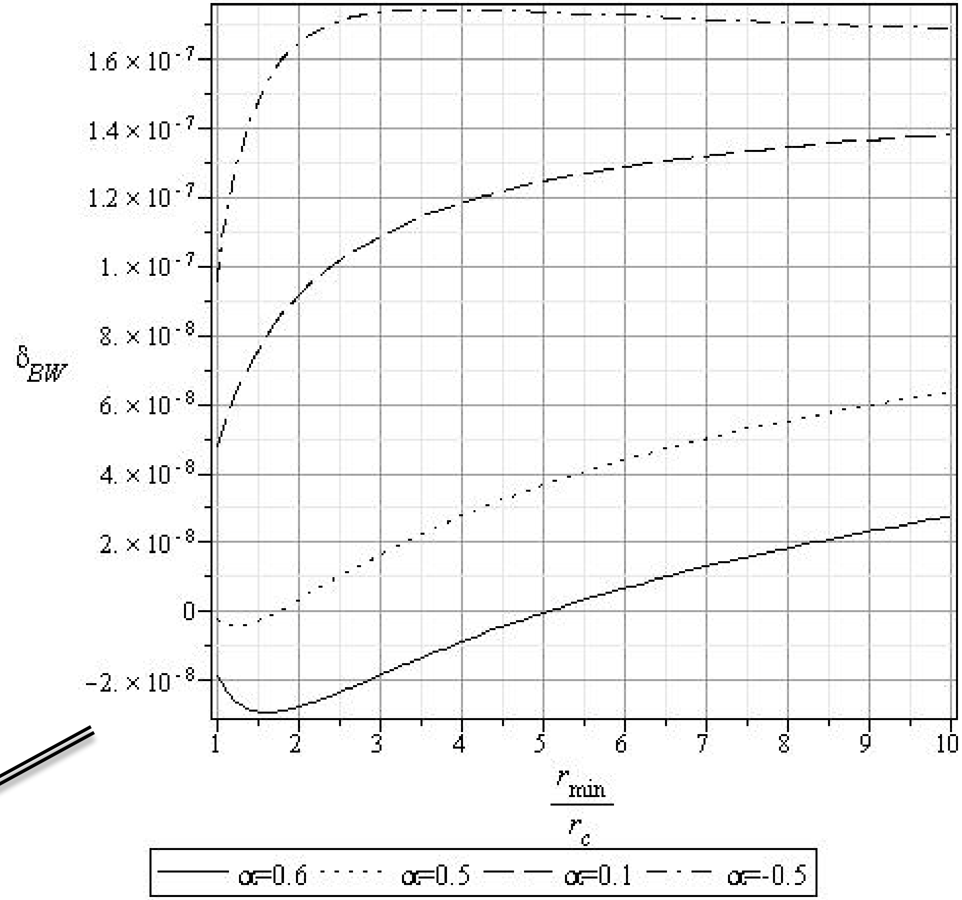
$$\delta_{\rm BW} = \frac{2GM_b}{c^2 r_{\rm min}} + \gamma \left[\frac{\pi}{2} + \left(\frac{r_c}{r_{\rm min}} \right)^{1-\alpha} g(\alpha) \right]$$

$$g(\alpha) = \int_1^\infty \frac{du}{u} \sqrt{\frac{1}{u^2-1}} \left[\frac{(1-\alpha)u^{\alpha-1} + (1+\alpha)u^{\alpha+1} - 2u^2}{(1-\alpha)(u^2-1)} \right]$$

