Duality cascade of softly broken supersymmetric theories

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1.Introduction

Supersymmetry (SUSY): good candidate for new physics.

Ex. Matter content of Minimal Supersymmetric Standard Model (MSSM)

Gauge group : $SU(3)_C \times SU(2)_L \times U(1)_Y$

From hep-ph/9709356

SUSY

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \ \ \widetilde{d}_L)$	$(u_L \ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
($\times$3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \ \ \widetilde{e}_L)$	$(\nu \ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
($\times$3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H^+_u \ \ H^0_u)$	$(\widetilde{H}^+_u \ \ \widetilde{H}^0_u)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H^0_d \ \ H^-_d)$	$(\widetilde{H}^0_d \ \ \widetilde{H}^-_d)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

1.Introduction

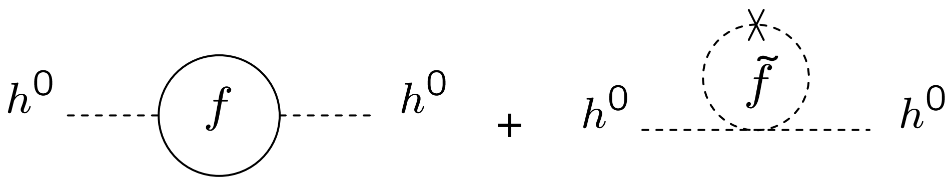
Supersymmetry (SUSY): good candidate for new physics.

Ex. Matter content of Minimal Supersymmetric Standard Model (MSSM)

Gauge group : $SU(3)_C \times SU(2)_L \times U(1)_Y$

- Solution of fine tuning problem $m^2_{Higgs} = (m^2_{Higgs})_0 + \delta m^2_{Higgs}$

1-loop correction to Higgs mass δm^2_{Higgs} :



$\sim m^2_{soft\ breaking} = \mathcal{O}(100GeV)^2$

$-\Lambda^2_{UV}$ $\qquad$ $\Lambda^2_{UV} + m^2_{soft\ breaking}$

- Coupling unification, dark matter (with R-parity).

For this talk,

- Approach to strong coupling gauge theory

Holomorphy, duality. (Calculability)

Motivation (why we want to look at duality)

Our original motivation is to build models of particle physics based on the string theory. Heckman, Vafa, Verlinde, Wijnholt

Because Ramond-Ramond tadpole (anomaly cancellation) condition often requires many D-branes, we tend to have very large gauge group naturally.

Example : D3-brane tadpole in F-theory compactification

$$N_{D3} + N_{flux} = \frac{\chi(Y_4)}{24}$$

$\chi(Y_4)$: Euler number of Calabi-Yau 4-fold

N_{flux}: background 3-form flux

$\mathcal{O}(10-10^3)$

Naturally, we may find SU(100) gauge group. $\longleftrightarrow$ Standard model (SM) SU(3)×SU(2)×U(1).

How can we get small rank of gauge group naturally?

Ref. probability (open string landscape) to get SM gauge group with proper matter content in toroidal intersecting D6-brane system;

$\sim 10^{-9}$.

Gmeiner et al.

Seiberg duality (we will see later again) $\boxed{3N/2 < N_f < 3N}$

It is known that $\mathcal{N}=1$ SU(N) SUSY QCD (SQCD) with Nf flavor have a non-trivial fixed point in the infrared regime and a dual description below the scale where coupling becomes strong: we have effective action of original SQCD.

Banks and Zaks
Seiberg

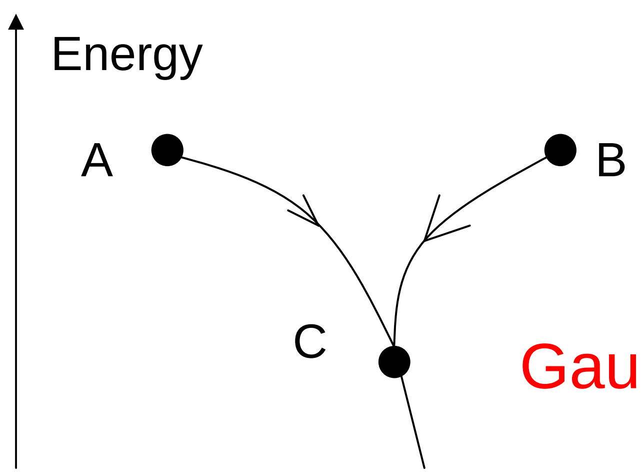
Ref. QCD ⇔ Chiral Lagrangian

A: $\underline{SU(N)}$ SQCD with N_f flavors $\qquad Q_i,\ \bar{Q}_j,\ i,j=1,2,\ldots N_f$

$\updownarrow$ Dual

B: $\underline{SU(N_f-N)}$ SQCD with N_f flavors + Mesons（∃fixed point, too）

$q_i,\ \bar{q}_j,\ M_{ij},\ i,j=1,2,\ldots N_f$



We will find the same “physics” below a fixed point C.
(gauge invariant operator)

Gauge group can be different from each other!

Seiberg duality cascade (non-conformal but have fixed points)

Klebanov and Strassler

Consider $\mathcal{N}=1$ SUSY gauge theory whose gauge groups are

$$SU(kN)\times SU((k-1)N),$$

Representation of quarks for gauge group of this model

	$SU(kN)$	$SU((k-1)N)$
Q_r	kN	$\overline{(k-1)N}$
$\bar{Q}_s$	$\overline{kN}$	$(k-1)N$

$r,s=1,2$

with quarks and superpotential which becomes mass term of meson in low energy scale $(W\sim(Q\bar{Q})^2)$.

SU(kN) can be asymptotically free (2(k-1)N flavor),
while SU((k-1)N) can be asymptotically non-free (2kN flavor).

$$g_k \;\to g_k\approx g_k^*,\;\; g_{k-1}\;\to\; 0.$$

More than 2-loop.

Suppose that we can take Seiberg duality for SU(kN).
(We need k >4 for non-abelian Coulomb phase, otherwise we will see free magnetic phase or confinement phase.)

After one takes Seiberg dual of SU(kN) theory, we get

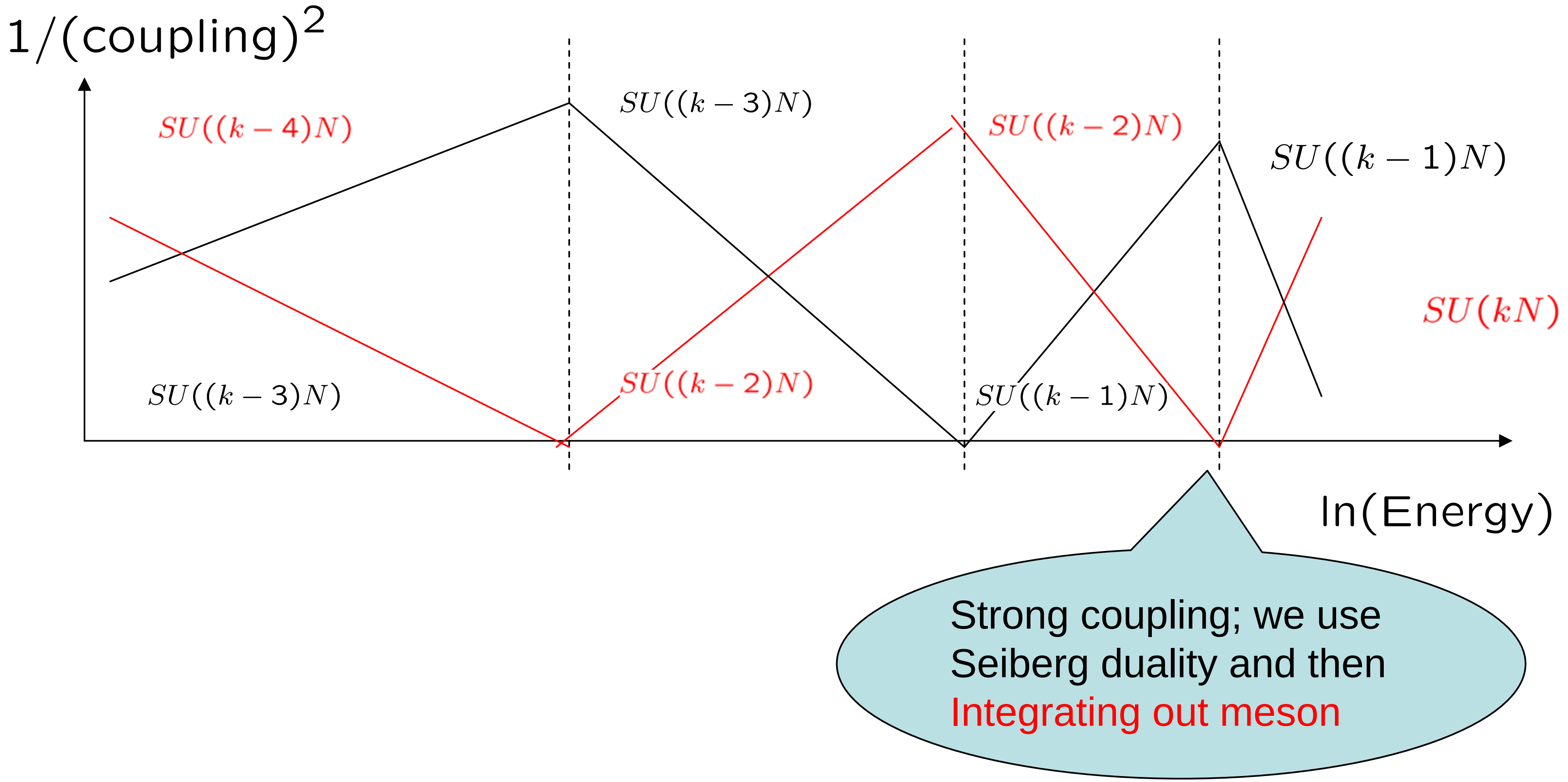
$$SU((k-2)N) \times SU((k-1)N).$$

This is similar to the previous model but we have smaller gauge group. (Meson are massive. $\longrightarrow W_{Low} \sim (q\bar{q})^2$)
This will continue as (we will see later)

Energy scale

$$SU(kN) \times SU((k-1)N),$$
$$\longrightarrow SU((k-2)N) \times SU((k-1)N).$$
$$\longrightarrow SU((k-2)N) \times SU((k-3)N).$$
$$\cdots$$
$$\longrightarrow SU(2N) \times SU(N).$$
$$\longrightarrow SU(N).$$
$$\longrightarrow \text{confined}$$

Rough illustration of running of gauge couplings



What is important is that we have smaller gauge group in low energy scale based on a duality cascade!

Perhaps, more complicated duality cascade leads to the standard model. (But no one still find it.)

$$\prod_i SU(N_i) \overset{?}{\rightarrow} SU(3) \times SU(2) \times U(1) \times \ldots$$

Many theories of Rank = O(100)

Toy model:

$$U(3) \times USp(6)^2_{L/R} \times U(1) \quad \rightarrow \quad U(3) \times USp(2)^2_{L/R} \times U(1).$$

2 times duality

This is a motivation that we want to study duality cascade.

Furthermore, when one adds SUSY breaking terms to this model, it may become more realistic because it seems that SUSY is broken in our world.

Trial for an explicit model: Uranga et al., Heckman et al. (Seiberg dual: Abel et al., Oz et al.)

Because we want to know just qualitative properties of duality cascade, in this talk we will consider

SU(kN) $\times$ SU((k-1)N) model with soft SUSY breaking terms

as a first step.

(This might be a field theoretic dual of supergravity on Klebanov-Strassler warped throat with SUSY breaking effects (anti-D3? (IASD flux?)).) Dewolfe et al., Kachru et al.

Using a spurion method (physical parameters are treated as external superfields), we will introduce soft SUSY breaking terms by hand.

What we did :

・ We study evolution of renormalization group of both supersymmetric terms and SUSY breaking terms in SU(kN)×SU((k-1)N) model of cascade under 1-loop anomalous dimension (=1-loop or 2-loop beta function).

What we found :

・Almost SUSY breaking terms are suppressd and converge to weak coupling gaugino mass in the infrared regime.

・Because of SUSY breaking term (holomorphic mass term (B-term) or non-holomorphic scalar mass term), we could find gauge symmetry breaking; the cascade could end.

Plan of talk

1 Introduction (and summary)

2.Beta functions (preparation)

3.Seiberg dual (warming up)

4.Duality cascade

5.Summary

2.Beta functions for soft SUSY breaking terms from spurion method (preparation)

Many people contributed;
Yamada, Jack et al., Avdeev et al., Kobayashi et al., Arkani-Hamed et al.,…

(For example, see review by Terao (hep-ph/0112021) references therein.)

