Francis Bernardeau IPhT Saclay

New insights into cosmological gravitational clustering

Crocce, Scoccimarro, astro-ph/0509418-9, arXiv:0704.2783

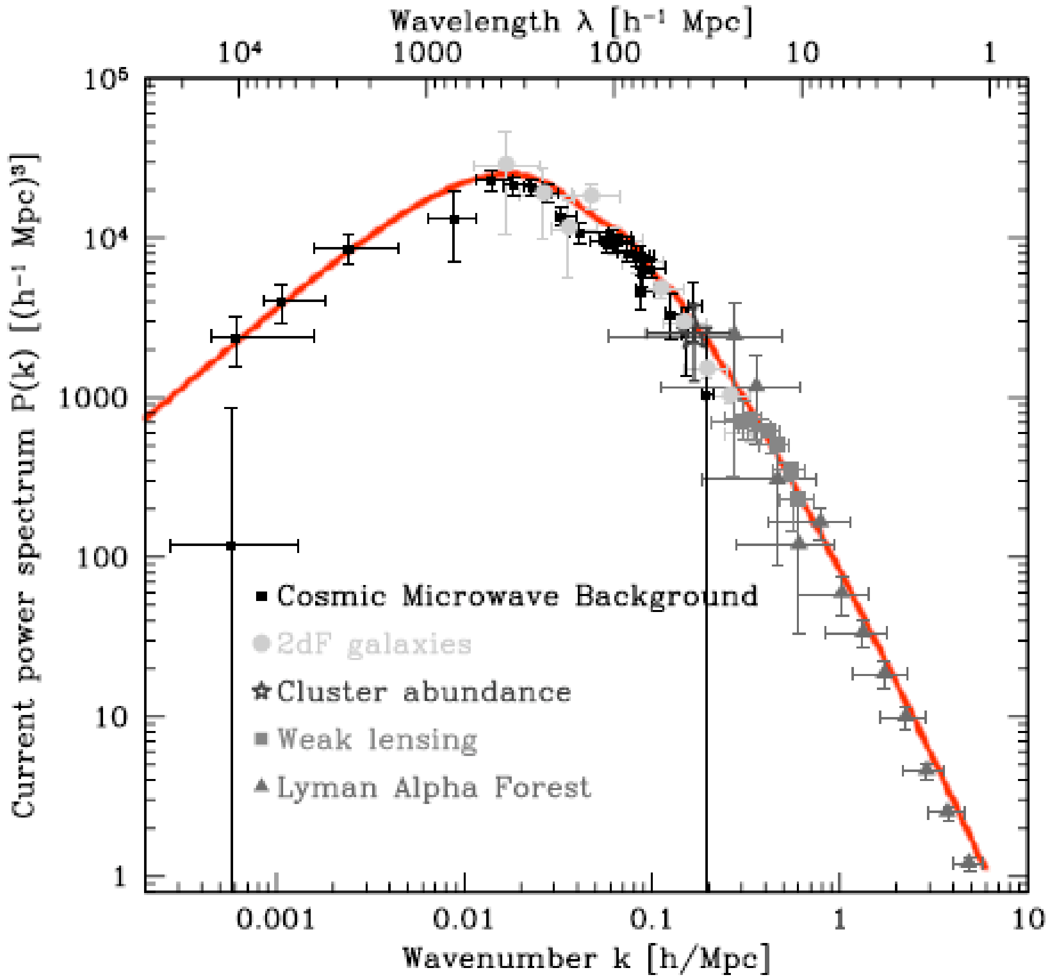
FB, Crocce, Scoccimarro, arXiv:0806.2334

FB, Valageas, arXiv:0805.0805

Gravitational instability is the driver to large-scale structure formation

- Data show that large-scale structure has formed from small density inhomogeneities since time of matter dominated universe with a dominant cold dark matter component
- inflation provides us with a compelling framework for the origin of such density fluctuations with specific statistics (Gaussian) and spectrum (nearly scale invariant before horizon crossing)

Correspondence between "initial conditions" and local large-scale structure is now observed



Tegmark et al. 2003

A self-gravitating expanding dust fluid

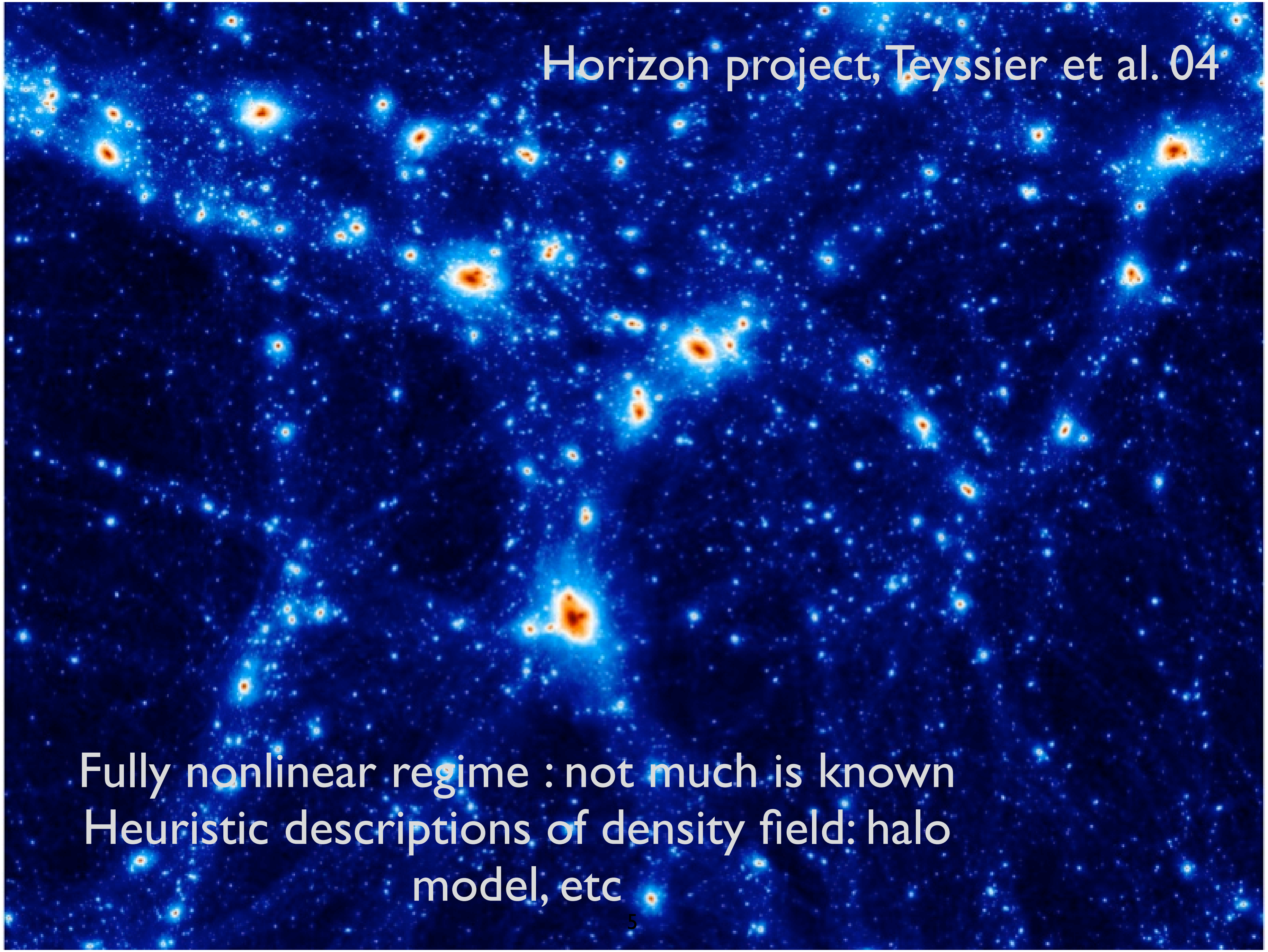
The Vlasov equation (collisionless Boltzmann equation) - f(x,p) is the phase space density distribution -

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial}{\partial t} f(\mathbf{x},\mathbf{p},t) + \frac{\mathbf{p}}{ma^2}\frac{\partial}{\partial \mathbf{x}} f(\mathbf{x},\mathbf{p},t) - m\nabla_{\mathbf{x}}.\Phi(\mathbf{x})\frac{\partial}{\partial \mathbf{p}} f(\mathbf{x},\mathbf{p},t) = 0$$

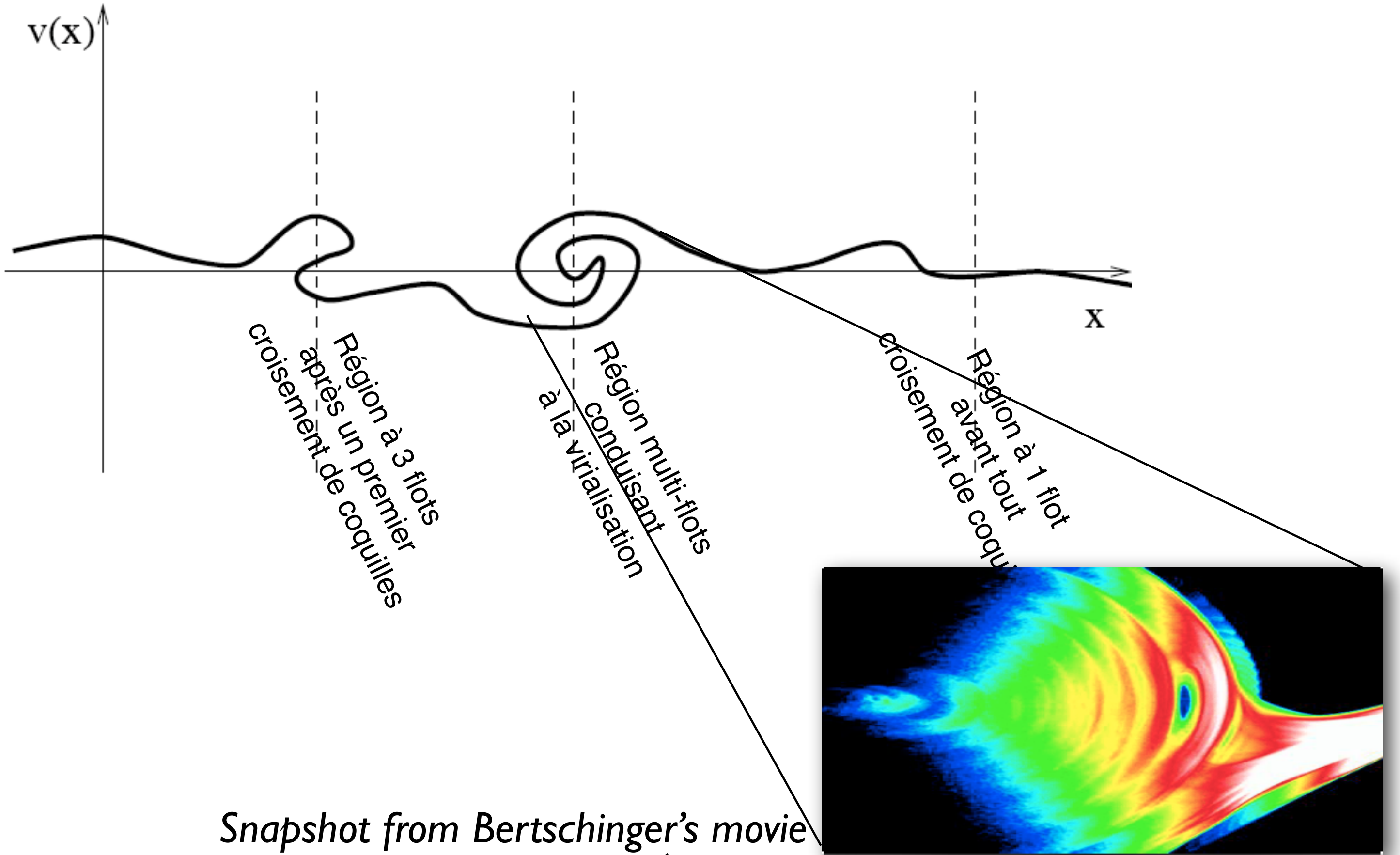
$$\Delta\Phi(\mathbf{x}) = \frac{4\pi G m}{a}\left(\int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p} - \overline{n}\right),$$

This is what N-body codes aim at simulating...