Braneworld contribution

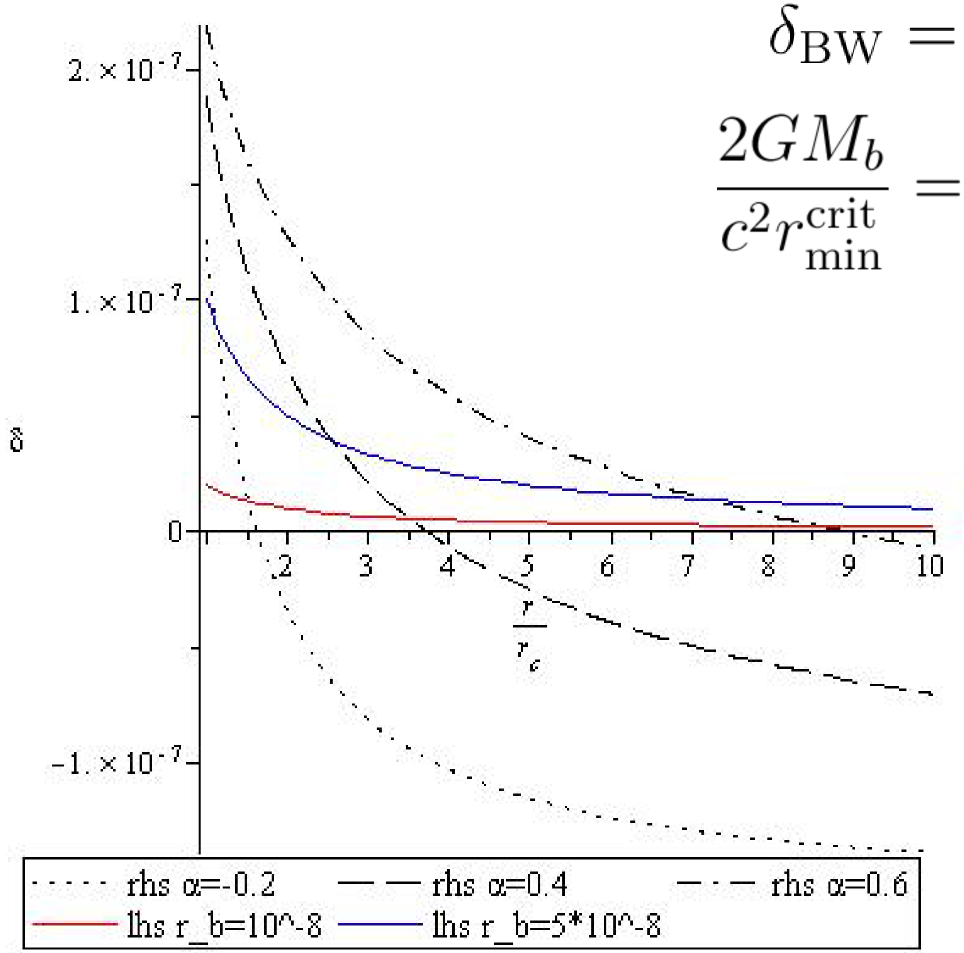
- The form of numerical integration g(α) and δ_{BW}





$$\gamma = 10^{-7}, M_b = 10^{-7} c^2 r_c / G$$

Critical approach radius



$$\delta_{\mathrm{BW}} = 0$$

$$\frac{2GM_b}{c^2 r_{\mathrm{min}}^{\mathrm{crit}}} = -\gamma\left[\frac{\pi}{2} + \left(\frac{r_c}{r_{\mathrm{min}}^{\mathrm{crit}}}\right)^{1-\alpha} g(\alpha)\right]$$

Intersecti
on $r_{\mathrm{min}}^{\mathrm{crit}}$ is the

Lensing profile comparison

- Use the parameters fixed by rotation curves
- Compare LSB braneworld lensing with dark matter lensing
- To see if there could be discriminative effect on the lensing profile
- View dark matter as a correction to baryonic mass lensing

The lensing of Dark Matter

- Pseudo isothermal halo vs NFW model
- Both can explain rotation curve of LSB
- Pseudo isothermal

$$\rho_{\mathrm{iso}}(r) = \frac{\rho_0}{1+\left(\frac{r}{r_s}\right)^2} \Longrightarrow M_{\mathrm{iso}}(r) = 4\pi r_s^2 \rho_0 \left(r - r_s \arctan\left(\frac{r}{r_s}\right)\right)$$

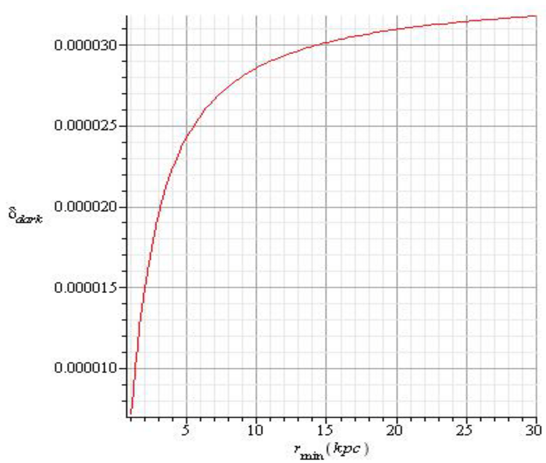
- NFW motivated by N body simulations

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_i}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2} \Longrightarrow M_{\mathrm{NFW}}(r) = 4\pi r_s^3 \rho_i \left[\frac{\left(1+\frac{r}{r_s}\right)\ln\left(1+\frac{r}{r_s}\right) - \frac{r}{r_s}}{1+\frac{r}{r_s}}\right]$$