・Physical coupling as a spurion (external superfield)

Consider $\mathcal{N}=1$ softly broken perturbative SUSY gauge theory with matter fields;

$$\mathcal{L}=\mathcal{L}_{susy}+\mathcal{L}_{soft}$$
$$=\int d^4\theta \sum_i [(1-m_i^2\theta^4)Q_i^\dagger e^{2V^{(Q)}}Q_i]+\int d^2\theta(y_{i_1\ldots i_n}-a_{i_1\ldots i_n}\theta^2)Q_{i_1}\ldots Q_{i_n}$$

$\tilde{Z}_i$: Wave function superfield

$\tilde{y}_{i_1\ldots i_n}$: n-point coupling superfield

$$+\frac{1}{4}\int d^2\theta\,(1-2M_\lambda\theta^2)\mathcal{W}^\alpha\mathcal{W}_\alpha+c.c.$$

$\dfrac{1}{\tilde{g}_c^2}$: Gauge coupling superfield

$$-\mathcal{L}_{soft}=m_i^2|\tilde{Q}_i|^2+(a_{i_1\ldots i_n}\tilde{Q}_{i_1}\ldots\tilde{Q}_{i_n}+\frac{1}{2}M_\lambda\lambda^\alpha\lambda_\alpha+c.c.).$$

$$Q=\tilde{Q}+\theta^\alpha\psi_\alpha^Q+\ldots,\ \ \mathcal{W}_\alpha=-\frac{1}{4}\bar{D}^2e^{-2V}D_\alpha e^{2V}=-i\lambda_\alpha-\frac{i}{2}F_{\mu\nu}(\sigma^{\mu\nu})_\alpha{}^\beta\theta_\beta+\ldots$$

$\tilde{Q}$: scalar, ψ_α^Q : fermion, $F_{\mu\nu}$: gauge field strength, λ_α : gaugino, θ : fermionic coordinate.

Remind that we have $\int d^4\theta \tilde{Z}_i Q_i^\dagger Q_i, \ \int d^2\theta \frac{1}{\tilde{g}_c^2} \mathcal{W}^\alpha \mathcal{W}_\alpha,$ at generic scale;

we need renormalizations of fields, then we will find

$$\frac{\tilde{Z}_i|_{\theta^4}}{Z_i} = \ln(\tilde{Z}_i)|_{\theta^4} \ \leftrightarrow \ -m_i^2$$:Non-holomorphic scalar mass,

$$\tilde{y}_{i_1 \ldots i_n}\Big|_{\theta^2} \ \leftrightarrow \ -a_{i_1 \ldots i_n}$$:holomorphic n-point scalar coupling,

$$\tilde{g}_c^{-2}|_{\theta^2} g_c^2 = -\ln(\tilde{g}_c^2)|_{\theta^2} \ \leftrightarrow \ -M_\lambda$$:gaugino mass.

Note that superfield propagator in softly broken theory is given by supersymmetric one;

$$\langle Q_i Q_i^\dagger \rangle_{soft} = (1 + \frac{1}{2} m_i^2 \theta^4) \langle Q_i Q_i^\dagger \rangle_{susy} (1 + \frac{1}{2} m_i^2 \theta^4).$$

This factor can be absorbed by redefining couplings (g_c, y) with a wave function.
Then will find (spurious) supersymmetric propagator and

$$\tilde{Z}_i = Z_i(\tilde{g}_c, \tilde{y}, \tilde{y}^\dagger).$$

We can express couplings as spurion;

$$\tilde{Z}_i(\theta,\bar{\theta}) = \tilde{Z}_{Q_i}(\theta)\tilde{Z}^{\dagger}_{Q_i}(\bar{\theta})(1 - \underline{m_i^2\theta^4}).$$

$\tilde{Z}_{Q_i}(\theta)$: holomorphic Z factor spurion superfield
(relate to renormalization of holomorphic variables
(irrelevant for this talk))

$$\boxed{\begin{aligned} &\tilde{\alpha}^{-1} = \alpha^{-1}(1 - M_\lambda\theta^2 - \bar{M}_\lambda\bar{\theta}^2 - \Delta_g\theta^4), \quad \alpha = \frac{g_c^2}{8\pi^2}. \\ &\tilde{y}_{i_1\ldots i_n} = y_{i_1\ldots i_n} - a_{i_1\ldots i_n}\theta^2 + \frac{1}{2}(m_i^2 + \ldots m_{i_n}^2)y_{i_1\ldots i_n}\theta^4 \end{aligned}}$$

$$\Delta_g = -\frac{\alpha}{1 - T_G\alpha}(\sum_i T_i m_i^2 - T_G|M_\lambda|^2) \sim \mathrm{Str}[T_i(\mathrm{mass}^2)_i].$$

We can determine this from θ^4-term in Novikov-Shifman-Vainshtein-Zakharov formula.
(This Δ relates to ε-scalar's soft mass counter term in DERD regularization)

$$\frac{8\pi^2}{\tilde{g}_c^2} = \mathrm{Re}\left(\frac{1}{\tilde{g}_h^2}\right) - \sum_i T_i \ln(\tilde{Z}_i) - T_G \ln(\tilde{g}_c^2).$$

Beta function for supersymmetric coupling

- Physical gauge coupling $\alpha = \frac{g_c^2}{8\pi^2}$

$$\frac{d\alpha}{d\ln(\mu)} = \beta_\alpha(\alpha, y, y^*) = -\frac{\alpha^2}{1 - T_G\alpha}[3T_G - \sum_i T_i(1 - \gamma_i)]$$

For SU(N), T_G = N : Dynkin index for adjoint representation,
T_i =1/2 : Dynkin index for (anti-)fundamental representation.

$$\gamma_i(g_c, y, y^*) = -\frac{d\ln(Z_i)}{d\ln(\mu)}$$: anomalous dimension for Q_i $(\overline{Q}_i)$

$$\frac{8\pi^2}{g_c^2} = \mathrm{Re}\left(\frac{8\pi^2}{g_h^2}\right) + b\ln(\Lambda/\mu) - \sum_i T_i \ln(Z_i) - T_G \ln(g_c^2)$$

- Physical n-point coupling y

$$\frac{dy_{i_1 \ldots i_n}}{d\ln(\mu)} = \beta_y(g_c, y, y^*) = \frac{1}{2}(\gamma_{i_1} + \gamma_{i_2} + \ldots \gamma_{i_n})y_{i_1 \ldots i_n}$$

μ : renormalization energy scale $\quad y_{i_1 \ldots i_n} = Z_{i_1}^{-1/2} \ldots Z_{i_n}^{-1/2} Y_{i_1 \ldots i_n} = e^{-1/2\sum_i \ln(Z_i)} y^h_{i_1 \ldots i_n}.$

・Beta functions for soft terms

We obtain them from θ^2 -terms of beta function for $\tilde{g}_c$, $\tilde{y}$ and θ^4 -terms of anomalous dimensions of superfields;

・Gaugino mass

$$\frac{dM_\lambda}{d\ln(\mu)} = D_1\left(\frac{\beta_\alpha}{\alpha}\right)$$

・Holomorphic scalar n-point coupling

$$\frac{da_{i_1\ldots i_n}}{d\ln(\mu)} = \frac{1}{2}(\gamma_{i_1} + \cdots + \gamma_{i_n})a_{i_1\ldots i_n} - [(D_1\gamma_{i_1}) + \cdots + (D_1\gamma_{i_n})]y_{i_1\ldots i_n}$$

Or $$\frac{d}{d\ln(\mu)}\left(\frac{a_{i_1\ldots i_n}}{y_{i_1\ldots i_n}}\right) = -[(D_1\gamma_{i_1}) + \cdots + (D_1\gamma_{i_n})]$$

・Non-holomorphic scalar mass

$$\frac{dm_i^2}{d\ln(\mu)} = D_2\gamma_i$$

$$D_1 = \alpha M_\lambda \frac{\partial}{\partial\alpha} - a\frac{\partial}{\partial y}$$: projection operator on θ^2 -component

$$D_2 = \bar{D}_1 D_1 + \alpha(|M_\lambda|^2 + \Delta_g)\frac{\partial}{\partial\alpha} + \frac{1}{2}(m_{i_1}^2 + \cdots + m_{i_n}^2)\left[y\frac{\partial}{\partial y} + y^*\frac{\partial}{\partial y^*}\right]$$

: projection operator on θ^4 –component

3.Seiberg dual:
N=1 SQCD and its dual theory
(warming up for duality cascade)

For example, see review by Terao (hep-ph/0112021) references therein, too.

Seiberg duality for $\mathcal{N}=1$ SQCD

A: $SU(N)$ SQCD with N_f flavors $\boxed{3N/2 < N_f < 3N}$

	$SU(N)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)$	$U(1)_R$
Q	N	N_f	$\mathbf{1}$	1	$(N_f-N)/N_f$
$\bar{Q}$	$\bar{N}$	$\mathbf{1}$	$\bar{N}_f$	-1	$(N_f-N)/N_f$

B: $SU(N_f-N)$ SQCD with N_f flavors + Mesons

	$SU(N_f-N)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)$	$U(1)_R$
q	N_f-N	$\bar{N}_f$	$\mathbf{1}$	$N/(N_f-N)$	N/N_f
$\bar{q}$	$\overline{N_f-N}$	$\mathbf{1}$	N_f	$-N/(N_f-N)$	N/N_f
M	$\mathbf{1}$	N_f	$\bar{N}_f$	0	$2(N_f-N)/N_f$

$M \sim Q\bar{Q}$ $\qquad \mathrm{dim}_{classical}[M] \equiv 1$

Lagrangian of both A and B theory at UV scale

A: $\mathcal{L}_A = \int d^4\theta [Q^\dagger e^{2V} Q + \bar{Q}^\dagger e^{-2V} \bar{Q}] + \frac{1}{4}\int d^2\theta f_A \mathcal{W}^\alpha \mathcal{W}_\alpha + c.c.$

$$W_A = 0. \qquad f = \frac{1}{g^2} - i\frac{\vartheta}{8\pi^2}$$

B: $$\mathcal{L}_B = \int d^4\theta [q^\dagger e^{2V} q + \bar{q}^\dagger e^{-2V} \bar{q} + \underline{M^\dagger M}] + \int d^2\theta \underline{y M \bar{q} q} + \frac{1}{4}\int d^2\theta f_B \mathcal{W}^\alpha \mathcal{W}_\alpha + c.c.$$

$$W_B = y M \bar{q} q.$$

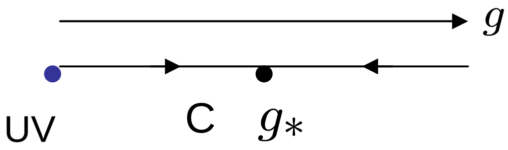
Because of SUSY, flavor symmetry and charge conjugation,all Q and $\bar{Q}$ (or q, $\bar{q}$ and Ms in dual theory) have the same anomalous dimension.

A: at an infrared fixed point C,

For example, see Strassler's review (hep-th/0505135)

$$\beta_{\alpha_A} \propto (3N - N_f(1-\gamma_Q^*)) = 0 \;\rightarrow\; \gamma_Q^* = 1 - 3N/N_f < 0.$$

$$\mathrm{dim}[\mathrm{Meson}] = \mathrm{dim}[Q\bar{Q}] = 2 + \gamma_Q^* = 3(1 - N/N_f)$$



B: at an infrared fixed point C' with $y \ll 1$,

$$\beta_{\alpha_B} \propto (3(N_f - N) - N_f(1-\gamma_q^*)) = 0 \;\rightarrow\; \gamma_q^* = -2 + 3N/N_f < 0.$$

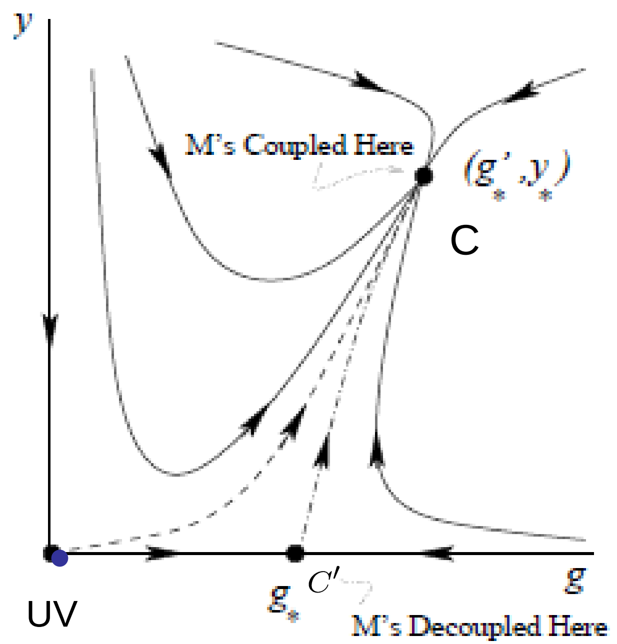
$$\beta_y = \frac{1}{2}(2\gamma_q^* + \gamma_M(y, y^*))y \approx \frac{1}{2}(2\gamma_q^*)y < 0.$$

Therefore we will find a fixed point with fixed value of y (≠0) in an infrared regime.

$$\beta_y = \frac{1}{2}(2\gamma_q^* + \gamma^*)y_* = 0$$

$$\rightarrow\; \gamma_M^* = -2\gamma_q^* = 4 - 6N/N_f > 0.$$

$$\mathrm{dim}[M] = 1 + \gamma_M^*/2 = 3(1 - N/N_f)$$



Therefore A and B can describe the same physics at fixed point C.
(Baryon, anomaly of global symmetry, moduli space, integration of heavy mode)

Consider that one adds soft SUSY breaking terms to both A and B theory at UV scale by hand (spurion scheme).