How much do we understand those equations ?


Horizon project, Teyssier et al. 04
Fully nonlinear regime : not much is known
Heuristic descriptions of density field: halo
model, etc

Early stages of growth of instability for CDM



Snapshot from Bertschinger's movie

Newton = a self-gravitating expanding dust fluid

Peebles '80 ;Fry '84
FB, Colombi, Gaztañaga, Scoccimarro, '02

The Vlasov equation (collisionless Boltzmann equation) - f(x,p) is the phase space density distribution - are fully nonlinear.

This is what N-body codes aim at simulating...

$$\frac{\mathrm{d}f}{\mathrm{d}t}=\frac{\partial}{\partial t}f(\mathbf{x},\mathbf{p},t)+\frac{\mathbf{p}}{ma^2}\frac{\partial}{\partial \mathbf{x}}f(\mathbf{x},\mathbf{p},t)-m\nabla_{\mathbf{x}}.\Phi(\mathbf{x})\frac{\partial}{\partial \mathbf{p}}f(\mathbf{x},\mathbf{p},t)=0$$

$$\Delta\Phi(\mathbf{x})=\frac{4\pi Gm}{a}\left(\int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p}-\overline{n}\right),$$

The rules of the game: single flow equations + expansion with respect to initial density fields

$$\begin{aligned}
\frac{\partial}{\partial t}\delta(\mathbf{x},t)+\frac{1}{a}\nabla_i.\left[(1+\delta(\mathbf{x},t))\mathbf{u}_i(\mathbf{x},t)\right] &= 0\\
\frac{\partial}{\partial t}\mathbf{u}_i(\mathbf{x},t)+\frac{\dot a}{a}\mathbf{u}_i(\mathbf{x},t)+\frac{1}{a}\mathbf{u}_j(\mathbf{x},t)\,\mathbf{u}_{i,j}(\mathbf{x},t) &= -\frac{1}{a}\nabla_i\Phi(\mathbf{x},t)\\
\nabla^2\Phi(\mathbf{x},t)-4\pi G\overline{\rho}(t)a^2\,\delta(\mathbf{x},t) &= 0.
\end{aligned}$$

- Motion equations in Fourier space in the single flow approximation

$$
\begin{aligned}
a\frac{\partial\delta(\mathbf{k},a)}{\partial a}+\theta(\mathbf{k},a) &= -\frac{1}{(2\pi)^{3/2}}\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\,\delta_D(\mathbf{k}-\mathbf{k}_{12}) \\
&\times \quad \alpha(\mathbf{k}_1,\mathbf{k}_2)\theta(\mathbf{k}_1,a)\delta(\mathbf{k}_2,a) \\
a\frac{\partial\theta(\mathbf{k},a)}{\partial a}+\frac{1}{2}\theta(\mathbf{k},a)+\frac{3}{2}\delta(\mathbf{k},a) &= -\frac{1}{(2\pi)^{3/2}}\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\,\delta_D(\mathbf{k}-\mathbf{k}_{12}) \\
&\times \quad \beta(\mathbf{k}_1,\mathbf{k}_2)\theta(\mathbf{k}_1,a)\theta(\mathbf{k}_2,a),
\end{aligned}
$$

$$
\alpha(\mathbf{k}_1,\mathbf{k}_2)\equiv\frac{\mathbf{k}_{12}\cdot\mathbf{k}_1}{k_1^2},\qquad \beta(\mathbf{k}_1,\mathbf{k}_2)\equiv\frac{k_{12}^2(\mathbf{k}_1\cdot\mathbf{k}_2)}{2k_1^2k_2^2}.
$$

- linear order = growth of structure
- higher order terms = mode couplings
- equations can be solved to any arbitrary order

$$
\begin{aligned}
\delta^{(n)}(\mathbf{x}) &= \int\frac{\mathrm{d}^3\mathbf{k}_1}{(2\pi)^{3/2}}\delta(\mathbf{k}_1)\ldots\frac{\mathrm{d}^3\mathbf{k}_n}{(2\pi)^{3/2}}\delta(\mathbf{k}_n)\,a^n\,F_n(\mathbf{k}_1,\ldots,\mathbf{k}_n), \\
\theta^{(n)}(\mathbf{x}) &= \int\frac{\mathrm{d}^3\mathbf{k}_1}{(2\pi)^{3/2}}\delta(\mathbf{k}_1)\ldots\frac{\mathrm{d}^3\mathbf{k}_n}{(2\pi)^{3/2}}\delta(\mathbf{k}_n)\,a^n\,G_n(\mathbf{k}_1,\ldots,\mathbf{k}_n).
\end{aligned}
$$

To be more explicit we have the following recursion

$$F_n(\mathbf{q}_1,\ldots,\mathbf{q}_n) = \sum_{m=1}^{n-1} \frac{G_m(\mathbf{q}_1,\ldots,\mathbf{q}_m)}{(2n+3)(n-1)}\Big[-(2n+1)\alpha(\mathbf{k}_1,\mathbf{k}_2)F_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n) + 2\beta(\mathbf{k}_1,\mathbf{k}_2)G_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n)\Big], \tag{4.83}$$

$$G_n(\mathbf{q}_1,\ldots,\mathbf{q}_n) = \sum_{m=1}^{n-1} \frac{G_m(\mathbf{q}_1,\ldots,\mathbf{q}_m)}{(2n+3)(n-1)}\Big[3\alpha(\mathbf{k}_1,\mathbf{k}_2)F_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n) - 2n\beta(\mathbf{k}_1,\mathbf{k}_2)G_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n)\Big], \tag{4.84}$$

où $\mathbf{k}_1 \equiv \mathbf{q}_1 + \ldots + \mathbf{q}_m$, $\mathbf{k}_2 \equiv \mathbf{q}_{m+1} + \ldots + \mathbf{q}_n$, $\mathbf{k} \equiv \mathbf{k}_1 + \mathbf{k}_2$, et $F_1 = -G_1 \equiv 1$.

Ainsi par exemple, pour n=2,

$$F_2 = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2}{k_1^2} + \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2}{k_2^2} + \frac{2}{7}\frac{(\mathbf{k}_1.\mathbf{k}_2)^2}{k_1^2\,k_2^2}, \tag{4.85}$$

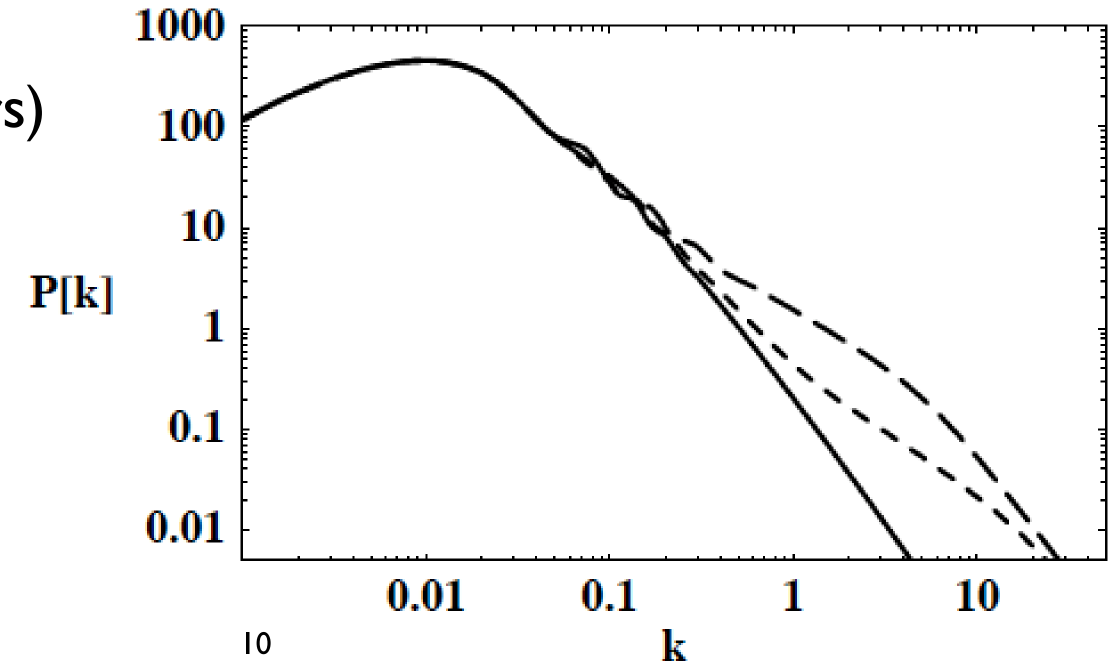
$$G_2 = -\left[\frac{3}{7} + \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2}{k_1^2} + \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2}{k_2^2} + \frac{4}{7}\frac{(\mathbf{k}_1.\mathbf{k}_2)^2}{k_1^2\,k_2^2}\right], \tag{4.86}$$

In this regime (matter dominated, sub-horizon scale) all coupling functions are thus explicitly known

Goals are multifold

Higher order statistical quantities (bispectrum, etc...)

Evolution of the power spectrum (with a controlled dependence on the cosmological parameters)



Gravity induced mode couplings have been computed and observed!