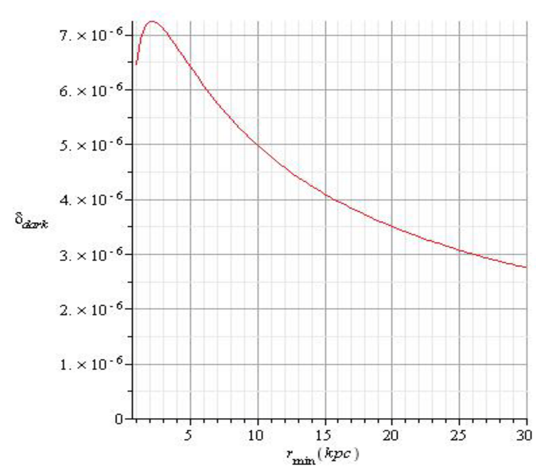
Isothermal halo lensing

- Assume only mass inside radius of approach cause deflection
- Recall that braneworld deflection angle converge to a constant, it is more simi isothermal hal

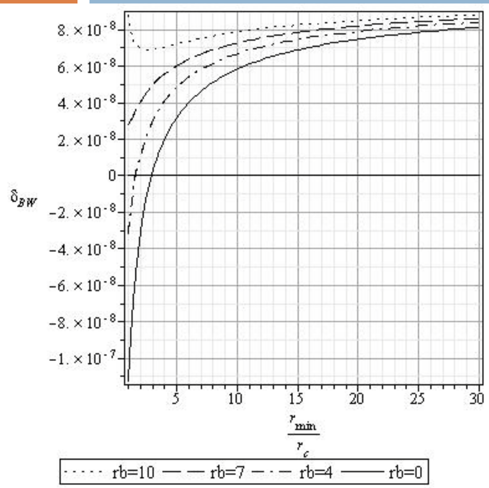
Pseudo Isothermal



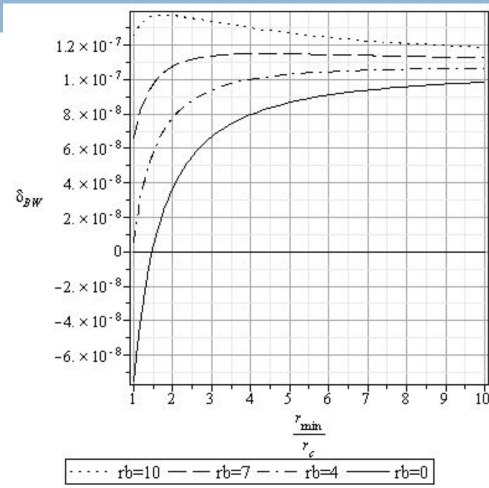
NFW



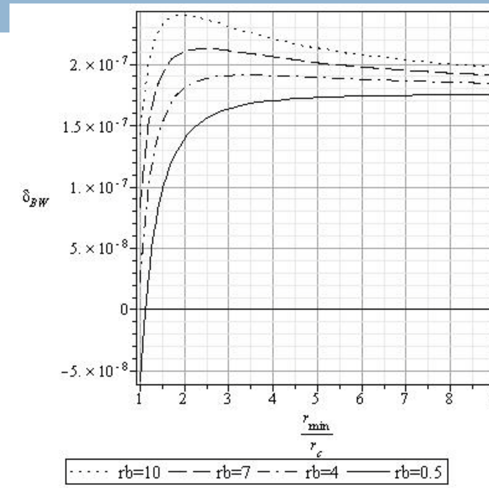
Isothermal Halo like cases



DDO 189

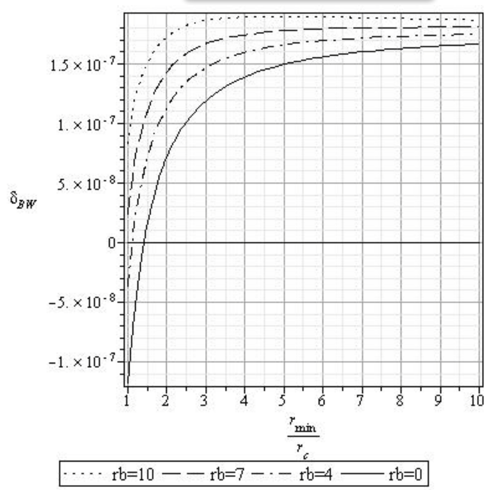


NGC3274

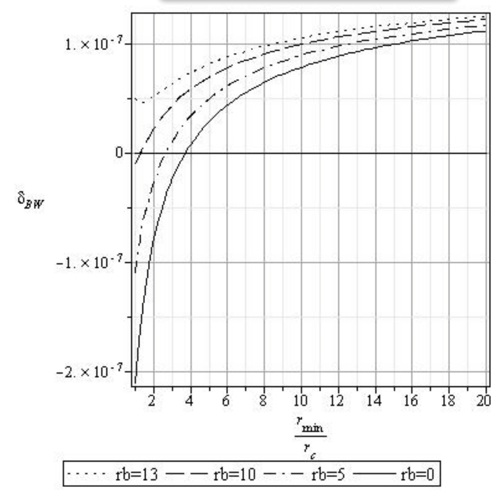


NGC3274

UGC3137



UGC10310



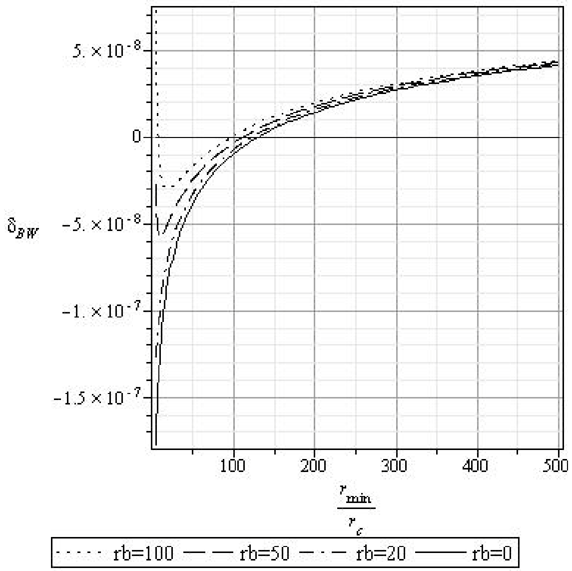
$$r_b = \frac{GM_b}{c^2}$$

Unit: $10^{-8} r_c$

Unusual Cases

- Negative contribution occurs at large radius