A: $$\mathcal{L}_{soft} = \int d^4\theta [(-m_Q^2\theta^4)Q^\dagger Q + (-m_{\bar{Q}}^2\theta^4)\bar{Q}^\dagger \bar{Q}] + \frac{1}{4}\int d^2\theta(-2\frac{M_A}{g_A^2}\theta^2)\,\mathcal{W}^\alpha \mathcal{W}_\alpha + c.c.$$

B: $$\mathcal{L}_{soft} = \int d^4\theta [(-m_q^2\theta^4)q^\dagger q + (-m_{\bar{q}}^2\theta^4)\bar{q}^\dagger \bar{q} + \underline{(-m_M^2\theta^4)M^\dagger M}] + \int d^2\theta \underline{(-a\theta^2)M\bar{q}q} + \frac{1}{4}\int d^2\theta(-2\frac{M_B}{g_B^2}\theta^2)\,\mathcal{W}^\alpha \mathcal{W}_\alpha + c.c.$$

We added global symmetric soft SUSY breaking terms and supposed that

$$\mu \gg m_{soft}.$$

Beta functions of A-theory: $\beta_{\alpha_A} = -\frac{\alpha_A^2}{1 - N\alpha_A}[3N - N_f(1 - \gamma_Q)]$

- Gaugino mass

$$\frac{dM_A}{d\ln(\mu)} = D_1\left(\frac{\beta_{\alpha_A}}{\alpha_A}\right) = \alpha_A M_A \frac{d}{d\alpha_A}\left(\frac{\beta_{\alpha_A}}{\alpha_A}\right)$$

- non-holomorphic soft scalar mass

$$\frac{dm_Q^2}{d\ln(\mu)} = \left(\alpha_A^2 \frac{d^2\gamma_Q}{d\alpha_A^2} + 2\alpha_A \frac{d\gamma_Q}{d\alpha_A}\right)|M_A|^2 + \Delta_A \alpha_A \frac{d\gamma_Q}{d\alpha_A}$$

$$\Delta_A = \frac{\alpha_A}{1 - N\alpha_A}\left[N|M_A|^2 - N_f \frac{(m_Q^2 + m_{\bar{Q}}^2)}{2}\right]$$

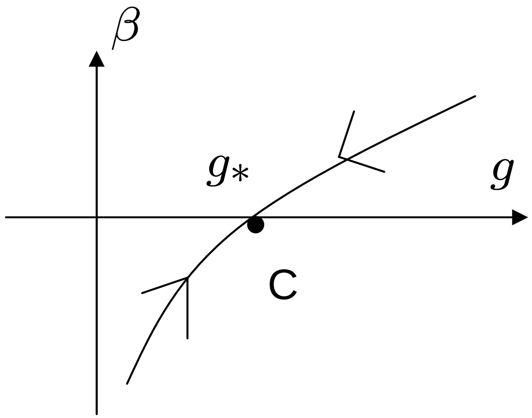
Sum of non-holomorphic soft scalar mass

$$\frac{d(m_Q^2 + m_{\bar{Q}}^2)}{d\ln(\mu)} = 2\left(\alpha_A^2 \frac{d^2\gamma_Q}{d\alpha_A^2} + 2\alpha_A \frac{d\gamma_Q}{d\alpha_A}\right)|M_A|^2 + 2\Delta_A \alpha_A \frac{d\gamma_Q}{d\alpha_A}$$

Around an infrared attractive fixed point C at scale μ ,
we will find $\beta_{\alpha_A}(\alpha_A \approx \alpha_A^*) \approx 0$, then also obtain

$$\frac{dM_A}{d\ln(\mu)} = \alpha_A M_A \frac{d}{d\alpha_A}\left(\frac{\beta_{\alpha_A}}{\alpha_A}\right) \approx M\left(\left.\frac{d\beta_{\alpha_A}}{d\alpha_A}\right|_{\alpha_A \approx \alpha_A^*}\right) \equiv \Gamma M$$

$$\Gamma = \left.\frac{d\beta_{\alpha_A}}{d\alpha_A}\right|_{\alpha_A \approx \alpha_A^*} = -\frac{N_f(\alpha_A^*)^2}{1 - N\alpha_A^*}\left(\left.\frac{d\gamma_Q}{d\alpha_A}\right|_{\alpha_A \approx \alpha_A^*}\right) > 0.$$



$$M_A(\mu) = M_A(\mu_0)\left(\frac{\mu}{\mu_0}\right)^{\Gamma} \xrightarrow{\mu \to 0 \text{ (low energy)}} 0\ !$$

Note that $\alpha M/\beta_\alpha$ is RG-invariant.

Hisano and Shifman

$$\frac{d(m_Q^2+m_{\bar{Q}}^2)}{d\ln(\mu)} = 2\left(\alpha_A^2\frac{d^2\gamma_Q}{d\alpha_A^2}+2\alpha_A\frac{d\gamma_Q}{d\alpha_A}\right)|M_A|^2+2\Delta_A\alpha_A\frac{d\gamma_Q}{d\alpha_A}$$
$$\approx \Gamma(m_Q^2+m_{\bar{Q}}^2)-2\Omega|M_A|^2$$

$$\Omega = -\left(2\alpha_A^*+\frac{N(\alpha_A^*)^2}{1-N\alpha_A^*}\right)\left(\left.\frac{d\gamma_Q}{d\alpha_A}\right|_{\alpha_A\approx\alpha_A^*}\right)-(\alpha_A^*)^2\left(\left.\frac{d^2\gamma_Q}{d\alpha_A^2}\right|_{\alpha_A\approx\alpha_A^*}\right)$$

Around the region where $|M_A|^2 \ll (m_Q^2+m_{\bar{Q}}^2)$, we will find

$$(m_Q^2+m_{\bar{Q}}^2)(\mu) = (m_Q^2+m_{\bar{Q}}^2)(\mu_0)\left(\frac{\mu}{\mu_0}\right)^{\Gamma} \xrightarrow{\mu\,\to 0} 0\,!$$

$$(\ \Delta_A \supset \sum_i T_i m_i^2 \ \longrightarrow\ 0.\ )$$

The assumption that there is the region can be justified.

$$\because\ \frac{d\sigma}{d\ln(\mu)} = -\Gamma\sigma-2\Omega, \quad \sigma\equiv\left(\frac{(m_Q^2+m_{\bar{Q}}^2)}{|M_A|^2}\right)$$

$$\sigma \sim \left(\frac{\mu}{\mu_0}\right)^{-\Gamma}+\text{constant} \xrightarrow{\mu\,\to 0} \infty$$

Beta functions of B-theory:

$$\beta_{\alpha_B} = -\frac{\alpha_B^2}{1 - N\alpha_B}[3(N_f - N) - N_f(1-\gamma_q)], \qquad \beta_y = \frac{y}{2}(2\gamma_q + \gamma_M)$$

For convenience, we see the following beta functions

$$\frac{d(\alpha_B M_B)}{d\ln(\mu)} = \beta_{\alpha_B} M_B + \alpha_B D_1\left(\frac{\beta_{\alpha_B}}{\alpha_B}\right), \quad D_1 = \left(\alpha_B M_B \frac{\partial}{\partial \alpha_B} - a\frac{\partial}{\partial y}\right)$$

$$\frac{d(y^\dagger a)}{d\ln(\mu)} = a y^\dagger(2\gamma_q + \gamma_M) - |y|^2 D_1(2\gamma_q + \gamma_M)$$

and define a square of Yukawa coupling $\alpha_y = |y|^2/8\pi^2$.
Around a infrared attractive fixed point C, we find

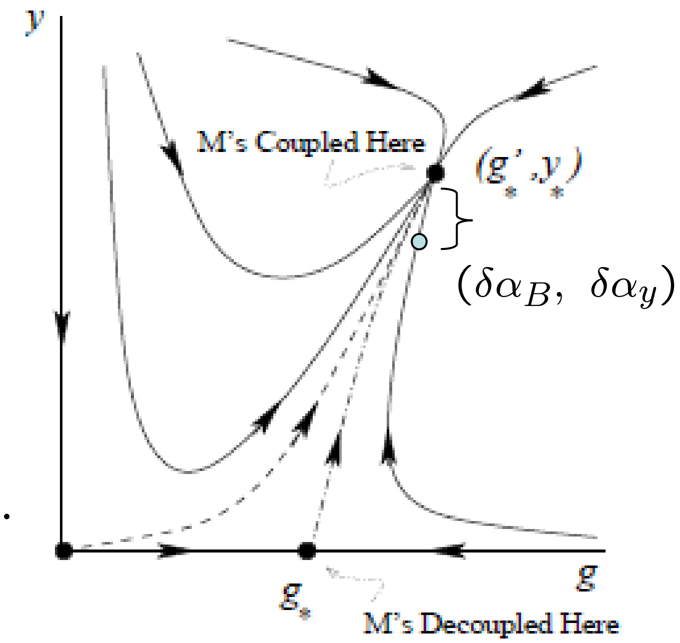
$$\beta_{\alpha_B}(\alpha_B \approx \alpha_B^*) \approx \beta_y(y \approx y_*) \approx 0,$$

$$\frac{d}{d\ln(\mu)}\begin{pmatrix} \alpha_B^* M_B \\ -y_*^\dagger a \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial\beta_{\alpha_B}}{\partial\alpha_B}\right|_* & \left.\frac{\partial\beta_{\alpha_B}}{\partial\alpha_y}\right|_* \\ \left.\frac{\partial\beta_{\alpha_y}}{\partial\alpha_B}\right|_* & \left.\frac{\partial\beta_{\alpha_y}}{\partial\alpha_y}\right|_* \end{pmatrix}\begin{pmatrix} \alpha_B^* M_B \\ -y_*^\dagger a \end{pmatrix}.$$

Notice that when one sees small deviation from a fixed point

$$\delta\alpha \equiv \alpha_B - \alpha_B^*, \qquad \delta\alpha_y \equiv \alpha_y - \alpha_y^*$$

these deviation satisfy the same equation as soft terms.
(Of course, A-theory, too.)
Infrared attractive nature implies that matrix is positive definite.



$$\delta\alpha_B,\ \delta\alpha_y \xrightarrow{\mu\ \to 0} 0$$

$$\updownarrow \qquad \begin{pmatrix} \tilde{\alpha}_B|_{\theta^2} \\ \tilde{\alpha}_y|_{\theta^2} \end{pmatrix} = \begin{pmatrix} \alpha_B M_B \\ -y^\dagger a \end{pmatrix}.$$

$$M_B,\ a \xrightarrow{\mu\ \to 0} 0!$$

Note that $\dfrac{\alpha M}{\beta_\alpha} = -\dfrac{\alpha_y}{\beta_{\alpha_y}}\left(\dfrac{a}{y}\right) = \mathrm{RG-invariant.}$

When one looks at θ^4 –components of (α , α y),
we can find that these components also satisfy the same differential equation. Therefore

$$\tilde{\alpha}|_{\theta^4} = \Delta_B + \mathcal{O}(|M_B|^2) \sim \sum_i T_i m_i^2 \propto m_q^2 + m_{\bar{q}}^2$$

$$\tilde{\alpha}_y|_{\theta^4} = \alpha_y^*(m_q^2 + m_{\bar{q}}^2 + m_M^2) + \mathcal{O}(|a|^2) \sim m_q^2 + m_{\bar{q}}^2 + m_M^2.$$

$\longrightarrow$ 0 !

This means

$$(m_q^2 + m_{\bar{q}}^2),\ m_M^2 \ \longrightarrow\ 0.$$

$$\left(\ \sum_i T_i m_i^2, \quad m_q^2 + m_{\bar{q}}^2 + m_M^2 \longrightarrow\ 0.\ \right)$$

How about duality cascade case?: not conformal
but almost conformal.

4.Duality cascade

Strassler's review (hep-th/0505135)

Duality cascade

Anomaly free symmetries:

$$SU(kN)\times SU((k-1)N)\times SU(2)\times SU(2)\times U(1)_B\times \mathbf{Z}_{2N}^{(R)}$$

Global symmetries

	$SU(kN)$	$SU((k-1)N)$	$SU(2)$	$SU(2)$	$U(1)_B$	$\mathbf{Z}_{2N}$
Q_r	kN	$\overline{(k-1)N}$	2	1	$\frac{1}{(N^2k(k-1))}$	1/2
$\bar{Q}_s$	$\overline{kN}$	$(k-1)N$	1	2	$\frac{-1}{(N^2k(k-1))}$	1/2

$r,s=1,2$

Superpotential:

$$W=h\,\underset{r,s}{\mathrm{trdet}}(Q_r\bar{Q}_s)$$
$$=h\left[(Q_1)_a^\alpha(\bar{Q}_1)_\beta^a(Q_2)_b^\beta(\bar{Q}_2)_\alpha^b-(Q_1)_a^\alpha(\bar{Q}_2)_\beta^a(Q_2)_b^\beta(\bar{Q}_1)_\alpha^b\right]$$

$\alpha,\beta: SU(kN)$ index, $\quad a,b: SU((k-1)N)$ index

This preseves above symmetries and relates to string (supergravity) theoretic geometry (or N=2 SUSY gauge multiplet + 2 hypermultiplet + chiral adjoint mass).

Beta functions : all anomalous dimension γ are the same because of symmetry and charge conjugation.