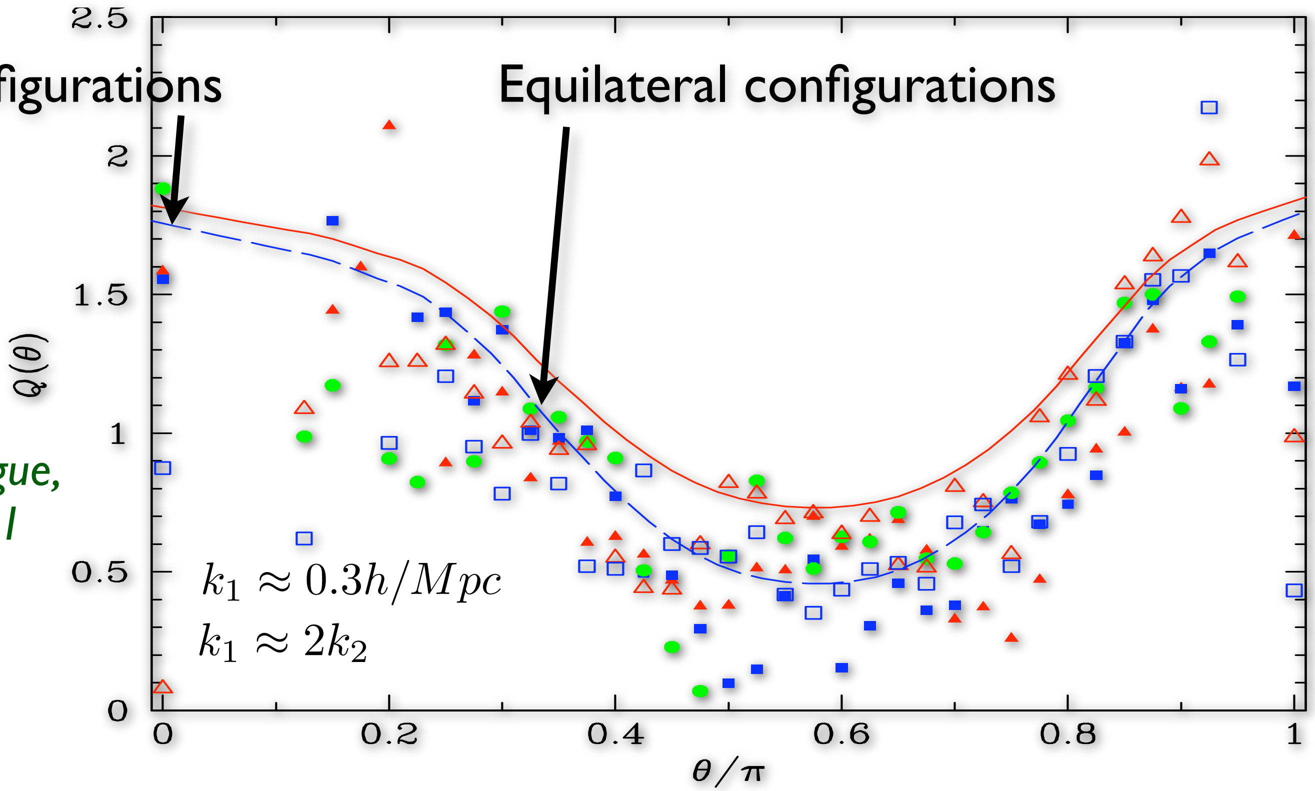
Tree order "fNL" for the density field:

$$F_2^{(s)} = \left(\frac{3\nu_2}{4} - \frac{1}{2}\right) + \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2}{k_1^2} + \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2}{k_2^2} + \left(\frac{3}{2} - \frac{3\nu_2}{4}\right)\frac{(\mathbf{k}_1.\mathbf{k}_2)^2}{k_1^2\, k_2^2}$$

$$\nu_2(\Omega_m, \Omega_\Lambda, \omega, \dots) = 34/21 + \dots$$

Flattened configurations

Equilateral configurations



From PSCz catalogue, Feldman et al. '01

- A lot is known at tree order with a very good matching with Nbody simulations

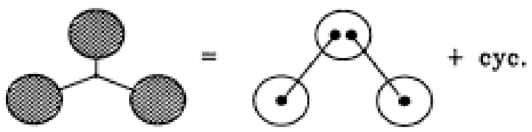


FIG. 3

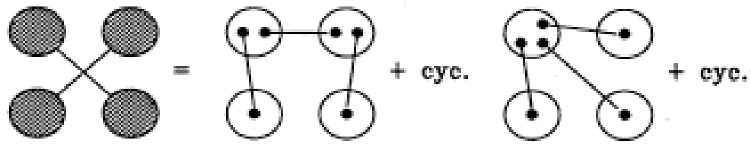
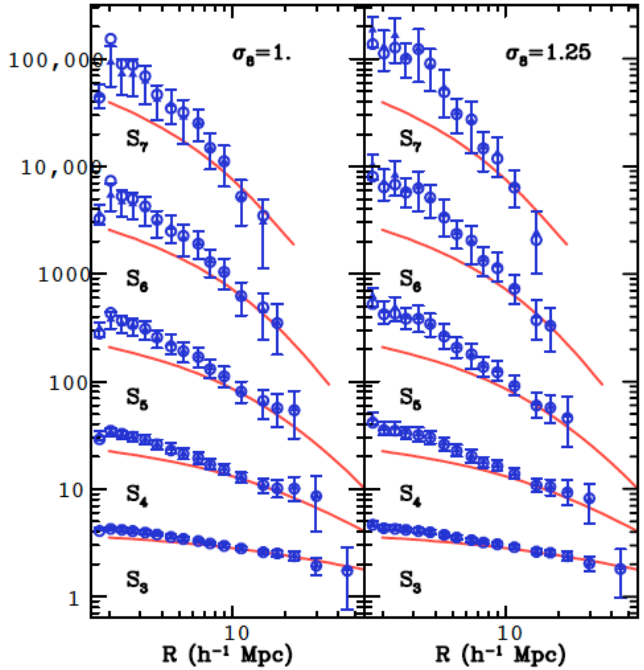
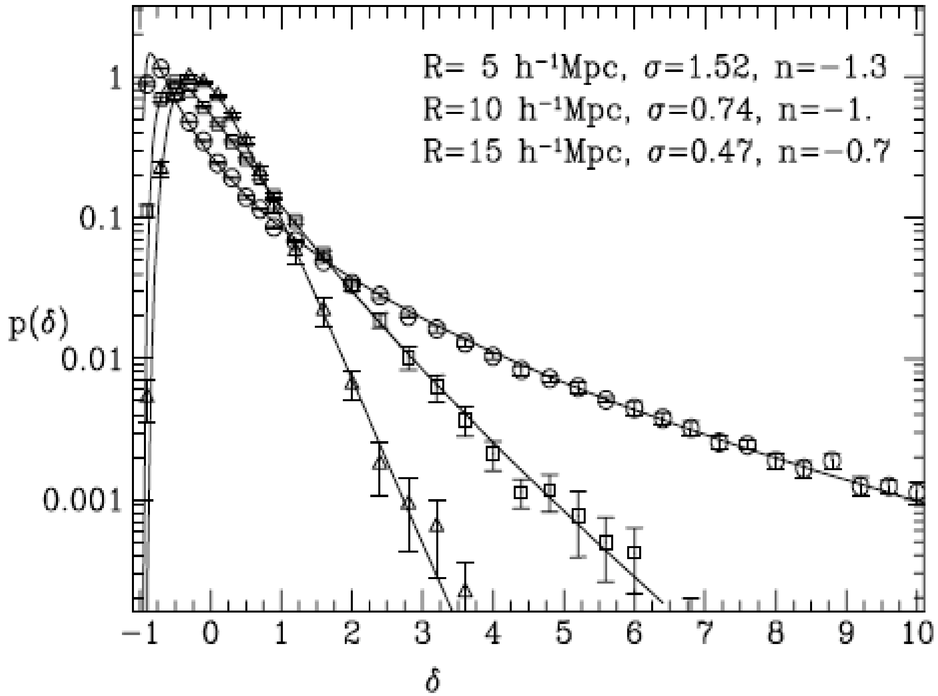


FIG. 4

From FB '92





From FB '94

- But things get not as nice when one wants to include loops

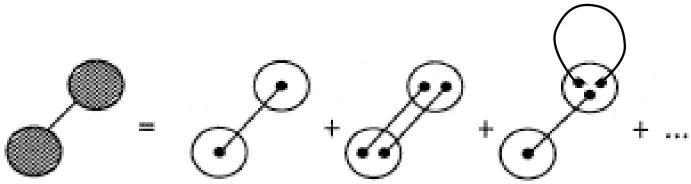


FIG. 2

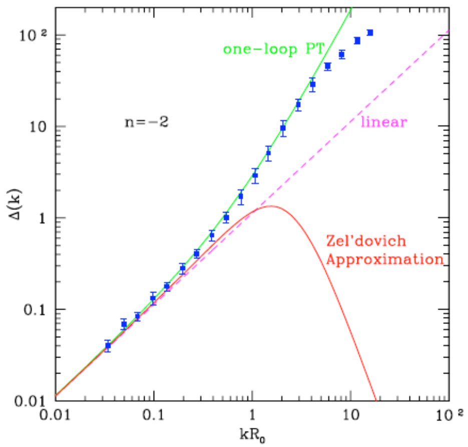
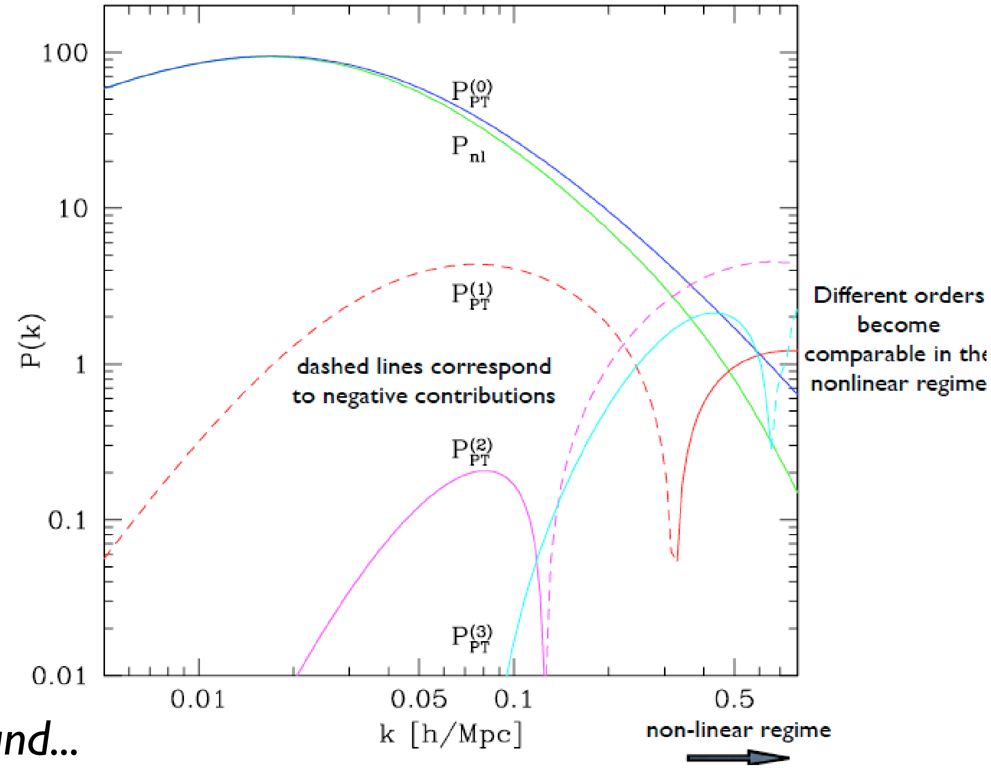


Fig. 13. The power spectrum for $n = -2$ scale-free initial conditions. Symbols de measurements in numerical simulations from [560]. Lines denote linear PT, one- PT [Eq. (169)] and the Zel'dovich Approximation results [Eq. (181)], as labele



Not necessarily the best way to expand...