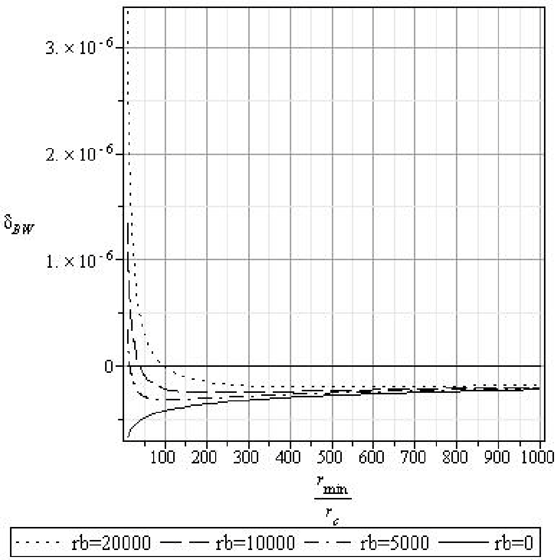
UGC3137



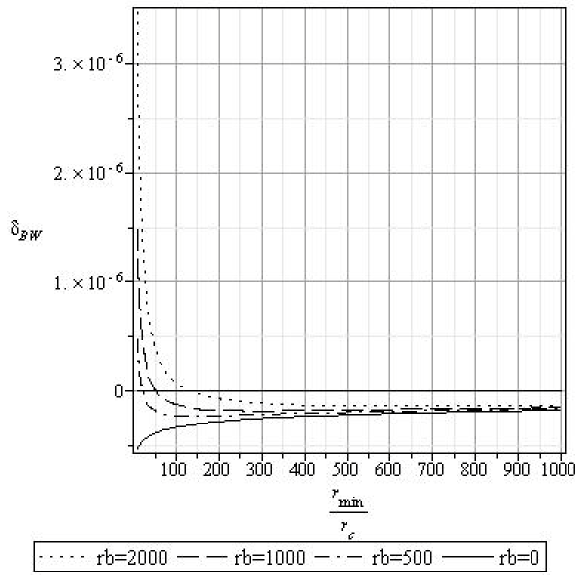
Unusual Cases

- Braneworld contribution remain negative at large impact

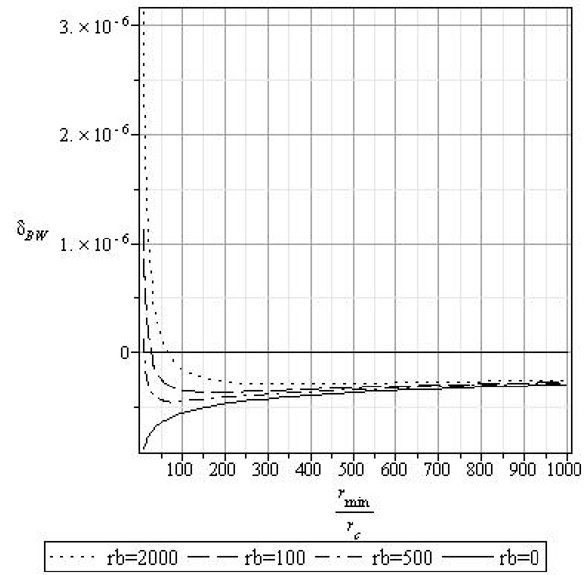
NGC4395



NGC4455

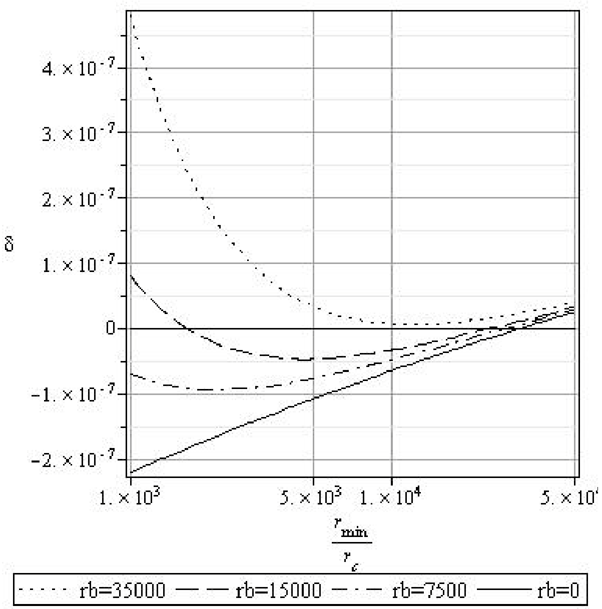


NGC5023

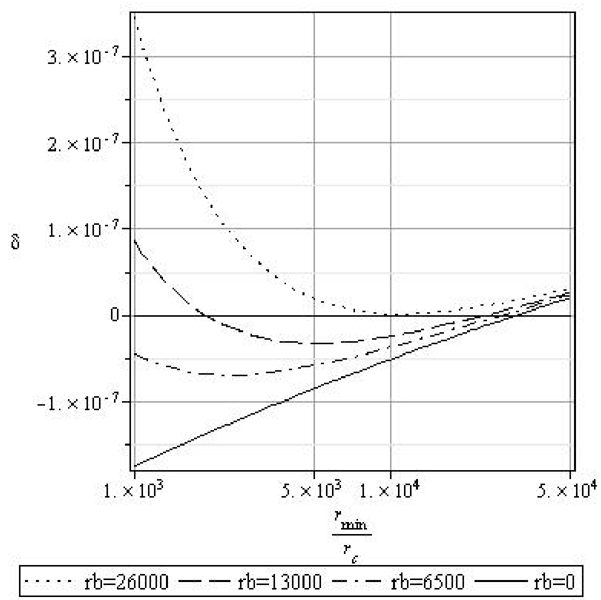


Scattering at Imaginative range

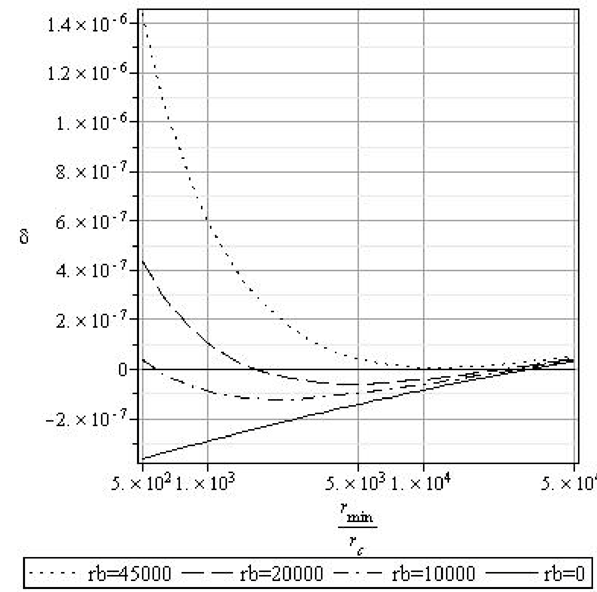
NGC4395



NGC4455



NGC5023



- α~1 → $$P = U + \frac{B}{r^2}$$

Another way of comparison

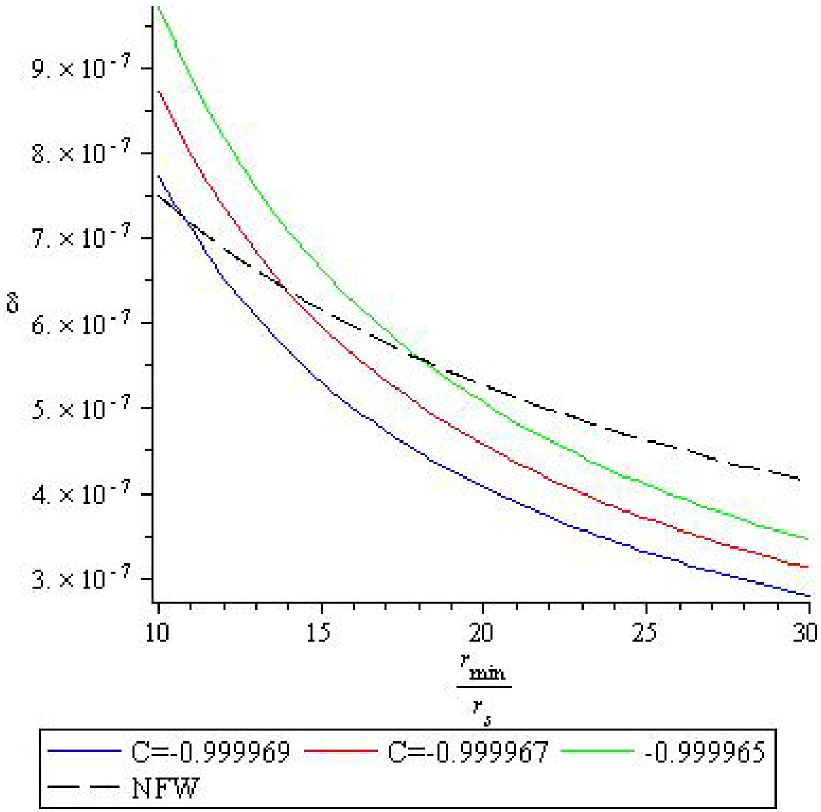
- Assume a well description of rotation curves by braneworld and dark matter model
- The metric will be determined up to a integration constant