- SU(kN) gauge coupling

$$\beta_k = \frac{-(\alpha_k)^2}{1-kN\alpha_k} N[k+2+2(k-1)\gamma_Q] \equiv -F(\alpha_k) N[k+2+2(k-1)\gamma_Q]$$

- SU((k-1)N) gauge coupling

$$\beta_{k-1} = \frac{-(\alpha_{k-1})^2}{1-(k-1)N\alpha_{k-1}} N[k-3+2k\gamma_Q] \equiv -F(\alpha_{k-1}) N[k-3+2k\gamma_Q]$$

- Dimensionless quartic coupling $\eta \equiv h\mu$

$$\beta_\eta = \eta(1+2\gamma_Q)$$

μ : renormalization scale

We can find two fixed points when k $\geqq 5$ (conformal window).

$$A: \quad -\beta_{k-1} \propto k - 3 + 2k\gamma_Q = 0, \quad \alpha_k = \eta = 0.$$

$$(\alpha_{k-1} = \alpha^*_{k-1} \neq 0)$$

$$B: \quad -\beta_k \propto k + 2 + 2(k-1)\gamma_Q = 0, \quad \alpha_{k-1} = \eta = 0.$$

$$(\alpha_k = \alpha^*_k \neq 0)$$

At a vicinity of fixed point "A" with $\alpha_{k-1} \approx \alpha^*_{k-1}, \ (\alpha_k, \ \eta) \ll 1$, (region I) we find

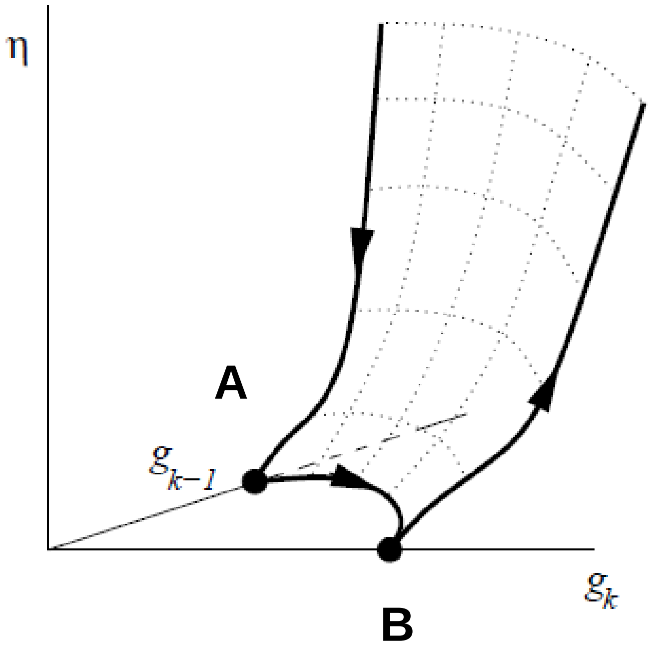
$$\gamma_Q \approx -\frac{1}{2} + \frac{3}{2k}, \ \underline{\beta_k \approx -F(\alpha_k)6N < 0}, \ \beta_\eta \approx 3\eta/k > 0.$$

These mean that α_k increases and η decreases towards the infrared direction. On the other hand, at a vicinity of "B" with $\alpha_k \approx \alpha^*_k, \ (\alpha_{k-1}, \ \eta) \ll 1$ (region II), we find

$$\gamma_Q \approx -\frac{1}{2} - \frac{3}{2(k-1)}, \ \beta_{k-1} \approx F(\alpha_{k-1})6N > 0, \ \underline{\beta_\eta \approx -3\eta/(k-1) < 0}.$$

Thus α_{k-1} decreases and η increases towards the infrared direction.
It can be natural that suppose that we have a renormalization flow from UV (repulsive) fixed point "A" to (attractive) IR fixed point "B" (except for η).

Renormalizarion trajectory



The section along η =0 surface

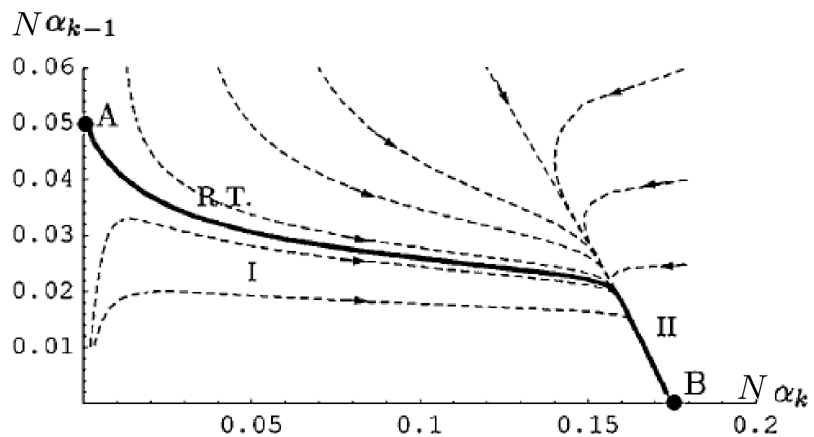


FIG. 1. RG flows in the coupling space (α_k, α_{k-1}) in the case of $k = 5$. The points A and B represent the UV and IR fixed points, respectively. The renormalized trajectory connecting these fixed points is shown by the bold line.

We used 1-loop anomalous dimension with neglecting O(1/(kN)) for k=5

$$\gamma_Q \approx -N[k\alpha_k + (k-1)\alpha_{k-1}] + \mathcal{O}(|\eta|^2).$$

Large η →large g_k
for fixed anomalous dimension. (quasi fixed point)

Around fixed point “B”, theory of SU(kN) (with 2(k-1)N flavor) is strongly coupled and would be well-described by Seiberg duality.
(We can evade to deal large η . $\because$ around $B,\ \beta_\eta < 0.$)

Dual gauge group becomes

$$SU(kN) \underset{\text{dual}}{\rightarrow} SU(2(k-1)N - kN) = SU((k-2)N)$$

with 2(k-1)N flavor. Then total gauge group becomes

$$SU((k-2)N) \times SU((k-1)N).$$

Then we will obtain the following matter content and superpotential in dual theory.

- Symmetries

$$SU((k-2)N) \times SU((k-1)N) \times SU(2) \times SU(2) \times U(1)_B \times \mathbf{Z}_{2N}^{(R)}$$

- matter content

	$SU((k-2)N)$	$SU((k-1)N)$	$SU(2)$	$SU(2)$	$U(1)_B$	$\mathbf{Z}_{2N}^{(R)}$
q_r	$(k-2)N$	$\overline{(k-1)N}$	**2**	**1**	$\frac{1}{(N^2(k-2)(k-1))}$	**1/2**
$\bar{q}_s$	$\overline{(k-2)N}$	$(k-1)N$	**1**	**2**	$\frac{-1}{(N^2(k-2)(k-1))}$	**1/2**
M_{rs}	1	Adjoint	**2**	**2**	**0**	**1**

$r, s = 1, 2$

$$(M_{rs})_a{}^b = (M_{rs}^{adj})_a{}^b + \frac{\delta_a{}^b}{(k-1)N} M_{rs}^0 \sim (Q_r)_a^\alpha (\bar{Q}_s)_\alpha^b$$

We also have gauge singlet meson $M_{rs}^0 = (M_{rs})_a{}^a$, but we will not write explicitly here because they can be similar to adjoint ones.

mass term for M

- Superpotential in dual theory

$$W_{dual} = y\bar{q}_r M_{rs} q_s + m \underset{r,s}{\mathrm{trdet}}(M_{rs}).$$

$$= y(\bar{q}_r)^a_\alpha (M_{rs})_a{}^b (q_s)^\alpha_b + m[(M_{11})_a{}^b (M_{22})_b{}^a - (M_{12})_a{}^b (M_{21})_b{}^a]$$

$\alpha, \beta : SU((k-2)N)$ index, $\quad a, b : SU((k-1)N)$ index

Supersymmetric quartic term and mass term would be related as

$$h(\Lambda_k)\Lambda_k \sim \frac{m(\Lambda_{k-2})}{\Lambda_{k-2}} \qquad \Lambda_k \simeq \Lambda_{k-2}$$

Λ_k: scale where gauge coupling of SU(kN) becomes strong (at fixed point)
Λ_{k-2}: scale where gauge coupling of SU((k-2)N) becomes strong (at fixed point)

Then, we want to think as $\eta = h\mu \sim \hat{\eta} = m/\mu.$

Beta functions for dual theory

- SU((k-2)N) gauge coupling

$$\beta_{k-2} = \frac{-(\alpha_{k-2})^2}{1-(k-2)N\alpha_{k-2}} N[k-4+2(k-1)\gamma_q] \equiv -F(\alpha_{k-2})N[k-4+2(k-1)\gamma_q].$$

- SU((k-1)N) gauge coupling

$$\beta_{k-1} = \frac{-(\alpha_{k-1})^2}{1-(k-1)N\alpha_{k-1}} N[-3k+5+2(k-2)\gamma_q+4(k-1)\gamma_M]$$
$$\equiv -F(\alpha_{k-1})N[-3k+5+2(k-2)\gamma_q+4(k-1)\gamma_M]$$

- Yukawa coupling

$$\beta_y = \frac{y}{2}(2\gamma_q+\gamma_M)$$

- dimensionless supersymmetric mass $\widehat{\eta} = m/\mu$

$$\beta_{\widehat{\eta}} = (-1+\gamma_M)\widehat{\eta}$$

At a vicinity of fixed point “B” with

$$\alpha_{k-2} \approx \alpha^*_{k-2}, \;\; y \approx y_*, \;\; (\alpha_{k-1}, \hat{\eta}) \ll 1,$$

We can see

$$\beta^{(dual)}_{k-1} = \beta^{(original)}_{k-1} \approx F(\alpha_{k-1}) 6N > 0, \;\; \underline{\beta_{\hat{\eta}} \approx -3\hat{\eta}/(k-1) < 0.}$$

$$(\beta_\eta/\eta = \beta_{\hat{\eta}}/\hat{\eta} \approx -3/(k-1) < 0)$$

$$\gamma_q \approx -\frac{1}{2} + \frac{3}{2(k-1)} \approx -\frac{\gamma_M}{2} < 0,$$

$$\mathrm{dim}[\bar{Q}Q] \approx \frac{3}{2}\left[1 - \frac{3}{k-1}\right] \approx \mathrm{dim}[M] \qquad \left(\gamma_Q \approx -\frac{1}{2} - \frac{3}{2(k-1)}\right)$$

Then mass of meson increases towards the infrared direction like a quartic term in the original theory, so for $\hat{\eta} \gg 1 \;\; (m \gg \mu)$, we integrate out mesons around a fixed point B.

$$\text{but } \alpha_{k-2} \lesssim \alpha^*_{k-2}, \;\; y \lesssim y_*, \;\; \alpha_{k-1} \to 0.$$

Theory can be in quasi fixed point for fixed value of anomalous dims.

$$W_{dual} = y_* \bar{q}_r M_{rs} q_s + m \underset{r,s}{\mathrm{trdet}}(M_{rs}).$$

$$\Big\downarrow \; \partial_{M_{rs}} W_{dual} = 0$$

$$W_{low} = -\frac{y_*^2}{m} \underset{r,s}{\mathrm{trdet}}(q_r \bar{q}_s)$$
$$= \tilde{h} \underset{r,s}{\mathrm{trdet}}(q_r \bar{q}_s)$$

$$\tilde{h} = -\frac{y_*^2}{m} \sim \frac{1}{h}.$$

Thus we could obtain small $\tilde{h}$ as $\tilde{h} \to 0$ when $(h, m) \to \infty$.

Then we obtain finally SU((k-1)N) × SU((k-2)N) gauge theory and matter content as following

	$SU((k-2)N)$	$SU((k-1)N)$	$SU(2)$	$SU(2)$	$U(1)_B$	$\mathbf{Z}_{2N}^{(R)}$
q_r	$(k-2)N$	$\overline{(k-1)N}$	**2**	**1**	$\frac{1}{(N^2(k-2)(k-1))}$	**1/2**
$\bar{q}_s$	$\overline{(k-2)N}$	$(k-1)N$	**1**	**2**	$\frac{-1}{(N^2(k-2)(k-1))}$	**1/2**

$r, s = 1, 2$

$$W = \tilde{h}\,\underset{r,s}{\mathrm{trdet}}(q_r \bar{q}_s) \qquad \tilde{h} = -\frac{y_*^2}{m}.$$

around the region where

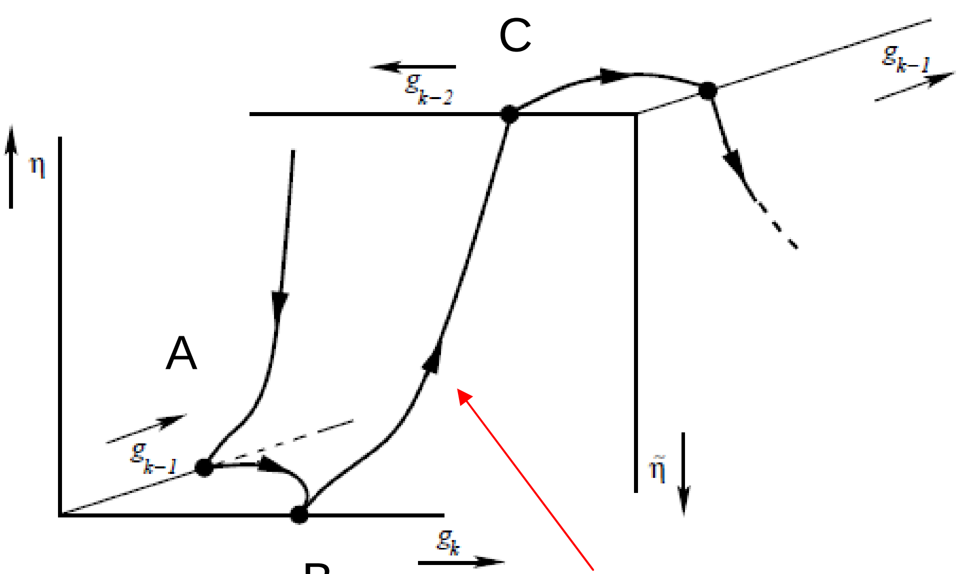
$$\alpha_{k-2} \approx \alpha_{k-2}^*, \quad (\alpha_{k-1}, \tilde{\eta}) \ll 1. \qquad \tilde{\eta} = \tilde{h}\mu$$