The RPT reformulation

Scoccimarro and Crocce '05

A reformulation of the theory in a QFT like manner

Scoccimarro '97

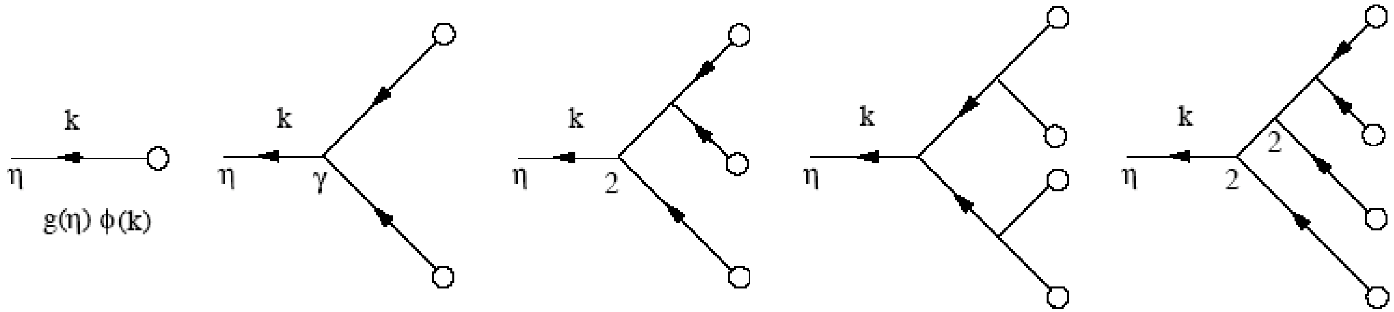
$$\Psi_i(\mathbf{k},a) = g_{ij}(a)\ \phi_j(\mathbf{k}) + \int_0^a \frac{\mathrm{d}a'}{a'}\ g_{ij}(a/a')\ \gamma_{jkl}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\ \Psi_k(\mathbf{k}_1,a')\Psi_l(\mathbf{k}_2,a')$$

density-div v doublet

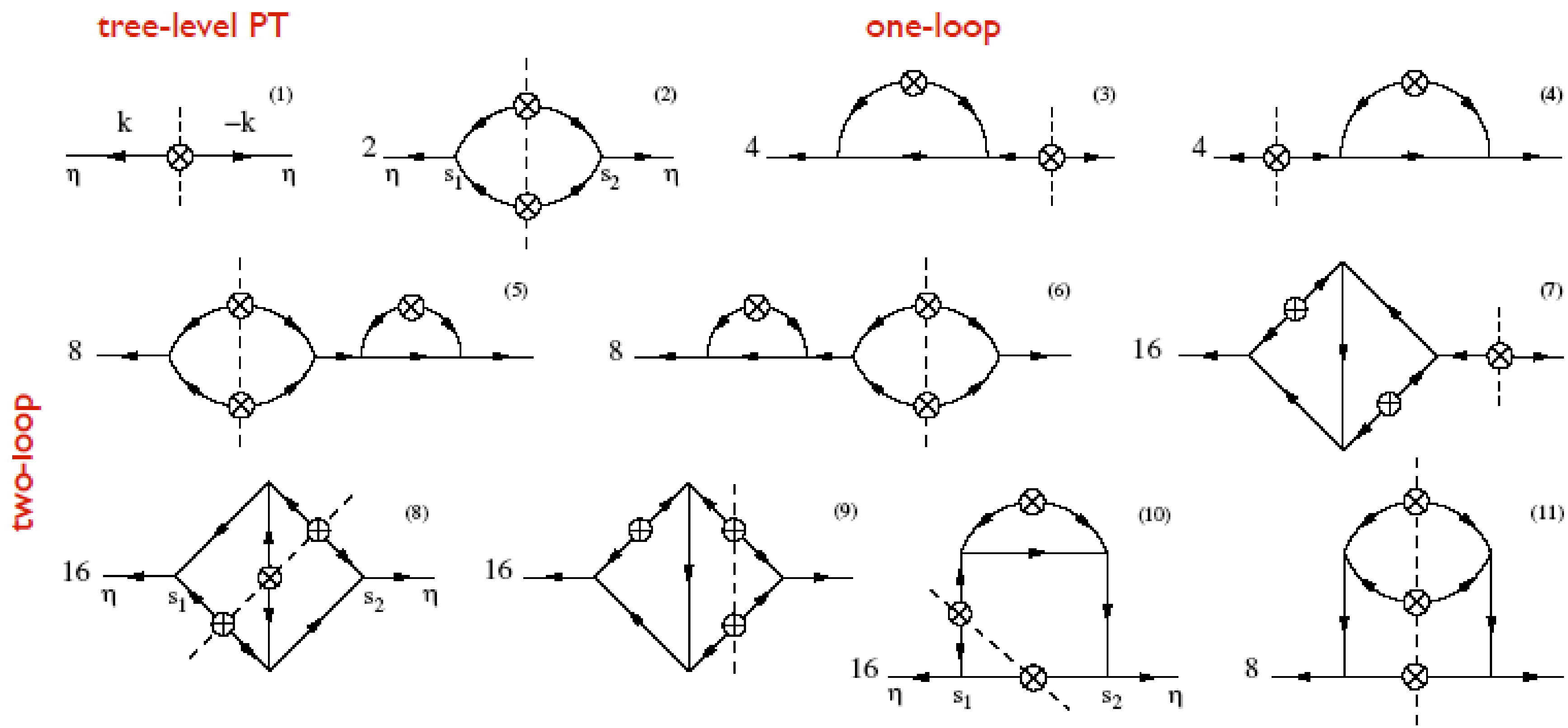
$$\left(\begin{array}{c}\delta(\mathbf{k})\\ \theta(\mathbf{k})\end{array}\right)$$

$$\phi_a(\mathbf{k}) = \Psi_a(\mathbf{k},\eta=0)$$

$$g_{ab}(\eta) = \frac{\mathrm{e}^\eta}{5}\left[\begin{array}{cc}3 & 2\\ 3 & 2\end{array}\right] - \frac{\mathrm{e}^{-3\eta/2}}{5}\left[\begin{array}{cc}-2 & 2\\ 3 & -3\end{array}\right]$$



A new diagrammatic expansion



One key ingredient : the propagator

Final density / velocity div.

$$G_{ab}(k,\eta)\ \delta_{\rm D}(\mathbf{k}-\mathbf{k}') \equiv \left\langle \frac{\delta\Psi_a(\mathbf{k},\eta)}{\delta\phi_b(\mathbf{k}')} \right\rangle$$

Initial Conditions

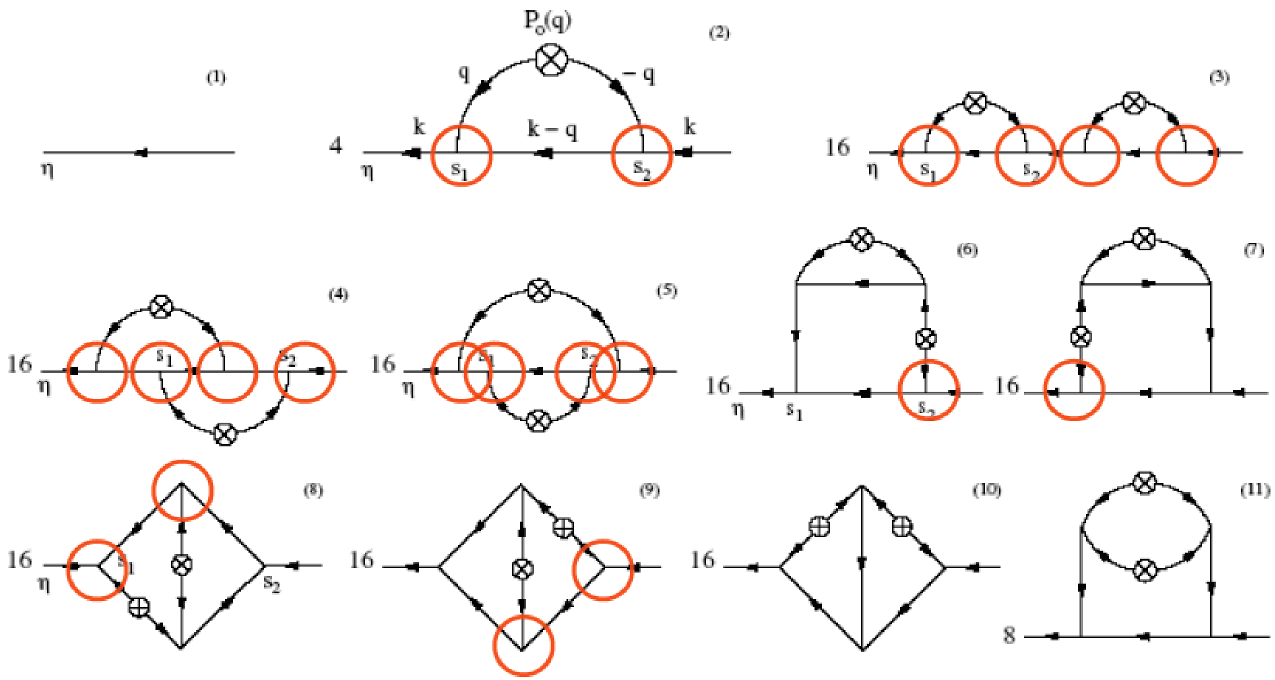


FIG. 2: Diagrams for the non linear propagator $G(k,\eta)$ up to two loops.

- The dominant contributions can be resummed exactly in the high k limit.