- Compare the lensing effect predicted by braneworld versus dark matter

Formulation

- Assume a correction on Schwarzschild metric $$ds^2 = -\left[1 - \frac{r_b}{r} + \nu(r)\right] dt^2 + \left[1 - \frac{r_b}{r} + \mu(r)\right]^{-1} dr^2 + r^2 d\Omega$$
- Assume ν fixed by tangential velocity profile in rotation curves $$v_{tg}^2 = \frac{r g'_{tt}}{2 g_{tt}}$$
- Obtain the differential equation of μ by traceless ke lowest order, e. g. $$\frac{d\mu(r)}{dr} = -\frac{\mu(r)\left[1 + M'(r)\right]}{r} - \frac{M'(r)}{r}$$
- Calculate the braneworld contribution to

Results

- If the rotation curves is well described by a Newtonian motion in NFW density profile
- It is general for Braneworld predict different deflection against dark matter



Summary

- Braneworld contributions to the brane come from bulk geometry
- Galactic rotation curves as a probe to bulk geometry
- Braneworld lensing indicate unique features as compared to the dark matter case
- Future Studies
  - Determining more precisely the equation of state
  - "Corresponding" mass profile in braneworld, so to obtain 2D lensing image by programming code like glafic (Oguri, 2010)
  - HSB lensing study would allow comparison with more observations
  - What kind of high energy processes in the early Universe may have determined the form of F? Large