This situation is very similar to original fixed point “A”.
(at UV fixed point of smaller gauge group)
As now there are no meson, this fixed point is unstable; $\beta_{k-1} < 0.$

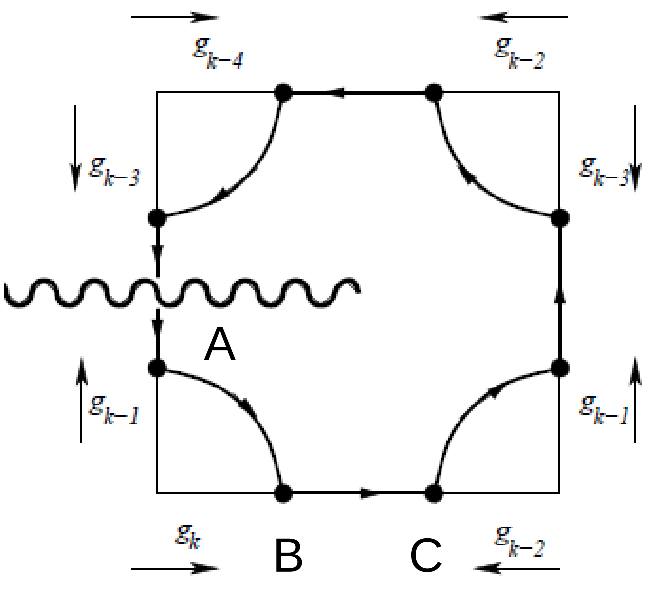
We finished one period of cascade.

$$
\begin{array}{llcl}
\text{A:} & (g_k \approx 0, g_{k-1} \approx g^*_{k-1}, \eta \approx 0) & & \\
 & \downarrow & & \\
\text{B:} & (g_k \approx g^*_k, g_{k-1} \approx 0, \eta \approx 0) & \leftrightarrow & (g_{k-2} \approx g^*_{k-2}, g_{k-1} \approx 0, y \approx y^*, m/\mu \approx 0) \\
 & \downarrow & \text{dual} & \downarrow \\
 & (g_k \approx g^*_k, g_{k-1} \approx 0, \eta \gg 1) & \leftrightarrow & (g_{k-2} \approx g^*_{k-2}, g_{k-1} \approx 0, y \approx y^*, m/\mu \gg 1) \\
 & & & \downarrow \text{ integrating out } M_{rs} \\
\text{C:} & & & (g_{k-2} \approx g^*_{k-2}, g_{k-1} \approx 0, \tilde{\eta} \approx 0).
\end{array}
$$

Renormalization trajectory



View from “above”



Quasi fixed point for gk and (gk-2, y) for fixed γ

In the end of cascade, we will find

$$\cdots \to \underline{SU(4N)} \times SU(3N)$$ ← SU(4N): Free magnetic phase

$$\to \underline{SU(3N)} \times SU(2N)$$ ← Below Λ_{4N} ,
IR free SU(2) but $g_{2N} \neq 0$
SU(3N): Free magnetic phase

$$\to SU(2N) \times SU(N)$$

Below Λ_{3N}, we will see SU(2N) × SU(N). Especially, below Λ_{2N} SU(2N) can be confined and have quantum deformed moduli space.
(we suppose that we have IR free SU(N) but g_N <<1 $\neq$ 0.)

$$W = h \underset{r,s}{\mathrm{tr}} \underset{2N \times 2N}{\det} M_{rs} + X(\det M - B\bar{B} - (\Lambda_{2N})^{4N})$$

X: Lagrange multiplier, B ($\bar{B}$): SU(2N) (anti-)baryon (singlet for SU(N)),
M: SU(2N) meson (adjoint + singlet for SU(N)), Λ_{2N}: dynamical scale of SU(2N)

For baryonic branch, we have solution

$$B = \bar{B} = i(\Lambda_{2N})^{2N}, \;\; M = 0.$$

Meson fields which are charged under SU(2N) become massive.

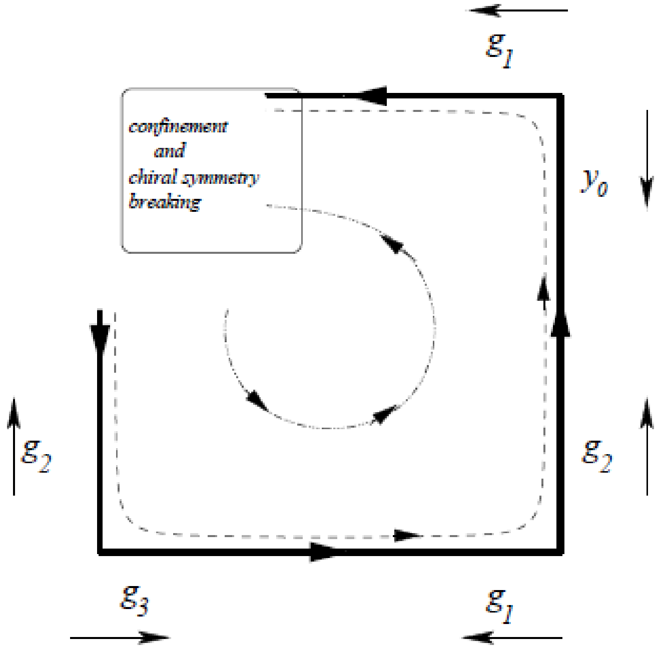
(We have $U(1)_B$ Nambu-Goldstone boson multiplet which are neutral for SU(N) and have irrelevant couplings to SU(N) sector suppressed by $\Lambda_{2N} \gg \Lambda_N$)

Below the mass scale, finally we obtain pure SU(N) SUSY gauge theory. Then theory is confined at Λ_N.

$$\langle \lambda^\alpha \lambda_\alpha \rangle \propto e^{2\pi i k/N} \Lambda_N^3 \neq 0$$

$$(\mathbf{Z_{2N}^{(R)}} \to \mathbf{Z_2^{(R)}} : \; \mathrm{N} \text{ vacua})$$

Chiral R-symmetry breaking



Corner implies theory include IR free phase.

We will introduce soft terms with a spurion method.

・SU(kN) × SU((k-1)N) theory

$$\mathcal{L} = \int d^4\theta K + \int d^2\theta W + \frac{1}{4}\int d^2\theta \frac{1}{g_k^2}\mathcal{W}_k^\alpha \mathcal{W}_{\alpha\ k} + \frac{1}{4}\int d^2\theta \frac{1}{g_{k-1}^2}\mathcal{W}_{k-1}^\alpha \mathcal{W}_{\alpha\ k-1} + c.c.$$

$$K = Q^\dagger Q + \bar{Q}^\dagger \bar{Q}, \quad W = h\,\mathrm{trdet}(Q\bar{Q})$$

$$\downarrow$$

$$\mathcal{L} = \int d^4\theta K + \int d^2\theta W + \frac{1}{4}\int d^2\theta \frac{1}{g_k^2}\left(1 - \underline{2M_k\theta^2}\right)\mathcal{W}_k^\alpha \mathcal{W}_{\alpha\ k} + \frac{1}{4}\int d^2\theta \frac{1}{g_{k-1}^2}\left(1 - \underline{2M_{k-1}\theta^2}\right)\mathcal{W}_{k-1}^\alpha \mathcal{W}_{\alpha\ k-1} + c.c.$$

$$K = (1 - \underline{m_Q^2\theta^4})Q^\dagger Q + (1 - \underline{m_{\bar{Q}}^2\theta^4})\bar{Q}^\dagger \bar{Q},$$

$$W = (h - \underline{a_h\theta^2})\mathrm{trdet}(Q\bar{Q})$$

Gaugino mass (general expression)

$$\mu \frac{dM_{1/2}^{(k)}}{d\mu} = -N(k+2+2(k-1)\gamma_Q)H'(\alpha_k)\alpha_k M_{1/2}^{(k)} \longleftarrow$$

< 0 at fixed point A
Vanish at fixed point B

$$- 2(k-1)NH(\alpha_k)\frac{\partial \gamma_Q}{\partial \alpha_k}\alpha_k M_{1/2}^{(k)} \longleftarrow$$

irrelevant at A
Damping factor at B

$$- 2(k-1)NH(\alpha_k)\frac{\partial \gamma_Q}{\partial \alpha_{k-1}}\alpha_{k-1} \underline{M_{1/2}^{(k-1)}}, \longleftarrow$$

Convergence factor
around B
(variation is very slow)

$$\mu \frac{dM_{1/2}^{(k-1)}}{d\mu} = -N(k-3+2k\gamma_Q)H'(\alpha_{k-1})\alpha_{k-1} M_{1/2}^{(k-1)} \longleftarrow$$

Vanish at A
>0 other regime

$$- 2kNH(\alpha_{k-1})\frac{\partial \gamma_Q}{\partial \alpha_{k-1}}\alpha_{k-1} M_{1/2}^{(k-1)} \longleftarrow$$

Damping factor at A
irrelevant other regime

$$- 2kNH(\alpha_{k-1})\frac{\partial \gamma_Q}{\partial \alpha_k}\alpha_k M_{1/2}^{(k)}, \longleftarrow$$

>0 around A (M_{k-1} to be negative)
irrelevant around B

where $H(\alpha) = F(\alpha)/\alpha \approx \alpha$ and $H'(\alpha) = dH/d\alpha$.

For explicit calculation, we used 1-loop anomalous dimension (2-loop beta function) for the moment

$$\gamma_Q \approx -N[k\alpha_k + (k-1)\alpha_{k-1}] + \mathcal{O}(|\eta|^2),$$

(We do not know how correct this is, around isolated fixed point…)

as we want to know qualitative behavior of SUSY breaking terms.

From $dM_k/d\ln(\mu) \sim 0,$ we find around around fixed point B

$$\frac{\partial \gamma_Q}{\partial \alpha_k}\alpha_k M_k \to -\frac{\partial \gamma_Q}{\partial \alpha_{k-1}}\alpha_{k-1} M_{k-1} \xrightarrow{\text{1-loop}} k\alpha_k^* M_k \to -(k-1)\alpha_{k-1} M_{k-1}$$

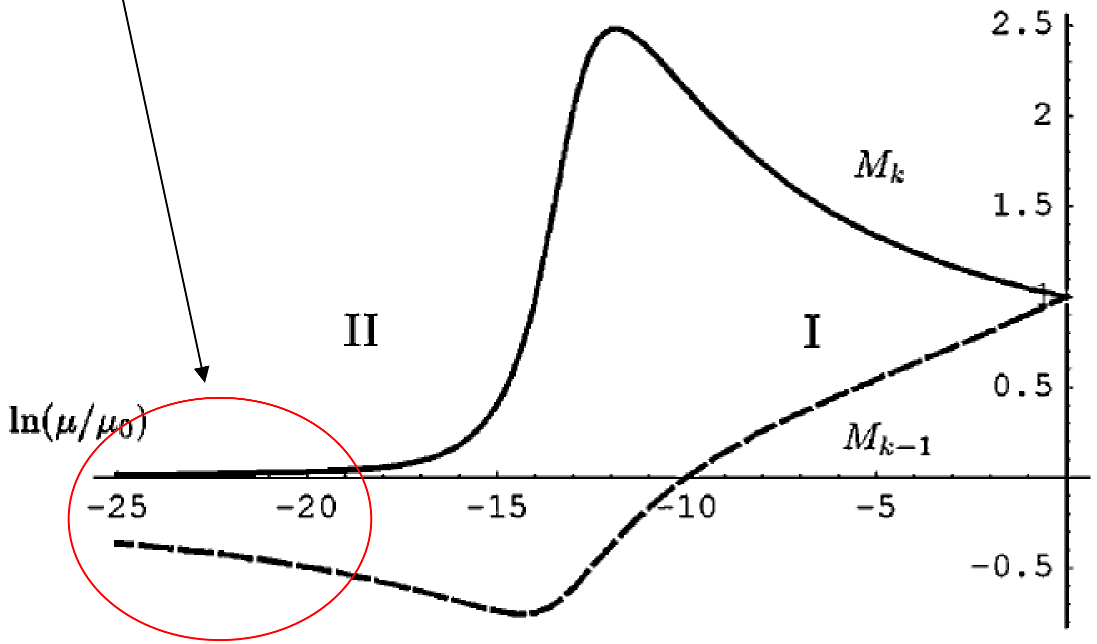


FIG. 2. RG running of the gaugino masses $M_{1/2}^{(k-1)}(\mu)$ and $M_{1/2}^{(k)}(\mu)$ with respect to $\ln(\mu/\mu_0)$. The gauge couplings are given at $\mu = \mu_0$ as $(\alpha_k, \alpha_{k-1}) = (0.0128, 0.04)$ and run along the renormalized trajectory. $k = 5.$

Beta function for soft scalar mass with 1-loop anomalous dimension

$$\mu \frac{dm_Q^2}{d\mu} = -k\alpha_k(2|M_{1/2}^{(k)}|^2 + \Delta_k) - (k-1)\alpha_{k-1}(2|M_{1/2}^{(k-1)}|^2 + \Delta_{k-1}),$$

where

$$\Delta_k = H(\alpha_k)[3k|M_{1/2}^{(k)}|^2 - 2(k-1)m_Q^2],$$

$$\Delta_{k-1} = H(\alpha_{k-1})[3(k-1)|M_{1/2}^{(k-1)}|^2 - 2km_Q^2].$$

Note that we took $m_Q^2 = m_{\bar{Q}}^2,\ N\alpha \to \alpha.$
Therefore m_Q^2 means $(m_Q^2 + m_{\bar{Q}}^2)/2$ here.