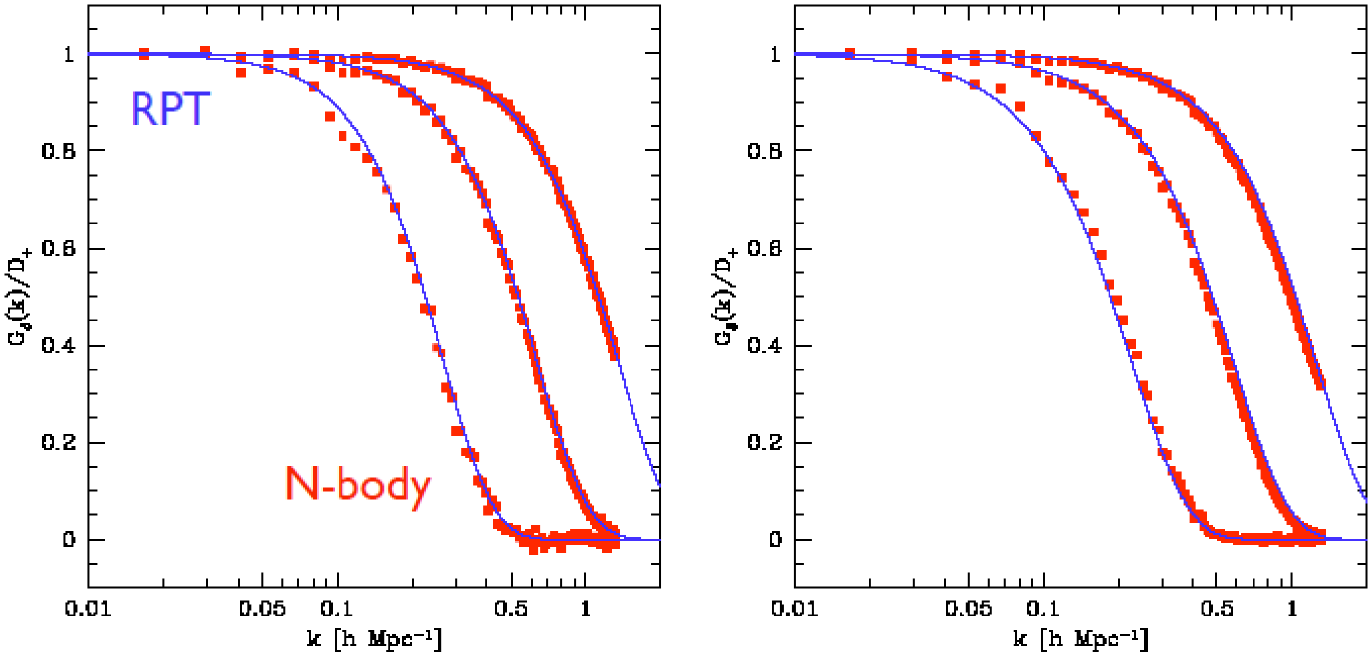
Crocce and Scoccimarro 05

$$G_{ab}(k,\eta) \simeq g_{ab}(\eta)\ \exp\left(-\frac{1}{2}k^2\sigma_v^2(e^\eta-1)^2\right)$$

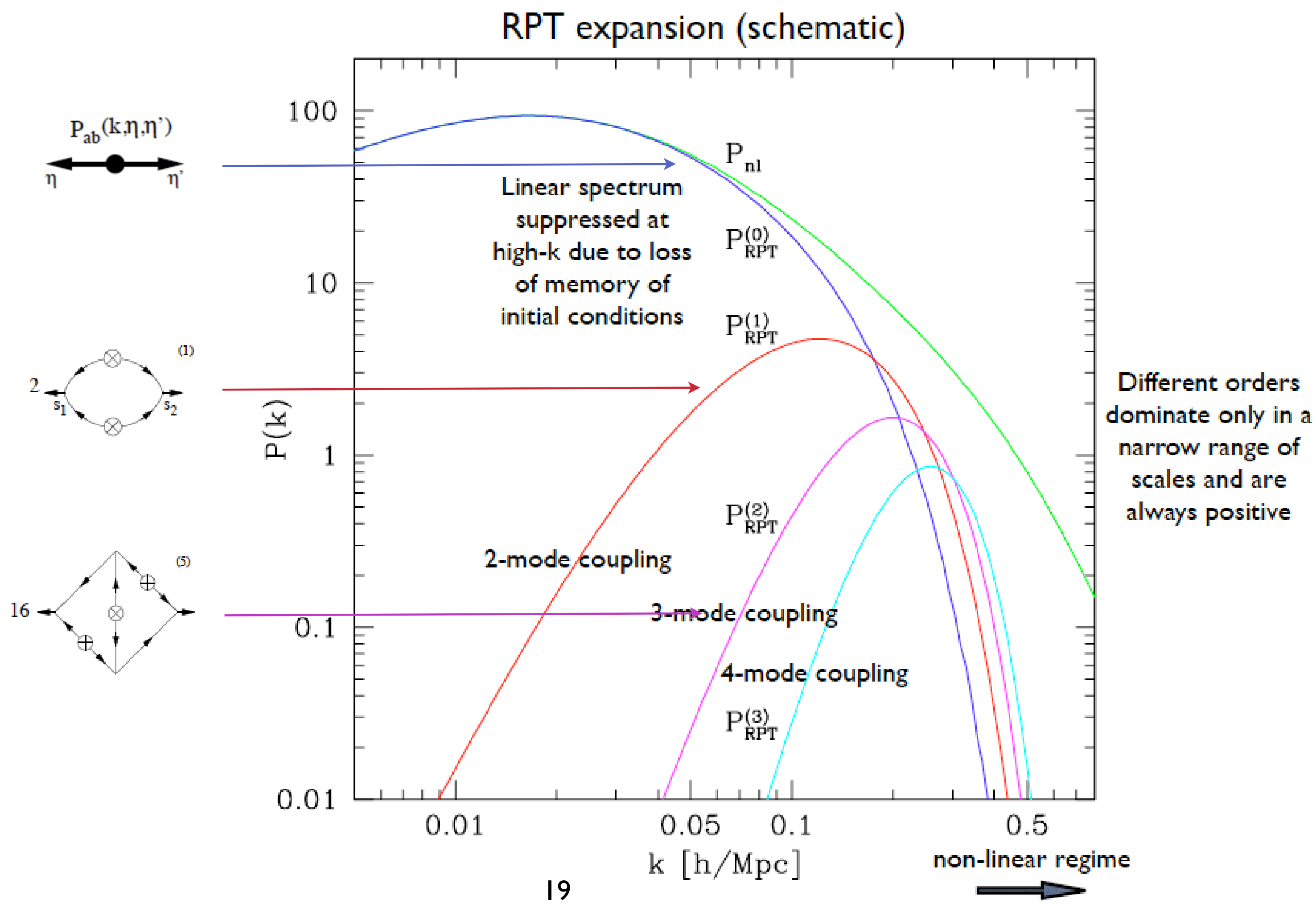
(high-k limit)

$$\sigma_v^2 \equiv \frac{1}{3}\int d^3q\frac{P(q)}{q^2}$$

Comparison between RPT and N-Body Simulations
(z = 0, 2, 5)



- RPT (Scoccimarro and Crocce) consists in standard PT when $g \to G$

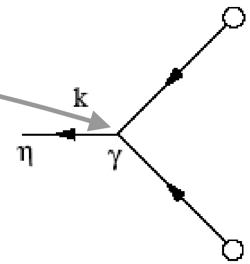


Insights into higher order propagators

FB, Crocce, Scoccimarro, '08

▸Towards a complete "renormalisation" of PT ?

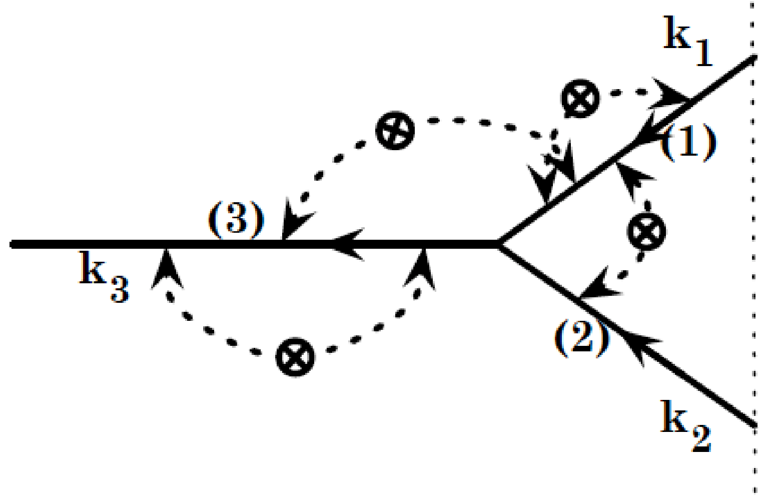
The next thing to look at is the vertex ...



What we found is that these are the "p-point propagator" that can be "renormalized"

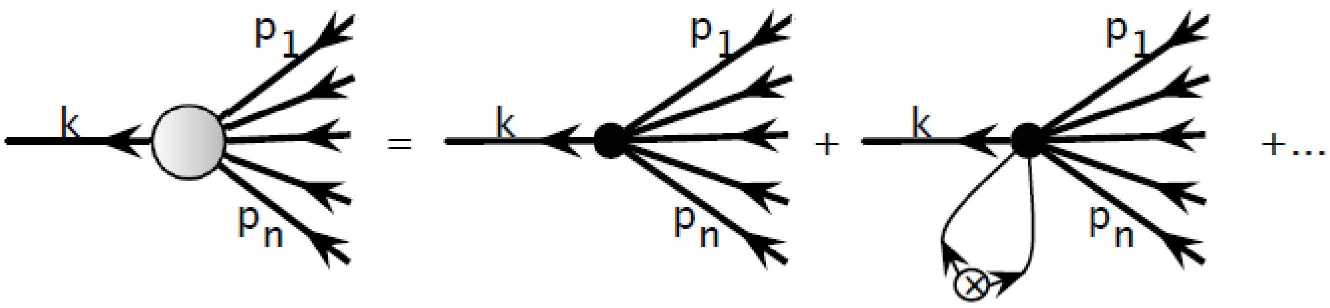
$$\frac{1}{p!}\left\langle\frac{\delta^p\Psi_a(\mathbf{k},\eta)}{\delta\phi_{b_1}(\mathbf{k}_1)\dots\delta\phi_{b_p}(\mathbf{k}_p)}\right\rangle = \delta_{\rm D}(\mathbf{k}-\mathbf{k}_{1\dots p})\Gamma^{(p)}_{ab_1\dots b_p}(\mathbf{k}_1,\dots,\mathbf{k}_p,\eta)$$

$\Gamma^{(2)}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$



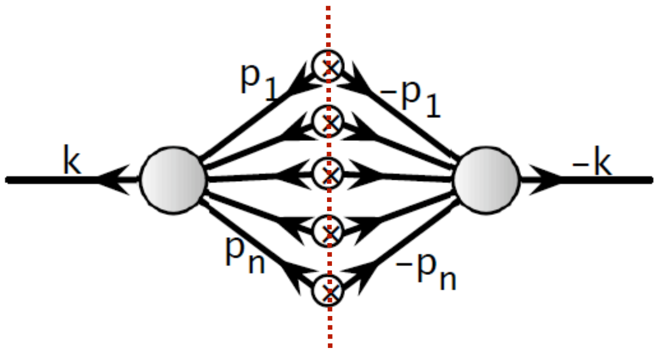
- This suggests another scheme : use the n-point propagators as the building blocks

$$\Gamma^{(n)}(k,p_1,\dots,p_n)=$$



- The reconstruction of the power spectrum :

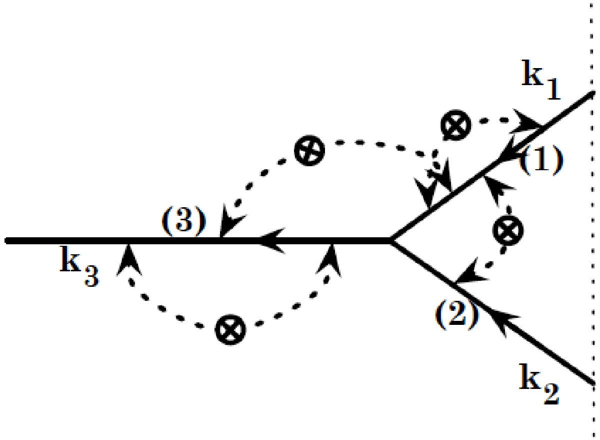
$P(k) = \Sigma_n$



Sum of positive terms

FIG. 3: Reconstruction of the power spectrum out of transfer functions. The crossed circles represent the initial power spectrum. The sum runs over the number of internal connecting lines, e.g. the number of such circles. It is to be noted that each term of this sum is positive.