Similarly we can also find convergent values of scalar mass around each fixed point from $dm_Q^2/d\ln(\mu) \sim 0$.

$$m_Q^2 \to \frac{1}{k(\alpha_k^*)^2}\alpha_{k-1}|M_{1/2}^{(k-1)}|^2$$

$$m_Q^2 \to \frac{1}{(k-1)(\alpha_{k-1}^*)^2}\alpha_k|M_{1/2}^{(k)}|^2$$

(but very weak)

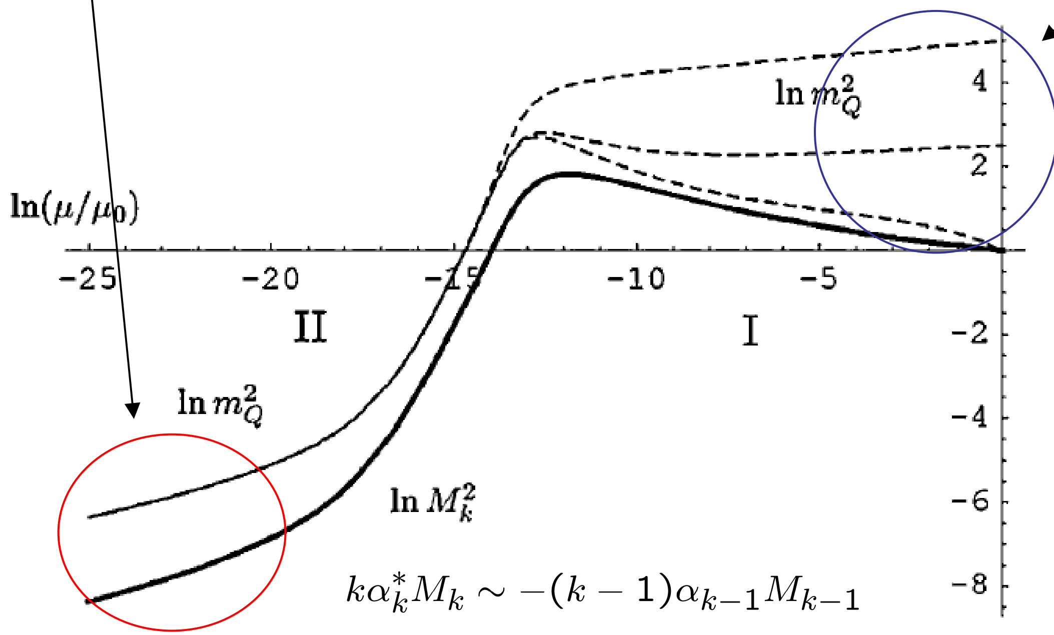


FIG. 3. RG running behaviors of the scalar mass $\ln m_Q^2$ and the gaugino mass $2\ln M_{1/2}^{(k)}$ are shown by dotted lines and the bold line, respectively.

initial values are taken as $\ln m_Q^2 = 0,\ 2.5,\ 5.0$

Holomorphic quartic term on a surface of $\eta = \mu h << 1$

Ratio $A_h \equiv a_h / h$

$$\frac{dA_h}{d\ln(\mu)} = 4\left(-\alpha_a M_a \frac{\partial}{\partial \alpha_a} + \eta A_h \frac{\partial}{\partial \eta}\right)\gamma_Q$$
$$= 4Nk\alpha_k M_k + 4N(k-1)\alpha_{k-1}M_{k-1} + \mathcal{O}(|\eta|^2)A_h$$

Excluding around fixed point B:

$$\frac{dA_h}{d\ln(\mu)} \sim 4Nk\alpha_k M_k > 0.$$

A_h can decrease.
(and could become negative.)

Around fixed point B: $k\alpha_k^* M_k \sim -(k-1)\alpha_{k-1}M_{k-1}$

$$\frac{dA_h}{d\ln(\mu)} \sim \mathcal{O}(|\eta|^2)A_h \sim 0.$$

Therefore magnitude of A_h would not change drastically if gaugino mass are not much larger than A_h at the initial condition.

We will look around the region η >>1 and want to evade dealing awkward non-renormalizable coupling and its soft SUSY breaking term, with using a Seiberg dual.
(We supposed that μ >> m_{soft} to make theory approximately supersymmetric.)

But we may lose matching condition for coupling and soft terms between original theory and dual theory, though the followings may be natural.

$$\eta(\Lambda_k) = h(\Lambda_k)\Lambda_k \sim \hat{\eta}(\Lambda_{k-2}) = m(\Lambda_{k-2})/\Lambda_{k-2}$$

$$M_k(\Lambda_k) \sim M_{k-2}(\Lambda_{k-2}) \qquad \Lambda_k \simeq \Lambda_{k-2}$$

$$A_h(\Lambda_k) \sim B(\Lambda_{k-2}) \qquad \text{etc.}$$

Dual theory:

$$\mathcal{L} = \int d^4\theta K + \int d^2\theta W + \frac{1}{4}\int d^2\theta \frac{1}{g_{k-2}^2}\mathcal{W}_{k-2}^{\alpha}\mathcal{W}_{\alpha\ k-2} + \frac{1}{4}\int d^2\theta \frac{1}{g_{k-1}^2}\mathcal{W}_{k-1}^{\alpha}\mathcal{W}_{\alpha\ k-1} + c.c.$$

$$K = q^\dagger q + \bar{q}^\dagger \bar{q} + M^\dagger M, \quad W = m\,\mathrm{trdet}(M)$$

$$\downarrow$$

$$\mathcal{L} = \int d^4\theta K + \int d^2\theta W + \frac{1}{4}\int d^2\theta \frac{1}{g_k^2}\left(1 - \underline{2M_{k-2}\theta^2}\right)\mathcal{W}_{k-2}^{\alpha}\mathcal{W}_{\alpha\ k-2} + \frac{1}{4}\int d^2\theta \frac{1}{g_{k-1}^2}\left(1 - \underline{2M_{k-1}\theta^2}\right)\mathcal{W}_{k-1}^{\alpha}\mathcal{W}_{\alpha\ k-1} + c.c.$$

$$K = (1 - \underline{m_q^2\theta^4})q^\dagger q + (1 - \underline{m_{\bar{q}}^2\theta^4})\bar{q}^\dagger \bar{q} + (1 - \underline{m_M^2\theta^2})M^\dagger M,$$

$$W = (m - \underline{b\theta^2})\mathrm{trdet}(M)$$

1-loop anomalous dimension

$$\begin{aligned}\gamma_q &= -(k-2)\alpha_{k-2} - (k-1)\alpha_{k-1} + 2(k-1)\alpha_y \\ \gamma_M &= -2(k-1)\alpha_{k-1} + (k-2)\alpha_y\end{aligned}$$

We used $N\alpha \to \alpha$.

Gaugino mass and trilinear holomorphic scalar (θ^2 -component) coupling around fixed point B

$$\frac{d}{d\ln(\mu)}\begin{pmatrix}\alpha^*_{k-2}M_{k-2}\\ -y^\dagger_* a\\ \alpha_{k-1}M_{k-1}\end{pmatrix}=\begin{pmatrix}\left.\frac{\partial\beta_{k-2}}{\partial\alpha_{k-2}}\right|_* & \left.\frac{\partial\beta_{k-2}}{\partial\alpha_y}\right|_* & \left.\frac{\partial\beta_{k-2}}{\partial\alpha_{k-1}}\right|_*\\ \left.\frac{\partial\beta_{\alpha y}}{\partial\alpha_{k-2}}\right|_* & \left.\frac{\partial\beta_{\alpha y}}{\partial\alpha_y}\right|_* & \left.\frac{\partial\beta_{\alpha y}}{\partial\alpha_{k-1}}\right|_*\\ \left.\frac{\partial\beta_{k-1}}{\partial\alpha_{k-2}}\right|_* & \left.\frac{\partial\beta_{k-1}}{\partial\alpha_y}\right|_* & \left.\frac{\partial\beta_{k-1}}{\partial\alpha_{k-1}}\right|_*\end{pmatrix}\begin{pmatrix}\alpha^*_{k-2}M_{k-2}\\ -y^\dagger_* a\\ \alpha_{k-1}M_{k-1}\end{pmatrix}.$$

$*$ stands for substitution of $g_{k-2}\approx g^*_{k-2},\ y\approx y_*,\ g_{k-1}\ll 1.$

Deviation from fixed point B which will satisfy similar equation can shrink to almost zero because of a infrared attractive-like nature as previous usual (dual of) SQCD.

But soft mass terms converge to M_{k-1} whose variation can be very slow.

$$dm_{soft}/d\ln(\mu)\sim 0. \qquad \longrightarrow \qquad \alpha^*_{k-2}M_{k-2}\sim -y^\dagger_* a\sim \alpha_{k-1}M_{k-1}.$$

(Sum of) scalar mass around fixed point B: θ^4 -component

As previous usual dual of SQCD, θ^4-component of $\alpha_{k-2}|_{\theta^4},\ \alpha_y|_{\theta^4},\ \alpha_{k-1}|_{\theta^4}$ also satisfy the same equations. Therefore

$$\alpha_y^*(m_q^2 + m_{\bar{q}}^2 + m_M^2) \sim \alpha_{k-2}^*(m_q^2 + m_{\bar{q}}^2) \sim \alpha_{k-1}|M_{k-1}|^2.$$

$$(\alpha_y^* m_M^2 \sim \alpha_{k-1}|M_{k-1}|^2)$$

Holomorphic quadratic term on a surface of $\hat{\eta} = m/\mu \ll 1$

Ratio $B \equiv b/m$ and $a = yA_y$

$$\frac{dB}{d\ln(\mu)} = 2\left(-\alpha_{k-1}M_{k-1}\frac{\partial}{\partial\alpha_{k-1}} + a\frac{\partial}{\partial y}\right)\gamma_M$$
$$= 4N(k-1)\alpha_{k-1}M_{k-1} + N(k-2)\alpha_y A_y$$

Excluding around fixed point B:

$$\frac{dB}{d\ln(\mu)} \sim N(k-2)\alpha_y A_y > 0.$$

B can decrease.
(and could become negative.)

Around fixed point B:

$$\frac{dB}{d\ln(\mu)} \sim \alpha_{k-1}M_{k-1} \sim 0.$$

Therefore magnitude of B would not change drastically if gaugino mass and A_y are not much larger than A_h at the initial condition.

For m >> μ >> B (and other soft terms), we can integrate out meson which can be adjoint representation for SU((k-1)N) in an approximately supersymmetric manner. Then

$$W_{low} = -\frac{(y_* - a\theta^2)^2}{(m - b\theta^2)} \underset{r,s}{\mathrm{trdet}}(q_r \bar{q}_s)$$

$$= \tilde{h}' \underset{r,s}{\mathrm{trdet}}(q_r \bar{q}_s)$$

$$\tilde{h}' = -\frac{y_*^2}{m}[1 + (B - 2A_y)\theta^2] = (\tilde{h} - a_{\tilde{h}}\theta^2).$$

We have threshold effect (~ gauge mediation) from meson, too.

$$\Delta M_{k-1} \sim \alpha_{k-1} B, \;\; \Delta m_q^2 = \Delta m_{\bar{q}}^2 \sim \alpha_{k-1}^2 |B|^2.$$

Duality cascade will continue.

After many times of duality cascade, that is, for $\mu \gtrsim$ m ~ B ~ $\sqrt{m_M^2}$, we obtain mass of meson, which are adjoint (and singlet) for weakly interacting gauge theory;

$$|m|^2 + m_M^2 \pm |b|.$$

Off-diagonal part of scalar mass

We could find tachyonic mode and then weakly interacting gauge group may break by $\langle M \rangle \neq 0.$ ref. EWSB in MSSM

(In this case (magnetic) quarks can gain supersymmetric mass through a superpotential $W \sim yM\bar{q}q \to y\langle M \rangle \bar{q}q.$)

Then cascade would be terminated.

(Of course we must see whole potential and the number of flavor)

We have another possibilities which we did not study.

Soft masses for singlet mesons M_0 may be driven to be negative because of the Yukawa couplings.

Similarly, the singlet meson fields M_0 may develop their VEVs depending on values of their various mass terms. Their VEVs induce mass terms of dual quarks. If such masses are large enough, the dual quarks would decouple and the flavor number would reduce to be outside of the conformal window. Then, the cascade could end. In addition, scalar components of q_r and $\bar{q}_s$ may develop their VEVs depending on values the A-terms and their soft scalar masses as well as other parameters in the scalar potential. Their VEVs break gauge symmetry and the cascade would end.

If the quartic A-term is comparable with SUSY breaking scalar masses m_Q, the origin of the scalar potential of Q would be unstable and similar symmetry breaking would happen. Such gauge symmetry breaking with reducing the flavor number may correspond to the symmetry breaking by VEVs of M with inducing dual quark masses.

And so on…

Beta function for scalar mass m_q^2 under 1-loop approximation

$$
\begin{aligned}
\frac{dm_q^2}{d\ln\mu} &= \left(\frac{\partial\gamma_q}{\partial\alpha_{k-2}}\tilde{\alpha}_{k-2} + \frac{\partial\gamma_q}{\partial\alpha_{k-1}}\tilde{\alpha}_{k-1} + \frac{\partial\gamma_q}{\partial\alpha_y}\tilde{\alpha}_y\right)\Bigg|_{\theta^2\bar{\theta}^2} \\
&= -(k-2)\alpha_{k-2}\left(2|M_{k-2}|^2 + \Delta'_{k-2}\right) - (k-1)\alpha_{k-1}\left(2|M_{k-1}|^2 + \Delta'_{k-1}\right) \\
&\quad +2(k-1)\alpha_y(\Sigma^2 + |A_y|^2) \\
\frac{dm_M^2}{d\ln\mu} &= \left(\frac{\partial\gamma_M}{\partial\alpha_{k-1}}\tilde{\alpha}_{k-1} + \frac{\partial\gamma_M}{\partial\alpha_y}\tilde{\alpha}_y\right)\Bigg|_{\theta^2\bar{\theta}^2} \\
&= -2(k-1)\alpha_{k-1}\left(2|M_{k-1}|^2 + \Delta'_{k-1}\right) + (k-2)\alpha_y(\Sigma^2 + |A_y|^2)
\end{aligned}
$$

$$
\begin{aligned}
\Delta'_{k-2} &= \frac{\alpha_{k-2}}{1-(k-2)\alpha_{k-2}}\left[3(k-2)|M_{k-2}|^2 - 2(k-1)m_q^2\right] \\
\Delta'_{k-1} &= \frac{\alpha_{k-1}}{1-(k-1)\alpha_{k-1}}\left[3(k-1)|M_{k-1}|^2 - 2(k-2)m_q^2 - 4m_M^2\right] \\
\Sigma^2 &= 2m_q^2 + m_M^2 \qquad (m_q^2 = m_{\tilde{q}}^2), \quad a = yA_y.
\end{aligned}
$$

$$
\begin{aligned}
\gamma_q &= -(k-2)\alpha_{k-2} - (k-1)\alpha_{k-1} + 2(k-1)\alpha_y \\
\gamma_M &= -2(k-1)\alpha_{k-1} + (k-2)\alpha_y
\end{aligned}
$$

Around fixed point B

$$
\gamma_q^* = -\frac{k-4}{2(k-1)}, \qquad \gamma_M^* = \frac{k-4}{k-1}
$$

$$
\alpha_y^* = \frac{k-4}{(k-1)(k-2)}, \qquad \alpha_{k-2}^* = \frac{5k-6}{2(k-2)}\alpha_y^*
$$

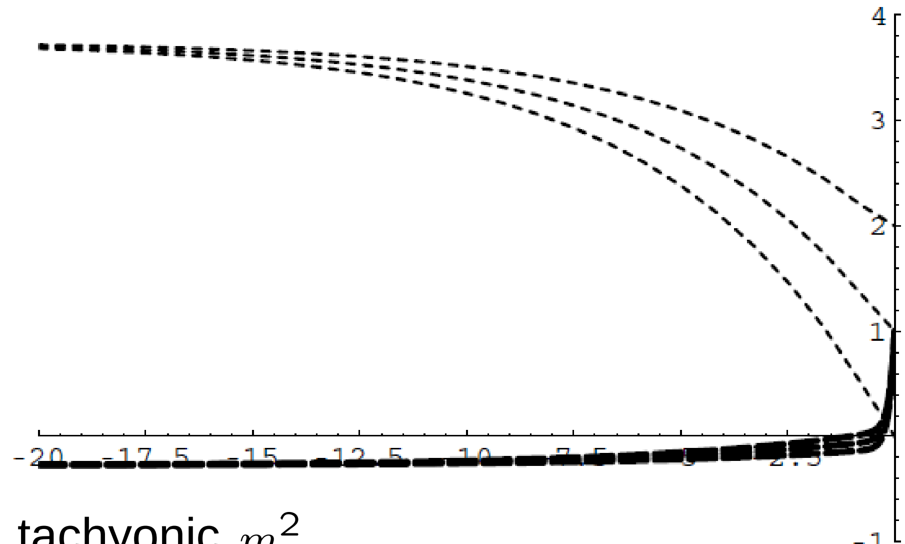
$$
\begin{aligned}
\frac{dm_q^2}{d\ln\mu} &\simeq 2(k-1)(k-2)\alpha_{k-2}^* m_q^2 + 2(k-1)\alpha_y^*\Sigma^2 - 2(k-1)\alpha_{k-1}|M_{k-1}|^2 \\
\frac{dm_M^2}{d\ln\mu} &\simeq +(k-2)\alpha_y^*\Sigma^2 - 4(k-1)\alpha_{k-1}|M_{k-1}|^2
\end{aligned}
$$

$m_q^2 = m_{\tilde{q}}^2$

m_M^2

$\alpha_{k-1}|M_{k-1}|^2 = 0.05 = \text{constant.}$

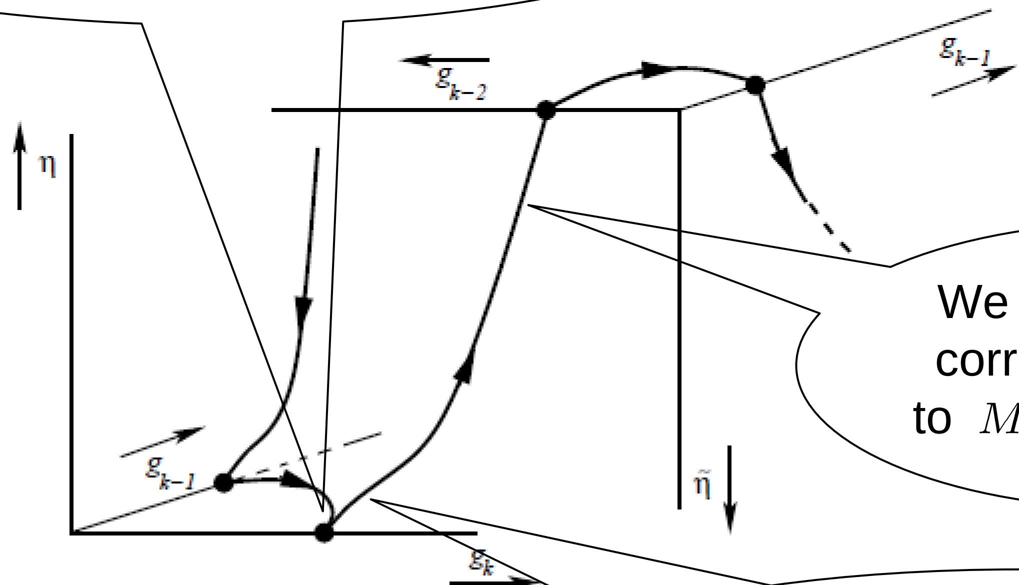
k=5
(We do not know for large k, large ‘tHooft coupling)



tachyonic m_q^2

Summary of model with SUSY breaking terms

Soft terms except for Ah are suppressed and converge to weakly interacting gaugino mass



We have threshold correction B from M to $M_{k-1},\ m_q^2,\ m_{\bar{q}}^2$.

Soft terms except for B are suppressed and converge to weakly interacting gaugino mass

After many times…

We could have gauge symmetry breaking by tachyonic modes.
The cascade could end.

5.Summary

What we did :

・ We study evolution of renormalization group of both supersymmetric terms and SUSY breaking terms in SU(kN)×SU((k-1)N) model of cascade under 1-loop anomalous dimension (=1-loop or 2-loop beta function).

What we found :

・Almost SUSY breaking terms are suppressd and converge to weak coupling gaugino mass in the infrared regime.

・Because of SUSY breaking term (holomorphic mass term (B-term) or non-holomorphic scalar mass term), we could find gauge symmetry breaking; the cascade could end.

Future direction:

・explicit model for supersymmetric standard model

・model dependent analysis

(e.g. D-term contribution, length of running, the magnitude of M_{k-1}, B, flavor…)

Illustrating model : Breaking of L-R symmetry by L-R Higgs

U(3) × USp(6)L × USp(6)R × U(1)

3 × $\tilde{Q}_L$: $(3, 6, 1, 0)$, $\tilde{Q}_R$: $(\bar{3}, 1, 6, 0)$, $\tilde{L}_L$: $(1, 6, 1, -1)$, $\tilde{L}_R$: $(\bar{1}, 1, 6, 1)$

$$W = h\tilde{Q}_L\tilde{Q}_R\tilde{L}_L\tilde{L}_R.$$

U(3) × USp(2)L × USp(2)R × U(1)

3 × $\hat{Q}_L$: $(\bar{3}, 2, 1, 0)$, $\hat{Q}_R$: $(3, 1, 2, 0)$, L_L: $(1, 2, 1, 1)$, and L_R: $(1, 1, 2, -1)$.

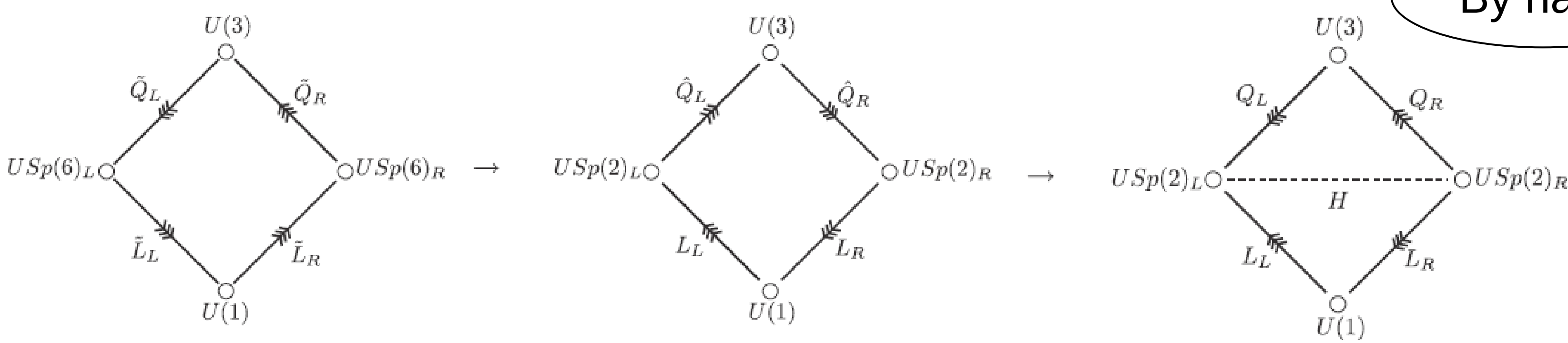
$$W = \hat{h}\hat{Q}_L\hat{Q}_R L_L L_R.$$

U(3) × USp(2)L × USp(2)R × U(1)

3 × Q_L: $(3, 2, 1, 0)$, Q_R: $(\bar{3}, 1, 2, 0)$, L_L: $(1, 2, 1, 1)$, L_R: $(1, 1, 2, -1)$

9 × H: $(1, 2, 2, 0)$ (Meson of above SU(3)) $W = y_Q Q_L Q_R H + y_L L_L L_R H + mHH.$

By hand



End

And Appendix

・Physical coupling and holomorphic coupling

Consider $\mathcal{N}=1$ SUSY gauge theory with matter fields which have superpotential of n-point coupling Y at ultraviolet scale Λ.

$$\mathcal{L}=\int d^4\theta \sum_i [Q_i^\dagger e^{2V^{(Q)}} Q_i] + \int d^2\theta Y_{i_1 i_2 \ldots i_n} Q_{i_1} Q_{i_2} \ldots Q_{i_n} + \frac{1}{4}\int d^2\theta f\, \mathcal{W}^\alpha \mathcal{W}_\alpha + c.c.$$

$$f=\frac{1}{g^2}-i\frac{\vartheta}{8\pi^2}$$

at low energy scale μ
(perturbative)

$$\downarrow$$

$$\mathcal{L}=\int d^4\theta \sum_i [Z_i(\mu) Q_i^\dagger e^{2V^{(Q)}} Q_i] + \int d^2\theta Y_{i_1 i_2 \ldots i_n} Q_{i_1} Q_{i_2} \ldots Q_{i_n} + \frac{1}{4}\int d^2\theta \left[f+\frac{b}{8\pi^2}\ln(\Lambda/\mu)\right] \mathcal{W}^\alpha \mathcal{W}_\alpha + c.c.$$

b: coefficient of 1-loop beta-function of gauge coupling

$$Q=\tilde{Q}+\theta^\alpha \psi_\alpha^Q+\ldots,\ \mathcal{W}_\alpha=-\frac{1}{4}\bar{D}^2 e^{-2V} D_\alpha e^{2V}=-i\lambda_\alpha-\frac{i}{2}F_{\mu\nu}(\sigma^{\mu\nu})_\alpha{}^\beta \theta_\beta+\ldots$$

$\tilde{Q}$: scalar, ψ_α^Q: fermion, $F_{\mu\nu}$: gauge field strength, λ_α: gaugino, θ: fermionic coordinate.

By holomorphy and shift symmetry: $\vartheta \;\to\; \vartheta + 2\pi$, we will have wave function renormalization Zi and 1-loop correction (bLog(μ)) to holomorphic gauge coupling f perturbatively.

By rescaling matter superfields and gauge superfield to canonical form,

$$Q_i \;\to\; Z_i^{-1/2} Q_i, \qquad V \to\; g_c V,$$

we will find physical couplings (g_c, y) through a supersymmetric rescaling (Konishi) anomaly.

Arkani-Hamed and Murayama

$$y_{i_1 \dots i_n} = Z_{i_1}^{-1/2} \dots Z_{i_n}^{-1/2} Y_{i_1 \dots i_n} = e^{-1/2 \sum_i \ln(Z_i)} Y_{i_1 \dots i_n}.$$

$$\frac{8\pi^2}{g_c^2} = \frac{8\pi^2}{g^2} + b \ln(\Lambda/\mu) - \sum_i T_i \ln(Z_i) - T_G \ln(g_c^2)$$

$$\left(\int d^2\theta \frac{1}{g_c^2} \mathcal{W}^\alpha(g_c V) \mathcal{W}_\alpha(g_c V) \right)$$

Novikov-Shifman-Vainshtein-Zakharov (NSVZ) form

T_i :Dynkin index of matter i

T_G: Dynkin index for adj.