- *Calculation of renormalized vertex in high k limit*



if p_{ij} is the number of lines connecting the segment (i) to (j)

$$\Gamma^{(2)}_{abc,\{p_{ij}\}} = \frac{s_{\{p_{ij}\}}}{\mathcal{M}_{\{p_{ij}\}}}\left(-\frac{\sigma_v^2}{4}\right)^{\sum_{i\leq j} p_{ij}} \prod_i k_i^{2p_{ii}} \prod_{i<j} (\mathrm{k}_i.\mathrm{k}_j)^{p_{ij}} \int_0^s \mathrm{d}s' g_{ad}(s-s')\gamma_{def}(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k}_3) g_{eb}(s') g_{fc}(s')$$
$$\times \left(e^{s'}-1\right)^{2p_{11}+2p_{22}+2p_{12}+p_{13}+p_{23}} \left(e^s - e^{s'}\right)^{2p_{33}+p_{13}+p_{23}} .$$

$$s_{\{p_{ij}\}} = 2^{2\sum_{i\leq j} p_{ij}}$$

$$\mathcal{M}(p_{ii}) = 2^{p_{ii}} p_{ii}!, \text{ and } \mathcal{M}(p_{ij}) = p_{ij}! \text{ if } i \neq j$$

$$\boxed{\Gamma^{(2)}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \exp\left(-\frac{\sigma_v^2 k_3^2}{2}\left(e^s-1\right)^2\right)\Gamma^{(2)}_{abc,\mathrm{tree}}}$$

- *It implies that the vertex cannot be "renormalized" (into an operator which is local in time)*

Comparison with numerical simulations

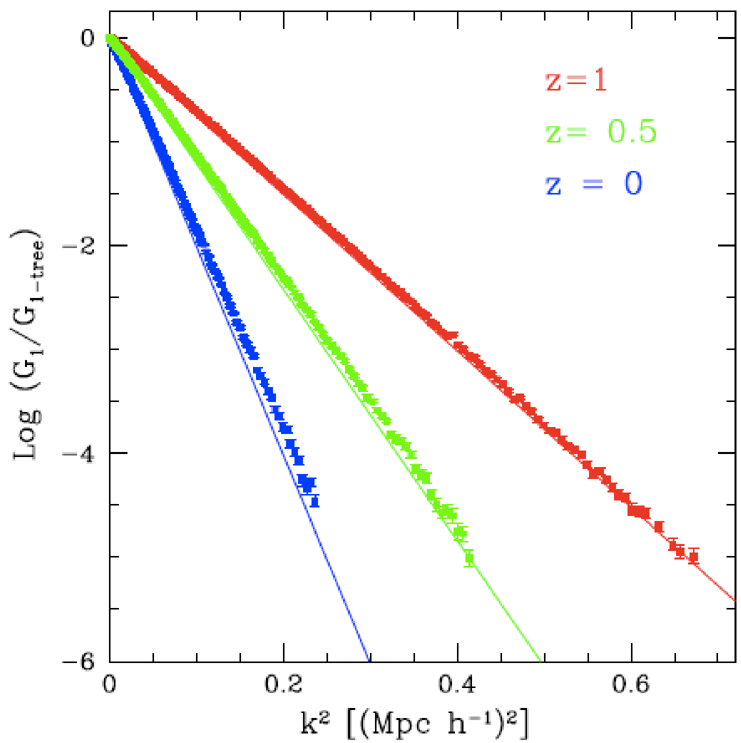


FIG. 4: The large-k limit of the two-point density propagator $\Gamma^{(1)}$. Symbols correspond to measurements in numerical simulations at redshifts $z = 1, 0.5$ and $z = 0$ (top to bottom), see text for details. The solid lines correspond to the large-k limit expression given in Eq. (25). The linear relation obtained by plotting $\log G$ vs. k^2 makes evident that the suppression of G is indeed Gaussian in the high-k limit. Moreover, the slope is very well predicted by Eqs. (25,26).

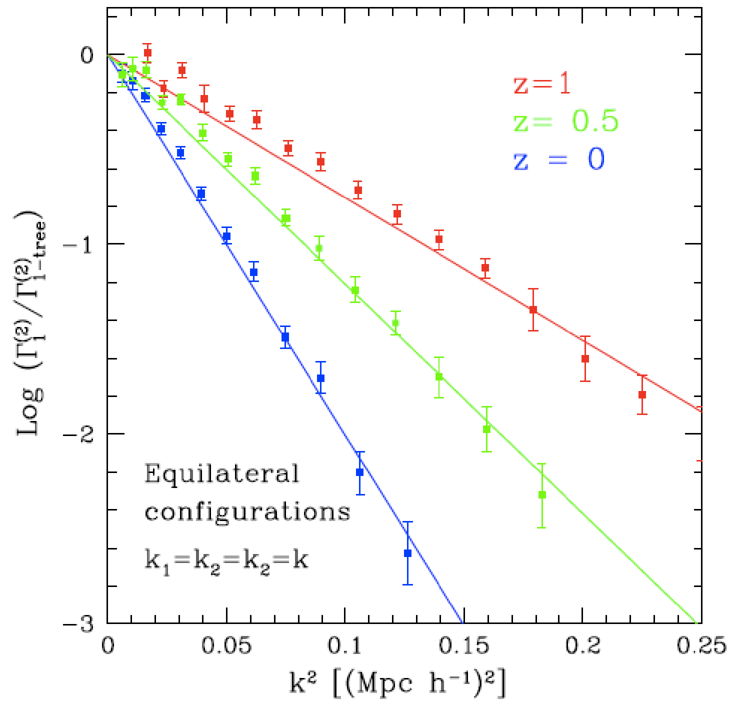
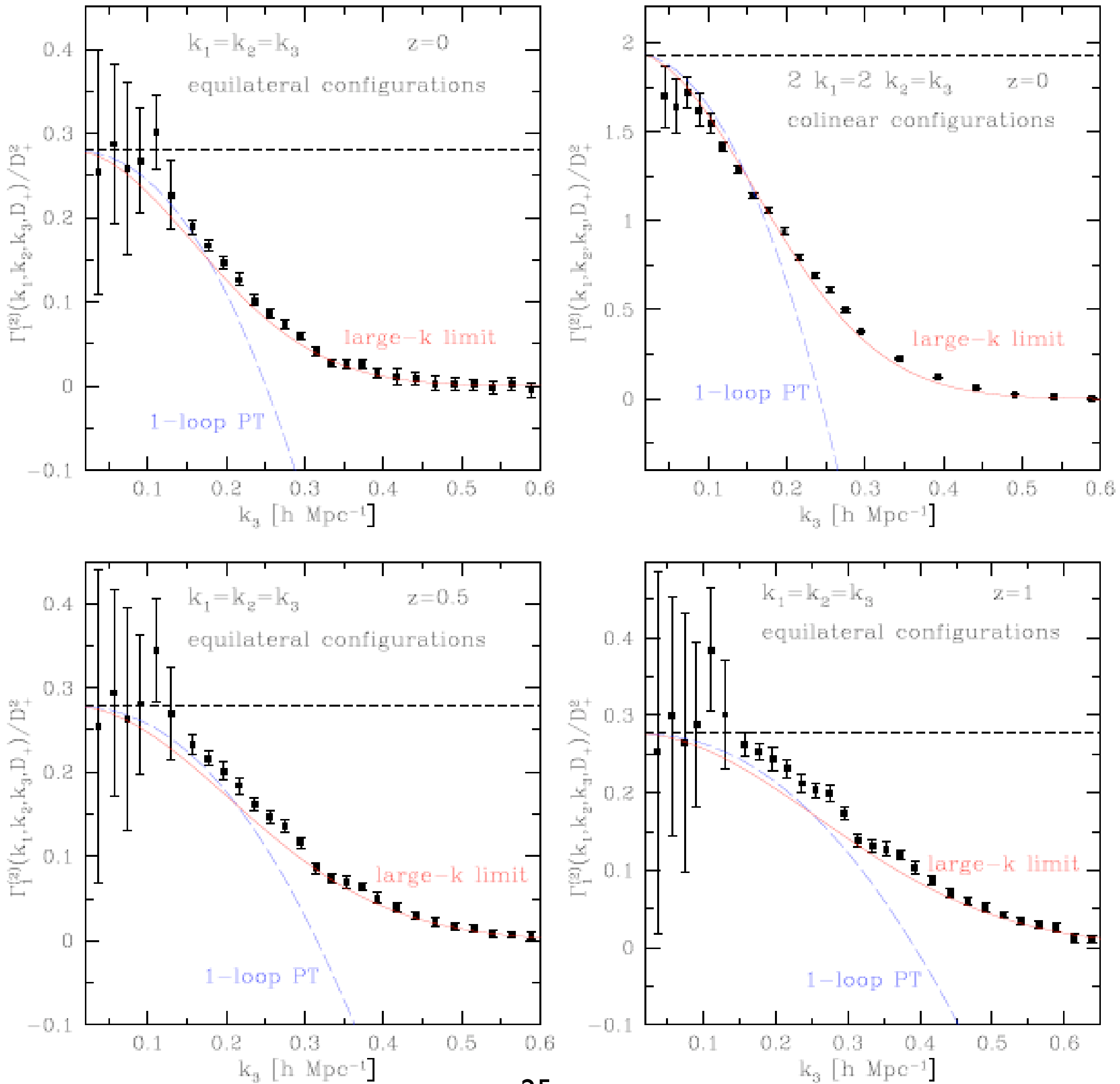
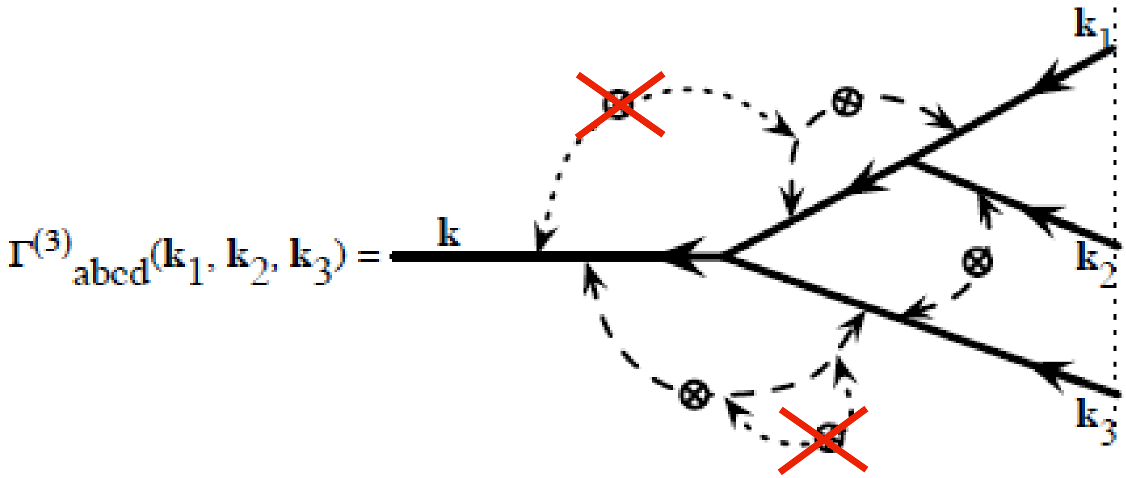


FIG. 8: The large-k limit of the three-point density propagator $\Gamma_1^{(2)} \equiv \Gamma_{1bc}^{(2)} u_b u_c$, the only density contraction that can be measured for growing mode initial conditions, $u_b = (1, 1)$. The symbols in the figure correspond to equilateral configurations at redshifts $z = 1, 0.5, 0$ (from top to bottom). We have normalized these measurements to its low-k limit $\Gamma_{1,\mathrm{tree}}^{(2)}$ given by Eq. (20). The figure clearly shows that the measured propagator closely follows the large-k limit given by Eq. (37) represented by solid lines, once $\Gamma_1^{(2)}$ decays by $\approx e^{-1}$ from its tree-level value.

Comparison with numerical simulations



- Re-summation can be extended to any order



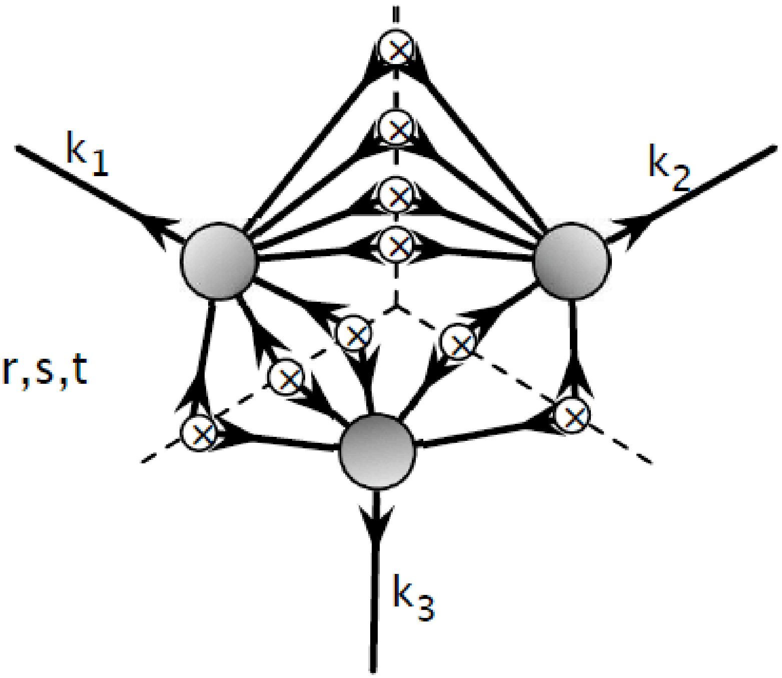
In the large k limit we have :

$$\Gamma^{(p)} = \exp\left[-\frac{|\mathbf{k}_1 + \ldots \mathbf{k}_p|^2 \sigma_v^2}{2}(e^s - 1)^2\right] \Gamma^{(p)}_{\rm tree}$$

Conclusions (1)

- Does it speed up the convergence for the reconstruction of P(k) ?
- Also provide the building blocks for higher order moments...

$$B(k_1,k_2,k_3) = \Sigma_{r,s,t}$$



- Is this exponential cutoff really physical ?

From Eulerian to Lagrangian space

FB, Valageas, '08

Are the large-scale modes disrupting the forming halos or simply moving them away?

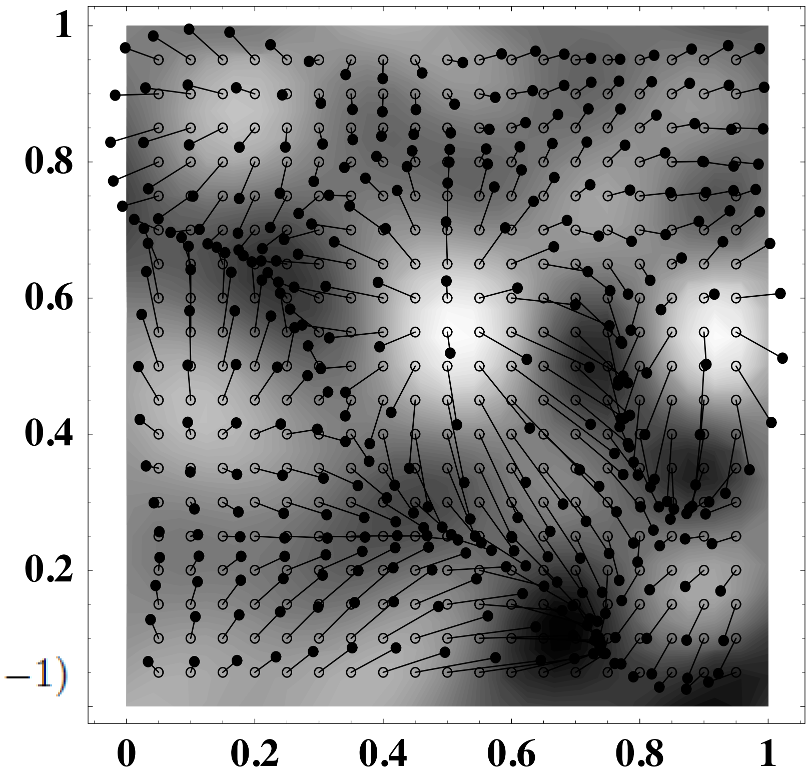
Insights into the hidden math..

FB, Valageas, '08

- The same thing in Lagrangian space ...

$$\mathbf{x} = \mathbf{q} + \Psi(\mathbf{q}, t),$$

Notably more difficult because displacement is not potential beyond 3rd order



$$\frac{\partial^2 \mathbf{x}(\mathbf{q})}{\partial \tau^2} + \mathcal{H} \frac{\partial \mathbf{x}(\mathbf{q})}{\partial \tau} = -\nabla_{\mathbf{x}} \phi(\mathbf{q})$$

$$J(\mathbf{q})\, \nabla_{\mathbf{x}} . \left[\Psi''(\mathbf{q}) + \left(\frac{3\Omega_{\rm m}}{2f^2} - 1 \right) \Psi'(\mathbf{q}) \right] = \frac{3\Omega_{\rm m}}{2f^2} (J(\mathbf{q}) - 1)$$

$$\nabla_{\mathbf{x}} \times \left[\frac{\partial^2 \Psi(\mathbf{q})}{\partial \tau^2} + \mathcal{H} \frac{\partial \Psi(\mathbf{q})}{\partial \tau} \right] = 0$$

Motion equations for the displacement divergence and vector component (2D dynamics)

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\partial \chi}{\partial q_1} + \frac{\partial \lambda}{\partial q_2} \\ \frac{\partial \chi}{\partial q_2} - \frac{\partial \lambda}{\partial q_1} \\ 0 \end{pmatrix} = \nabla_{\mathbf{q}}.\chi + \nabla_{\mathbf{q}} \times (\lambda\, \mathbf{e}_3).$$

convergence : $\kappa = -\nabla_{\mathbf{q}}^2 \chi$ vorticity : $\omega = -\nabla_{\mathbf{q}}^2 \lambda$

- motion equations (no shell crossing)

$$\kappa'' + \frac{1}{2}\kappa' - \frac{3}{2}\kappa = \int \mathrm{d}\mathbf{k}_1 \mathrm{d}\mathbf{k}_2\, \delta_{\mathrm{D}}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k})$$
$$\times \left\{ \alpha(\mathbf{k}_1, \mathbf{k}_2) \left[\kappa_1 (\kappa_2'' + \frac{1}{2}\kappa_2' - \frac{3}{4}\kappa_2) + \omega_1 (\omega_2'' + \frac{1}{2}\omega_2' - \frac{3}{4}\omega_2) \right] \right.$$
$$\left. + \beta(\mathbf{k}_1, \mathbf{k}_2) \left[\omega_1 (\kappa_2'' + \frac{1}{2}\kappa_2') - \kappa_1 (\omega_2'' + \frac{1}{2}\omega_2') + \frac{3}{2}\kappa_1 \omega_2 \right] \right\} \quad (32)$$

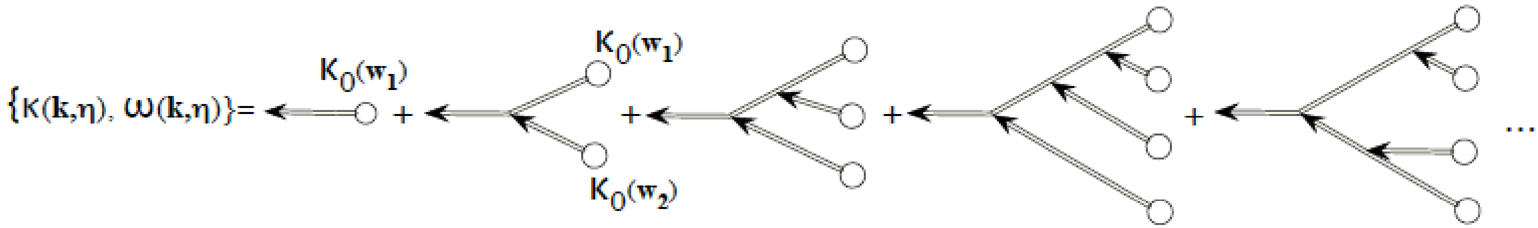
$$\omega' = \int \mathrm{d}\mathbf{k}_1 \mathrm{d}\mathbf{k}_2 \delta_{\mathrm{D}}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \left\{ \alpha(\mathbf{k}_1, \mathbf{k}_2) [\kappa_1 \omega_2' - \omega_1 \kappa_2'] + \beta(\mathbf{k}_1, \mathbf{k}_2) [\kappa_1 \kappa_2' + \omega_1 \omega_2'] \right\}.$$