(from gaugino)

Spurion method including soft SUSY breaking terms

We can treat physical couplings as a external superfield when we have a (spurious) supersymmetric regularization.

$$Z_i \to \tilde{Z}_i(\theta, \bar{\theta}), \ \ \underline{Y_{i_1 \ldots i_n} \to \tilde{Y}_{i_1 \ldots i_n}(\theta), \ \ f \to \tilde{f}(\theta)}$$

holomorphic

$$\mathcal{L} = \int d^4\theta \sum_i [\tilde{Z}_i(\mu) Q_i^\dagger e^{2V^{(Q)}} Q_i] + \int d^2\theta \tilde{Y}_{i_1 i_2 \ldots i_n} Q_{i_1} Q_{i_2} \cdots Q_{i_n}$$

$$+ \frac{1}{4} \int d^2\theta \tilde{f}(\mu) \mathcal{W}^\alpha \mathcal{W}_\alpha + c.c.$$

$f(\mu) = f + \frac{b}{8\pi^2} \ln(\Lambda/\mu)$

Lagrangian for soft SUSY breaking terms:

$$\mathcal{L}_{soft} = -m_i^2|\tilde{Q}_i|^2 - (a_{i_1\ldots i_n}\tilde{Q}_{i_1}\ldots\tilde{Q}_{i_n} + \frac{1}{2}M_\lambda\lambda^\alpha\lambda_\alpha + c.c.)$$
$$= \int d^4\theta(\underline{-m_i^2\theta^4})Q_i^\dagger Q_i + \int d^2\theta(\underline{-a_{i_1\ldots i_n}\theta^2})Q_{i_1}\ldots Q_{i_n}$$
$$-\frac{1}{4}\int d^2\theta(\underline{2M_\lambda\theta^2})\mathcal{W}^\alpha\mathcal{W}_\alpha + c.c.$$

$$Q = \tilde{Q} + \mathcal{O}(\theta),\ \mathcal{W}^\alpha\mathcal{W}_\alpha = \lambda^\alpha\lambda_\alpha + \mathcal{O}(\theta)$$

$\tilde{Q}$: scalar componet of Q, λ_α : gaugino

Note that Lsoft is written by canonically normalized fields.

$$Q_i \rightarrow \tilde{Z}_i^{-1/2}Q_i, \quad V \rightarrow \tilde{g}_c V. \qquad V \propto \lambda_\alpha$$

More precisely, we can express spurion Z factor as

$$\tilde{Z}_i(\theta, \bar{\theta}) = \tilde{Z}_{Q_i}(\theta)\tilde{Z}^{\dagger}_{Q_i}(\bar{\theta})(1 - m_i^2\theta^4).$$

$$\tilde{Z} = Z + \theta^2 Z^{(1)} + \bar{\theta}^2 \bar{Z}^{(1)} + \theta^4 Z^{(2)} \to \tilde{Z}_{Q_i}(\theta) = Z_i^{1/2}(1 + Z_i^{-1} Z^{(1)}\theta^2)$$

$\tilde{Z}_{Q_i}(\theta)$: holomorphic Z factor spurion superfield
(relate to renormalization of holomorphic SUSY breaking term)

Then we can find the physical coupling superfields.

- Physical n-point coupling superfield y :

$$\begin{aligned}\tilde{y}_{i_1 \ldots i_n} &= \tilde{Z}^{-1}_{Q_i}(1 - \theta^4 m^2_{i_1})^{-1/2} \ldots \tilde{Z}^{-1}_{Q_n}(1 - \theta^4 m^2_{i_n})^{-1/2} \tilde{Y}_{i_1 \ldots i_n} \\ &= y_{i_1 \ldots i_n} - a_{i_1 \ldots i_n}\theta^2 + \frac{1}{2}(m_i^2 + \ldots m^2_{i_n}) y_{i_1 \ldots i_n}\theta^4\end{aligned}$$

This is also supported by the fact that superpropagor is renormalized as

$$\langle Q_i Q_i^{\dagger}\rangle_{soft} = (1 + \frac{1}{2}m_i^2\theta^4)\langle Q_i Q_i^{\dagger}\rangle_{susy}(1 + \frac{1}{2}m_i^2\theta^4).$$

Physical gauge coupling superfield g_c

$$\frac{8\pi^2}{\tilde{g}_c^2} = 8\pi^2 \frac{(\tilde{f} + \tilde{f}^\dagger)}{2} + b \ln(\Lambda/\mu) - \sum_i T_i \ln(\tilde{Z}_i) - T_G \ln(\tilde{g}_c^2)$$

$$= \alpha^{-1}(1 - M_\lambda \theta^2 - \bar{M}_\lambda \bar{\theta}^2 - \Delta_g \theta^4), \quad \alpha = \frac{g_c^2}{8\pi^2}.$$

We should define physical gaugino mass not in holomorphic coupling but in physical gauge coupling. From θ^4 -term, we can determine Δ_g

$$\Delta_g = -\frac{\alpha}{1 - T_G \alpha} (\sum_i T_i m_i^2 - T_G |M_\lambda|^2).$$

This can relate to the counter term (radiative correction) for soft mass squared of ε -scalar in dimensional reduction scheme.

In summary,

$$\boxed{\begin{aligned}\tilde{\alpha}^{-1} &= \alpha^{-1}(1 - M_\lambda\theta^2 - \bar{M}_\lambda\bar{\theta}^2 - \Delta_g\theta^4), \quad \alpha = \frac{g_c^2}{8\pi^2}. \\ \tilde{y}_{i_1\ldots i_n} &= y_{i_1\ldots i_n} - a_{i_1\ldots i_n}\theta^2 + \frac{1}{2}(m_i^2 + \ldots m_{i_n}^2)y_{i_1\ldots i_n}\theta^4\end{aligned}}$$

$$\tilde{\alpha} = \alpha(1 + M_\lambda\theta^2 + \bar{M}_\lambda\bar{\theta}^2 + (2|M_\lambda|^2 + \Delta_g)\theta^4).$$

$$\Delta_g = -\frac{\alpha}{1 - T_G\alpha}(\sum_i T_i m_i^2 - T_G|M_\lambda|^2).$$

This is supported by anomalous global symmetry :

$$Q_i \;\to\; Q_i e^t, \quad \tilde{Z}_i \;\to\; e^{-t^\dagger}\tilde{Z}_i e^{-t}, \quad \tilde{Y}_{i_1\ldots i_n} \;\to\; e^{-nt}\tilde{Y}_{i_1\ldots i_n}, \quad f \;\to\; f - \frac{t}{4\pi^2}\sum_i T_i.$$

t: chiral superfield

$\tilde{Z}^{-n}\tilde{Y}\tilde{Y}^\dagger, \quad 4\pi^2(\tilde{f} + \tilde{f}^\dagger) - \sum_i T_i \ln(\tilde{Z}_i) = F(\tilde{g}_c^2)$: Physical coupling is invariant

Supercovariant derivative $D_\alpha,\ \bar{D}_{\dot\alpha}$ acting on spurion superfield $(\tilde{g}_c, \tilde{y})$ will not appear in renormalized superfield at least perturbatively, because in that case power of momentum (divergence) will not be sufficient to produce results.
In otherwords, final integral of superspace will have

$$\int d^4\theta\theta^\alpha(\theta^4)\bar{D}^2 D_\alpha(\theta^4),\ \int d^4\theta(\theta^4)\bar{D}^2(\theta^4), \cdots = 0$$

$$(\bar{D}^2 D^2 \bar{D}^2 = 16p^2\bar{D}^2.)$$

instead of

$$\int d^4\theta(\theta^4)\bar{D}^2 D^2(\theta^4) = 16 \neq 0.$$

Beta function for spurion superfield

$$\frac{d\ln(\tilde{\alpha})}{d\ln(\mu)} = \frac{\beta_\alpha(\tilde{\alpha},\tilde{y},\tilde{y}^\dagger)}{\tilde{\alpha}} = -\frac{\tilde{\alpha}}{1-T_G\tilde{\alpha}}[3T_G - \sum_i T_i(1-\tilde{\gamma}_i)]$$

$$= \frac{\beta_\alpha}{\alpha} + \underline{\theta^2\frac{d(M_\lambda)}{d\ln(\mu)}} + \cdots = \frac{\beta_\alpha}{\alpha} + \underline{\theta^2\left.\frac{\beta_{\tilde{\alpha}}}{\tilde{\alpha}}\right|_{\theta^2}} + \ldots$$

$$\tilde{\alpha} = \alpha(1 + M_\lambda\theta^2 + \bar{M}_\lambda\bar{\theta}^2 + (2|M_\lambda|^2 + \Delta_g)\theta^4). \quad \Delta_g = -\frac{\alpha}{1-T_G\alpha}(\sum_i T_i m_i^2 - T_G|M_\lambda|^2).$$

Here

$$\tilde{\gamma}_i = -\frac{d\ln(\tilde{Z}_i)}{d\ln(\mu)} \equiv \gamma_i + \theta^2\tilde{\gamma}_i|_{\theta^2} + \bar{\theta}^2(\tilde{\gamma}_i|_{\theta^2})^\dagger + \underline{\theta^4\tilde{\gamma}_i|_{\theta^4}}$$

$$= \gamma_i + \theta^4\underline{\frac{dm_i^2}{d\ln(\mu)}} + \ldots$$

$$\tilde{Z}_i(\theta,\bar{\theta}) = \tilde{Z}_{Q_i}(\theta)\tilde{Z}^\dagger_{Q_i}(\bar{\theta})(1 - m_i^2\theta^4).$$

$$\frac{d\tilde{y}_{i_1\ldots i_n}}{d\ln(\mu)} = \beta_y(\tilde{g}_c, \tilde{y}, \tilde{y}^\dagger) = (\tilde{\gamma}_{Q_{i_1}} + \tilde{\gamma}_{Q_{i_2}} + \ldots \tilde{\gamma}_{Q_{i_n}})\tilde{y}_{i_1\ldots i_n}$$

$$= \beta_y - \theta^2 \underline{\frac{da_{i_1\ldots i_n}}{d\ln(\mu)} +} \ldots .$$

$$= \beta_y - \theta^2 [\underline{\frac{1}{2}(\gamma_{i_1} + \ldots \gamma_{i_n})a_{i_1\ldots i_n} - (\tilde{\gamma}_{i_1}|_{\theta^2} + \cdots + \tilde{\gamma}_{i_n}|_{\theta^2})y_{i_1\ldots i_n}}] + \ldots$$

Here we used

$$\tilde{y}_{i_1\ldots i_n} = \tilde{Z}^{-1}_{Q_i}(1 - \theta^4 m^2_{i_1})^{-1/2} \ldots \tilde{Z}^{-1}_{Q_n}(1 - \theta^4 m^2_{i_n})^{-1/2}\tilde{Y}_{i_1\ldots i_n}$$

$$= y_{i_1\ldots i_n} - a_{i_1\ldots i_n}\theta^2 + \frac{1}{2}(m^2_i + \ldots m^2_{i_n})y_{i_1\ldots i_n}\theta^4$$

$$\tilde{\gamma}_{Q_i} = -\frac{d\ln[Z_{Q_i}(\theta)(1 - m^2_i\theta^4/2)^{1/2}]}{d\ln(\mu)} = \frac{1}{2}\gamma_i + \theta^2\tilde{\gamma}_i|_{\theta^2} + \ldots$$

$$\tilde{Z} = Z + \theta^2 Z^{(1)} + \bar{\theta}^2\bar{Z}^{(1)} + \theta^4 Z^{(2)} \to \tilde{Z}_{Q_i}(\theta) = Z_i^{1/2}(1 + Z_i^{-1}Z^{(1)}\theta^2)$$

Beta functions

- Physical gauge coupling $\alpha = \frac{g_c^2}{8\pi^2}$

$$\frac{d\alpha}{d\ln(\mu)} = \beta_\alpha(\alpha, y, y^*) = -\frac{\alpha^2}{1 - T_G\alpha}[3T_G - \sum_i T_i(1-\gamma_i)]$$

- Physical n-point coupling y

$$\frac{dy_{i_1 \dots i_n}}{d\ln(\mu)} = \beta_y(g_c, y, y^*) = \frac{1}{2}(\gamma_{i_1} + \gamma_{i_2} + \dots \gamma_{i_n})y_{i_1 \dots i_n}$$

- Gaugino mass

$$\frac{dM_\lambda}{d\ln(\mu)} = D_1\left(\frac{\beta_\alpha}{\alpha}\right) \qquad D_1 = \alpha M_\lambda \frac{\partial}{\partial\alpha} - a\frac{\partial}{\partial y}$$

- Holomorphic scalar n-point coupling

$$\frac{d}{d\ln(\mu)}\left(\frac{a_{i_1 \dots i_n}}{y_{i_1 \dots i_n}}\right) = -[(D_1\gamma_{i_1}) + \cdots + (D_1\gamma_{i_n})]$$

- Non-holomorphic scalar mass

$$\frac{dm_i^2}{d\ln(\mu)} = D_2\gamma_i$$

$$D_2 = \bar{D}_1 D_1 + \alpha(|M_\lambda|^2 + \Delta_g)\frac{\partial}{\partial\alpha} + \frac{1}{2}(m_{i_1}^2 + \cdots + m_{i_n}^2)\left[y\frac{\partial}{\partial y} + y^*\frac{\partial}{\partial y^*}\right]$$