Kernels are homogeneous in k

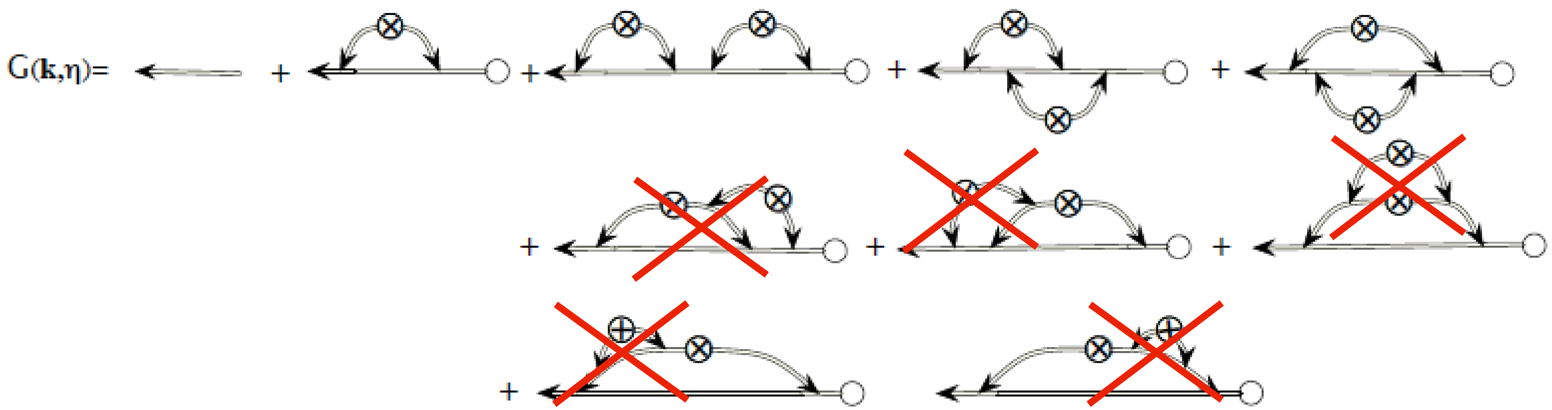
$$\alpha(\mathbf{k}_1, \mathbf{k}_2) = \frac{\det(\mathbf{k}_1, \mathbf{k}_2)^2}{k_1^2 k_2^2},$$
$$\beta(\mathbf{k}_1, \mathbf{k}_2) = \frac{(\mathbf{k}_1.\mathbf{k}_2) \det(\mathbf{k}_1, \mathbf{k}_2)}{k_1^2 k_2^2},$$

→ ω *vanishes at linear and second order*

Expansion for the fields



Expansion for the propagator



- *incoming modes are all in linear regime, therefore of κ type only;*
- *there are two types of kernels, α or β*

- Direct re-summation seems impossible...
- but it is possible to map this statistical problem (with ensemble average to be done with a set of continuous variables) into a problem with only a finite set of variables (2 in 2D, 5 in 3D) assuming the incoming modes are in linear regime.
- Incoming modes correspond to a collection of modes that are Gaussian distributed

$$\hat{\alpha}(\mathbf{k}) = \int \mathrm{dw}\ \kappa_0(\mathbf{w})\alpha(\mathbf{k},\mathbf{w})$$
$$\hat{\beta}(\mathbf{k}) = \int \mathrm{dw}\ \kappa_0(\mathbf{w})\beta(\mathbf{k},\mathbf{w})$$

$$\langle\hat{\alpha}^2\rangle = 3\sigma_2^2,\quad \langle\hat{\beta}^2\rangle = \sigma_2^2,\quad \langle\hat{\alpha}\hat{\beta}\rangle = 0$$

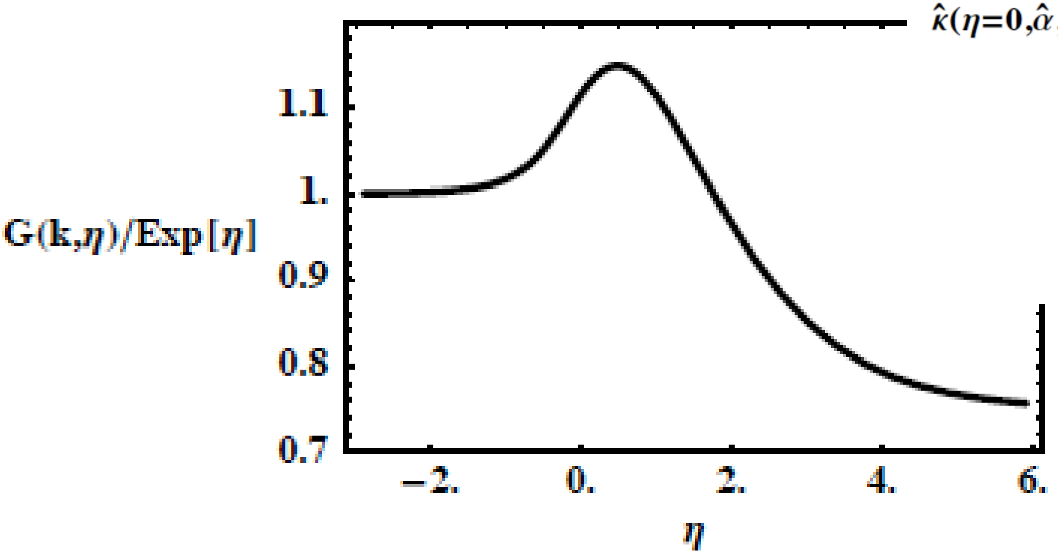
- For the diagrams we want to compute, κ and ω depend on the other modes through α and β only,

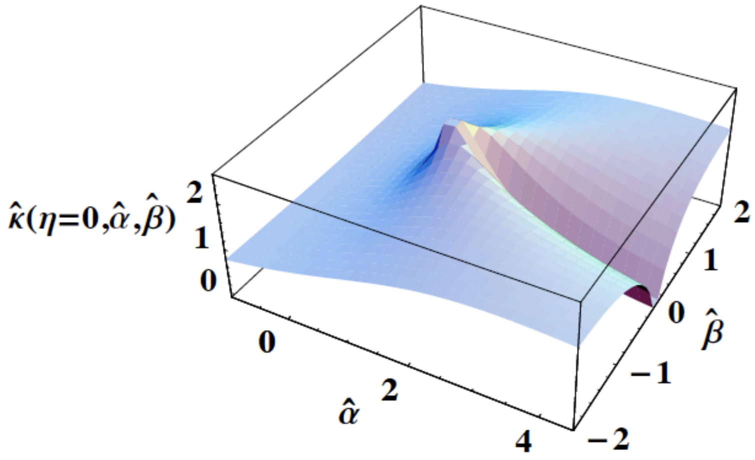
$$\begin{aligned}\kappa''(\mathbf{k},\eta) + \frac{1}{2}\kappa'(\mathbf{k},\eta) - \frac{3}{2}\kappa(\mathbf{k},\eta) &= e^{\eta}\hat{\alpha}(\mathbf{k})\left(\kappa''(\mathbf{k},\eta) + \frac{1}{2}\kappa'(\mathbf{k},\eta)\right) + e^{\eta}\hat{\beta}(\mathbf{k})\left(\omega''(\mathbf{k},\eta) + \frac{1}{2}\omega'(\mathbf{k},\eta)\right)\\ \omega'(\mathbf{k},\eta) &= -e^{\eta}\hat{\beta}(\mathbf{k})(\kappa'(\mathbf{k},\eta) - \kappa(\mathbf{k},\eta)) + e^{\eta}\hat{\alpha}(\mathbf{k})(\omega'(\mathbf{k},\eta) - \omega(\mathbf{k},\eta)).\end{aligned}$$

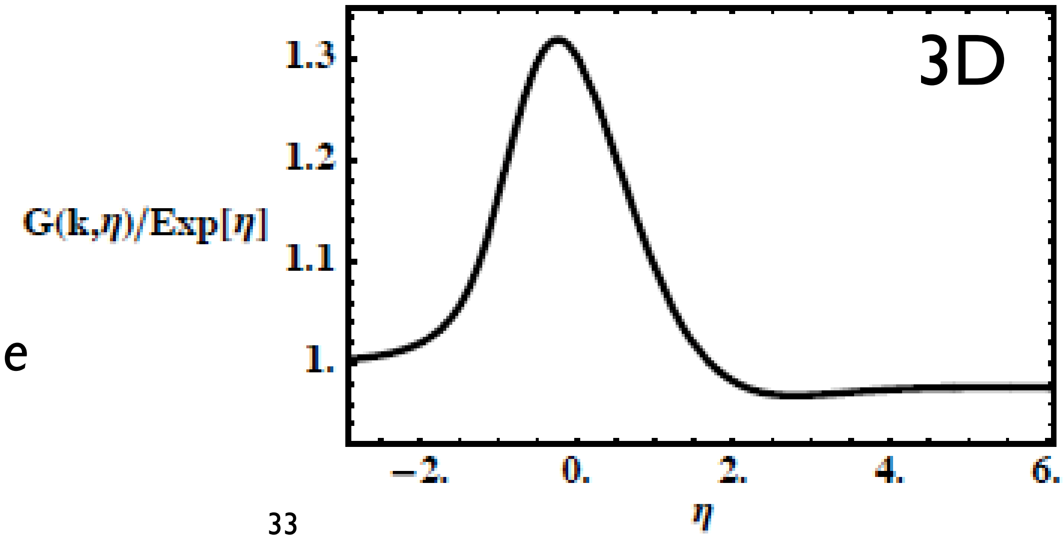
- More specifically the propagator is given by,

$$\hat{G}(\eta) = \int_{-\infty}^{\infty} \hat{\kappa}(\eta;\hat{\alpha},\hat{\beta})\mathcal{P}(\hat{\alpha},\hat{\beta})\,\mathrm{d}\hat{\alpha}\,\mathrm{d}\hat{\beta},$$

Behavior of G(k,η)







➪ *How large scale inhomogeneities affect the growth of structure*

Conclusions (2)

- New "exact" results and new computational technique;
- Many ways of implementing PT calculations (RPT, this approach, closure theory, etc.) but what is the most effective way is yet unclear;
- Lagrangian versus Eulerian calculations give new insights but the validity regime of the former needs to be assess.